Leading order formalism for spin and transverse momenta dependent fracture functions

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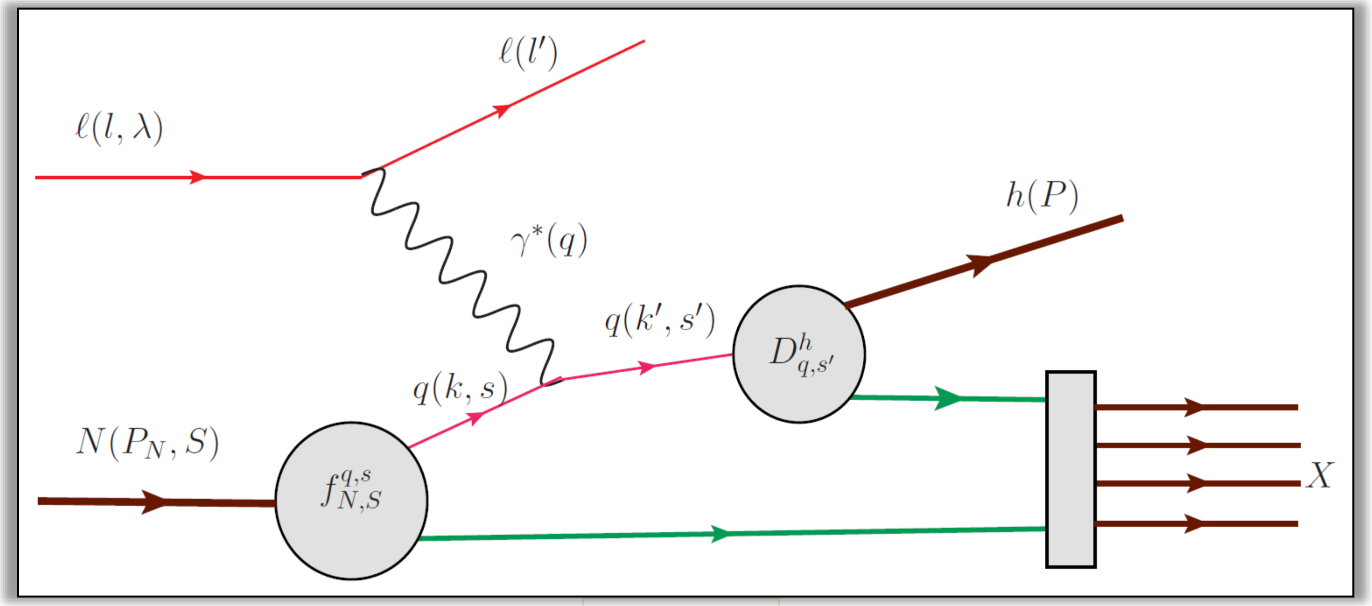
- **Processes to access the non-perturbative inputs Spin and Transverse Momentum Dependent (STMDs)**
  - Parton Distribution Functions in nucleon STMD PDF: SIDIS, DY
  - Parton Fragmentation Functions STMD FF: Hadron production in e^+e^- annihilation (SIA), SIDIS, high p_T hadron production in pp collisions
  - STMD Fracture Functions: SIDIS, DY
  - String Fragmentation: LEPTO, PYTHIA

SIDIS: CFR



$$x_F > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s'}_T}{m_h} H_1(z,p_T^2)$$

Measured in e^+e^- semi inclusive annihilation (SIA) to 2 back-to-back jets $e^+e^- \to h_1 h_2 + X$

Twist-2 TMD PDFs

Nucleon Polarization	Quark polarization: U	Quark polarization: L	Quark polarization: T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{[\mathbf{k}_T \times \mathbf{S}_T]_3}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

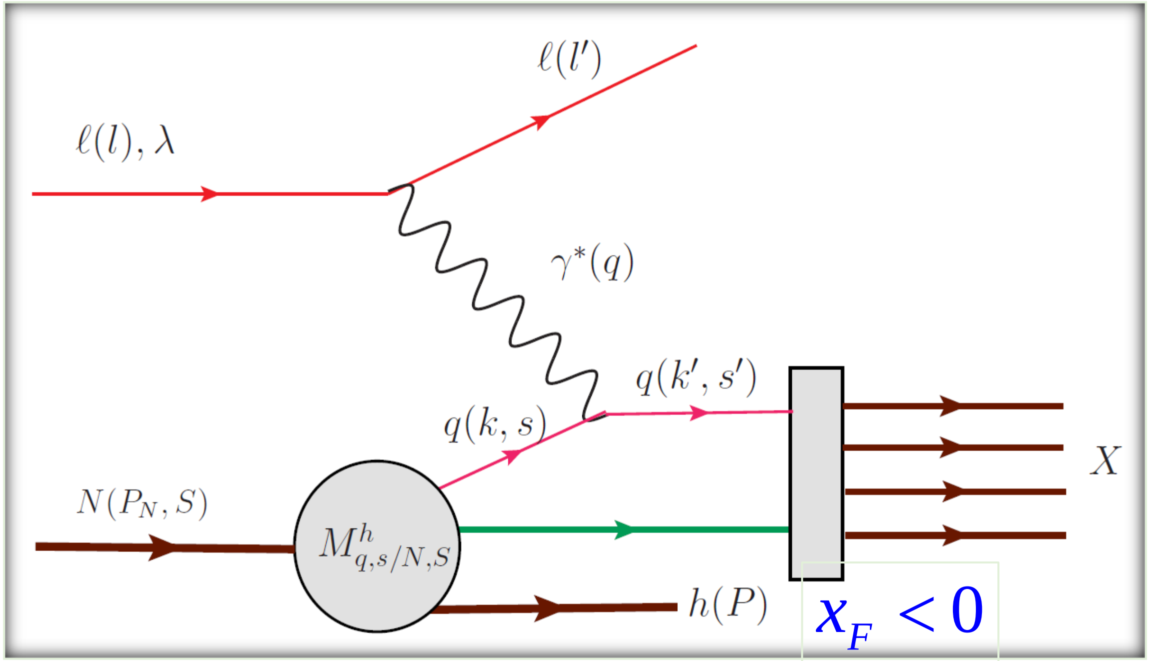
$$\times\Big[F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+$$
$$S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+$$
$$S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\boxed{\sin(\phi_h-\phi_S)}+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\boxed{\sin(\phi_h+\phi_S)}+\\F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+$$
$$\lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\Big]$$

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

At LO only 8 terms out of 18 Structure Functions entering in the general expression of SIDIS cross section

6 azimuthal modulations, 4 terms are generated by Collins effect in fragmentation

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
....

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\zeta} = M^h_{q/N}\left(x, Q^2, \zeta\right) \otimes \frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Fracture function *M* is a Conditional Probability Distribution Function (CPDF) to observe the hadron h produced in target nucleon momentum direction in γ^*P CMS when hard probe interacts with parton carrying fraction x of nucleon momentum.

Collinear Frac.Func.: application to HERA data, 1

D. de Florian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$\frac{d^3\sigma^p_{target}}{d\beta dQ^2 dx_{\mathrm{P}}} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right) M^h_p\left(\beta, Q^2, x_{\mathrm{P}}\right), \quad \beta = \frac{x}{1-\zeta}, \quad \zeta = \frac{p_h^+}{p_N^+} \qquad x_{\mathrm{P}} = \zeta$$

$$x M_q^{p/p}(\beta, Q_0^2, x_{\mathrm{P}}) = N_s \beta^{a_s}(1-\beta)^{b_s}\{C_{\mathrm{P}}\beta x_{\mathrm{P}}^{\alpha_{\mathrm{P}}} + C_{\mathrm{LP}}(1-\beta)^{\gamma_{\mathrm{LP}}}[1 + a_{\mathrm{LP}}(1-x_{\mathrm{P}})^{\beta_{\mathrm{LP}}}]\}$$

