Jet as a probe to nucleon tomography (at small-x)

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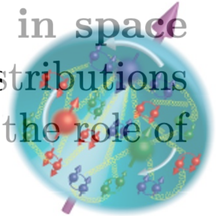
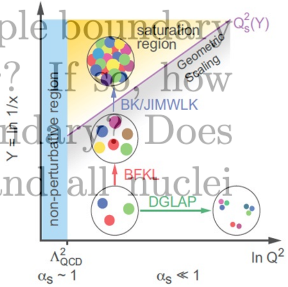
Big questions for EIC

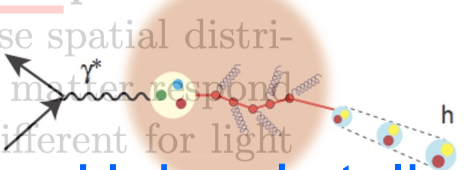
- **How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?** How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction? What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?



- **Where does the saturation of gluon densities set in?** Is there a simple boundary that separates this region from that of more dilute quark-gluon matter? If so, how do the distributions of quarks and gluons change as one crosses the boundary? Does this saturation produce matter of universal properties in the nucleon and all nuclei viewed at nearly the speed of light?



- **How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?** How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks?



Unified view of the Nucleon

❑ Wigner distributions (Belitsky, Ji, Yuan)

5D

$W(x, b_T, k_T)$
Wigner distributions

$\int d^2 b_T$ $\quad$ $\int d^2 k_T$

3D

$f(x, k_T)$
transverse momentum distributions (TMDs)
semi–inclusive processes

$f(x, b_T)$
impact parameter distributions

Fourier trf.
$b_T \leftrightarrow \Delta$

$H(x, 0, t)$
$t = -\Delta^2$

$\xi = 0$

$H(x, \xi, t)$
generalized parton distributions (GPDs)
exclusive processes

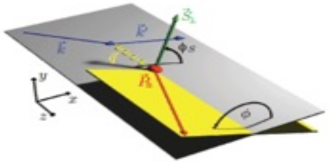
1D

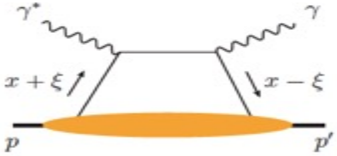
$\int d^2 k_T$ $\quad$ $\int d^2 b_T$ $\quad$ $\int dx$ $\quad$ $\int dx x^{n-1}$

$f(x)$
parton densities
inclusive and semi–inclusive processes

$F_1(t)$
form factors
elastic scattering

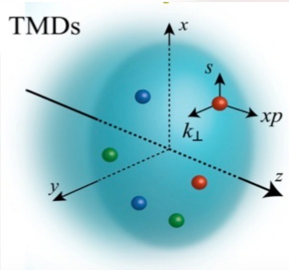
$A_{n,0}(t) + 4\xi^2 A_{n,2}(t) + ...$
generalized form factors
lattice calculations

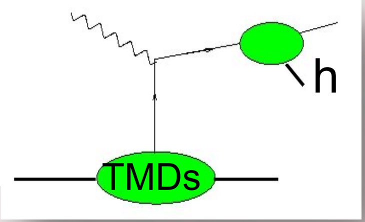




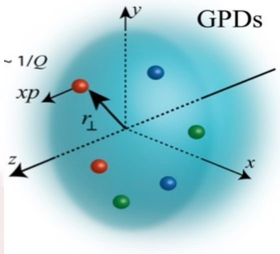
Zoo of TMDs & GPDs

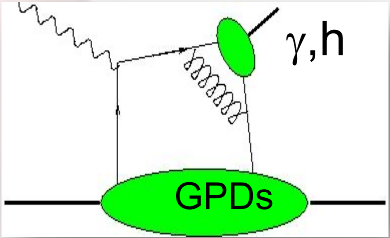
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$





	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$





- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of x_B, t, Q^2** (GPD) and **x_B, P_T, Q^2 , z** (TMD)

