*TRANSVERSITY 2022, May 26, 2022, Pavia, Italy*

# Collinear twist-3 observables in eN - [and $e^+e^-$ -] collisions

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Experimental transverse hadron spin observables
in (semi - ) inclusive high-energy reactions

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

or

$$\Delta\sigma_T = \frac{\sigma^{\rightarrow\uparrow} - \sigma^{\rightarrow\downarrow}}{\sigma^{\rightarrow\uparrow} + \sigma^{\rightarrow\downarrow}}$$

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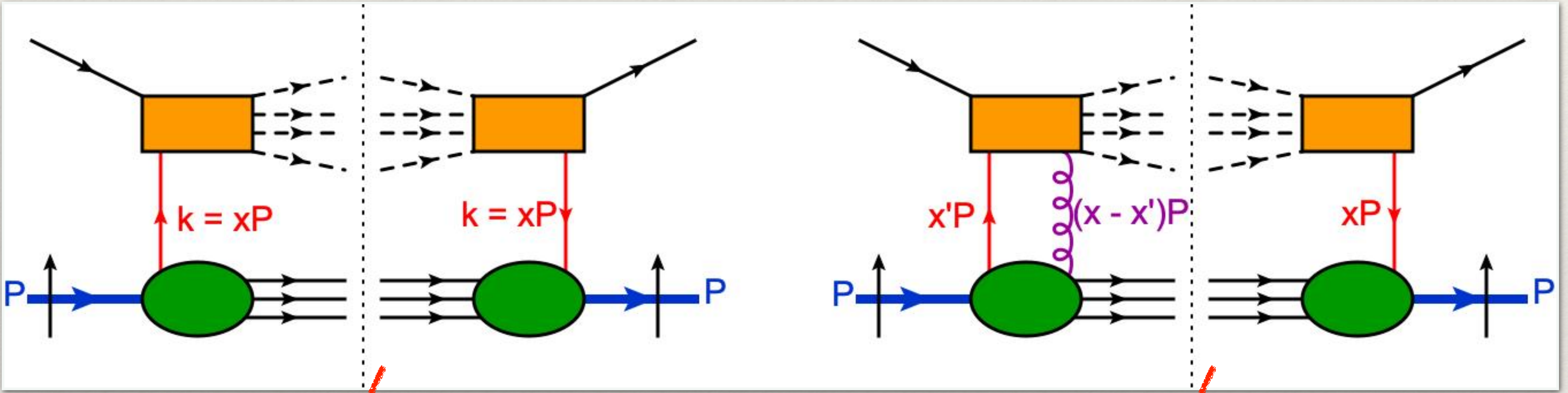
Sketch of coll. tw-3 factorization: eN collisions




2-parton correlations:
Unpolarized cross section (twist-2)
Intrinsic & kinematic twist-3

3-parton correlations:
Dynamical twist-3

**Related through QCD EoM & LIR relations**

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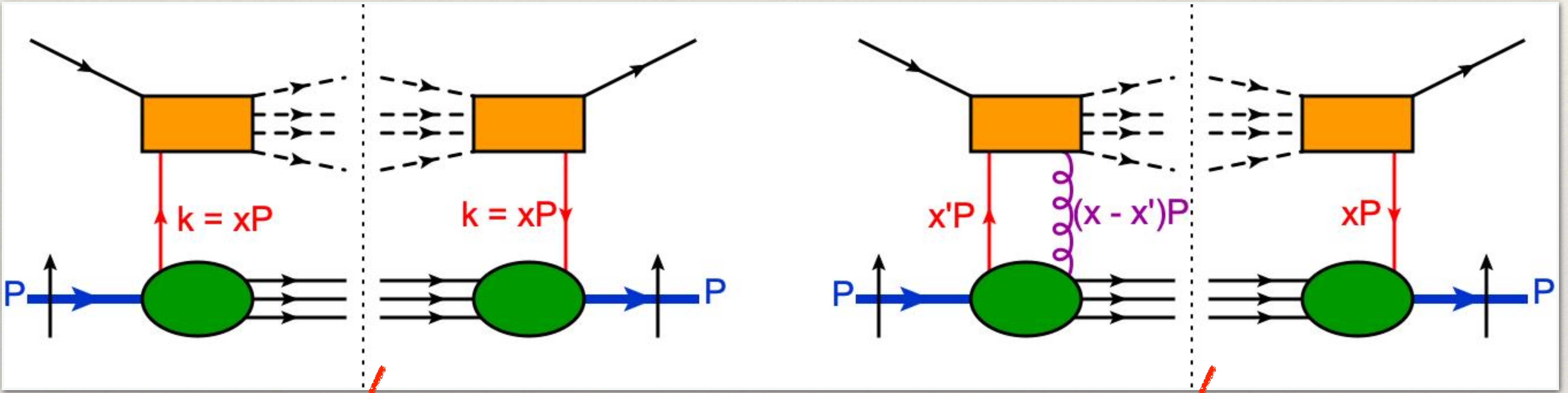


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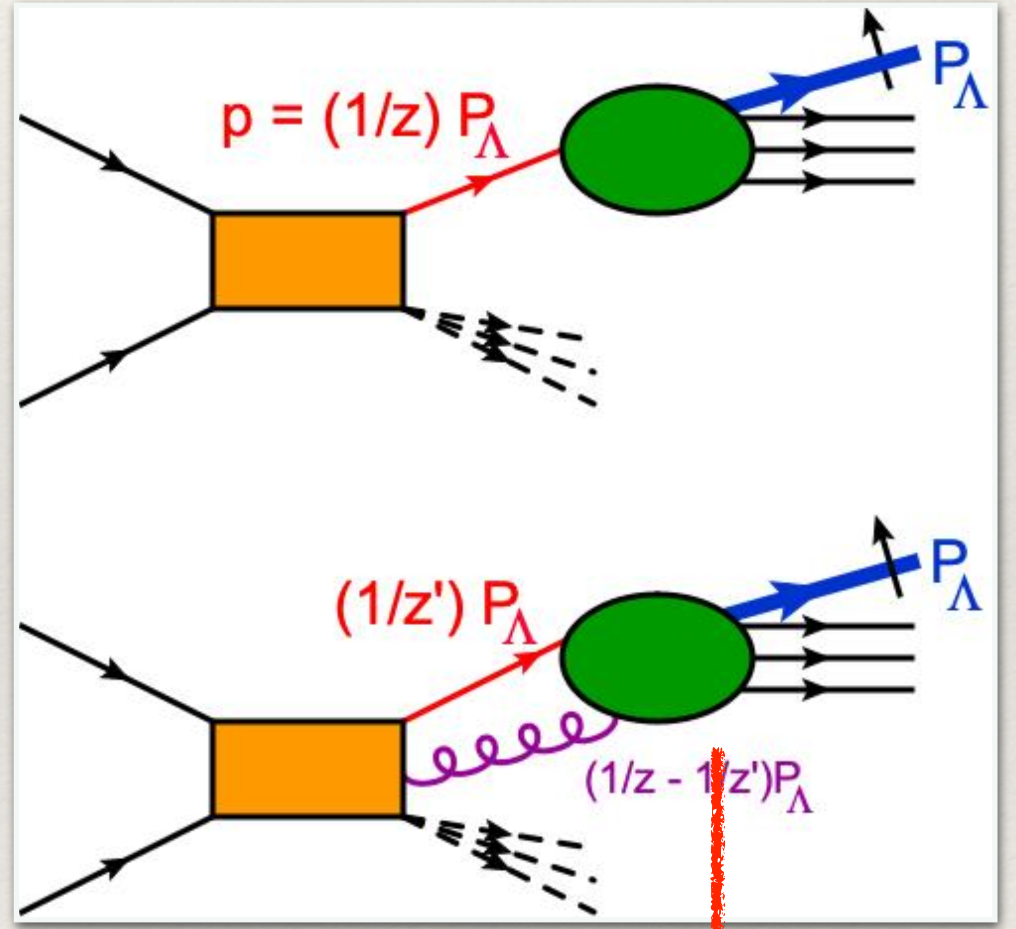
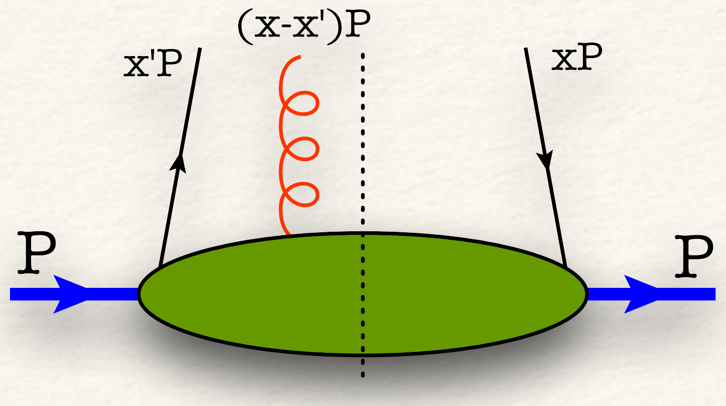
$e^+e^-$ collisions ($\Lambda$ production)




2 & 3-parton fragmentation:
Intrinsic & kinematical
& dynamical twist-3

# Quark-gluon-quark correlations in a transversely polarized nucleon

*Dynamical* twist-3:
Quark - Gluon - Quark Correlations




→ dependence on two parton momenta x, x':
2-dimensional support: |x|, |x'|, |x - x' | < 1

→ Symmetry: $F_{FT}(x,x') = + F_{FT}(x',x)$ , $G_{FT}(x,x') = -G_{FT}(x',x)$
Anti-quarks: $F^{\text{anti-quark}}(x,x') = +/- F^{\text{quark}}(-x,-x')$

$$2M\, i\epsilon^{Pn\rho S} F^q_{FT}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x'} e^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

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→ Relation to Sivers function: $\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$
constraint by data
& Evolution of Sivers function

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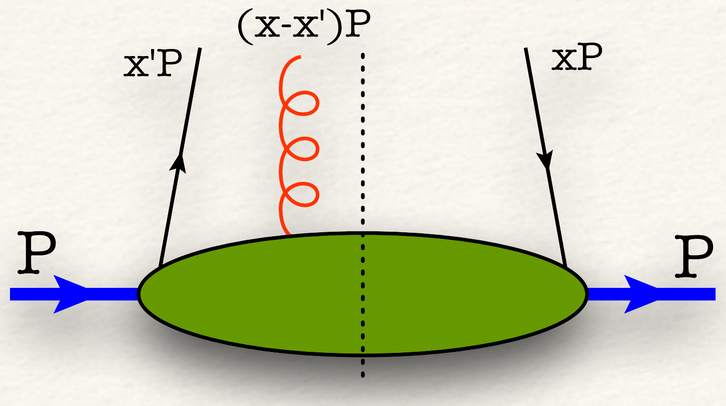


