Probing (gluon) TMDs with quarkonia production

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Outline

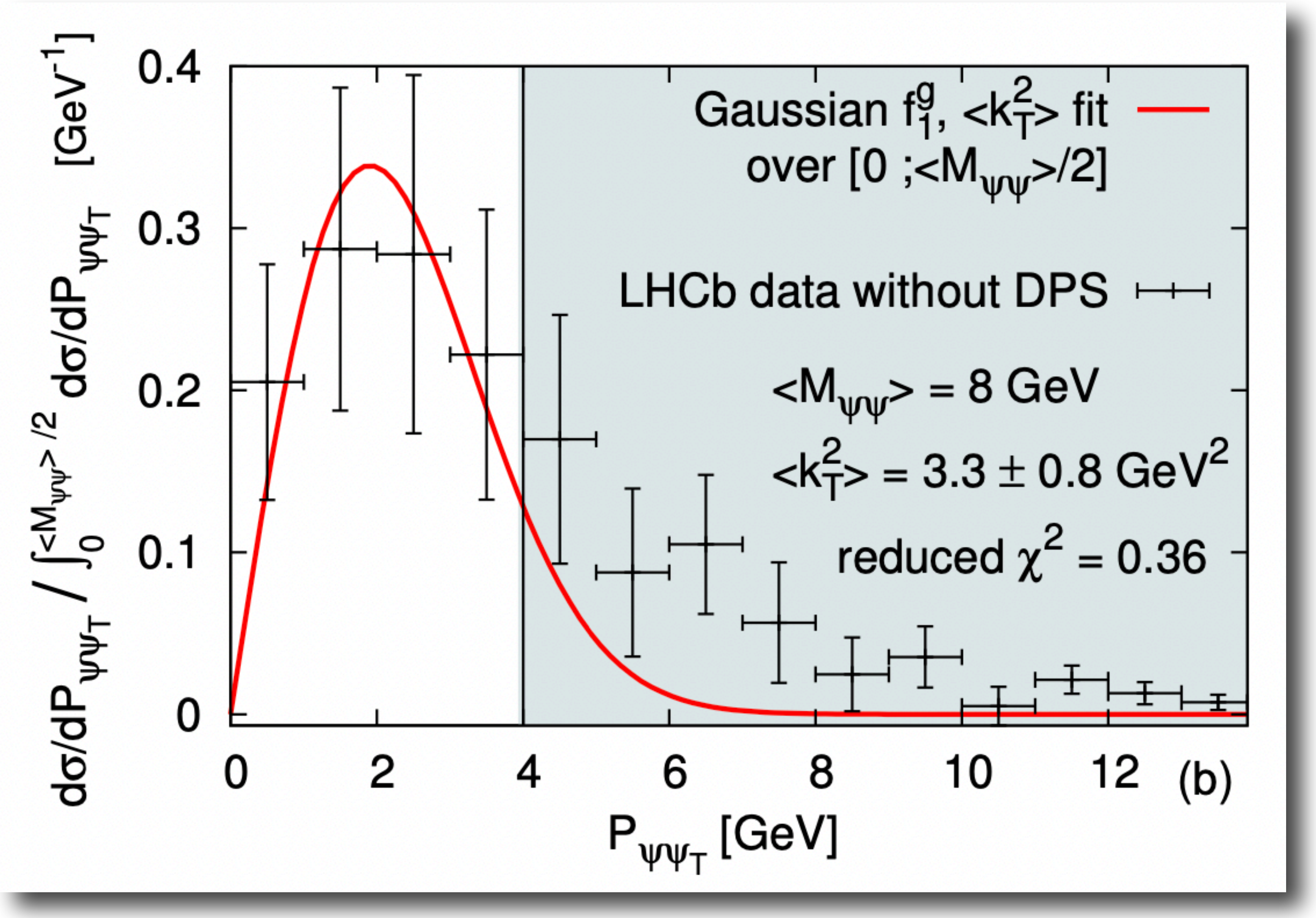
- **Motivation**
- **TMDShFs in eta_c hadroproduction**
- **TMDShFs in Jpsi leptoproduction**
- **Quarkonium TMD Fragmentation Functions**
- **Conclusions**

Motivation

Gluon TMDs: not much known yet…

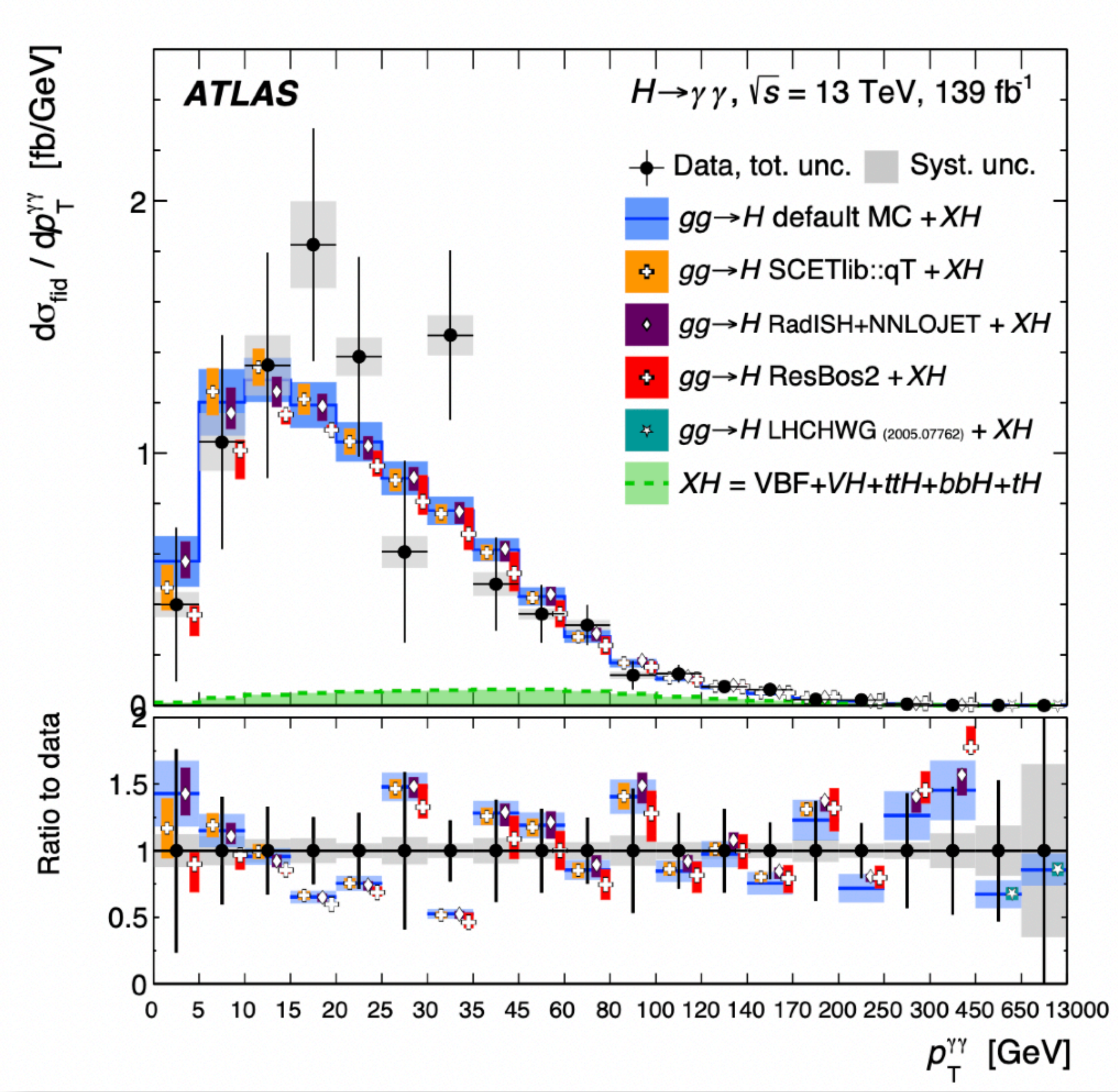
- **Quark TMDs are fairly well known**: advanced theory, different measurements and processes, several groups…
- Gluon TMDs: not easy to probe them!