What do we learn

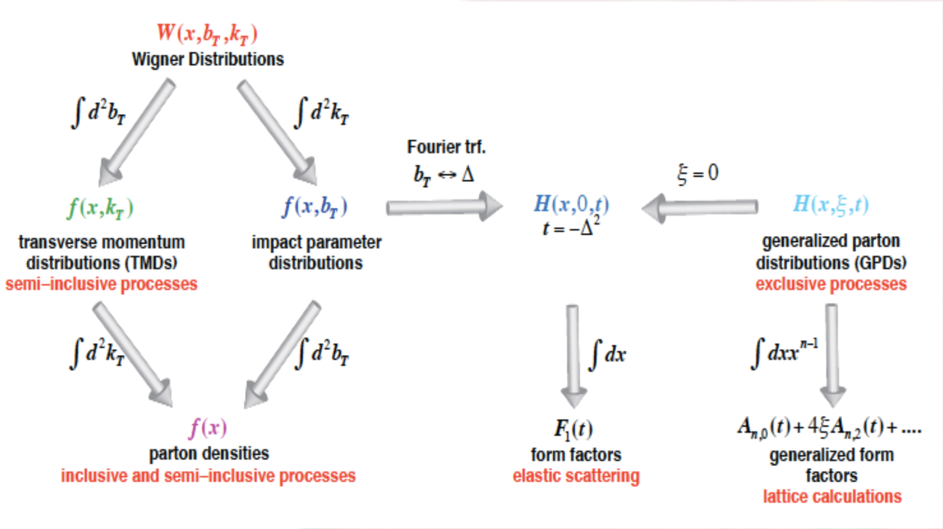
- 3D Imaging of partons inside the nucleon (non-trivial correlations)
  - Try to answer more detailed questions as Rutherford was doing for atomic matter more than 100 years ago
- QCD dynamics involved in these processes
  - Transverse momentum distributions: universality, factorization, evolutions,…
  - Small-x resummation: BFKL and Sudakov

Jet @EIC has been very active in recent years

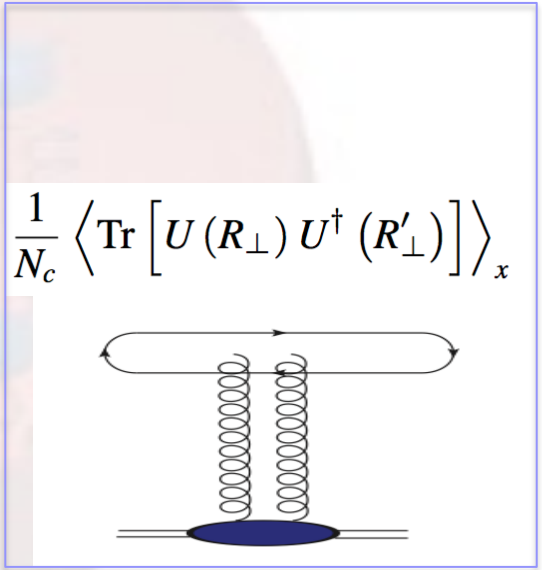
- Contribute to explore
  - Spin/tomography of nucleon
  - Small-x gluon saturation
  - Hard probe interaction with cold nuclei matter
- QCD dynamics in precision study
  - E.g., jet substructure to measure α_s, jet algorithms, jet angularity, hadronization, energy-energy correlators, etc.
- Observables:
  - Leading jet/hadron, dijet/dihadron, jet substructure, …

Felix and Iganizo’s talks

This talk will focus: small-x gluon tomography



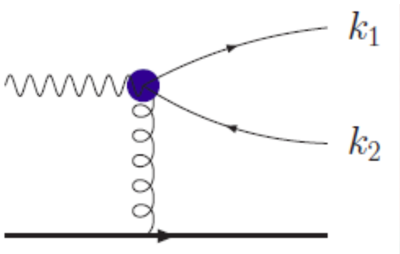
Small-x

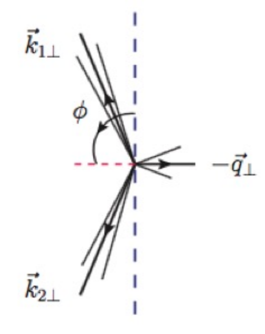


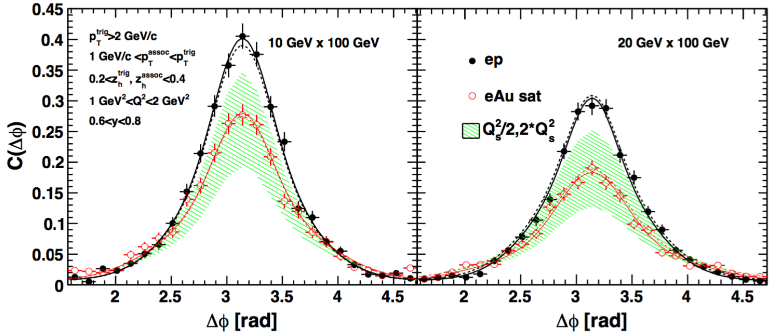
Hatta-Xiao-Yuan,1601.01585
earlier: Mueller, NPB 1999

