## **GTMDs & Wigner functions: Recent Developments**



Shohini Bhattacharya

BNL

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## Outline

- Generalized TMDs (GTMDs)
- Wigner functions
- Observables for GTMDs: State of the art
- Summary





**Parameterization of correlator through GTMDs:** 

$$X^q(x,\xi,\vec{k}_{\perp}^2,\vec{\Delta}_{\perp}^2,\vec{k}_{\perp}\cdot\vec{\Delta}_{\perp})$$









**General results:** 

- i. <u>16</u> leading-twist GTMDs for quarks (Meissner, Metz, Schlegel, arXiv: 0906.5323)
- ii. <u>16</u> leading-twist GTMDs for gluons (Lorce, Pasquini, arXiv: 1307.4497)
- iii. GTMDs are complex functions



## Why are GTMDs interesting?







Why are GTMDs interesting?











#### Wigner functions & connection to parton Orbital Angular Momentum

• Recap from NRQM:

**Expectation value of observables** 
$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$$



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OAM as a moment of Wigner distribution

: (Lorce, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

$$L_{z}^{q} = \int dx \int d^{2}k_{\perp} d^{2}b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_{z} W^{q} (x, \vec{b}_{\perp}, \vec{k}_{\perp})$$
  
Intuitive definition of OAM





• OAM as a moment of Wigner distribution/GTMD: (Lorce, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

$$L_z^{q,g} = -\int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp^2)$$

Relation between GTMD  $F_{1,4}^{q,g}$  & OAM



• OAM as a moment of Wigner distribution/GTMD: (Lorce, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

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Relation between GTMD  $F_{1,4}^{q,g}$  & OAM





# **Observables for GTMDs: State of the art**



**Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)** 















#### Main result:

$$\frac{d\sigma}{dy_1 dy_2 d^2 \vec{\Delta}_{\perp} d^2 \vec{P}_{\perp}} \propto z(1-z)[z^2 + (1-z)^2] \int d^2 k_{\perp} d^2 k'_{\perp} S(k_{\perp}, \Delta_{\perp}) S(k'_{\perp}, \Delta_{\perp}) \\ \times \left[ \frac{\vec{P}_{\perp}}{P_{\perp}^2 + \epsilon^2} - \frac{\vec{P}_{\perp} - \vec{k}_{\perp}}{(P_{\perp} - k_{\perp})^2 + \epsilon^2} \right] \cdot \left[ \frac{\vec{P}_{\perp}}{P_{\perp}^2 + \epsilon^2} - \frac{\vec{P}_{\perp} - \vec{k}'_{\perp}}{(P_{\perp} - k'_{\perp})^2 + \epsilon^2} \right]$$

 $\approx d\sigma_0 + 2\cos 2\left(\phi_{P_\perp} - \phi_{\Delta_\perp}\right)d\tilde{\sigma}$ 

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## **Observables for GTMDs**





## **Observables for GTMDs**





## **Observables for GTMDs**







#### Exclusive double quarkonium production (SB, Metz, Ojha, Tsai, Zhou, arXiv: 1802.10550)



Main result:

$$\frac{1}{2} \left( \tau_{XY} - \tau_{YX} \right) \approx 2 \operatorname{Re.} \left\{ -\frac{\varepsilon_{\perp}^{ij} \Delta_{a\perp}^{j}}{M} C \left[ \frac{k_{a\perp}^{i}}{M} F_{1,4}(x_{a}, \vec{k}_{a\perp}) F_{1,1}(x_{b}, \vec{k}_{b\perp}) \right] C \left[ F_{1,1}^{*}(x_{a}, \vec{p}_{a\perp}) F_{1,1}^{*}(x_{b}, \vec{p}_{b\perp}) \right] \right\}$$



