

# TMD Fragmentation Functions

**Transversity 2022**  
**May 24, 2022**

**Ralf Seidl (RIKEN)**

## Single hadron FF

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
<p>Single hadron cross sections:  <math>e^+e^- \rightarrow hX</math>  <math>D_{1,\textcolor{red}{q}}^{\textcolor{blue}{h}}(\textcolor{violet}{z}, Q^2)</math></p> <p><a href="#">PRL111 (2013) 062002</a>  <a href="#">PRD101(2020) 092004</a></p>	<p>Azimuthal asymmetries:  <math>e^+e^- \rightarrow (h)(h)X</math>,  <math>\cos(\phi_1 + \phi_2)</math>  <math>H_{1,\textcolor{red}{q}}^{\perp(1)\textcolor{blue}{h}}(\textcolor{violet}{z}, Q^2)</math></p> <p><a href="#">PRL 96 (2006) 232002</a>  <a href="#">PRD 78 (2008) 032011</a></p>	<p>Unpol SIDIS, pp: <math>\frac{d\sigma}{dz}</math>  <math>e^+e^- \rightarrow (h)(h)X</math></p> <p><a href="#">PRD92 (2015) 092007</a>  <a href="#">PRD101(2020) 092004</a></p> <p>and scale dependence</p>
<p>Transverse momentum dependent FFs:  <math>e^+e^- \rightarrow (h)X</math>  <math>D_{1,\textcolor{red}{q}}^{\textcolor{blue}{h}}(\textcolor{violet}{z}, \textcolor{brown}{k}_T, Q^2)</math></p> <p><a href="#">PRD 99 (2019) 112006</a></p>	<p>Transverse momentum dependent asymmetries  <math>e^+e^- \rightarrow (h)(h)X</math>,  <math>\cos(\phi_1 + \phi_2), Q_t</math>  <math>H_{1,\textcolor{red}{q}}^{\perp h}(\textcolor{violet}{z}, \textcolor{brown}{k}_T, Q^2)</math></p> <p><a href="#">PRD100 (2019) 92008</a></p>	<p>Polarizing <math>\Lambda</math> fragmentation</p> <p><a href="#">PRL 122 (2019), 042001</a></p> <p><math>D_{1,\textcolor{red}{q}}^{\perp h}(\textcolor{violet}{z}, \textcolor{brown}{k}_T, Q^2)</math></p>

## Dihadron FF (IFF)

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## Single hadron FF

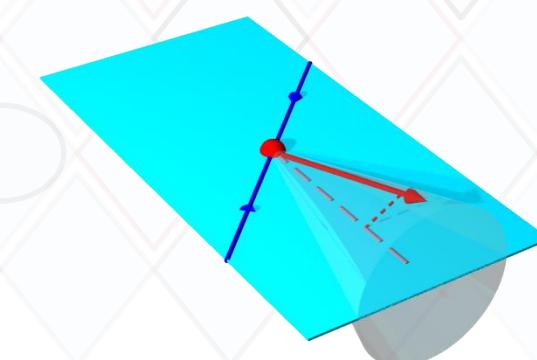
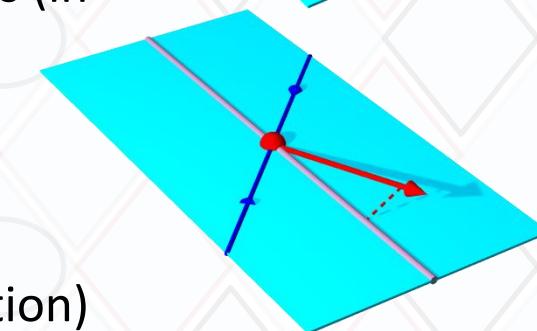
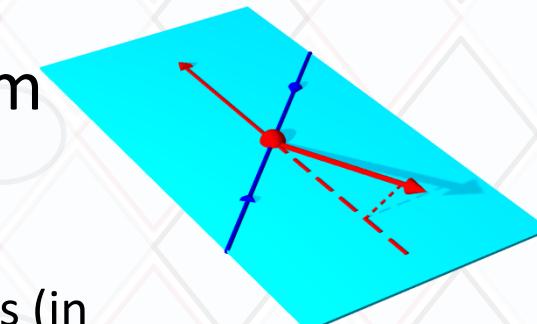
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# $K_T$ Dependence of FFs in $e^+e^-$

- Gain also sensitivity into transverse momentum generated in fragmentation
- Two ways to obtain transverse momentum dependence
  - Traditional 2-hadron FF
    - use transverse momentum between two hadrons (in opposite hemispheres)
    - Usual convolution of two transverse momenta
  - Single-hadron FF wrt to Thrust
    - No convolution
    - Need correction for  $q\bar{q}$  axis (similar to a Jet function)
  - Single-hadron FF wrt jet axis
    - No convolution
    - Need Jet function



Ongoing  
(nearly finished)

Published

Ongoing  
(started)

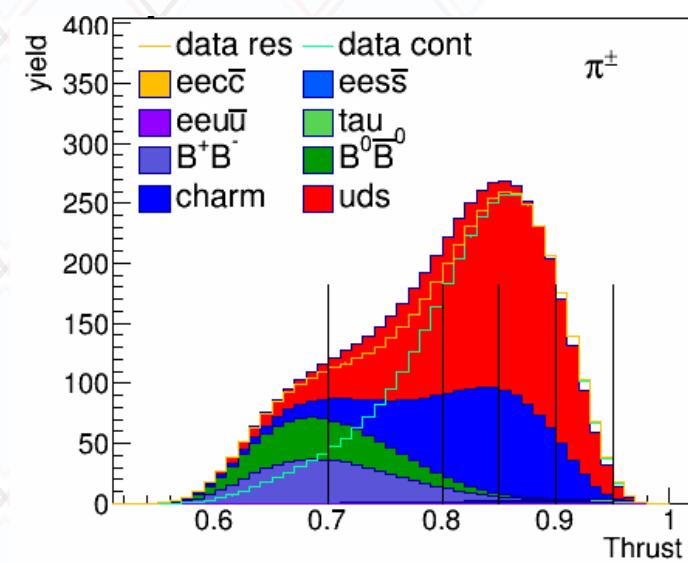
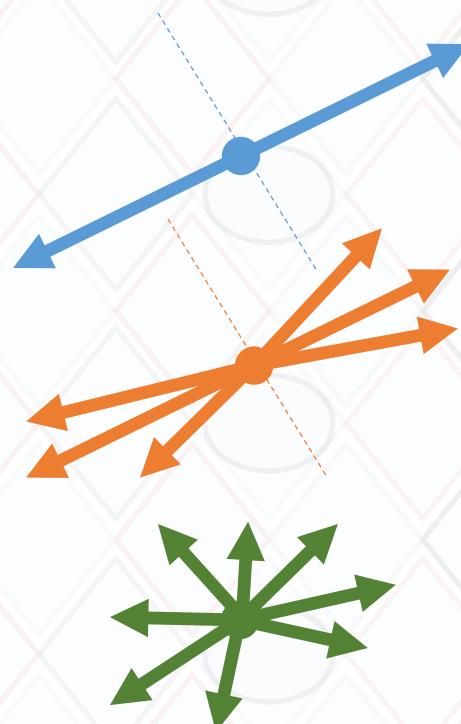
# Thrust definition

- Event shape variable thrust is defined as:

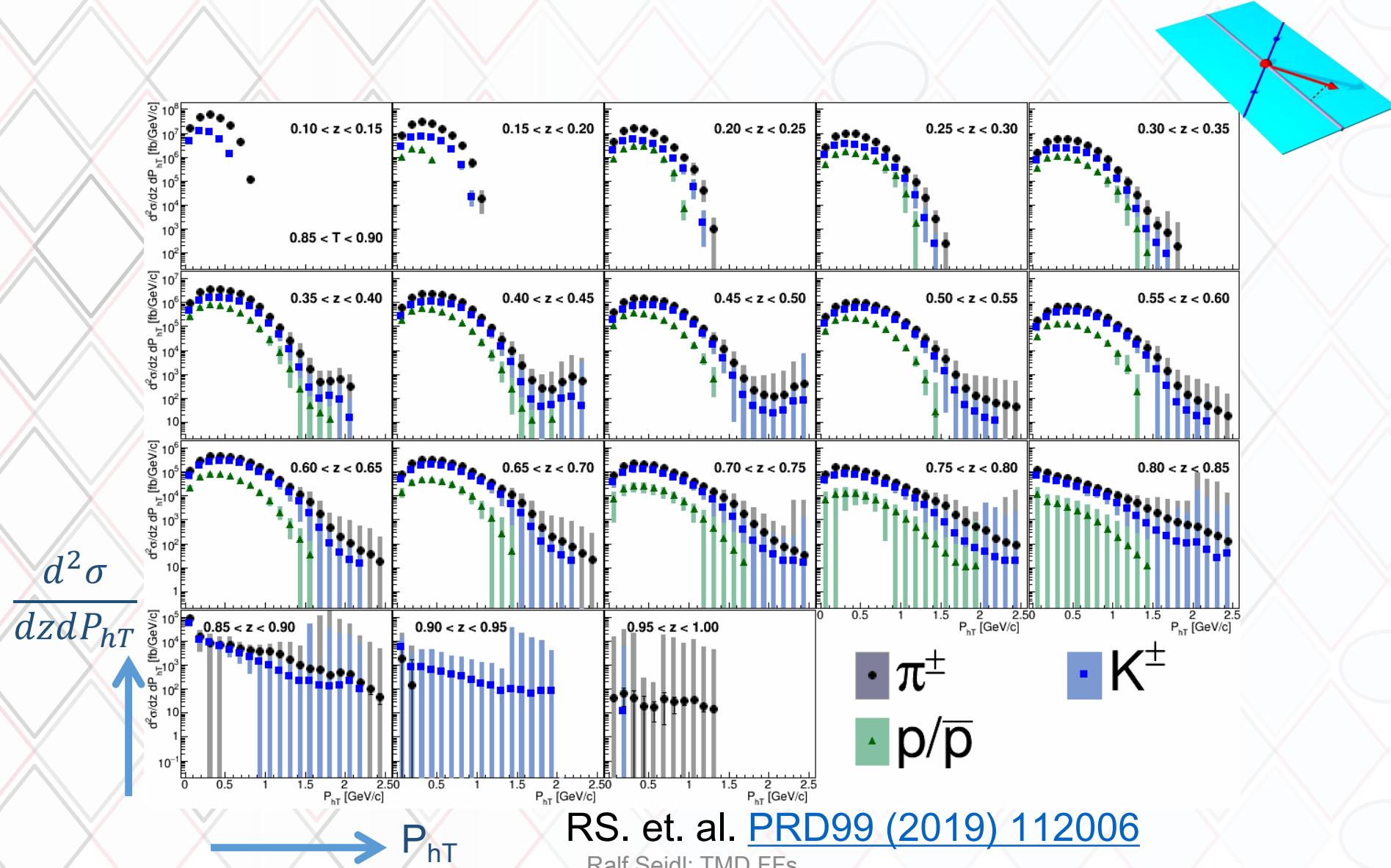
$$T \stackrel{max}{=} \frac{\sum_h |\mathbf{P}_h \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h|}$$