Semi-inclusive process: DIS dijet probes gluon TMDs

$$\gamma_T^* A \to q(k_1) + \bar{q}(k_2) + X$$





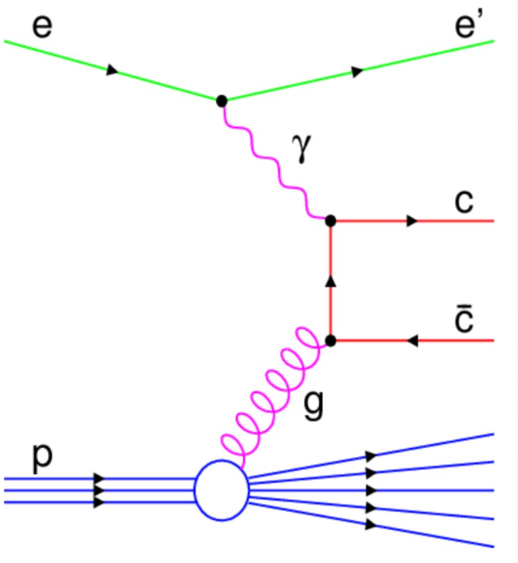


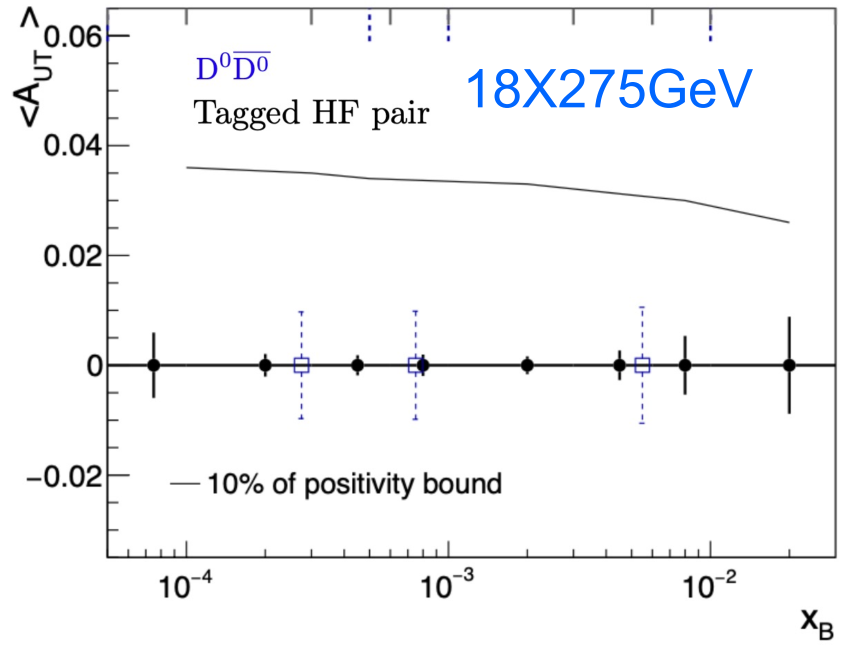
Zheng, et al., 1403.2413

- q_t-dependence measure the gluon distribution
  - Weizsacker-Williams gluon distribution in nucleus (CGC predictions)
- Various channels at the EIC: heavy flavor production, real and virtual photon

