The role of positivity in QCD global analysis

Nobuo Sato

In collaboration with J. Collins, T. Rogers and the JAM collaboration (W. Melnitchouk, C. Coccuza, Y. Zhou, A. Metz)

Transversity (May 2022)

Jefferson Lab

Can $\overline{\text{MS}}$ parton distributions be negative?

Alessandro Candido, Stefano Forte & Felix Hekhorn

Journal of High Energy Physics **2020**, Article number: 129 (2020) | Cite this article

It is common lore that Parton Distribution Functions (PDFs) in the $\overline{\text{MS}}$ factorization scheme can become negative beyond leading order due to the collinear subtraction which is needed in order to define partonic cross sections. We show that this is in fact not the case and next-to-leading order (NLO) $\overline{\text{MS}}$ PDFs are actually positive in the perturbative regime. In order to prove this, we modify the subtraction prescription, and perform the collinear subtraction in such a way that partonic cross sections remain positive. This defines a factorization scheme in which PDFs are positive. We then show that positivity of the PDFs is preserved when transforming from this scheme to $\overline{\text{MS}}$, provided only the strong coupling is in the perturbative regime, such that the NLO scheme change is smaller than the LO term.

Positivity and renormalization of parton densities

John Collins, Ted C. Rogers, and Nobuo Sato
Phys. Rev. D **105**, 076010 – Published 14 April 2022

Track A

- Start from an operator definition for pdfs
  - Only UV divergence
  - Use renormalization

$$f_{j/H}^{\mathrm{bare,A}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-} \langle p|\, \bar{\psi}_{j,0}(0, w^-, \mathbf{0}_{\mathrm{T}}) \frac{\gamma^+}{2} W[0, w^-] \psi_{j,0}(0, 0, \mathbf{0}_{\mathrm{T}})\, |p\rangle \,.$$

$$f^{\mathrm{renorm,A}}(\xi) \equiv Z^A \otimes f^{\mathrm{bare,A}}$$

- Factorization
  - Region analysis (Libby Sterman)
  - Higher order corrections via nested subtractions

$$F(Q, x_{\mathrm{bj}}) = \mathcal{C}^A \otimes f^{\mathrm{renorm,A}} + \mathrm{error}$$
$$= \sum_j \int_{x_{\mathrm{bj}}}^1 \mathrm{d}\xi\, \mathcal{C}_j^A(x_{\mathrm{bj}}/\xi, \alpha_s(Q))\, f_j^{\mathrm{renorm,A}}(Q, \xi) + \mathrm{error}$$

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“bare factorization”
Curci, Furmanski, Petronzio (1980)

Track B

- Assert(?) a factorization in terms of bare pdf.

$$F(Q, x_{\rm bj}) = F^{\rm partonic} \otimes \boxed{f^{\rm bare,B}}$$

- Isolate collinear divergences from partonic structure function.

$$F^{\rm partonic} = \mathcal{C}^B \otimes Z^B$$

- Reabsorb singularities inside bare pdf.

$$\begin{aligned} F(Q, x_{\rm bj}) &= \left(\mathcal{C}^B \otimes Z^B\right) \otimes f^{\rm bare,\ b} \\ &= \mathcal{C}^B \otimes \left(Z^B \otimes f^{\rm bare,B}\right) \\ &= \mathcal{C}^B \otimes f^{\rm renorm,B}\,, \end{aligned}$$

Positivity argument

$$
\begin{aligned}
F(Q, x_{\mathrm{bj}}) &= (\mathcal{C}^B \otimes Z^B) \otimes f^{\text{bare, b}} \\
&= \mathcal{C}^B \otimes (Z^B \otimes f^{\text{bare,B}}) \\
&= \mathcal{C}^B \otimes f^{\text{renorm,B}} ,
\end{aligned}
$$

If Z^B **is positive**

If $f^{\text{bare,B}}$ **is positive**

then $f^{\text{renorm,B}}$ **is positive**

Can $\overline{\text{MS}}$ parton distributions be negative?

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$$f_i(\xi) = \frac{1}{4\pi}\int dy^- e^{-i\xi P^+ y^-}\langle P|\bar{\psi}_i(0, y^-, \vec{0}_T)\gamma^+ \mathcal{P}\exp\left[ig_s\int_0^{y^-} d\bar{y}^- A_a^+(0, \bar{y}^-, \vec{0}_T)\frac{1}{2}\lambda_a\right]\psi_i(0)|P\rangle, \tag{2}$$

where $\mathcal{P}$ denotes path-ordering; P is the four-momentum of the parent hadron in light-cone components and g_s is the strong coupling, with analogous expressions for antiquarks and gluons [8]. It can be shown (see e.g. Sect. 6.7 of Ref. [10]) that the expression Eq. (2) is a number density, and as such before subtraction of divergences it is positive.

⋮

positivity of the partonic cross section at the regularized level. If all contributions which are factored away from the partonic cross section and into the PDF remain positive, then the latter also stays positive.

Track B: “bare factorization”

$$F(Q, x_{\rm bj}) = F^{\rm partonic} \otimes \boxed{f^{\rm bare,B}}$$

$$I^{\rm ren.\ BPHZ'} = \int \frac{d^{2-2\epsilon}\boldsymbol{k}_{\rm T}}{(2\pi)^{2-2\epsilon}} \left(\frac{1}{k_{\rm T}^2 + C(x)} - \frac{1}{k_{\rm T}^2} \right)$$

$$- \int_{k_{\rm T}<\Lambda} \frac{d^{2-2\epsilon}\boldsymbol{k}_{\rm T}}{(2\pi)^{2-2\epsilon}} \frac{1}{k_{\rm T}^2} = - \int_0^{\Lambda^2} \frac{dk_{\rm T}^2 (k_{\rm T}^2)^{-\epsilon}}{\Gamma(1-\epsilon)\,(4\pi)^{1-\epsilon}} \frac{1}{k_{\rm T}^2}$$

$$= \frac{\Lambda^{-2\epsilon}}{\epsilon\,\Gamma(1-\epsilon)\,(4\pi)^{1-\epsilon}}$$

Given that $\epsilon < 0$ to regulate the collinear divergence, the collinear divergence in the counterterm is actually negative. Therefore the supposed positivity of the “bare” track-B pdfs is actually violated.

