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MW determination at hadron colliders

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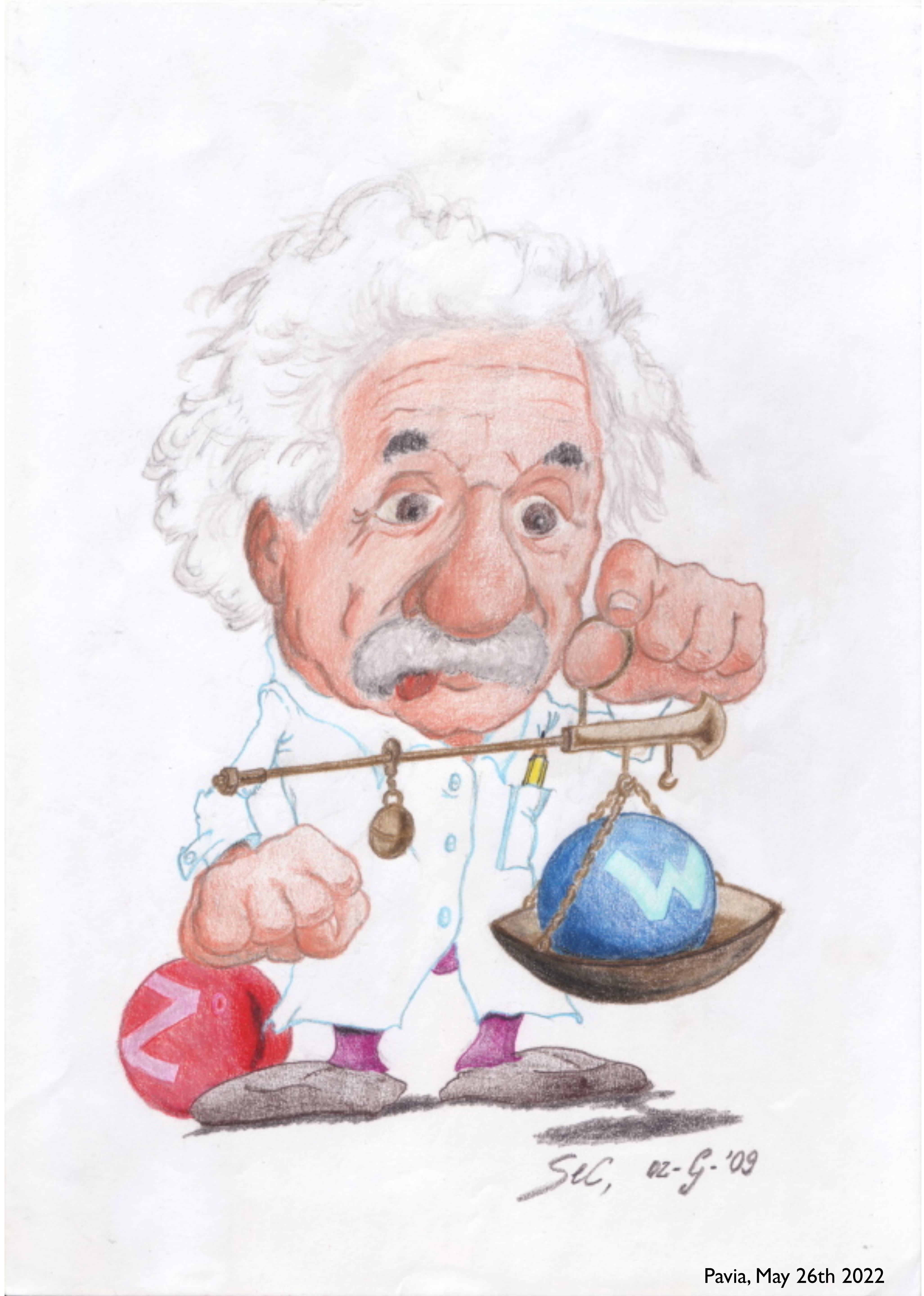


Outline

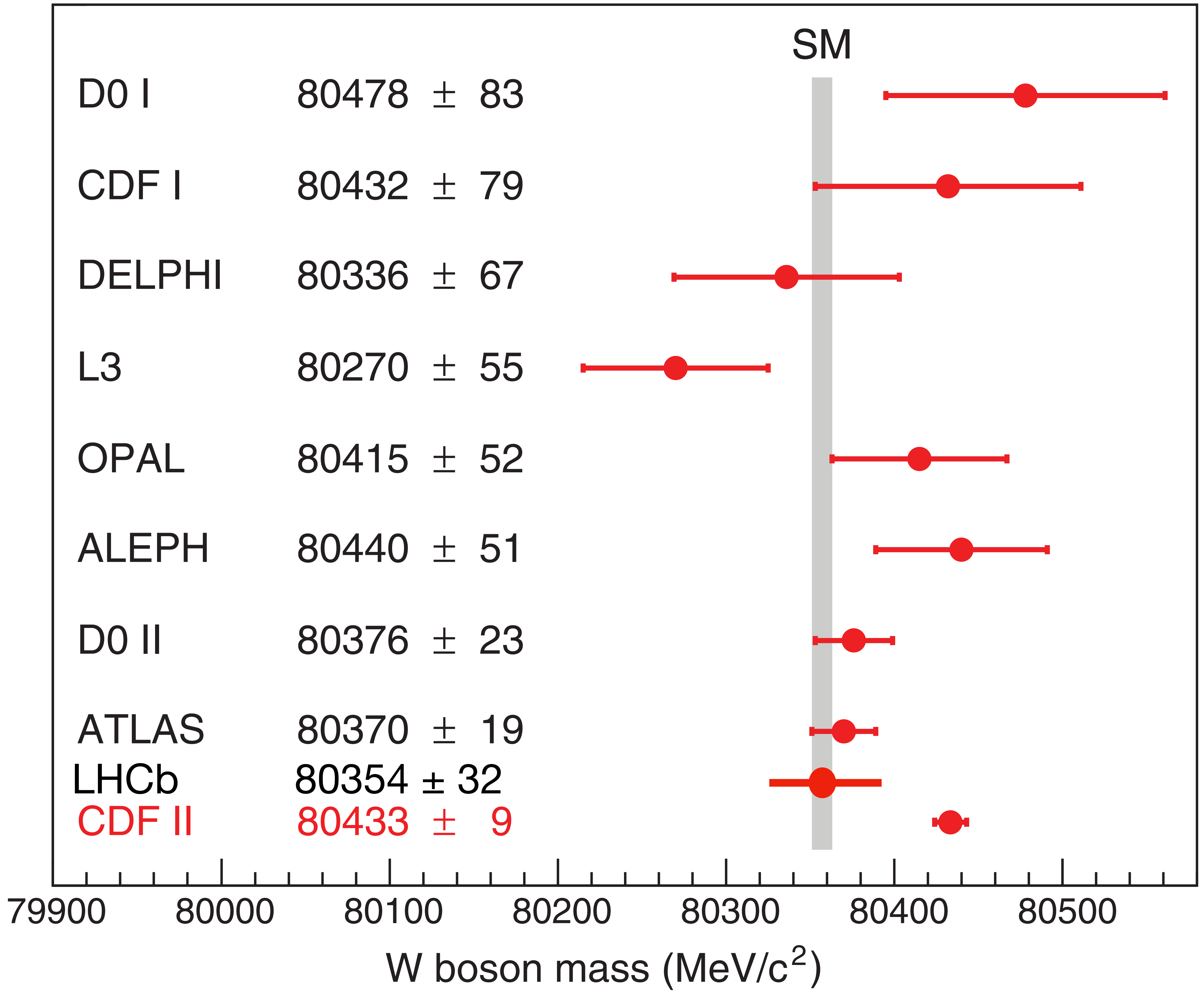
- precision tests of the Standard Model and searches for New Physics signals

- MW determinations at hadron colliders → can we properly estimate the theoretical systematic errors ?
  is the methodology robust for an unbiased determination of the central value ?

- the gauge boson transverse momentum distribution

- heavy-quark initiated processes and impact on the MW extraction

- collinear PDF uncertainties on MW



Precision tests of the SM

The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, m_Z, m_H)$ minimises the parametric uncertainty of the predictions

$$
\begin{aligned}
\alpha(0) &= 1/137.035999139(31) \\
G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\
m_H &= 125.09(24)\ \mathrm{GeV}/c^2
\end{aligned}
$$

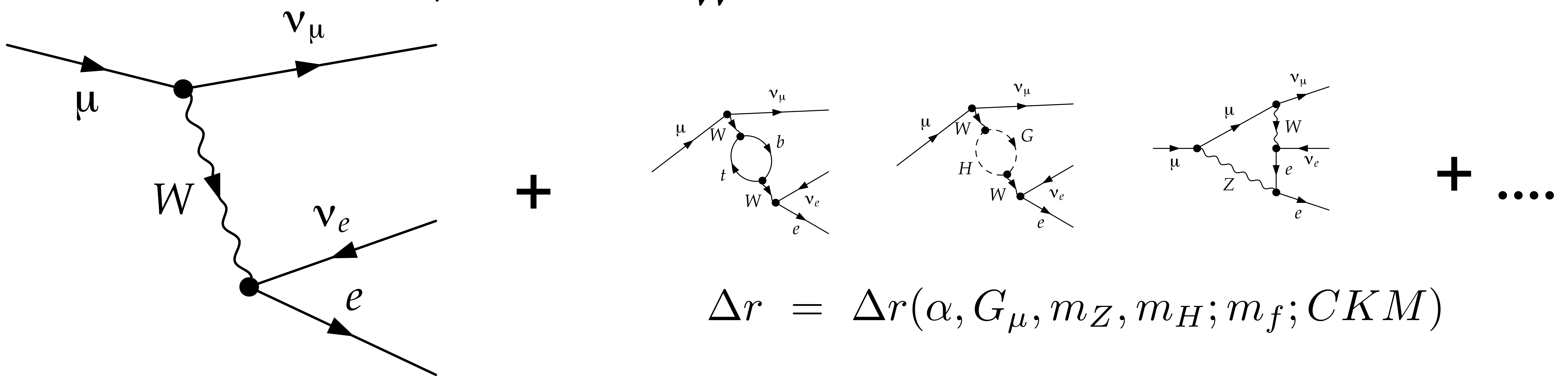
- with these inputs, m_W and the weak mixing angle are predictions of the SM, to be tested against the experimental data

The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$\rightarrow$ we can compute m_W

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r)$$



$$\Delta r \;=\; \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

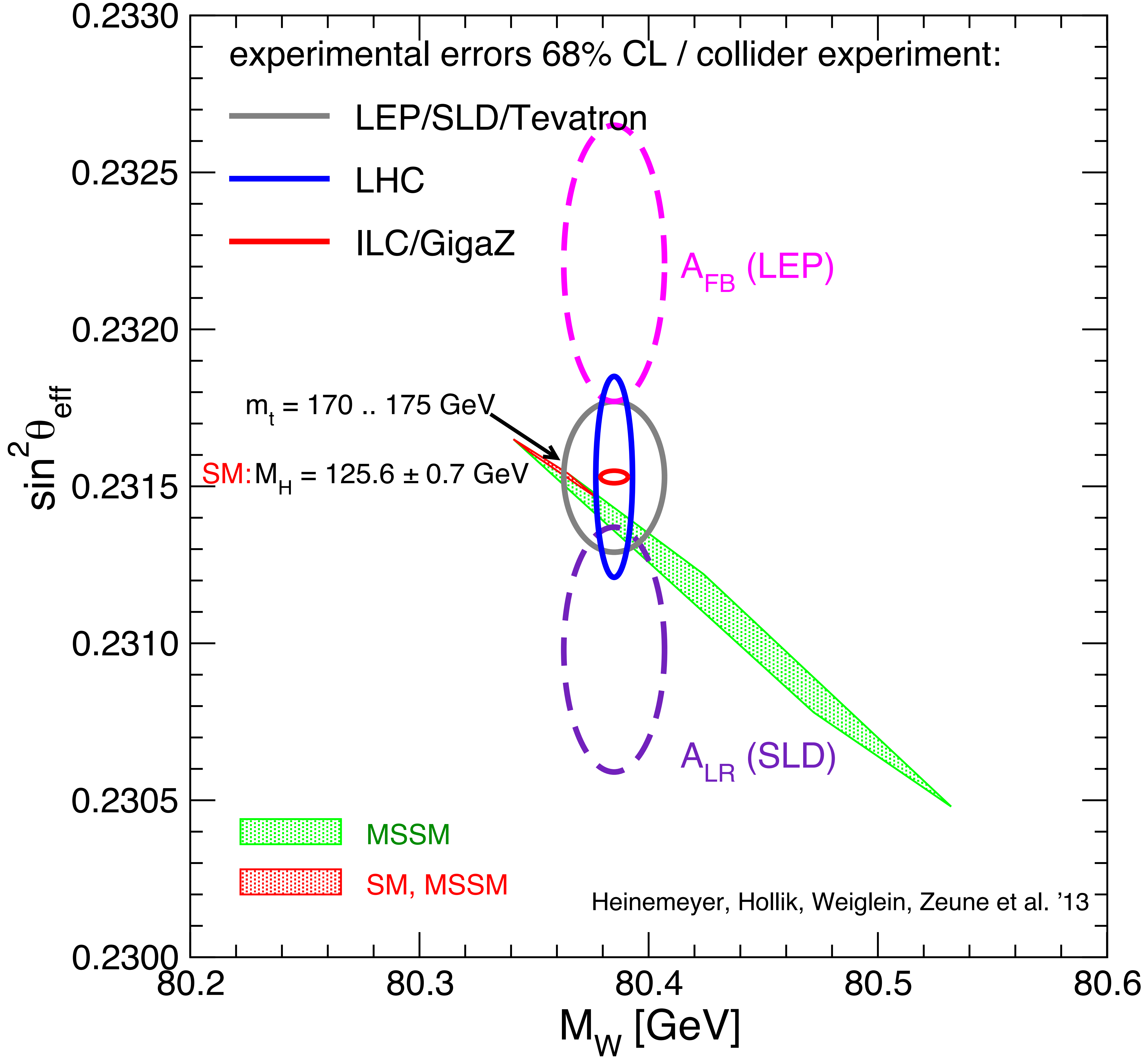
$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

$m_W = 80.357 \pm 0.009 \pm 0.003$ GeV

G.Degrassi, P.Gambino, P.Giardino, arXiv:1411.7040

Relevance of new high-precision measurement of EW parameters

Baak et al., arXiv:1310.6708, Snowmass 2013, EW WG



The precision measurement of m_W and $\sin^2\theta_{eff}$

with an error of 5 MeV and 0.000100 at a hadron collider

(0.7 MeV and 0.000004 at e+e- collider)

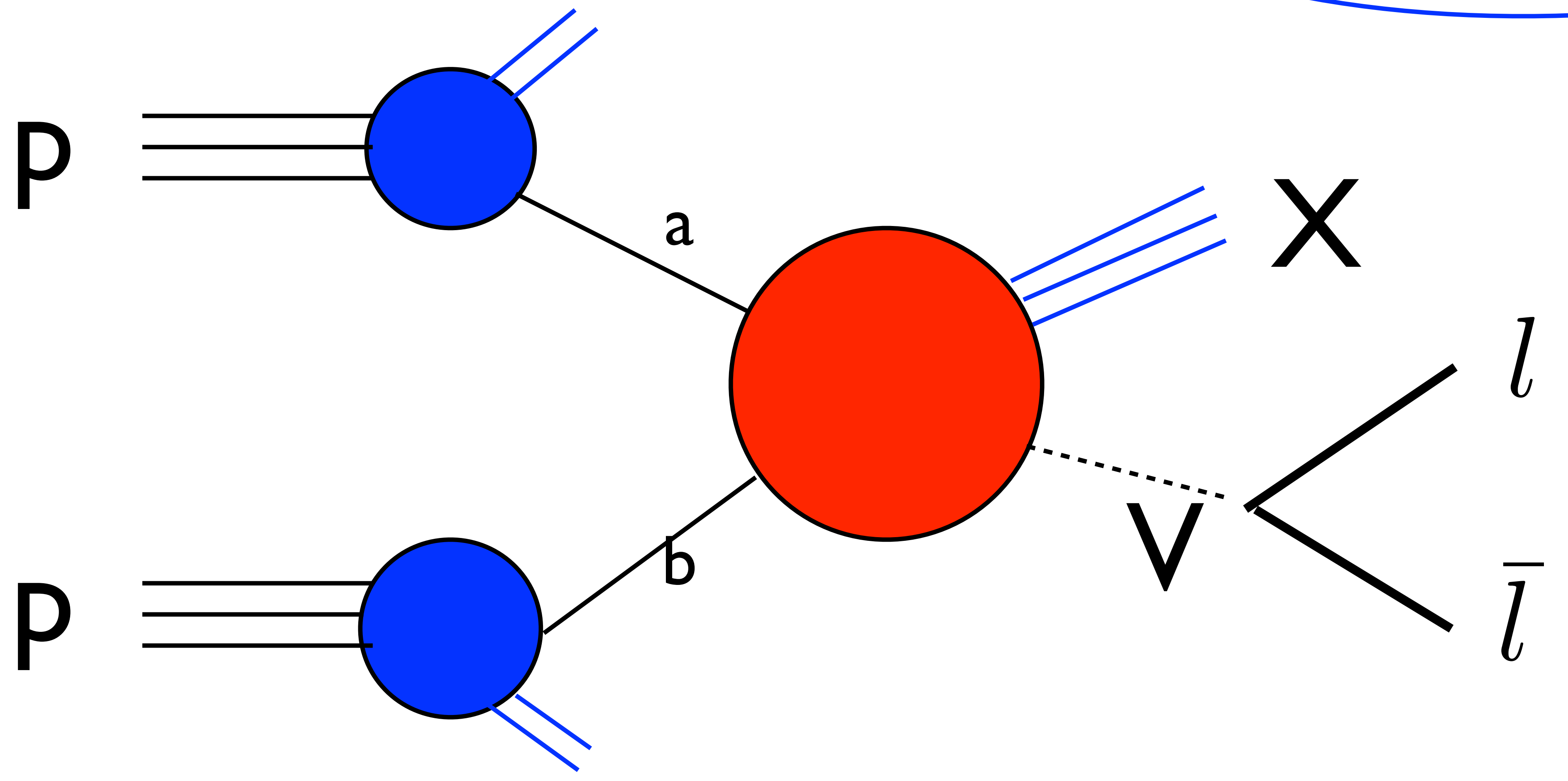
(formidable challenges!)

would offer a very stringent test of the SM likelihood

MW determination at hadron colliders

Lepton-pair production at hadron colliders (theory breakdown)

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



We need

- best description of the partonic cross section including fixed- and all-orders radiative corrections QCD, EW, mixed QCDxEW

- accurate and consistent description of the QCD environment including PDFs, intrinsic partonic $k_\perp$, QED DGLAP PDF evolution

▻ QCD modelling both perturbative and non-perturbative QCD contributions

transverse d.o.f. → gauge bosons $p_\perp^V$ spectra; dependent on non-perturbative contributions at low $p_\perp^Z$

longitudinal d.o.f. → rapidity distributions ; affected by PDF uncertainties

▻ EW and mixed QCDxEW effects

important QED/EW corrections (mostly FSR) modulated by the underlying QCD dynamics

are our current tools adequate for the precision determination of EW parameters ?

MW determination at hadron colliders

In charged-current DY, it is NOT possible to reconstruct the lepton-pair invariant mass
Full reconstruction is possible (but not easy) only in the transverse plane

MW extracted from the study of the shape of the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions in CC-DY
thanks to the jacobian peak that enhances the sensitivity to m_W

$$\frac{d}{dp_\perp^2} \to \frac{2}{s} \frac{1}{\sqrt{1-4p_\perp^2/s}} \frac{d}{d\cos\theta}$$

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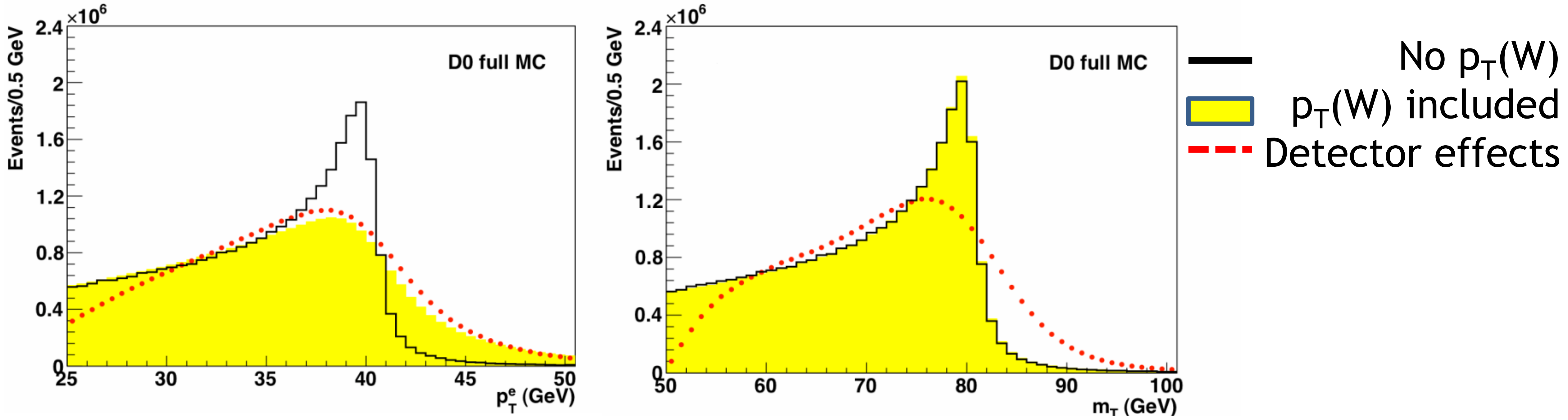
problems are due to
- the smearing of the distributions due to difficult neutrino reconstruction
- strong sensitivity to the modelling of initial state QCD effects



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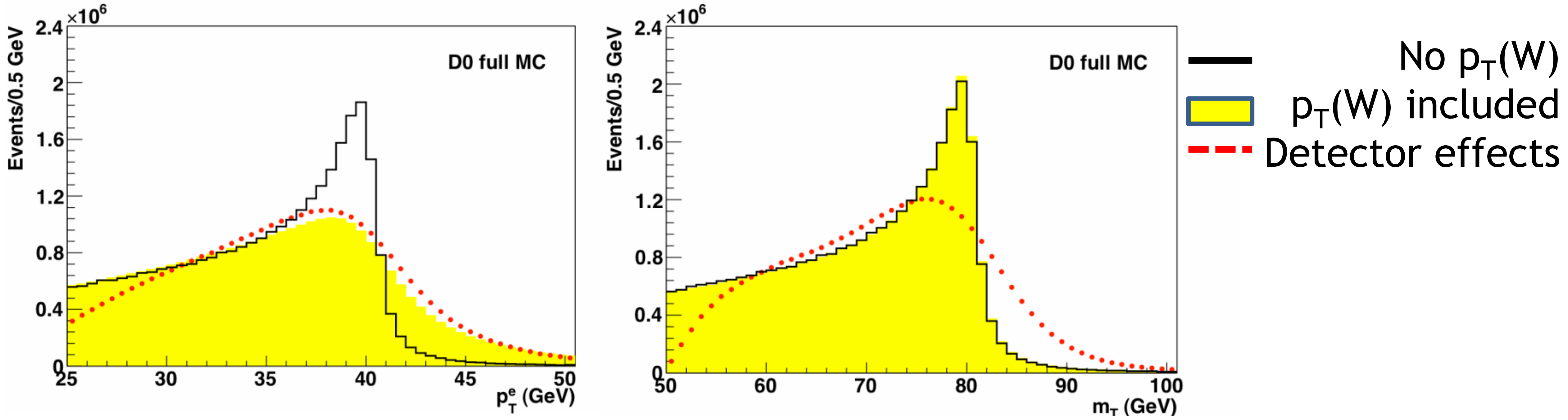

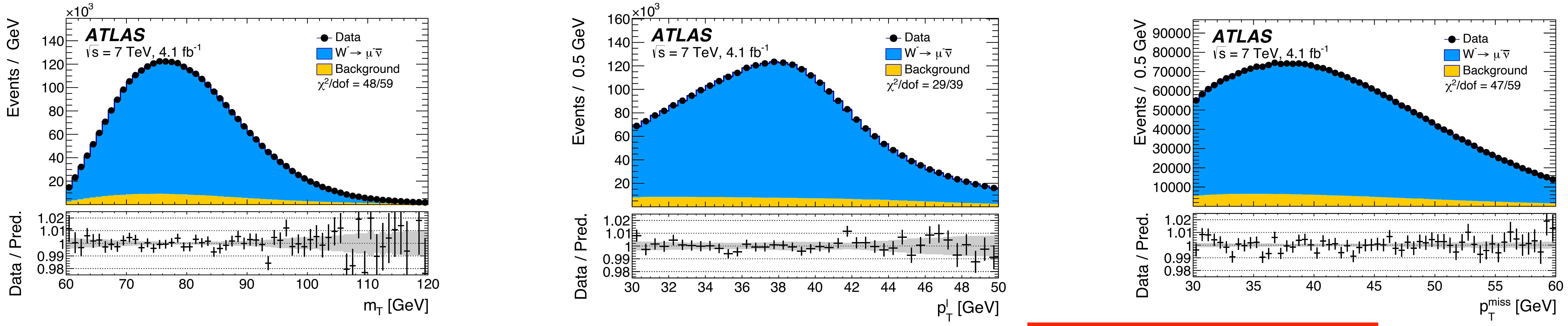

$$\begin{aligned} m_W &= 80369.5 \pm 6.8\ \text{MeV(stat.)} \pm 10.6\ \text{MeV(exp. syst.)} \pm \boxed{13.6\ \text{MeV(mod. syst.)}} \\ &= 80369.5 \pm 18.5\ \text{MeV}, \end{aligned}$$

ATLAS error dominated by modelling systematics

Determination of SM parameters at colliders: methodology

We experimentally measure only cross sections (kinematical distributions) and asymmetries $\rightarrow$ observables

Any interpretation requires that we choose a model (e.g. the SM) and
 we compute the observables as functions of the model input parameters
 in the SM gauge sector we can only determine (g, g', v) expressed in terms e.g. of (α, m_W, m_Z)

At a hadron collider, an analytic fit is often impossible $\rightarrow$ we need to compute templates as a function of e.g. (α, m_W, m_Z)
The templates will be compared to the data, looking for the best-fit values of the input parameters