Only existing "fit" of gluon TMD



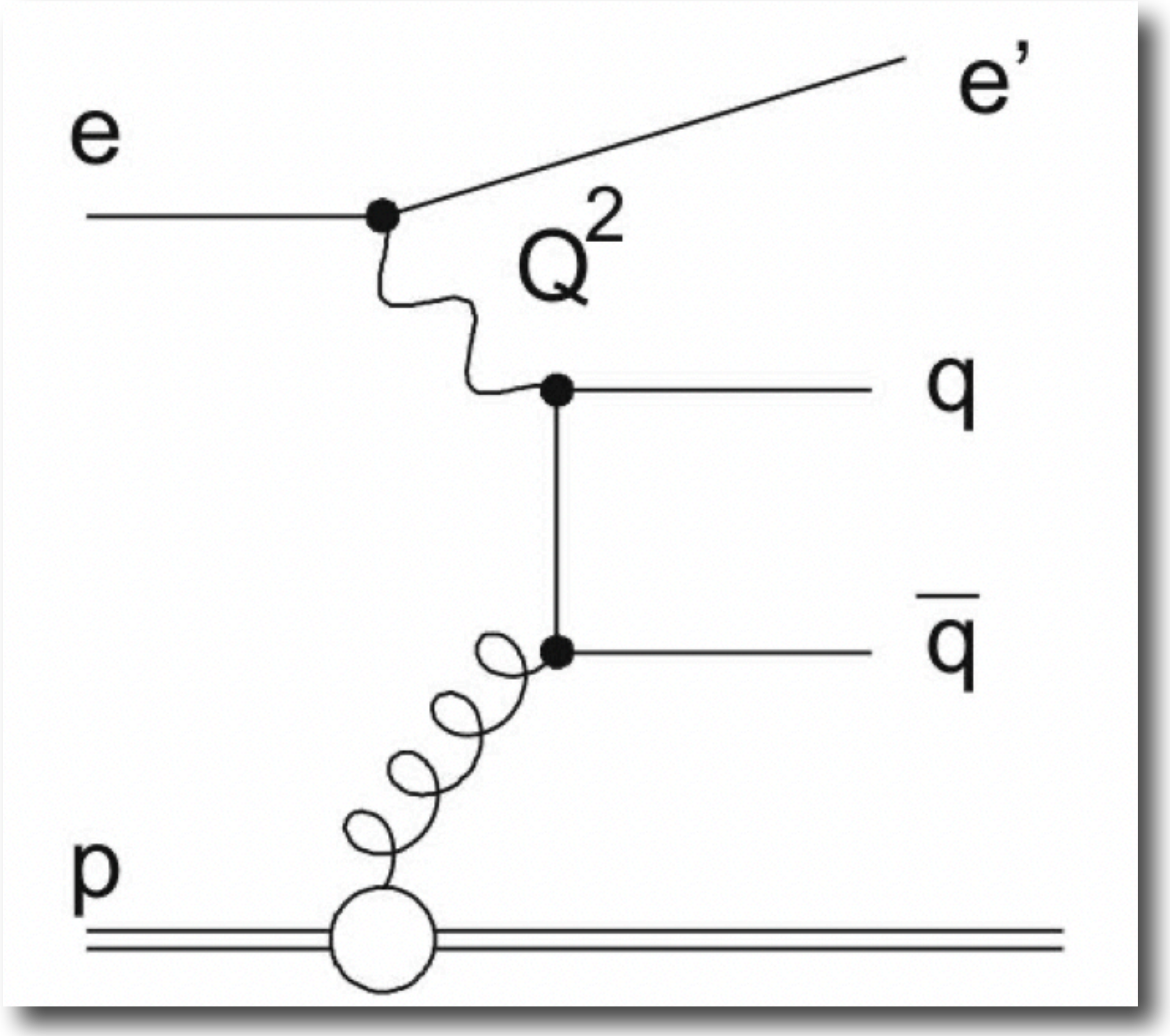
[Lansberg, Pisano, Scarpa, Schlegel 1710.01684]

Simple gaussian model
No TMD evolution, LO
DPS…



[ATLAS 2202.00487]

H^0 not useful yet to constrain gluon TMDs…



Jets might be useful, but have their own issues…

Quarkonium production as a tool to probe (gluon) TMDs

- Several processes proposed to access (un)polarized gluon TMDs
- Several measurements exist (e.g. di-onium@LHC), data on tape (di-onium asymmetries at LHC), future experiments promising (EIC, FT@LHC,…)

—> See talks by Cristian, Luca & Raj

$p+p\to\eta_{c,b}+X$

$p+p\to\chi_{c,b}+X$

$p+p\to H^0+X$

$p+p\to\gamma+\gamma+X$

$p+p\to J/\psi+\gamma^*+X$

$p+p\to J/\psi+Z+X$

$p+p\to J/\psi+J/\psi+X$

$p+p\to\eta_c+\eta_c+X$

$e+p\to e+c+\bar{c}+X$

$e+p\to e+J/\psi+jet+X$

$e+p\to e+J/\psi+\pi+X$

$e+p\to e+J/\psi+X$

$e^++e^-\to J/\psi+\pi+X$

+ more…

Factorization proven

Ansatz: factorization of 2 soft mechanisms in the processes: soft gluon resummation and formation of bound state

Quarkonium production is both an opportunity and a challenge!!
Need to improve the theory to properly deal with TMDs…

TMD factorization for

$$pp \to \eta_{c,b} X$$

[MGE 1907.06494]

NRQCD in a nutshell

- Non-Relativistic QCD: an effective theory of QCD
- Expansion of the QCD lagrangian in powers of the relative velocity of the heavy-quark pair

$$d\sigma(i+j \to \mathcal{Q}+X) = \sum_n d\hat{\sigma}(i+j \to Q\bar{Q}(n)+X)\,\langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

Perturbative
(apart from PDFs)

LDMEs

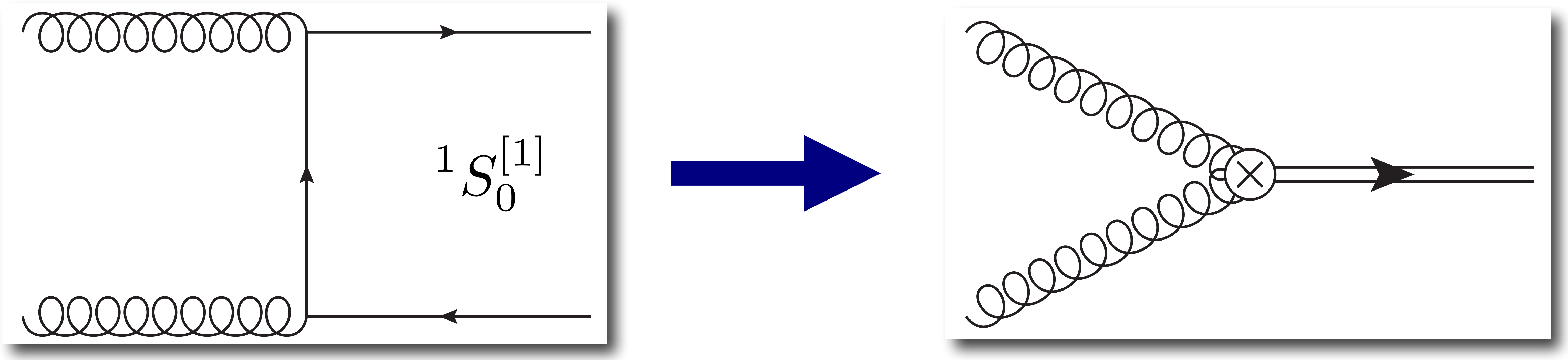
$$\mathcal{O}_n^{\mathcal{Q}} = (\psi^\dagger \kappa_n \chi)^\dagger \sum_X |\mathcal{Q}\,X\rangle\langle\mathcal{Q}\,X|\,(\psi^\dagger \kappa_n \chi)$$

$$n \equiv {}^{2S+1}L_J^{[c]}$$

X-section is a double expansion in α_s and v

	$1,{}^1S_0$	$1,{}^3S_1$	$8,{}^1S_0$	$8,{}^3S_1$	$1,{}^1P_1$	$1,{}^3P_0$	$1,{}^3P_1$	$1,{}^3P_2$	$8,{}^1P_1$	$8,{}^3P_0$	$8,{}^3P_1$	$8,{}^3P_2$
					NRQCD Factorization							
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Effective operator in SCET+NRQCD