Track A

Object	Hadron structure function, F	Partonic hard part $\mathcal{C}^A$	Bare hadronic pdf $f^{\rm bare,A}$	Renormalized hadronic pdf $f^{\rm renorm,A}$
Ultraviolet behavior	*Standard Lagrangian counterterms* $\Longrightarrow$ UV finite	*Lagrangian counterterms & operator counterterms* $\Longrightarrow$ UV finite	*Bare pdf, so no counterterms* $\Longrightarrow$ UV divergent	*Lagrangian counterterms & operator counterterms* $\Longrightarrow$ UV finite
Collinear behavior	*Non-massless, finite range theory* $\Longrightarrow$ Collinear finite	*Double counting subtractions* $\Longrightarrow$ Collinear finite	*Non-massless, finite range theory* $\Longrightarrow$ Collinear finite	*Non-massless, finite range theory* $\Longrightarrow$ Collinear finite

Track B

Object	Partonic structure function $F^{\rm partonic}$	Partonic hard part $\mathcal{C}^B$	Bare hadronic pdf $f^{\rm bare,B}$
Ultraviolet behavior	*Standard Lagrangian counterterms* $\Longrightarrow$ UV finite	*Lagrangian counterterms & operator counterterms* $\Longrightarrow$ UV finite	*Track B bare pdf must be UV finite to be consistent with hadronic structure function* $\Longrightarrow$ UV finite
Collinear behavior	*Massless partons* $\Longrightarrow$ Collinear divergent	*Collinear divergence absorbed into pdf redefinition* $\Longrightarrow$ Collinear finite	*Track B bare pdf must be collinear divergent to cancel collinear divergence in partonic structure function* $\Longrightarrow$ Collinear divergent

Positivity argument

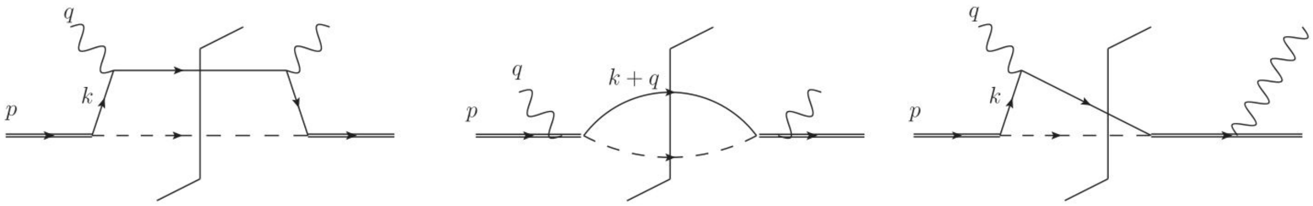
$$
\begin{aligned}
F(Q, x_{\mathrm{bj}}) &= \left(\mathcal{C}^B \otimes Z^B\right) \otimes f^{\text{bare, b}} \\
&= \mathcal{C}^B \otimes \left(Z^B \otimes f^{\text{bare,B}}\right) \\
&= \mathcal{C}^B \otimes f^{\text{renorm,B}} \, ,
\end{aligned}
$$

If Z^B **is positive**

If $f^{\text{bare,B}}$ **is positive** ✗

then $f^{\text{renorm,B}}$ **is positive** ✗

<u>An example:</u> factorization in a Yukawa model

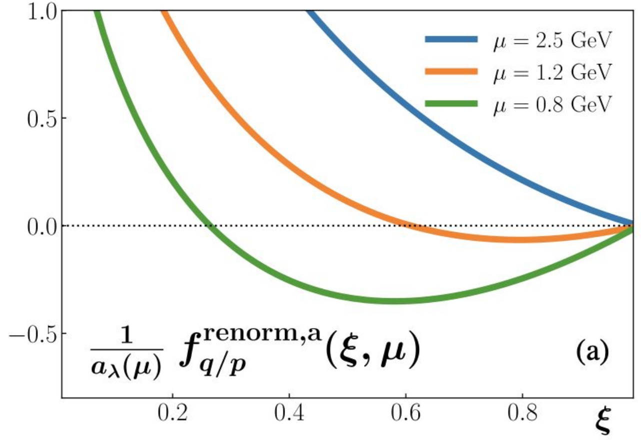


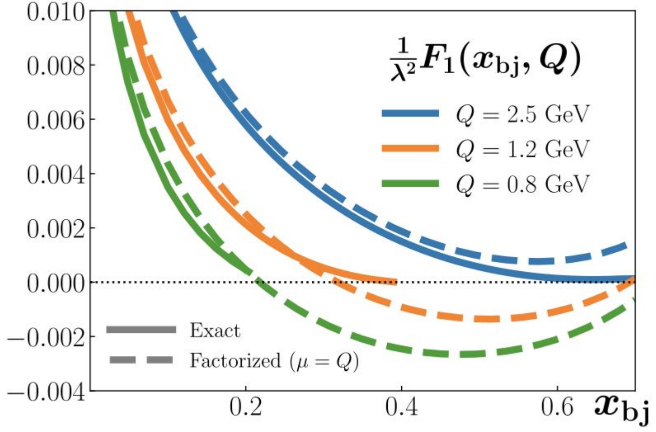
(a) (b) (c)

$$F_1(x_{\mathrm{bj}}, Q) \overset{x_{\mathrm{bj}}<1}{=} \sum_f \int_{x_{\mathrm{bj}}}^1 \frac{\mathrm{d}\xi}{\xi}\frac{1}{2}\left\{\delta\left(1-\frac{x_{\mathrm{bj}}}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x_{\mathrm{bj}}}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x_{\mathrm{bj}}}{\xi}\right)^2 - 3\,\frac{x_{\mathrm{bj}}}{\xi} + \frac{3}{2}}{\left(1-\frac{x_{\mathrm{bj}}}{\xi}\right)^2} - \ln\frac{4x_{\mathrm{bj}}\mu^2}{Q^2(\xi - x_{\mathrm{bj}})}\right]\delta_{pf}\right\}$$

$$\times \underbrace{\left\{\delta(1-\xi)\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)} + \ln\left(\frac{\mu^2}{\Delta(\xi)}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} + O(a_\lambda(\mu)^2) + O(m^2/Q^2).$$