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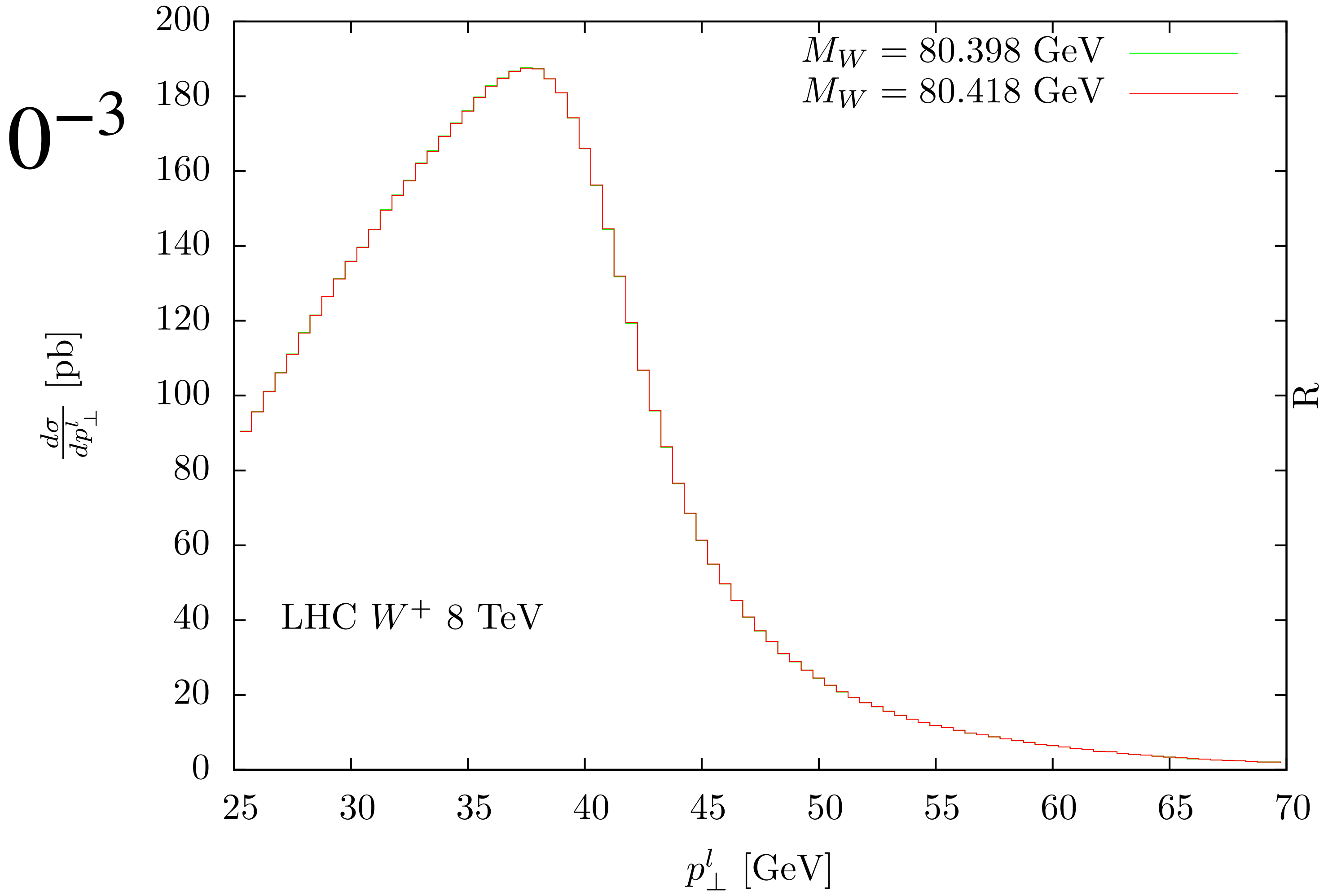
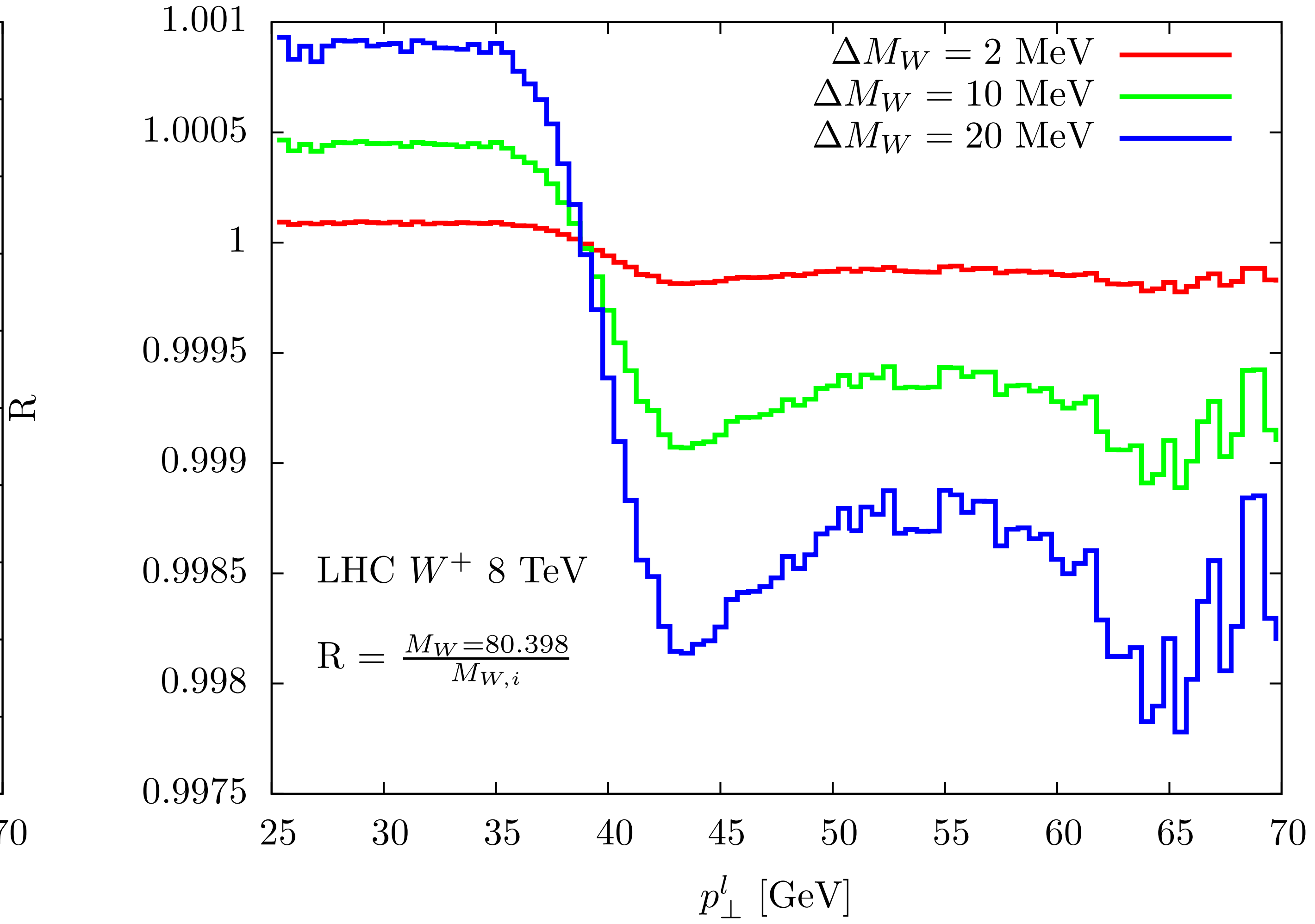
The templates are perturbative predictions.
Their residual theoretical uncertainties will propagate as theoretical systematic errors on the determination of (α, m_W, m_Z)

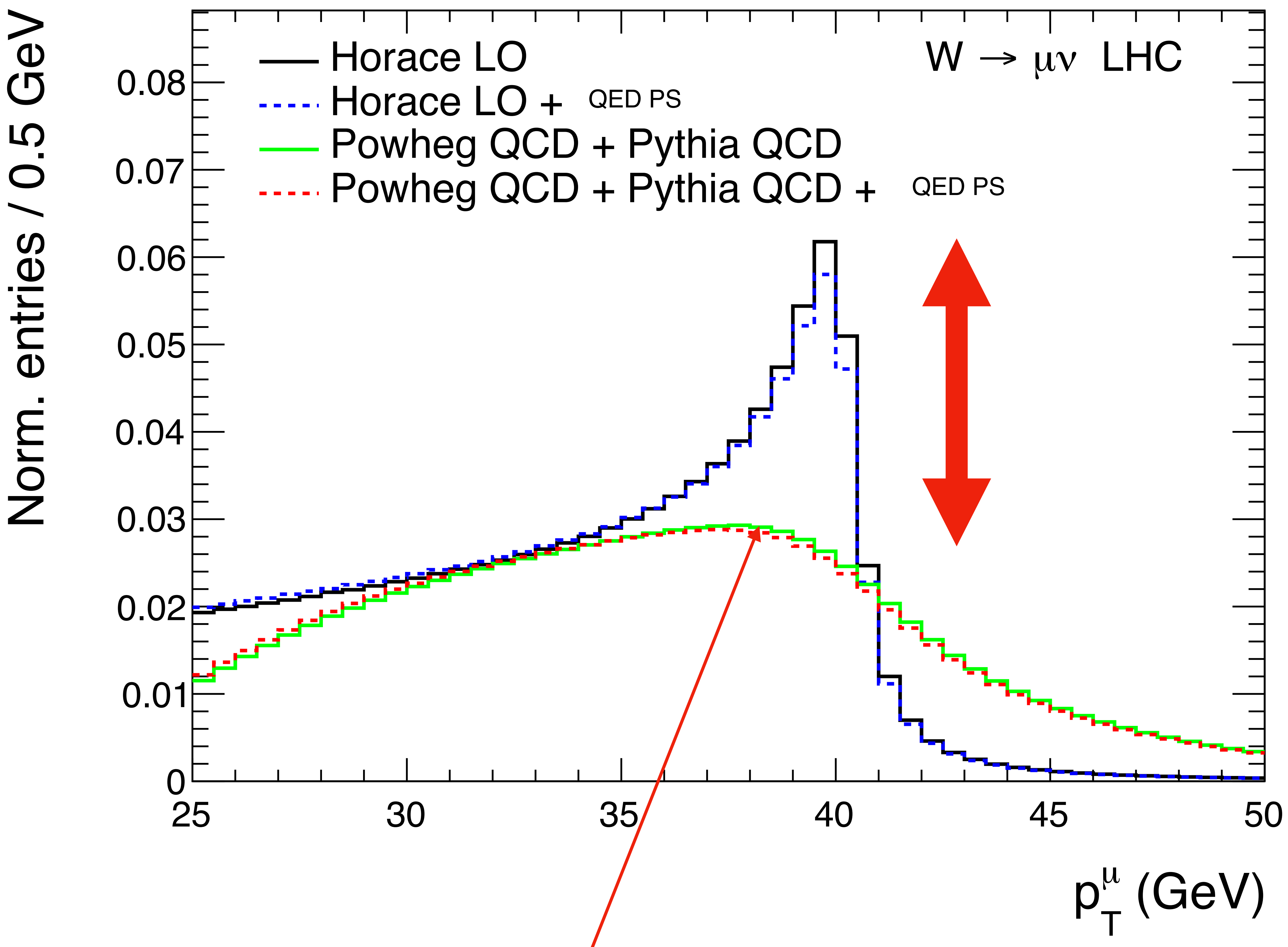
Given the very high precision goal
$\delta m_W / m_W \sim 1 \cdot 10^{-4}, \qquad \delta \sin^2\theta_{eff} / \sin^2\theta_{eff} \sim 1 \cdot 10^{-3}$
control on the shape of the distributions
at the sub-percent level is needed,
at a hadron collider…





Basic ingredients of the simulation tools needed for the MW determination

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841



- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

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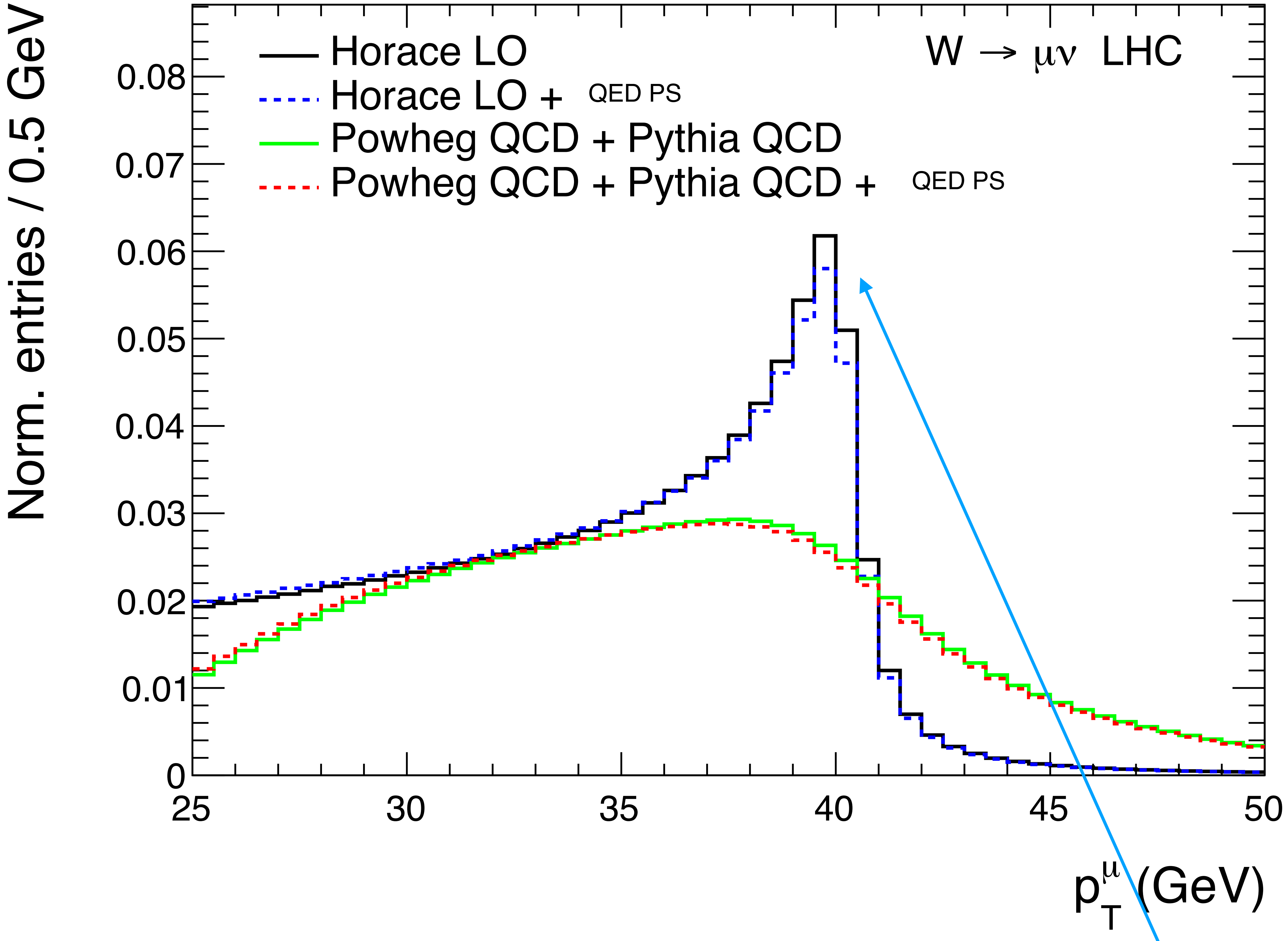

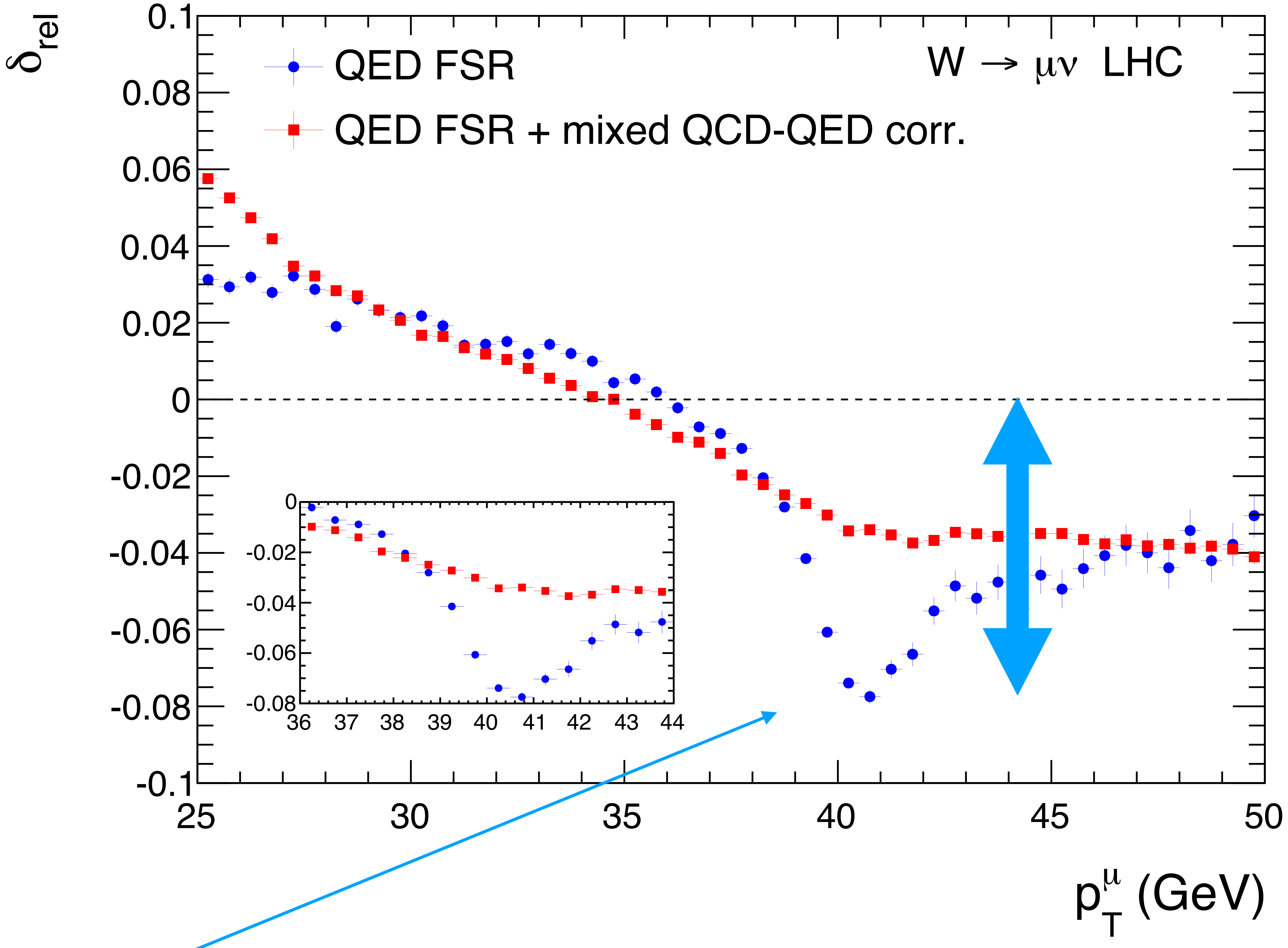

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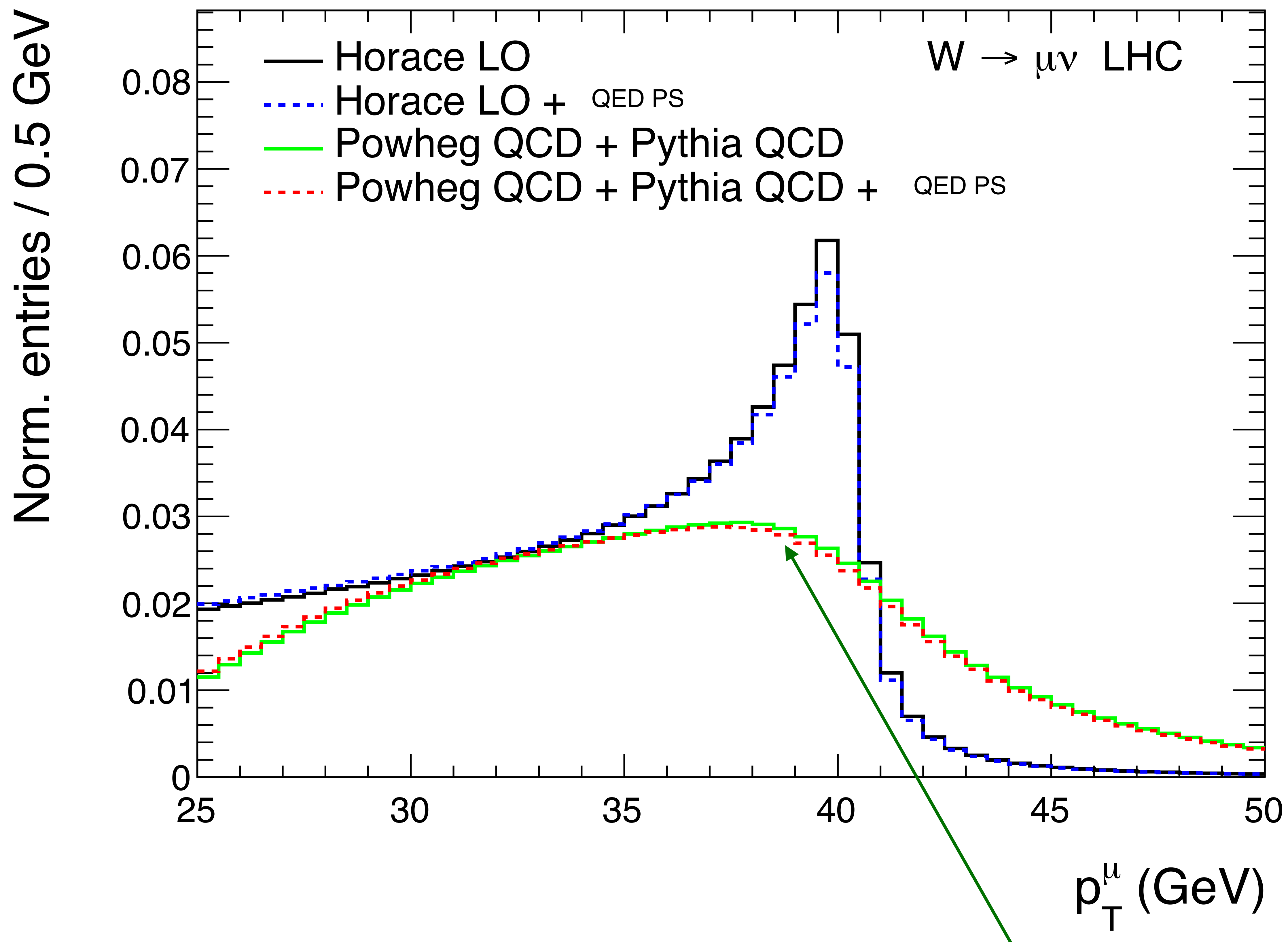