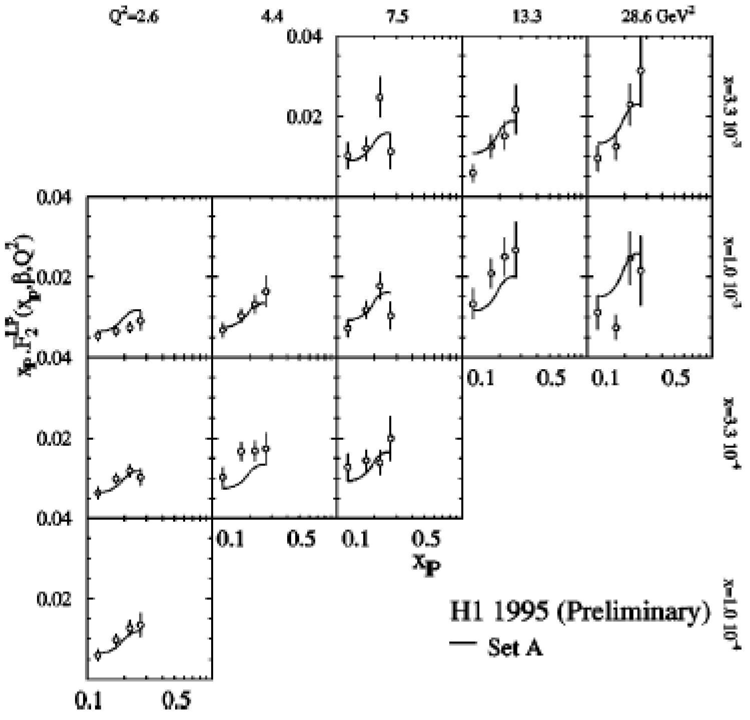


FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

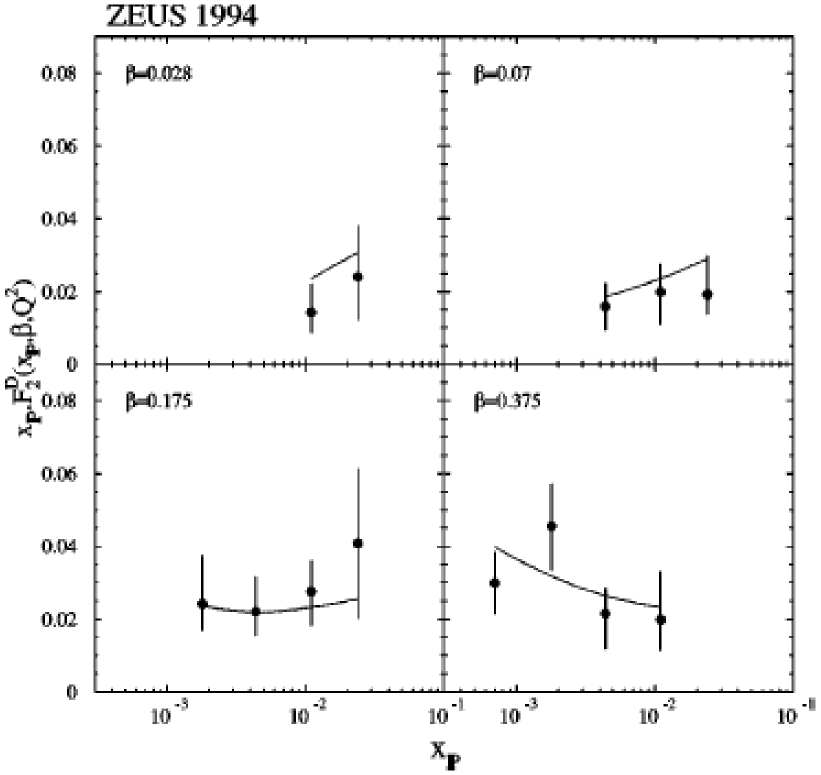
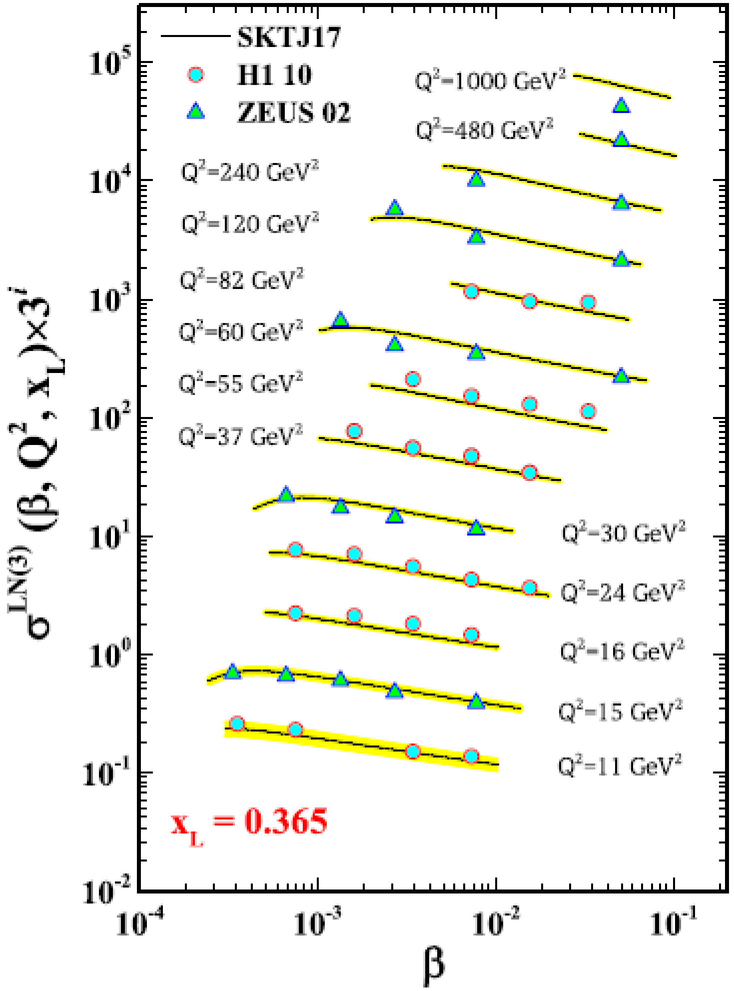


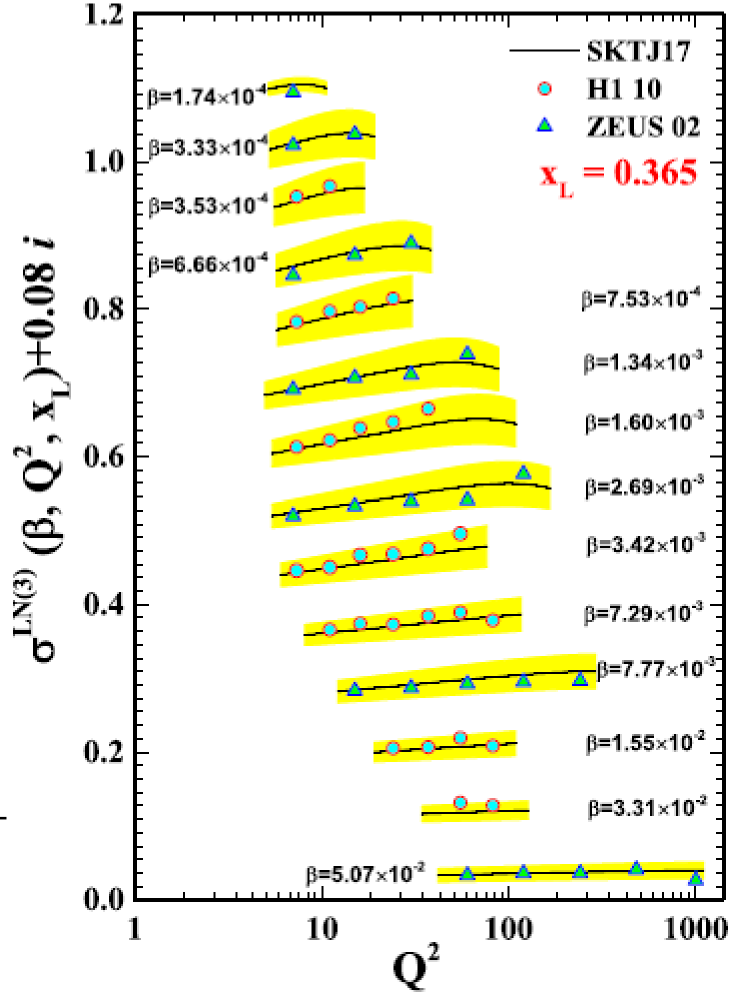
FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

Collinear Frac.Func.: application to HERA data, 2

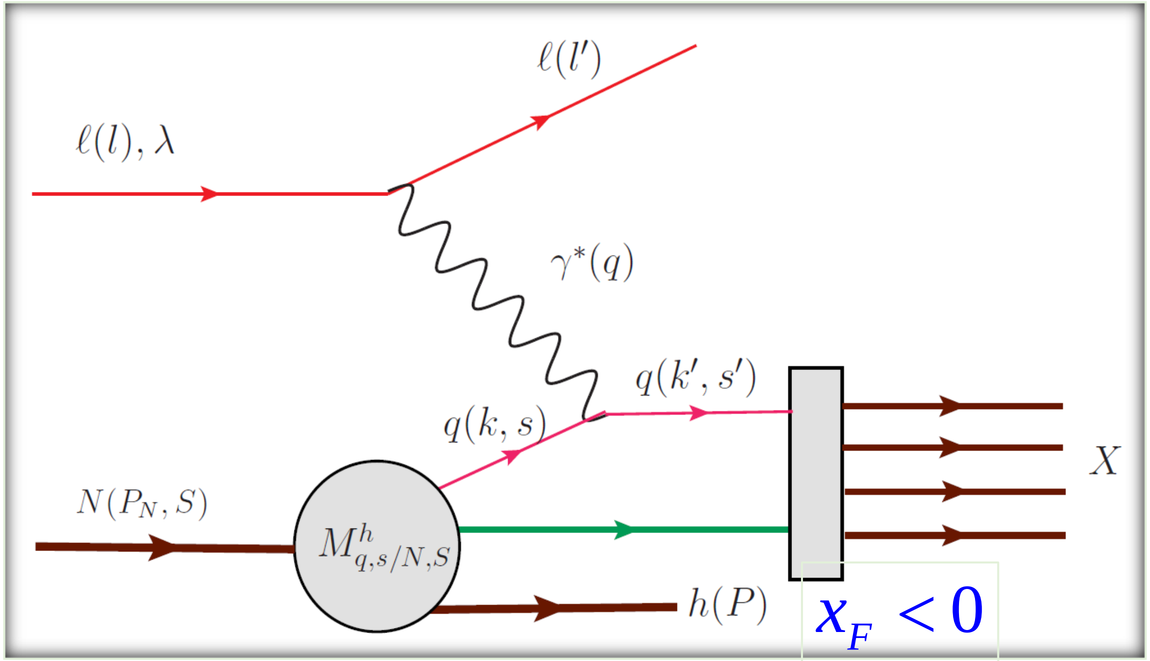
Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)



$$x_L \simeq \frac{E_B}{E_P}, \quad \beta = \frac{x}{1-x_L}$$



SIDIS TFR: Spin & TMD dependent Fracture Functions



Anselmino, Barone and AK, PL B 699 (2011)108**; 706** (2011)46**; 713** (2012)317

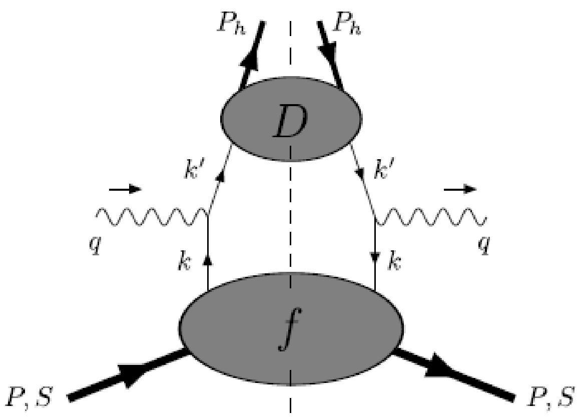
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

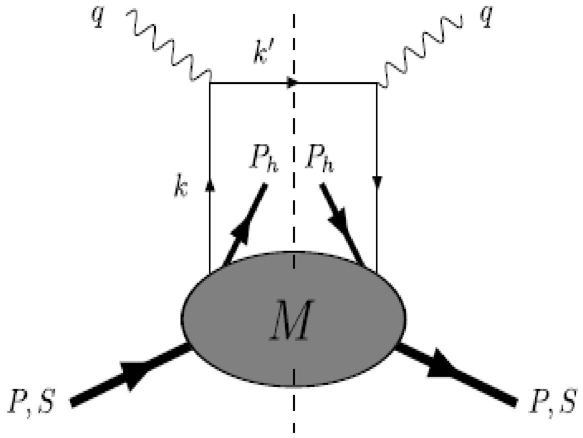
$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Quark correlator

SIDIS



CFR



TFR

$$\mathcal{M}^{[\Gamma]}(x_B, \vec{k}_\perp, \zeta, \vec{P}_{h\perp}) = \frac{1}{4\zeta}\int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{i(x_B P^- \xi^+ - \vec{k}_\perp \cdot \vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$

$$\times \langle P, S | \bar{\psi}(0)\Gamma | P_h, S_h; X\rangle \langle P_h, S_h; X | \psi(\xi^+, 0, \vec{\xi}_\perp) | P, S\rangle$$