- All final-state particles are included in the sum (not on parton level!)
- A two-jet-like event has a high thrust value
- A completely spherical event has a thrust value of 0.5

- Thrust axis  $\mathbf{n}$  also defines the hemispheres

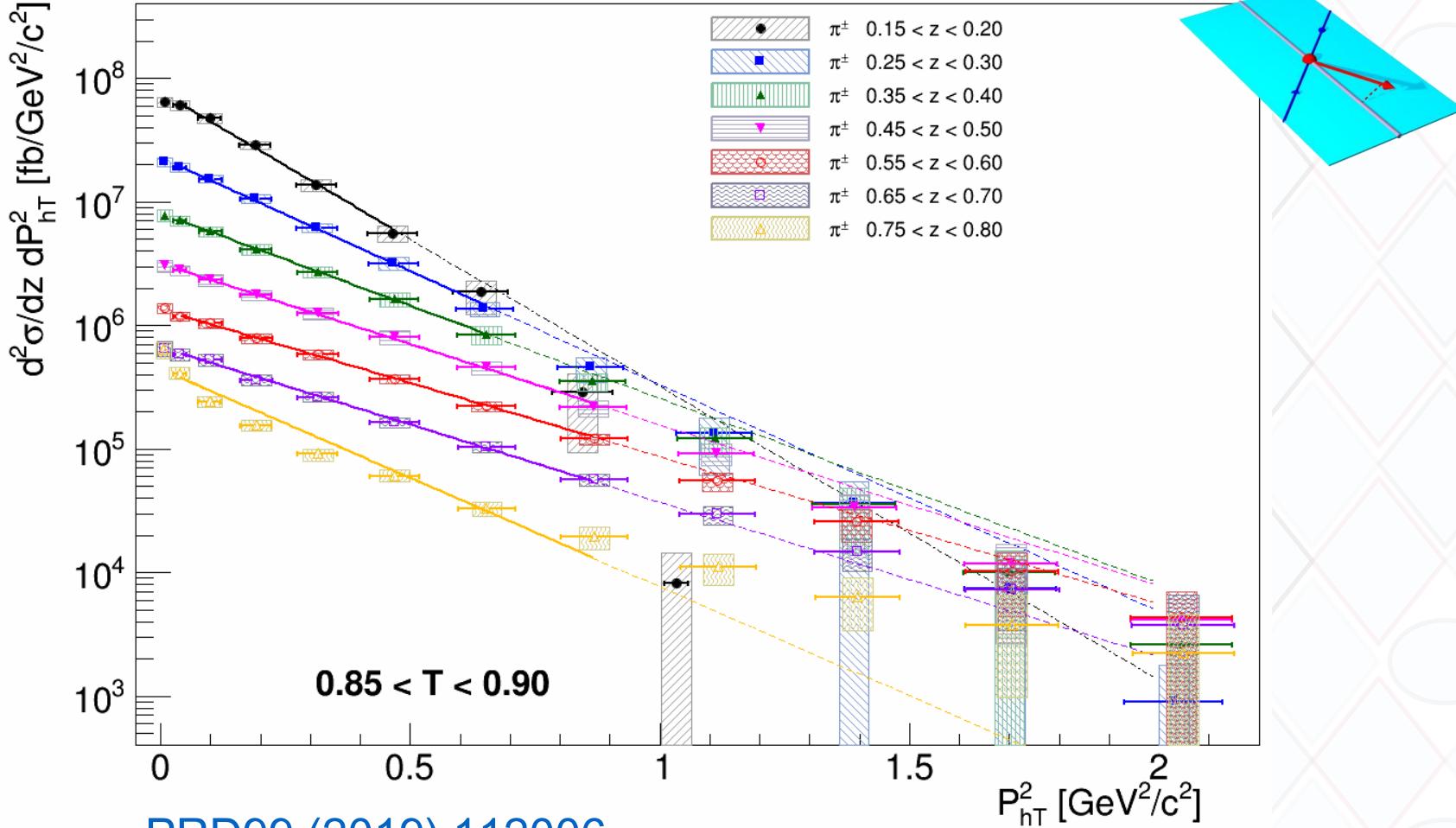


# Cross sections various hadrons



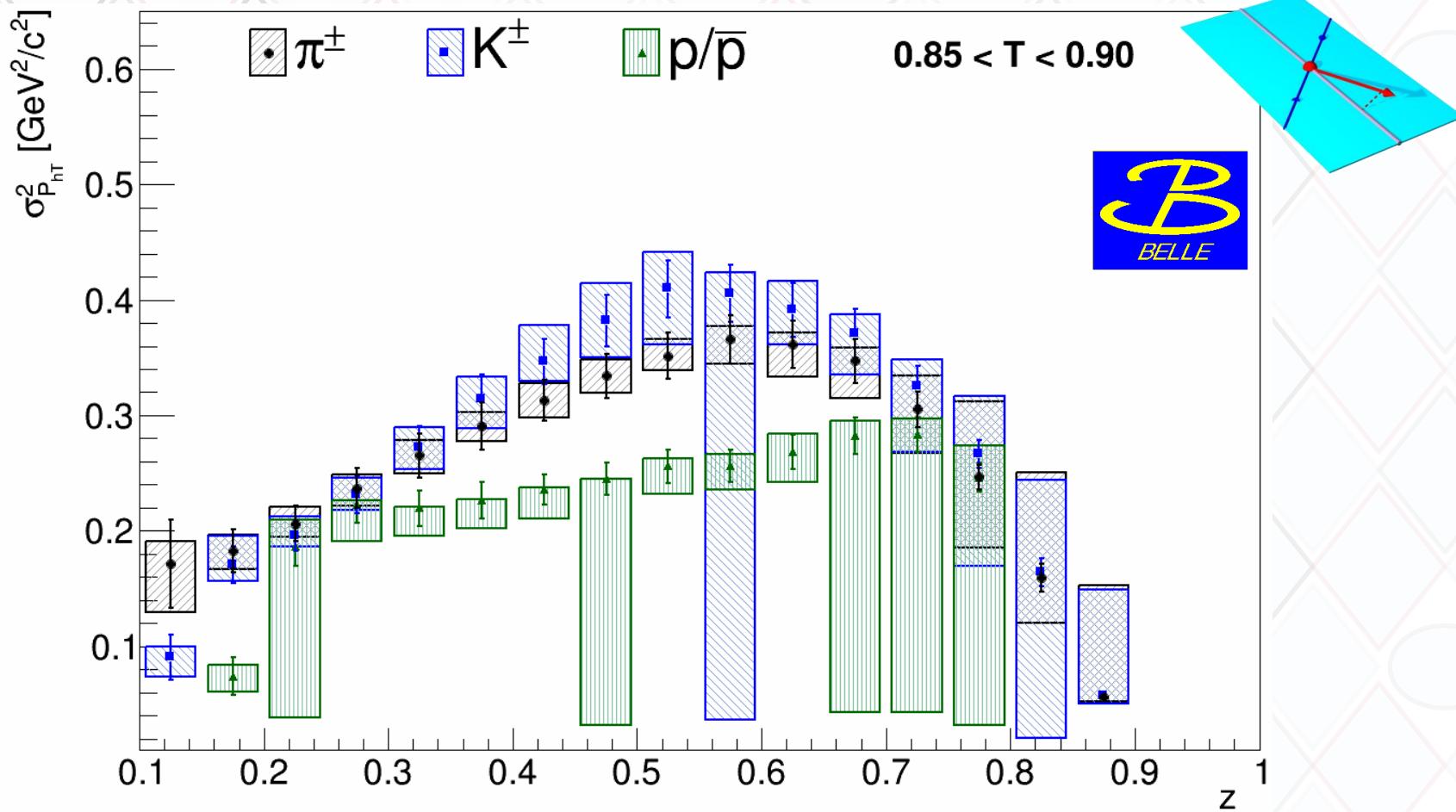
# Fits vs $P_{hT}^2$

Fit exponential to smaller transverse momenta for Gaussian  $P_{hT}$  dependence and power low at higher  $P_{hT}$