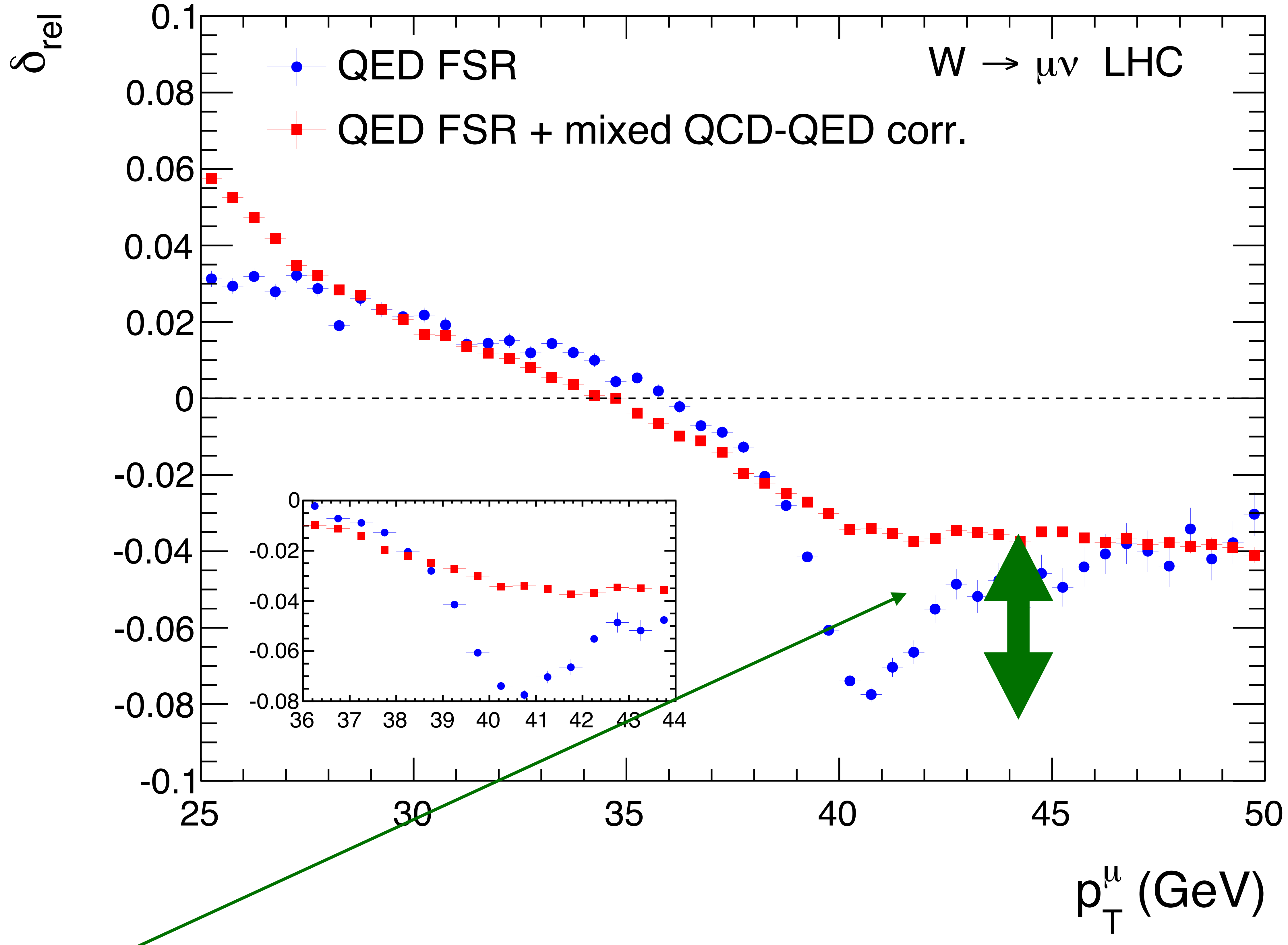

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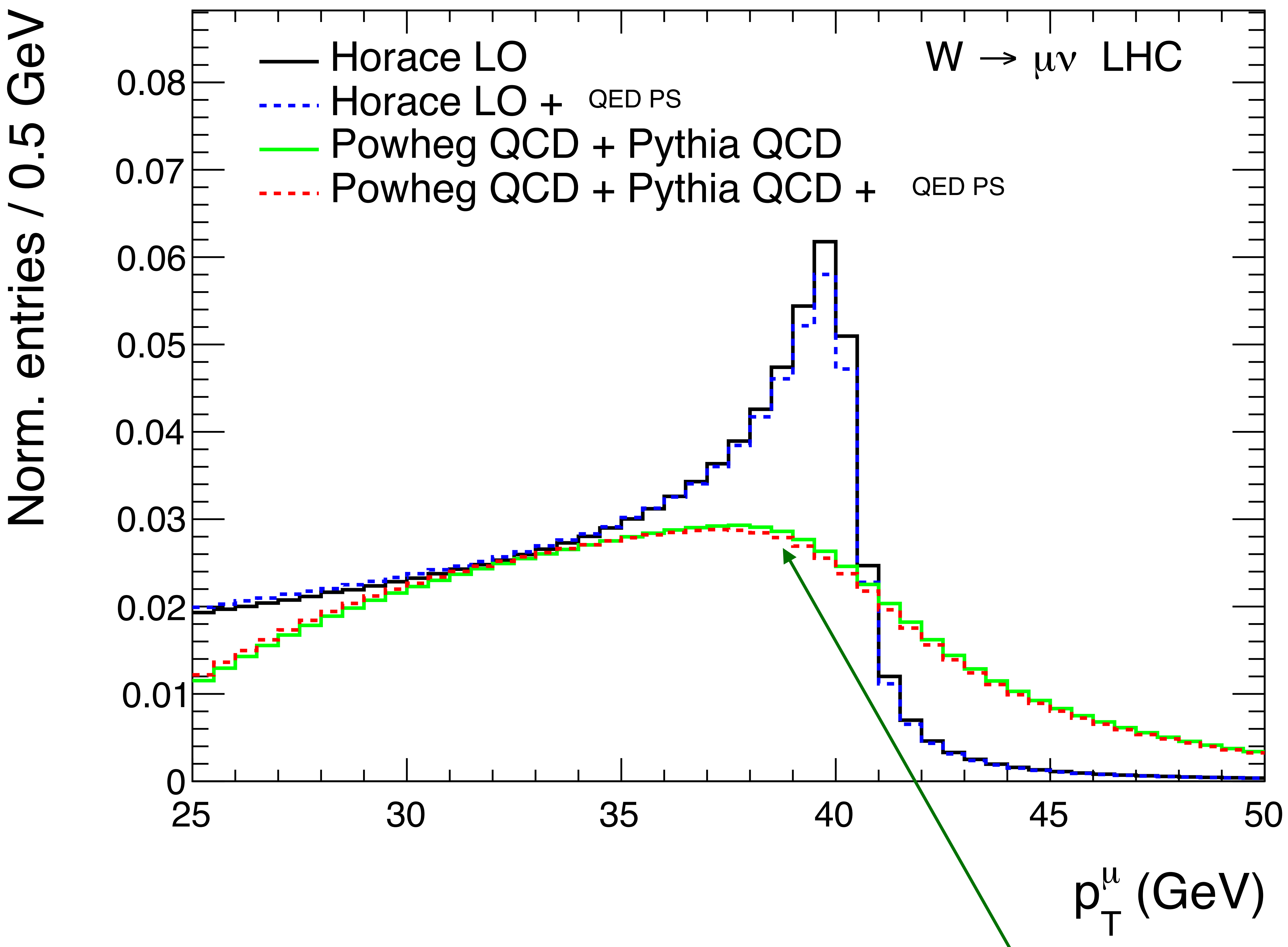

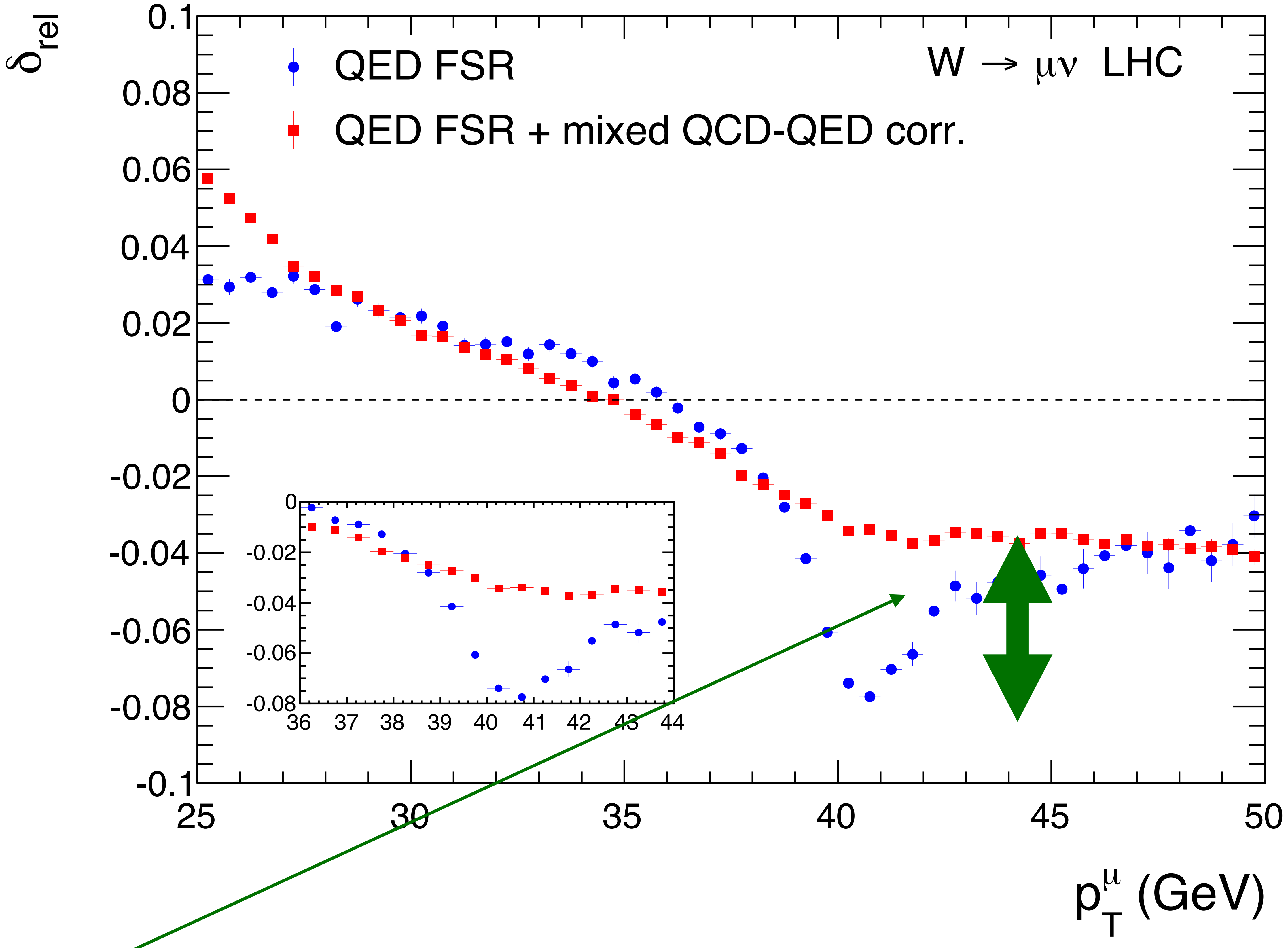

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NLO-QCD + QCDPS + QEDPS is the lowest order meaningful approximation of this observable

m_W determination at colliders: open problems

- Perturbative QCD modelling: qt resummation and matching with fixed order results
- Heavy-quark-induced processes: collinear-log resummation, mass effects, differences between W and Z production
- QED radiation and the transverse momentum of the lepton pair
- QED DGLAP evolution of the proton PDFs
- PDF uncertainties on the m_W determination

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m_W is extracted from observables defined in the transverse plane $\rightarrow$ all the uncertainties affecting $p_\perp^l$ matter

the finite rapidity acceptance of the detectors induces a collinear PDF effect on the $p_\perp^l$ distribution

at this level of precision QCD, EW and mixed QCD-EW corrections are necessary

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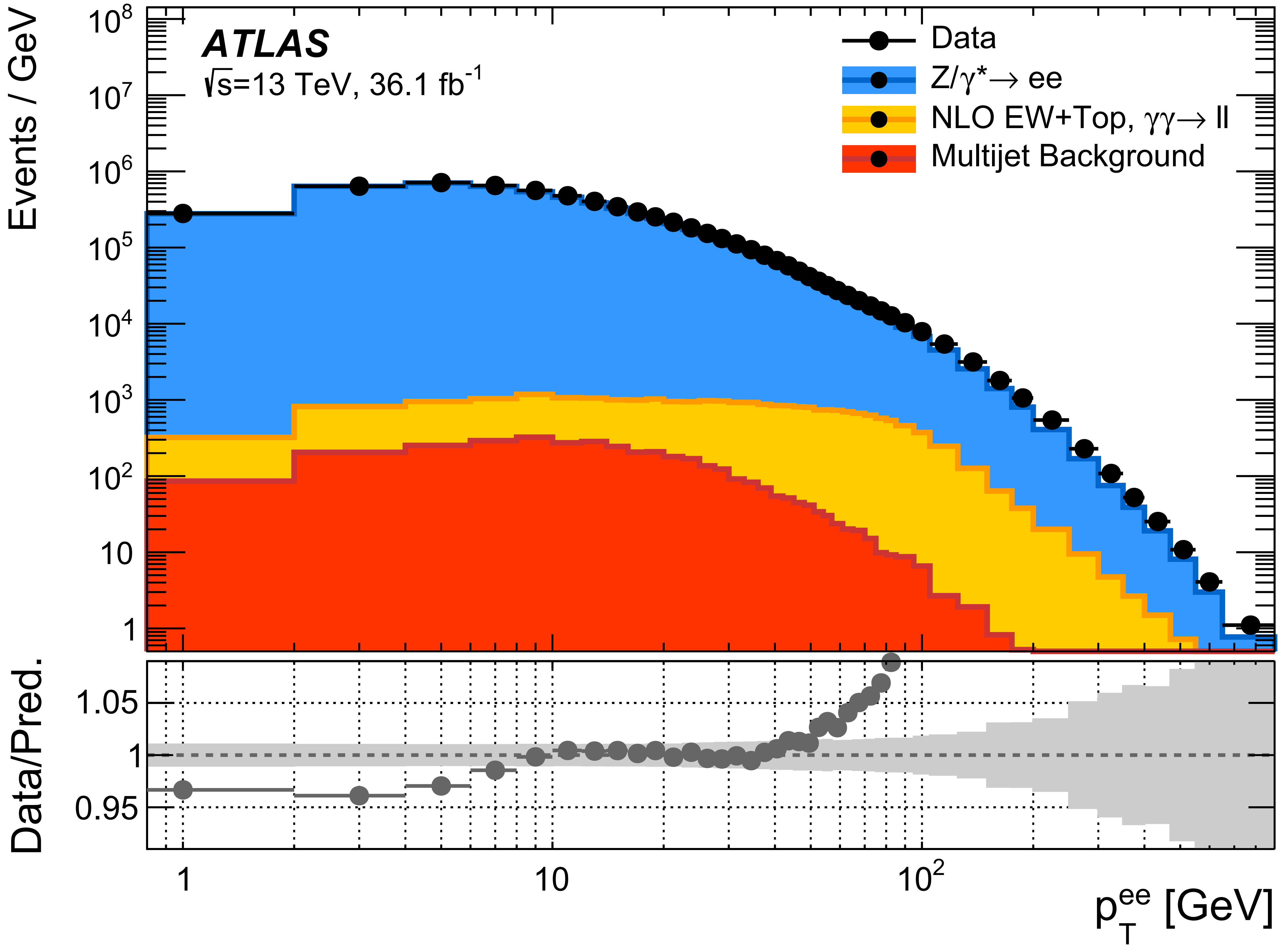
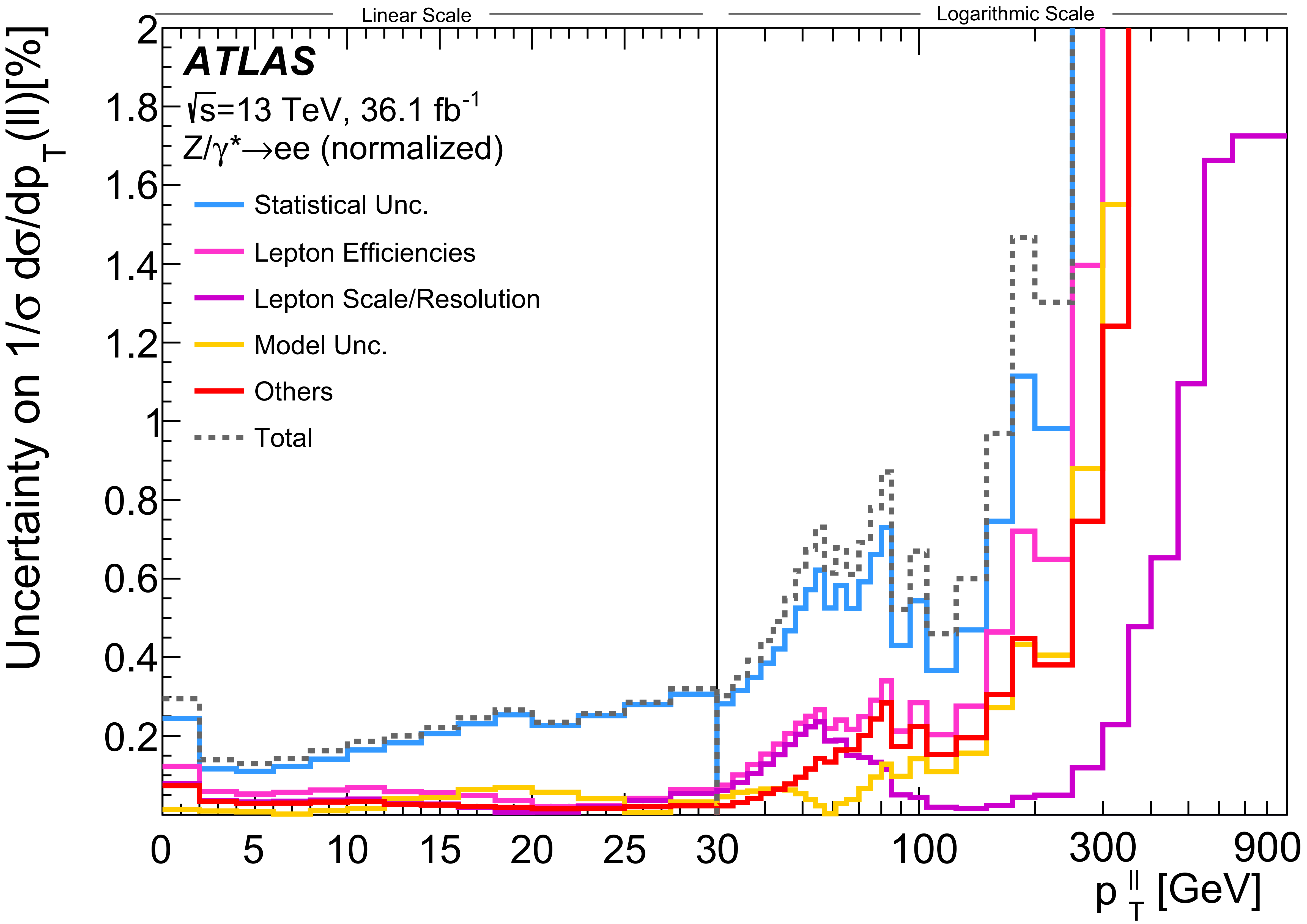
Lepton-pair transverse momentum distribution

Lepton-pair transverse momentum distribution

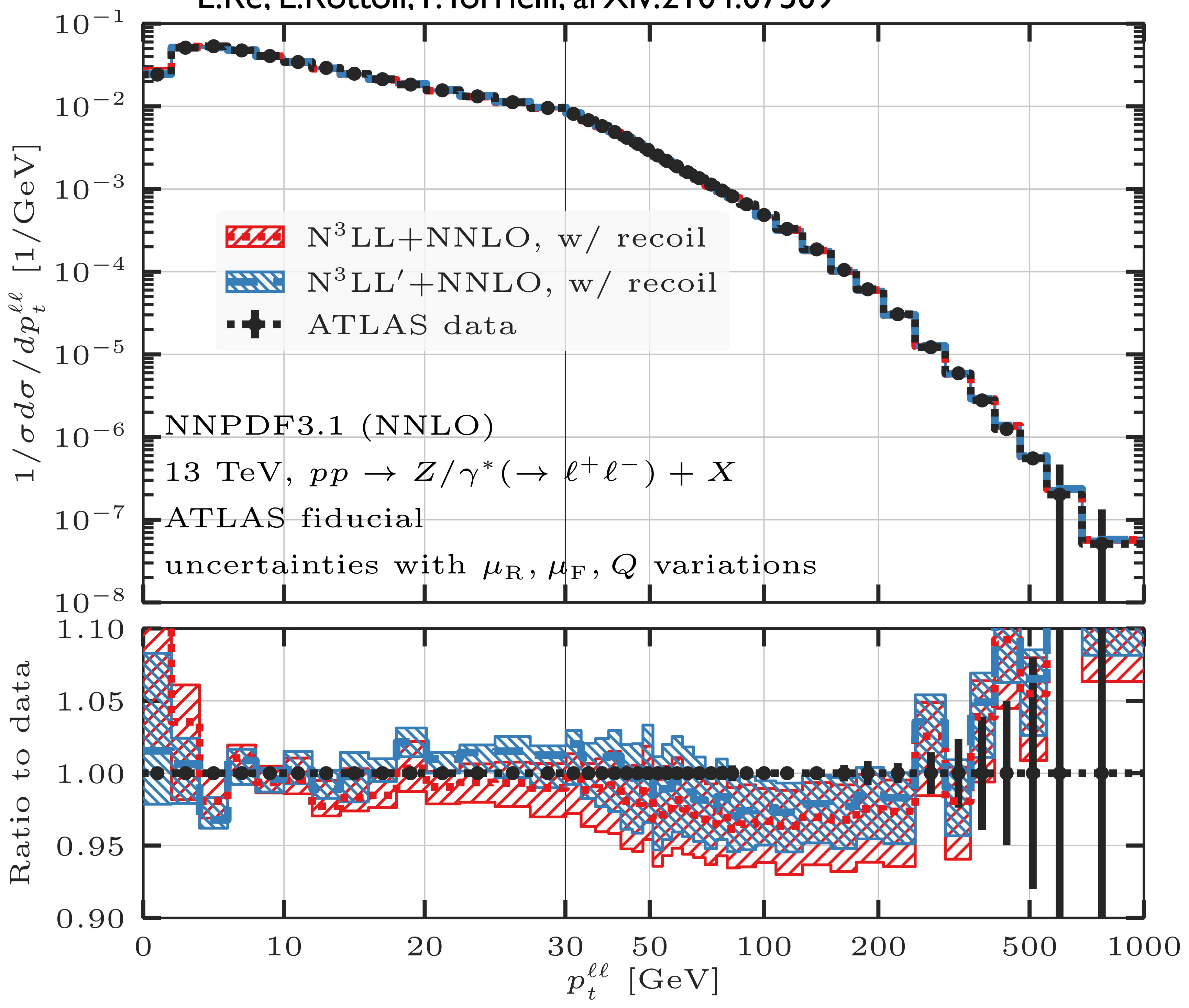
- A crucial role in precision EW measurements (m_W in particular) is played by the $p_\perp^Z$ distribution
  - ▻ m_W is extracted from the fit to the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions
  - ▻ the $p_\perp^l$ and $p_\perp^\nu$ simulation strongly depends on a precise knowledge of the $p_\perp^W$ distribution
  - ▻ a precise $p_\perp^W$ measurement is not yet available → we rely on $p_\perp^Z$ and extrapolate from it   are W and Z identical ?
  - ▻ $p_\perp^Z$ is used to calibrate Monte Carlo tools (Parton Shower at low-$p_\perp^Z$)



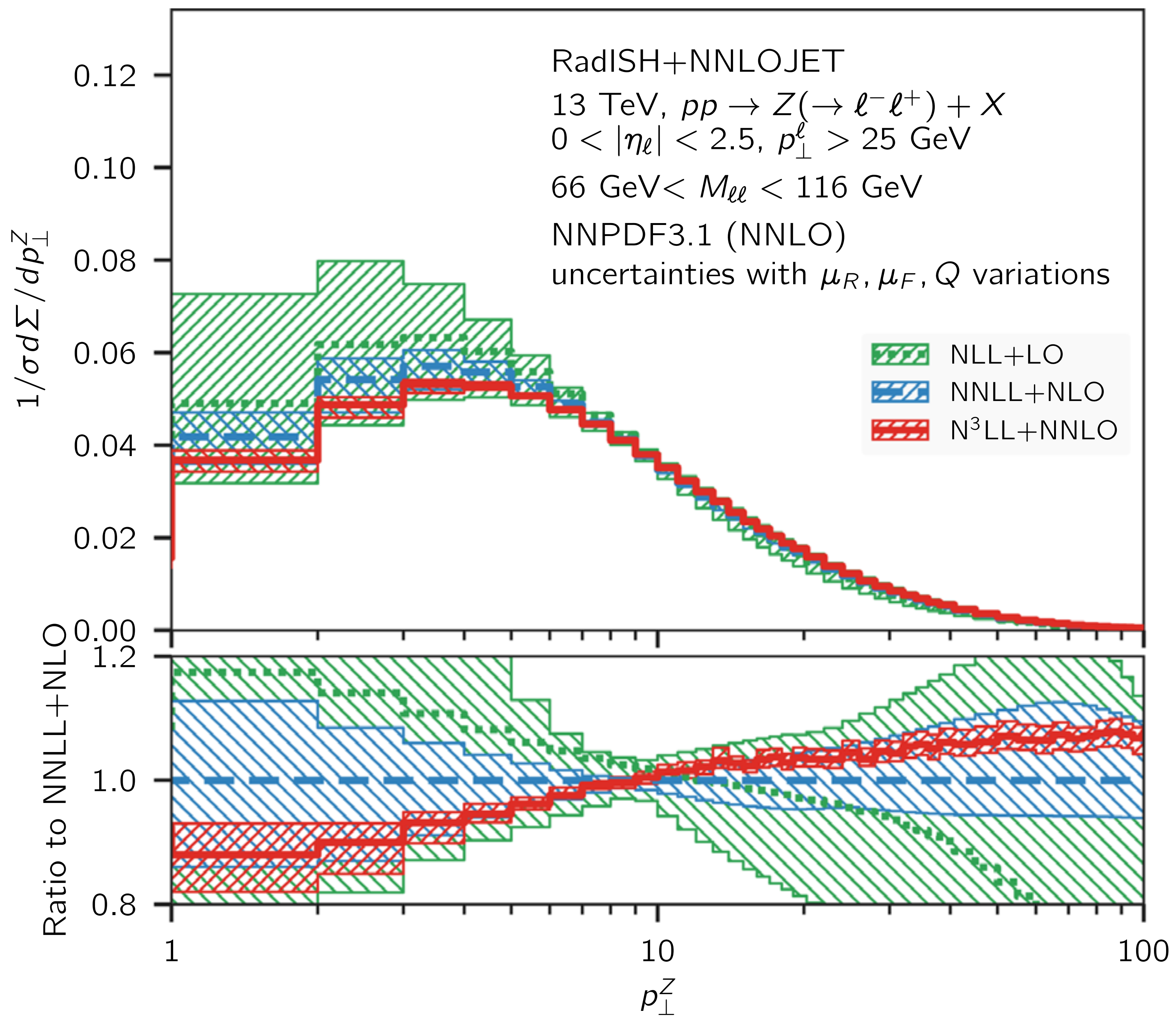


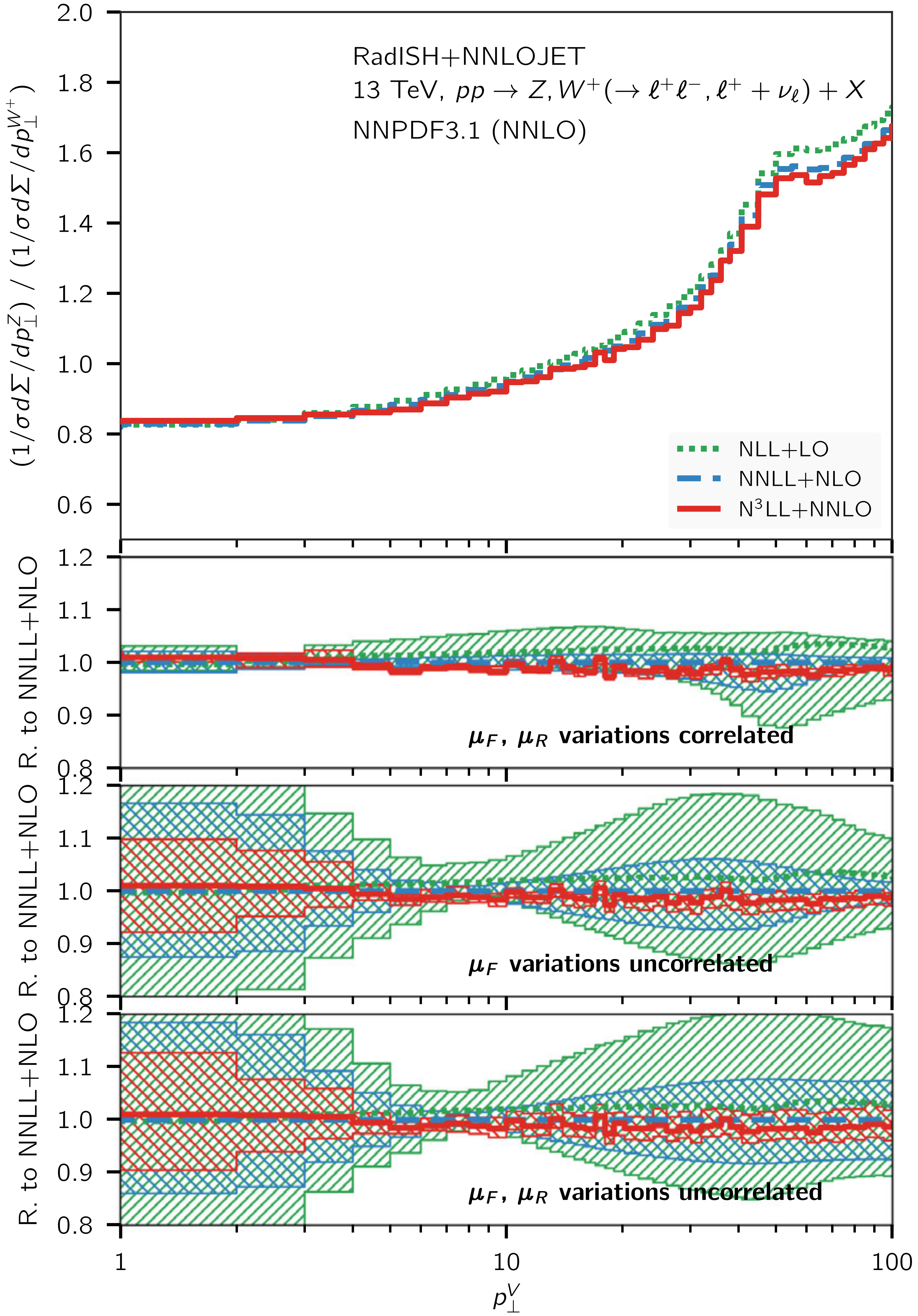
Progress in the QCD calculations and simulations: lepton-pair transverse momentum