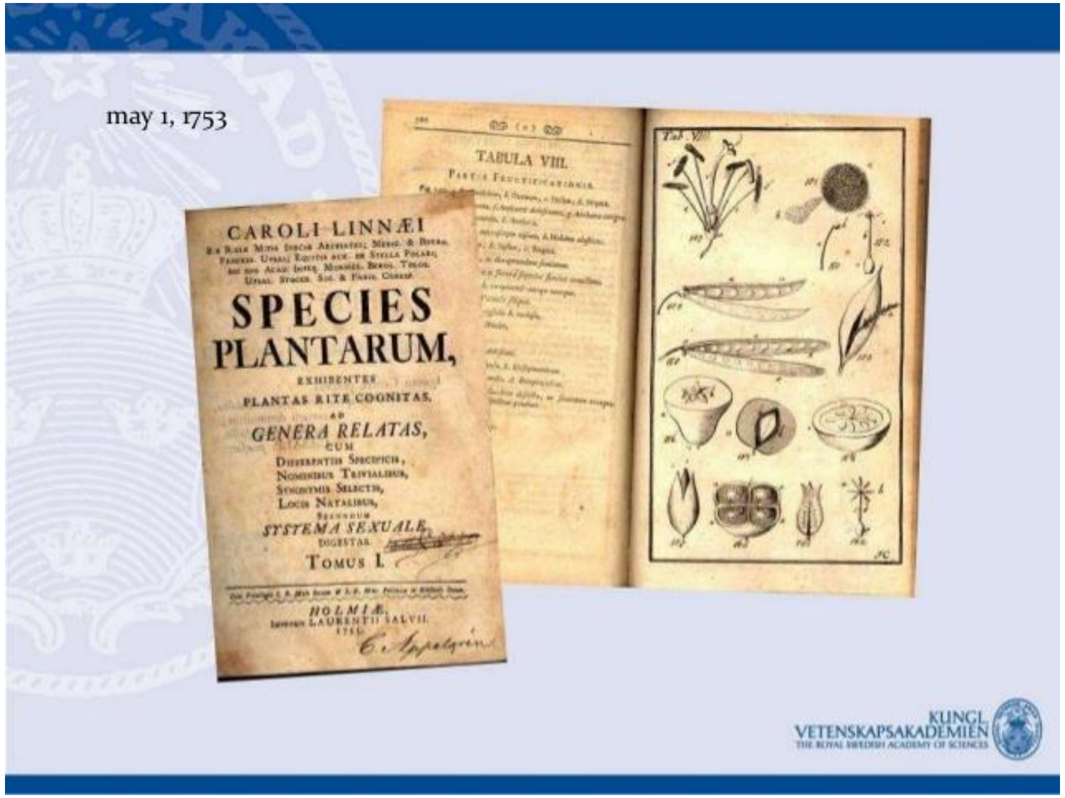
$$\Gamma = \gamma^-, \quad \gamma^-\gamma_5, \quad i\sigma^{i-}\gamma_5$$

Probabilistic interpretation at LO:

the conditional probabilities to find an unpolarized ($\Gamma = \gamma^-$), a longitudinally polarized ($\Gamma = \gamma^- \gamma_5$) or a transversely polarized ($\Gamma = \sigma^{i-} \gamma_5$) quark with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction ζ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

Karl Linney: plants classification

Plants were divided by it into 24 classes and 116 groups on the basis of features of a structure of their reproductive organs.



[9]

Plate IV. *Simple Leaves.*

Fig. 1. Orbicular.
2. Roundiſh.
3. Ovate.
4. Oval.
5. Oblong.
6. Lanceolate.
7. Linear.
8. Subulate or awl-ſhaped.
9. Reniform or kidney ſhaped.
10. Cordate or heart-ſhaped.
11. Lunate or creſcent-ſhaped.
12. Triangular.
13. Sagittate or arrow-ſhaped.
14. Heart-arrow ſhaped.
15. Haſtate or halbert-ſhaped.
16. Obcordate or inverſely heart-ſhaped.
17. 3-lobed.
18. Premorſe or as if bitten.
19. Lobed.
20. 5-angled.
21. Eroded or gnawed.
22. Palmate.
23. Pinnatifid or wing-cleft.
24. Laciniate or jagged.
25. Sinuate or indented.
26. Tooth-ſinuate.
27. Runcinate or barbed.
28. Parted or divided.
29. Repand or ſerpentine.
30. Toothed.
31. Serrate.
32. Doubly ſerrate.
33. Doubly crenate or ſcalloped.
34. Cartilaginous.

Fig. 35. Acutely crenate or ſcalloped.
36. Obtuſely crenate.
37. Plaited.
38. Panduræform or fiddle-ſhaped.
39. Spatulate or ſhaped like a battledore.
40. Obtuſe.
41. Acute.
42. Acuminate or pointed.
43. Obtuſe with a point.
44. Acutely emarginate or notched.
45. Cuneiform or wedge-ſhaped.
46. Retuſe.
47. Hairy.
48. Downy.
49. Hiſpid or covered with ſtiffiſh briſtles.
50. Ciliate or fringed.
51. Rhombic.
52. Veined.
53. Nerved.
54. Papillous or pimpled.
55. Parabolic.
56. Acinaciform or ſcymetar-ſhaped.
57. Dolabriform or hatched-ſhaped.
58. Deltoid.
59. Triangular.
60. Channeled.
61. Furrowed or grooved.
62. Cylindrical or without angles.

VOL. V.—B Plate

For STMD Fracture Functions I was expecting
32 (Trentadue) independent structures.
Fortunately, we end up with only 16 of them at twist-2

STMD Fracture Functions for spinless hadron production

Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^{\perp}$
L	$\frac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

STMD fracture functions

depend on

$x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$

$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$

azimuthal dependence

in fracture functions

TMD Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{u}_1 = (1-x)\, f_1(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x)\, f_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{l}_{1L} = (1-x) g_{1L}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x, k_T^2)$$

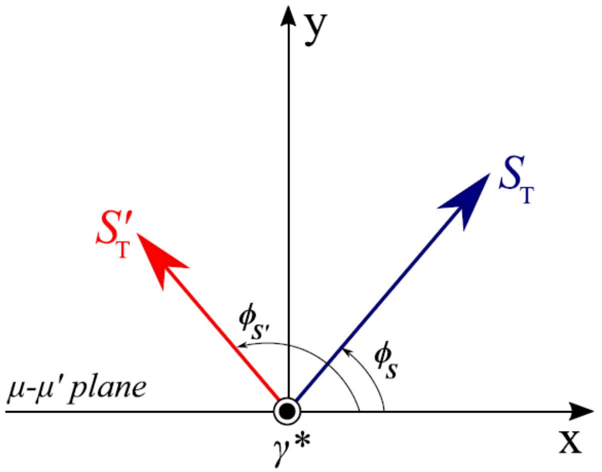
$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x, k_T^2)$$

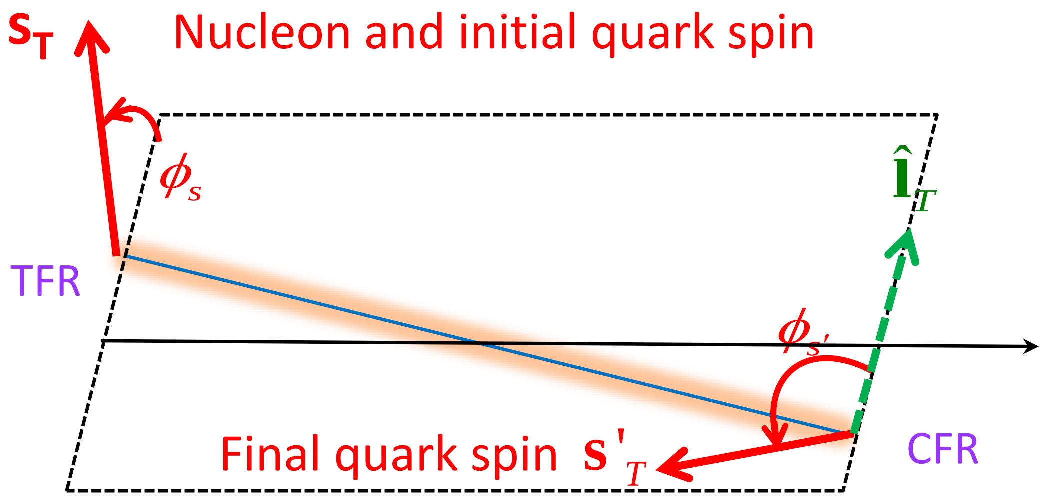
$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}\right)^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x, k_T^2)$$

Quark transverse spin in hard *l-q* scattering

AK, Transversity workshop,
Yerevan, 2009





$$\text{QED: } lq \to l'q' \;\Rightarrow\; s'_T = D_{nn}(y) s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

$$\text{CFR: } \left[\mathbf{s}'_T \times \mathbf{p}_T\right] \propto \sin\left(\phi_h - \phi_{s'}\right) = -\sin\left(\phi_h + \phi_s\right)$$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.
→ No access to quark transverse polarization dependent fracture functions.
No Collins like modulation.

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = \frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2 \times$$

$$\times\left[\tilde{u}_1\left(x,\zeta,P_T^2\right) - S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S) + \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right) + S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{u}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{l}_{1L}$$

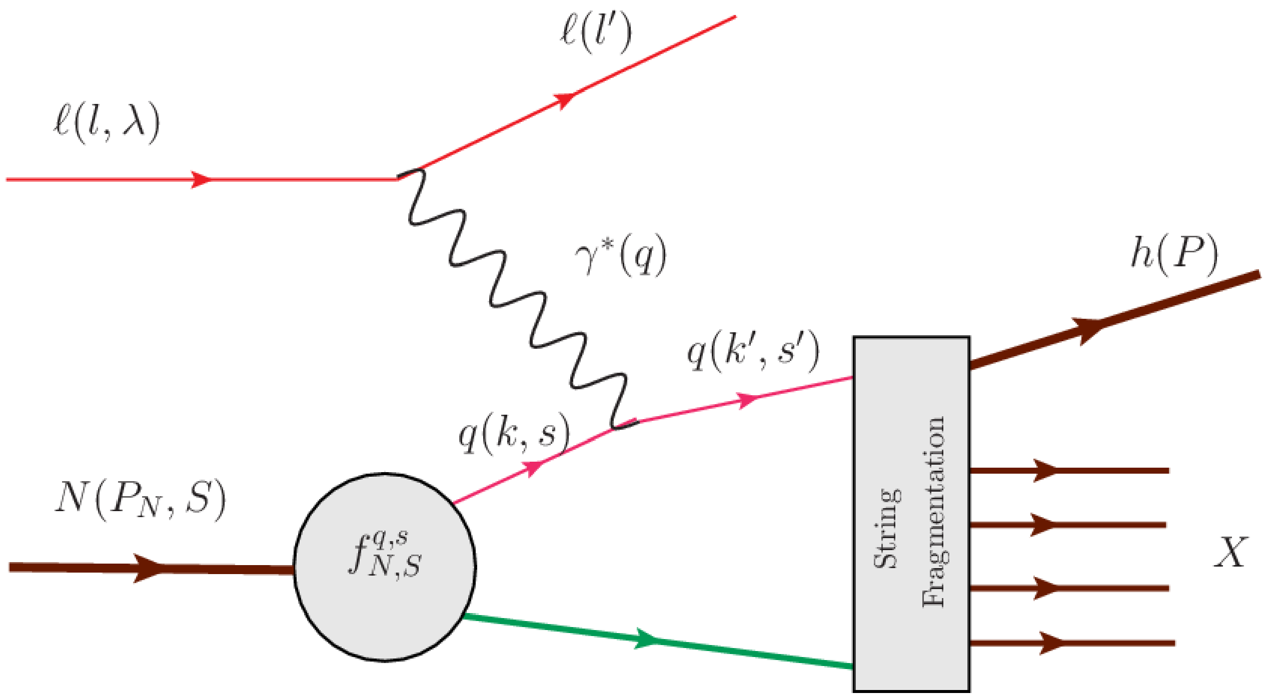
$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{l}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO (twist 2) only 4 terms out of 18 Structure Functions in SIDIS,
Only 2 azimuthal modulations