# Transverse momentum dependent unpol FFs:

- First direct (no convolutions) measurement of z and kt dependence
- Extraction of Gaussian kt widths



[PRD99 \(2019\) 112006](#)

# Gaussian widths comparison to MC

first direct (no convolutions) measurement of z dependence of Gaussian widths

Pythia6  
MSTP(21):

0.28

0.325

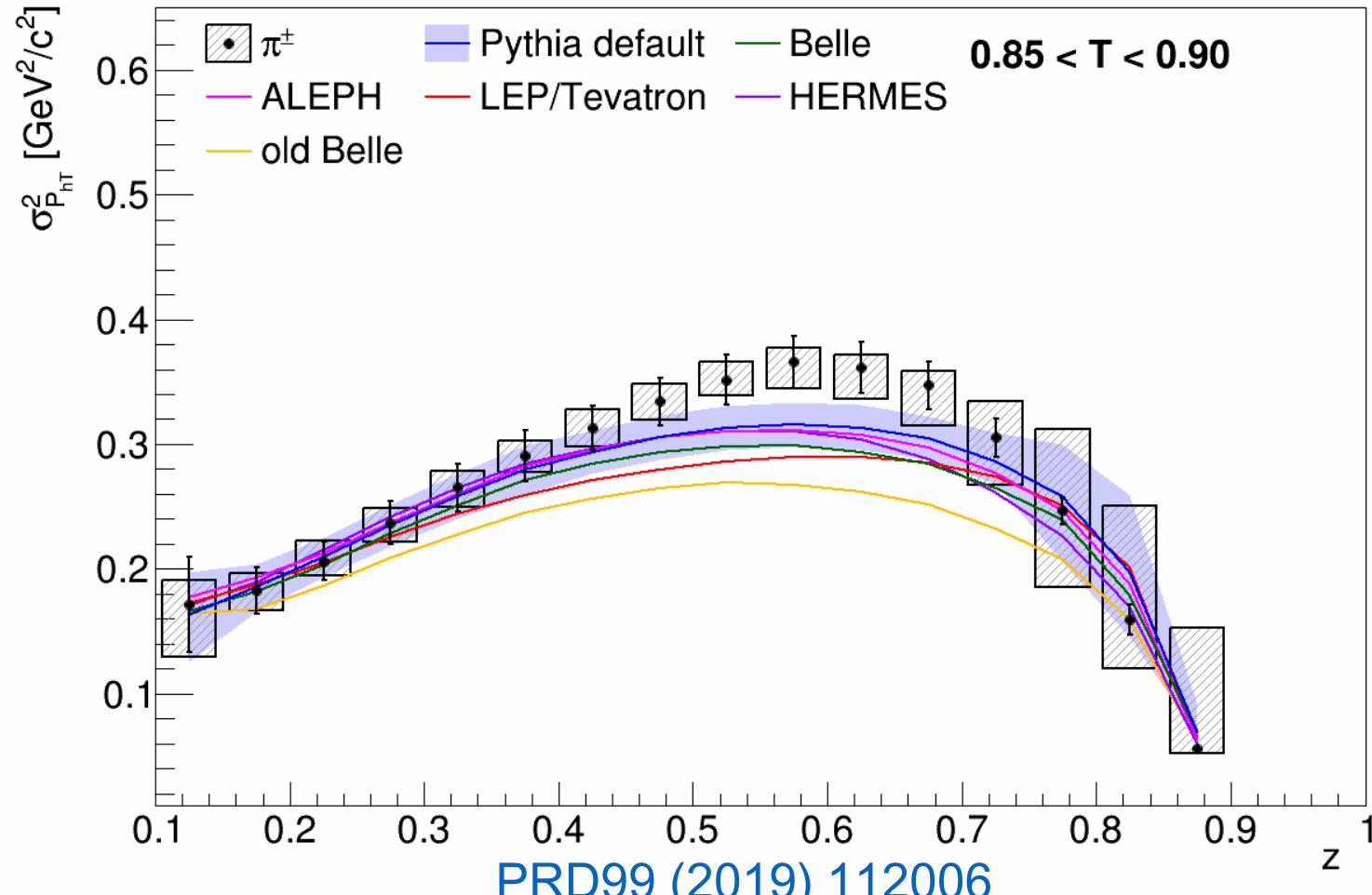
0.36

0.36

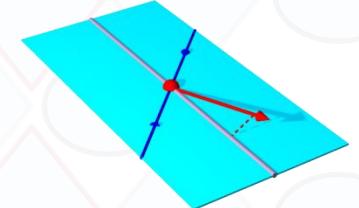
0.37

0.40

5/24/2022



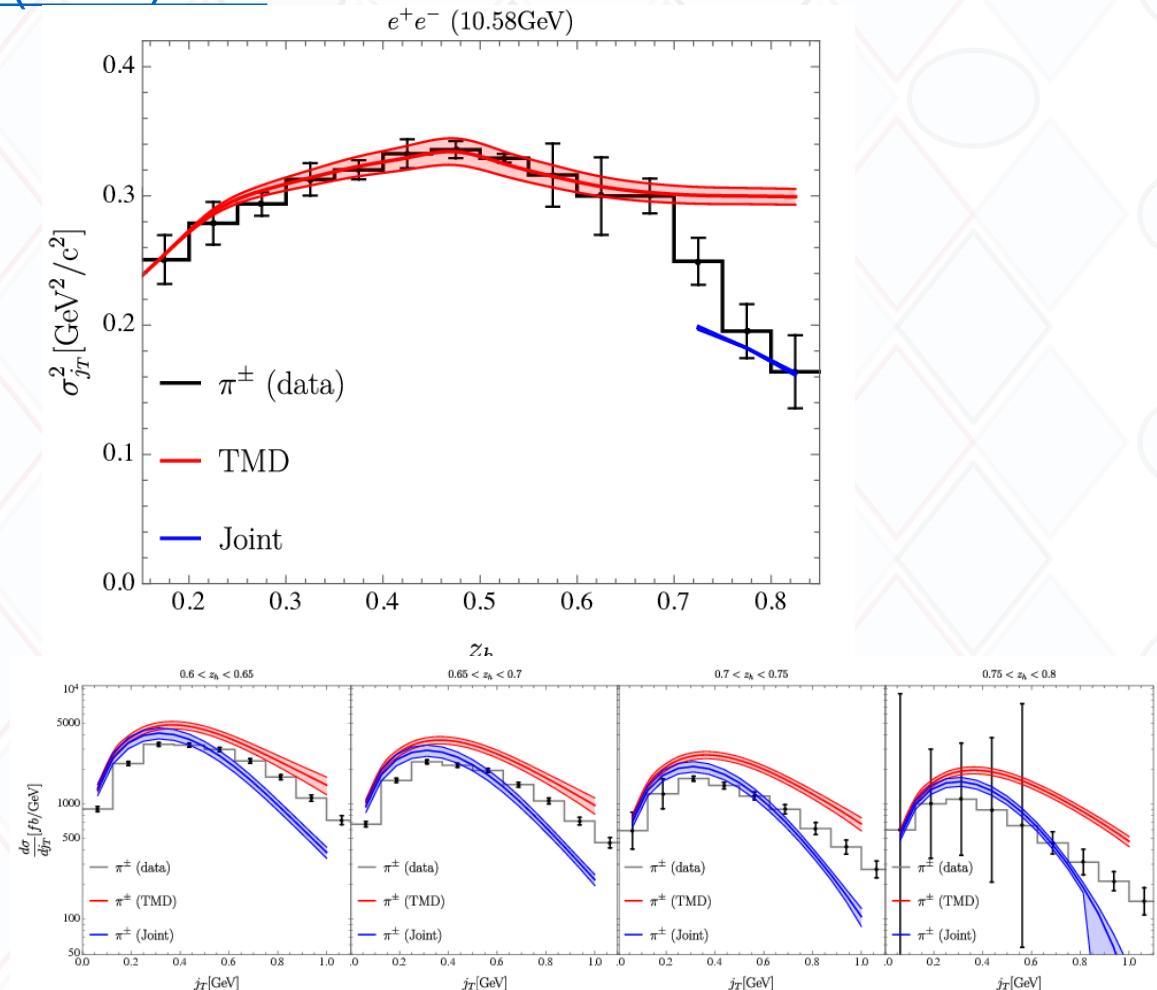
Ralf Seidl: TMD FFs



# Phenomenological Fits of cross sections I

Kang, et. al. JHEP 12 (2020) 127

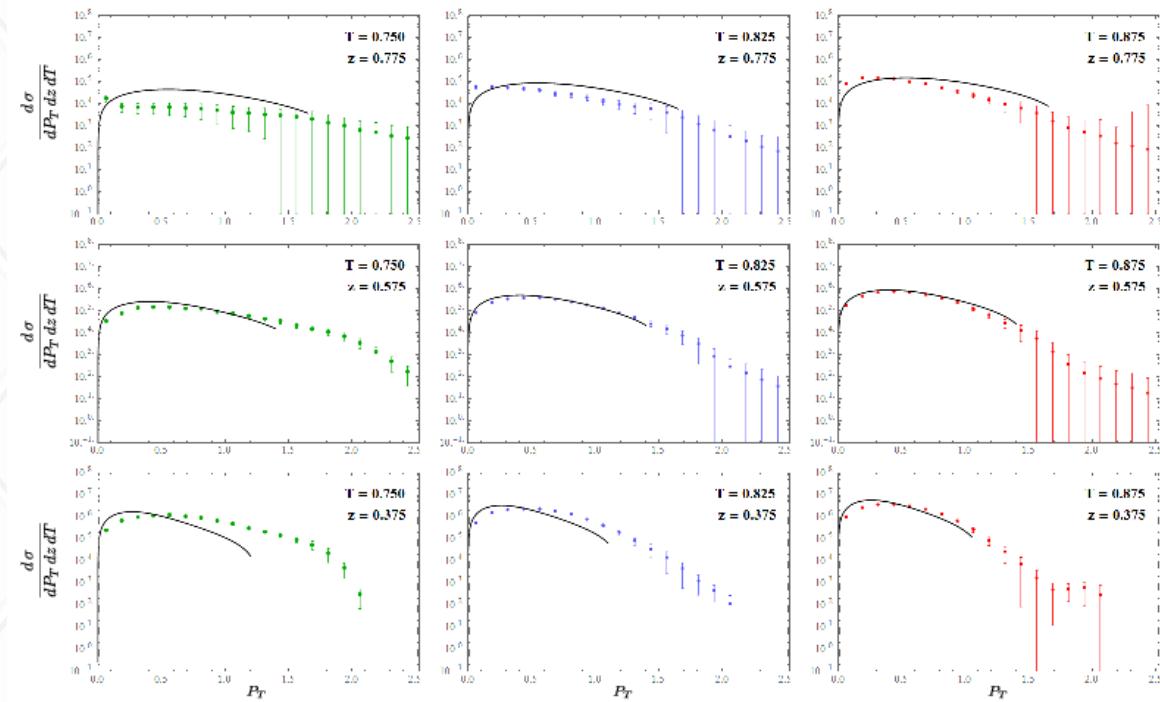
- SCET formalism
- Inclusion of Thrust axis possible in similar way to Jet functions  
TMD and threshold resummation needed