E.Re, L.Rottoli, P.Torrielli, arXiv:2104.07509



W.Bizon, A.Gehrmann-De Ridder, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, D.Walker, arXiv:1905.05171

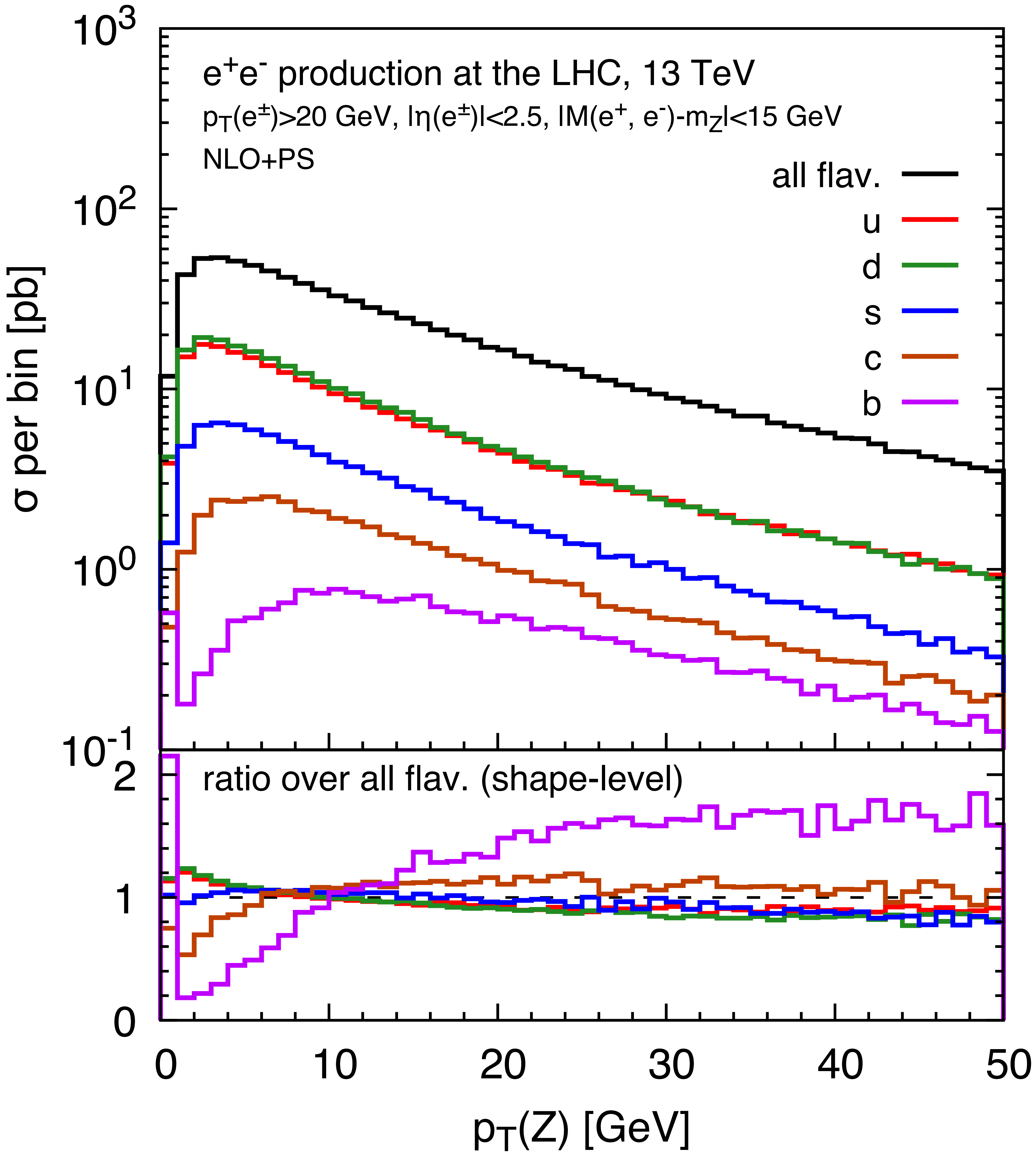




- the ptV distribution is responsible for a large component of the lepton transverse momentum, mostly 0< ptV < 10 GeV
- the resummation at N3LL and the matching with NNLO results has a strong impact on both central value and uncertainty band
- the description of the data based on a perturbative-QCD description at N3LL is acceptable, with uncertainties in the 2-3% range
  → the need for a non-perturbative contribution is correspondingly reduced
  → the reweighing factor needed to "reach the data" should become closer to 1

Bottom quark contributions to the ptZ distribution in the 5FS

E.Bagnaschi, F.Maltoni, AV, M.Zaro, arXiv:1803.04336



· in the 5FS the bottom quark is treated as a massless parton

· the bottom density in the proton resums via DGLAP eqs large collinear logs

· the masslessness of the bottom may affect some kinematical distributions where the quark mass acts as a natural regulator of the transverse d.o.f. e.g. the ptZ distribution with ptZ ~ O(mb) ~ O(5 - 20 GeV)

· the PDF evolution starts for the heavy quarks at Q ~ mq

→ in the 5FS the bottom contribution to the ptZ spectrum is harder than the one of light quarks

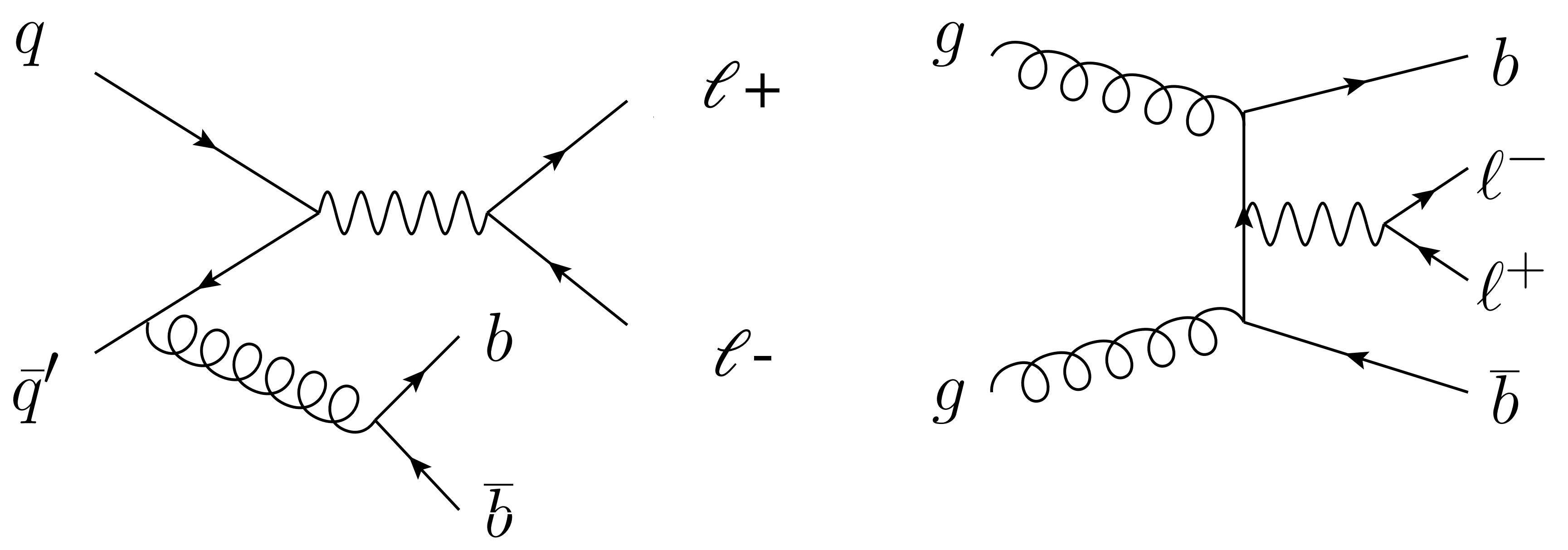
initial state quark	cross section (pb)	%
u	374.44 ± 0.62	35.0
d	391.15 ± 0.63	36.5
c	91.44 ± 0.34	8.6
s	170.43 ± 0.45	15.9
b	43.13 ± 0.26	4.0
total	1070.58 ± 0.86	100.0

· given the exp error below 0.5% in a large range the bottom contribution of O(4%)

→ we need a prediction of the b contribution precise at the O(10%) level

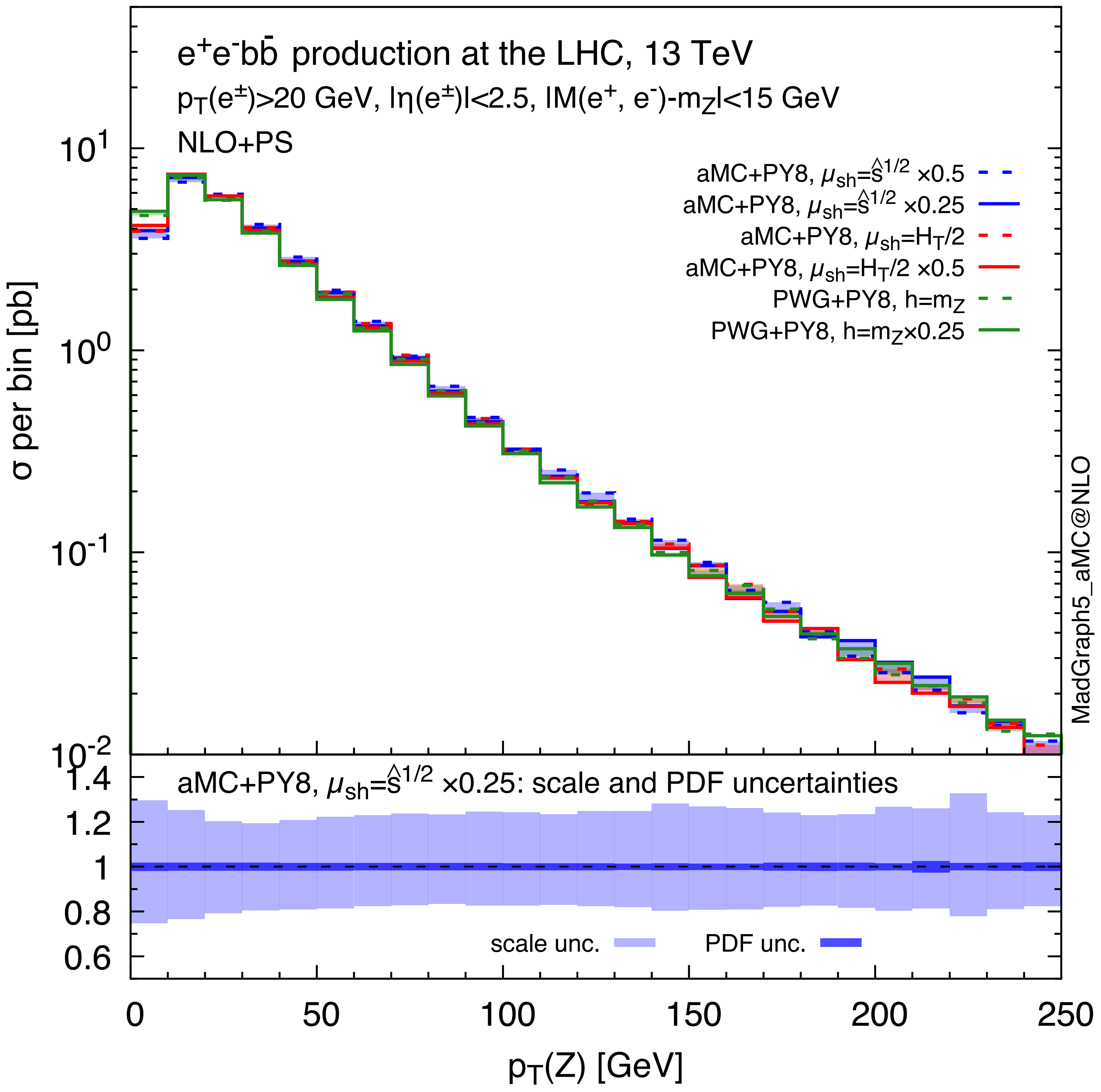
Z b bbar associated production in the 4FS ($pp \to e^+e^- b\,\bar{b}$): ptZ distribution, inclusive over b quarks

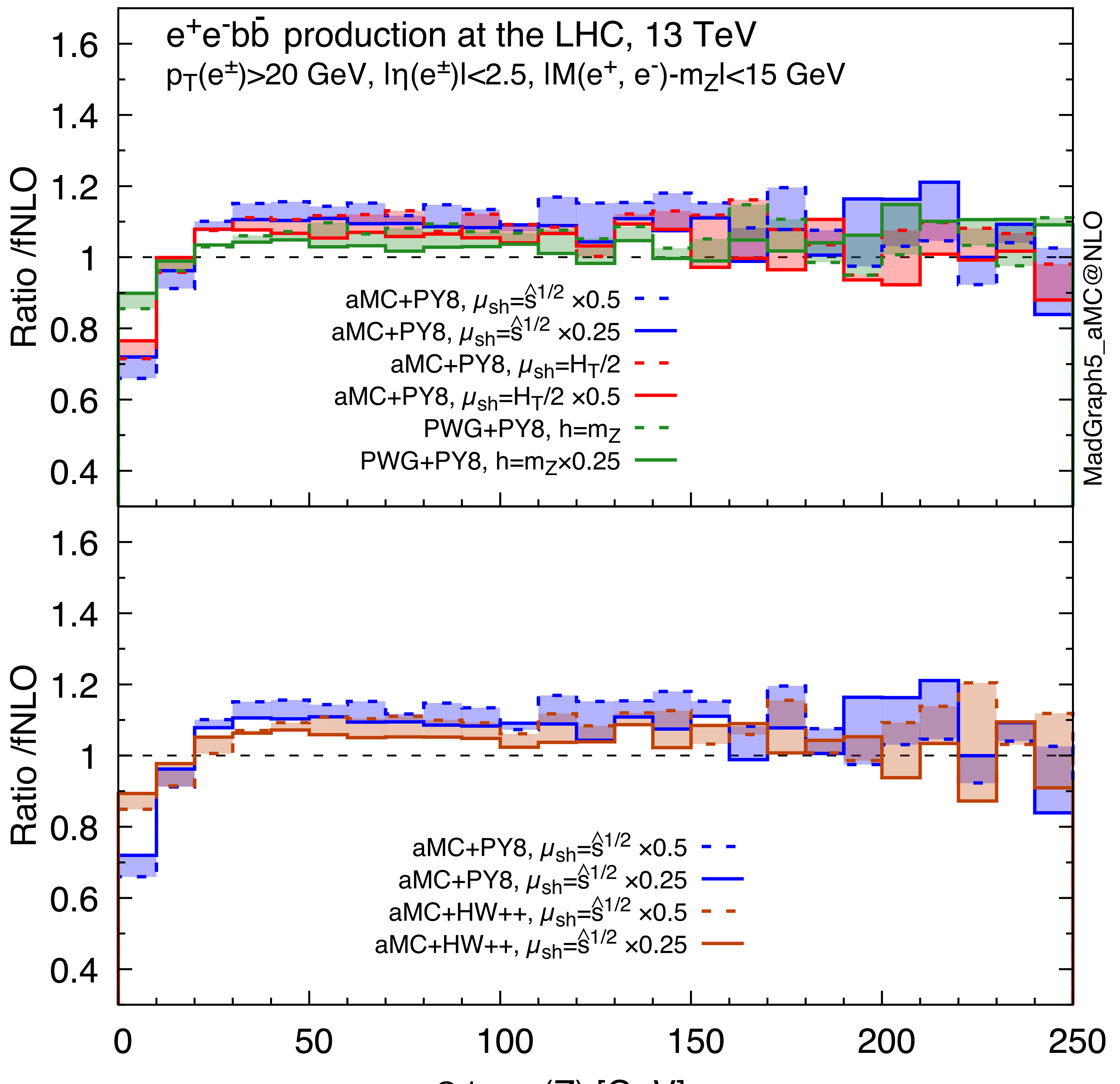
E.Bagnaschi, F.Maltoni, AV, M.Zaro, arXiv:1803.04336



in the 4FS the bottom quark

- is absent in the proton
- it can be produced in the final state as a massive particle
  → improved description of the kinematical distributions
- at LO the collinear logs are included only at fixed order





perturbative uncertainties:

- scale variations
- choice of the shower scale

- Parton Shower radiation model

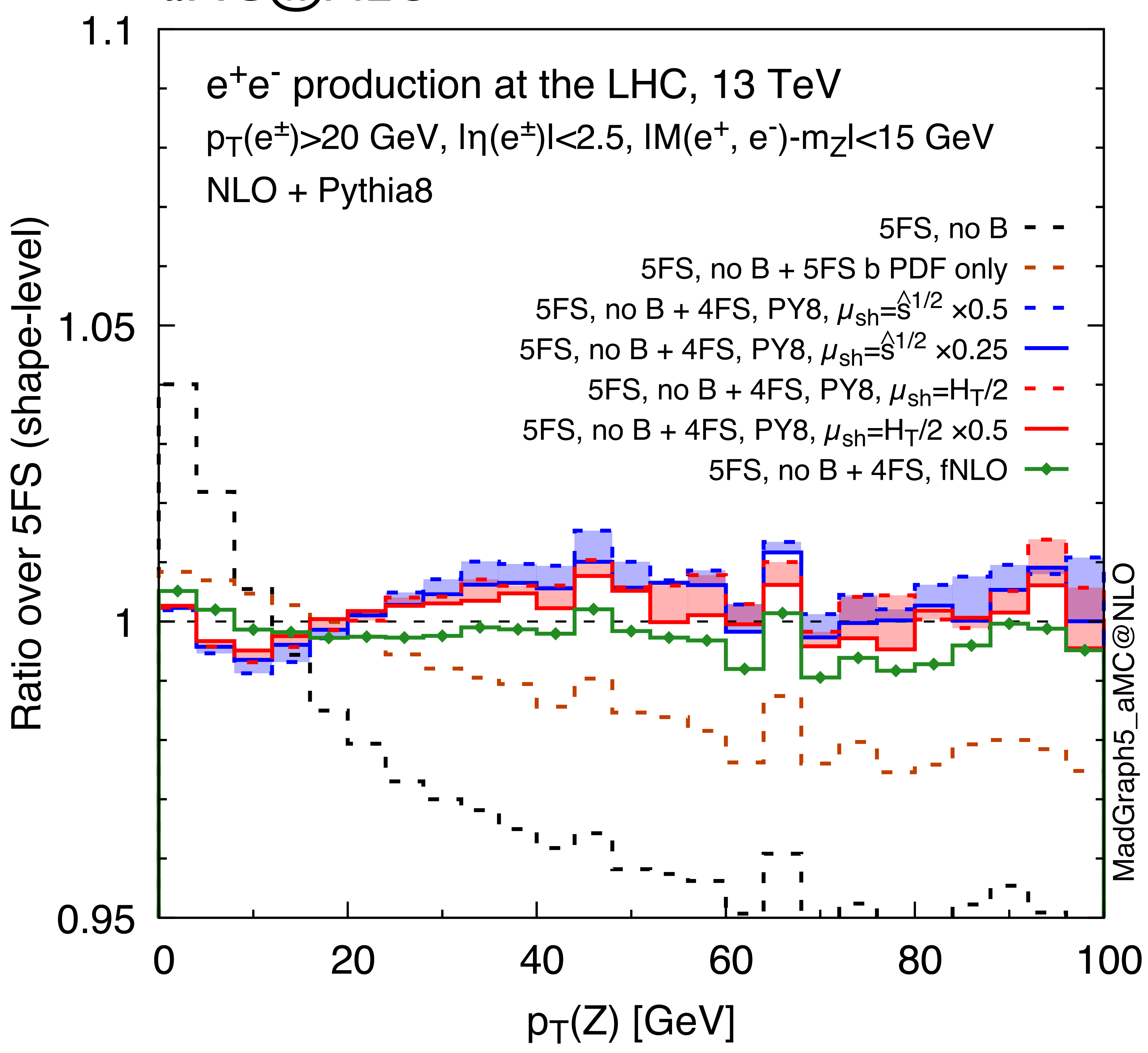
Improved prediction of the ptZ distribution

E.Bagnaschi, F.Maltoni, AV, M.Zaro, arXiv:1803.04336

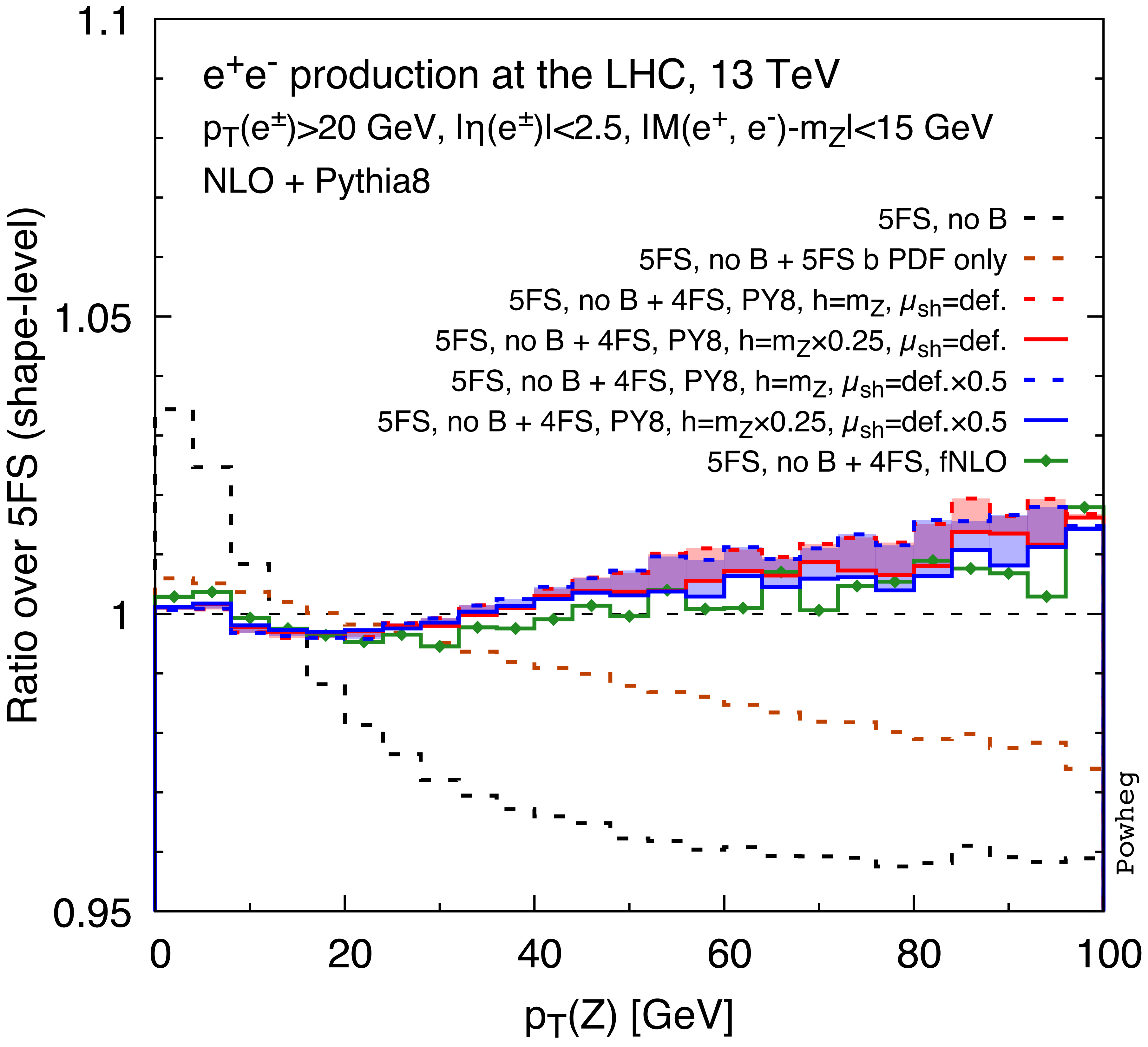
$$\frac{d\sigma^{best}}{dp_\perp^{l^+l^-}} = \frac{d\sigma^{\mathtt{5FS-Bveto}}}{dp_\perp^{l^+l^-}} + \frac{d\sigma^{4FS}}{dp_\perp^{l^+l^-}}$$