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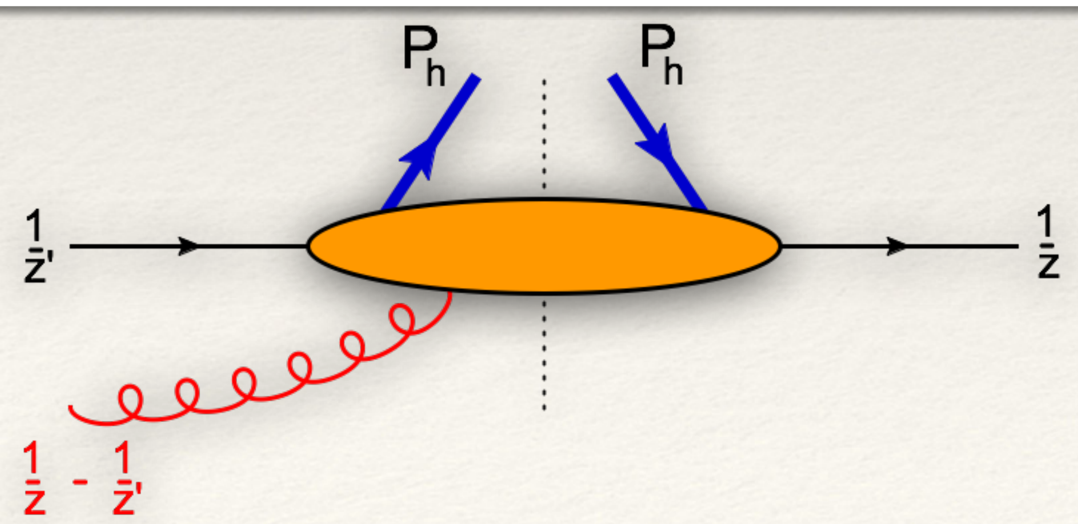
# Quark-gluon-quark correlations in fragmentation

*dynamical* twist-3 FF with transverse spin:

$$\Delta^\alpha_F(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) | P_\Lambda, S_\Lambda; X\rangle \langle P_\Lambda, S_\Lambda; X |\, \bar{q}(0)\, |0\rangle$$
$$\Longrightarrow \hat{D}^{\Lambda/q}_{FT}(z,z')\,,\ \hat{G}^{\Lambda/q}_{FT}(z,z')$$




Same operator as for dynamical twist-3 PDFs, but:

- No gluonic or fermonic poles

$FF(z,z) = 0$ $\quad FF(z,0) = 0$ $\quad \frac{\partial}{\partial z'} FF(z,z')\Big|_{z'=z} = 0$

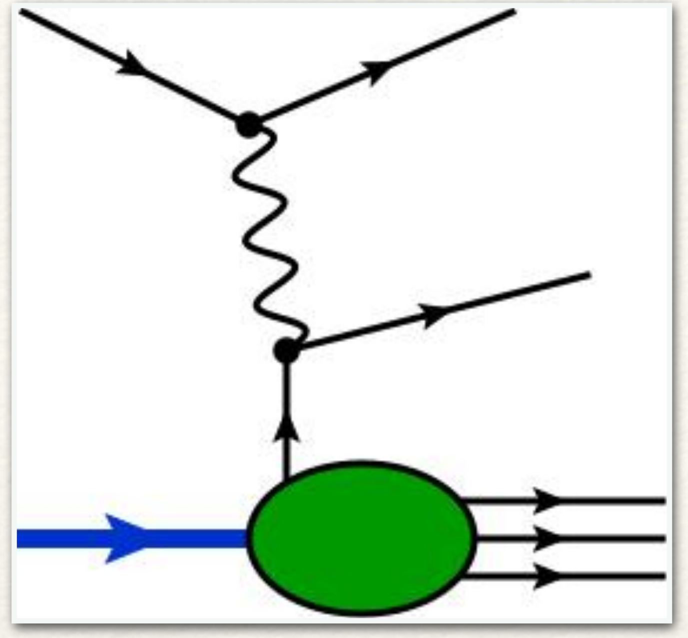
- Support properties different

$z \leq z' < \infty$

- No time reversal: dynamical twist-3 FFs are complex

# Transverse Spin Asymmetries
# in
# Semi-inclusive $\gamma$ production in l+N collisions

# Deep-inelastic scattering: e(l) + N(P) $\longrightarrow$ e(l') + X

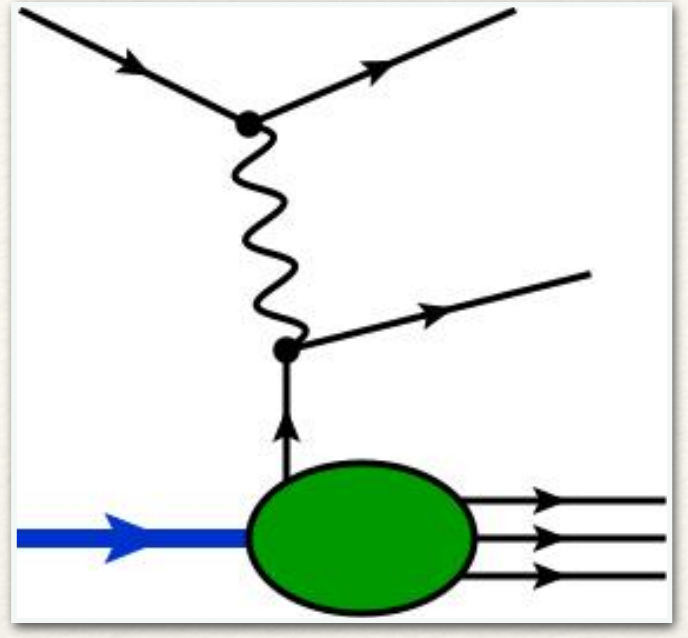


hard scale: $\tilde{Q}^2 = -(l - l')^2$

scaling variable: $\tilde{x}_B = \frac{Q^2}{2P \cdot (l - l')}$

Unpolarized cross section (LO parton model): $E' \frac{d\sigma}{d^3 l'} \propto \sum_q e_q^2 \, f_1^q(\tilde{x}_B)$

# Deep-inelastic scattering: e(l) + N(P) ⟶ e(l') + X



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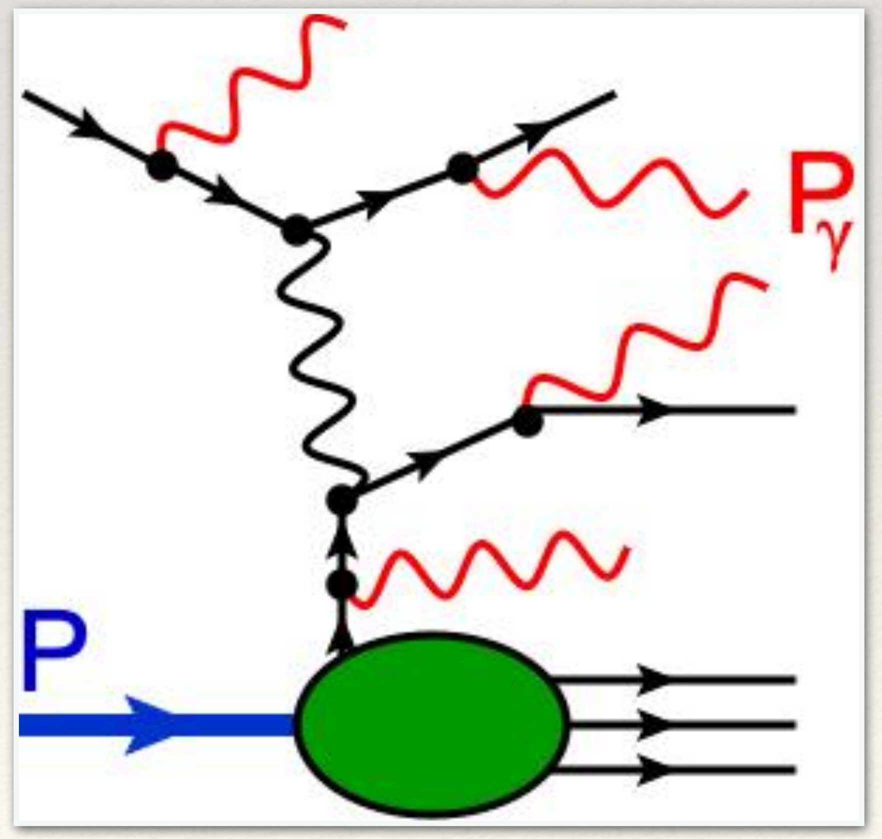
# Semi-inclusive γ production in DIS: e(l) + N(P) ⟶ e(l') + γ($P_\gamma$)+ X

[Albaltan, Prokudin, M.S., PLB 804 (2020), 135367]

unpolarized cross section in the parton model

[Brodsky, Gunion, Jaffe, PRD 1972; see also works by Metz et al; Pisano, Mukherjee; de Rujula, Vogelsang; …]




- avoid photon fragmentation: isolated photons
- collinear factorization: information on final quark is integrated out
  - two scales: $Q^2 = -(l-l'-P_\gamma)^2$    $\tilde{Q}^2 = -(l-l')^2$
  - two scaling variables: $x_B = \dfrac{Q^2}{2P\cdot(l-l'-P_\gamma)}$    $\tilde{x}_B = \dfrac{\tilde{Q}^2}{2P\cdot(l-l')}$

$$E'E_\gamma \frac{d\sigma}{d^3\vec{l}'\, d^3\vec{P}_\gamma} = \hat{\sigma}_{BH}\, f^{BH}(x_B) + \hat{\sigma}_C\, f^C(x_B) + \hat{\sigma}_I\, f^I(x_B)$$