- TMD region of  $j_T \ll Q$
- Additional description for high-z region



# Phenomenological Fits of cross sections II

[Boglione, Simonelli JHEP 02 \(2021\) 076](#)

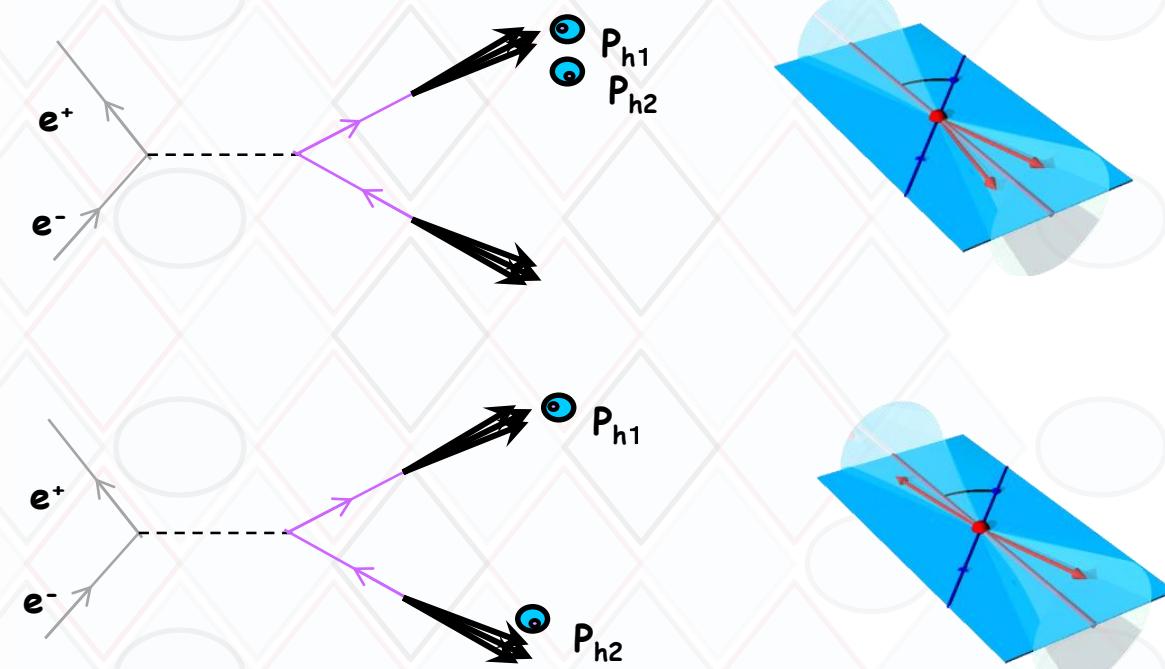
- NLO and NLL description of cross sections, based on NNFF1.0\_NLO
- Collinear parts of phase space need to be cut out (esp. high  $P_T$ )
- Intermediate Thrust range can be described well
- High thrust and high z range would need different pheno treatment



# Di-hadron fragmentation functions

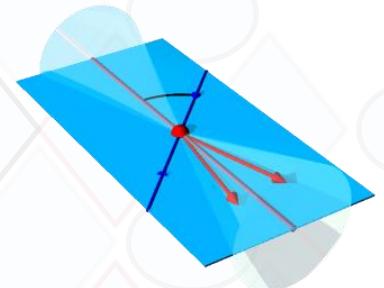
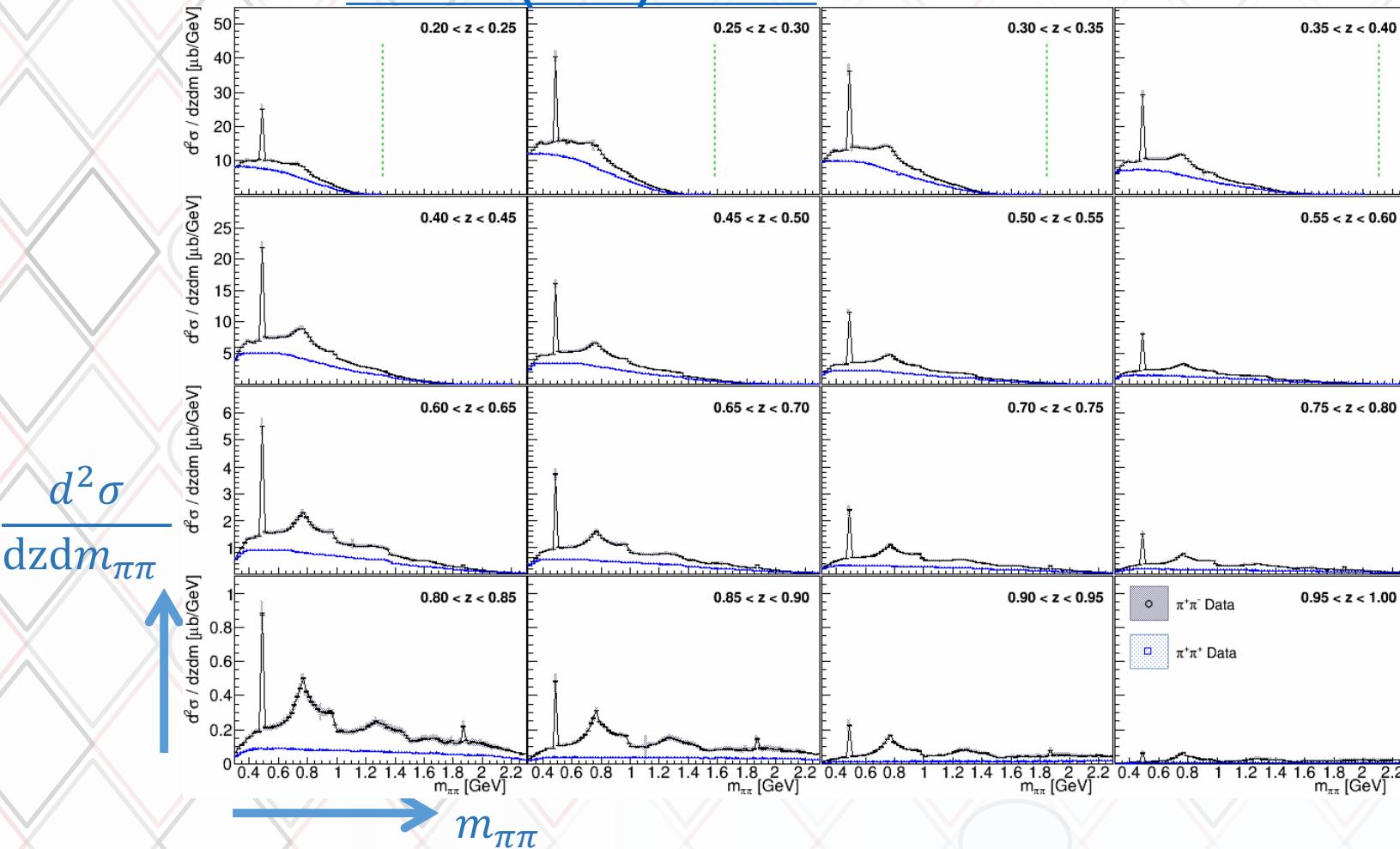
$$D_{1,q}^{h_1 h_2}(z, m, Q^2)$$

$$D_{1,q}^h(z_1, Q^2) D_{1,q}^h(z_2, Q^2)$$



# Di-hadron mass dependence

Belle: RS et.al. [PRD96 \(2017\) 032005](#)