Dominguez-Marquet-Xiao-Yuan 2011

Example: Gluon Sivers function at EIC



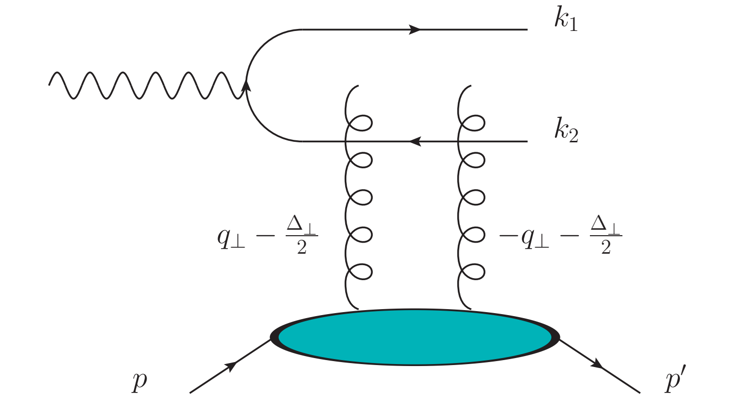


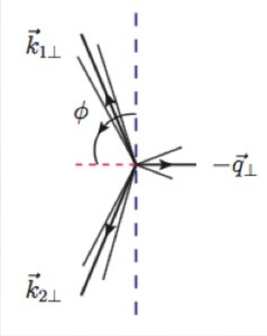
Sooraj Radhakrishnan
DIS2022

see also, inclusive dijet study,
L. Zheng et al, 1805.05290

Nucleon/Nucleus Tomography at Small-x

Hatta-Xiao-Yuan,1601.01585





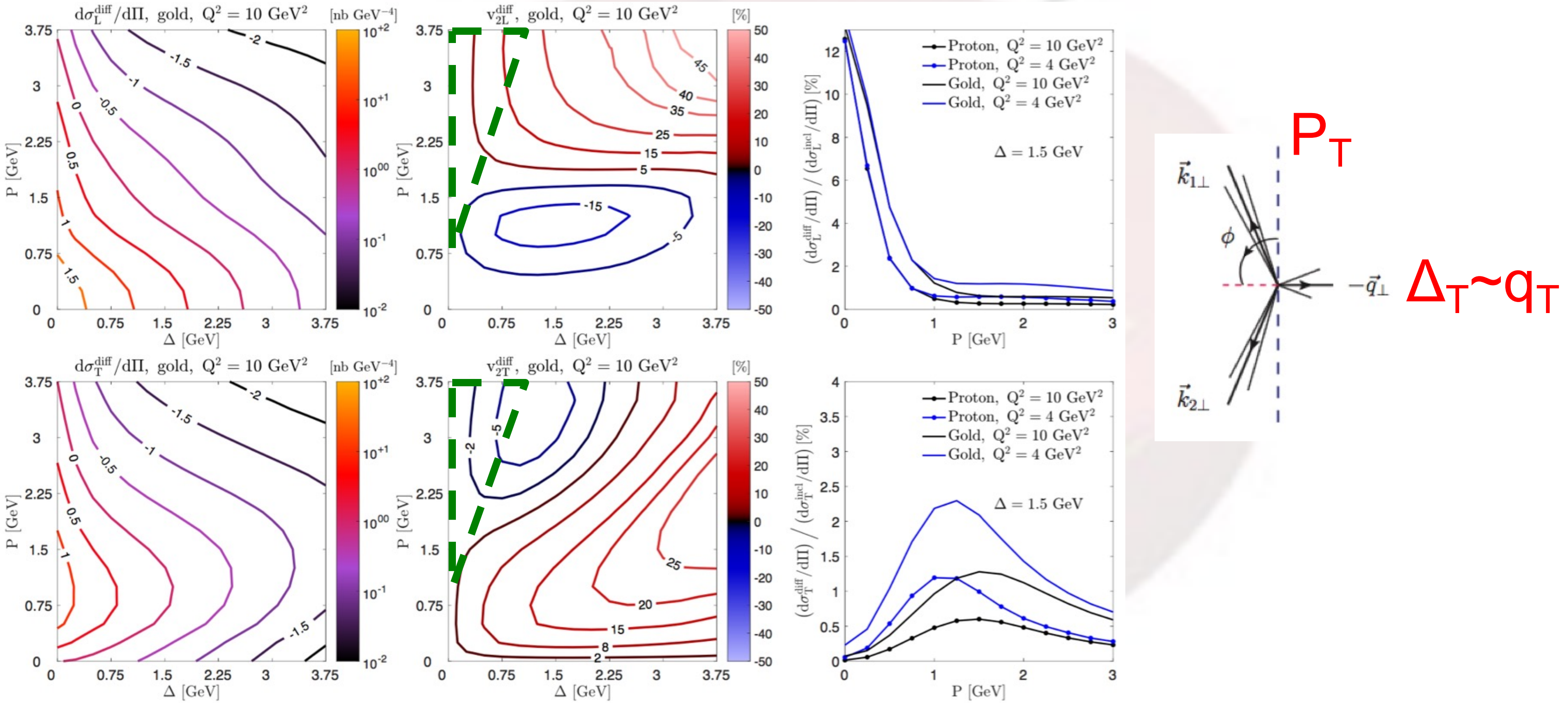
cos(2ϕ) anisotropy

- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T. By measuring jets momenta, one can approximately access q_T.
- The diffractive dijet cross section is proportional to the square of the Wigner distribution → nucleon/nucleus tomography