#### Exclusive double quarkonium production (SB, Metz, Ojha, Tsai, Zhou, arXiv: 1802.10550)



Main result:

$$\frac{1}{2} \left( \tau_{XY} - \tau_{YX} \right) \approx 2 \operatorname{Re.} \left\{ -\frac{\varepsilon_{\perp}^{ij} \Delta_{a\perp}^{j}}{M} C \left[ \frac{k_{a\perp}^{i}}{M} F_{1,4}(x_{a}, \vec{k}_{a\perp}) F_{1,1}(x_{b}, \vec{k}_{b\perp}) \right] C \left[ F_{1,1}^{*}(x_{a}, \vec{p}_{a\perp}) F_{1,1}^{*}(x_{b}, \vec{p}_{b\perp}) \right] \right\}$$

This linear combination of polarization observables is sensitive to gluon OAM



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#### arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target: GTMD distributions and the Odderons

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>1</sup> Lech Szymanowski,<sup>2</sup> and Samuel Wallon<sup>3, 4</sup>





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The CMS Collaboration

#### Michael Murray's talk, DIS 2022

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV



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Michael Murray's talk, DIS 2022



#### arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou












### More developments ...









#### First & only process sensitive to quark GTMDs



# **Our recent work**

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,<br/>1,\* Renaud Boussarie,<br/>2,† and Yoshitaka Hatta<br/>1,3,‡

In Collaboration with: **Renaud Boussarie** (CPHT, CNRS) **Yoshitaka Hatta** (BNL)

Based on:

#### PRL 128, 182002 (arXiv: 2201.08709)































**Twist expansion:** 

• Twist-2 amplitude: Proportional to gluon GPD



Braun, Ivanov, 0505263

$$\begin{split} A_T^2 &= \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{q_\perp^2 + \mu^2} \big( \bar{u}(q_1) \not \in_\perp v(q_2) \big) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \\ & \times \left( 1 + \frac{2\xi^2(1 - 2\beta)}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \boxed{\int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)} \\ A_L^2 &= \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{(q_\perp^2 + \mu^2)^2} \, 4\xi z \bar{z} Q W \big( \bar{u}(q_1) \gamma^- v(q_2) \big) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \\ & \times \left( 1 + \frac{4\xi^2 \bar{\beta}}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \boxed{\int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)} \end{split}$$



**Twist expansion:** 

• Twist-3 amplitude: Proportional to gluon OAM

$$\begin{split} A_{T}^{3} &= -\frac{ig_{s}^{2}e_{em}e_{q}}{N_{c}}\frac{2(\overline{z}-z)}{(q_{\perp}^{2}+\mu^{2})^{2}}\bar{u}(q_{1})\epsilon_{\perp}\cdot\gamma_{\perp}v(q_{2})\int dx\frac{x}{(x^{2}-\xi^{2}+i\xi\varepsilon)^{2}}\left(2\xi+\frac{(2\xi)^{3}(1-2\beta)}{(x^{2}-\xi^{2}+i\xi\varepsilon)}\right)\int d^{2}k_{\perp}q_{\perp}\cdot\boldsymbol{k}_{\perp} xf_{g}(x,\xi,k_{\perp},\Delta_{\perp}) \\ &-\frac{ig_{s}^{2}e_{em}e_{q}}{N_{c}}\frac{2(2\xi)^{2}z\overline{z}W}{(q_{\perp}^{2}+\mu^{2})^{2}}\bar{u}(q_{1})\gamma^{-}v(q_{2})\int dx\frac{x}{(x^{2}-\xi^{2}+i\xi\varepsilon)^{2}}\int d^{2}k_{\perp}\epsilon_{\perp}\cdot\boldsymbol{k}_{\perp} xf_{g}(x,\xi,k_{\perp},\Delta_{\perp}) \end{split}$$

$$A_{L}^{3} = \frac{ig_{s}^{2}e_{em}e_{q}}{N_{c}} \frac{16\xi^{2}(\overline{z}-z)z\overline{z}QW}{(q_{\perp}^{2}+\mu^{2})^{3}} \bar{u}(q_{1})\gamma^{-}v(q_{2}) \int dx \frac{x}{(x^{2}-\xi^{2}+i\xi\varepsilon)^{2}} \left(1 + \frac{8\xi^{2}(1-\beta)}{(x^{2}-\xi^{2}+i\xi\varepsilon)}\right) \int d^{2}k_{\perp} \, q_{\perp} \cdot \mathbf{k}_{\perp} \, xf_{g}(x,\xi,k_{\perp},\Delta_{\perp})$$