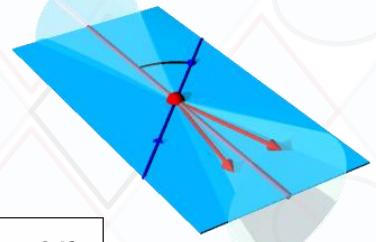
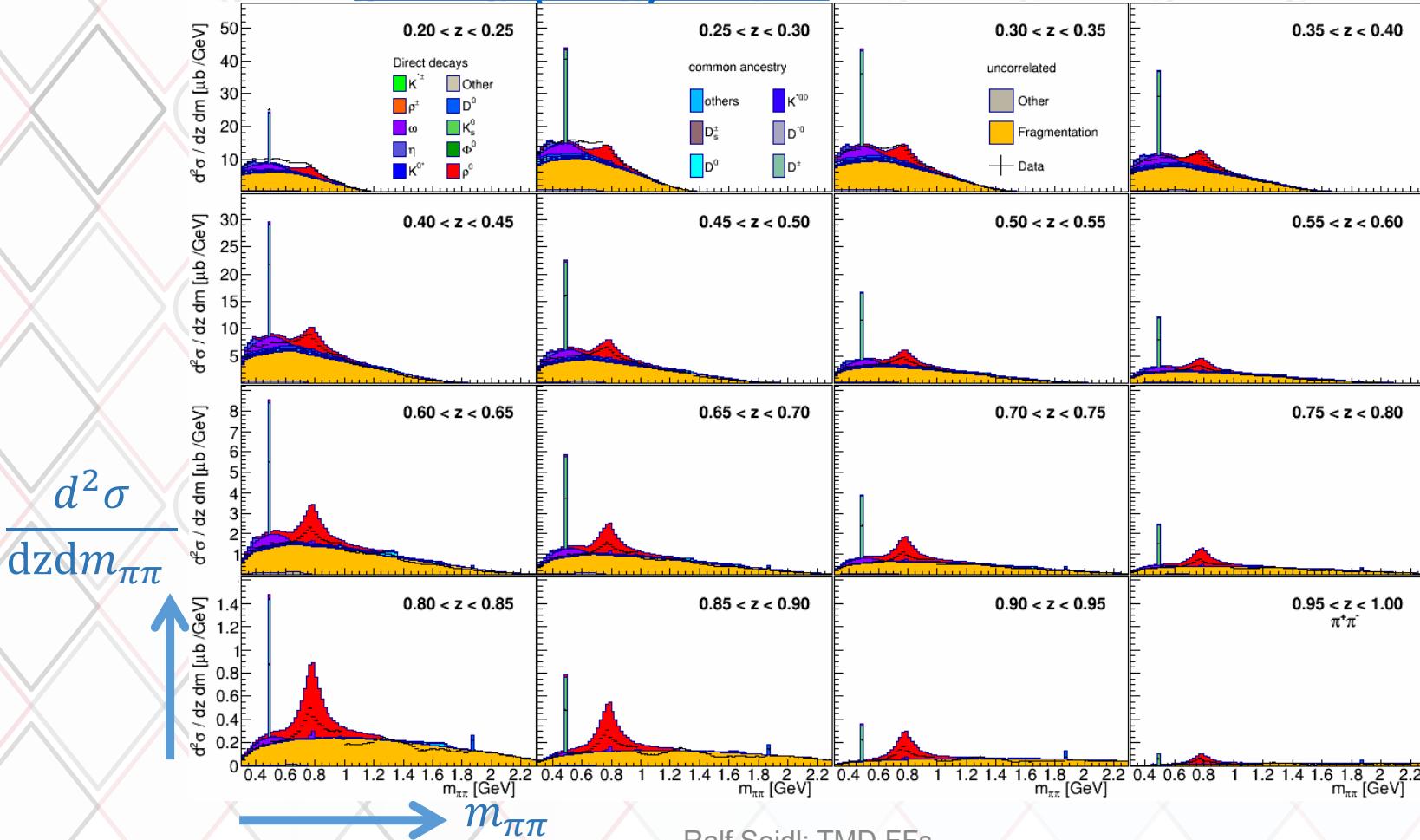


- Important input for IFF based transversity global analysis
- Individual resonances, etc quite visible; interesting for FF in itself

# Di-pion individual contributions

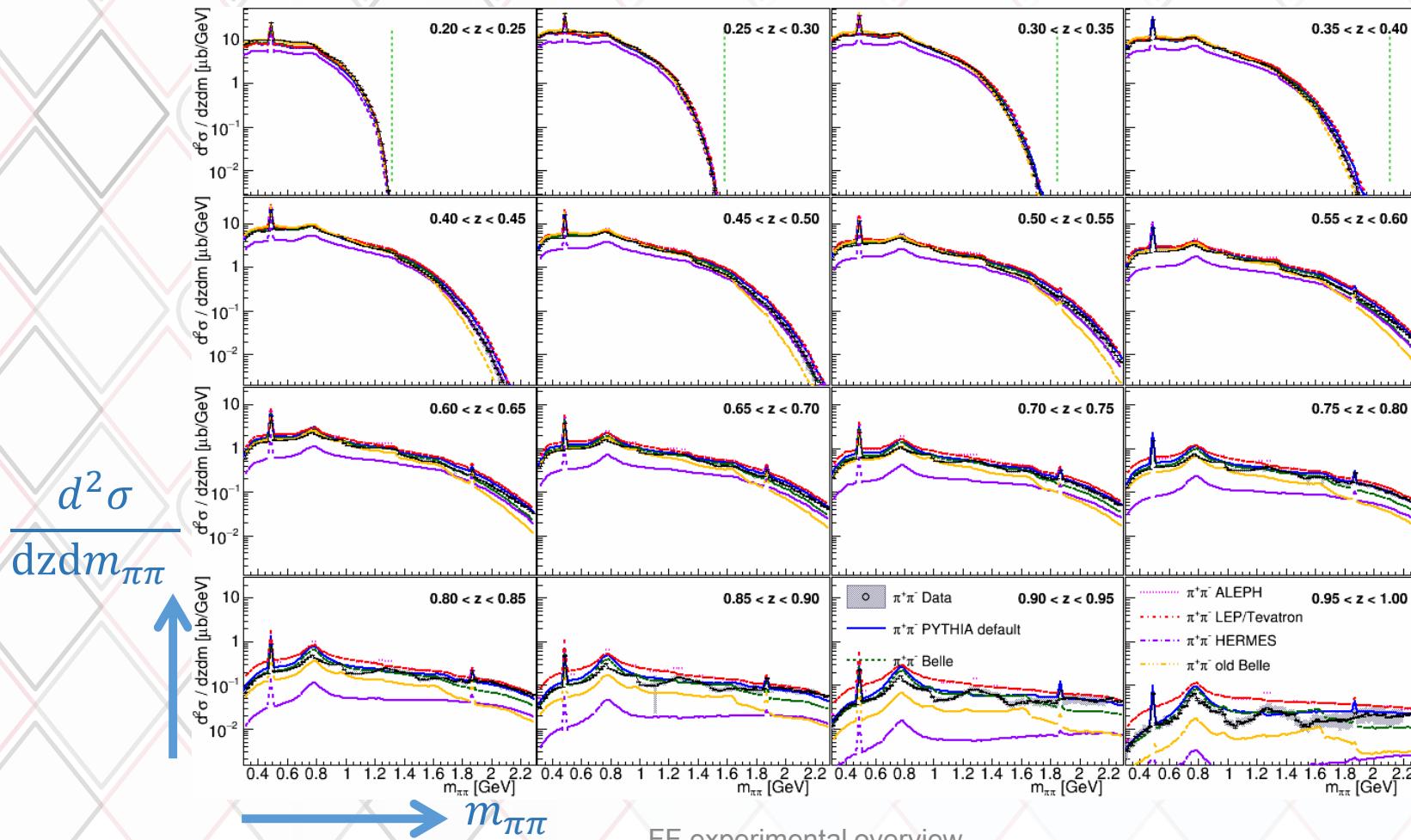
Contributions from various resonances and direct fragmentation

Belle: RS et.al. [PRD96 \(2017\) 032005](#)