$$\mathcal{R}(p_\perp^{l^+l^-}) = \left(\frac{1}{\sigma_{fid}^{best}}\frac{d\sigma^{best}}{dp_\perp^{l^+l^-}}\right)\cdot\left(\frac{1}{\sigma_{fid}^{5FS}}\frac{d\sigma^{5FS}}{dp_\perp^{l^+l^-}}\right)^{-1}$$

aMC@NLO



POWHEG



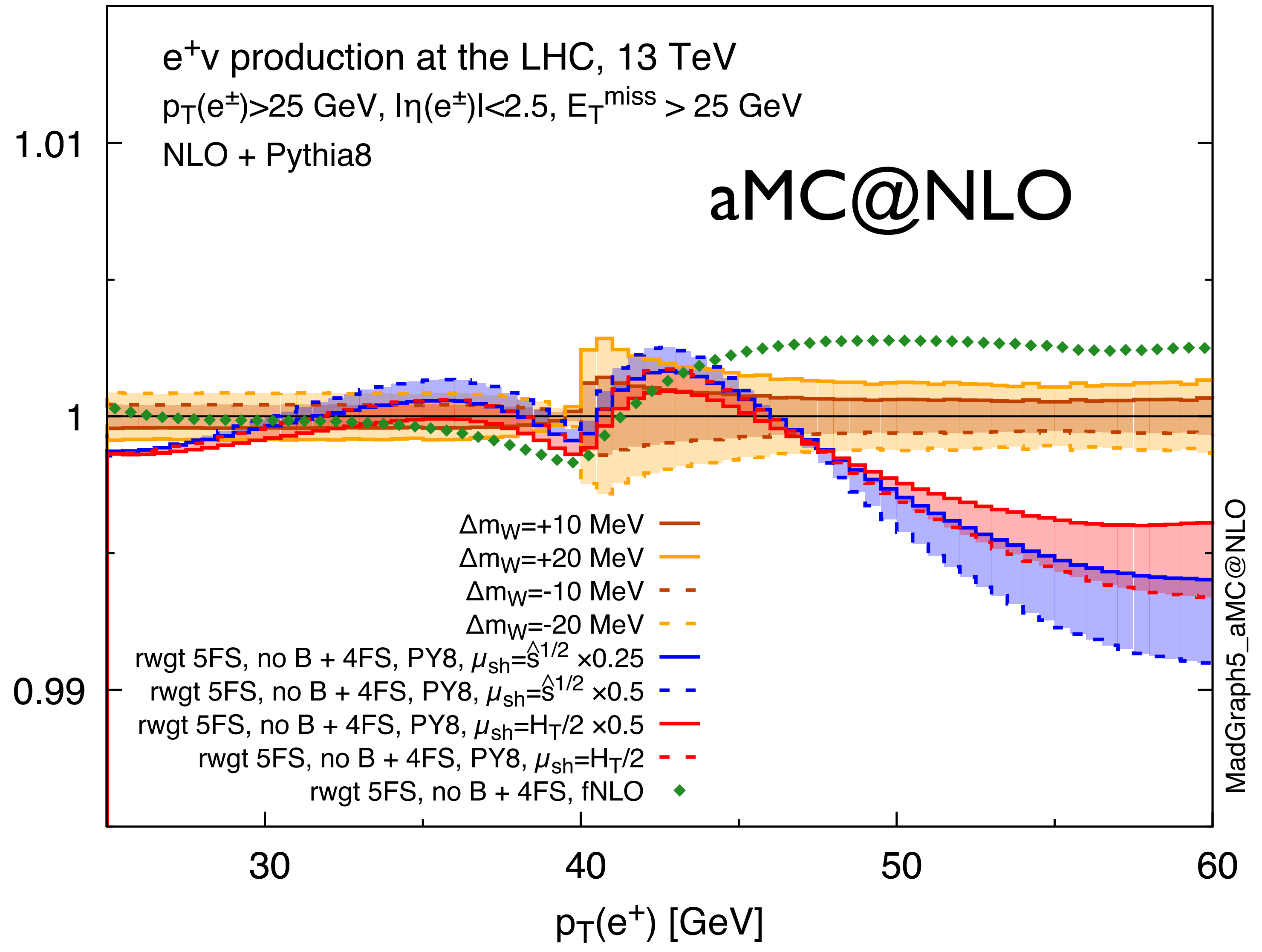
- distortion with a non trivial shape for ptZ<50 GeV
- in aMC@NLO effects at the ±1% level, in POWHEG effects at the ±0.5% level

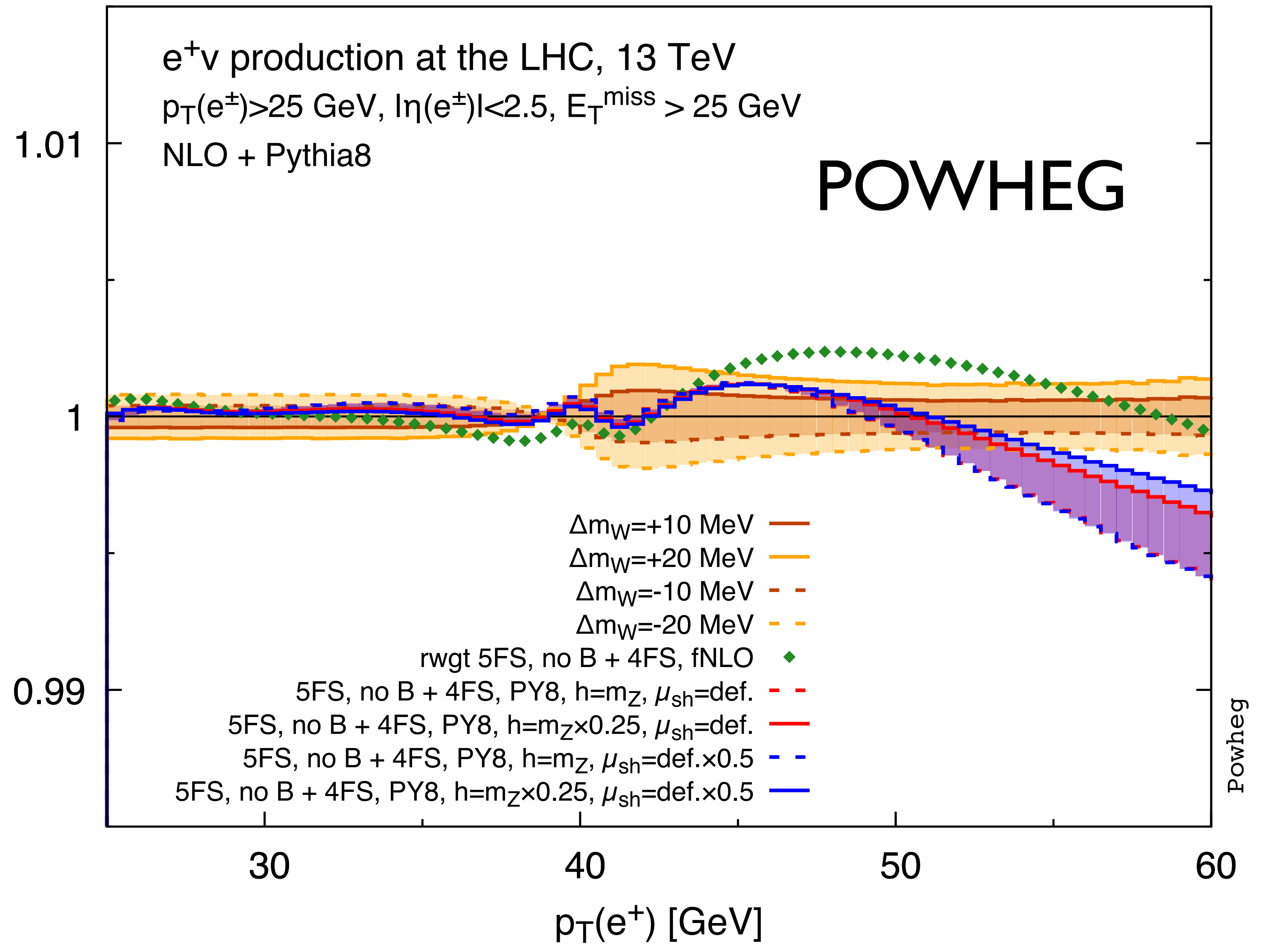
Impact on the CC-DY observables of b-quark effects

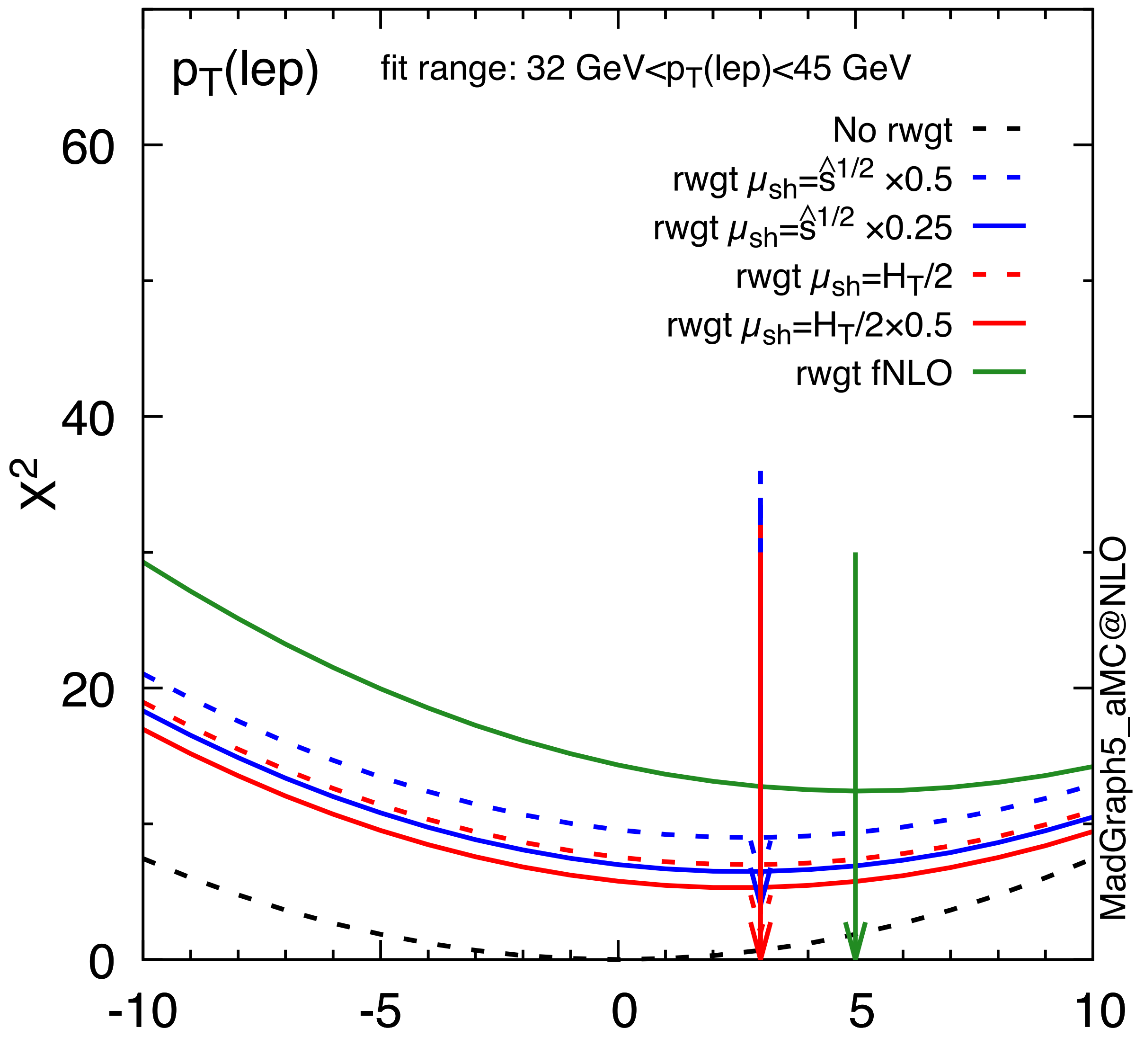
E.Bagnaschi, F.Maltoni, AV, M.Zaro, arXiv:1803.04336

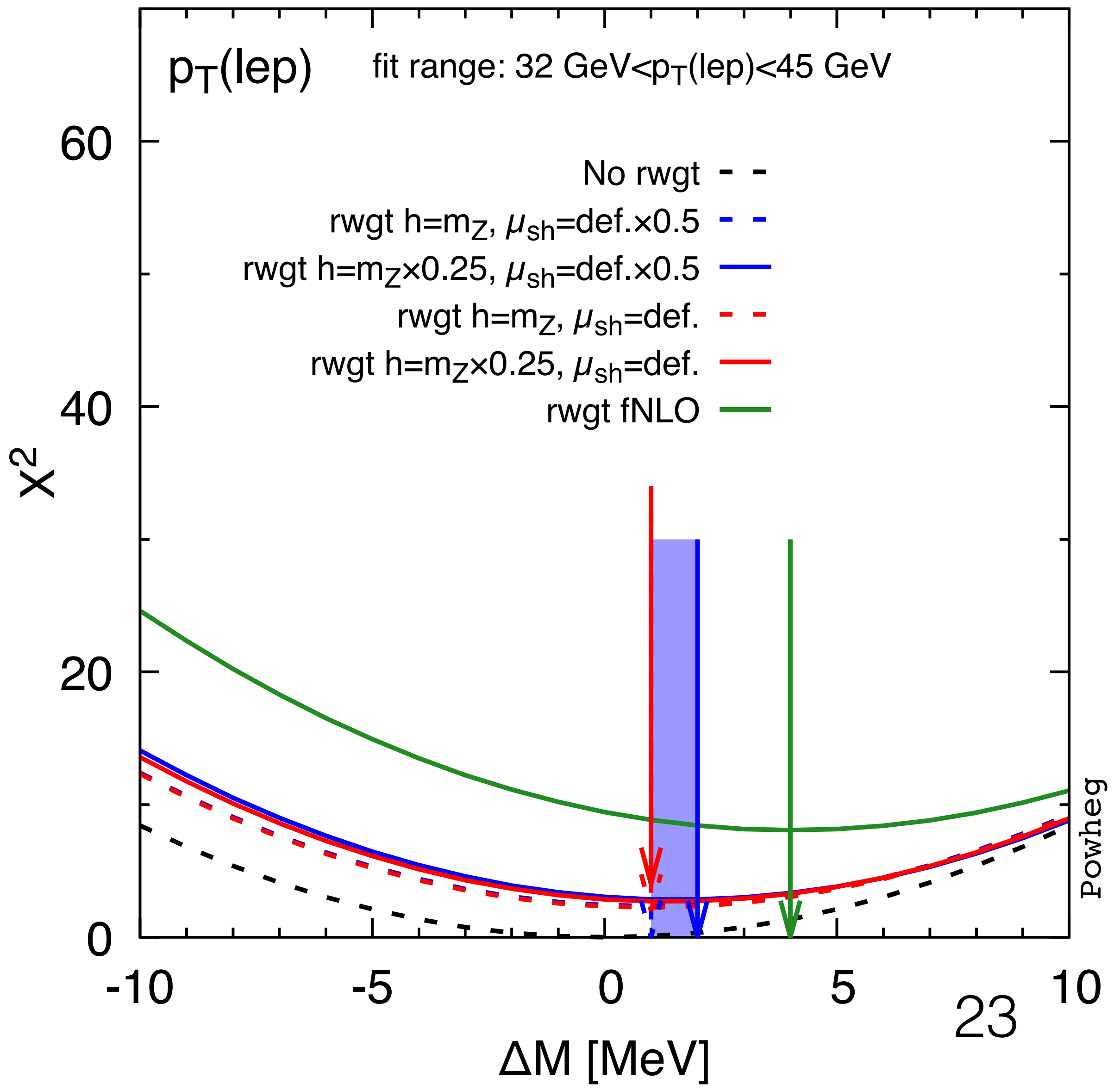
The CC-DY observables are evaluated in the plain 5FS

The new Parton Shower tune accounting for the improved treatment of bottom contributions is mimicked by reweighing the events with $1/\mathscr{R}(p_\perp)$









The impact on MW is estimated by
template fit of the reweighed distributions (RGB),
with templates in the plain 5FS (light brown)

Fixed-order results → $\Delta m_W \sim 5$ MeV
Additional radiation reduces this effect down to $\Delta m_W \sim 3$ MeV

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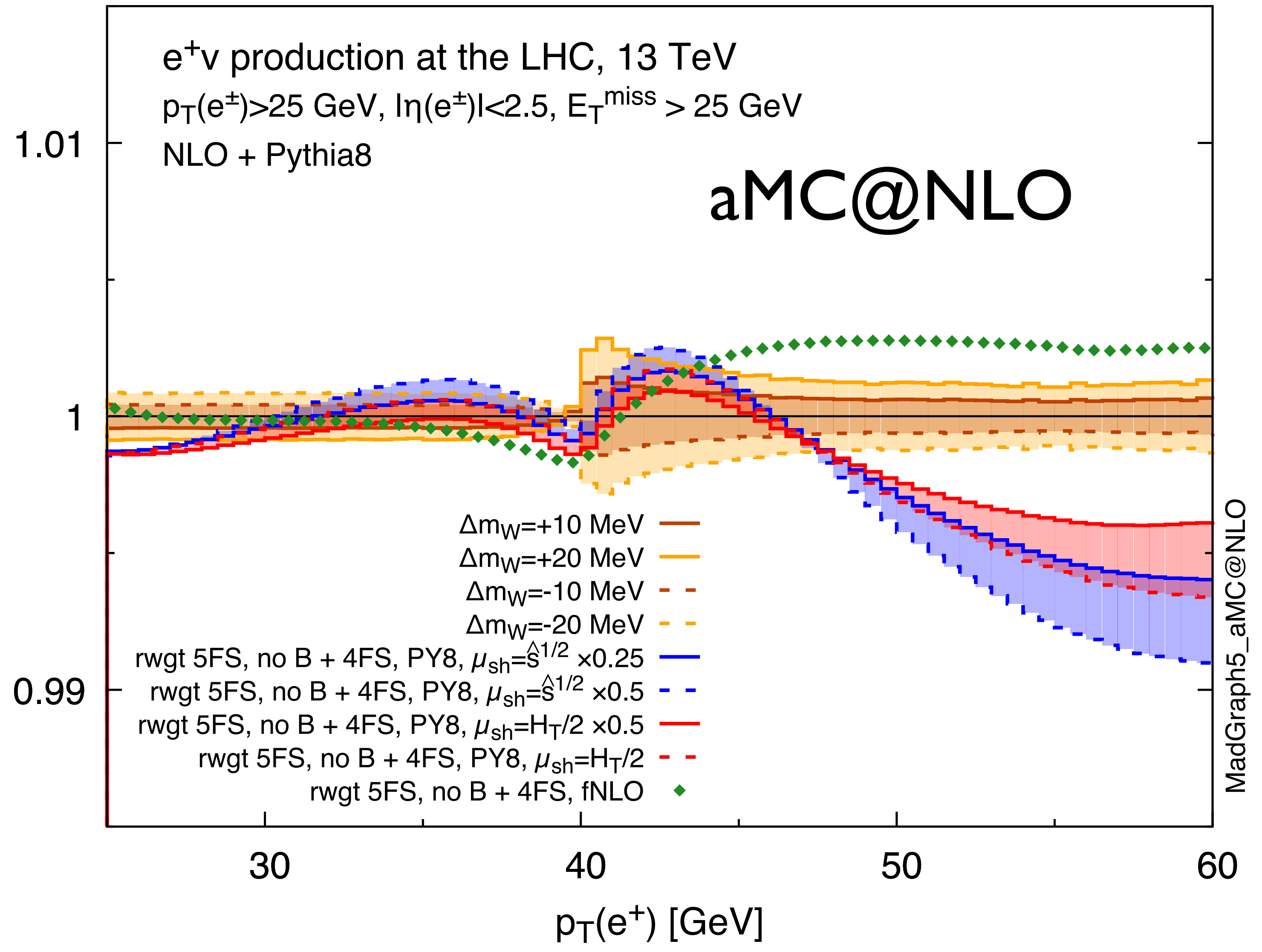
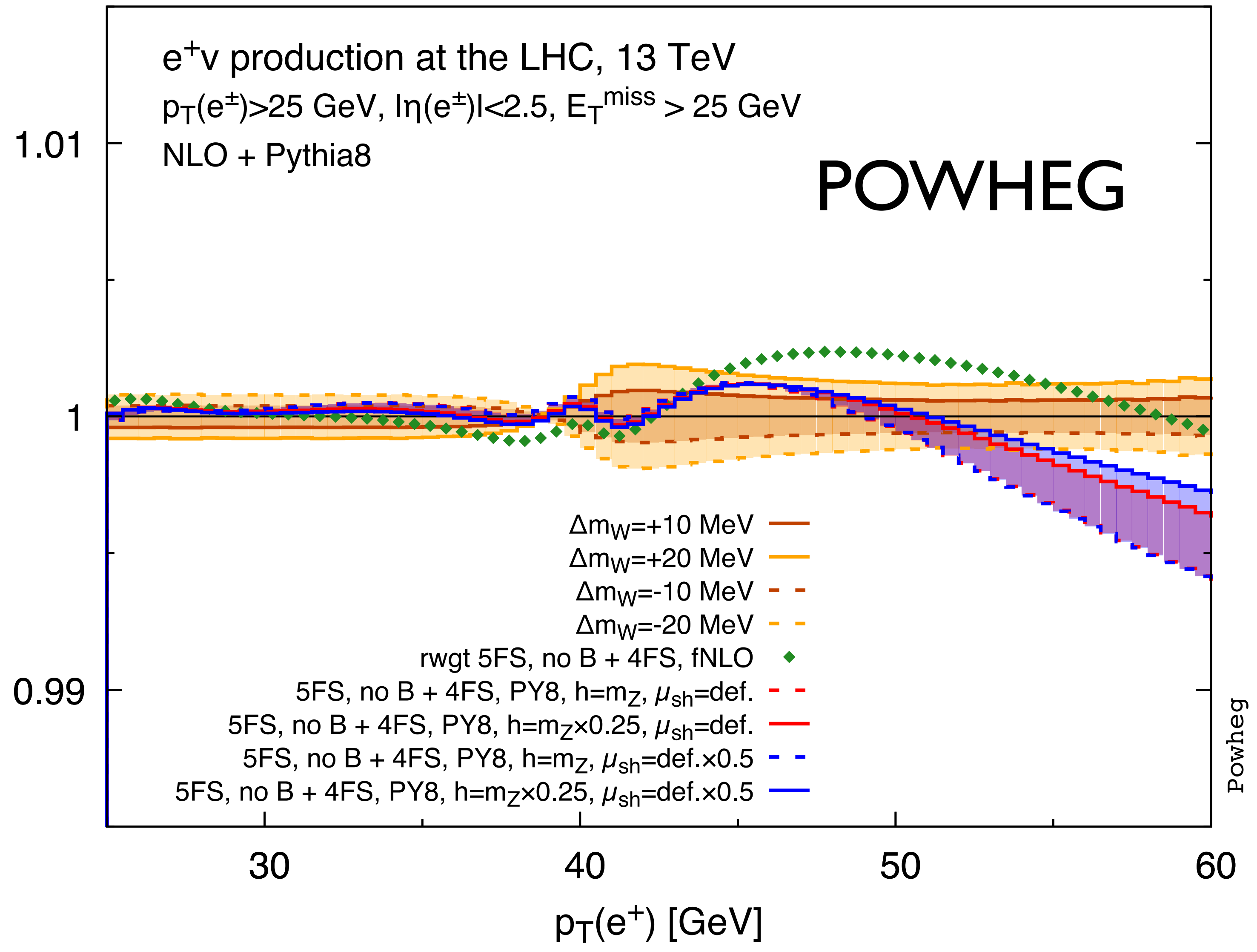
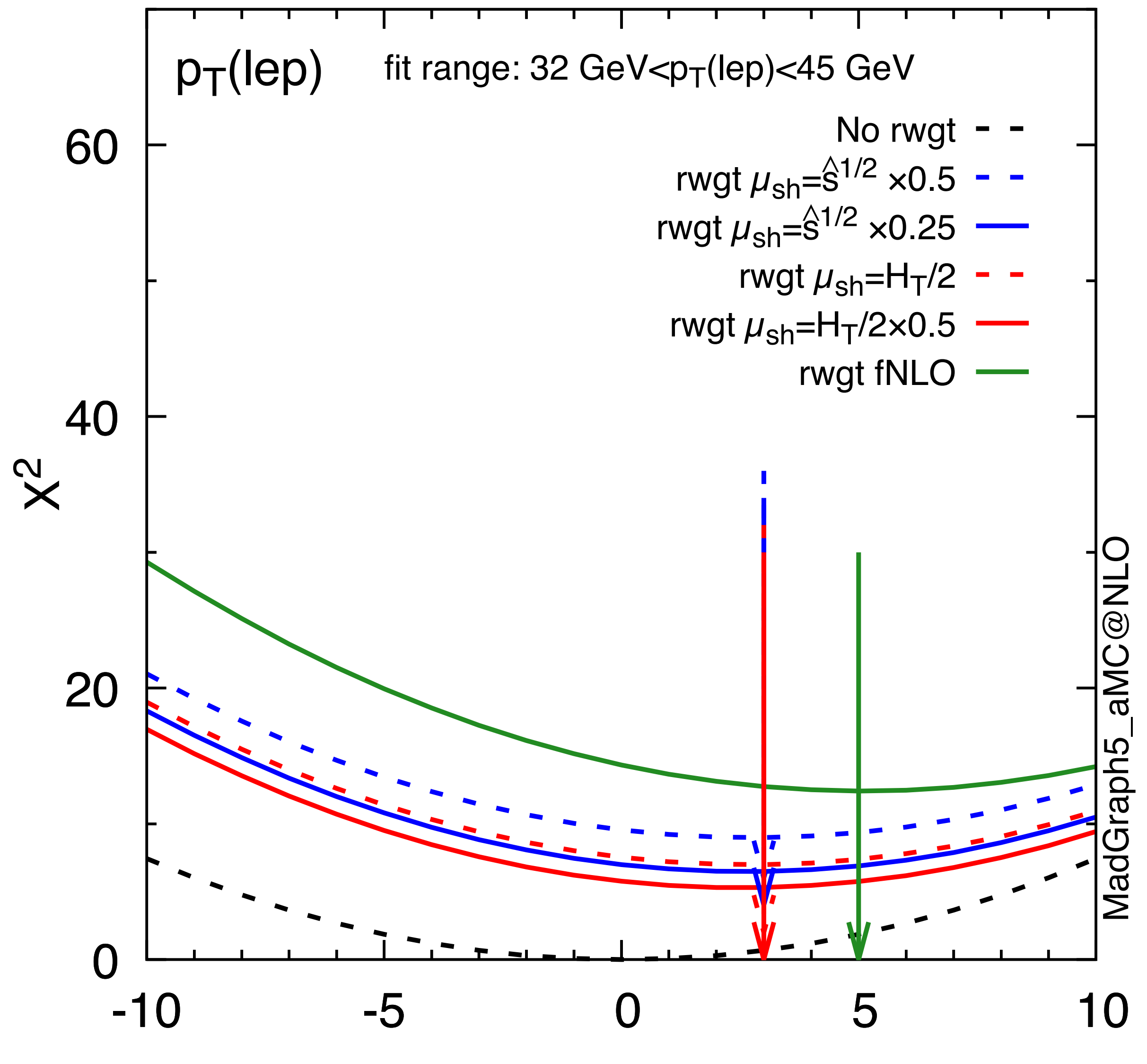