Note that mu dependence cancels



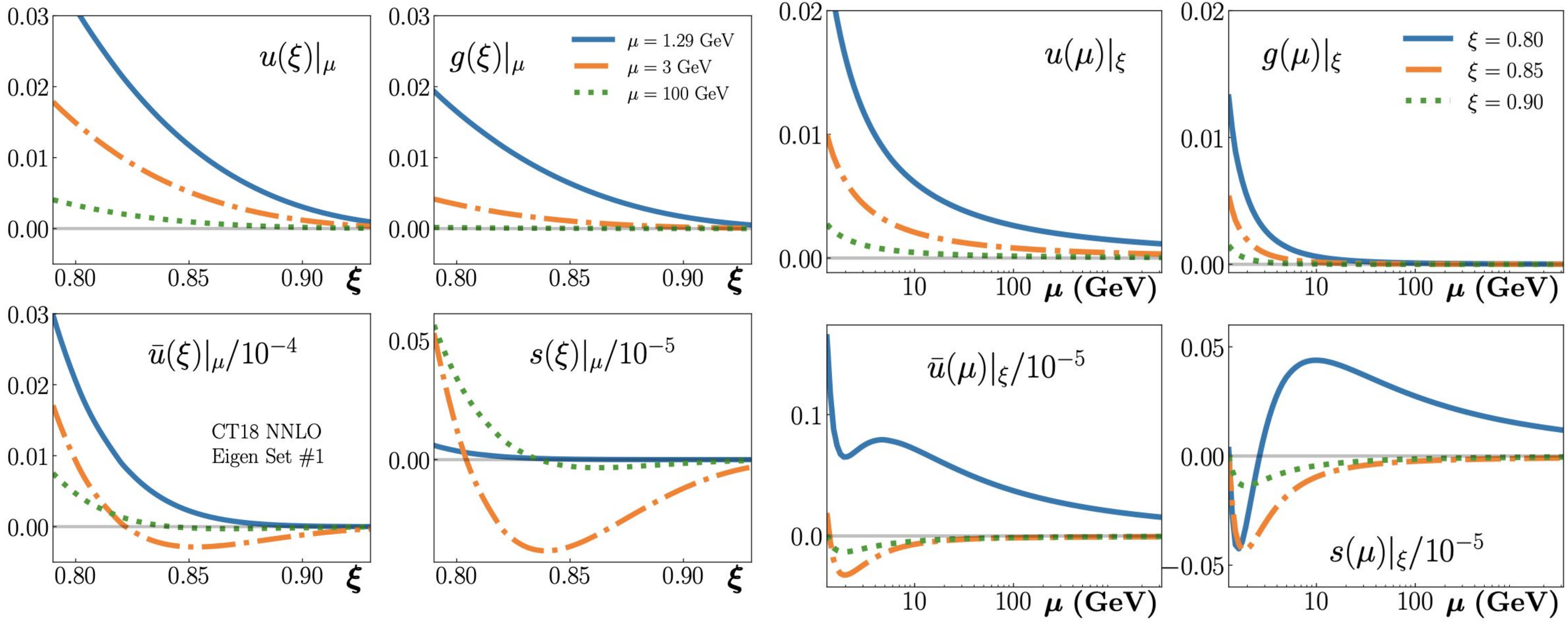


Observations

- F1 is mu independent
- This means once can choose any scale for the pdf and it will give the same F1
- Even a negative pdf will give a good approximation to F1
- Factorization in a Yukawa theory does not excludes negative MSbar pdfs

Ok, so is this issue a purely academic point or does it have practical consequences?

Some examples of negative pdfs



Edinburgh 2021/12
Nikhef-2021-013
TIF-UNIMI-2021-11

The Path to Proton Structure at One-Percent Accuracy

The NNPDF Collaboration:

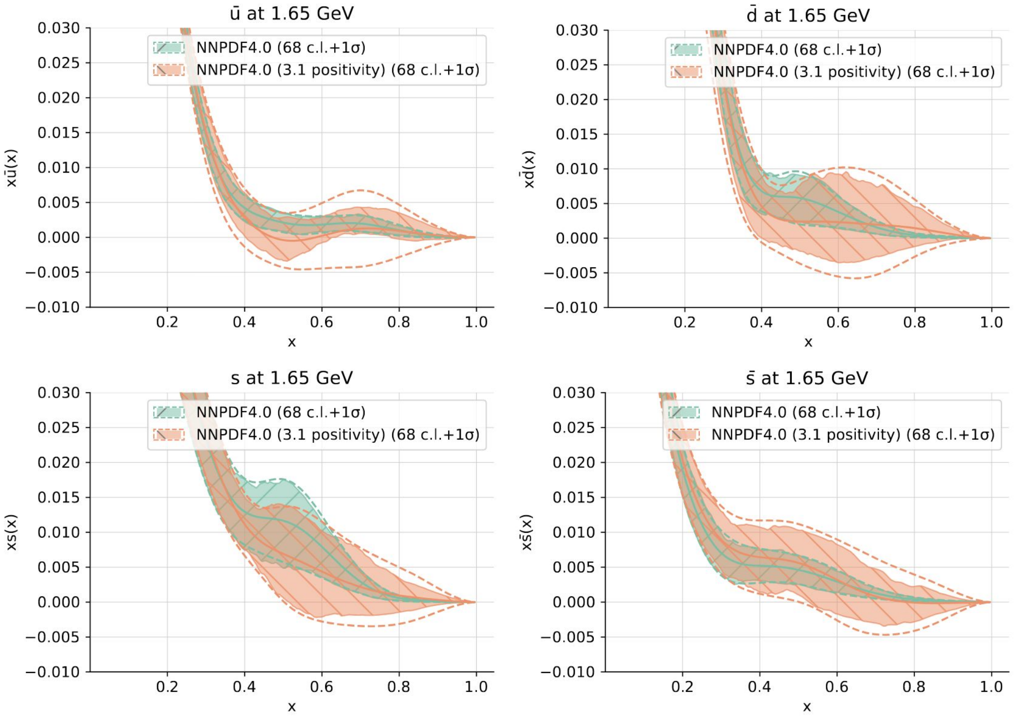
Richard D. Ball,[1] Stefano Carrazza,[2] Juan Cruz-Martinez,[2] Luigi Del Debbio,[1] Stefano Forte,[2] Tommaso Giani,[1,8] Shayan Iranipour,[3] Zahari Kassabov,[3] Jose I. Latorre,[4,5,6] Emanuele R. Nocera,[1,8] Rosalyn L. Pearson,[1] Juan Rojo,[7,8] Roy Stegeman,[2] Christopher Schwan,[2] Maria Ubiali,[3] Cameron Voisey,[9] and Michael Wilson[1]

It was recently shown in Ref. [21] that PDFs for individual quark flavors and the gluon in the $\overline{\text{MS}}$ factorization scheme are non-negative. We thus now also impose this positivity condition along with the constraint of positivity of physical cross-sections discussed above. Indeed, note that the positivity of $\overline{\text{MS}}$ PDFs is neither necessary nor sufficient in order to ensure cross-section positivity [21]: they are independent (though of course related) constraints that limit the space of acceptable PDFs.

Why? Because cross section predictions in the unmeasured region can become negative

NNPDF

The Path to Proton Structure at One-Percent Accuracy



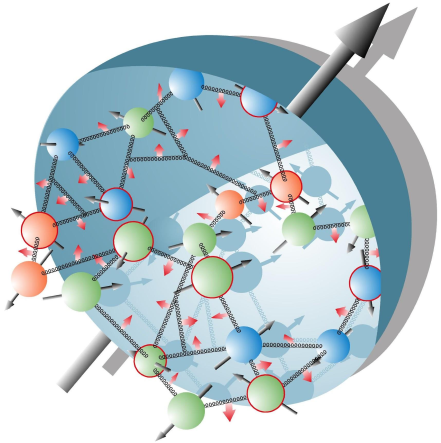
Pros

- Uncertainties on PDFs are smaller
- Cross section predictions at extreme kinematics are positive

Cons

- How do we interpret negative cross sections? Negative PDFs or failure of factorization/pQCD?