+ crossed diagram

$$\mathcal{O}(\xi) = -2q^2 C_H(-q^2;\mu^2)\left[\psi^\dagger(\xi)\,\Gamma_{\mu\nu}\,\chi(\xi)\right]\left[B^{\mu,b}_{\bar{n}\perp}(\xi)\,\mathcal{Y}^{\dagger ba}_{\bar{n}}(\xi)\,\mathcal{Y}^{ac}_{n}(\xi)\,B^{\nu,c}_{n\perp}(\xi)\right]$$

$$B^\mu_{n\perp} = B^{\mu,a}_{n\perp}t^a = \frac{1}{g}[W^\dagger_n i D^{\perp\mu}_n W_n] = \frac{1}{\bar{n}\cdot\mathcal{P}} i\bar{n}_\alpha g^\mu_{\perp\beta} W^\dagger_n F^{\alpha\beta}_n W_n = \frac{1}{\bar{n}\cdot\mathcal{P}} i\bar{n}_\alpha g^\mu_{\perp\beta} t^a (\mathcal{W}^\dagger_n)^{ab} F^{\alpha\beta,b}_n$$

$$W_n(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\,\bar{n}\cdot A^a_n(\xi+\bar{n}s)t^a\right]$$

$$Y_n(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\,n\cdot A^a_s(\xi+ns)t^a\right]$$

Caligraphic means:

$(t^a)^{bc} = -if^{abc}$

Factorization

- Cross-section given by:

$$d\sigma = \frac{1}{2s}\frac{d^3q}{(2\pi)^3 2E_q}\int d^4\xi\, e^{-iq\cdot\xi}\sum_X \left\langle PS_A, \bar{P}S_B\right| \mathcal{O}^\dagger(\xi)\left|X\eta_Q\right\rangle \left\langle \eta_Q X\right| \mathcal{O}(0)\left|PS_A, \bar{P}S_B\right\rangle$$

Factorization in EFT
=
decoupling of modes!

$m_Q v \sim q_T$

$$\left|X\eta_Q\right\rangle = \left|X_n\right\rangle \otimes \left|X_{\bar{n}}\right\rangle \otimes \left|X_s\eta_Q\right\rangle$$

$$\left|PS_A, \bar{P}S_B\right\rangle = \left|PS_A\right\rangle \otimes \left|\bar{P}S_B\right\rangle$$

$$d\sigma = \frac{1}{2s}\frac{d^3q}{(2\pi)^3 2E_q}\, 4M^4\, H(M^2,\mu^2)\,\Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}\int d^4\xi\; e^{-iq\xi}$$
$$\times \sum_{X_n}\left\langle PS_A\right| B^{\sigma,c'}_{n\perp}(\xi)\left|X_n\right\rangle \left\langle X_n\right| B^{\nu,c}_{n\perp}(0)\left|PS_A\right\rangle \sum_{X_{\bar{n}}}\left\langle \bar{P}S_B\right| B^{\rho,b'}_{\bar{n}\perp}(\xi)\left|X_{\bar{n}}\right\rangle \left\langle X_{\bar{n}}\right| B^{\mu,b}_{\bar{n}\perp}(0)\left|\bar{P}S_B\right\rangle$$
$$\times \sum_{X_s}\left\langle 0\right| \left[\mathcal{Y}^{\dagger c'a'}_n \mathcal{Y}^{a'b'}_{\bar{n}}\chi^\dagger\psi\right](\xi)\left|X_s\eta_Q\right\rangle \left\langle \eta_Q X_s\right| \left[\mathcal{Y}^{\dagger ba}_{\bar{n}} \mathcal{Y}^{ac}_n\psi^\dagger\chi\right](0)\left|0\right\rangle$$

$$H(M^2,\mu^2) = |C_H(-q^2,\mu^2)|^2$$

TMDs with eta_c hadroproduction

- After some algebraic manipulations…

$$\frac{d\sigma}{dy d^2 q_\perp} = \hat{\sigma}_{\mu\nu\rho\sigma}\, H \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times J_n^{(0)\sigma\nu}(x_A, \boldsymbol{k}_{n\perp}, S_A; \mu; \eta_n)\, J_{\bar{n}}^{(0)\rho\mu}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \mu; \eta_{\bar{n}})\, S_Q^{(0)}(\boldsymbol{k}_{s\perp}; \mu)$$

$$x_{A,B} = \sqrt{\tau}\, e^{\pm y}$$

- Pure collinear and bare TMD shape functions defined as:

$$\tau = (M^2 + q_T^2)/s$$

$$J_n^{(0)\mu\nu} = \frac{x_A P^+}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{-i\left(\frac{1}{2}x_A \xi^- P^+ - \boldsymbol{\xi}_\perp \boldsymbol{k}_{n\perp}\right)} \langle PS_A|\, B_{n\perp}^{\mu,a}(\xi^-, \boldsymbol{\xi}_\perp)\, B_{n\perp}^{\nu,a}(0)\, |PS_A\rangle$$

$$J_{\bar{n}}^{(0)\mu\nu} = \frac{x_B \bar{P}^-}{2} \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{-i\left(\frac{1}{2}x_B \xi^+ \bar{P}^- - \boldsymbol{\xi}_\perp \boldsymbol{k}_{\bar{n}\perp}\right)} \langle \bar{P}S_B|\, B_{\bar{n}\perp}^{\mu,a}(\xi^+, \boldsymbol{\xi}_\perp)\, B_{\bar{n}\perp}^{\nu,a}(0)\, |\bar{P}S_B\rangle$$