# Mass dependence comparisons to Pythia tunes

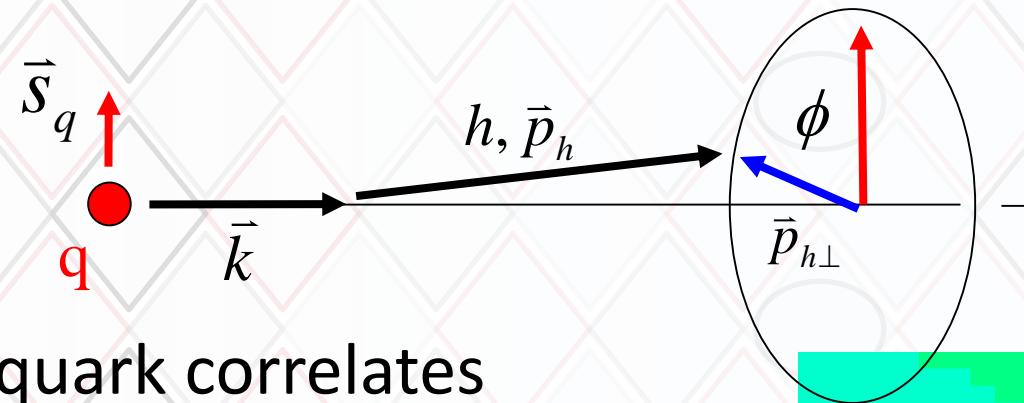
Magnitude and z dependence reasonable in Pythia 6.4 default,  
Intermediate mass structure better described by LEP tunes (higher spin mesons)



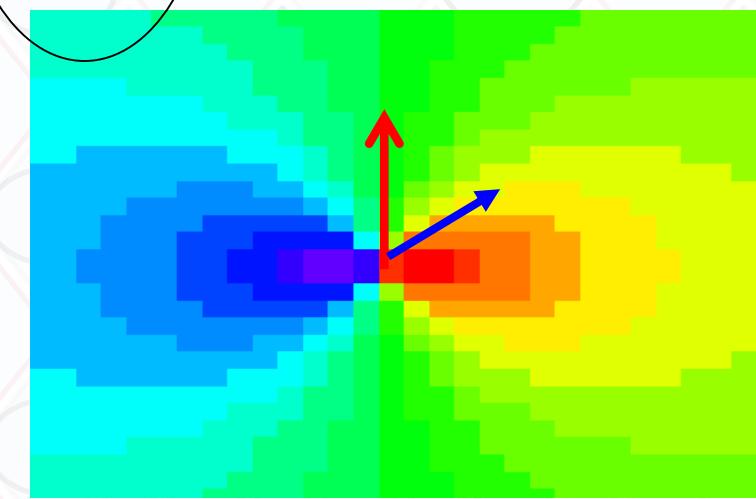
# Collins fragmentation function

J. Collins, Nucl. Phys. B396, (1993) 161

$$D_{q\uparrow}^h(z, P_{h\perp}) = D_{1,q}^h(z, P_{h\perp}^2) + H_{1,q}^{\perp h}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{z M_h}$$



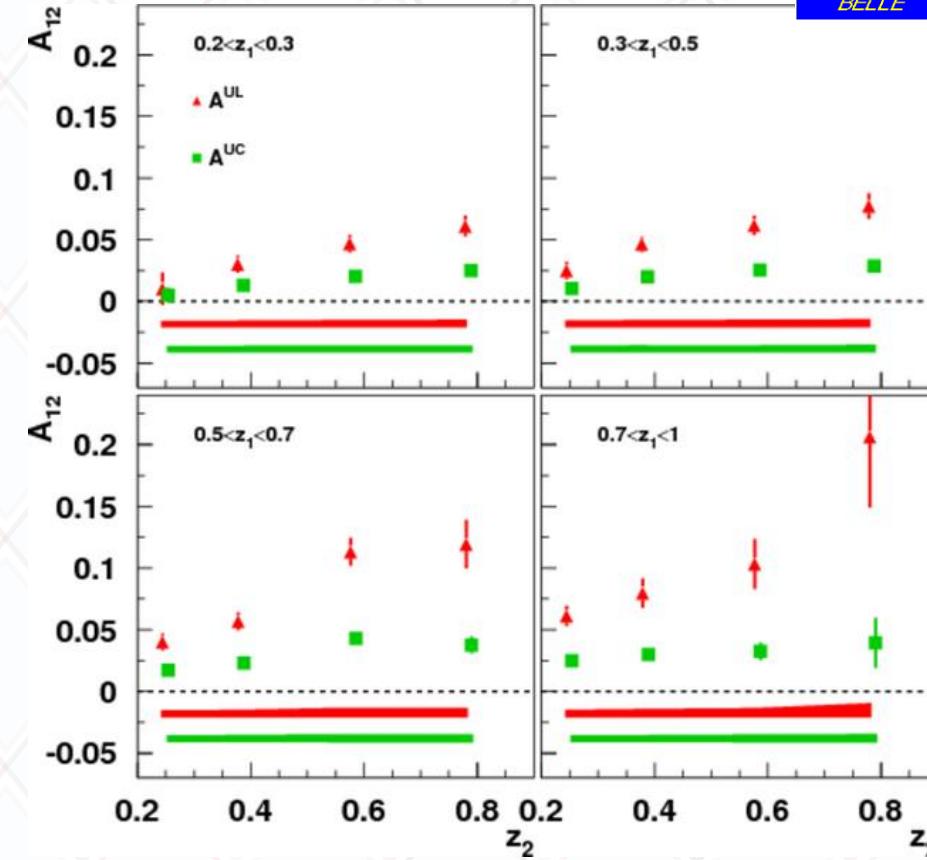
- Spin of quark correlates with hadron transverse momentum
- translates into azimuthal anisotropy of final state hadrons



# Belle Collins asymmetries



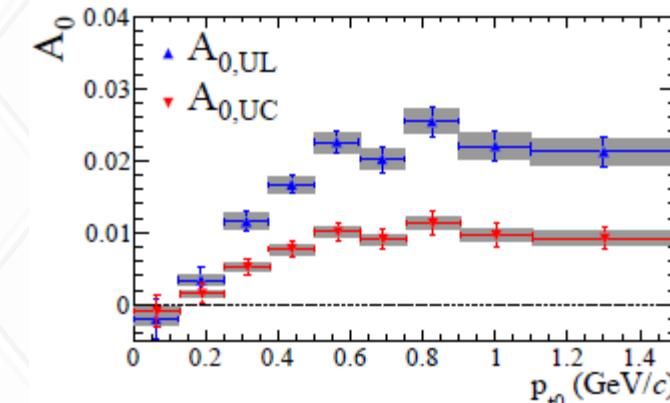
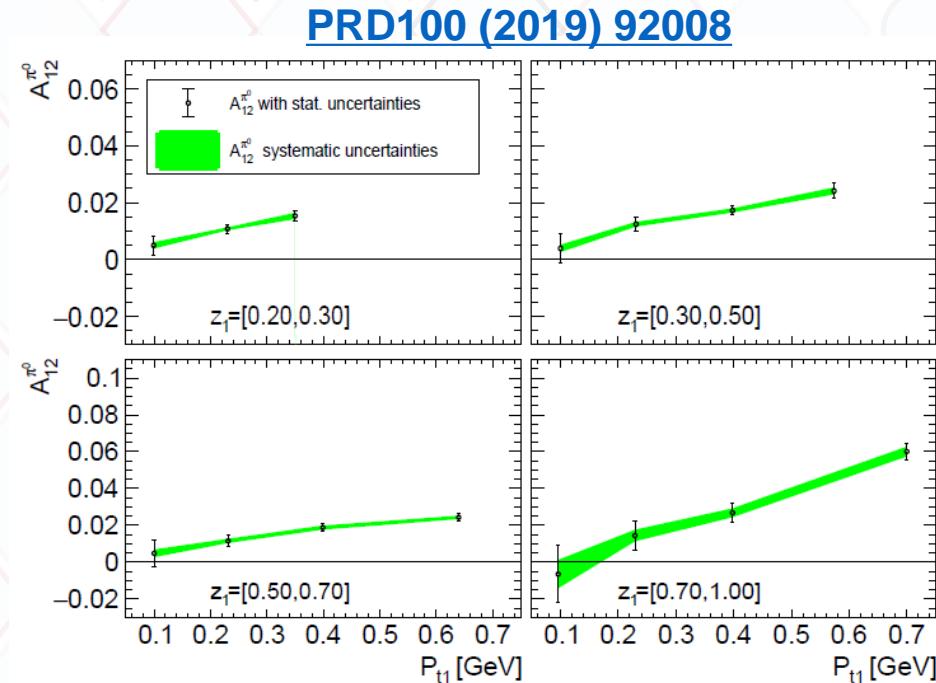
- Red points :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over **like** sign pion pair ratio :  $A^{UL}$
- Green points :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over **any charged** pion pair ratio :  $A^{UC}$
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF



RS et al (Belle), PRL96: 232002  
PRD 78:032011, Erratum D86:039905

# Transverse momentum

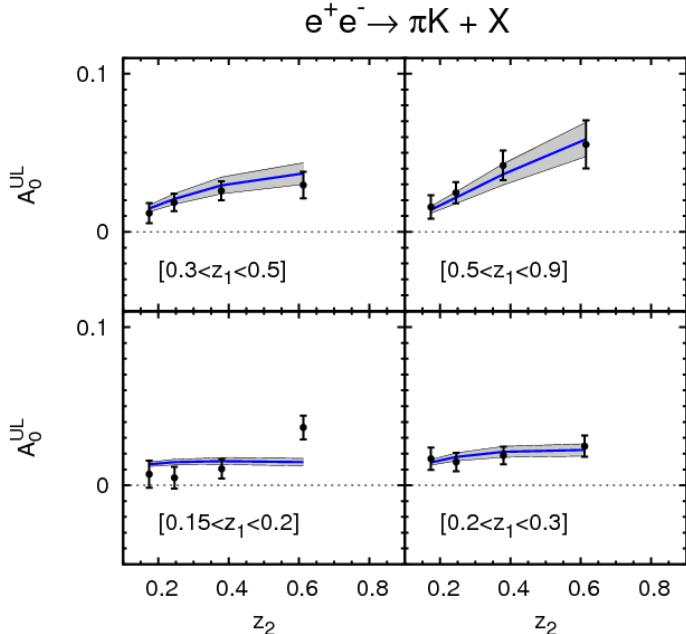
- Add transverse momentum to Collins asymmetries' z dependence
- Currently only 1 or 2-dimensional extractions available ( $q_t$ ,  $z_1 \times z_2$ ,  $p_{t1} \times p_{t2}, z_1 \times p_{t1}$ )
- Increasing asymmetries with both z and pt, but pt reach limited
- Multidimensional extractions needed