Ok, so are there more consequences?
-> what about the "dark spin"?



$$|\Delta f_i(x, Q^2)| \leq f_i(x, Q^2)$$

How well do we know the gluon polarization in the proton?

Y. Zhou, N. Sato, and W. Melnitchouk (Jefferson Lab Angular Momentum (JAM) Collaboration)

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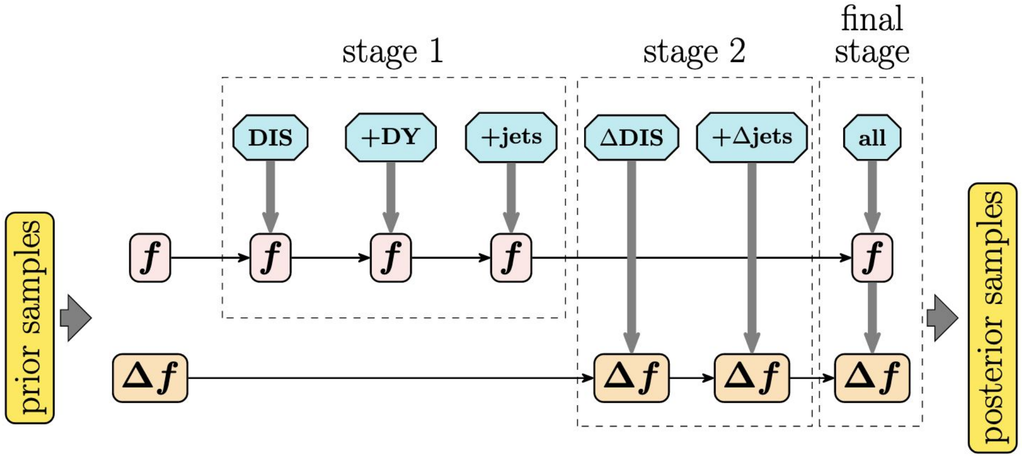
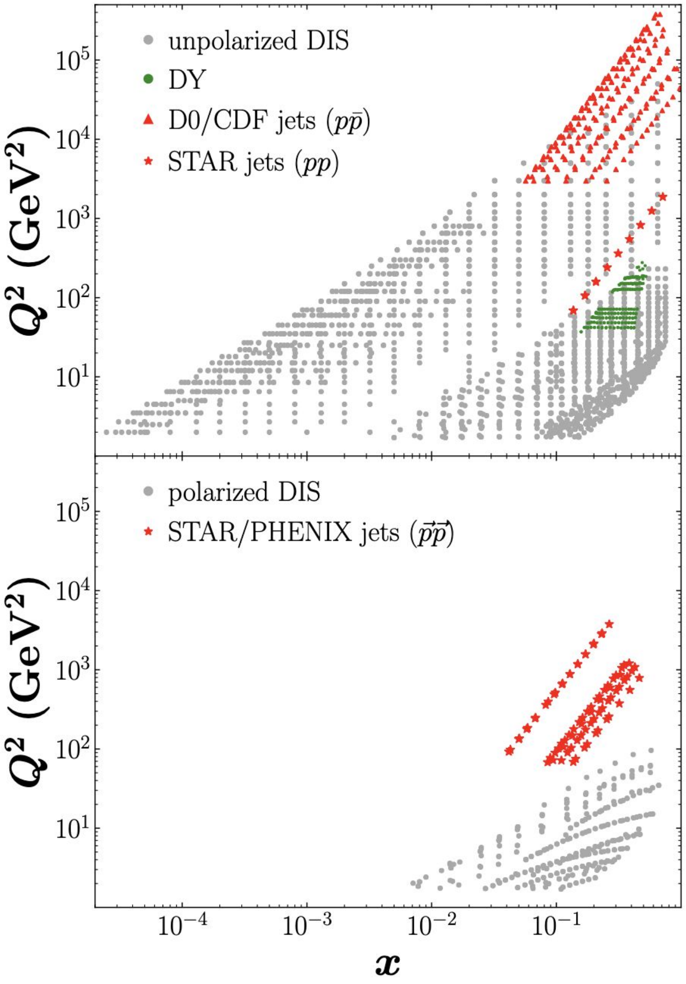




Theory/external constraints

SU(2)

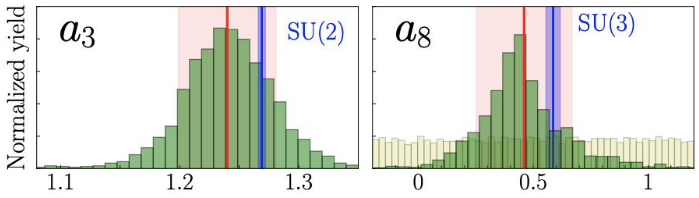
$$\int_0^1 \mathrm{d}x \left[\Delta u^+ - \Delta d^+\right] = g_A$$

SU(3)

$$\int_0^1 \mathrm{d}x \left[\Delta u^+ + \Delta d^+ - 2\Delta s^+\right] = a_8,$$