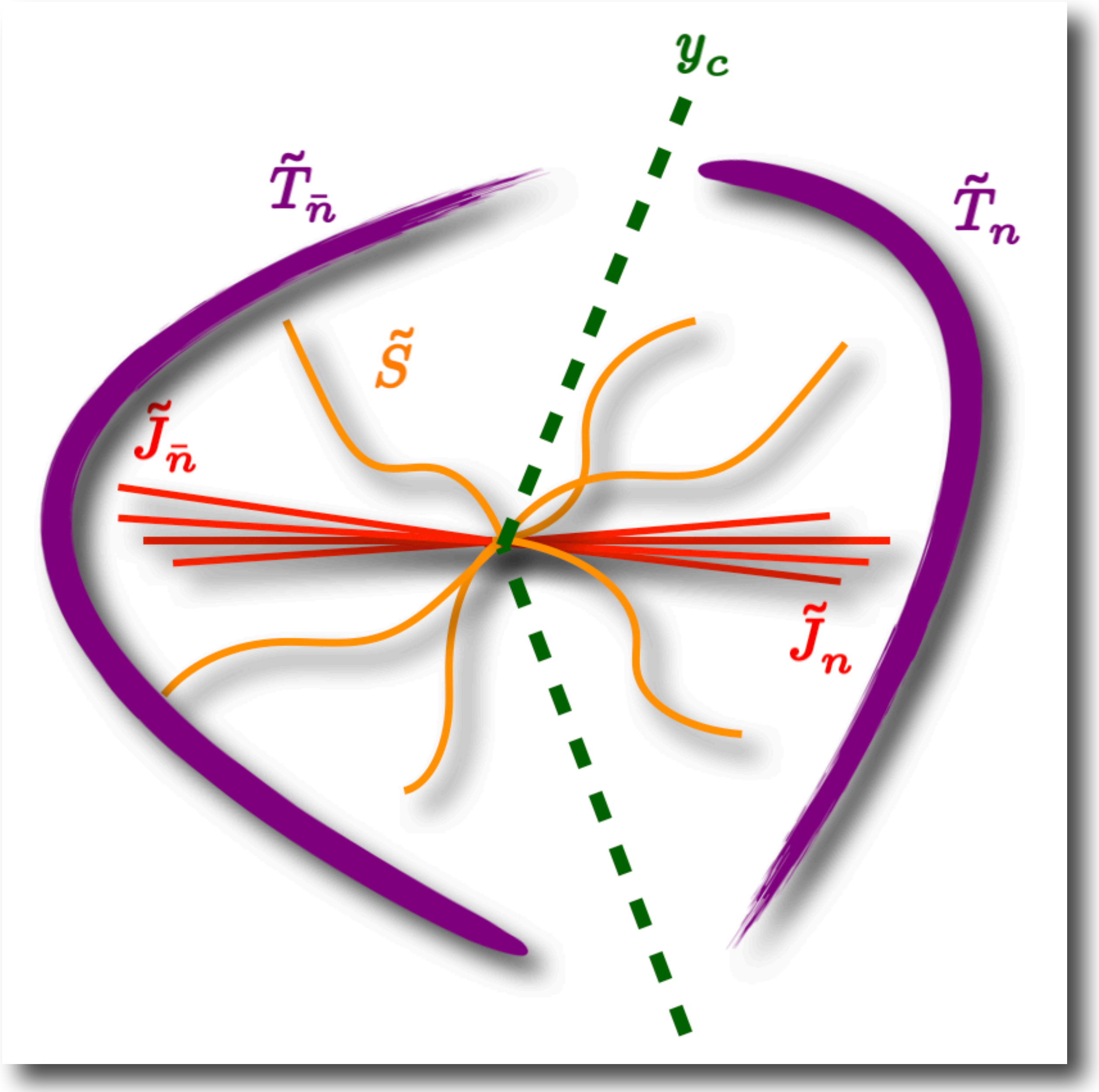
$$S_Q^{(0)} = \frac{1}{N_c^2 - 1} \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}} \langle 0|\left[\mathcal{Y}_n^{\dagger ab} \mathcal{Y}_{\bar{n}}^{bc} \chi^\dagger \psi\right](\boldsymbol{\xi}_\perp)\, a_{\eta_Q}^\dagger a_{\eta_Q} \left[\mathcal{Y}_{\bar{n}}^{\dagger cd} \mathcal{Y}_n^{da} \psi^\dagger \chi\right](0)\, |0\rangle$$

All of them have
rapidity divergences!

Definition of gluon TMDs

- We need to introduce the relevant soft function (same as for Higgs production):

$$S = \frac{1}{N_c^2 - 1} \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2} e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}} \langle 0| \left[\mathcal{Y}_n^{\dagger ab} \mathcal{Y}_{\bar{n}}^{bc}\right](\boldsymbol{\xi}_\perp) \left[\mathcal{Y}_{\bar{n}}^{\dagger cd} \mathcal{Y}_n^{da}\right](0) |0\rangle$$



$\tilde{T}_n \equiv G_{g/A}$

Soft function can be split to all orders in pQCD:

$$\tilde{S}(b_T;\mu;\eta_n,\eta_{\bar{n}}) = \tilde{S}_-\left(b_T;\mu;\eta_n\right)\tilde{S}_+\left(b_T;\mu;\eta_{\bar{n}}\right)$$

TMDs are defined as (Rapidity divergences free):

$$\tilde{G}^{\mu\nu}_{g/A}(x_A,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = \tilde{J}_n^{(0)\mu\nu}(x_A,\boldsymbol{b}_\perp,S_A;\mu;\eta_n)\,\tilde{S}_-(b_T;\mu;\eta_n)$$

$$\tilde{G}^{\mu\nu}_{g/B}(x_B,\boldsymbol{b}_\perp,S_B;\zeta_B,\mu) = \tilde{J}_{\bar{n}}^{(0)\mu\nu}(x_B,\boldsymbol{b}_\perp,S_B;\mu;\eta_{\bar{n}})\,\tilde{S}_+(b_T;\mu;\eta_{\bar{n}})$$

[MGE, Idilbi, Scimemi 1111.4996, 1211.1947, 1211.1947]
[MGE, Kasemets, Mulders, Pisano 1502.05354]
[Collins' book 2011]

Final factorization theorem

$$\frac{d\sigma}{dy\, d^2q_\perp} = \hat{\sigma}_{\mu\nu\rho\sigma}\, H \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times\, G^{\sigma\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, G^{\rho\mu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_{s\perp}; \mu)$$

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\tilde{S}(\boldsymbol{y}_\perp)}$$

TMD Shape Function!

TMDShF

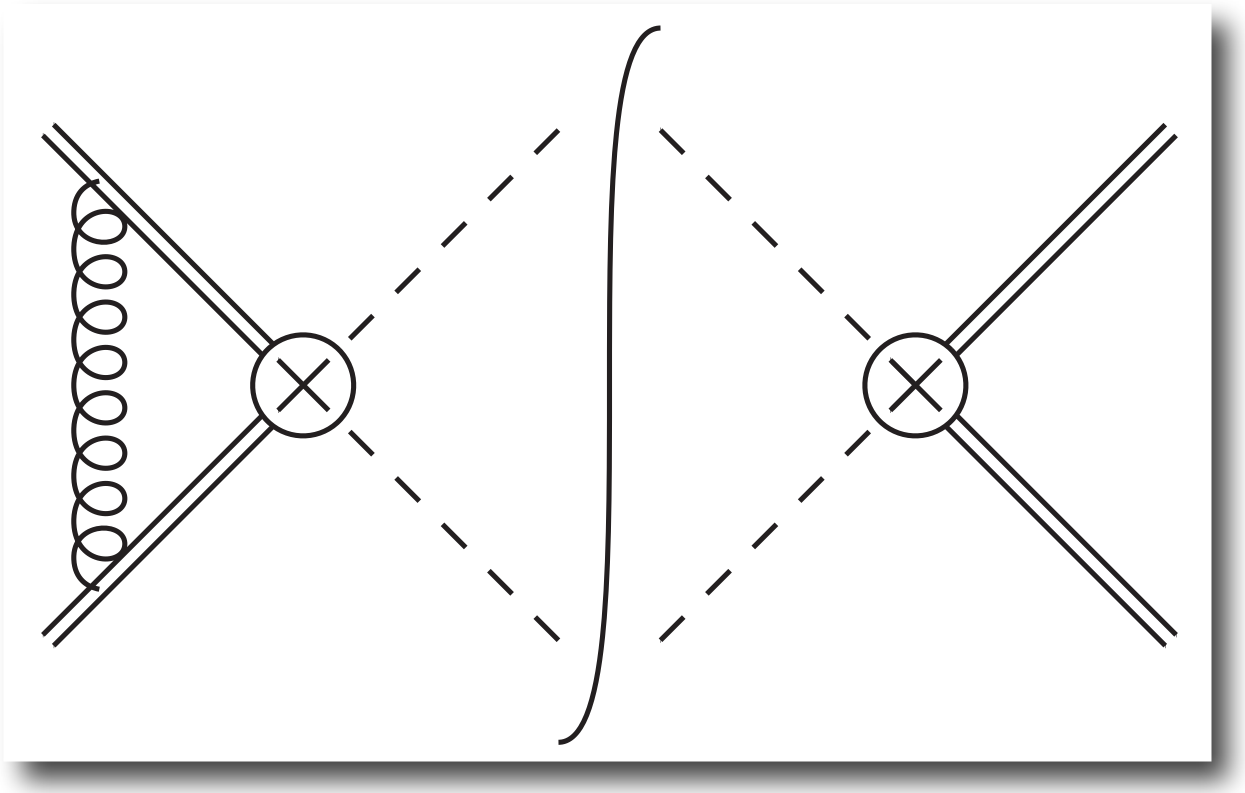
[MGE 1907.06494]