No Collins-like $\sin(\phi_h+\phi_S)$ modulation

No access to quark transverse polarization

MC event generators (LEPTO, PYTHIA): Hadronization Function



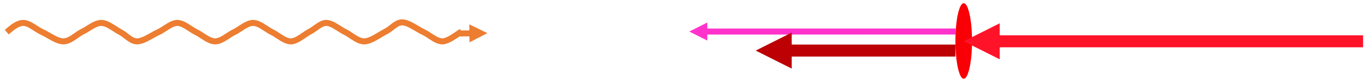
$$d\sigma^{lN \to lhX} = \sum_q f_q\left(x, k_T^2\right) \otimes d\sigma^{lq \to lq} \otimes H^q_{h/N}\left(x, \mathbf{k}_\mathbf{T}; x_F, \mathbf{p}^h_\mathbf{T}\right)$$

Hadronization Function modeled by Lund String Fragmentation

Aram Kotzinian

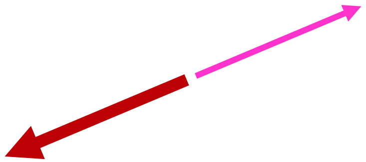
Quark dynamics in MC even generators

- Before



- After hard scattering

- Include k_T with isotropic azimuth



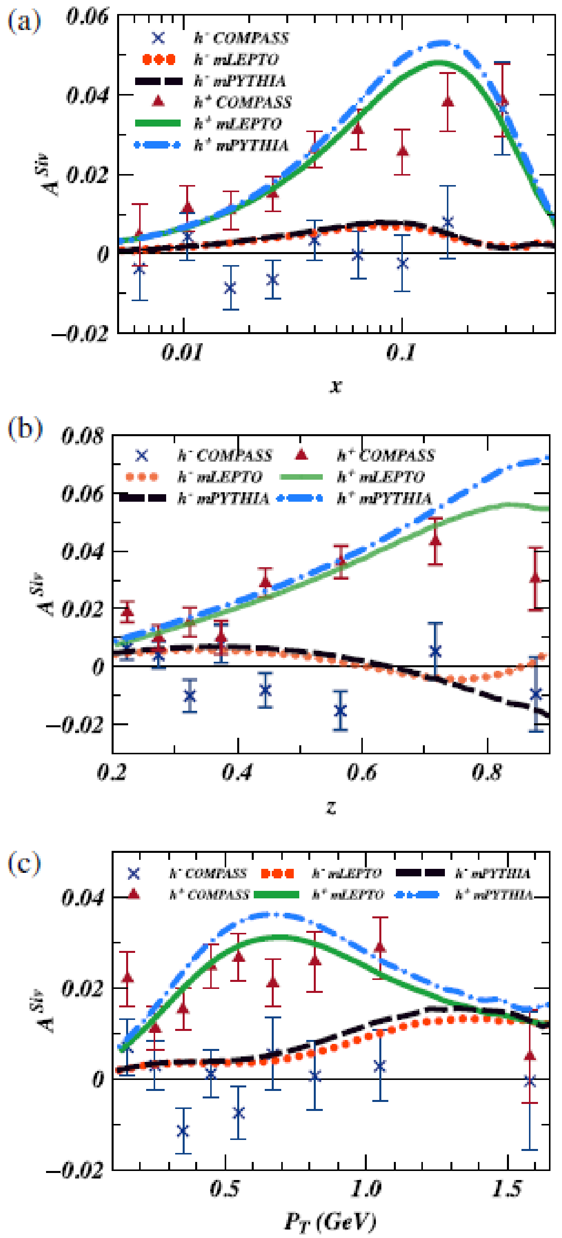
Modified, mLEPTO and mPYTHIA

Include k_T with anisotropic azimuthal modulation according Sivers function

$$d\sigma^{lN\to lhX} = \sum_q \left(f_q(x,k_T^2) + \frac{\mathbf{k_T}\times\mathbf{S_T}}{M} f_{qT}^{\perp}(x,k_T^2)\right)\otimes d\sigma^{lq\to lq}\otimes H_{h/N}^{q}(x,\mathbf{k_T};x_F,\mathbf{p}_\mathbf{T}^h)$$

Sivers effect in the event generators

Matevosyan, AK, Aschenauer, Avakian, Thomas, PRD 92, 054028 (2015)



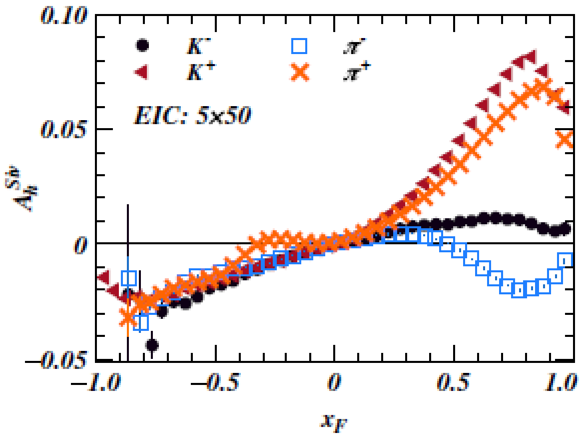


FIG. 13 (color online). EIC model SSAs for 5×50 SIDIS kinematics for charged pions and kaons versus x_F. The Sivers asymmetry is present both in the current and target fragmentation regions.

Only correlation of target $\mathbf{S_T}$ and struck quark $\mathbf{k_T}$ is explicitly parametrized using Sivers PDFs. Then this correlation is transferred to produced hadrons via unpolarized string fragmentation .

$$\tilde{u}_{1T}^{h}(x_B, \zeta_2, P_{T2}^2) = \int d^2k_T \left\{ \hat{u}_{1T}^{h} + \frac{m_2}{m_N} \frac{\mathbf{k}_T \cdot \mathbf{P}_{T2}}{P_{T2}^2} \hat{u}_{1T}^{\perp} \right\}$$

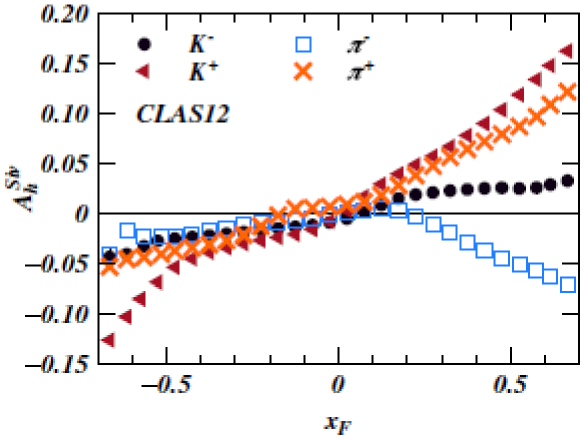
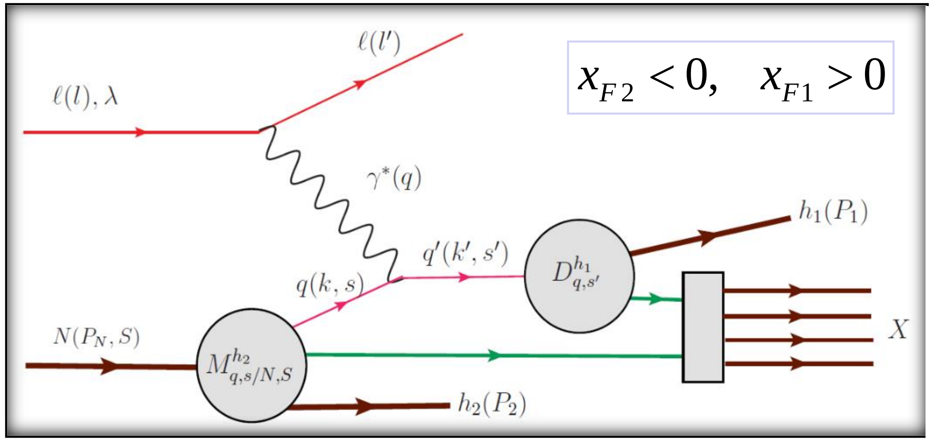
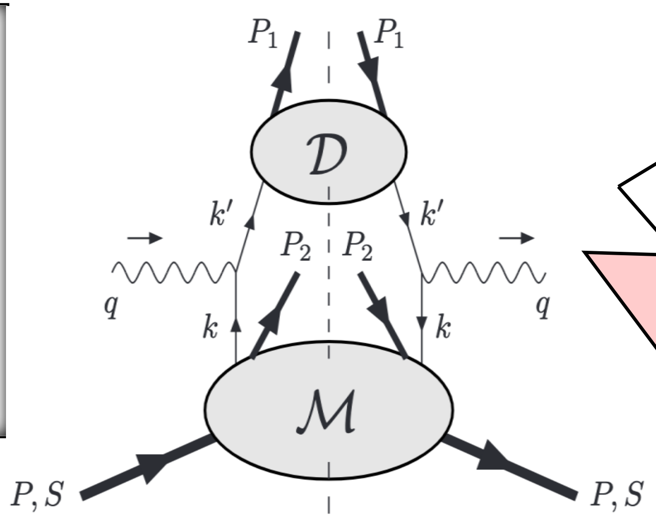