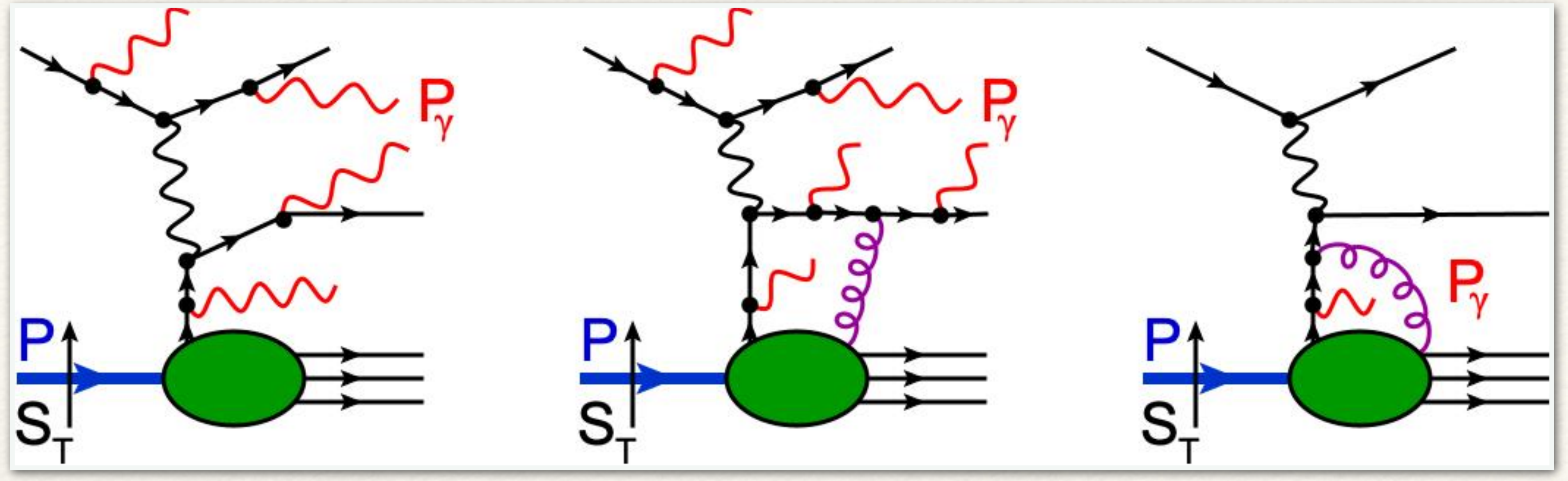
$$f^{BH} = \sum_{q=u,d,\ldots} e_q^2(f^q + f^{\bar{q}})$$

$$f^C = \sum_{q=u,d,\ldots} e_q^4(f^q + f^{\bar{q}})$$

$$f^I = \sum_{q=u,d,\ldots} e_q^3(f^q - f^{\bar{q}})$$

# Transverse spin effects in photon SIDIS

Include *intrinsic*, *kinematical* & *dynamical* twist - 3 contributions




SSA: need a phase
(propagator poles)

1) Soft Gluon Poles: $F_{FT}(x_B,x_B)$
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DSA:
principal value of propagator

EoM & LIR relations

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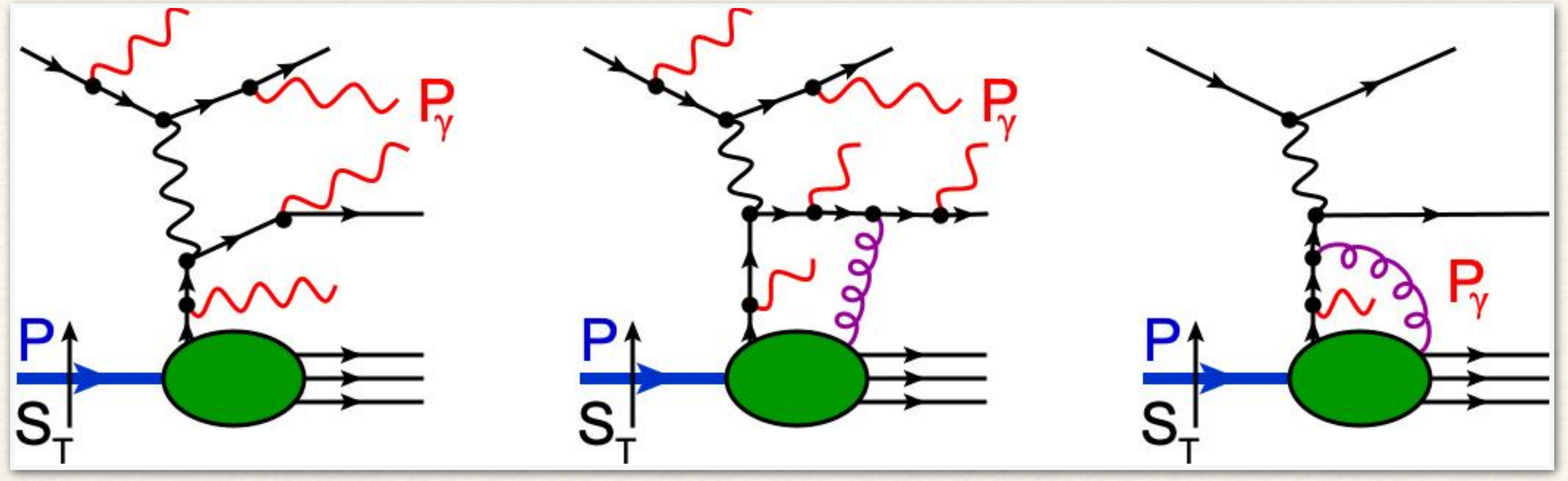


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Single-Spin Asymmetry: $$A_{UT} \propto \frac{M}{Q}\left[\epsilon^{Pll'S}A + \epsilon^{PlP_\gamma S}B + (l'_T \cdot S_T)\epsilon^{Pll'P_\gamma}C + (P_{\gamma T}\cdot S_T)\epsilon^{Pll'P_\gamma}D\right]$$ Last two structures redundant

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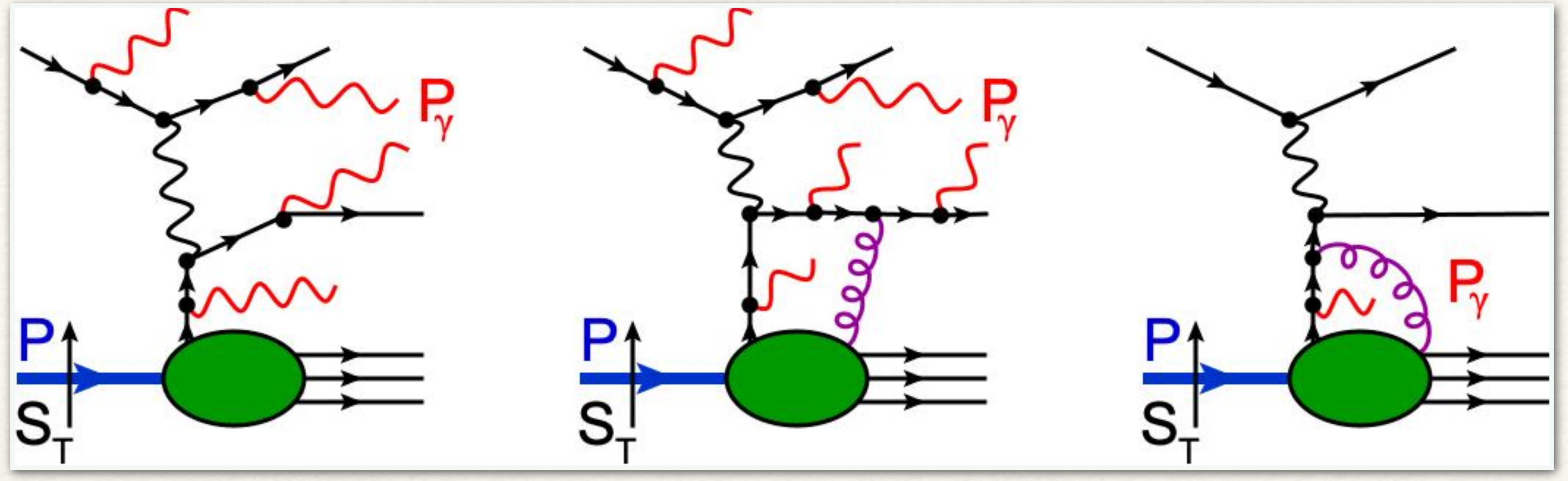


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Update:

$$A_{UT} \propto \frac{M}{Q}\left[\epsilon^{Pll'S}\hat{\sigma}^1_{UT} + \epsilon^{PlP_\gamma S}\hat{\sigma}^2_{UT}\right] \propto \left[\sin(\phi_S-\phi_{l'})\hat{\sigma}^1_{UT} + \sin(\phi_S-\phi_\gamma)\hat{\sigma}^2_{UT}\right]$$

$$\hat{\sigma}^{i=1,2}_{UT} = \sum_{n=C,I}\left[\hat{\sigma}^{n;i=1,2}_{HP,F}\,F^n_{FT}(x_B,\tilde{x}_B) + \hat{\sigma}^{n;i=1,2}_{HP,G}\,G^n_{FT}(x_B,\tilde{x}_B) + \hat{\sigma}^{n;i=1,2}_{SFP,F}\,F^n_{FT}(x_B,0) + \hat{\sigma}^{n;i=1,2}_{SFP,G}\,G^n_{FT}(x_B,0)\right]$$