$$xW_g^T(x,|\vec{q}_\perp|,|\vec{b}_\perp|)+2\cos(2\phi)xW_g^\epsilon(x,|\vec{q}_\perp|,|\vec{b}_\perp|)$$

Anisotropy ~ few %

This has generated a lot of interests…



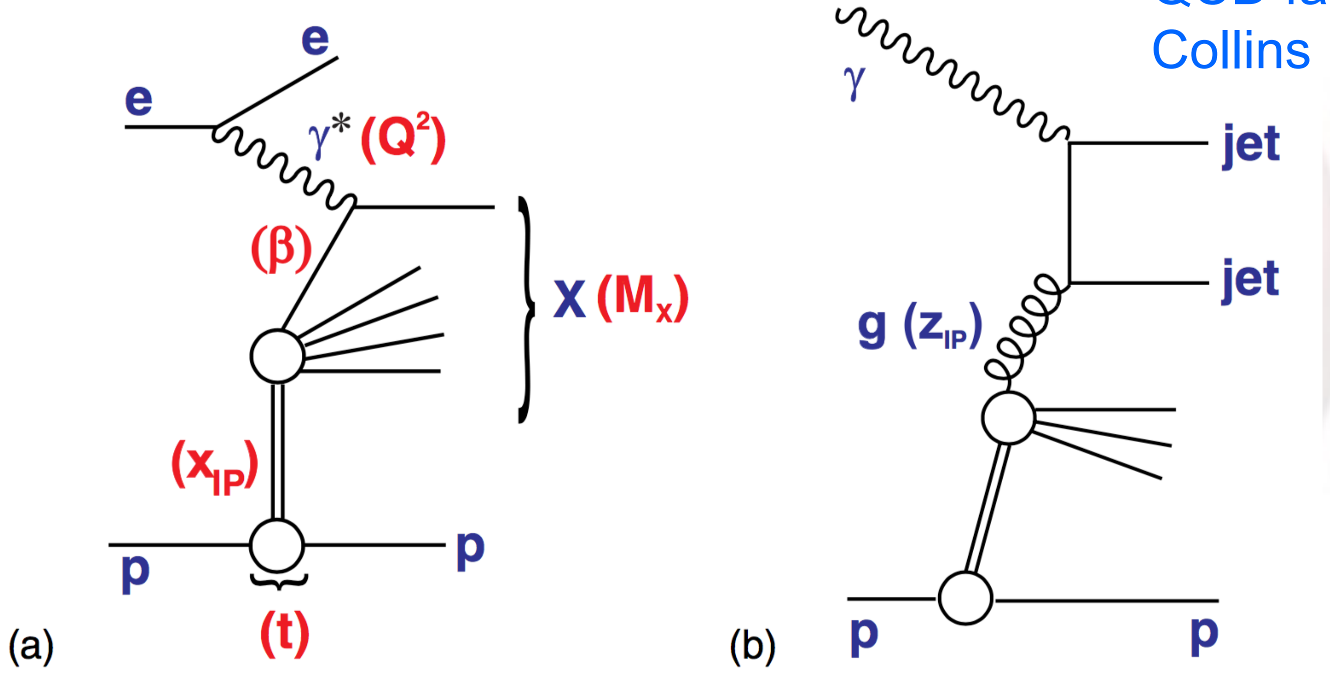
CGC calculations: Mäntysaari-Mueller-Salazar-Schenke, 1912.05586, 1902.05087; Mäntysaari-Roy-Salazar-Schenke 2011.02464