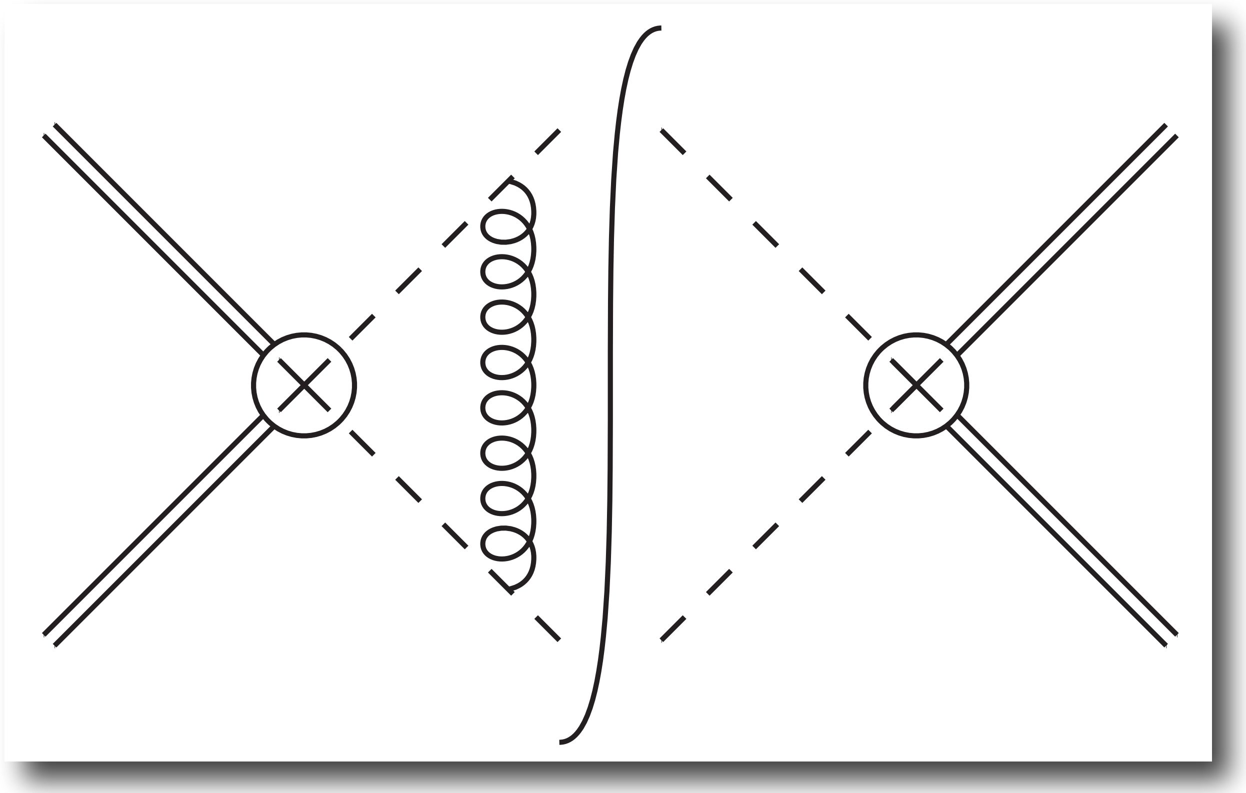
[Fleming, Makris, Mehen 1910.03586]

$$\tilde{S}^{(0)}_{\mathcal{Q}}(\xi_\perp) = \frac{1}{N_c^2 - 1} \sum_{X_s} \langle 0| \left[\mathcal{Y}^{\dagger ab}_{n} \mathcal{Y}^{bc}_{\bar{n}} \chi^\dagger \psi\right](\boldsymbol{\xi}_\perp)\, |\eta_c X_s\rangle\, \langle X_s \eta_c|\, \left[\mathcal{Y}^{\dagger cd}_{\bar{n}} \mathcal{Y}^{da}_{n} \psi^\dagger \chi\right](0)\, |0\rangle$$

- Valid for all (un)polarized TMDs
- TMD Shape Function is spin-independent
- TMDShF depends on the quarkonium state

TMDShF: virtual part





+ crossed diagrams and self-energies…

$$\tilde{S}^{(0)}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v} + \tilde{S}\Big|^1_v$$

contains rapidity divergences

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\tilde{S}(\boldsymbol{y}_\perp)}$$

$$\tilde{S}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v}$$

Check of factorization: Hard part at NLO

$$\frac{d\sigma}{dy d^2 q_\perp} = \sigma_0(\mu) H(M^2, \mu^2) \left[\mathcal{C}[f_1^g \, f_1^g \, S_Q] - \mathcal{C}[w_{UU} h_1^{\perp g} \, h_1^{\perp g} \, S_Q] \right]$$

$$\left.\frac{d\sigma}{\sigma_0}\right|_v = \delta(1-x_A)\delta(1-x_B) \Big\{ 1 + \frac{\alpha_s}{2\pi} \left[C_F \frac{\pi^2}{v} - 2\left(\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} + \frac{1}{\varepsilon_{\mathrm{IR}}} \left(\frac{\beta_0}{2} + C_A \ln \frac{\mu^2}{M^2} \right) \right) - C_A \ln^2 \frac{\mu^2}{M^2} - C_A \frac{\pi^2}{6} + 2 B_{1\,S_0}^{[1]} \right] \Big\}$$

[Petrelli, Cacciari, Greco, Maltoni, Mangano hep-ph/9707223]

$$\left.\tilde{f}_1^g\right|_v = \delta(1-x) + \frac{\alpha_s}{2\pi} \left[-\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} - \frac{1}{\varepsilon_{\mathrm{IR}}} \left(\frac{\beta_0}{2} + C_A \ln \frac{\mu^2}{\zeta_A} \right) \right] \delta(1-x)$$

[MGE, Kasemets, Mulders, Pisano 1502.05354]

$$\zeta_A \zeta_B = M^4$$

$$\left.\tilde{S}_Q\right|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v}$$

$$H = 1 + \frac{\alpha_s}{2\pi} \left[- C_A \ln^2 \frac{\mu^2}{M^2} - C_A \frac{\pi^2}{6} + 2 B_{1\,S_0}^{[1]} \right]$$

(same double log as for H^0 in pp)

TMDShFs are an essential non-perturbative functions!

$$\tilde{S}_{\mathcal{Q}}^{(0)}(\xi_\perp) = \frac{1}{N_c^2-1} \sum_{X_s} \langle 0| \left[\mathcal{Y}_n^{\dagger ab} \mathcal{Y}_{\bar{n}}^{bc} \chi^\dagger \psi\right](\boldsymbol{\xi}_\perp) \, |\eta_c X_s\rangle \, \langle X_s \eta_c| \left[\mathcal{Y}_{\bar{n}}^{\dagger cd} \mathcal{Y}_n^{da} \psi^\dagger \chi\right](0) \, |0\rangle$$

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}_{\mathcal{Q}}^{(0)}(\boldsymbol{y}_\perp)}{\tilde{S}(\boldsymbol{y}_\perp)}$$

$$\tilde{S}_{\mathcal{Q}}(b;\mu) = C(b;\mu)\,\langle \mathcal{O}_{\mathcal{Q}}(\mu)\rangle\,\tilde{S}_{\mathcal{Q}}^{NP}(b)$$

$$C(b;\mu) = 1 + \mathcal{O}(\alpha_s^2)$$

Analogous to TMDs

- **Up to NLO** it turns out to be **just the LDME**
- But at **NNLO and beyond we don't know**!
- It anyway is a **non-perturbative function** on the same footing as the TMDs
- **One has to include (=model!) it in any pheno study!!**