- no BH contributions
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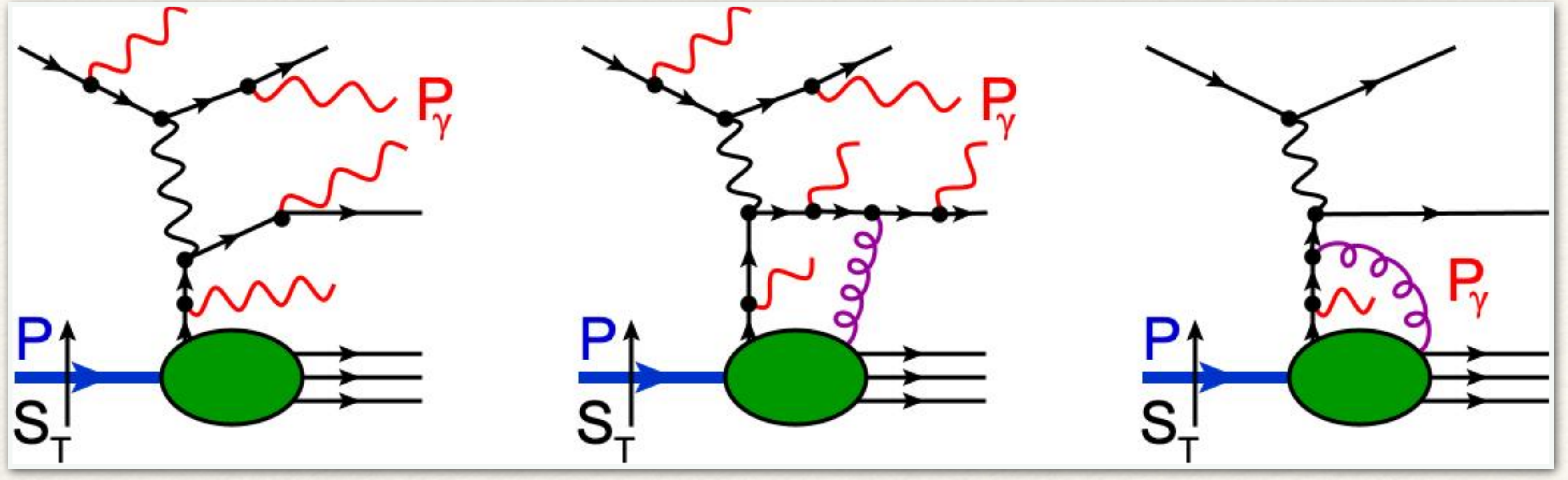


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**Double-Spin Asymmetry:**

$$\Delta\sigma_T \propto \frac{M}{Q}\left[(l\cdot S)\Delta\hat{\sigma}^1_T + (l'\cdot S)\Delta\hat{\sigma}^2_T + (P_\gamma\cdot S)\Delta\hat{\sigma}^3_T\right] \propto \left[\cos(\phi_S-\phi_{l'})\Delta\hat{\sigma}^2_T + \cos(\phi_S-\phi_\gamma)\Delta\hat{\sigma}^3_T\right]$$

$$\Delta\hat{\sigma}^{i=1,2,3}_T = \sum_{n=BH,C,I}\int_{x_B}^{1}dx\int_{x-1}^{1}dx'\left[\Delta\hat{\sigma}^{i,n}_{WW}(x)\,g^n_1(x) + \Delta\hat{\sigma}^{i,n}_F(x,x')\,F^n_{FT}(x,x') + \Delta\hat{\sigma}^{i,n}_G(x,x')\,G^n_{FT}(x,x')\right]$$

- also BH contributes
- EoM & LIR relations crucial
- WW contribution present

# Inclusive $\gamma$ production in l+N collisions

$$e + N \longrightarrow \gamma + X$$

$$(\text{cf. } p + p \longrightarrow \gamma + X)$$

# Inclusive (high-$p_T$) $\gamma$ production in DIS: $e(l) + N(P) \longrightarrow \gamma(P_\gamma) + X$

[D. Rein, M.S., W. Vogelsang, in progress]

Conceptually & Experimentally: simpler observable: single-inclusive $E_\gamma \frac{d\sigma}{d^3\vec{P}_\gamma}$ $\longrightarrow$ "Left - Right" Asymmetry

Mandelstam variables $s = (P+l)^2,\ t = (P-P_\gamma)^2,\ u = (l-P_\gamma)^2$ scaling fraction $x_0 = \frac{-u}{s+t}$ $\quad v = 1 + s/t$

Theoretically: Phase-space integration on lepton non-trivial: even LO parton model has elements of NLO calculation!

$\longrightarrow$ interesting on its own…

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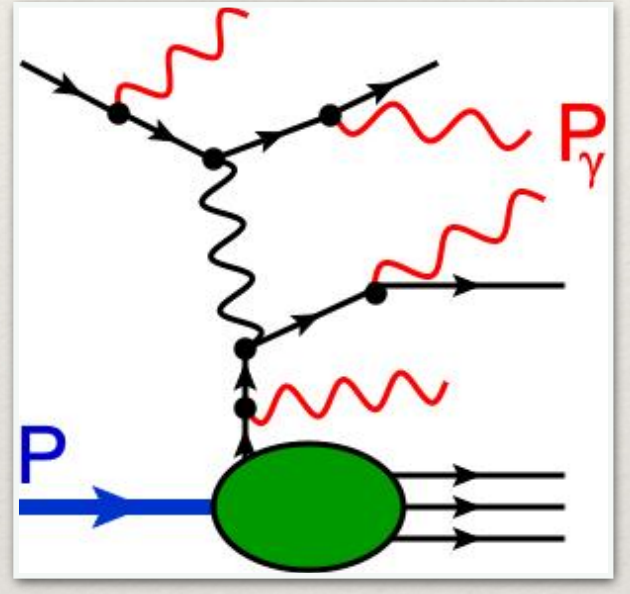


## unpolarized cross section in LO parton model

Bethe-Heitler Terms

$$E_\gamma \frac{d\sigma^{LO}_{BH}}{d^3\vec{P}_\gamma} = \frac{\alpha^3_{\rm em}}{\pi s u} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_{BH}(v, w, \mu^2)\, f_1^{BH}\left(\frac{x_0}{w}, \mu\right)$$

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collinear divergence in e$^-$ phase space

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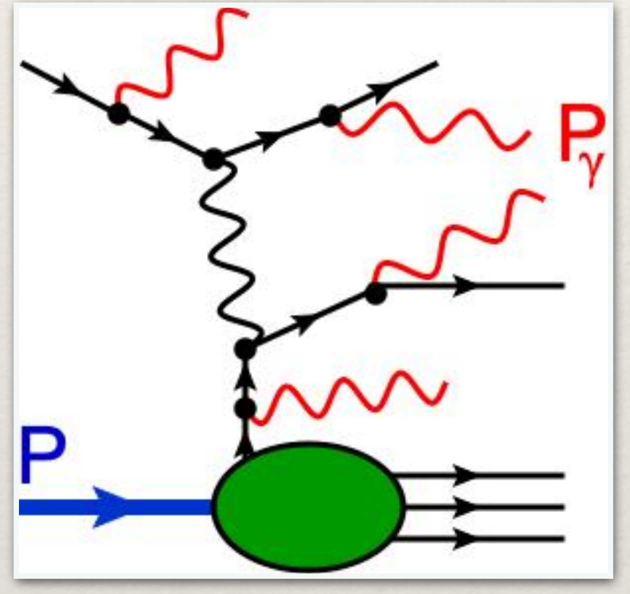


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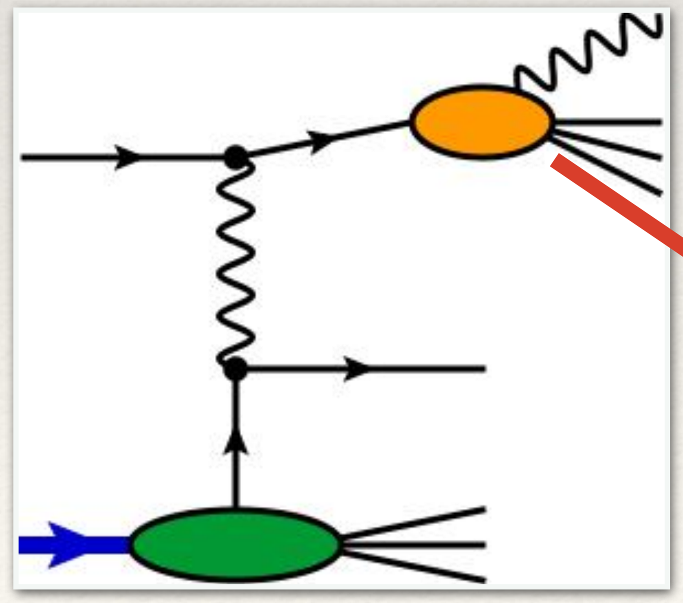

$$+ \frac{\alpha^2_{\rm em}}{su} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}^{WW}_{BH}(v, w)\, D_1^{l/\gamma}(1 - v(1-w), \mu)\, f_1^{BH}\left(\frac{x_0}{w}, \mu\right)$$

"Weizsäcker-Williams" fragmentation function, perturbative in QED

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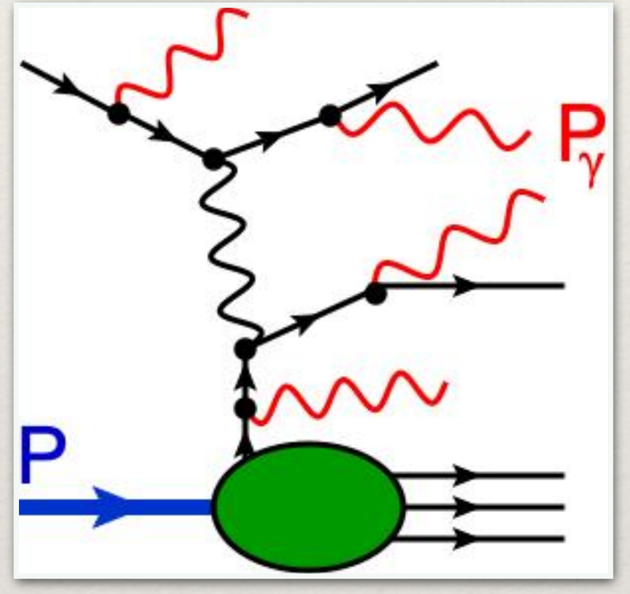


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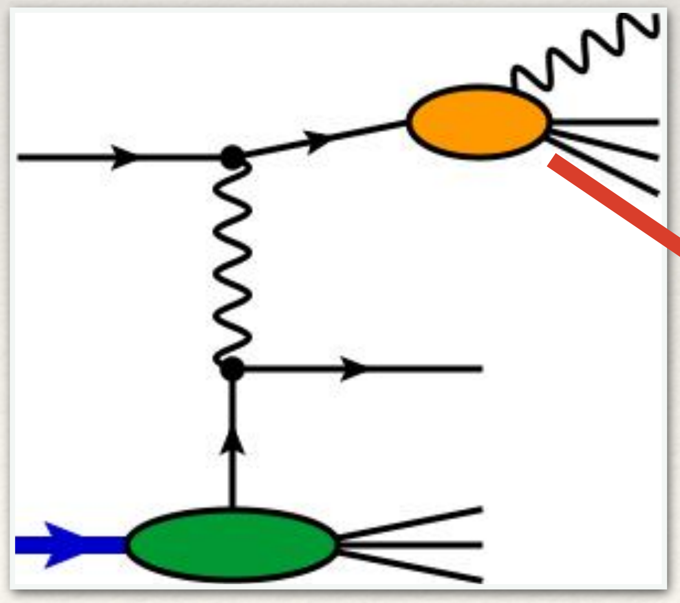
collinear divergence in $e^-$ phase space