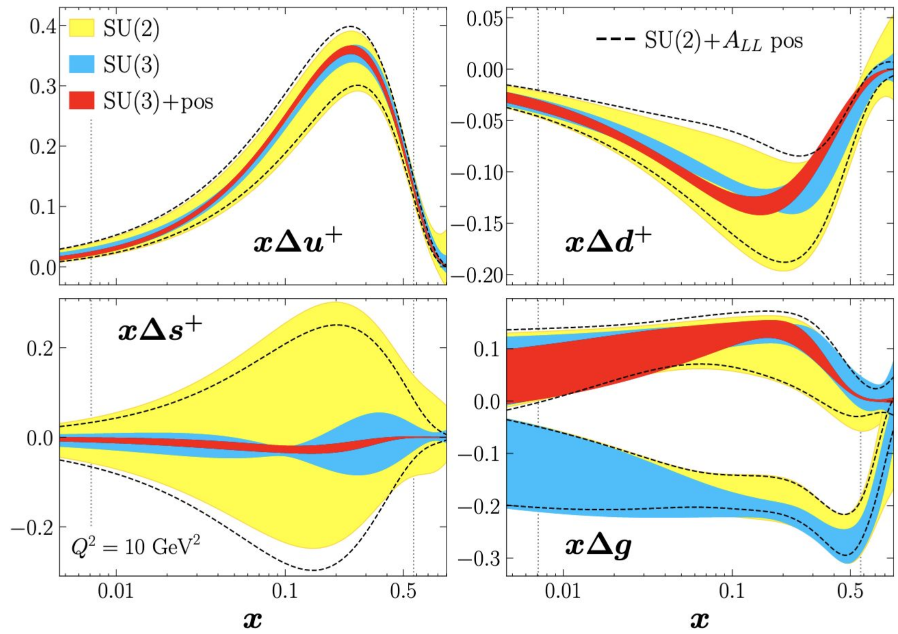
Hyperon beta-decays

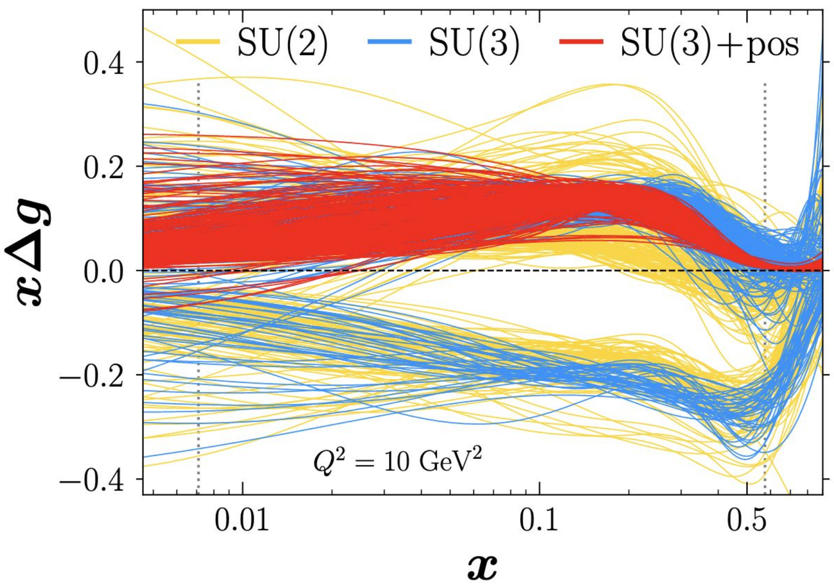
Constraints from
SIDIS with kaons
JAM17

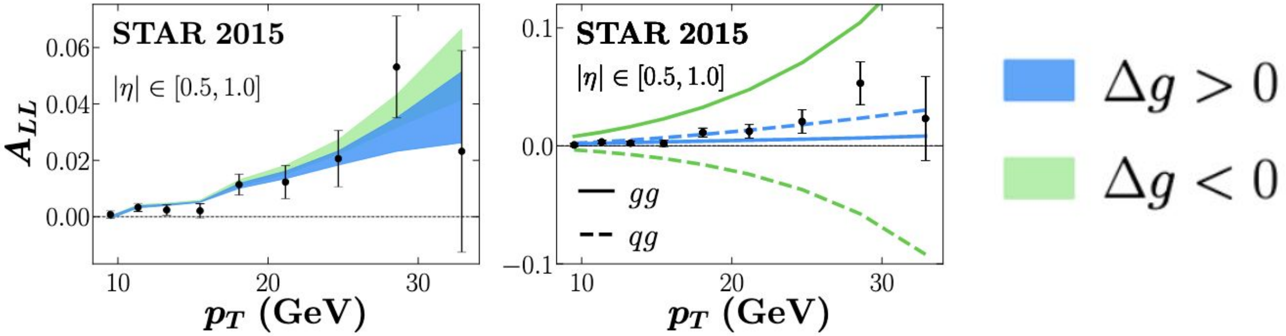


Positivity

$$\left|\Delta q(x, Q^2)\right| \leqslant q(x, Q^2)$$

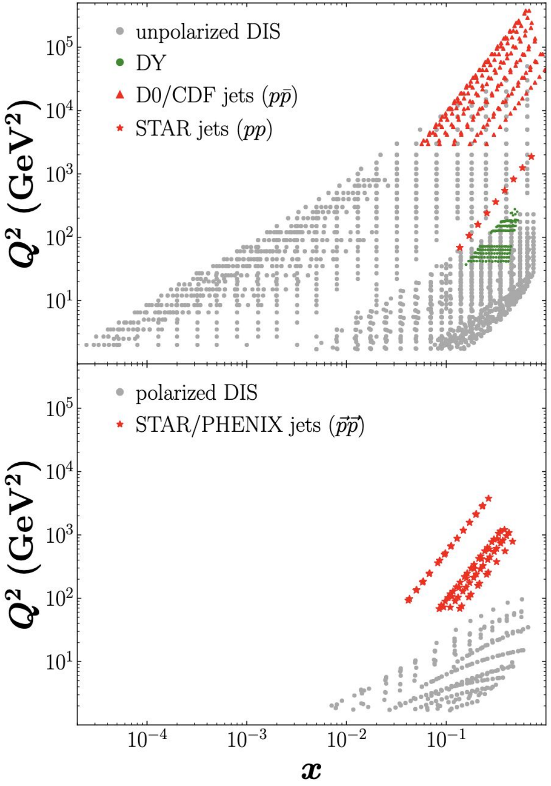


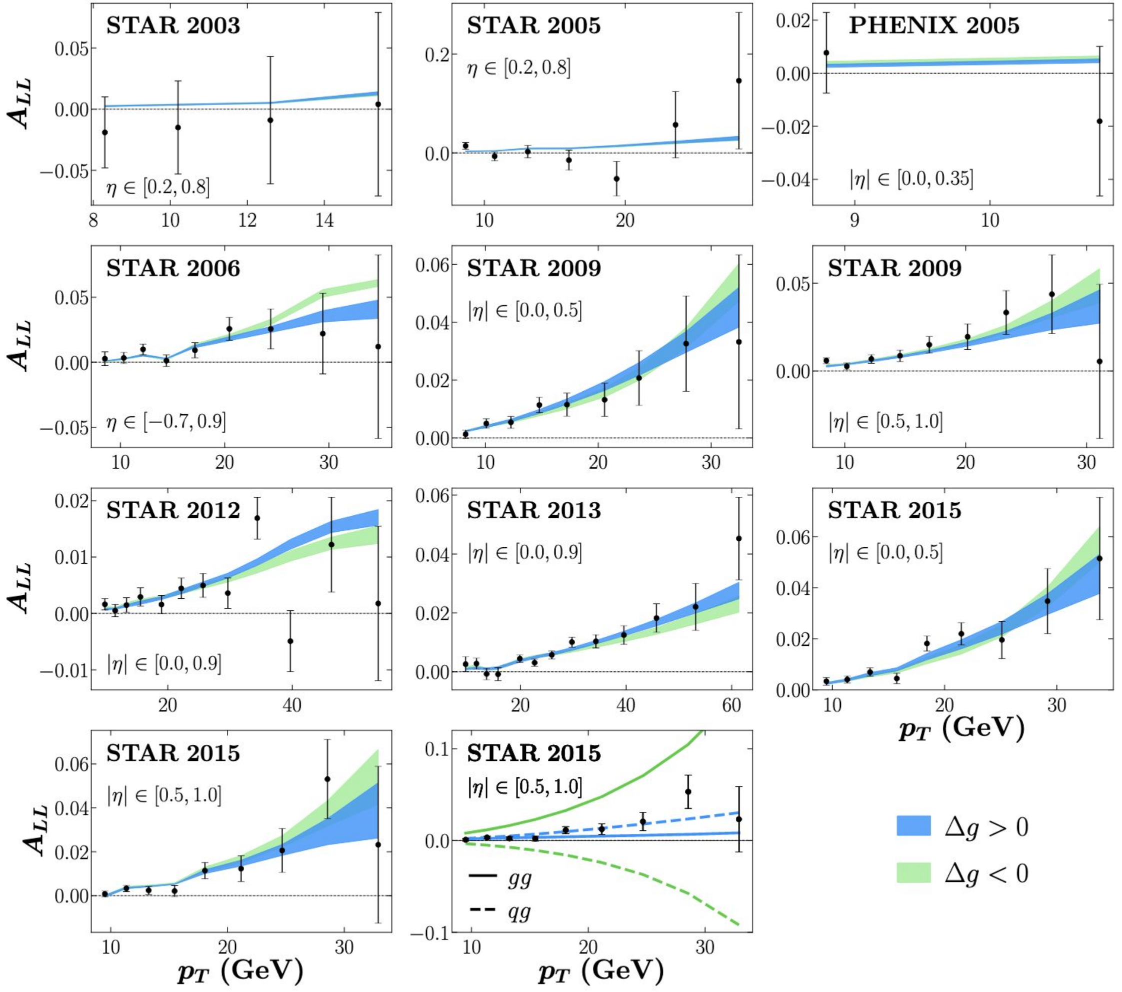


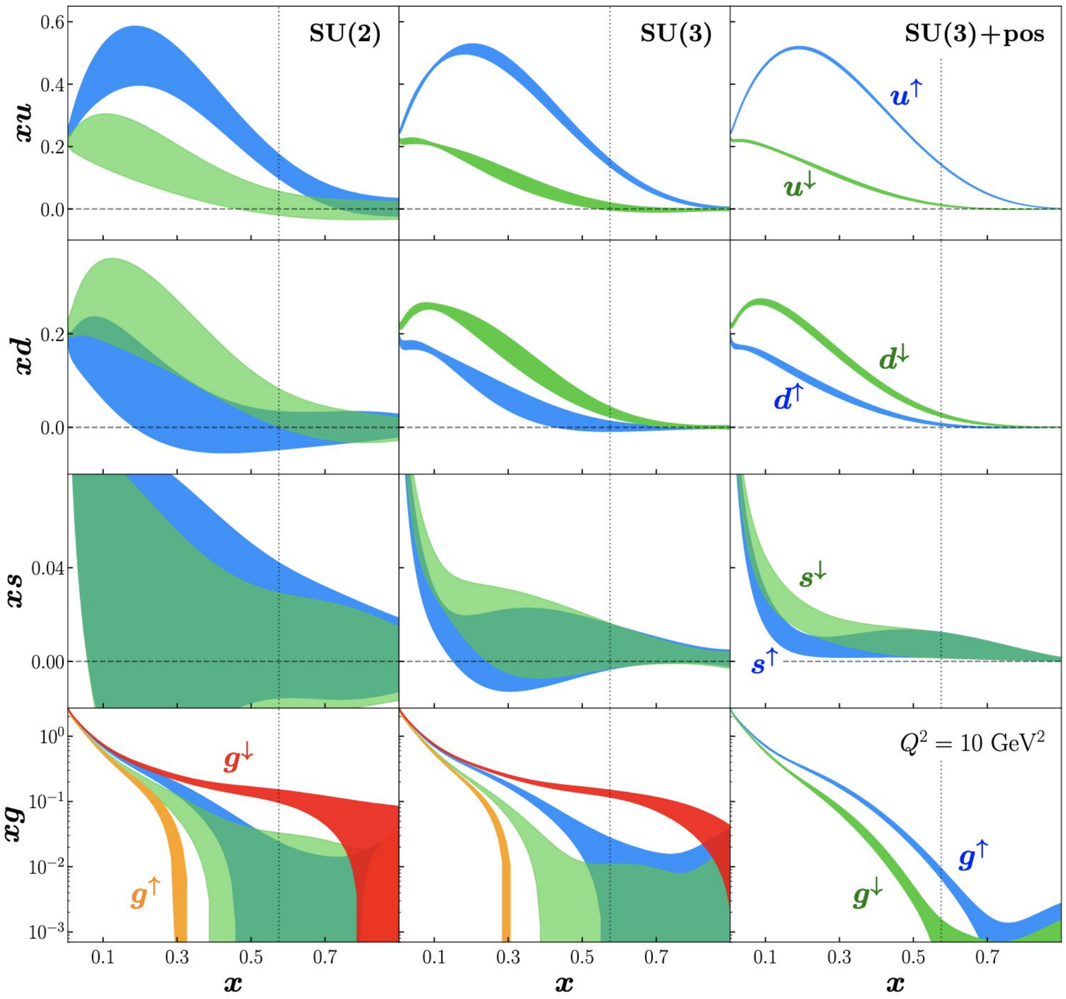


Polarized jet data cannot discriminate between positive & negative solutions

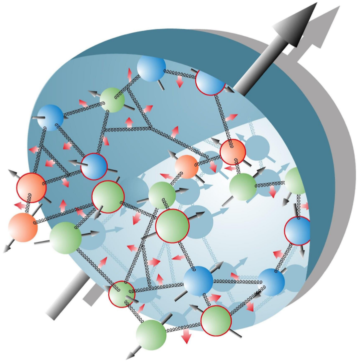
Polarized Jets







Helicity basis

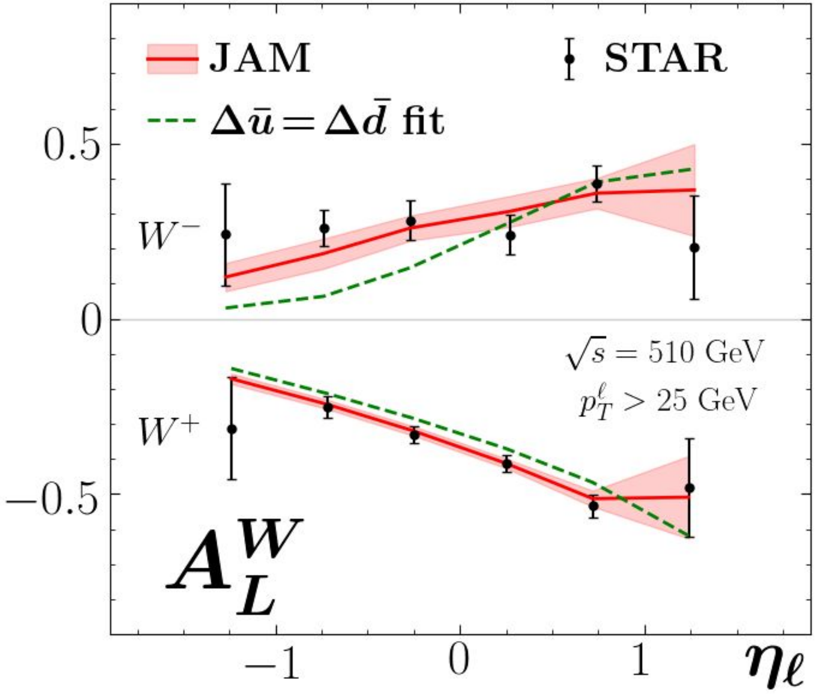