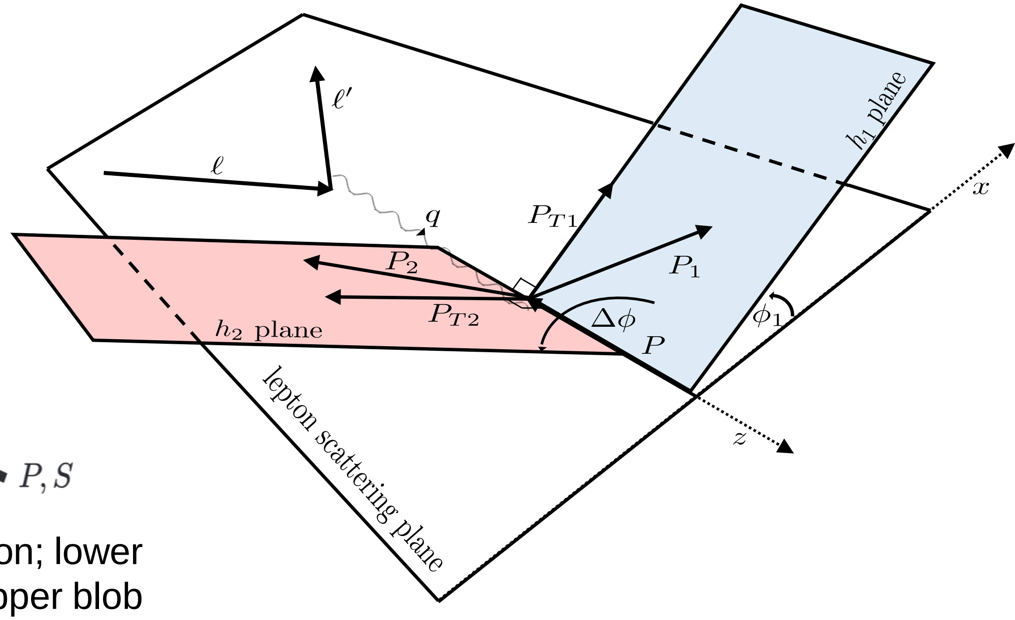
FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus x_F at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

Double hadron production in DIS (DSIDIS): TFR & CFR





Handbag diagram for dihadron production; lower blob contains Fracture Functions and upper blob contains the FFs.



$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

Unintegrated DSIDIS LO cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} \hat{u}^{h_2} \otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2} \otimes D_1^{h_1} \\ +\hat{t}^{h_2} \otimes \dfrac{\mathbf{p}_T \times \mathbf{s}'_T}{m_{h_1}} H_1^{h_1} \end{pmatrix}$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} \sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right) \end{pmatrix}$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms similarly to 1h SIDIS cross section.

$$D_{ll}(y) = \frac{y(2-y)}{1+(1-y)^2}$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1}) \hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2}) D_a(z,p_T^2) w$$

Structure functions $F_{...}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi),\ \text{with}\ \Delta\phi = \phi_1 - \phi_2$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2) + D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) - \left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S) \\
& + D_{nn}(y) \Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + \phi_S) \\
& \quad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2 + \phi_S) \\
& \quad + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S) \\
& \quad + \left(\frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\
& \quad + \left(\frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\
& \quad - \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\
& \quad - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\
& \quad + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \Bigg]
\end{aligned}
$$

σ_LU, σ_LL, σ_LT

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1 \cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

A_{LU} asymmetry

Anselmino, Barone and AK, PLB **713** (2012) 317

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

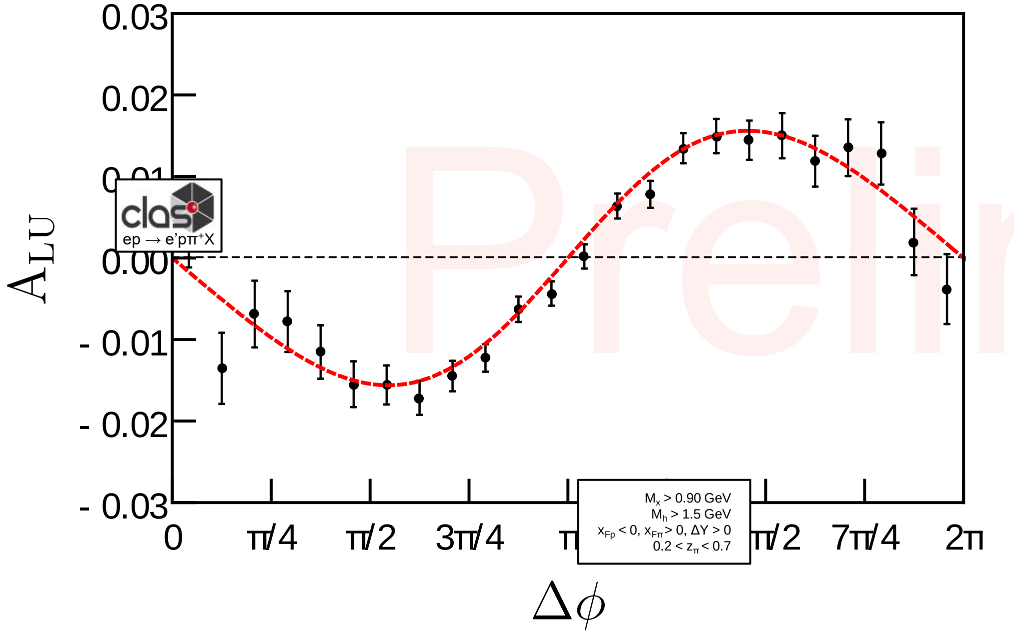
Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

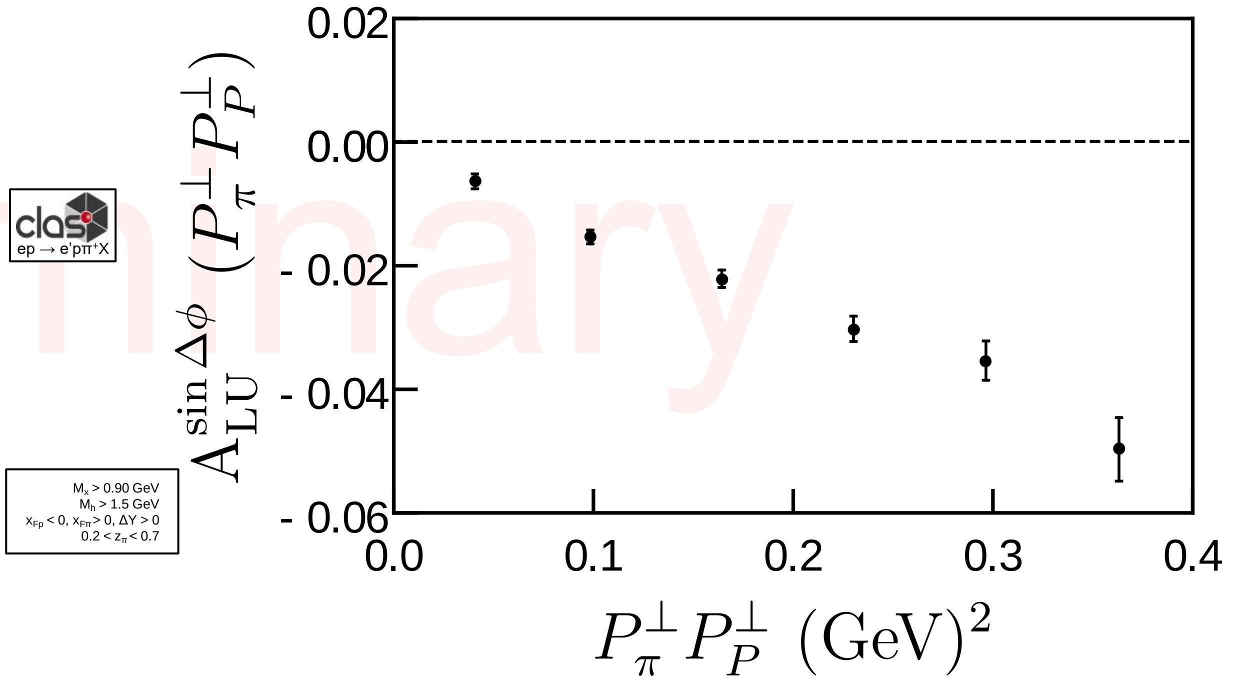
$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)} = p_1 \sin(\Delta\phi) + p_2 \sin(2\Delta\phi) + \cdots$$

A_{LU} @ CLAS

Timothy B. Hayward, H. Avakian and A.Kotzinian Talk at CPHI and

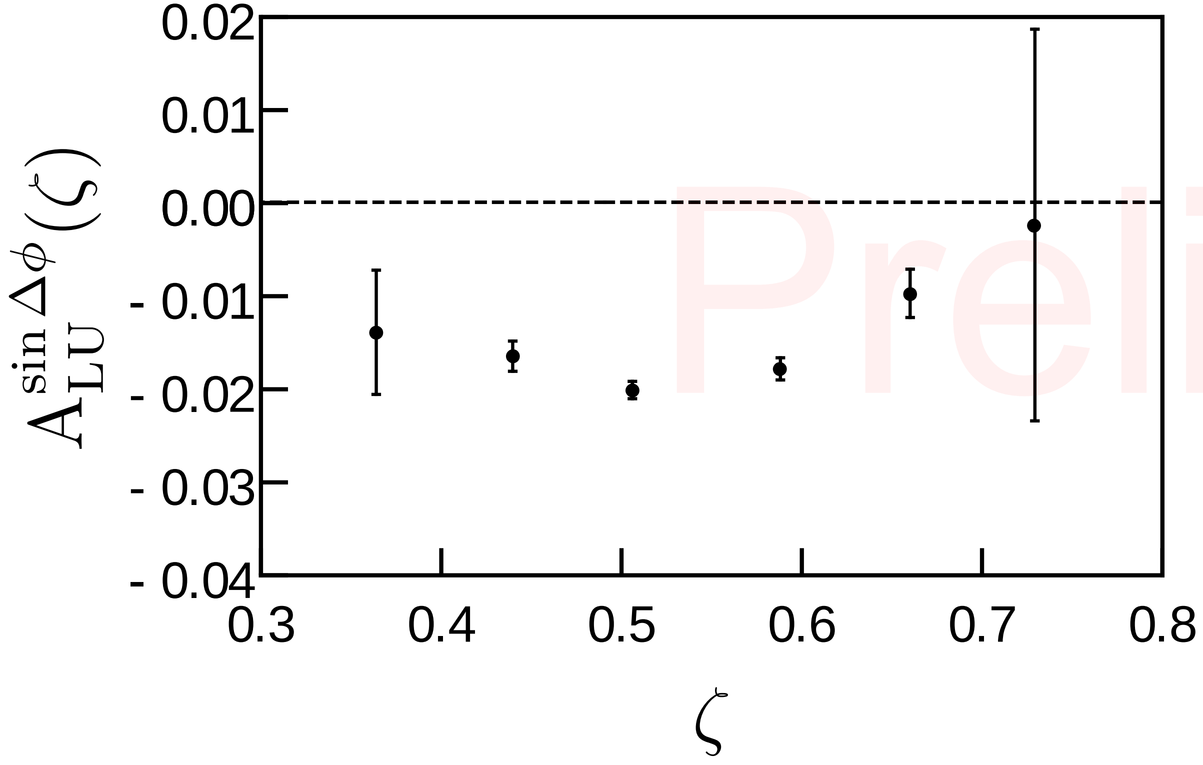
- Observed linear dependence on the product of transverse momenta is consistent with expectations.
- Non-zero asymmetries are the first experimental observation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.