However, life is more complicated, ☹

- Gluon radiation adds additional complications
  - Because final state jet carries color, soft gluon contribution will modify the intuitive and simple picture
  - Nontrivial azimuthal angular correlations can come from soft gluon radiation, Hatta-Xiao-Yuan-Zhou 2021
- On the other hand, gluon radiation offers a unique opportunity to study different perspective of gluon saturation, Iancu-Mueller-Triantafyllopoulos, 2112.06353
  - We have a direct way to compute the so-called diffractive parton distributions

Hard diffractive processes

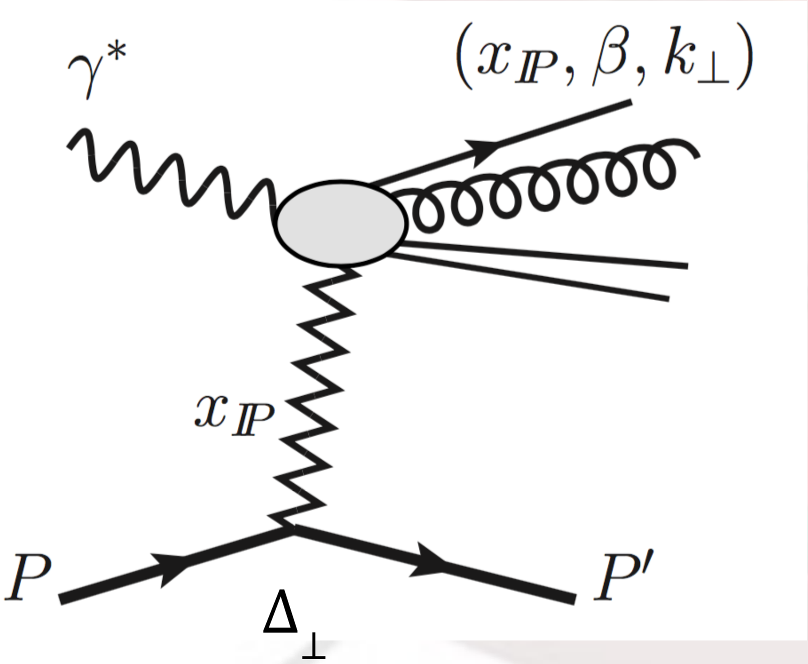
QCD factorization:
Collins 1998



HERA Legacy: Newman-Wing, Rev. Mod. Phys. 86, 1037(2014)

Semi-inclusive Diffractive DIS

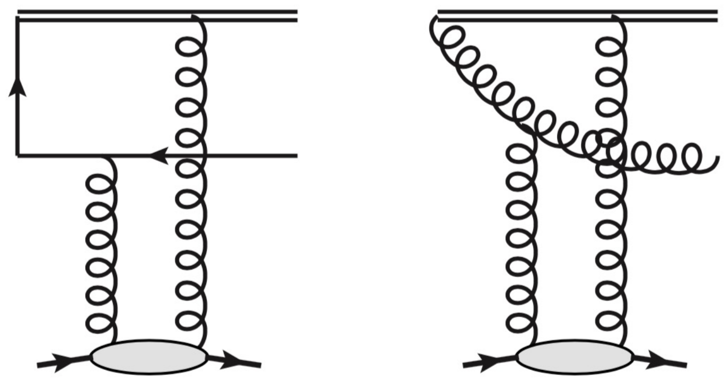
Hatta, Xiao, Yuan, 2205.08060



- Study the flavor dependence in the diffractive PDFs
- TMD dependence can be measured
  - Similar to TMDs in SIDIS (Piet et al 96 and 98), with additional correlation between $k_\perp$ and $\Delta_\perp$
- However, they differ from the GTMD introduced by Metz and friends

Compute the diffractive PDFs

- Definition is similar to TMDs for non-diffractive processes
- Requires large rapidity gap/color-singlet exchange

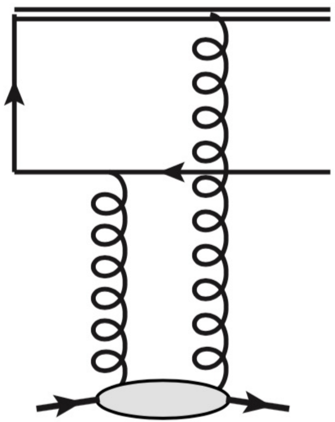


$$2E_{P'}\frac{df_q^D(x,k_\perp;x_{I\!P},t)}{d^3P'} = \int \frac{d\xi^- d^2\xi_\perp}{2(2\pi)^6} e^{-ix\xi^- P^+ + i\vec{\xi}_\perp\cdot\vec{k}_\perp}$$