Polarized Antimatter in the Proton from Global QCD Analysis

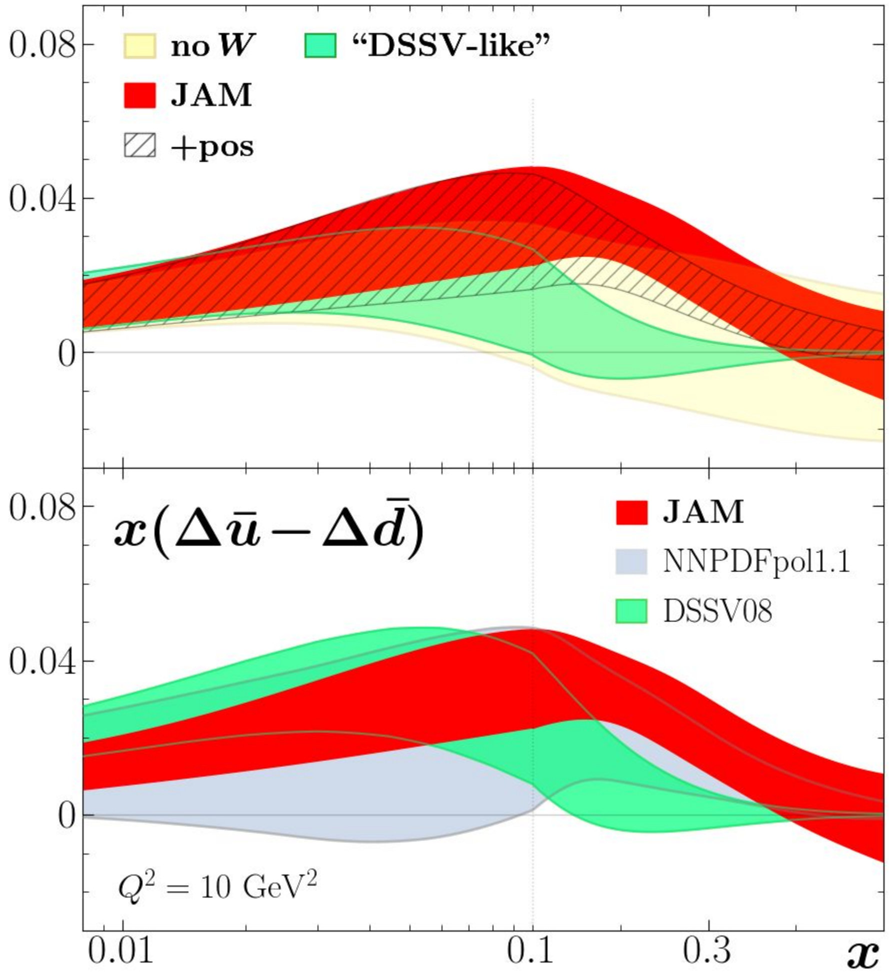
Jefferson Lab Angular Momentum (JAM) Collaboration • C. Cocuzza (Temple U.) et al. (Feb 7, 2022)

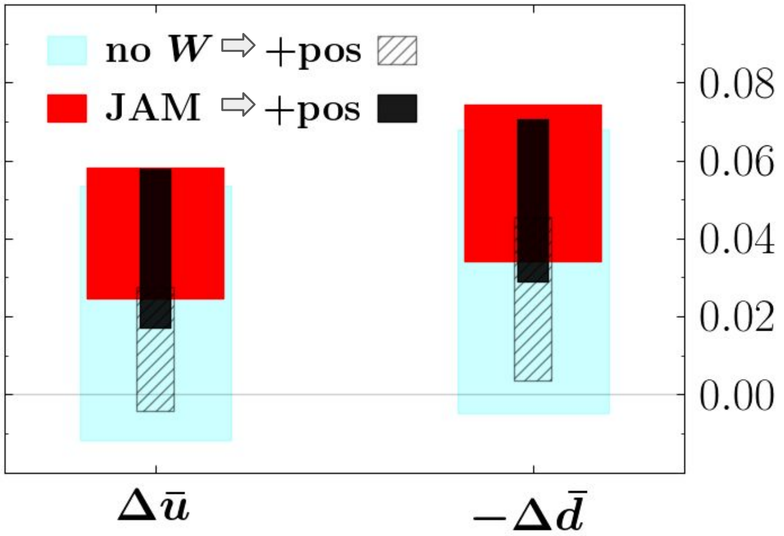
e-Print: 2202.03372 [hep-ph]

(updated version in progress)



process	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
polarized		
inclusive DIS	365	0.93
SIDIS (π^+, π^-)	64	0.93
SIDIS (K^+, K^-)	57	0.36
SIDIS (h^+, h^-)	110	0.93
inclusive jets	83	0.81
STAR $W^\pm$	12	0.53
PHENIX $W^\pm/Z$	6	0.63
total	**697**	**0.86**
unpolarized		
inclusive DIS	3908	1.11
SIDIS (π^+, π^-)	498	0.88
SIDIS (K^+, K^-)	494	1.01
SIDIS (h^+, h^-)	498	0.52
inclusive jets	198	1.11
Drell-Yan	205	1.19
W/Z production	153	0.99
total	**5954**	**1.03**
SIA $(\pi^\pm)$	231	0.85
SIA $(K^\pm)$	213	0.49
SIA $(h^\pm)$	120	1.09
total	**7215**	**0.99**





- First ever universal analysis of pol. & upol. PDFs and fragmentation functions
- Consistent UQ for polarized antiquarks in the nucleon
- Clear evidence of an asymmetry, with opposite sign compared to unpolarized sea asymmetry

Summary/Outlook

- Track B based positivity of $\overline{\text{MS}}$ PDFs are not really supported on formal grounds