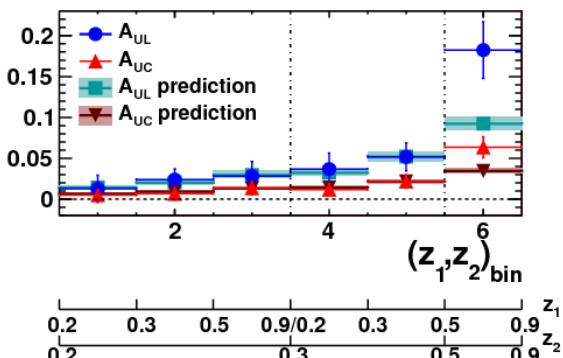
# Quark transversity via Collins: Kaons

BABAR: [PRD 92 \(2015\) 111101](#)

Anselmino et al: [PRD 93 \(2016\) 034025](#)



BESIII: [PRL 116 \(2016\) 042001](#)



- Addition of kaon Collins fragmentation strongly needed for flavor decomposition of quark transversity
- Large amount of potentially participating FFs well described by light and “heavy” favored and disfavored FFs
- Allows inclusion of HERMES and COMPASS kaon asymmetries (+eventually EIC) in fits
- Also: pion Collins at lower scale(BESIII) consistent with TMD evolution

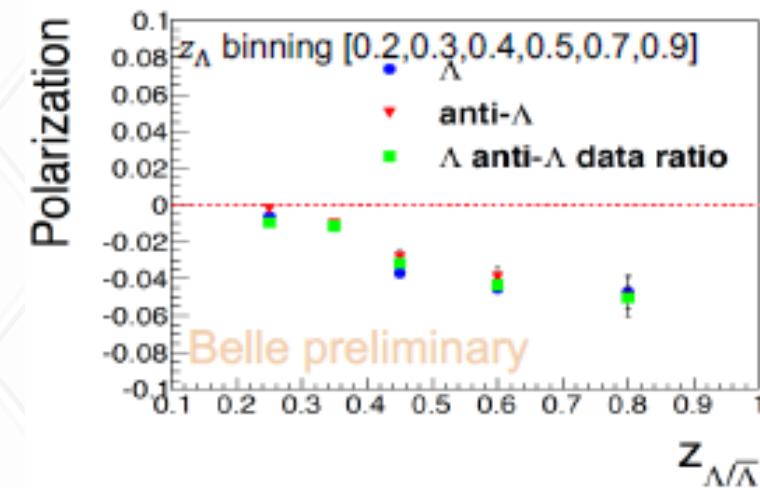
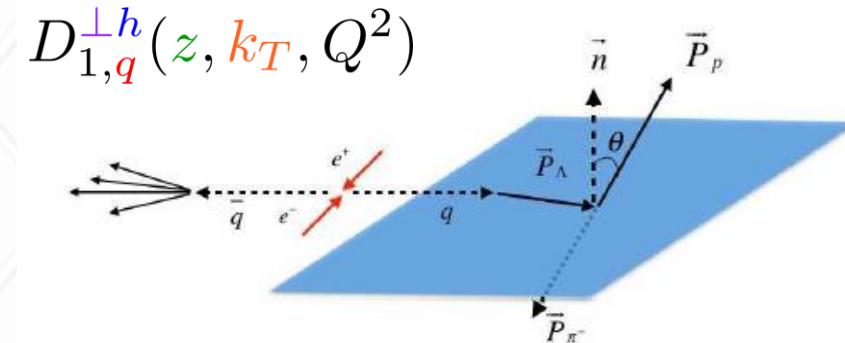
# Ongoing work: Collins multidimensional analysis and Kaon combinations

- Currently revisiting kaon combinations of the Collins asymmetries
- While doing so, try to perform a full multi-dimensional analysis:
  - Consider :
    - $6(z_1) \times 6(z_2) \times 5(k_{t1}) \times 5(k_{t2}) \times 1(\text{costheta}) \times 8(\text{phi})$  for  $A_{12}$  method
    - $6(z_1) \times 6(z_2) \times 10(q_t) \times 1(\text{costheta}) \times 8(\text{phi})$  for  $A_0$  method
- Perform most correction steps similar to recent analyses (PID, smearing)
  - Possibly simplified smearing unfolding as each  $z_1-z_2$  bin separately ( $z$  smearing almost nonexistent in such a binning)
  - non-qqbar removal, charm removal, ISR correction and acceptance might require introduction of nonzero MC asymmetries

# Single $\Lambda$ polarization measurements

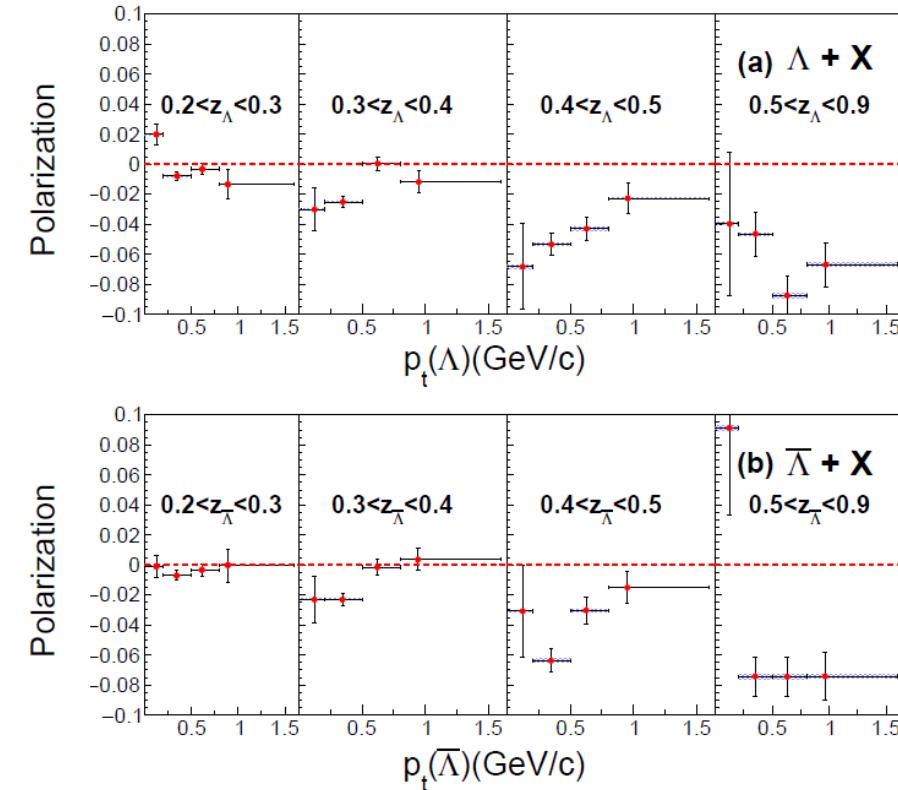
- Related to open question about  $\Lambda$  polarization in hadron collisions from 40 years ago!
- Fragmentation counterpart to the Sivers Function:
  - unpolarized parton fragments into transversely polarized baryon with transverse momentum wrt to parton direction
- Reconstruct  $\Lambda$ , its transverse momentum and polarization