Main result (z = 1/2):

DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\begin{split} \int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} &= -\frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_{\perp}^2 + \mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}}) \\ & \times \mathfrak{Re} \bigg[ \bigg\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \bigg( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \bigg) \bigg\} \mathcal{L}_g + \bigg( \mathcal{E}_g^{(1)*} + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{E}_g^{(2)*} \bigg) \frac{\mathcal{O}}{2} \bigg] \end{split}$$

**DSA** does not vanish for symmetric jet configurations  $z = \overline{z} = \frac{1}{2}$ 



Main result (z = 1/2):

DSA is sensitive to OAM through an interference between L & T amplitudes:



Signature of gluon OAM is cosine angular modulation



Main result (z = 1/2):

DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\begin{aligned} \int d\phi_{q_{\perp}} L^{\mu\nu} A^{*}_{\mu} A_{\nu} &= -\frac{2^{10} \pi^{4}}{N_{c}} h_{l} h_{p} \alpha^{2}_{s} \alpha_{em} e^{2}_{q} \frac{(1+\xi)\xi Q^{2}}{(q_{\perp}^{2}+\mu^{2})^{2}} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \\ &\times \mathfrak{Re} \left[ \left\{ \mathcal{H}^{(1)*}_{g} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}^{(1)*}_{g} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \left( \mathcal{H}^{(2)*}_{g} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}^{(2)*}_{g} \right) \right\} \mathcal{L}_{g} + \left( \mathcal{E}^{(1)*}_{g} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \mathcal{E}^{(2)*}_{g} \right) \frac{\mathcal{O}}{2} \right] \end{aligned}$$
"Compton Form Factors":
$$\mathcal{L}_{g}(\xi) = \int_{-1}^{1} dx \frac{x^{2} L_{g}(x,\xi)}{(x-\xi+i\epsilon)^{2}(x+\xi-i\epsilon)^{2}} \qquad \mathcal{H}^{(2)}_{g}(\xi) = \int_{-1}^{1} dx \frac{\xi^{2} H_{g}(x,\xi)}{(x-\xi+i\epsilon)^{2}(x+\xi-i\epsilon)^{2}} \end{aligned}$$



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DSA is sensitive to OAM through an interference between L & T amplitudes:

$$\int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu}^{*} A_{\nu} = -\frac{2^{10} \pi^{4}}{N_{c}} h_{l} h_{p} \alpha_{s}^{2} \alpha_{em} e_{q}^{2} \frac{(1+\xi)\xi Q^{2}}{(q_{\perp}^{2}+\mu^{2})^{2}} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \\ \times \mathfrak{Re} \bigg[ \bigg\{ \mathcal{H}_{g}^{(1)*} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}_{g}^{(1)*} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \bigg( \mathcal{H}_{g}^{(2)*} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}_{g}^{(2)*} \bigg) \bigg\} \mathcal{L}_{g} + \bigg( \mathcal{E}_{g}^{(1)*} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \mathcal{E}_{g}^{(2)*} \bigg) \frac{\mathcal{O}}{2} \bigg]$$
  
"Compton Form Factors": 
$$O(x,\xi) \equiv \int d^{2} \tilde{k}_{\perp} \frac{\tilde{k}_{\perp}^{2}}{M^{2}} F_{1,2}(x,\xi,\tilde{\Delta}_{\perp}=0)$$

 $\mathcal{O}(\xi) = \int_{-1}^{1} dx \frac{xO(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2}$ 



#### Scattering amplitude

Not the end of the story:



Not the end of the story:

• Interference between unpolarized & helicity GPD (z = 1/2):

Helicity GPD

$$\int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu} A_{\nu} = \frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_{\perp}^2+\mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \Re \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

Analogous contribution should enter SSA





Not the end of the story:

• Interference between unpolarized & helicity GPD (z = 1/2):

$$\int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu} A_{\nu} = \frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_{\perp}^2+\mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \Re\left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