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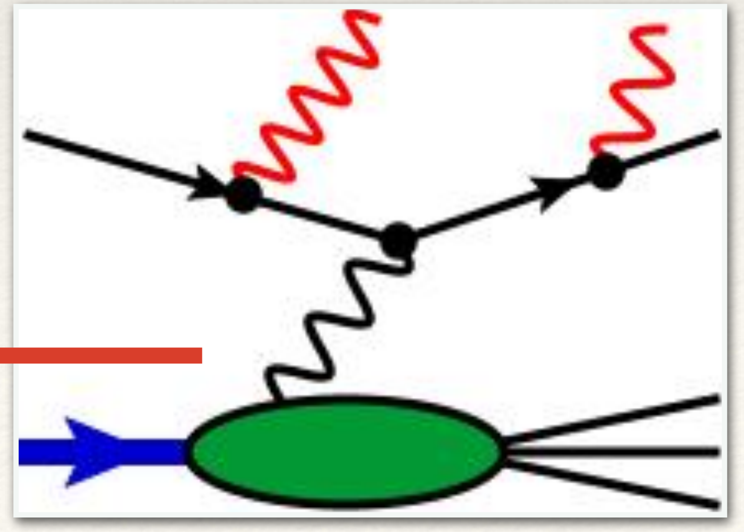
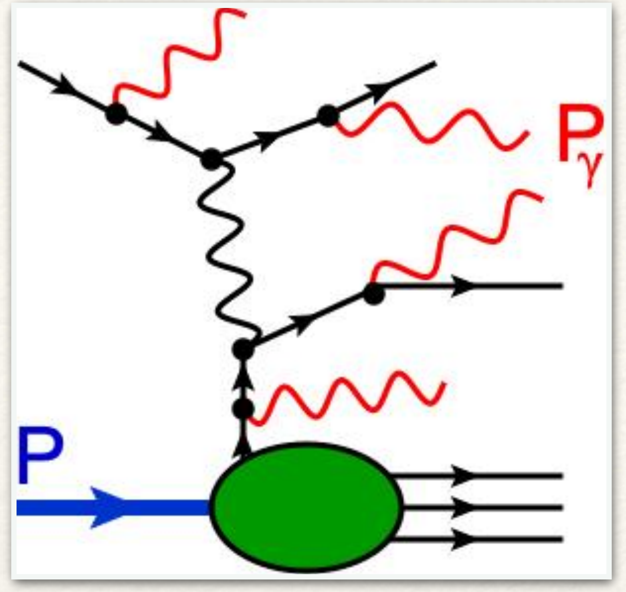
"Weizsäcker-Williams" fragmentation function, perturbative in QED

$$+ \frac{\alpha^2_{\text{em}}}{su} \hat{\sigma}_{BH}(v)\, f_1^{\gamma/N}(x_0, \mu)$$



$\gamma$/N PDF

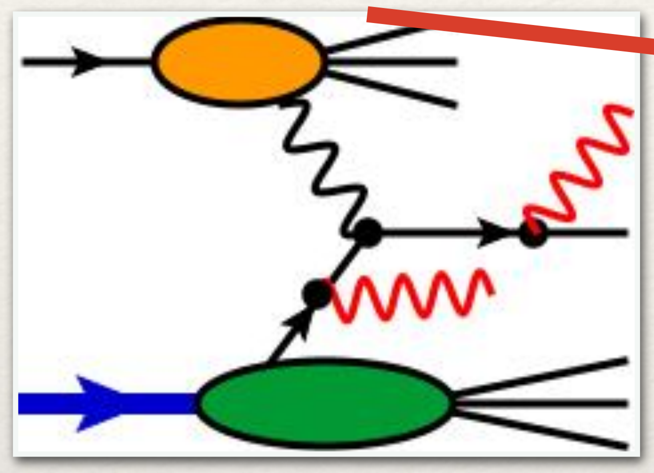
# Compton Terms




$$E_\gamma \frac{d\sigma_C^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{\rm em}^3}{\pi s u}\int_{x_0}^{1} \frac{dw}{w}\,\hat{\sigma}_C(v,w,\mu^2)\, f_1^C\left(\frac{x_0}{w},\mu\right)$$

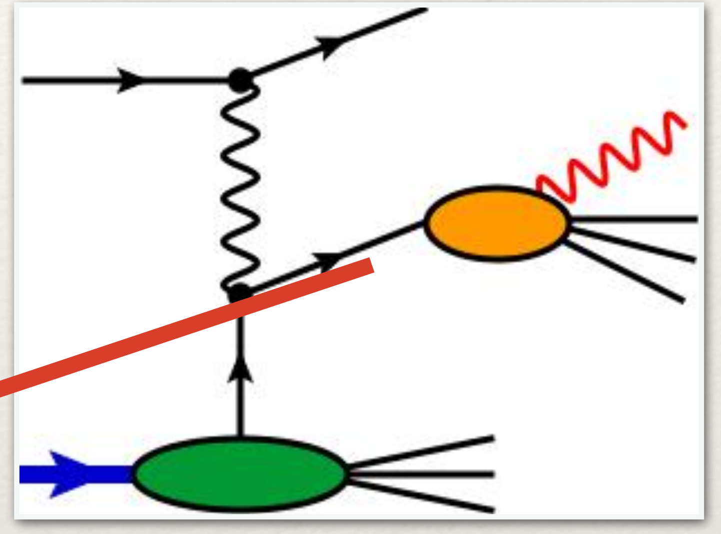
$$f^C = \sum_{q=u,d,\ldots} e_q^4\left(f^q + f^{\bar{q}}\right)$$

collinear divergence in e- phase space



$$+\frac{\alpha_{\rm em}^2}{s u}\int_{x_0}^{1} \frac{dw}{w}\,\hat{\sigma}_C^{WW}(v,w)\, f_1^{\gamma/l}\left(\frac{1-v}{1-vw},\mu\right) f_1^C\left(\frac{x_0}{w},\mu\right)$$

"Weizsäcker-Williams" distribution function, perturbative in QED

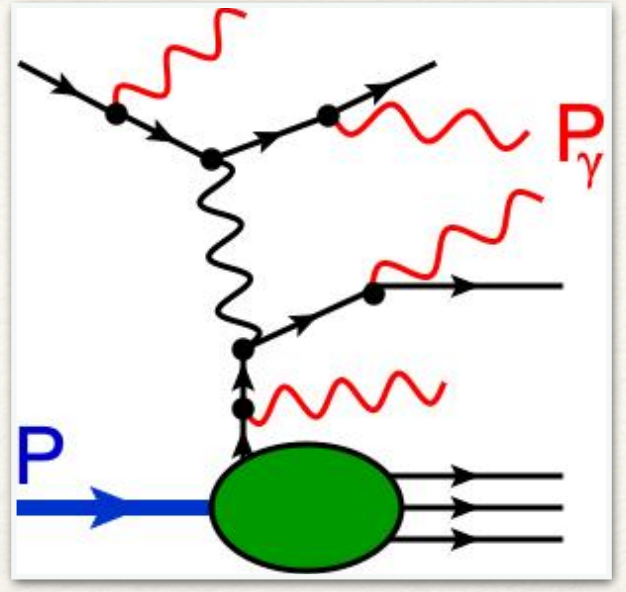


$$+\frac{\alpha_{\rm em}^2}{s u}\int_{x_0}^{1} \frac{dw}{w}\,\hat{\sigma}_C^{\gamma FF}(v,w) \sum_{f=u,d,\ldots} Q_f^2 D_1^{f/\gamma}(1-v(1-w),\mu)\,\left(f_1^{f/N}+f_1^{\bar{f}/N}\right)\left(\frac{x_0}{w},\mu\right)$$

q ⟶ γ Fragmentation function

could be eliminated by "isolation"

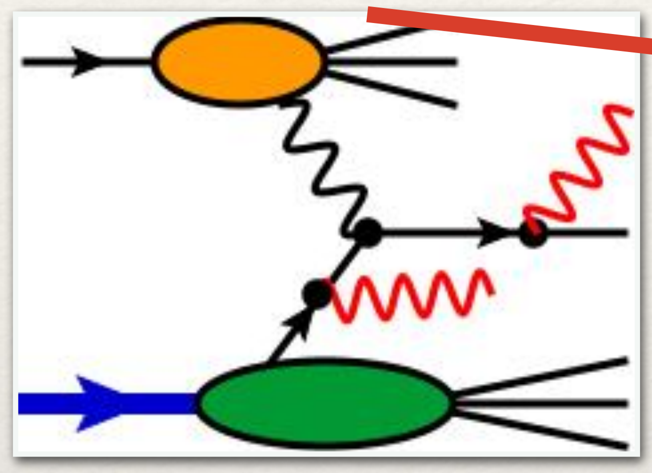
## Compton Terms




$$E_\gamma \frac{d\sigma_C^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{\rm em}^3}{\pi s u}\int_{x_0}^1 \frac{dw}{w}\,\hat{\sigma}_C(v,w,\mu^2)\, f_1^C(\tfrac{x_0}{w},\mu)$$

$$f^C = \sum_{q=u,d,\ldots} e_q^4 (f^q + f^{\bar{q}})$$

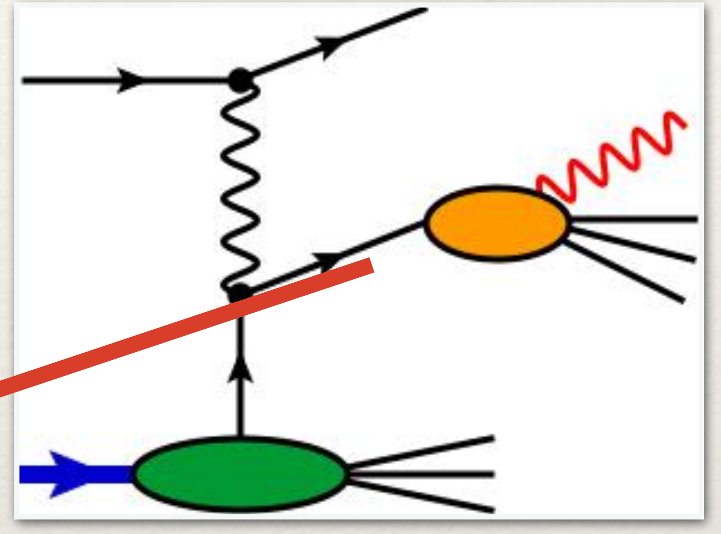
collinear divergence in e⁻ phase space



$$+\frac{\alpha_{\rm em}^2}{su}\int_{x_0}^1 \frac{dw}{w}\,\hat{\sigma}_C^{WW}(v,w)\, f_1^{\gamma/l}(\tfrac{1-v}{1-vw},\mu)\, f_1^C(\tfrac{x_0}{w},\mu)$$