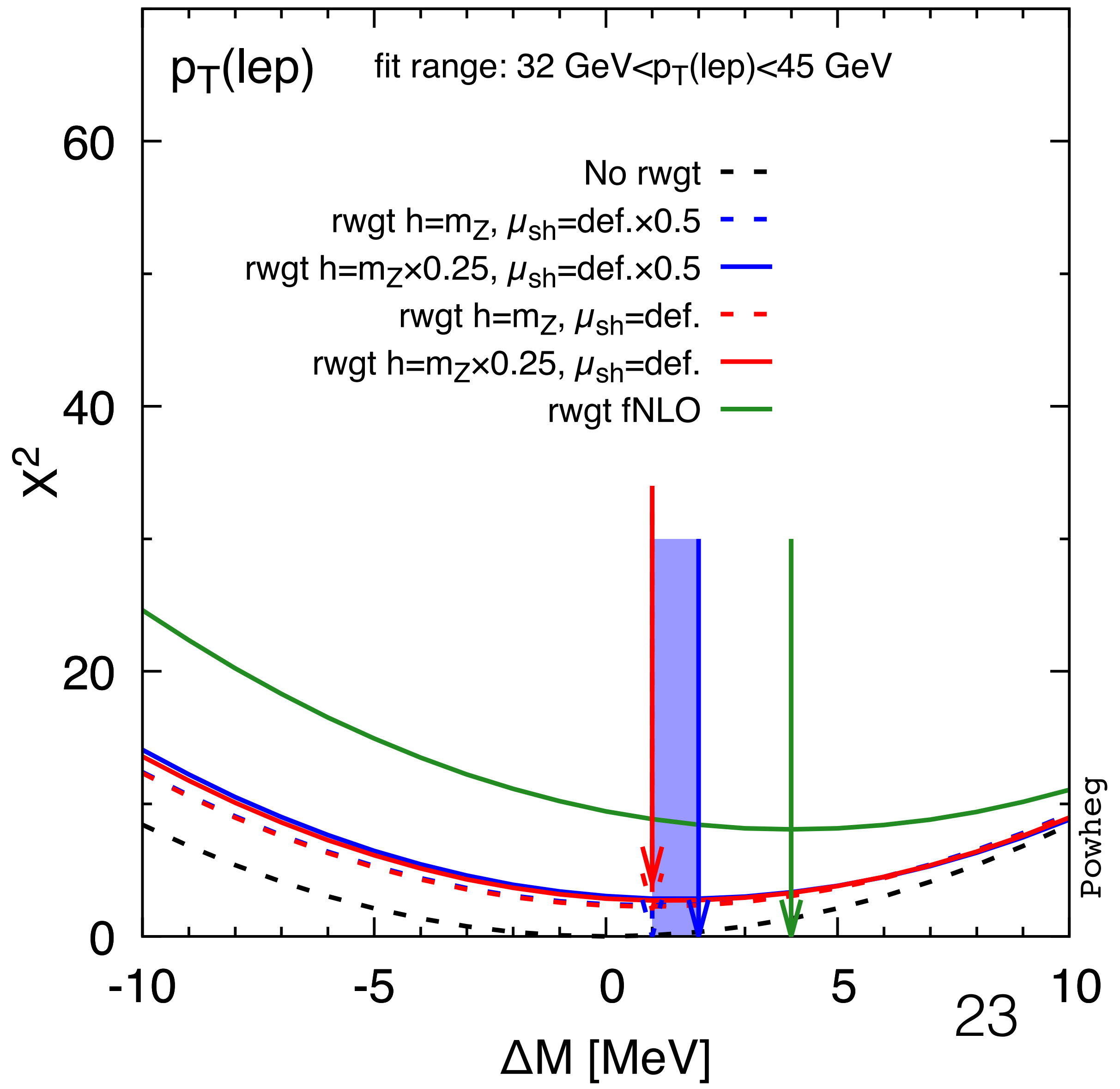


The explicit perturbative treatment of higher-order, flavour dependent effects yields a more universal, flavour independent PS tune

It is crucial for the m_W determination

- CC-DY and NC-DY have different b-quark contributions
- the missing QCD effects that we measure with ptZ and apply to ptW must be universal and flavour independent





The impact on MW is estimated by template fit of the reweighed distributions (RGB), with templates in the plain 5FS (light brown)

Fixed-order results → $\Delta m_W \sim 5$ MeV

Additional radiation reduces this effect down to $\Delta m_W \sim 3$ MeV

PDF uncertainties in the MW determination

PDF uncertainty on MW

- the proton parameterisation is fitted from experimental data and suffers from their error

- this uncertainty is represented by different parameterisations, in the hessian eigenvectors or Monte Carlo replicae language, all compatible with the data

- the templates used to fit m_W are computed using a given choice of the proton parameterisation
  changing this choice leads to different templates → different outcome of the fit (theoretical systematic error)

- the PDF uncertainty on m_W has 2 components:
  - the spread of the best values obtained with different PDF sets (due to methodological differences in the PDF fit)
  - for a given set, the uncertainty due to the experimental data

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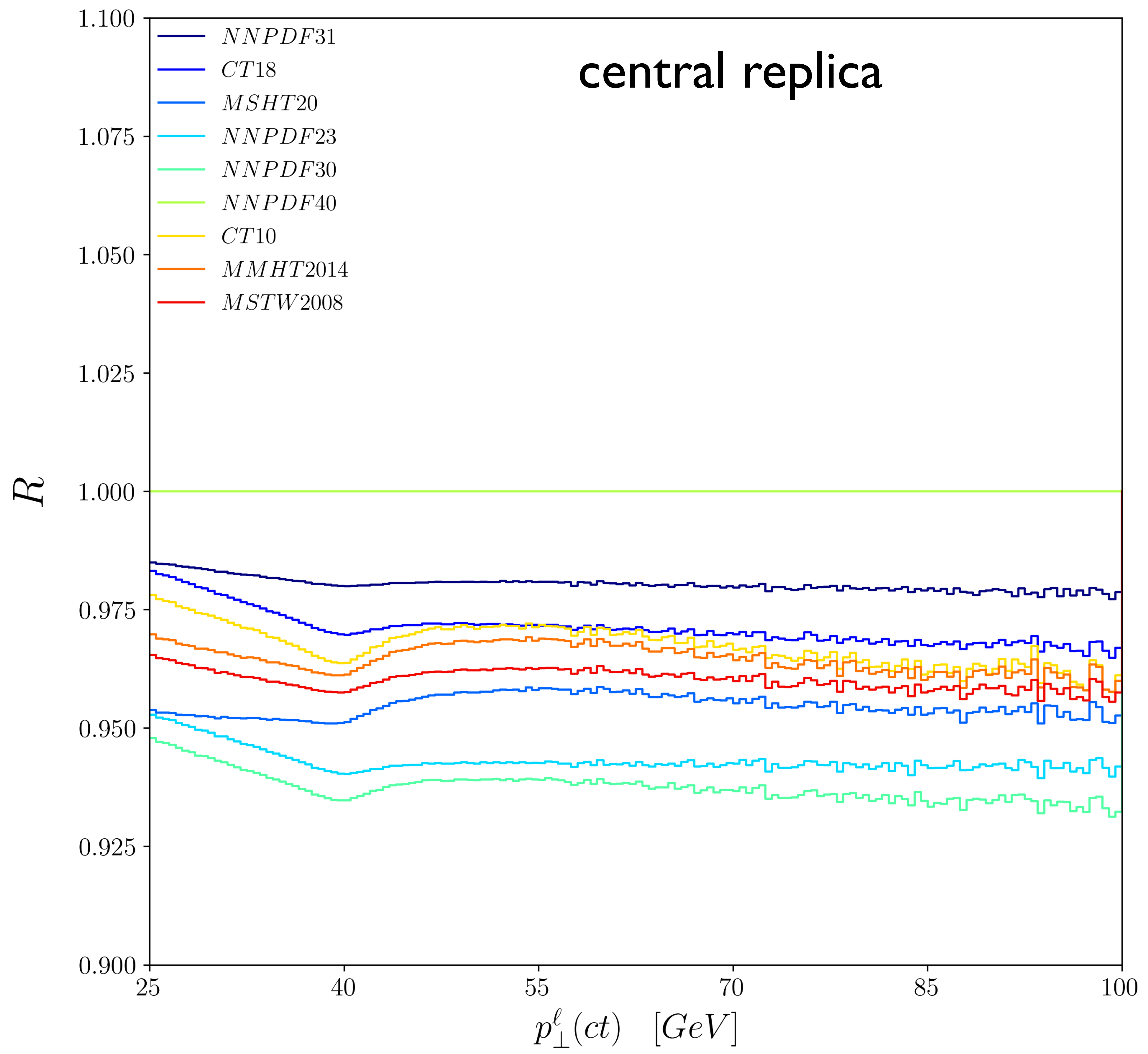
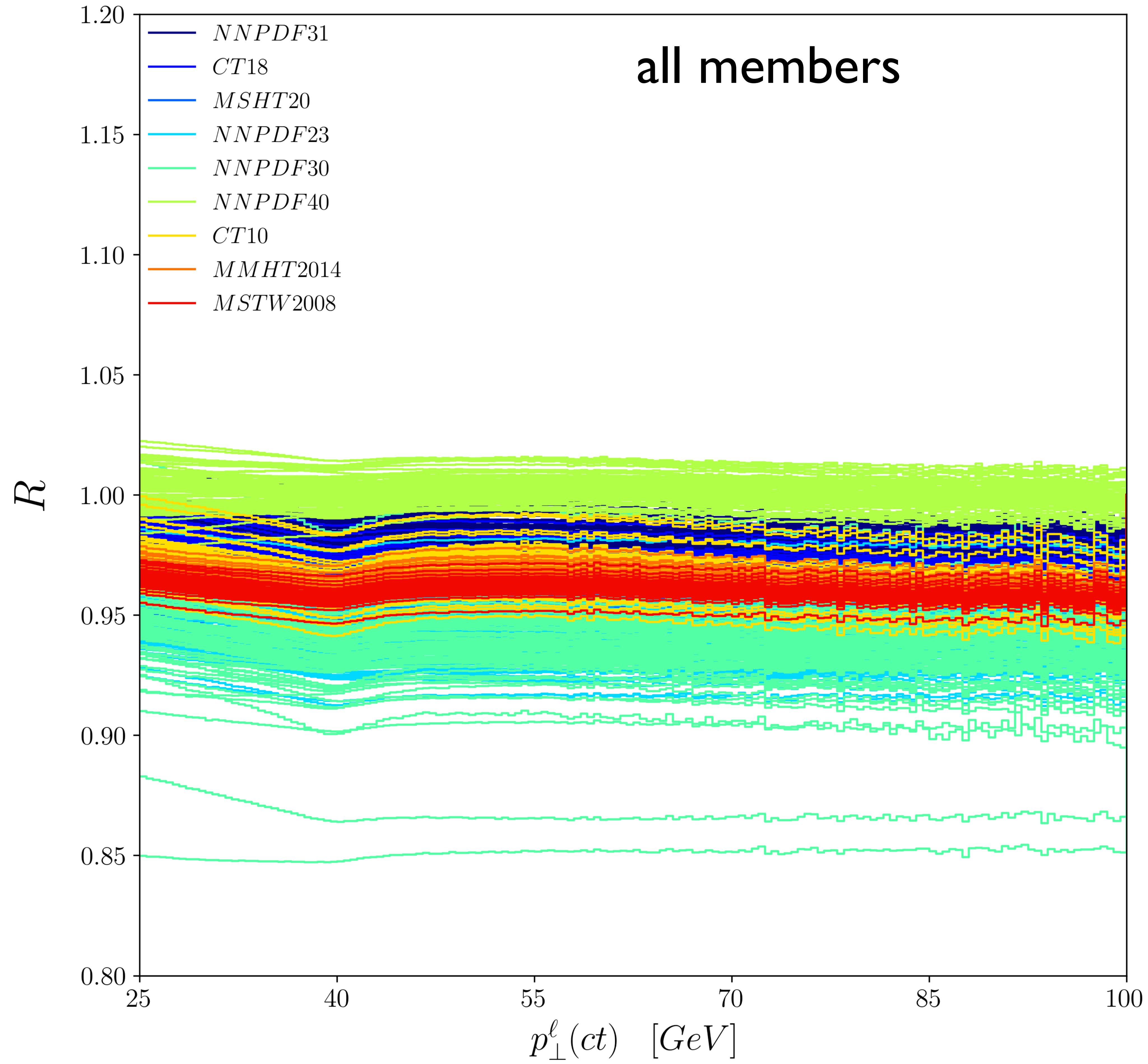
  - for a given set, the uncertainty due to the experimental data

- we are not discussing the propagation of the PDF error to the prediction of observables
  but
  how the different PDF alternatives affect the determination of a parameter (not an observable!)

- very important role of PDF correlations in the combination of the best-fit m_W values

Different predictions for the $p_\perp^l$ distribution





$p_\perp^l$ distribution simulated with POWHEG NLO-QCD + QCDPS
comparison taking as a reference the average replica of NNPDF4.0 differences in normalisation and shape

the distortion about $p_\perp^l \sim 40$ GeV is responsible for the m_W shifts

PDF uncertainty on MW: a conservative approach

G.Bozzi, L.Citelli, AV, arXiv:1501.05587   G.Bozzi, L.Citelli, M.Vesterinen, AV, arXiv:1508.06954

- choose the replica 0 of one set to compute all the templates, for different $m_{W,k}$ hypotheses
- generate one distribution j for each replica of a PDF set, using $m_{W,0}$ as a nominal value → pseudodata
- make a χ^2 fit of the pseudodata j using the templates k : call $\overline{m}_W^j$ the best fit result associated to the minimum $(\chi_k^2)_j^{min}$

  → the difference $\Delta m_W = \overline{m}_W^j - m_{W,0}$ expresses the impact of the distortion of the replica j compared to the PDF replica of the templates
- combine the N values $\overline{m}_W^j$ according to the prescription of the PDF collaboration (→ central value and dispersion)

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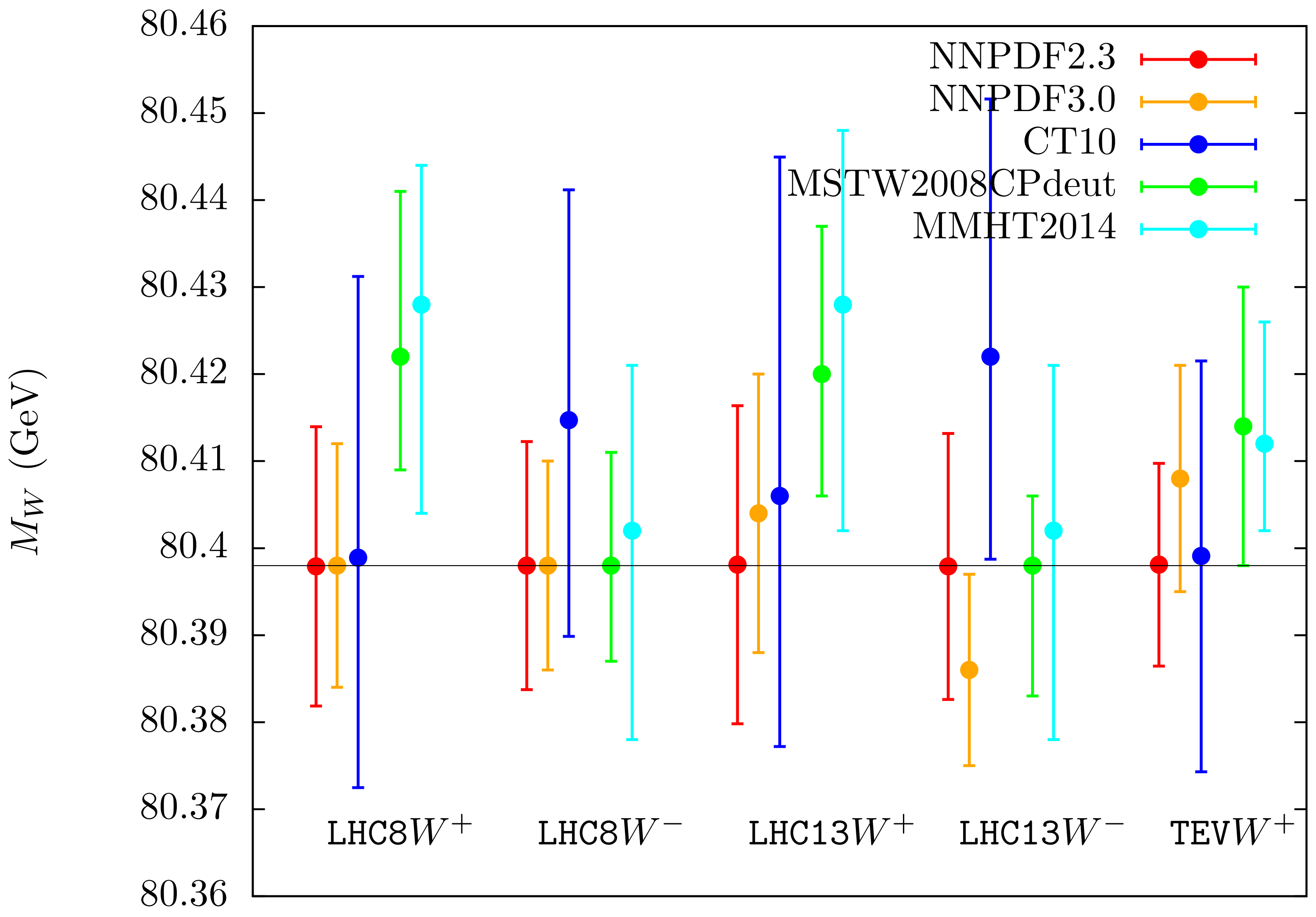

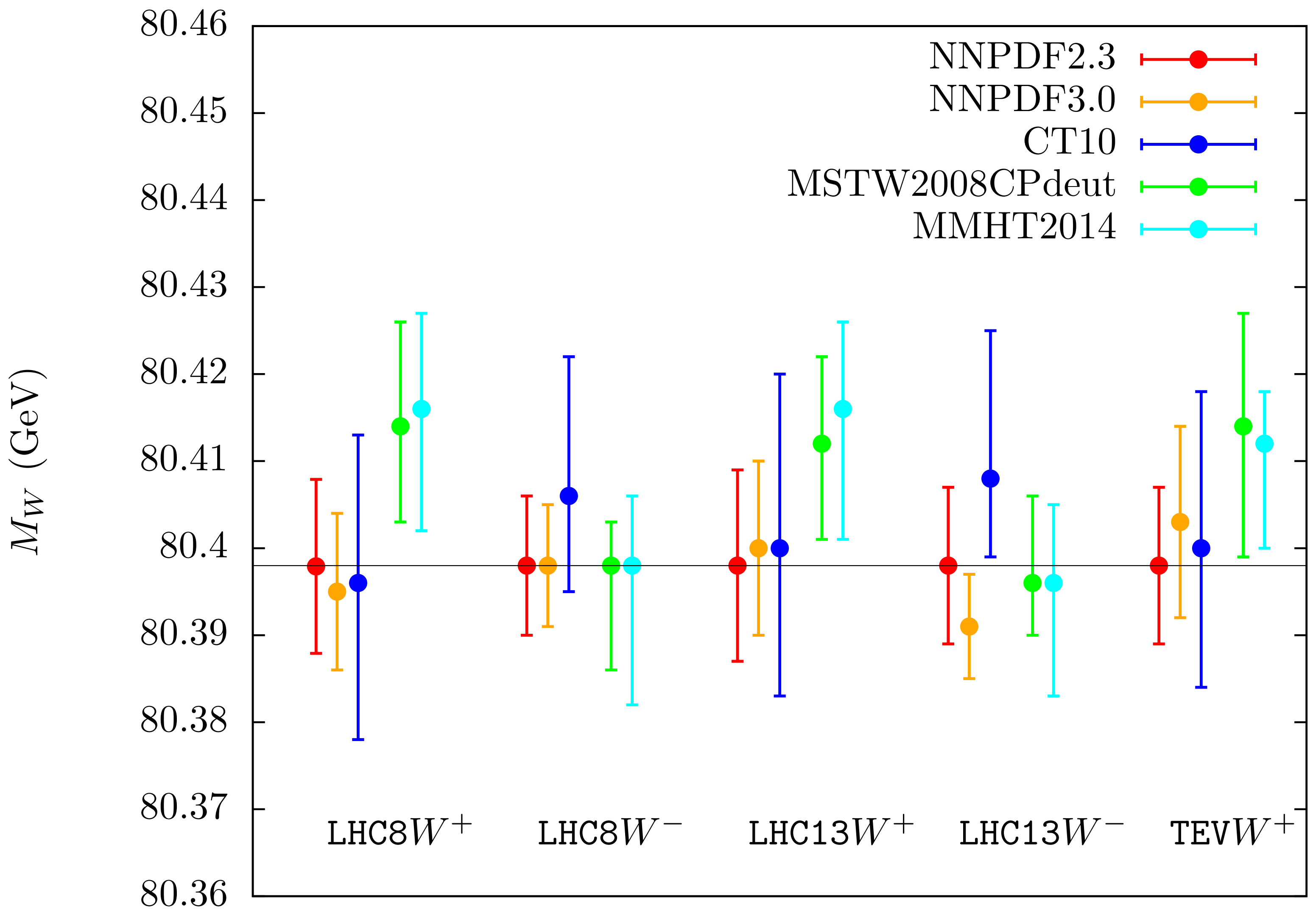

	no $p_\perp^W$ cut		$p_\perp^W < 15$ GeV	
	δ_{PDF} (MeV)	Δ_{sets} (MeV)	δ_{PDF} (MeV)	Δ_{sets} (MeV)
Tevatron 1.96 TeV	27	16	21	15
LHC 8 TeV W^+	33	26	24	18
W^-	29	16	18	8
LHC 13 TeV W^+	34	22	20	14
W^-	34	24	18	12

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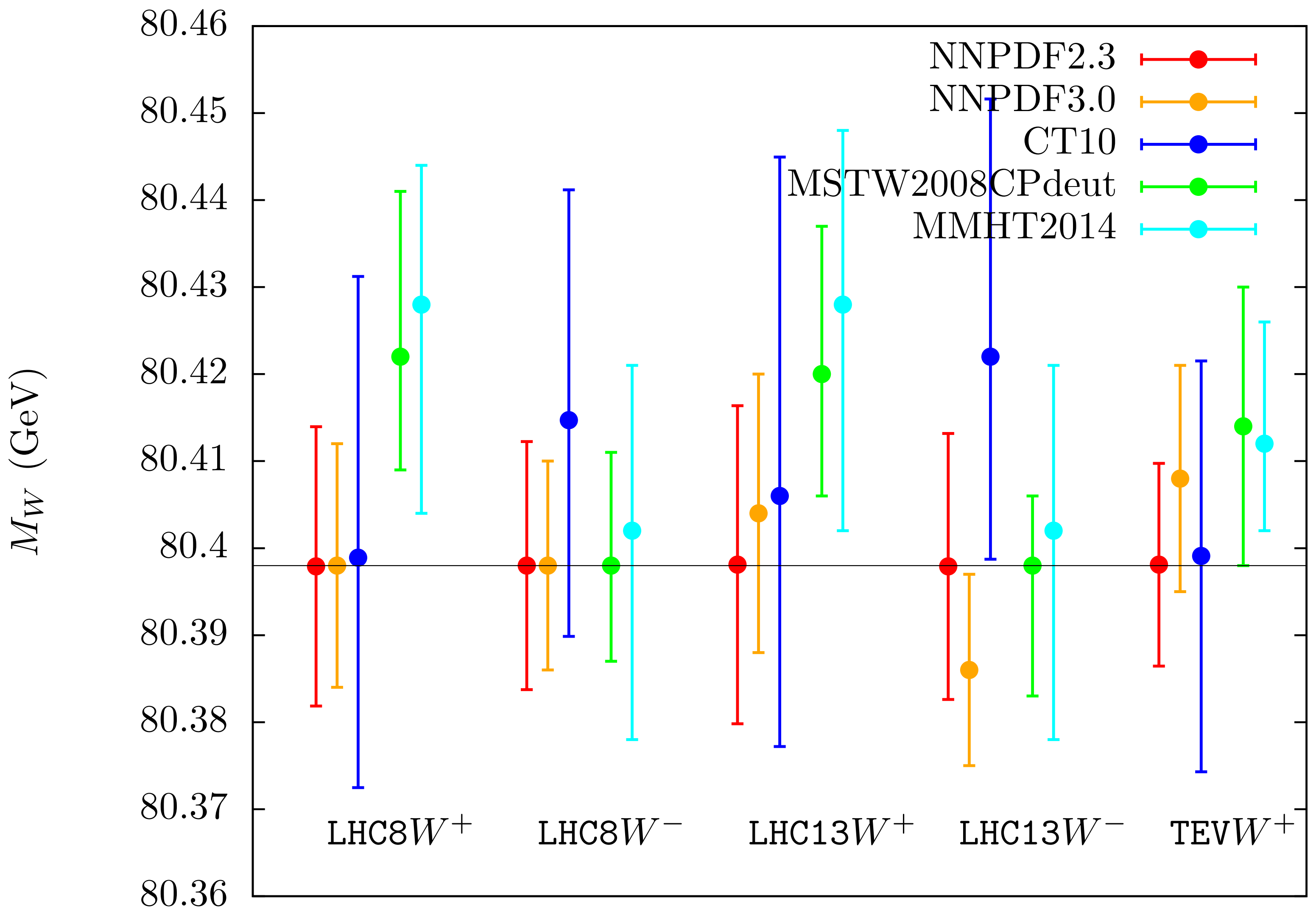