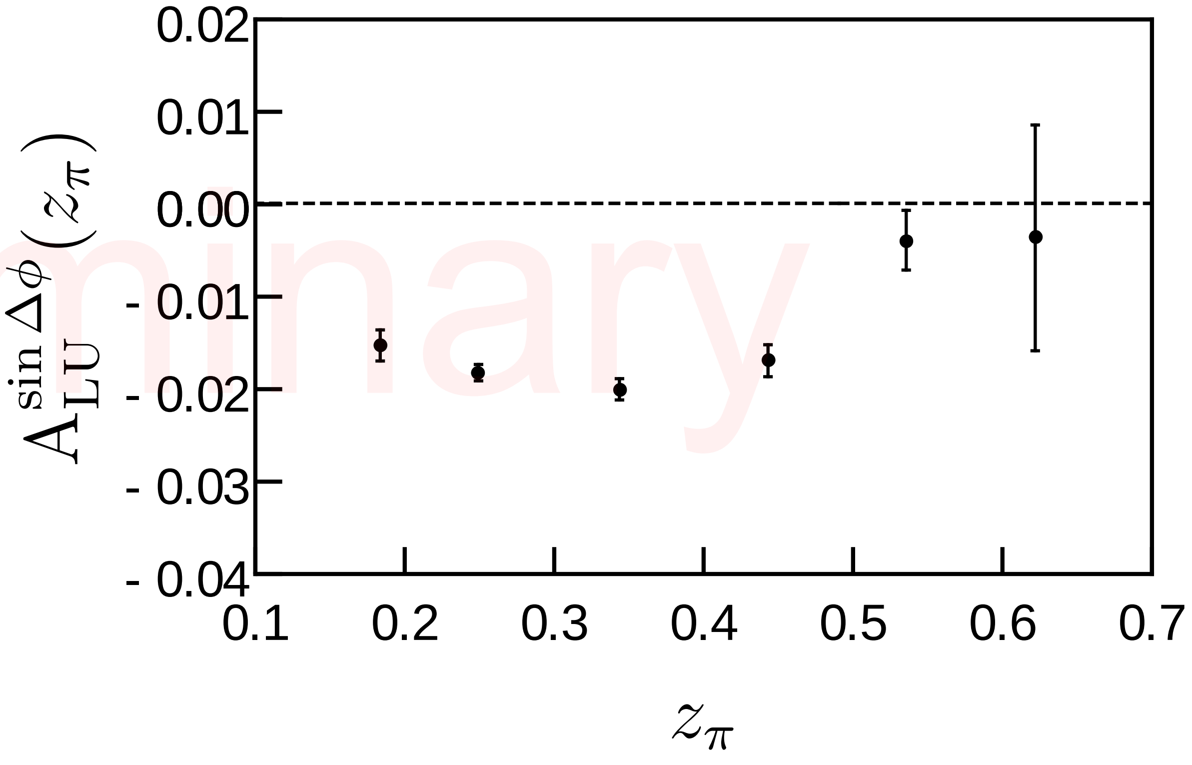




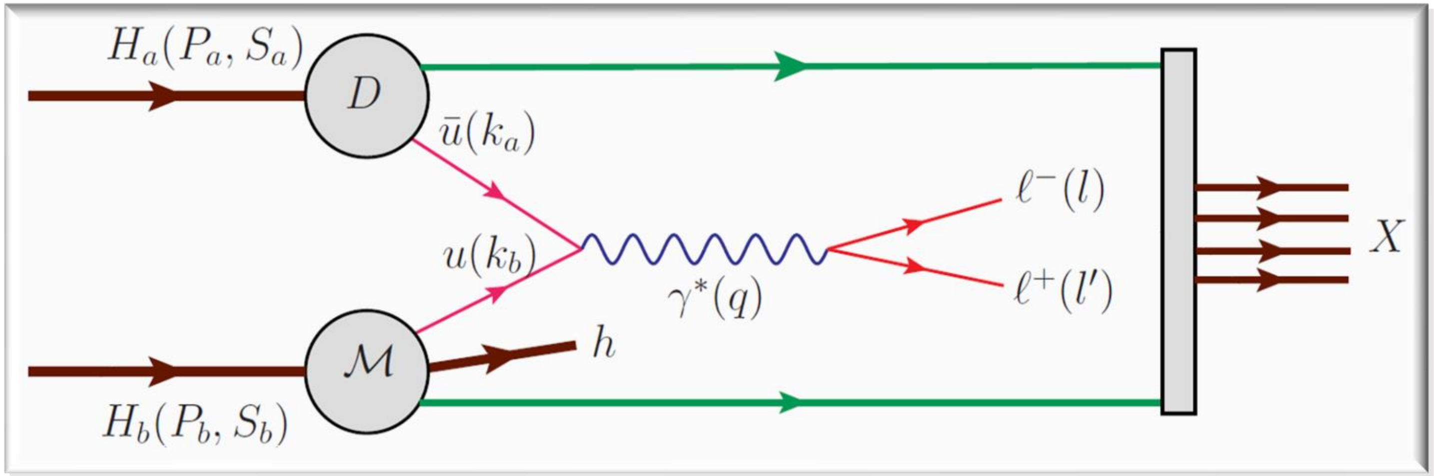
A_{LU} @ CLAS, 2

Timothy B. Hayward, H. Avakian and A.Kotzinian Talk at CPHI and



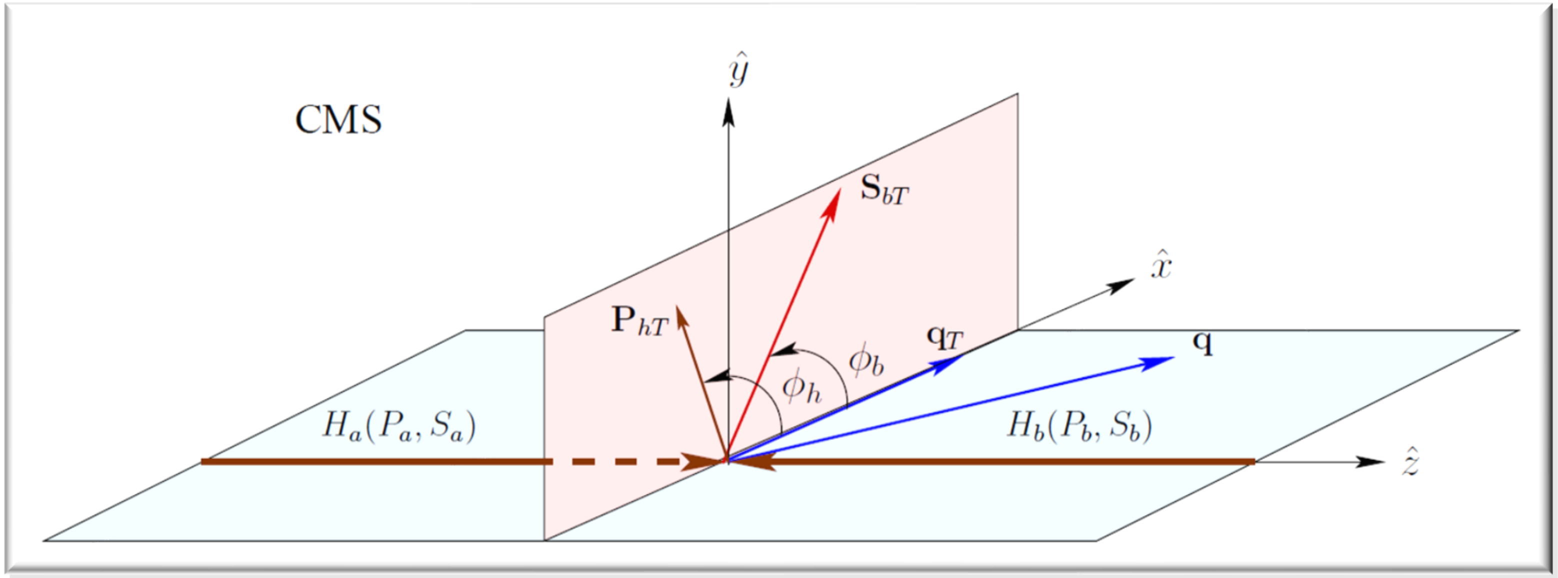


Polarized SIDY

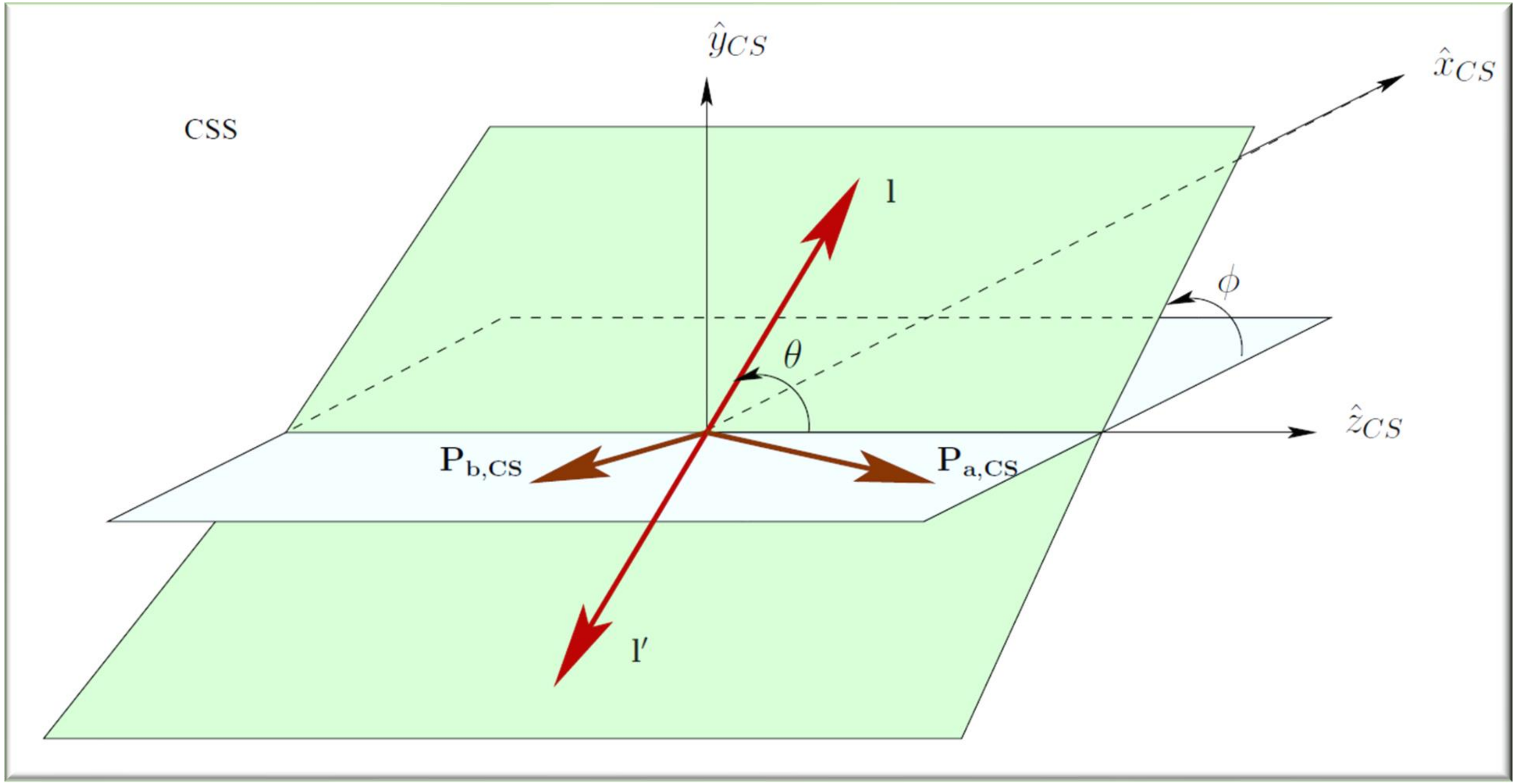


STAR@RHIC?

Kinematics as in Arnold.Metz.Schlegel, PhysRevD.79.034005



Collins-Soper system



SIDY cross section

$$\frac{d\sigma}{d^4q\,d\Omega\,d\zeta\,d^2P_T} = \frac{\alpha_{em}^2 x_a x_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\,\delta^{(2)}\left(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT}\right)\times$$

$$\times\left(\begin{array}{l} \left(1+\cos^2\theta\right)\left(\Phi^{q[\gamma^+]}\overline{\mathcal{M}}^{q[\gamma^-]} + \Phi^{q[\gamma^+\gamma_5]}\overline{\mathcal{M}}^{q[\gamma^-\gamma_5]}\right) \\ +\sin^2\theta\left(\begin{array}{l}\cos 2\phi\left(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2}\right) \\ +\sin 2\phi\left(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}\right)\end{array}\right)\Phi^{q[i\sigma^{i+}\gamma_5]}\overline{\mathcal{M}}^{q[i\sigma^{j-}\gamma_5]} \\ +\{\Phi \leftrightarrow \overline{\Phi}, \overline{\mathcal{M}} \leftrightarrow \mathcal{M}\} + \mathcal{O}(1/q) \end{array}\right)$$

$$= \frac{\alpha_{em}^2 x_a x_b}{2q^4}\left(\begin{array}{l} \sigma_{UU} + S_{bL}\sigma_{UL} + S_{bT}\sigma_{UT} \\ + S_{aL}\sigma_{LU} + S_{aL}S_{bL}\sigma_{LL} + S_{aL}S_{bT}\sigma_{LT} \\ + S_{aT}\sigma_{TU} + S_{aT}S_{bL}\sigma_{TL} + S_{aT}S_{bT}\sigma_{TT} \end{array}\right)$$

σ_{UU}

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[F_{UU}^{\cos(2\phi)} \cos(2\phi) + F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) + F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \right]$$

$$F_{UU} = F_0^{f_1 \cdot \hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U	*		*
	L			
	T			

σ$_{LU}$

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right)$$

$$-\sin^2\theta \left[F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) + F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) + F_{LU}^{\sin(2\phi)} \sin(2\phi) \right]$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}$$

Nucleon Polarization	Quark polarization: U	Quark polarization: L	Quark polarization: T
U		*	*
L			
T			

After integration over lepton scattering plane azimuth alangle ϕ

only first term survives and gives access to the fracture function $\hat{l}_1^{\perp h}$

Conclusions

- New members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are accessible at twist-two
  - SSA contains only a Sivers-type modulation $\sin(\phi_h-\phi_S)$ but no Collins-type $\sin(\phi_h+\phi_S)$ or $\sin(3\phi_h-\phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long-range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms. Access to all fracture functions.
- Polarized SIDY cross section ($p + p \to l^+l^- + h + X$) at LO contains 2 azimuthal independent, 20 lepton plane azimuthal angle independent and 52 lepton plane azimuthal angle dependent terms. In total – 74 terms.
- The ideal place to test the fracture functions formalism and measure these new nonperturbative objects are JLab12, EIC facilities and COMPASS using Camera detector

Additional slides : LO SIDY cross section

Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D] = F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i] = P_{T1}^i F_{k1}^{\hat{M}\cdot D} + P_{T2}^i F_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot D\,p^i] = P_{T1}^i F_{p1}^{\hat{M}\cdot D} + P_{T2}^i F_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i k^j] = P_{T1}^i P_{T1}^j F_{kk1}^{\hat{M}\cdot D} + P_{T2}^i P_{T2}^j F_{kk2}^{\hat{M}\cdot D} + \delta^{ij} F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i p^j] = P_{T1}^i P_{T1}^j F_{kp1}^{\hat{M}\cdot D} + P_{T2}^i P_{T2}^j F_{kp2}^{\hat{M}\cdot D} + \left(P_{T1}^i P_{T2}^j - P_{T1}^j P_{T2}^i\right) F_{kp3}^{\hat{M}\cdot D} + \delta^{ij} F_{kp4}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i k^j p^k] = P_{T1}^i P_{T1}^j P_{T1}^k F_{kkp1}^{\hat{M}\cdot D} + P_{T1}^i P_{T1}^j P_{T2}^k F_{kkp2}^{\hat{M}\cdot D} + P_{T2}^i P_{T2}^j P_{T2}^k F_{kkp3}^{\hat{M}\cdot D}$$
$$+ P_{T2}^i P_{T2}^j P_{T1}^k F_{kkp4}^{\hat{M}\cdot D} + P_{T1}^k \delta^{ij} F_{kkp5}^{\hat{M}\cdot D} + P_{T2}^k \delta^{ij} F_{kkp6}^{\hat{M}\cdot D}$$