"Weizsäcker-Williams" distribution function, perturbative in QED

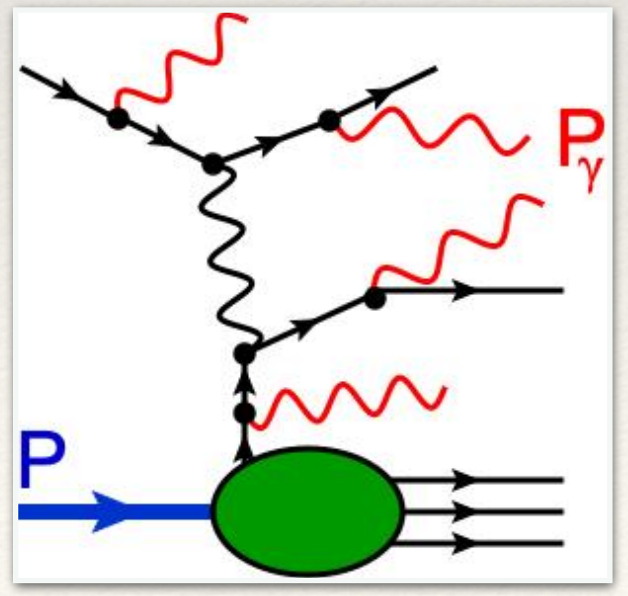
$$+\frac{\alpha_{\rm em}^2}{su}\int_{x_0}^1 \frac{dw}{w}\,\hat{\sigma}_C^{\gamma FF}(v,w)\sum_{f=u,d,\ldots} Q_f^2 D_1^{f/\gamma}(1-v(1-w),\mu)\,(f_1^{f/N}+f_1^{\bar{f}/N})(\tfrac{x_0}{w},\mu)$$



q ⟶ γ Fragmentation function

could be eliminated by "isolation"

## Interference Terms

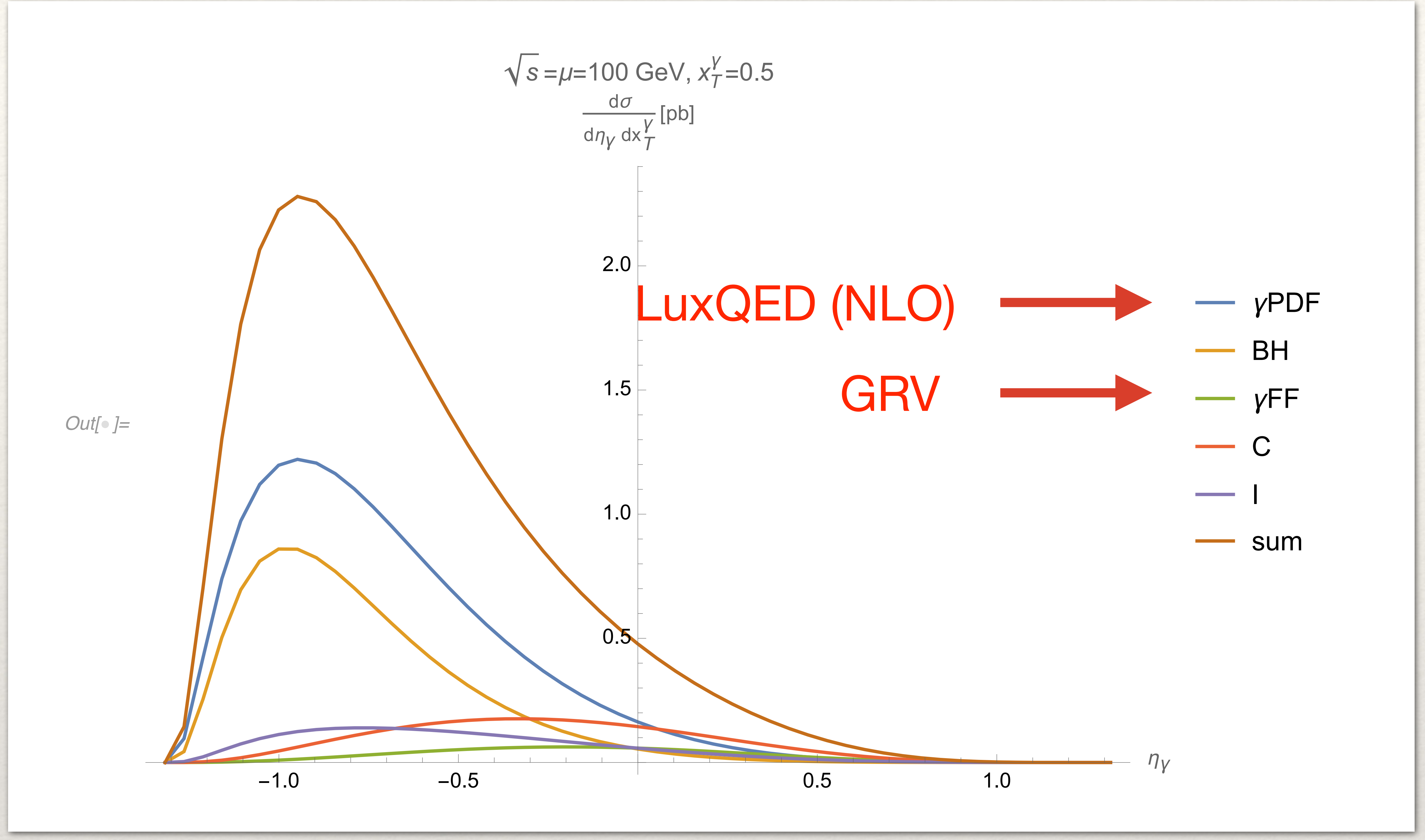


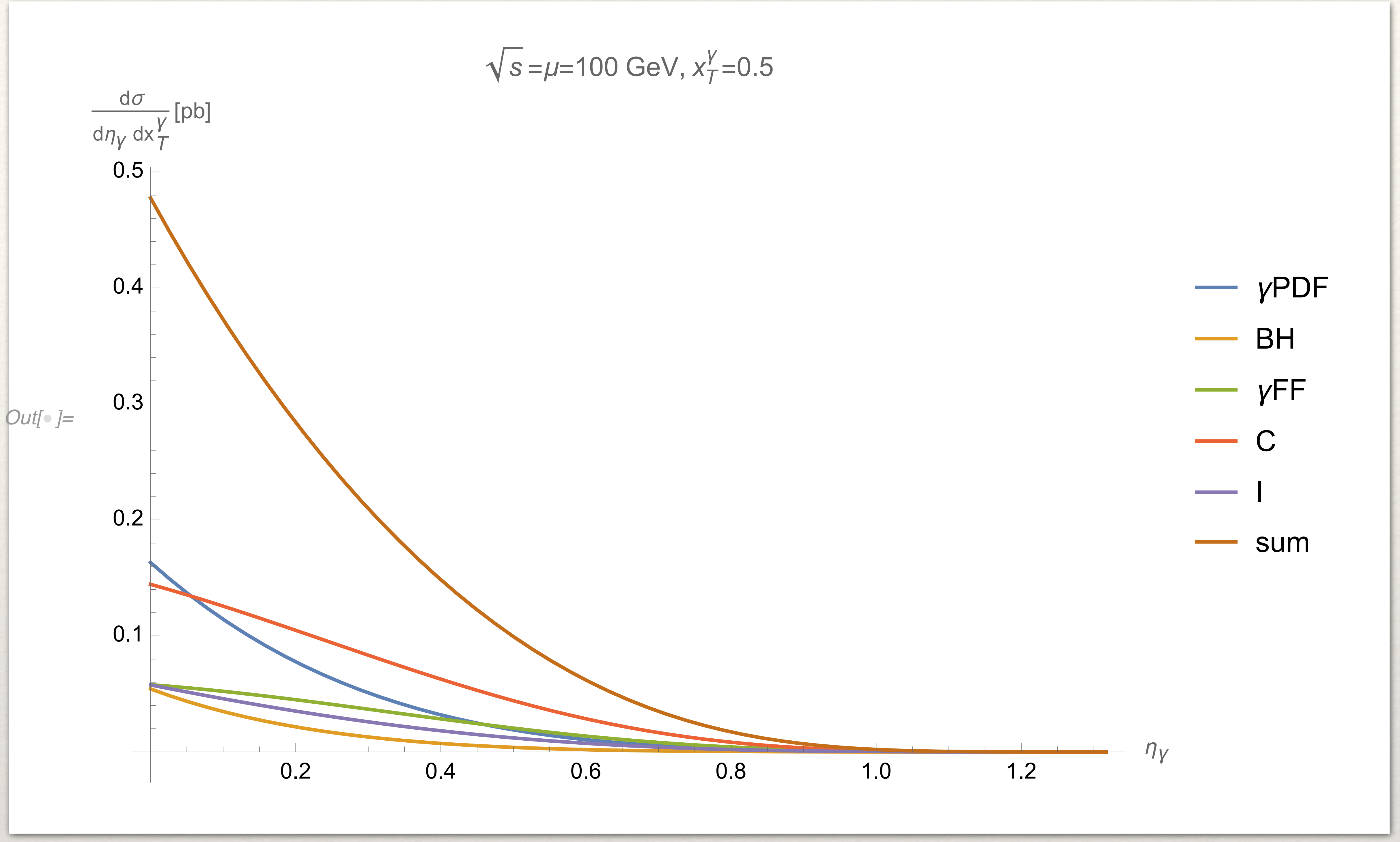

$$E_\gamma \frac{d\sigma_I^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{\rm em}^3}{\pi s u}\int_{x_0}^1 \frac{dw}{w}\,\hat{\sigma}_I(v,w)\, f_1^I(\tfrac{x_0}{w},\mu)$$

$$f^I = \sum_{q=u,d,\ldots} e_q^3 (f^q - f^{\bar{q}})$$

No collinear divergence for interference contributions,
but phase space more difficult…