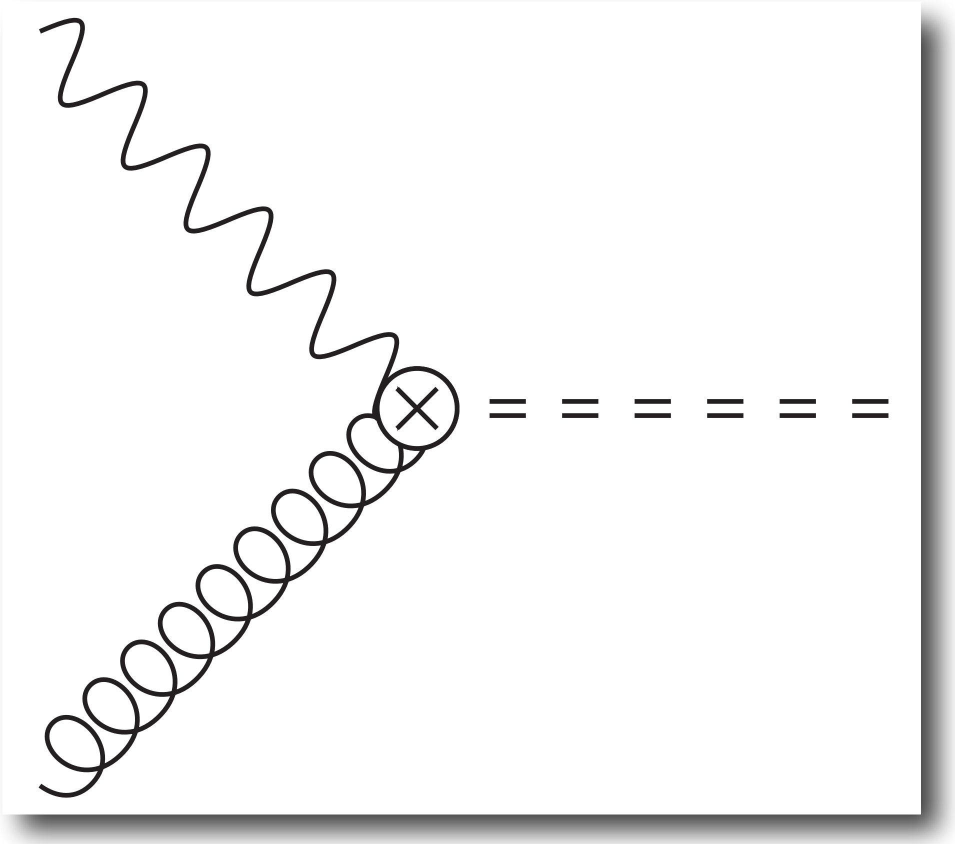
TMD factorization for

$$e^- \, p \to e^- \, J/\psi \, X$$

[Work in progress with
P. Gutierrez and P. Taels]

Effective operator

$e^- p \to e^- J/\psi$



$$\mathcal{O}^\mu(\xi) \sim \left[\psi^\dagger(\xi)\,\Gamma^\mu_\nu\, t_a\,\chi(\xi)\right]\left[\mathcal{Y}^{ac}_n(\xi)\,B^{\nu,c}_{n\perp}(\xi)\right]$$

Several color-octet LDMEs contribute

[Bacchetta, Boer, Pisano, Taels 1809.02056]

Time-like Wilson lines needed for gauge invariance!

$$\psi^\dagger Y_v^\dagger T^a Y_v \chi = \psi^\dagger \mathcal{Y}_v^{ab} T^b \chi$$

[Nayak, Qiu, Sterman hep-ph/0501235]
[Nayak, Qiu, Sterman hep-ph/0509021]
[Rothstein, Shrivastava, Stewart 1806.07398]

$$Y_v(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\, v\cdot A_s^a(\xi + vs)t^a\right]$$

$$v = (1,0,0,0)\,, \quad v^2 = 1$$

For color singlet there was no issue:

$$\psi^\dagger Y_v^\dagger Y_v \chi = \psi^\dagger \chi$$

Factorization

- Cross-section given by:

$$d\sigma \sim \hat{\sigma}_{\alpha\beta}\, H \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{s\perp}\right) G^{\alpha\beta}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_{s\perp}; \zeta_B, \mu)$$

There is actually a sum over states, with different TMDShFs, H and Lorentz structures, but let's simplify…

$$\tilde{S}^{(0)}_{\mathcal{Q}}(\xi_\perp) = \frac{1}{N_c^2 - 1} \sum_{X_s} \langle 0| \left[\mathcal{Y}^{\dagger ab}_n \mathcal{Y}^{bc}_v \chi^\dagger \psi\right](\boldsymbol{\xi}_\perp) \left|J/\psi X_s\right\rangle \left\langle X_s J/\psi\right| \left[\mathcal{Y}^{\dagger cd}_v \mathcal{Y}^{da}_n \psi^\dagger \chi\right](0) \left|0\right\rangle$$

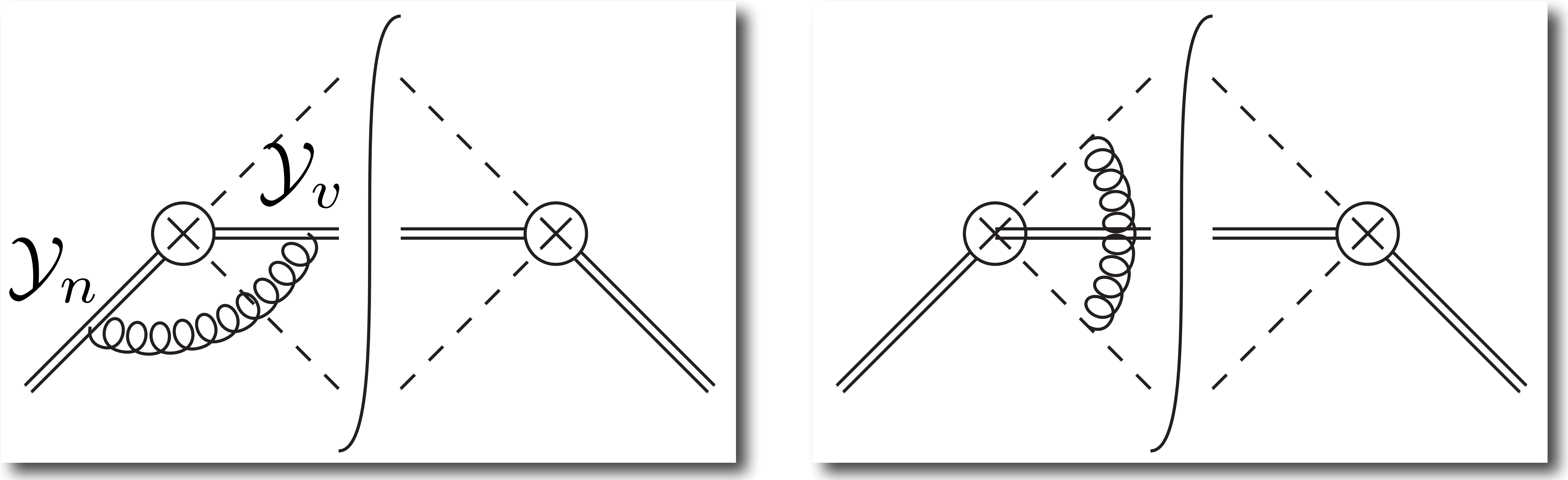
$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_\perp, S_A; \zeta_A, \mu) = \tilde{J}^{(0)\mu\nu}_n(x_A, \boldsymbol{b}_\perp, S_A; \mu; \eta_n)\, \tilde{S}_-(b_T; \mu; \eta_n)$$

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\tilde{S}_-(\boldsymbol{y}_\perp)}$$

Different from etac case!