where $F_{\dots}^{\hat{M}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2, \left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)$

Structure functions

$$F_{k1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^{2}}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}}\right]$$

$$F_{k2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^{2}}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}}\right]$$

$$F_{kk1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left(-2\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^{2}+\mathbf{k}^{2}\mathbf{P}_{T1}^{2}\right)\mathbf{P}_{T2}^{4}+\left(2\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^{2}-\mathbf{k}^{2}\mathbf{P}_{T2}^{2}\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^{2}+\mathbf{P}_{T1}^{2}\left(\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}-\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}\right)\mathbf{k}^{2}-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^{2}\mathbf{P}_{T1}^{4}}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}+\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)\mathbf{k}^{2}-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^{2}\mathbf{P}_{T1}^{2}-\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^{2}\mathbf{P}_{T2}^{2}}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}}\right]$$

$$F_{kp1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\left(\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{p})+(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T1}^{2}\right)\mathbf{P}_{T2}^{4}}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{4}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}+\frac{\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})(\mathbf{P}_{T2}\cdot\mathbf{p})-(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T2}^{2}\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{2}\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^{4}-\mathbf{P}_{T1}^{2}\mathbf{P}_{T2}^{2}\right)}\right)\right]$$

σ_{UU}

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[F_{UU}^{\cos(2\phi)} \cos(2\phi) + F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) + F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \right]$$

$$F_{UU} = F_0^{f_1 \cdot \hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

Nucleon Polarization \ Quark polarization	U	L	T
U	*		*
L			
T			

σ_{UL}

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{UL}^{\sin(\phi_h)} \sin(\phi_h) - \sin^2\theta \left[F_{UL}^{\sin(2\phi)} \sin(2\phi) + F_{UL}^{\sin(2\phi-\phi_h)} \sin(2\phi-\phi_h) + F_{UL}^{\sin(2\phi-2\phi_h)} \sin(2\phi-2\phi_h) \right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L	*		*
	T			

σ_{UT}

$$\sigma_{UT}=\left(1+\cos^2\theta\right)\left[\begin{array}{l}F_{UT}^{\sin(\phi_b-\phi_h)}\sin\left(\phi_b-\phi_h\right)\\+F_{UT}^{\sin(\phi_b)}\sin\left(\phi_b\right)\end{array}\right]$$

$$-\sin^2\theta\left[\begin{array}{l}F_{UT}^{\sin(2\phi+\phi_b)}\sin\left(2\phi+\phi_b\right)\\+F_{UT}^{\sin(2\phi-\phi_b+\phi_h)}\sin\left(2\phi-\phi_b+\phi_h\right)\\+F_{UT}^{\sin(2\phi+\phi_b-\phi_h)}\sin\left(2\phi+\phi_b-\phi_h\right)\\+F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)}\sin\left(2\phi+\phi_b-3\phi_h\right)\\+F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)}\sin\left(2\phi-\phi_b-2\phi_h\right)\\+F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)}\sin\left(2\phi+\phi_b-2\phi_h\right)\\+F_{UT}^{\sin(2\phi-\phi_b-\phi_h)}\sin\left(2\phi-\phi_b-\phi_h\right)\\+F_{UT}^{\sin(2\phi-\phi_b)}\sin\left(2\phi-\phi_b\right)\end{array}\right]$$

Nucleon Polarization \ Quark polarization	U	L	T
U			
L			
T	*		*

σ$_{LU}$

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right)$$

$$-\sin^2\theta \left[F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) + F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) + F_{LU}^{\sin(2\phi)} \sin(2\phi) \right]$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}$$

Nucleon Polarization \ Quark polarization	U	L	T
U		*	*
L			
T			

After integration over lepton scattering plane azimuth alangle

only first term survives and gives access to $\hat{l}_1^{\perp h}$ fracture function