**DSA** does not vanish for symmetric jet configurations  $z = \overline{z} = \frac{1}{2}$ 

Switch off the factorization-breaking third poles by setting  $z = \overline{z} = \frac{1}{2}$ 

$$\int dx \frac{H_g(x,\xi)}{(x^2 - \xi^2 + i\xi\epsilon)^3} \int dx \frac{x\tilde{H}_g(x,\xi)}{(x^2 - \xi^2 + i\xi\epsilon)^3}$$



Numerical estimate of cross section

## See backup slides for details on how we modelled GPDs and OAM



#### Numerical estimate of cross section

#### **Realistic EIC kinematics**

$\sqrt{s}~[{ m GeV}]$	$Q^2~[{ m GeV}^2]$	$oldsymbol{y}$	ξ
	2.7		
120	4.8	0.7	$\lesssim 10^{-3}$
	10.0		





#### Numerical estimate of cross section

### **Realistic EIC kinematics**





#### **Cross section:**

$$\frac{d\sigma}{dy dQ^2 d\phi_{l_{\perp}} dz dq_{\perp}^2 d^2 \Delta_{\perp}} = \frac{\alpha_{em} y}{2^{11} \pi^7 Q^4} \frac{\int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu}^* A_{\nu}}{(W^2 + Q^2)(W^2 - M_J^2) z\overline{z}}$$















 $\mathbf{DSA:} \quad \int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} \big|_{\delta\phi=0} \sim \mathfrak{Re} \bigg[ \mathcal{H}_{g}^{(1)*}(\xi) \, \tilde{\mathcal{H}}_{g}^{(2)}(\xi) \bigg] - \mathfrak{Re} \bigg[ \bigg\{ \mathcal{H}_{g}^{(1)*}(\xi) + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{H}_{g}^{(2)*}(\xi) \bigg\} \mathcal{L}_{g}(\xi) \bigg]$ 



$$\mathbf{DSA:} \quad \int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} \big|_{\delta\phi=0} \sim \mathfrak{Re} \bigg[ \mathcal{H}_g^{(1)*}(\xi) \, \tilde{\mathcal{H}}_g^{(2)}(\xi) \bigg] - \mathfrak{Re} \bigg[ \bigg\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \bigg\} \mathcal{L}_g(\xi) \bigg]$$



**DSA:** 
$$\int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} \Big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left( \tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_{\perp}^2 - Q^2/4}{q_{\perp}^2 + Q^2/4} \, \mathcal{L}_g(\xi) \right)$$

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## Summary of our work

• Gluon OAM related to the Wigner distribution



## Summary of our work

- Gluon OAM related to the Wigner distribution
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:

$$\int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} \sim - \Re \left[ \left\{ \mathcal{H}_{g}^{(1)*}(\xi) + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{H}_{g}^{(2)*}(\xi) \right\} \mathcal{L}_{g}(\xi) \right] \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}}) + \Re \left[ \mathcal{H}_{g}^{(1)*}(\xi) \tilde{\mathcal{H}}_{g}^{(2)}(\xi) \right] \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}})$$



## **Summary of our work**

- Gluon OAM related to the Wigner distribution
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:







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## Summary of our work

- Gluon OAM related to the Wigner distribution
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:

$$\int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} \sim - \Re \left[ \left\{ \mathcal{H}_{g}^{(1)*}(\xi) + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{H}_{g}^{(2)*}(\xi) \right\} \mathcal{L}_{g}(\xi) \right] \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}}) + \Re \left[ \mathcal{H}_{g}^{(1)*}(\xi) \tilde{\mathcal{H}}_{g}^{(2)}(\xi) \right] \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}})$$