This is not full S!

TMDShF: virtual part



+ crossed diagrams and self-energies…

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\tilde{S}_-(\boldsymbol{y}_\perp)}$$

Half the rapidity divergences

$$\tilde{S}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}}\ln\frac{1}{\zeta_B}$$

Remnant of S_-

No standard rapidity scale scales (no mixing of collinear and soft modes)

TMDs with JPsi leptoproduction

$$d\sigma \sim \hat{\sigma}_{\alpha\beta}\, H \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{s\perp}\right) G^{\alpha\beta}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_{s\perp}; \zeta_B, \mu)$$

$$\frac{d\sigma}{\sigma_0}\bigg|_v = \frac{\alpha_s}{2\pi}\bigg[\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \left(\frac{C_A}{\varepsilon_{\rm IR}^2} + \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{Q^2}\right)\right) - \frac{C_A}{\varepsilon_{\rm IR}}$$
$$- \frac{C_A}{2}\ln^2\frac{\mu^2}{Q^2} - C_A\ln\frac{\mu^2}{Q^2} - C_A\frac{\pi^2}{12} + 2D^{[8]}_{{}^1S_0}\bigg)\bigg]$$

[Maltoni, Mangano, Petrelli hep-ph/9708349]

$$\tilde{f}_1^g\bigg|_v = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A\ln\frac{\mu^2}{\zeta_A}\right)\right]\delta(1-x)$$

$$\tilde{S}_{\mathcal{Q}}\bigg|_v = 1 + \frac{\alpha_s}{2\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}}\ln\frac{1}{\zeta_B}$$

$$\zeta_A \zeta_B = Q^2$$

$$H = 1 + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{2}\ln^2\frac{\mu^2}{Q^2} - C_A\ln\frac{\mu^2}{Q^2} - C_A\frac{\pi^2}{12} + 2D^{[8]}_{{}^1S_0}\right]$$

(half the double log as for H^0 in pp + single log)

TMDShFs are an essential non-perturbative functions!

$$\tilde{S}^{(0)}_{\mathcal{Q}}(\xi_\perp) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\left[\mathcal{Y}^{\dagger ab}_n \mathcal{Y}^{bc}_v \chi^\dagger \psi\right](\boldsymbol{\xi}_\perp)|J/\psi X_s\rangle\langle X_s J/\psi|\left[\mathcal{Y}^{\dagger cd}_v \mathcal{Y}^{da}_n \psi^\dagger \chi\right](0)|0\rangle$$

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\tilde{S}_-(\boldsymbol{y}_\perp)}$$

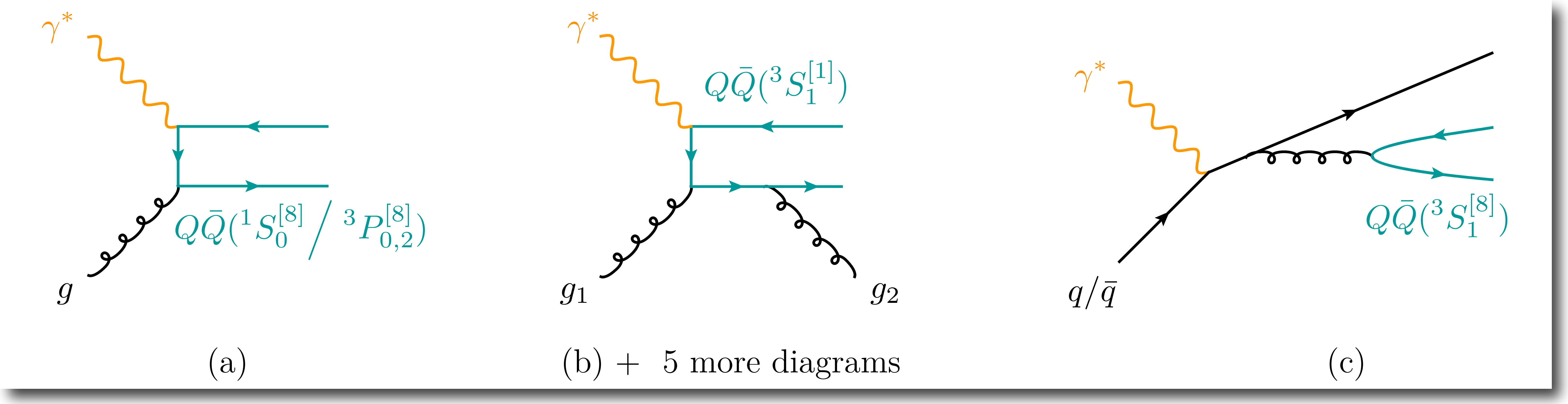
$$\tilde{S}_{\mathcal{Q}}(b;\mu,\zeta) = C(b;\mu,\zeta)\,\langle \mathcal{O}_{\mathcal{Q}}(\mu)\rangle\,\tilde{S}^{NP}_{\mathcal{Q}}(b)$$

$$C(b;\mu,\zeta) = 1+\mathcal{O}(\alpha_s)$$

Analogous to TMDs

- **Different** from the one(s) for etac hadroproduction
- **At NLO** it is already something different
- TMDShF universal: same as in Jpsi+pion in e+e-
- **One has to include (=model!) it in any pheno study!!**

Quarkonium TMDFFs



Photon-gluon fusion described in terms of gluon TMDPDFs and TMDShFs

Single-parton fragmentation

[MGE, Makris, Scimemi 2007.05547]

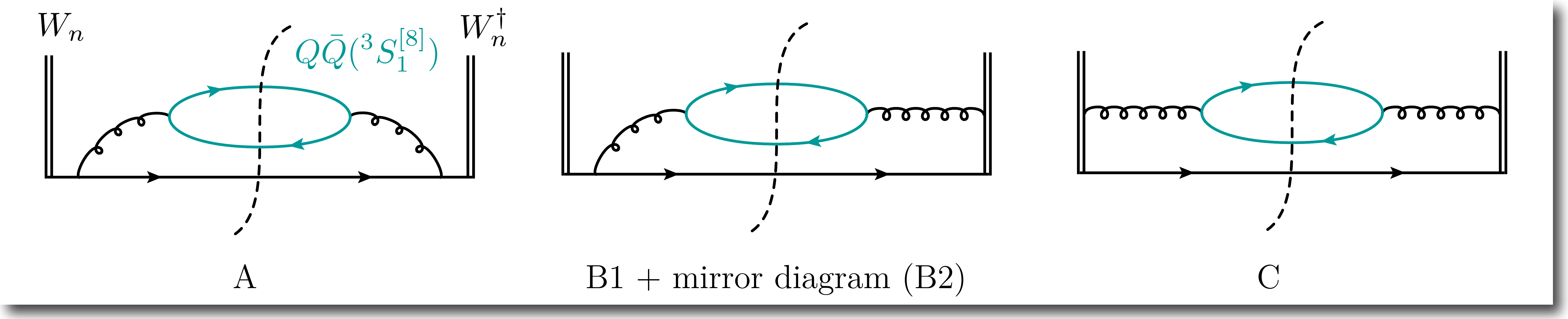
Gluon quarkonium TMDFF: [Work in progress with R. Fernandez del Castillo, S. Fernandez and I. Scimemi]

Quarkonium quark TMDFF (1/3)

$$\frac{d\sigma}{dxdzdQ^2d\vec{P}_\perp^2} = \sigma_0\,|C_V(Q^2,\mu^2)|^2 \int_0^\infty bdb\; J_0(b|\vec{q}_T|) f_{1,f\leftarrow h}\,(x_S, b;\mu,\zeta_1)\, D_{f\to H}\,(z_S, b;\mu,\zeta_2)$$