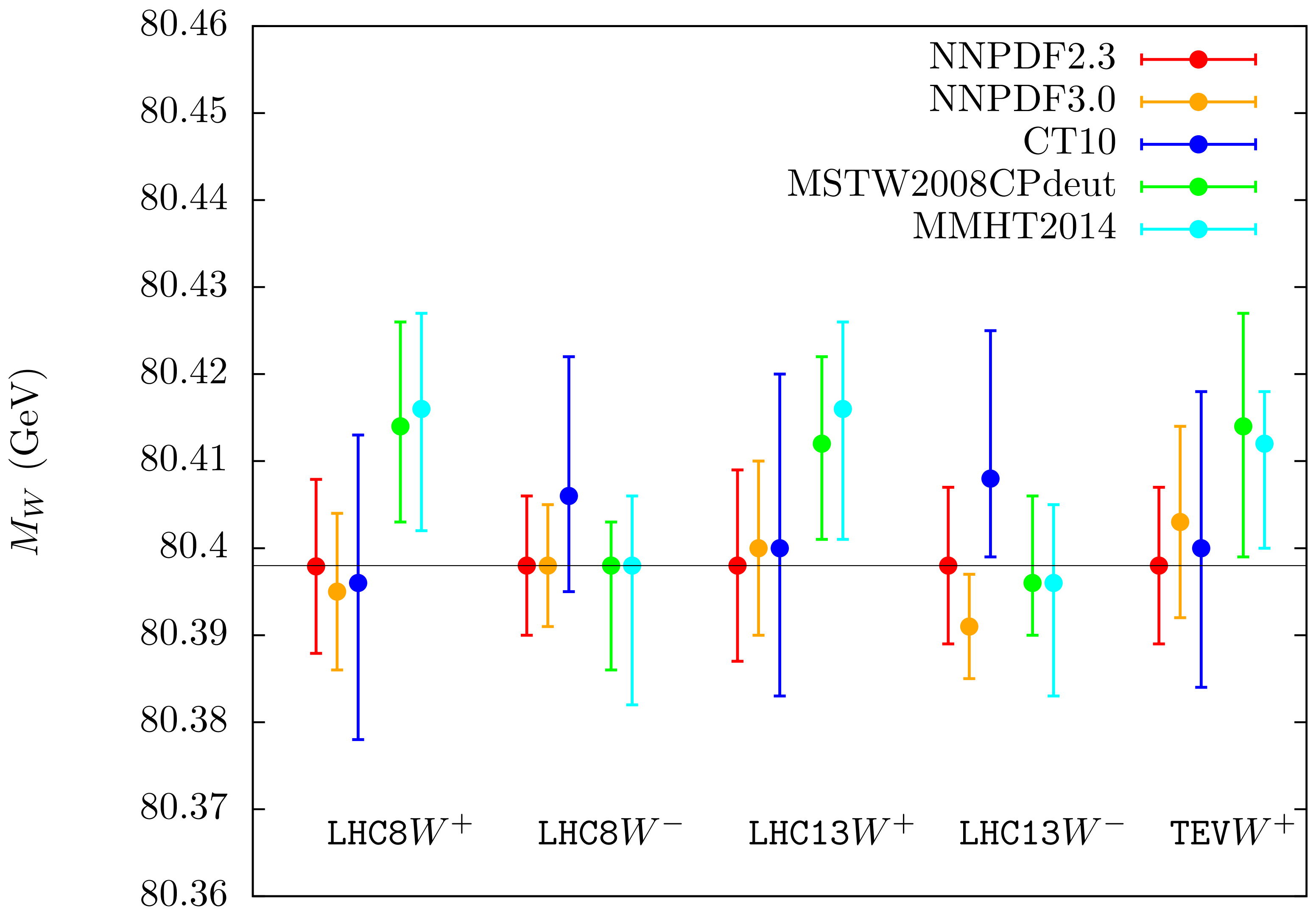

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Criticisms:

1) the different replicae do not describe the data with the same accuracy,
   i.e. we give equal weight to both good and bad fitting options, in the m_W combination used to compute the uncertainty
2) we do not exploit the available information encoded in the proton parameterisations

PDF uncertainty on MW: exploiting the available theoretical information

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

- the equivalence of the PDF replicae is a source of theoretical systematic error in the Drell-Yan simulation

- when writing a complete likelihood
  we choose the replica 0 as theoretical model called $\mathcal{T}_0$
  we treat the difference $\mathcal{S}_r = \mathcal{T}_r - \mathcal{T}_0$ w.r.t. the replica r as a systematic error, with the associated nuisance parameter α_r

$$\chi^2_k = \sum_{i \in \text{bins}} \frac{\left[(\mathcal{T}_{0,k})_i - (\mathcal{D}^{\text{exp}})_i - \sum_{r\in\mathcal{R}} \alpha_r (\mathcal{S}_{r,k})_i\right]^2}{\sigma_i^2} + \sum_{r\in\mathcal{R}} \alpha_r^2$$

- the minimisation, with respect to the nuisance parameters, leads to cast the χ^2 in terms of the inverse covariance matrix
  the covariance is defined with respect to variation of the systematic error, e.g. to the choice of different PDF replicae

$$\chi^2_{k,\text{min}} = \sum_{(r,s)\in\text{bins}} (\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}})_r (C^{-1})_{rs} (\mathcal{T}_{0,k} - \mathcal{D}^{\text{exp}})_s$$

$$C = \Sigma_{\text{PDF}} + \Sigma_{\text{stat}} + \Sigma_{\text{MC}} + \Sigma_{\text{exp,syst}}$$

$$(\Sigma_{\text{PDF}})_{rs} = \langle (\mathcal{T} - \langle \mathcal{T} \rangle_{\text{PDF}})_r (\mathcal{T} - \langle \mathcal{T} \rangle_{\text{PDF}})_s \rangle_{\text{PDF}}$$

- the data are fitted with the replica 0 only
  the PDF uncertainty is encoded in the PDF covariance matrix
  the width of the χ^2 parabola ($\Delta\chi^2 = 1$ rule) quantifies the PDF uncertainty

$$\langle \mathcal{O} \rangle_{\text{PDF}} \equiv \frac{1}{N_{\text{cov}}} \sum_{l=1}^{N_{\text{cov}}} \mathcal{O}^{(l)}$$

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

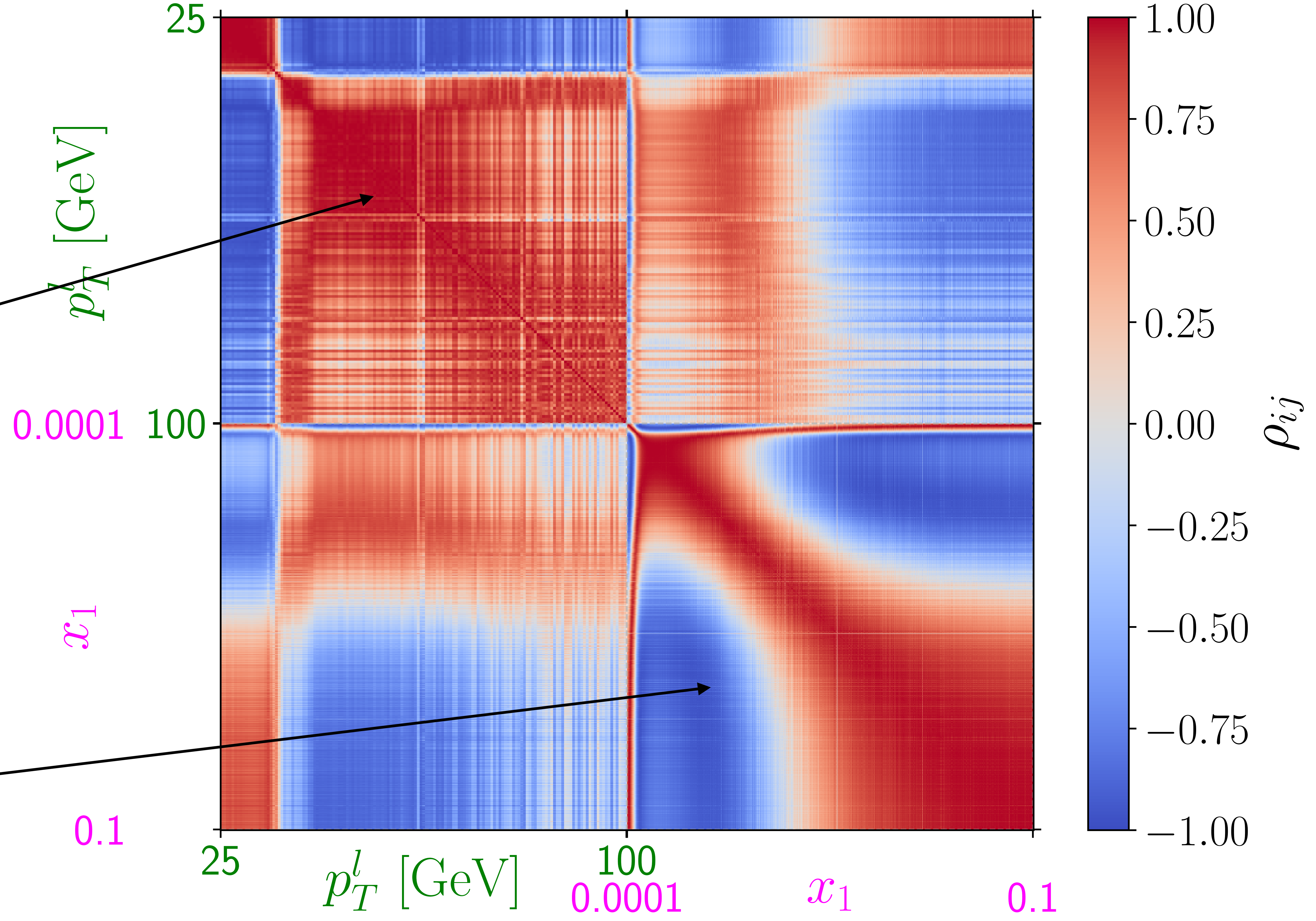
all PDF replicas are correlated because the parton densities are developed in the same QCD framework
1) obey sum rules, 2) satisfy DGLAP equations, 3) are based on the same data set

the "unitarity constraint" of each parton density affects the parton-parton luminosities, which, convoluted with the partonic xsec, in turn affect the hadron-level xsec

$$\rho_{ij} = \frac{\langle \left(\mathcal{O}_i - \langle \mathcal{O}_i \rangle_{PDF}\right) \left(\mathcal{O}_j - \langle \mathcal{O}_j \rangle_{PDF}\right) \rangle_{PDF}}{\sigma_i \sigma_j}$$

the tails of the $\frac{d\sigma}{dp_\perp^\ell}$ distribution
are strongly (anti)-correlated w.r.t. PDF variations

the tails of the $\frac{d\sigma}{dx}$ distribution
are strongly (anti)-correlated w.r.t. PDF variations

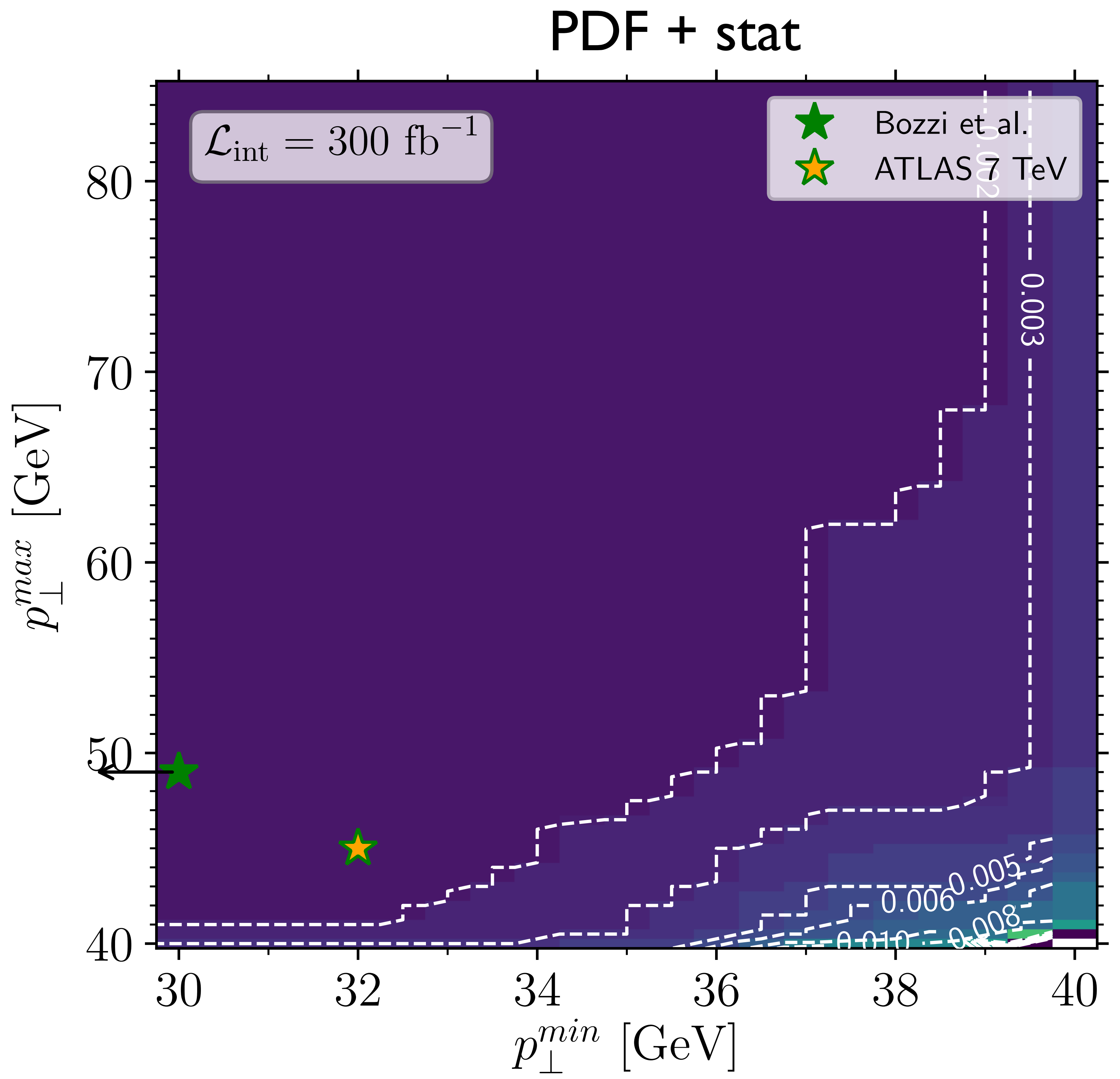


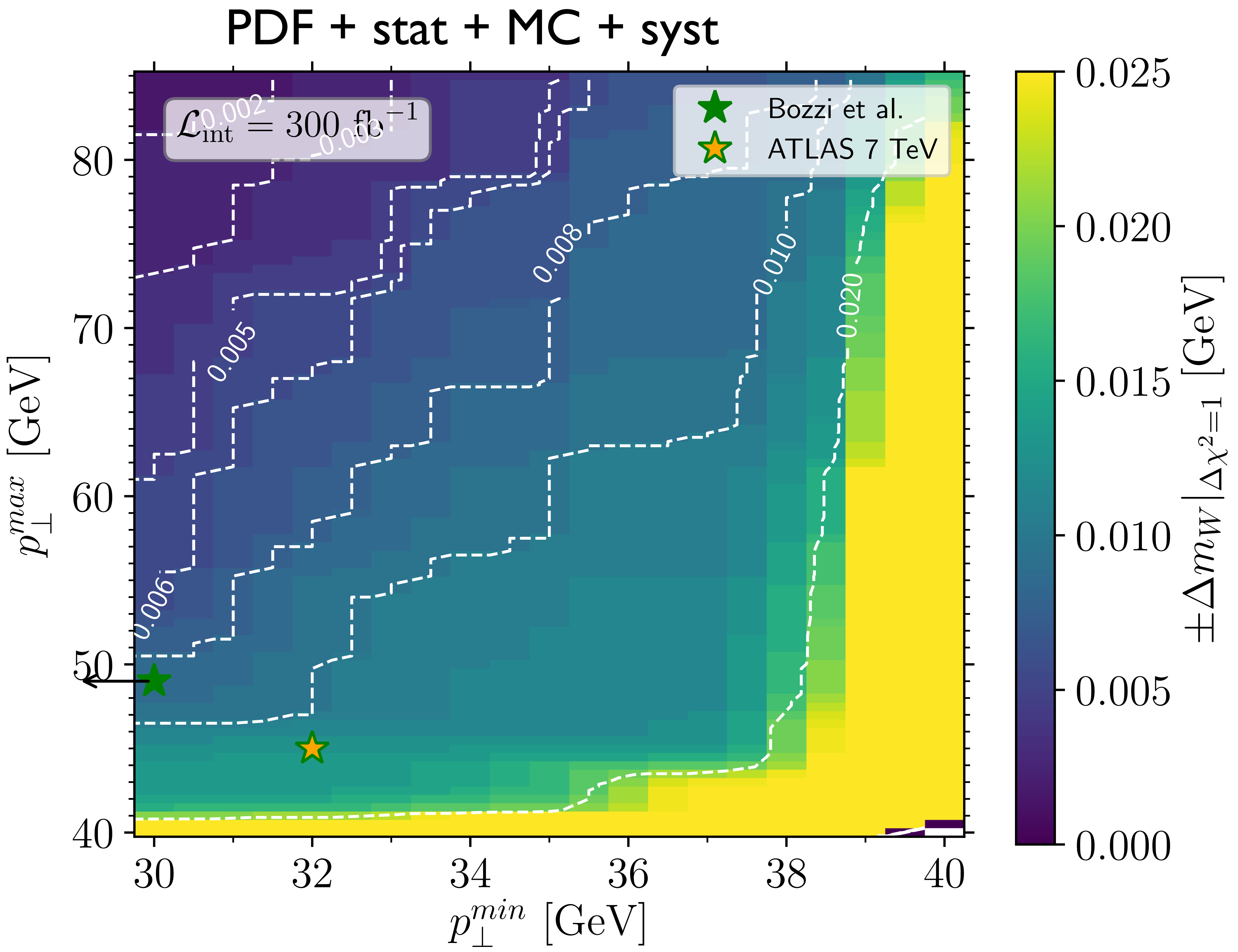
Non-trivial information is hidden in the distributions! We want to exploit it

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

scan over fitting windows for normalised distributions





$$\chi^2_{k,\,min} = \sum_{r,s\in bins} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_r C^{-1}_{rs} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_s$$

$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp\,syst}$ total covariance

total uncertainty determined with $\Delta\chi^2 = 1$ rule

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

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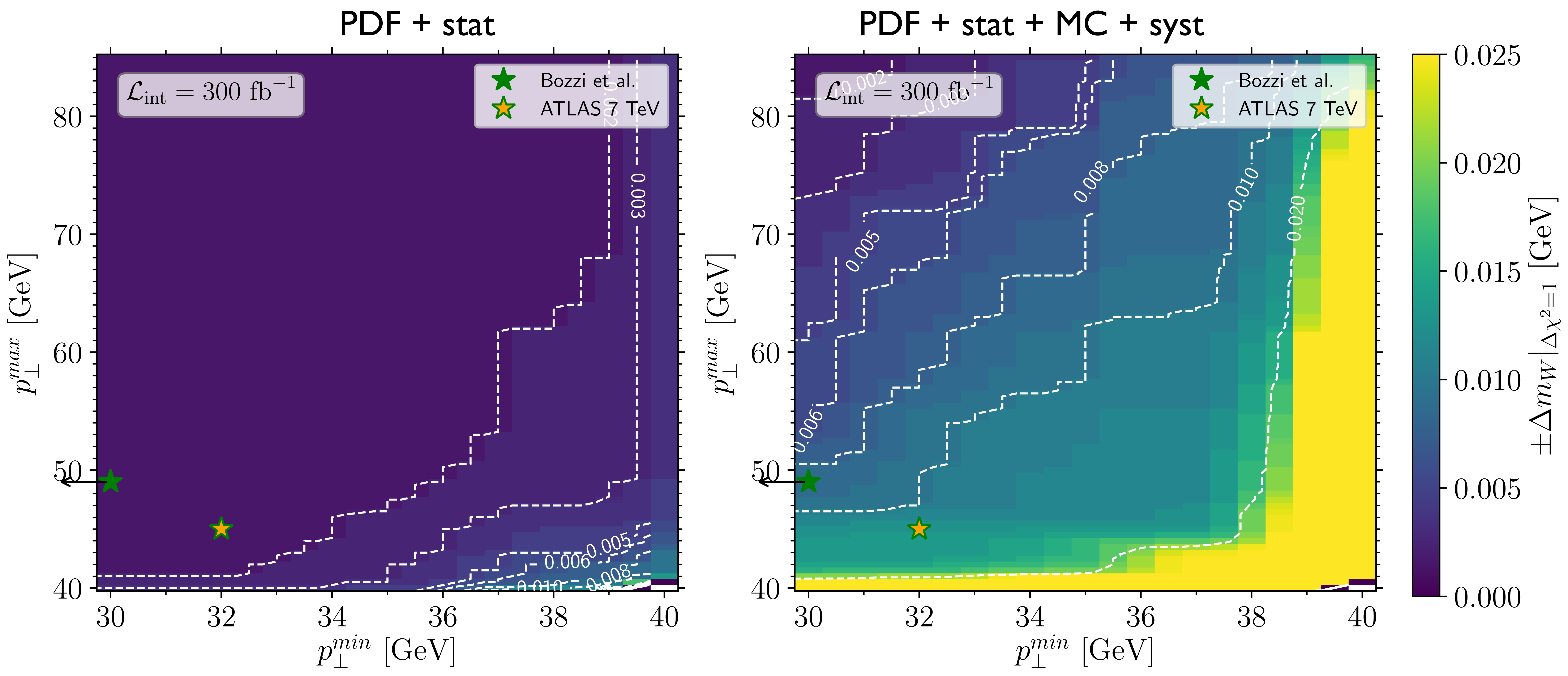

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total uncertainty determined with $\Delta\chi^2 = 1$ rule

$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp\,syst}$ total covariance

- The PDF uncertainty is not a limiting factor for MW with high luminosity and a "perfect" detector
- The MC statistics needed is of at least O(100B) of simulated events (several weeks on 1000 cores cluster)

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

- The full likelihood fit allows to exploit a very large of the information available in the replicae

- In the estimate of the PDF uncertainty, the perturbative QCD component plays a major role
  yielding the block structure of the covariance matrix
  (same results obtained with hessian PDF sets)

- In the estimate of the PDF uncertainty, little sensitivity to the non-perturbative input of the evolution equations (preliminary)
  the constraining power of the covariance matrix is always present

Conclusions

- The determination of m_W at hadron colliders strongly depends on our understanding of kinematical distributions such as $p_\perp^l$

- The perturbative description has reached in the last few years N3LL'+NNLO QCD for the gauge boson $p_\perp^V$ distribution with a residual uncertainty at the 2-3% level

- The flavour differences between W and Z production raise the issue of the universality of the QCD Parton Shower tunes
An explicit perturbative description of the heavy quark behaviour and of QED effects can lead to "more universal" results for the non-perturbative part of the tune

- The collinear PDF uncertainty affects m_W via the finite rapidity acceptance
the perturbative constraints of the collinear PDF formulation strongly affect the $p_\perp^l$ distribution,
largely reducing the estimated uncertainty on m_W (not a bottleneck!)
→ a full likelihood analysis is necessary for the proper estimate of the uncertainty on a fitted parameter

- The QED corrections have a large impact on m_W, the mixed QCD-EW corrections as well:
the "QCD model" modulates the size of the mixed corrections
→ a better QCD model and exact mixed QCD-EW corrections are needed to restrain this source of uncertainty

Back-up slides

The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, higher-order QCD corrections, resummation of reducible terms

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\mathrm{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\rm had}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\mathrm{GeV}}\right)$$

$$dh = [(m_H/125.15\ \mathrm{GeV})^2 - 1].$$

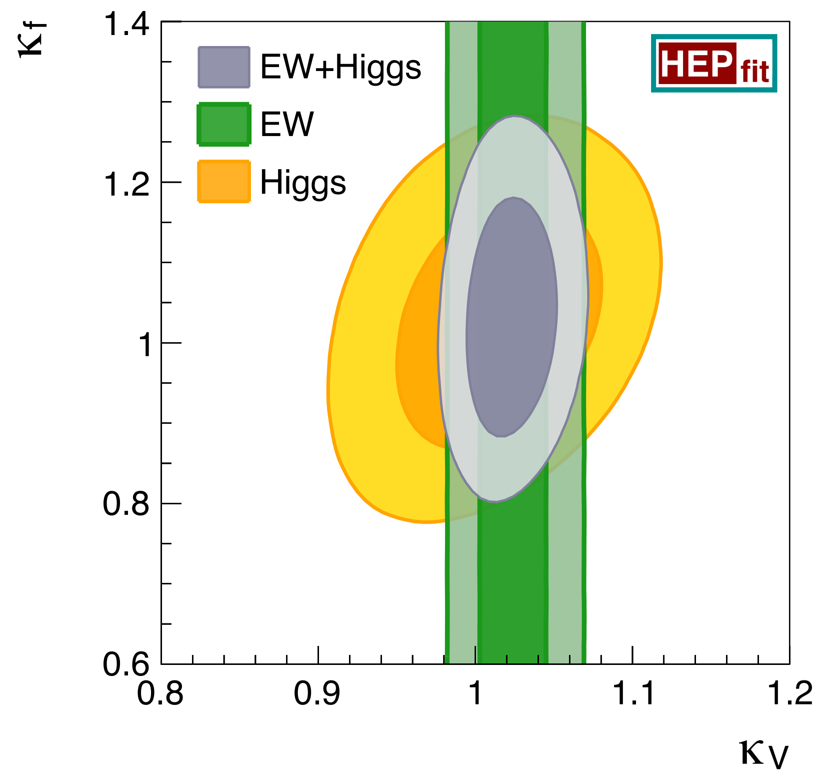
$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