# EIC c.m. frame (forward direction = proton direction)



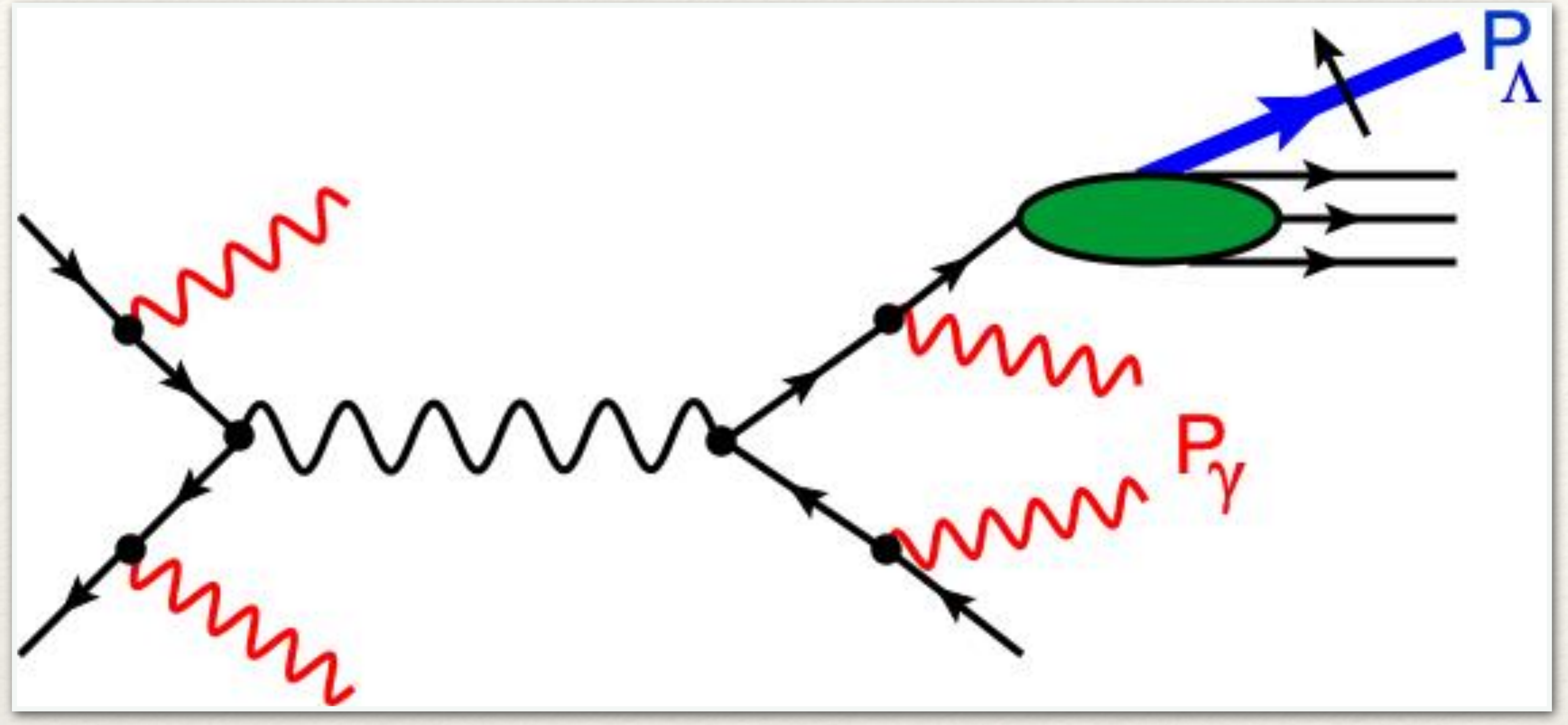




- γPDF/BH dominates in (proton) backward region & larger transverse γ momentum, EIC data may help to further constrain γPDF
- Compton / γFF dominates in (proton) forward region & smaller transverse γ momenta, we may learn about γFF
- Our goal: Calculate the longitudinal & transverse spin observables!

# Transverse Spin Asymmetries in semi-inclusive $\gamma$ production In $e^+e^-$ annihilation

# $\Lambda\ \gamma$ - pair production in lepton annihilation: $e^-(l) + e^+(l') \longrightarrow \Lambda(P_\Lambda) + \gamma(P_\gamma) + X$

[Albaltan, Prokudin, M.S., in preparation]




The 'crossed' or 'time-like' version: recover similar features as before

two scaling variables $z_\Lambda = \frac{2P_\Lambda \cdot (l + l' - P_\gamma)}{(l + l' - P_\gamma)^2}$ $\tilde{z}_\Lambda = \frac{2P_\Lambda \cdot (l + l')}{s}$

$$E_\Lambda E_\gamma \frac{d\sigma}{d^2 P_\Lambda\, d^3 P_\gamma} = \frac{N_c\, \alpha^3_{\text{e.m.}}}{4\pi^2 s^3\, z_\Lambda} \left( \hat{\sigma}^{BH}\, D_1^{BH}(z_\Lambda) + \hat{\sigma}^{C}\, D_1^{C}(z_\Lambda) + \hat{\sigma}^{I}\, D_1^{I}(z_\Lambda) \right)$$

# $\Lambda\ \gamma$ - pair production in lepton annihilation: $e^-(l) + e^+(l') \longrightarrow \Lambda(P_\Lambda) + \gamma(P_\gamma) + X$

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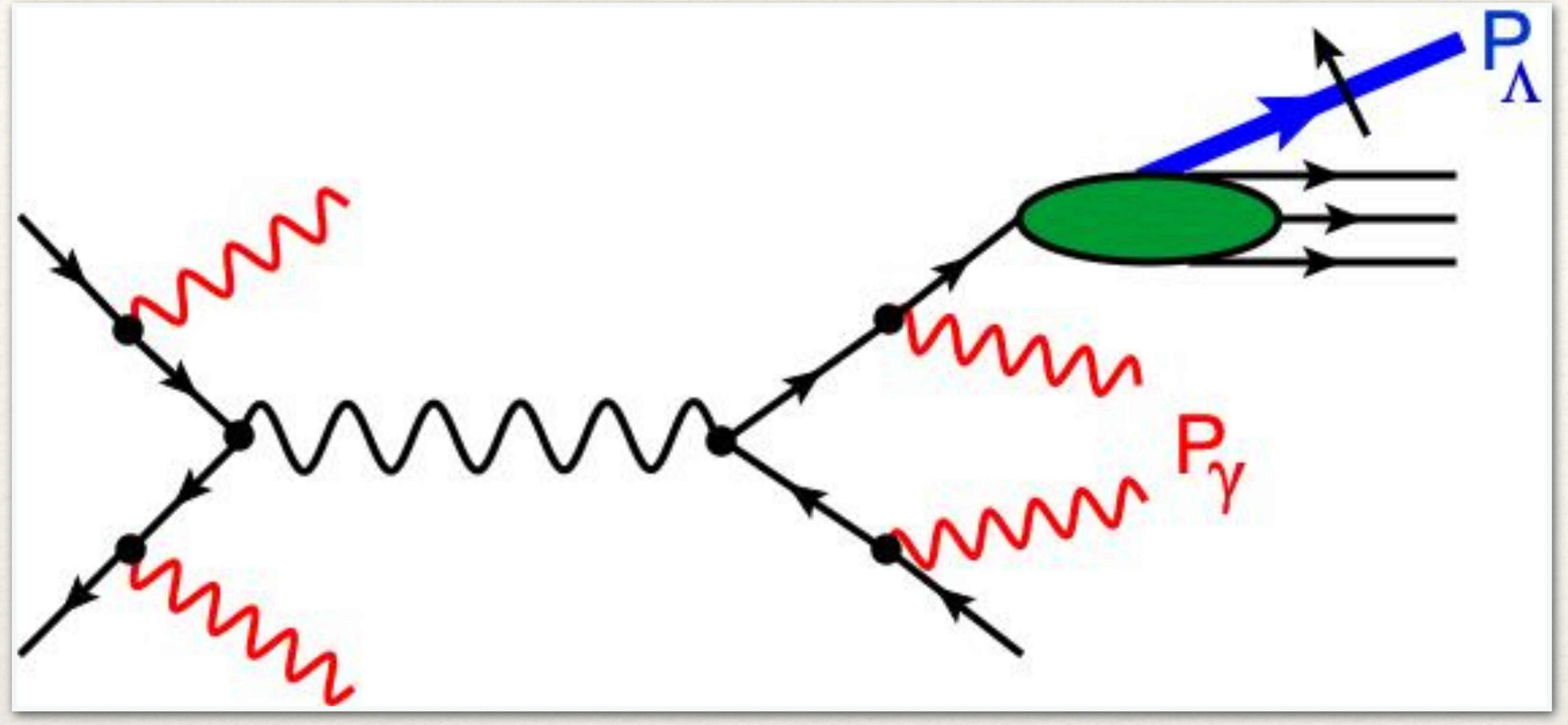