YingHui Guan (Indiana/KEK):  
PRL 122 (2019), 042001



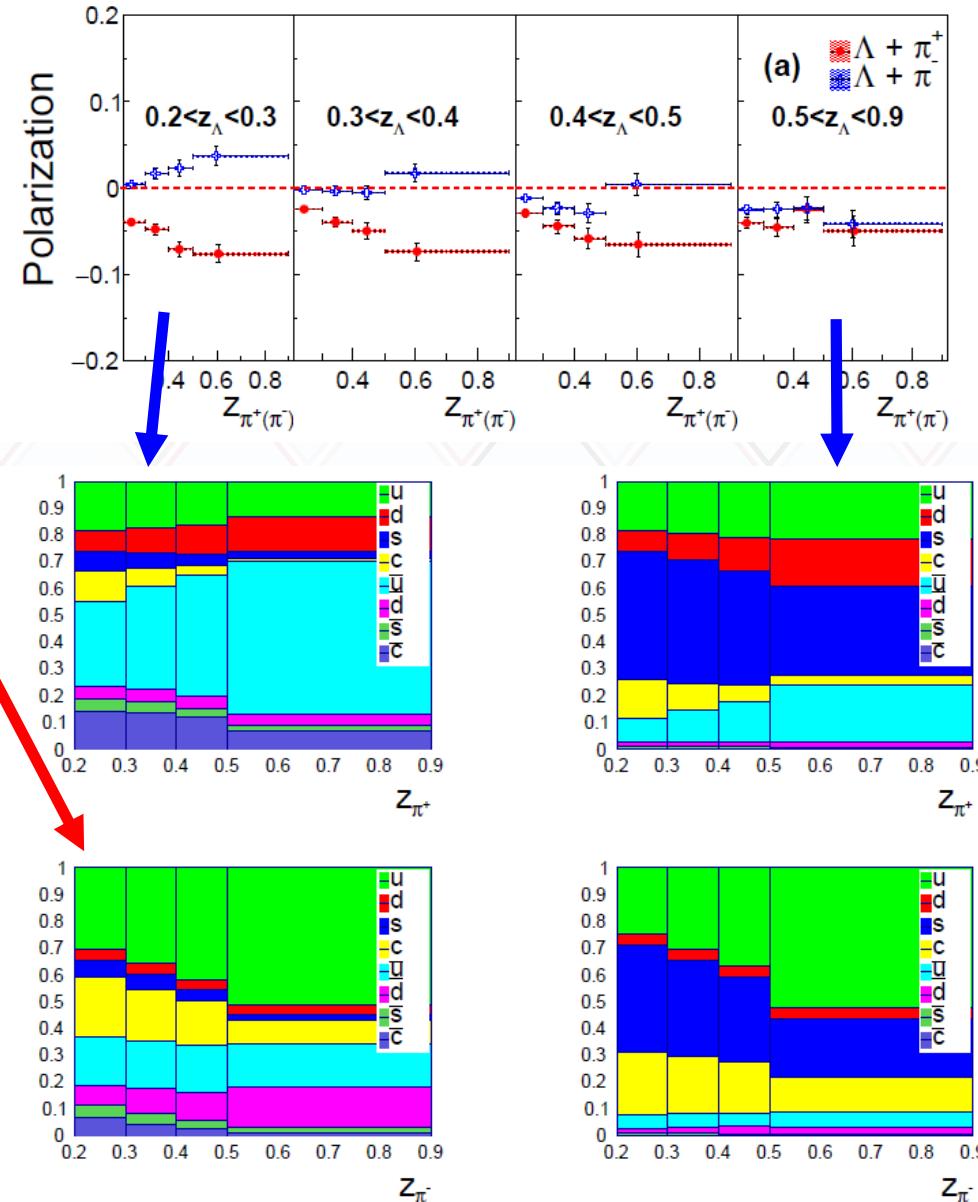
# Transverse momentum dependence

- Different behavior for low and high-z :
- At low z small
- At intermediate z falling Polarization with  $P_t$
- At high z increasing polarization with  $P_t$



# Opposite hemisphere pion correlation

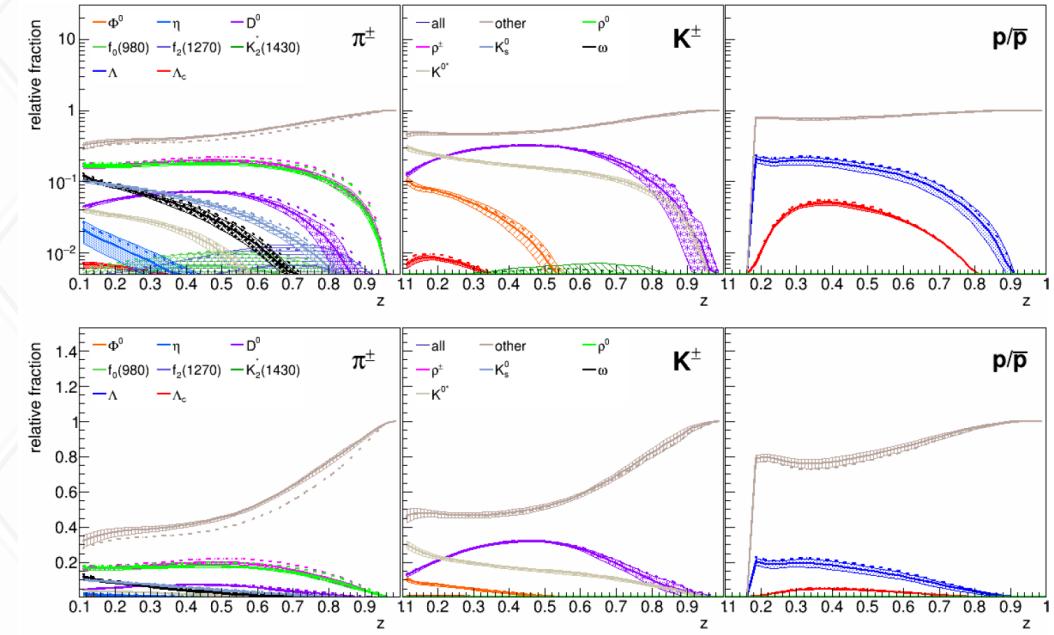
- Interesting  $z_\pi$  and  $z_\Lambda$  dependence :
- At low  $z_\Lambda$  light quark fragmentation dominant, some charm in  $\pi^- \rightarrow$  different signs
- At high  $z_\Lambda$  strange + charm fragmentation more relevant  $\rightarrow$  same signs
- Several fits to data with slightly different results



# Not TMD(yet) but indirectly related: Weak and strong decay feed-down

- Hadrons from Weak decays technically not part of FF definition, but often included
- Strong decays part of total sum over hadronic final state
- Both can affect the z (and transverse momentum) dependence of the detected hadrons:
  - naturally included in unpolarized MC,
  - in part added to polarized generators ( $\rightarrow$ Albi)
  - How does PHENO handle this (additional parameters?)

Decaying hadron fractions in light hadrons at  $\sqrt{s} = 10.58 \text{ GeV}$  (PYTHIA6):

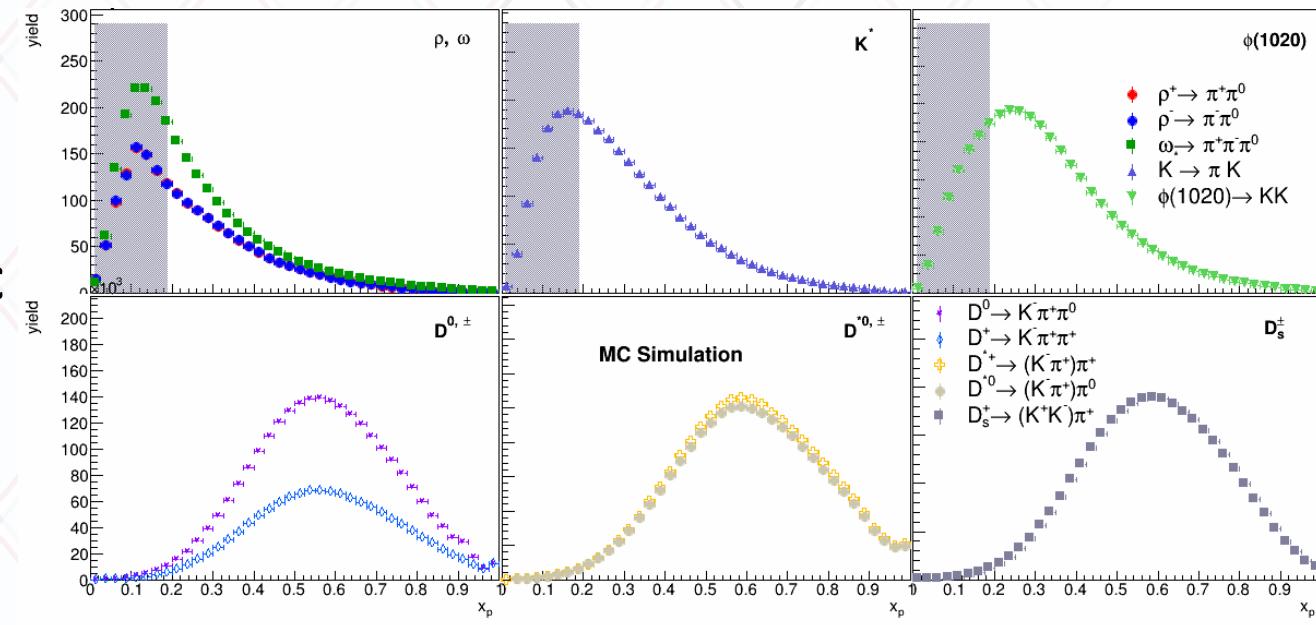


Bands: various Pythia tunes, including PARJ(11 VM to PS ratio) range from 0.3-0.55  
Dashed lines: default, but PARJ(11) = 0.6

# Ongoing: Decaying particle FFs

- Study the explicit differential cross sections for VMs, D mesons as a function of  $x_p$
- Mostly mass distributions and fits well-behaved, except for  $\rho$ – $\omega$  (interference) and more exotic resonances
- Also of interest for ultra high-energetic cosmic ray air shower research (muon problem)

- Example from MC at Belle energies (for  $4\pi$  acceptance):



# Summary

- Many Belle/Babar/BES3 TMD fragmentation related measurements available:
  - Unpolarized single hadrons wrt thrust axis
  - Collins asymmetries ( $k_t$  dependent, kaons)
  - Polarizing  $\Lambda$  fragmentation
- Detailed di-hadron cross section and asymmetry measurements
- More measurements on going for:
  - Other venues for unpolarized TMDs (opposite hemisphere dihadrons, hadron in jet)
  - Multi-dimensional extractions of Collins asymmetries
  - Other studies ongoing on the fragmentation of VMs and Ds
  - More to come from Belle and BelleII