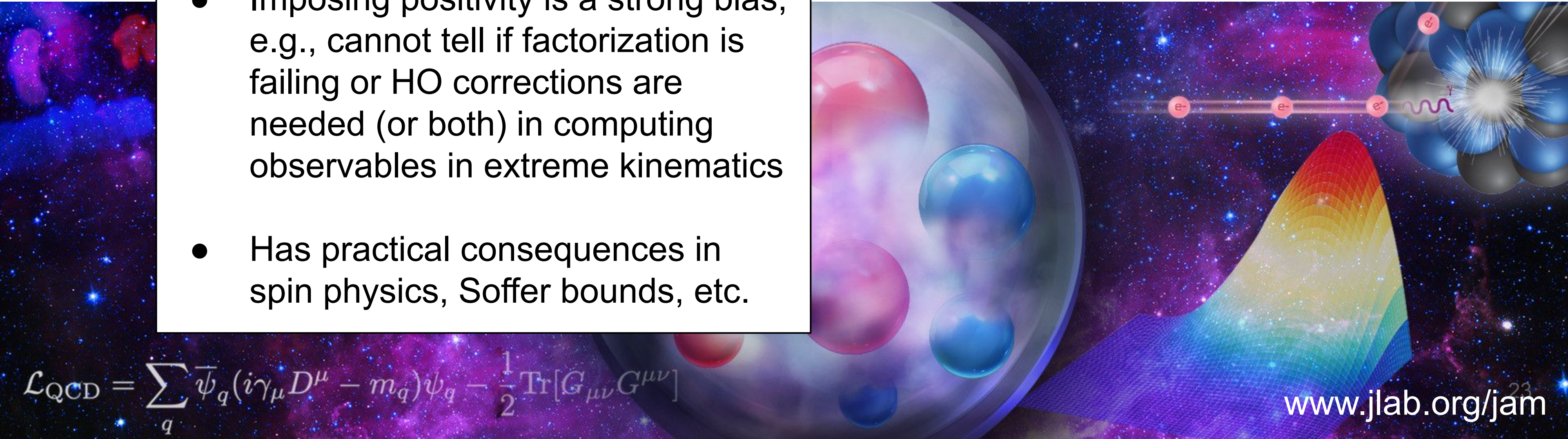
- Imposing positivity is a strong bias, e.g., cannot tell if factorization is failing or HO corrections are needed (or both) in computing observables in extreme kinematics

- Has practical consequences in spin physics, Soffer bounds, etc.

Special thanks to

C. Cocuzza

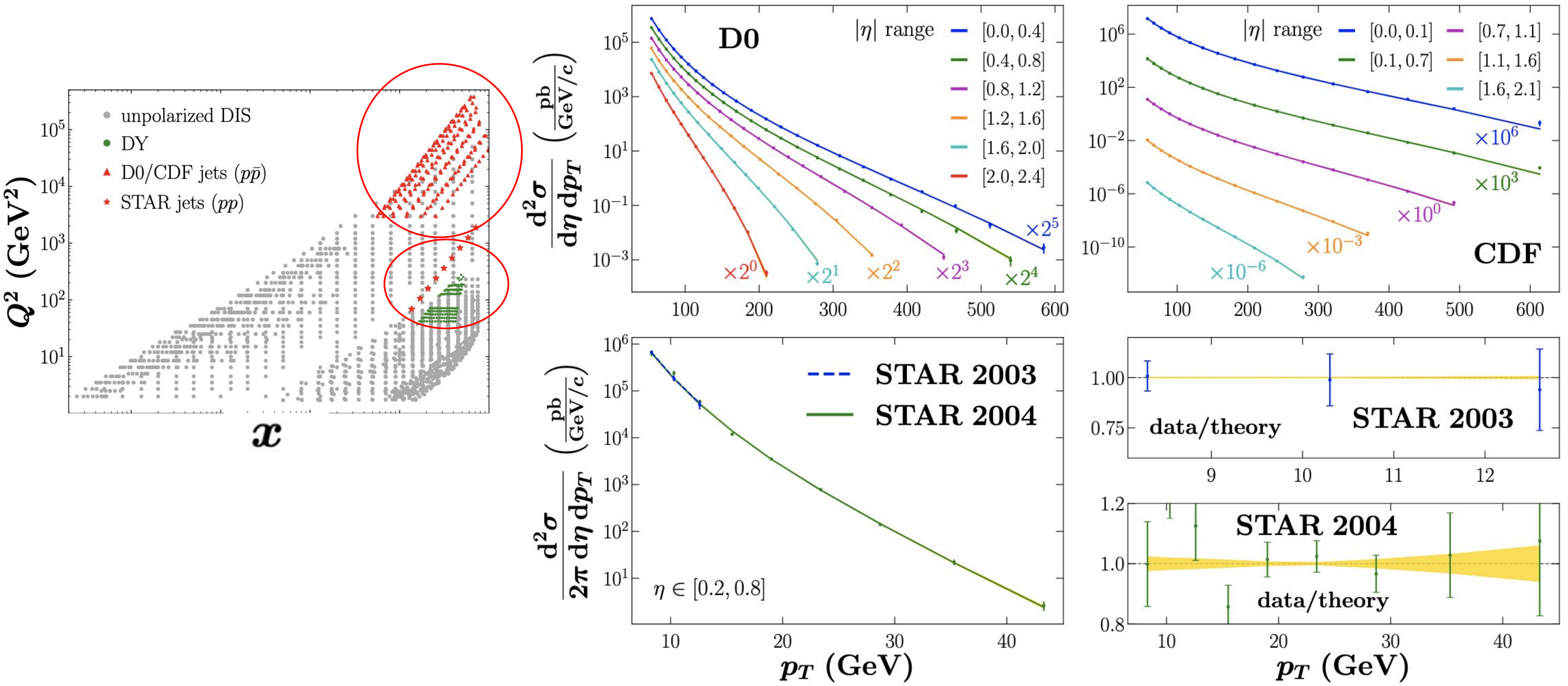
Y. Zhou



www.jlab.org/jam

Backup

Unpolarized Jets



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One of the arguments

$$\frac{1}{x}\underbrace{F_2^{\mathrm{NS}}(x,Q^2)}_{\text{If this is positive}} = \langle e_i^2\rangle \underbrace{\left[q^{\mathrm{NS}}\right]^{\mathrm{DIS}}}_{\text{...then this must be positive}}(x,Q^2)$$

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$$\underbrace{\left[q^{\mathrm{NS}}\right]^{\overline{\mathrm{MS}}}(\xi, Q^2)}_{\text{Is this positive?}} = \left[1 - \frac{\alpha_s}{2\pi}\Delta_q^{(1)\,\overline{\mathrm{MS}}} - \frac{\alpha_s}{2\pi}\bar{C}_q^{(1)\,\overline{\mathrm{MS}}} \otimes\right] \underbrace{\left[q^{\mathrm{NS}}\right]^{\mathrm{DIS}}(Q^2)}_{\text{If this is positive}} + O(\alpha_s^2)\,.$$

Yes, how about higher orders?

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$$\left[q^{\rm NS}\right]^{\overline{\rm MS}}(\xi,Q^2) = \frac{1}{1+\frac{\alpha_s}{2\pi}\Delta_q^{(1)\,\overline{\rm MS}}}\times$$
$$\left[1 - c_{\rm LL}\left[\frac{\ln(1-z)}{\left[1+c_{\rm LL}\ln^2(1-z)/2\right]^2}\frac{1}{1-z}\right]_+ \otimes\right]\left[q^{\rm NS}\right]^{\rm DIS}(Q^2) + {\rm NLL}(1-\xi)\,.$$

Yes, too! So pdfs in MSbar are positive!