- In NRQCD the TMDFF is factorized as:

$$D_{q\to\psi}(z,b;\mu,\zeta) = d_{q\to Q\bar{Q}(^3S_1^{[8]})}(z,b;\mu,\zeta)\frac{\langle\mathcal{O}^\psi(^3S_1^{[8]})\rangle}{(d-1)(N_c^2-1)}\Big(1+O(\alpha_s)\Big)$$



No rapidity divergences at this order, i.e., no need to include the soft function

$$D_{q\to\psi}(z,\boldsymbol{k}_T;\mu,\zeta) = 2\frac{\alpha_s^2\,C_F}{\pi}\frac{\bar{z}}{M^3z^3}\left[\frac{2z^2}{\bar{z}}\frac{\boldsymbol{k}_T^2}{(\boldsymbol{k}_T^2+\bar{z}M^2/z^2)^2}+\frac{4}{\boldsymbol{k}_T^2+\bar{z}M^2/z^2}\right]\frac{\langle\mathcal{O}^\psi(^3S_1^{(8)})\rangle}{3(N_c^2-1)}$$

Quarkonium quark TMDFF (2/3)

[Merabet, Mathiot, Mendez-Galain 1994]

- Photon-gluon fusion at fixed-order analyzed long ago. We take the low p_T limit:

$$\frac{d\sigma(\gamma^* g)}{dx dz dQ^2 d\boldsymbol{P}_\perp^2} \simeq \frac{\alpha_s^2(\mu)\,\alpha_{\rm em}^2 \pi}{(1-\varepsilon)s^2} \frac{64\, e_H^2}{27 z\bar{z}^3 (2-z)^2} \frac{1}{x^2} \frac{\langle \mathcal{O}^\psi(^3S_1^{[1]})\rangle}{Q^2 M^3} f_{g\leftarrow h}(x;\mu^2) F_g(\bar{z}, \frac{P_\perp}{M})$$

$$F_g(a,b) = \frac{a^4 + a^2 + b^2}{(1+b^2/a^2)^2}$$

- For a back-of-the-envelope comparison, we also take the large p_T limit of the TMD cross-section:

$$\frac{d\sigma(\gamma^* q)}{dx dz dQ^2 d\boldsymbol{P}_\perp^2} \simeq \frac{\alpha_s^2(\mu)\,\alpha_{\rm em}^2\, \pi}{(1-\varepsilon)s^2} \sum_f \frac{4\, e_f^2}{9x^2\, z} \frac{\langle \mathcal{O}^\psi(^3S_1^{[8]})\rangle}{M^5} f_{f\leftarrow h}(x;\mu^2) F_q(z, \frac{\boldsymbol{P}_\perp^2}{M^2 \bar{z}})$$

$$F_q(a,b) = \frac{a^2 b + 2(1-a)(1+b)}{(1-a)(1+b)^2}$$

- An order of magnitude comparison of these two contributions to the cross-section:

$$\frac{d\sigma(\gamma^* g)}{d\sigma(\gamma^* q)} \sim \left(\frac{M}{Qv^2}\right)^2$$

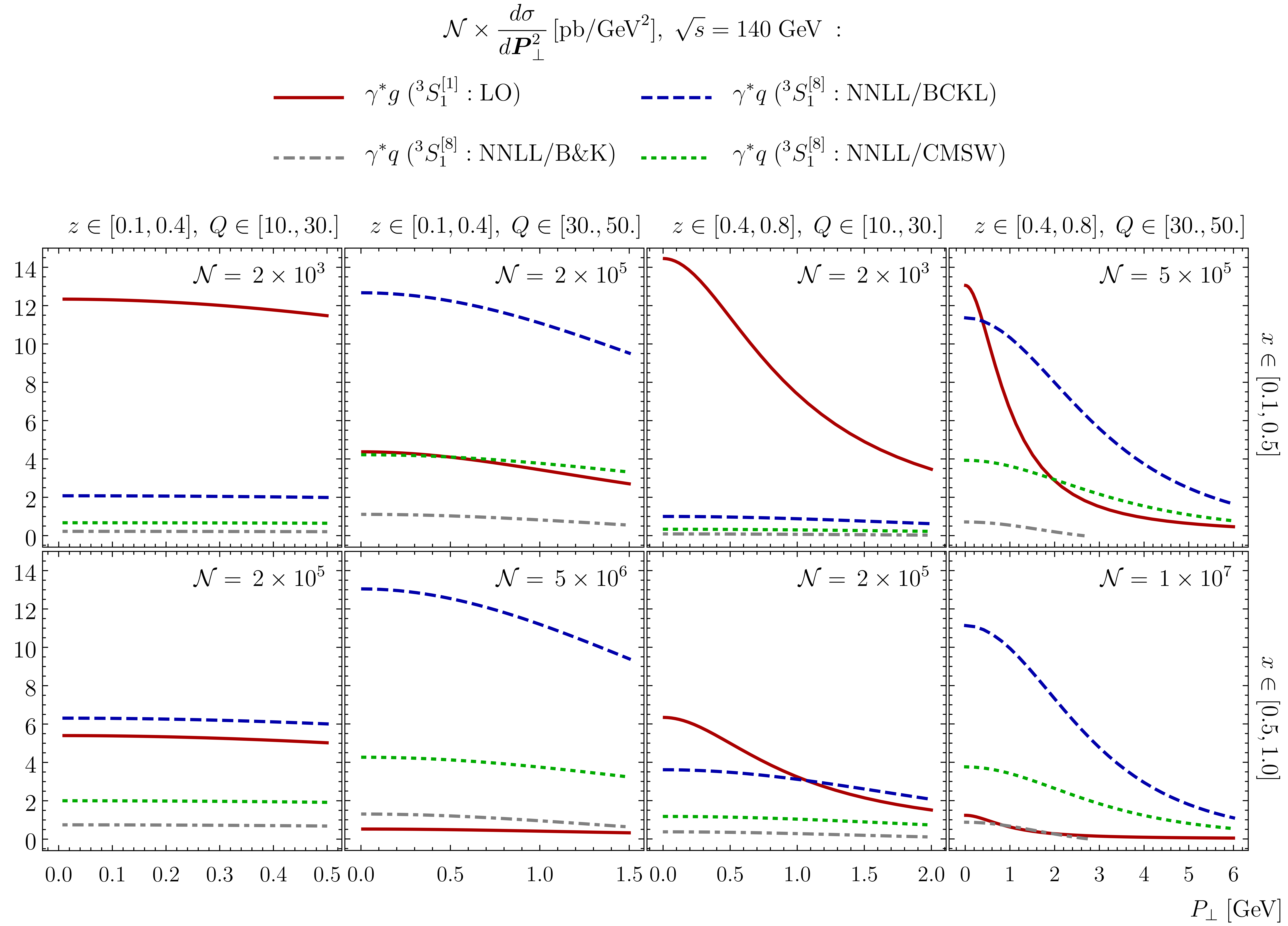
$M^2/(Q^2 v^4)$	$Q = 10$ [GeV]	$Q = 30$ [GeV]	$Q = 50$ [GeV]	$Q = 100$ [GeV]
charmonium ($v^2 \sim 0.3$)	1.0	0.1	0.04	0.01
bottomonium ($v^2 \sim 0.1$)	n.a.	10.0	3.8	1.0

Next we perform a proper comparison with full results

Quarkonium quark TMDFF (3/3)

Here we use the full results

- **Photon-gluon fusion** is in general **dominant at low values of Q and x**, while for some LDME sets quark fragmentation plays a major part in the opposite limit



Conclusions

- **Quarkonia production** is a **promising** way to probe gluon (and quark) **TMDs**
- New **non-perturbative** matrix elements appear: **TMD Shape Functions**
- **TMDShFs** are a new **zoo of functions:** one for each quarkonium state, channel and process
- **Fragmentation** mechanism **should be considered** too in pheno studies
- **ToDo**: factorization theorems, relations/constraints among TMDShFs, NLO, new pheno,…

Thank you!