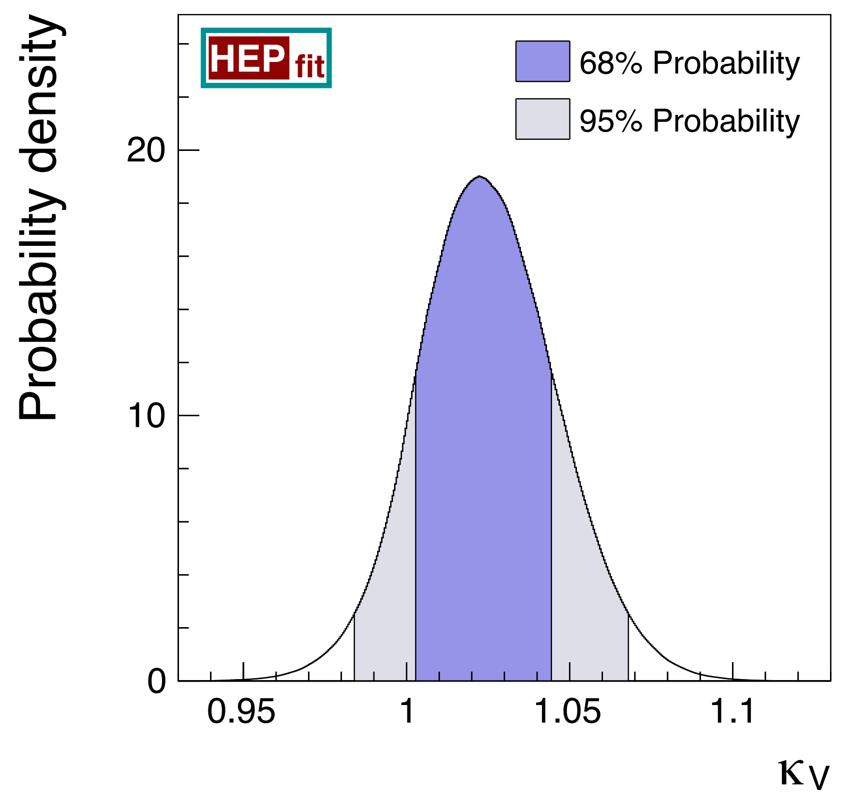
	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

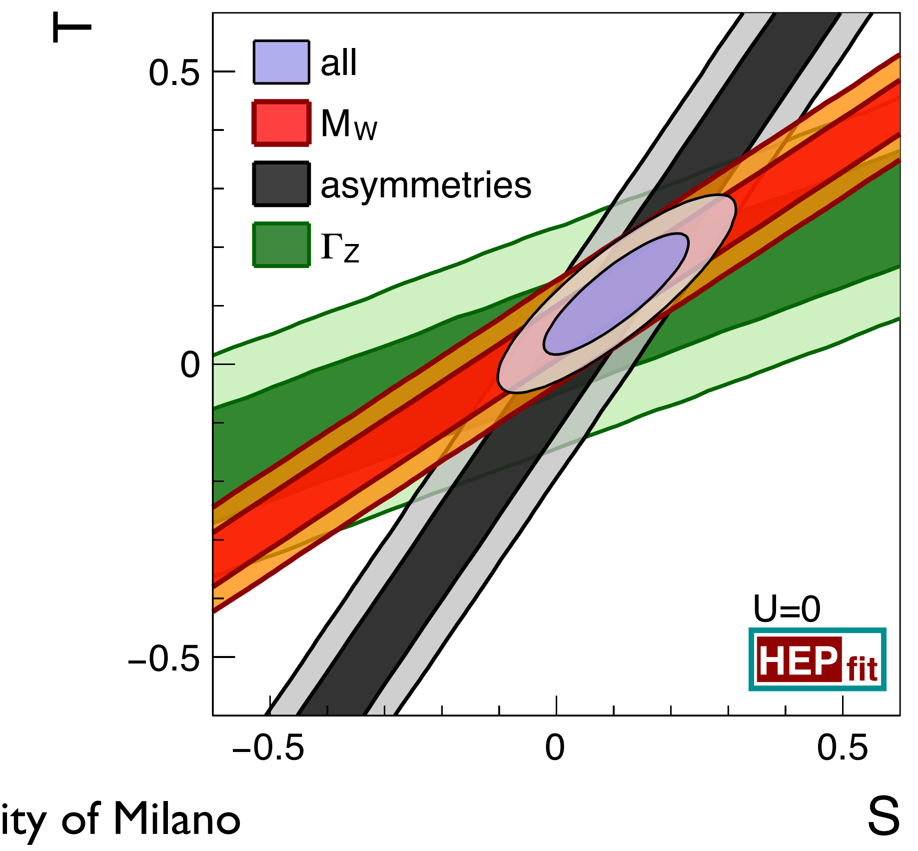
G.Degrassi, P.Gambino, P.Giardino, arXiv:1411.7040

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509







$$\mathcal{L}_{\text{Eff}} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \xrightarrow[\text{Effects suppressed by}]{} \left(\frac{q}{\Lambda}\right)^{d-4} \quad q = v, E < \Lambda$$

Λ: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu} \quad \text{EWSB} \quad \begin{cases} v^2 B^{\mu\nu} W^3_{\mu\nu} & \text{gauge boson masses} \\ v h B^{\mu\nu} W^3_{\mu\nu} & h \to ZZ, \gamma\gamma \end{cases}$$

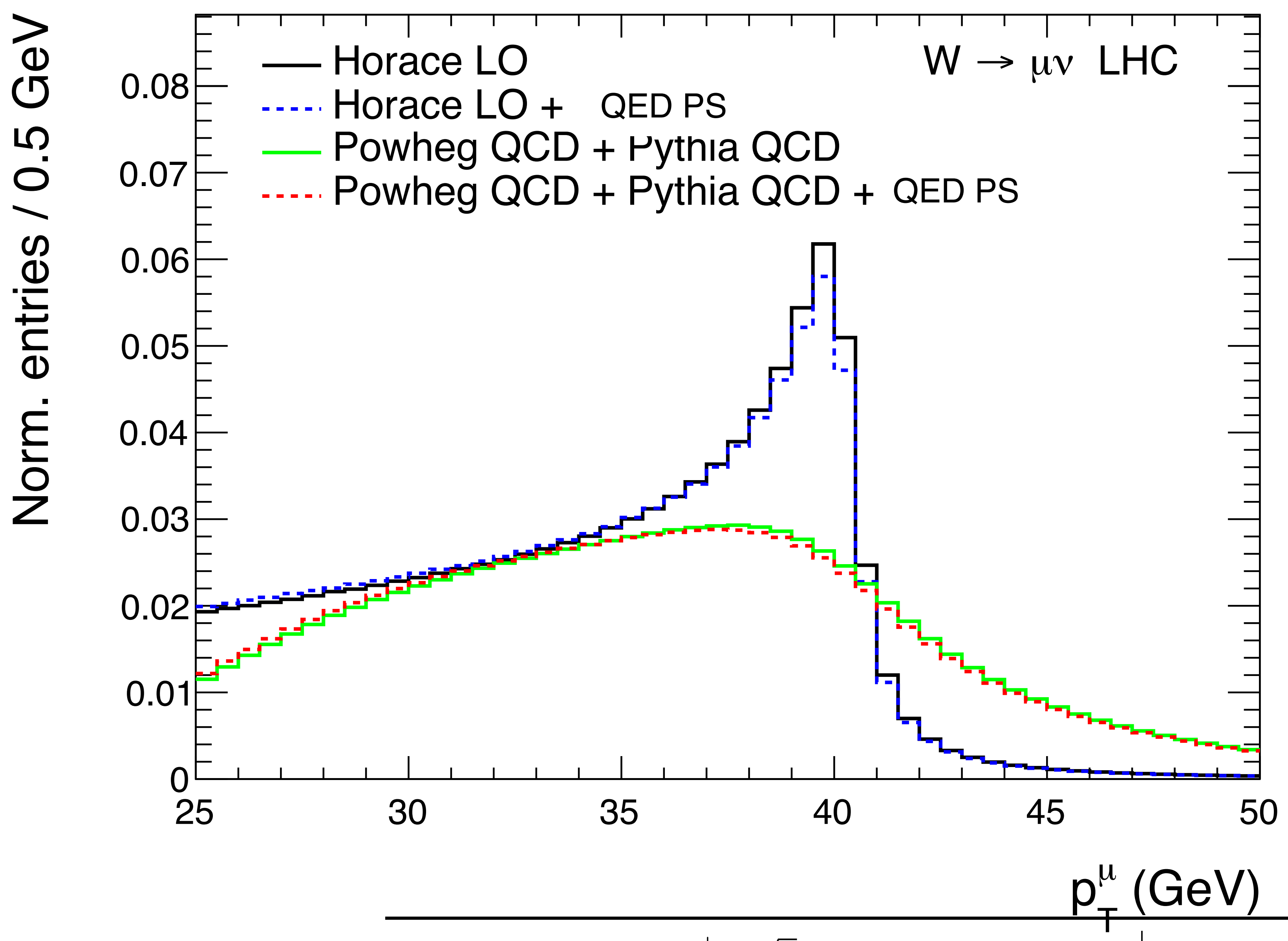
$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2} \left(\frac{1}{2} C_{\phi D} + 2\frac{s}{c} C_{\phi WB} + \frac{s^2}{c^2} \Delta_{G_\mu} \right) \frac{v^2}{\Lambda^2} \right]$$

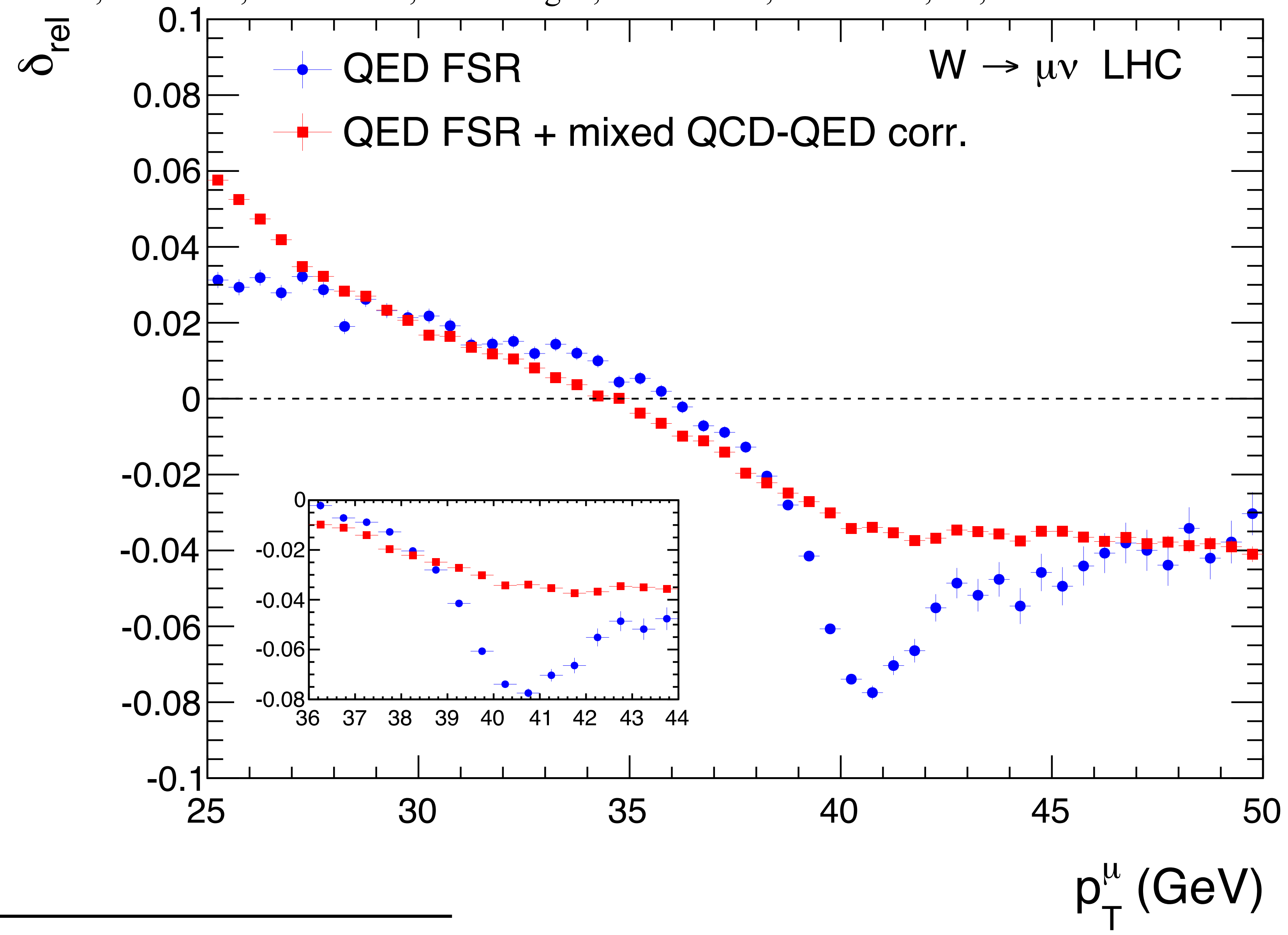
A precise measurement of m_W and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

Combined QCD-EW simulation tools: impact of QED-FSR on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841





	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
1	Horace only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	Horace FSR-LL	-89±1	-97±1	-179±1	-195±1
3	Horace NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	Horace FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	Photos FSR-LL	-92±1	$\boxed{-100\pm2}$	-182±1	-199±2

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{PS}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{PS}$	Photos	-88.0±0.6	$\boxed{-368\pm2}$	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{PS}$`two-rad`	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{PS}$`two-rad`	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

given that the bulk of the corrections is included in the analyses
- what is the associated uncertainty ?
- what happens if we change the underlying QCD model ?

can we constrain the formulation, for the $\alpha\alpha_s$ contribution ?
an exact NNLO QCD-EW calculation,
matched with QCD and QED Parton Shower
could push the ambiguities one order higher

A few comments on the MW determination at hadron colliders

The determination of m_W requires the calculation of templates for the shapes of $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distr. as a function of m_W

Within the Standard Model, the shape depends on several factors:

QCD ISR, QED FSR, PDF choice, matching algorithms in QCD, QED and mixed, EW effects, at least…

In the m_W studies, the available perturbative results are not yet sufficient to describe the data

The excellent knowledge of the $p_\perp^Z$ spectrum is then used to build a QCD-based model "reading from the data" the missing QCD higher orders.

This reweighing factor is corrected for the differences between Z and W, relying on theoretical results. $p_\perp^{W,'exp'} = p_\perp^{Z,exp} \dfrac{p_\perp^{W,th}}{p_\perp^{Z,th}}$

The corrected reweighing is applied in the calculation of the templates for the CC-DY process.

What can we quantitatively say about the theoretical uncertainties ?

classification of purely EW corrections and estimate of the missing higher orders is a quite robust procedure

the estimate of the size of the mixed QCD-QED corrections is possible, for a given QCD model

the QCD-based model tuned on the data has very reduced uncertainties, by construction

Improved prediction of the ptZ distribution: combining 5FS and 4FS

- the prediction of the ptZ distribution, inclusive over radiation, is split into two contributions with and without B hadrons in the final state

- we rely on the 5FS for the contributions without B hadrons (light quarks ~ massless partons)
  4FS for the contributions with B hadrons (exact massive kinematics +NLOPS acc.)
  and we combine the two results

- need to compare with analytical resummation in SCET, where a systematic handling of all large logarithmic corrections, at each ptZ value, is implemented

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  and we combine the two results

- in the 5FS B hadrons are generated by the QCD PS with two mechanisms:
  i) presence of a bottom quark in the initial state (b bbar and bg initiated subprocesses)
  ii) gluon splitting into b bbar

  → the contribution without B hadrons is computed in the 5FS imposing a veto on the presence of B hadrons in the event analysis

- the contribution with B hadrons is computed in the 4FS
  by definition the process $pp \to e^+e^- b\,\bar{b}$ contains bottom quarks in the final state
  additional b bbar pairs may be produced by gluon splitting

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by definition the process $pp \to e^+e^- b\,\bar{b}$ contains bottom quarks in the final state
additional b bbar pairs may be produced by gluon splitting

$$\frac{d\sigma^{best}}{dp_\perp^{l^+l^-}} = \frac{d\sigma^{\mathtt{5FS-Bveto}}}{dp_\perp^{l^+l^-}} + \frac{d\sigma^{4FS}}{dp_\perp^{l^+l^-}}$$

· need to compare with analytical resummation in SCET, where a systematic handling of all large logarithmic corrections, at each ptZ value, is implemented

Impact on CC-DY of the improvements in the ptZ description

Assumptions:

· it is possible in the 5FS to tune the QCD-PS to perfectly reproduce the experimental data (`tune1`)

· it is possible also in the improved approximation
to tune the QCD-PS to perfectly reproduce the experimental data (`tune2`)

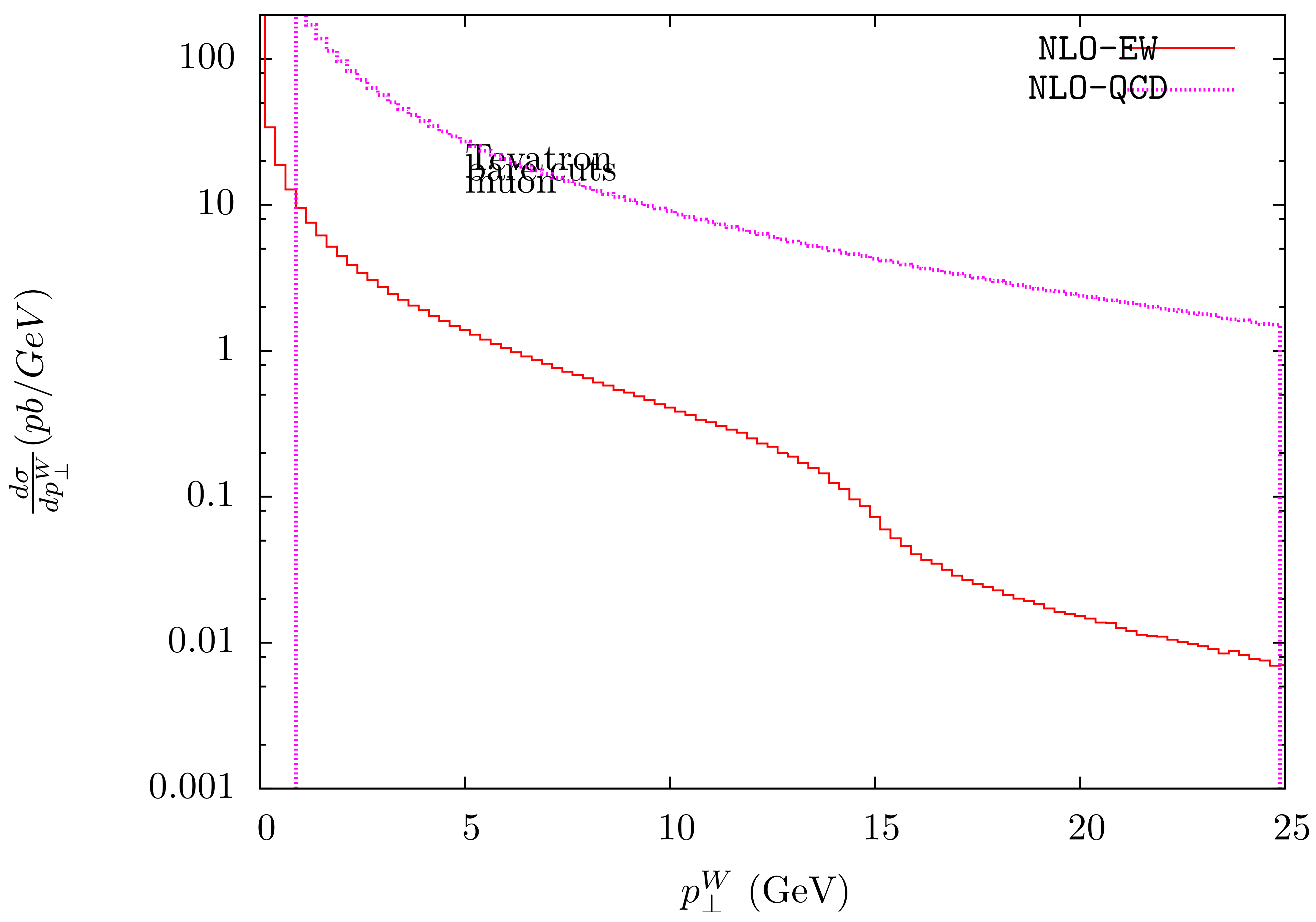
$$\frac{1}{\sigma_{fid}^{exp}}\frac{d\sigma^{exp}}{dp_\perp^{l^+l^-}} = \frac{1}{\sigma_{fid}^{5FS}}\left.\frac{d\sigma^{5FS}}{dp_\perp^{l^+l^-}}\right|_{\texttt{tune1}} = \frac{1}{\sigma_{fid}^{best}}\left.\frac{d\sigma^{best}}{dp_\perp^{l^+l^-}}\right|_{\texttt{tune2}} = \mathcal{R}(p_\perp^{l^+l^-})\frac{1}{\sigma_{fid}^{5FS}}\left.\frac{d\sigma^{5FS}}{dp_\perp^{l^+l^-}}\right|_{\texttt{tune2}}$$

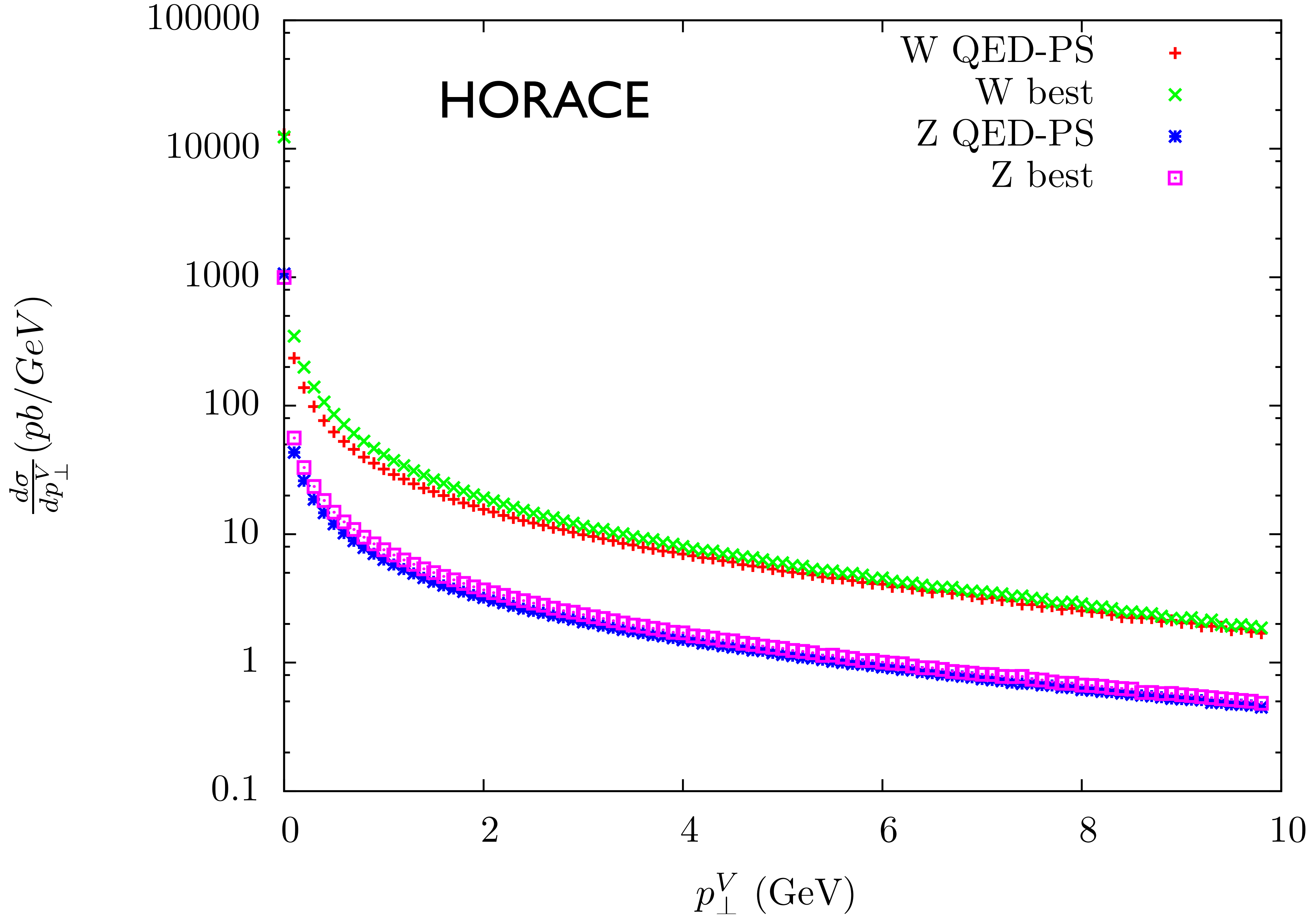
· $\mathcal{R}(p_\perp)$ expresses the difference of the predictions obtained in the best partonic approximation convoluted respectively with `tune1` and `tune2`

$$\frac{1}{\sigma_{fid}^{5FS}}\left.\frac{d\sigma^{5FS}}{dp_\perp^{l^+l^-}}\right|_{\texttt{tune2}} = \frac{1}{\mathcal{R}(p_\perp^{l^+l^-})}\frac{1}{\sigma_{fid}^{5FS}}\left.\frac{d\sigma^{5FS}}{dp_\perp^{l^+l^-}}\right|_{\texttt{tune1}}$$

· we use $\mathcal{R}(p_\perp)$ to reweigh the CC-DY events according to their ptW value

QED induced W(Z) transverse momentum





QED contribution to the PTV spectra is O(1%) of the QCD component

Differences between W and Z because of flavour structure

Bulk of the contribution due to QED-FSR,
but
matching with full NLO-EW adds more contributions,
again different between W and Z

$$\langle p_\perp^V \rangle =$$

Z FSR-PS	0.409	GeV
Z best	0.463	GeV
W FSR-PS	0.174	GeV
W best	0.207	GeV

Estimate of the "non-final state" component different in the 2 cases
$\Delta\langle p_\perp V\rangle$ = 54 (Z) - 33 (W) = 21 MeV