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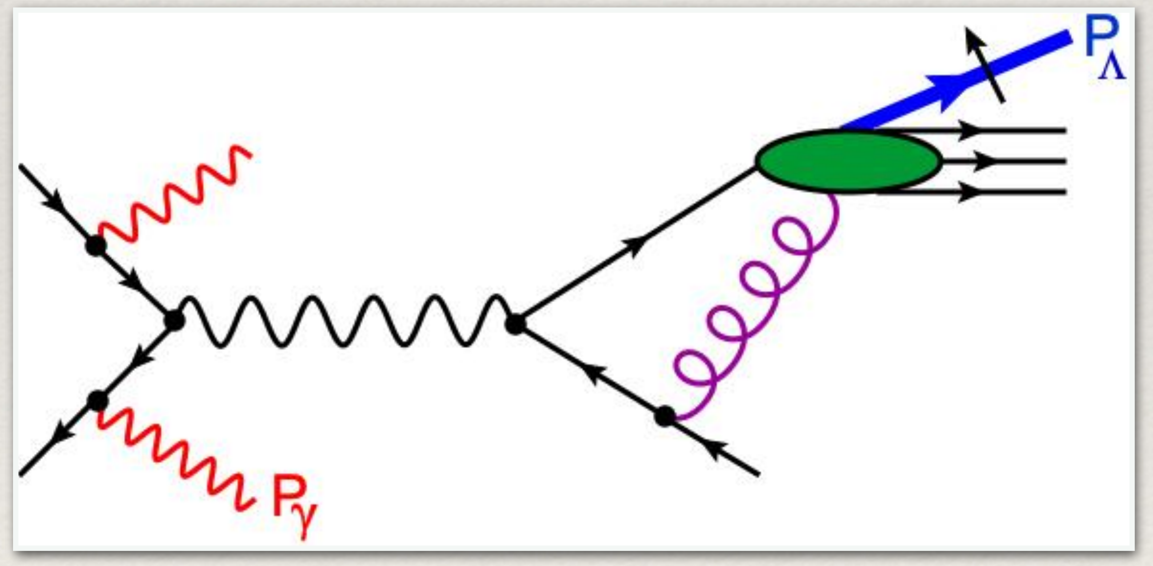
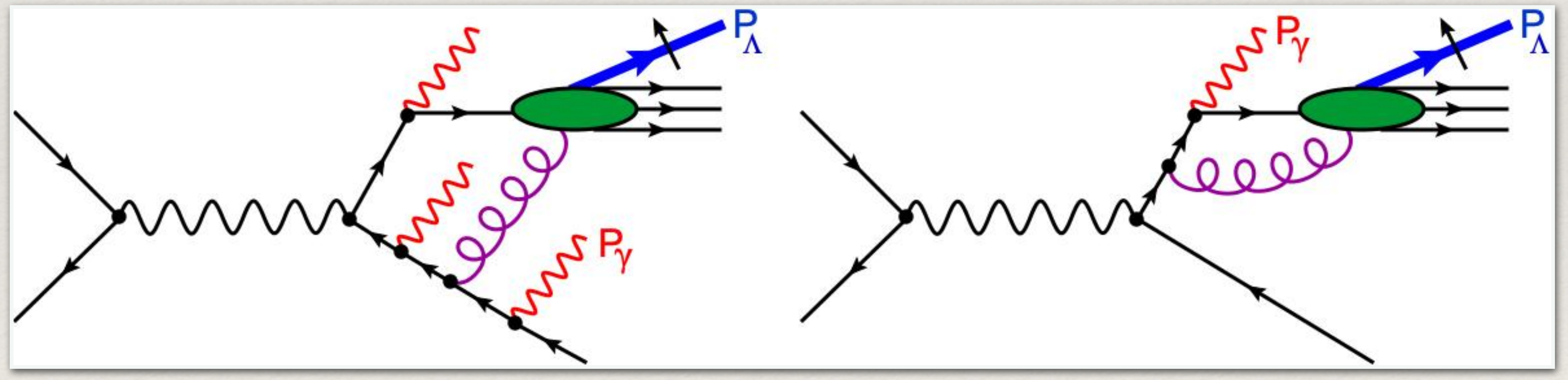
$$E_\Lambda E_\gamma \frac{d\sigma}{d^2 P_\Lambda\, d^3 P_\gamma} = \frac{N_c\, \alpha^3_{\mathrm{e.m.}}}{4\pi^2 s^3\, z_\Lambda} \left(\hat{\sigma}^{BH}\, D_1^{BH}(z_\Lambda) + \hat{\sigma}^{C}\, D_1^{C}(z_\Lambda) + \hat{\sigma}^{I}\, D_1^{I}(z_\Lambda)\right)$$

Transverse spin effects: include all twist-3 effects




Can we recover the interesting features of photon production in eN - collisions?
That is, SSA generated by hard & soft fermion pole contributions?

# Transverse Λ spin dependent cross section

$$E_\Lambda E_\gamma \frac{d\sigma_{UT}}{d^2P_\Lambda\, d^3P_\gamma}(S_\Lambda) = \frac{N_c\, \alpha_{\rm e.m.}^3}{4\pi^2 s^3\, z_\Lambda}\, \frac{M_\Lambda}{\sqrt{s}} \left( \frac{1}{s^{3/2}} \epsilon^{l P_\gamma P_\Lambda S_\Lambda}\, \sigma_{UT}^1 + \frac{1}{s^{3/2}} \epsilon^{l' P_\gamma P_\Lambda S_\Lambda}\, \sigma_{UT}^2 \right)$$

$\propto \sin(\phi_S - \phi_l)$  $\propto \sin(\phi_S - \phi_{l'})$  in $P_\Lambda$ - $P_\gamma$ c.m. frame

$$\sigma_{UT}^i = \sum_{n=C,I} \pi \left( \hat{\sigma}_{HP;D}^{i,n}\, {\rm Re}[\hat{D}_{FT}^n(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)] + \hat{\sigma}_{HP;G}^{i,n}\, {\rm Re}[\hat{G}_{FT}^n(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)\right) + \sum_{n=BH,C,I} \int_{z_\Lambda}^1 \frac{dz}{z} \int_0^1 dt \left( \hat{\sigma}_{dyn,1}^{i,n}(z,t)\, \frac{{\rm Im}[\hat{D}_{FT}^n - \hat{G}_{FT}^n](z,t)}{1-t} - 2\hat{\sigma}_{dyn,2}^{i,n}(z,t)\, \frac{{\rm Im}[\hat{D}_{FT}^n](z,t)}{(1-t)^2} \right)$$

Hard Pole contributions only!

“Principle Value” contributions
(Imaginary parts of qgq fragmentation functions)

# Transverse Λ spin dependent cross section

$$E_\Lambda E_\gamma \frac{d\sigma_{UT}}{d^2P_\Lambda\, d^3P_\gamma}(S_\Lambda) = \frac{N_c\, \alpha^3_{\text{e.m.}}}{4\pi^2 s^3\, z_\Lambda} \frac{M_\Lambda}{\sqrt{s}} \left( \frac{1}{s^{3/2}} \epsilon^{l P_\gamma P_\Lambda S_\Lambda}\, \sigma^1_{UT} + \frac{1}{s^{3/2}} \epsilon^{l' P_\gamma P_\Lambda S_\Lambda}\, \sigma^2_{UT} \right)$$

$\propto \sin(\phi_S - \phi_l)$ $\propto \sin(\phi_S - \phi_{l'})$ in $P_\Lambda$ - $P_\gamma$ c.m. frame

$$\sigma^i_{UT} = \sum_{n=C,I} \pi \left( \hat{\sigma}^{i,n}_{HP;D}\, \text{Re}[\hat{D}^n_{FT}(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)] + \hat{\sigma}^{i,n}_{HP;G}\, \text{Re}[\hat{G}^n_{FT}(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)] \right) + \sum_{n=BH,C,I} \int_{z_\Lambda}^1 \frac{dz}{z} \int_0^1 dt \left( \hat{\sigma}^{i,n}_{dyn,1}(z,t) \frac{\text{Im}[\hat{D}^n_{FT} - \hat{G}^n_{FT}](z,t)}{1-t} - 2\hat{\sigma}^{i,n}_{dyn,2}(z,t) \frac{\text{Im}[\hat{D}^n_{FT}](z,t)}{(1-t)^2} \right)$$

Hard Pole contributions only!

"Principle Value" contributions
(Imaginary parts of qgq fragmentation functions)

# Longitudinal Lepton spin - Transverse Λ spin dependent cross section

$$E_\Lambda E_\gamma \frac{d\sigma_{LT}}{d^2P_\Lambda\, d^3P_\gamma}(S_\Lambda) = \frac{N_c\, \alpha^3_{\text{e.m.}}}{4\pi^2 s^3\, z_\Lambda} \frac{M_\Lambda}{\sqrt{s}} \left( \frac{1}{s^{1/2}} (l \cdot S_\Lambda)\, \Delta\sigma^1_{UT} + \frac{1}{s^{1/2}} (l' \cdot S_\Lambda)\, \Delta\sigma^2_{UT} + \frac{1}{s^{1/2}} (P_\gamma \cdot S_\Lambda)\, \Delta\sigma^3_{UT} \right)$$

$$\Delta\sigma^i_{LT} = \sum_{n=C,I} \pi \left( \Delta\hat{\sigma}^{i,n}_{HP;D}\, \text{Im}[\hat{D}^n_{FT}(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)] + \Delta\hat{\sigma}^{i,n}_{HP;G}\, \text{Im}[\hat{G}^n_{FT}(z_\Lambda, z_\Lambda/\tilde{z}_\Lambda)] \right) + \sum_n \int_{z_\Lambda}^1 \frac{dz}{z} \int_0^1 dt \left( \Delta\hat{\sigma}^{i,n}_{WW}(z) \frac{G^n_1(z)}{z} + \Delta\hat{\sigma}^{i,n}_{dyn,1}(z,t) \frac{\text{Re}[\hat{D}^n_{FT} - \hat{G}^n_{FT}](z,t)}{1-t} - 2\Delta\hat{\sigma}^{i,n}_{dyn,2}(z,t) \frac{\text{Re}[\hat{G}^n_{FT}](z,t)}{(1-t)^2} \right)$$

Single - and Double spin asymmetries "almost" similar, reversed roles for Im and Re - part of dynamical FFs
Difference: WW - contribution for double spin asymmetry.

# Outlook

- Semi-inclusive $\gamma$s in eN: (hopefully) feasible at the EIC
  unpolarized cross section has been measured at HERA
- Goal: give theoretical estimates of the various contributions to SSA & DSA at the EIC (work in progress)
- Inclusive $\gamma$s in eN: simpler observables, but theoretically more challenging. Goal: work out transverse spin effects in collinear twist-3 formalism
- Need to constrain the quark-gluon-quark correlation functions $F_{FT}$ & $G_{FT}$:
  try to constrain these function with $g_2$ DIS data from SLAC, JLab,HERMES (work in progress)
- Once this is done, we can estimate the BH & C contributions at EIC.
- Semi-inclusive $\gamma$s in $e^+e^-$: (hopefully not just academic),
  could give an estimate of the unpolarized rate at the moment