$$\times \langle PS|\overline{\psi}(\xi)\mathcal{L}_n^\dagger(\xi)\gamma^+|P'X\rangle\langle P'X|\mathcal{L}_n(0)\psi(0)|PS\rangle \ , \quad (1)$$

$$\frac{d\left[xf_q^D(\beta,k_\perp;x_{I\!P})\right]}{dY_{I\!P}dt}=\int d^2k_{1\perp}d^2k_{2\perp}\mathcal{F}_{x_{I\!P}}(k_{1\perp},\Delta_\perp)$$

$$\times\mathcal{F}_{x_{I\!P}}(k_{2\perp},\Delta_\perp)\frac{N_c\beta}{2\pi}\mathcal{T}_q(k_\perp,k_{1\perp},k_{2\perp})\ . \qquad (3)$$



$$T_q(k_\perp,k_{1\perp},k_{2\perp})=$$

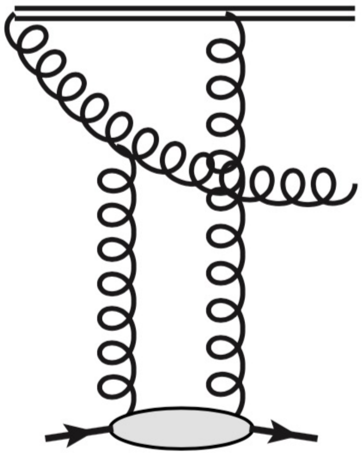
$$\frac{k'_{1\perp}\cdot k'_{2\perp}k_\perp^2}{\left[\beta k_\perp^2+(1-\beta)k'^2_{1\perp}\right]\left[\beta k_\perp^2+(1-\beta)k'^2_{2\perp}\right]}$$

$$\mathcal{F}_x(q_\perp,\Delta_\perp)=\int\frac{d^2b_\perp d^2r_\perp}{(2\pi)^4}e^{iq_\perp\cdot r_\perp+i\Delta_\perp\cdot b_\perp}$$

$$\times\frac{1}{N_c}\left\langle\mathrm{Tr}\left[U\left(b_\perp+\frac{r_\perp}{2}\right)U^\dagger\left(b_\perp-\frac{r_\perp}{2}\right)\right]\right\rangle_x$$

$$\frac{d\left[xf_g^D(\beta,k_\perp;x_{I\!P})\right]}{dY_{I\!P}dt}=\int d^2k_{1\perp}d^2k_{2\perp}\mathcal{G}_{x_{I\!P}}(k_{1\perp},\Delta_\perp)$$

$$\times\,\mathcal{G}_{x_{I\!P}}(k_{2\perp},\Delta_\perp)\frac{N_c^2-1}{\pi(1-\beta)}\mathcal{T}_g(k_\perp,k_{1\perp},k_{2\perp})\ , \qquad (6)$$