• First realistic numerical calculation of observable sensitive to OAM @ EIC



# Backup slides

# **Probing gluon OAM through exclusive dijet production**

#### Numerical estimate of cross section

**Ingredients for non-perturbative functions** 

$$\begin{aligned} \mathbf{OAM} \quad \int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu}^{*} A_{\nu} &= -\frac{2^{10} \pi^{4}}{N_{c}} h_{l} h_{p} \alpha_{s}^{2} \alpha_{em} e_{q}^{2} \frac{(1+\xi) \xi Q^{2}}{(q_{\perp}^{2}+\mu^{2})^{2}} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \\ & \times \Re \mathfrak{e} \Biggl[ \Biggl\{ \mathcal{H}_{g}^{(1)*} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}_{g}^{(1)*} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \Biggl( \mathcal{H}_{g}^{(2)*} - \frac{\xi^{2}}{1-\xi^{2}} \mathcal{E}_{g}^{(2)*} \Biggr) \Biggr\} \mathcal{L}_{g} + \Biggl( \mathcal{E}_{g}^{(1)*} + \frac{4q_{\perp}^{2}}{q_{\perp}^{2}+\mu^{2}} \mathcal{E}_{g}^{(2)*} \Biggr) \frac{\mathcal{O}}{2} \Biggr] \end{aligned}$$

$$\begin{aligned} \overbrace{\mathbf{Helicity}} & \int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu} A_{\nu} = \frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_{\perp}^2+\mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \\ & \times \Re \mathfrak{e} \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right] \end{aligned}$$

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**Ingredients for non-perturbative functions** 

• Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula

$$\begin{aligned} \mathbf{OAM} \qquad \int d\phi_{q_{\perp}} L^{\mu\nu} A^*_{\mu} A_{\nu} &= -\frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_{\perp}^2 + \mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}}) \\ & \times \mathfrak{Re} \bigg[ \bigg\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \bigg( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \bigg) \bigg\} \mathcal{L}_g + \bigg( \mathcal{E}_g^{(1)*} + \frac{4q_{\perp}^2}{q_{\perp}^2 + \mu^2} \mathcal{E}_g^{(2)*} \bigg) \frac{\mathcal{O}}{2} \bigg] \end{aligned}$$

$$\begin{aligned} \overbrace{\mathbf{Helicity}} & \int d\phi_{q_{\perp}} L^{\mu\nu} A_{\mu} A_{\nu} = \frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_{\perp}^2+\mu^2)^2} |l_{\perp}| |\Delta_{\perp}| \cos(\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}) \\ & \times \Re \mathfrak{e} \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right] \end{aligned}$$



- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)

$$\begin{pmatrix} H_g(x,\boldsymbol{\xi})\\ \tilde{H}_g(x,\boldsymbol{\xi}) \end{pmatrix} = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \,\delta(\beta + \boldsymbol{\xi}\alpha - x) \times \frac{15}{16} \frac{[(1-|\beta|)^2 - \alpha^2]^2}{(1-|\beta|)^5} \times \begin{cases} \beta \,G(\beta)\\ \beta \,\Delta G(\beta) \end{cases}$$



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- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
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- Model for OAM:



**Ingredients for non-perturbative functions** 

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:

1. "OAM density": (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\boldsymbol{x}) = x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{ genuine twist-three}$$



- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:

1. "OAM density": (Hatta, Yoshida, 1207.5332)





**Ingredients for non-perturbative functions** 

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:
  - 1. "OAM density": (Hatta, Yoshida, 1207.5332)

$$L_{can}^{g}(\boldsymbol{x}) \approx x \int_{x}^{1} \frac{dx'}{x'^{2}} (H_{g}(x') + E_{g}(x')) - 2x \int_{x}^{1} \frac{dx'}{x'^{2}} \Delta G(x') + \text{ genuine twist-three}$$

2. Use the Double distribution approach to construct  $xL_g(x, \boldsymbol{\xi})$  from  $xL_g(x)$  (GPD-like approach)



## Outlook



Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,<br/>1 Andreas  $\mathrm{Metz},^1$  and Jian  $\mathrm{Zhou}^2$ 



What about quark OAM?

- Exclusive double Drell-Yan is the only known process sensitive to quark OAM
- Low count rate (Amplitude  $\sim \alpha_{em}^2$ )
- Alternatively, access quark OAM through dijet production in ep collisions

(SB, Boussarie, Hatta, Work in progress)