σ_{LL}

$$\sigma_{LL} = \left(1 + \cos^2\theta\right) F_{LL}$$

$$+\sin^2\theta \left[F_{LL}^{\cos(2\phi - 2\phi_h)} \cos\left(2\phi - 2\phi_h\right) + F_{LL}^{\cos(2\phi - \phi_h)} \cos\left(2\phi - \phi_h\right) + F_{LL}^{\cos(2\phi)} \cos(2\phi) \right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{LT}

$$\sigma_{LT}=\left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{LT}^{\cos(\phi_b-\phi_h)}\cos\left(\phi_b-\phi_h\right)\\ +F_{LT}^{\cos(\phi_b)}\cos\left(\phi_b\right)\end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{LT}^{\cos(2\phi-\phi_b+\phi_h)}\cos\left(2\phi-\phi_b+\phi_h\right)\\ +F_{LT}^{\cos(2\phi+\phi_b)}\cos\left(2\phi+\phi_b\right)+\\ F_{LT}^{\cos(2\phi+\phi_b-\phi_h)}\cos\left(2\phi+\phi_b-\phi_h\right)\\ +F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)}\cos\left(2\phi+\phi_b-3\phi_h\right)\\ +F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)}\cos\left(2\phi-\phi_b-2\phi_h\right)\\ +F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)}\cos\left(2\phi+\phi_b-2\phi_h\right)\\ +F_{LT}^{\cos(2\phi-\phi_b-\phi_h)}\cos\left(2\phi-\phi_b-\phi_h\right)\\ +F_{LT}^{\cos(2\phi-\phi_b)}\cos\left(2\phi-\phi_b\right)\end{array}\right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L			
	T		*	*

σ_{TU}

$$
\begin{aligned}
\sigma_{TU} = \left(1+\cos^2\theta\right)&\left[\begin{array}{l}
F_{TU}^{\sin(\phi_a-\phi_h)}\sin(\phi_a-\phi_h)\\
+F_{TU}^{\sin(\phi_a)}\sin(\phi_a)\\
+F_{TU}^{\sin(\phi_a+\phi_h)}\sin(\phi_a+\phi_h)\\
+F_{TU}^{\sin(\phi_a-2\phi_h)}\sin(\phi_a-2\phi_h)
\end{array}\right]\\
+\sin^2\theta&\left[\begin{array}{l}
F_{TU}^{\sin(2\phi-\phi_a-\phi_h)}\sin(2\phi-\phi_a-\phi_h)\\
+F_{TU}^{\sin(2\phi-\phi_a)}\sin(2\phi-\phi_a)\\
+F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)}\sin(2\phi+\phi_a-3\phi_h)+\\
F_{TU}^{\sin(2\phi+\phi_a-\phi_h)}\sin(2\phi+\phi_a-\phi_h)\\
+F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)}\sin(2\phi+\phi_a-2\phi_h)\\
+F_{TU}^{\sin(2\phi+\phi_a)}\sin(2\phi+\phi_a)
\end{array}\right]
\end{aligned}
$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U	*	*	*
	L			
	T			

σ_{TL}

$$\sigma_{TL} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TL}^{\cos(\phi_a-\phi_h)}\cos(\phi_a-\phi_h) \\ +F_{TL}^{\cos(\phi_a)}\cos(\phi_a) \\ +F_{TL}^{\cos(\phi_a+\phi_h)}\cos(\phi_a+\phi_h) \\ +F_{TL}^{\cos(\phi_a-2\phi_h)}\cos(\phi_a-2\phi_h) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)}\cos(2\phi+\phi_a-3\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)}\cos(2\phi+\phi_a-2\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a)}\cos(2\phi+\phi_a) \\ +F_{TL}^{\cos(2\phi-\phi_a-\phi_h)}\cos(2\phi-\phi_a-\phi_h) \\ +F_{TL}^{\cos(2\phi-\phi_a)}\cos(2\phi-\phi_a) \end{array}\right]$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L		*	*
	T			

σ_{TT}

$$\sigma_{TT} = \left(1+\cos^2\theta\right)\begin{bmatrix} F_{TT}^{\cos(\phi_a-\phi_b)}\cos(\phi_a-\phi_b) \\ +F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)}\cos(\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)}\cos(\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)}\cos(\phi_a+\phi_b-\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b)}\cos(\phi_a+\phi_b) \\ +F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)}\cos(\phi_a-\phi_b-\phi_h) \end{bmatrix} + \sin^2\theta\begin{bmatrix} F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)}\cos(2\phi-\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)}\cos(2\phi-\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b)}\cos(2\phi+\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)}\cos(2\phi+\phi_a-\phi_b+\phi_h)+ \\ F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}\cos(2\phi-\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)}\cos(2\phi-\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)}\cos(2\phi+\phi_a+\phi_b-4\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)}\cos(2\phi+\phi_a-\phi_b-3\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}\cos(2\phi-\phi_a-\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)}\cos(2\phi+\phi_a-\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)}\cos(2\phi+\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)}\cos(2\phi+\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}\cos(2\phi+\phi_a-\phi_b) \end{bmatrix}$$

		Quark polarization		
		U	L	T
Nucleon Polarization	U			
	L			
	T	*	*	*

Structure functions of σ_{UU}, σ_{UL}, σ_{UT}

$$F_{UU} = F_0^{f_1\cdot\hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{u}_{1T}^\perp}$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1\cdot\hat{l}_1^{\perp h}}$$

$$F_{UL}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UL}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{l}_{1T}^h}$$

$$F_{UL}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{l}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UT}^{\sin(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{f_1\cdot\hat{t}_1^h} + \frac{P_T}{M_b} F_{b1}^{f_1\cdot\hat{t}_1^\perp}, \qquad F_{UT}^{\sin(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{f_1\cdot\hat{t}_1^\perp}$$

$$F_{UT}^{\sin(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_1^{\perp h}}, \qquad F_{UT}^{\sin(2\phi+\phi_b)} = \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi-\phi_b-\phi_h)} = \left(\frac{P_T}{M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}} + \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T}{m M_a M_b} F_{ab4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T}{M_a M_b^2} F_{abb5}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \right)$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = \left(\frac{q_T}{M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}} + \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} - \frac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T}{M_a M_b^2} F_{abb6}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \right)$$

Structure functions of σ_{LU}, σ_{LL}

$$F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{g_{1L} \cdot \hat{u}_{1L}^{\perp h}}$$

$$F_{LU}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp} \cdot \hat{u}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp} \cdot \hat{u}_{1T}^{\perp}}$$

$$F_{LU}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp} \cdot \hat{u}_{1T}^{h}}$$

$$F_{LU}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp} \cdot \hat{u}_{1T}^{\perp}}$$

$$F_{LL} = F_0^{g_{1L} \cdot \hat{l}_{1L}}$$

$$F_{LL}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp} \cdot \hat{l}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp} \cdot \hat{l}_{1T}^{\perp}}$$

$$F_{LL}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp} \cdot \hat{l}_{1T}^{h}}$$

$$F_{LL}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp} \cdot \hat{l}_{1T}^{\perp}}$$

Structure functions of σ_{LT}

$$F_{LT}^{\cos(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{g_{1L}\cdot\hat{t}_{1L}^{h}} + \frac{P_T}{M_b} F_{b1}^{g_{1L}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{g_{1L}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-\phi_h)} = \left(\begin{aligned} &\frac{P_T}{M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} \\ &+ \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T}{mM_a M_b} F_{ab4}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T}{M_a M_b^2} F_{abb5}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} \end{aligned} \right)$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \left(\begin{aligned} &\frac{q_T}{M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} - \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} \\ &+ \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T}{M_a M_b^2} F_{abb6}^{h_{1L}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} \end{aligned} \right)$$

Structure functions of σ_{TU}

$$F_{TU}^{\sin(\phi_a-\phi_h)} = -\frac{P_T}{M_a}F_{a1}^{f_{1T}^{\perp}\cdot\hat{u}_1} - \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}} - \frac{P_T}{mM_aM_b}F_{ab4}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a)} = -\frac{q_T}{M_a}F_{a2}^{f_{1T}^{\perp}\cdot\hat{u}_1} + \frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a-\phi_h)} = -\frac{P_T}{M_b}F_{b1}^{h_1\cdot\hat{u}_{1T}^{\perp}} - \frac{P_T}{m}F_0^{h_1\cdot\hat{u}_{1T}^{h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = -\frac{q_T F_{b2}^{h_1\cdot\hat{u}_{1T}^{\perp}}}{M_b}$$

$$F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)} = -\frac{P_T^3}{2mM_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{u}_{1T}^{h}} - \frac{P_T^3}{2M_a^2M_b}F_{aab1}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-\phi_h)} = -\frac{P_T q_T^2}{2mM_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{h}} - \frac{P_T q_T^2}{2M_a^2M_b}F_{aab4}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2M_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = -\frac{q_T^3}{2M_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

Structure functions of σ_{TL}

$$F_{TL}^{\cos(\phi_a-\phi_h)} = \frac{P_T}{M_a} F_{a1}^{g_{1T}\cdot\hat{l}_{1L}} - \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}} - \frac{P_T}{mM_a M_b} F_{ab4}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a)} = \frac{q_T}{M_a} F_{a2}^{g_{1T}\cdot\hat{l}_{1L}} + \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)} = \frac{P_T^3}{2mM_a^2} F_{aa1}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{h}} + \frac{P_T^3}{2M_a^2 M_b} F_{aab1}^{h_{1T}^{p}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)} = \frac{P_T^2 q_T}{2M_a^2 M_b} F_{aab2}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \frac{q_T^3}{2M_a^2 M_b} F_{aab3}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi-\phi_a-\phi_h)} = \frac{P_T}{M_b} F_{b1}^{h_1\cdot\hat{l}_{1T}^{\perp}} + \frac{P_T}{m} F_0^{h_1\cdot\hat{l}_{1T}^{h}}$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \frac{q_T}{M_b} F_{b2}^{h_1\cdot\hat{l}_{1T}^{\perp}}$$

Structure functions σ_{TT}

$$F_{TT}^{\cos(\phi_a-\phi_b)} = \left(\begin{array}{l} -\frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\Delta M_T^h} - \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\Delta M_T^p} \\ -\frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M_T^p} - \frac{1}{M_aM_b}F_{ab4}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{1}{M_aM_b}F_{ab4}^{g_{1T}\cdot\Delta M_T^p} \end{array} \right)$$

$$F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\Delta M_T^h} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\Delta M_T^h} + \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot M_T^{\perp}} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot M_T^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\Delta M_T^h}, \quad F_{TT}^{\cos(\phi_a+\phi_b)} = \frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot M_T^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\Delta M_T^p}$$

$$F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\Delta M_T^p} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot M_T^{\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp h}}, \quad F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2m^2}F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)} = \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)} = \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = F_0^{h_1\cdot\Delta_T\hat{M}_T} + \frac{P_T^2}{2m^2}F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{1}{M_b^2}F_{bb3}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)} = \frac{P_T^2}{2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2}{2M_a^2M_b^2}F_{aabb5}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}} - \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp h}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \frac{q_T^2}{2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}} + \frac{q_T^2}{2M_a^2M_b^2}F_{aabb6}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$