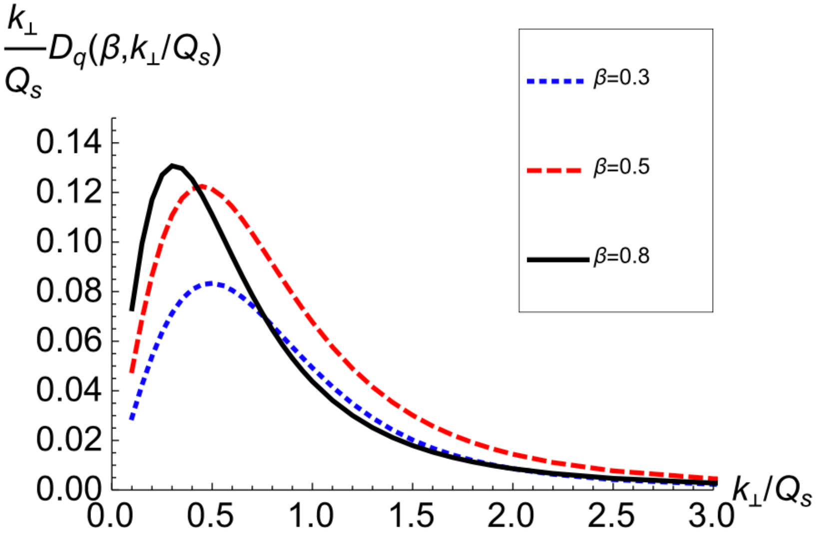


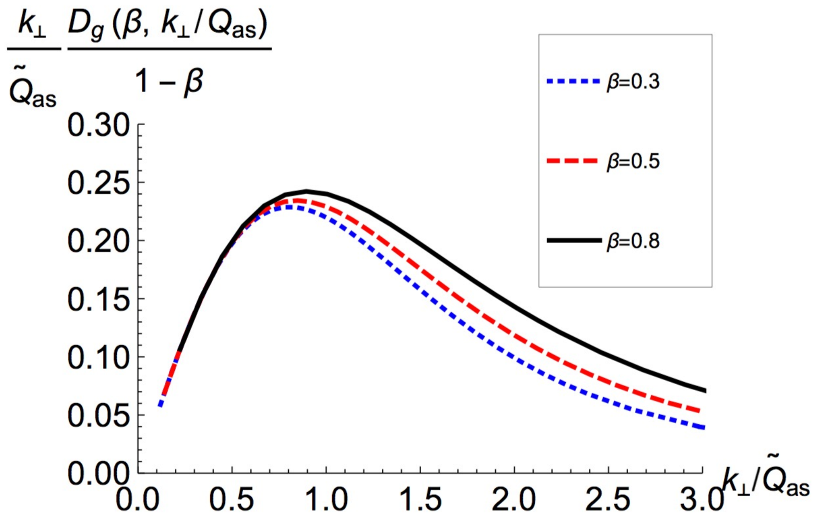
$$T_g(k_\perp,k_{1\perp},k_{2\perp})=\frac{1}{[\beta k_\perp^2+(1-\beta)k_{1\perp}'^2]}$$

$$\times\frac{1}{[\beta k_\perp^2+(1-\beta)k_{2\perp}'^2]}\left[\beta(1-\beta)\frac{k_{1\perp}'^2+k_{2\perp}'^2}{2}\right.$$

$$\left.+(1-\beta)^2(k_{1\perp}'\cdot k_{2\perp}')^2+\beta^2\frac{(k_\perp^2)^2}{2}\right]\ .$$

Transverse momentum dependence





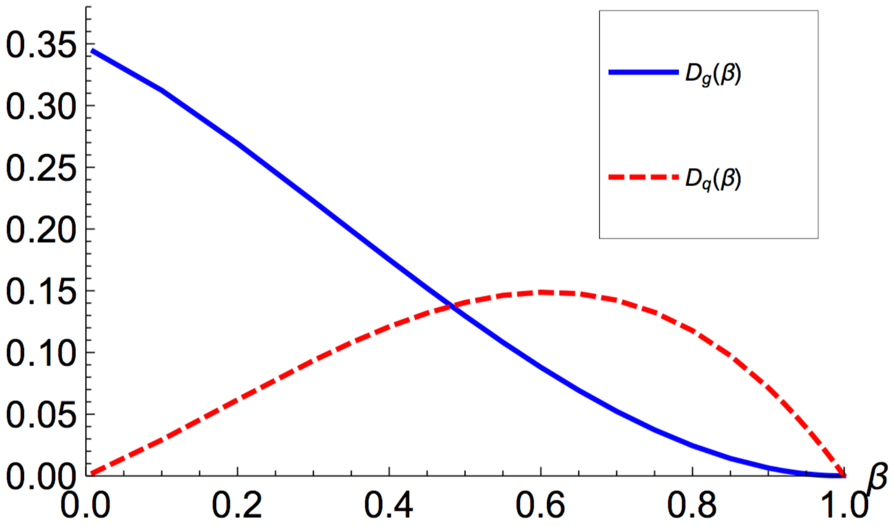
$$\widetilde{Q}^2_{(a)s} \equiv (1-\beta) Q^2_{(a)s}$$

Large kt:

$$x \frac{df^D_{q,g}(\beta, k_\perp; x_{I\!P})}{dY_{I\!P} dt}\bigg|_{k_\perp \gg Q_s} = \frac{\alpha_s^2}{2\pi} \frac{\mathcal{C}_{q,g}}{k_\perp^4} \underbrace{\left(H_g(x_{I\!P}, \Delta_\perp)\right)^2}_{\text{Gluon GPD}}$$

Integral over transverse momentum

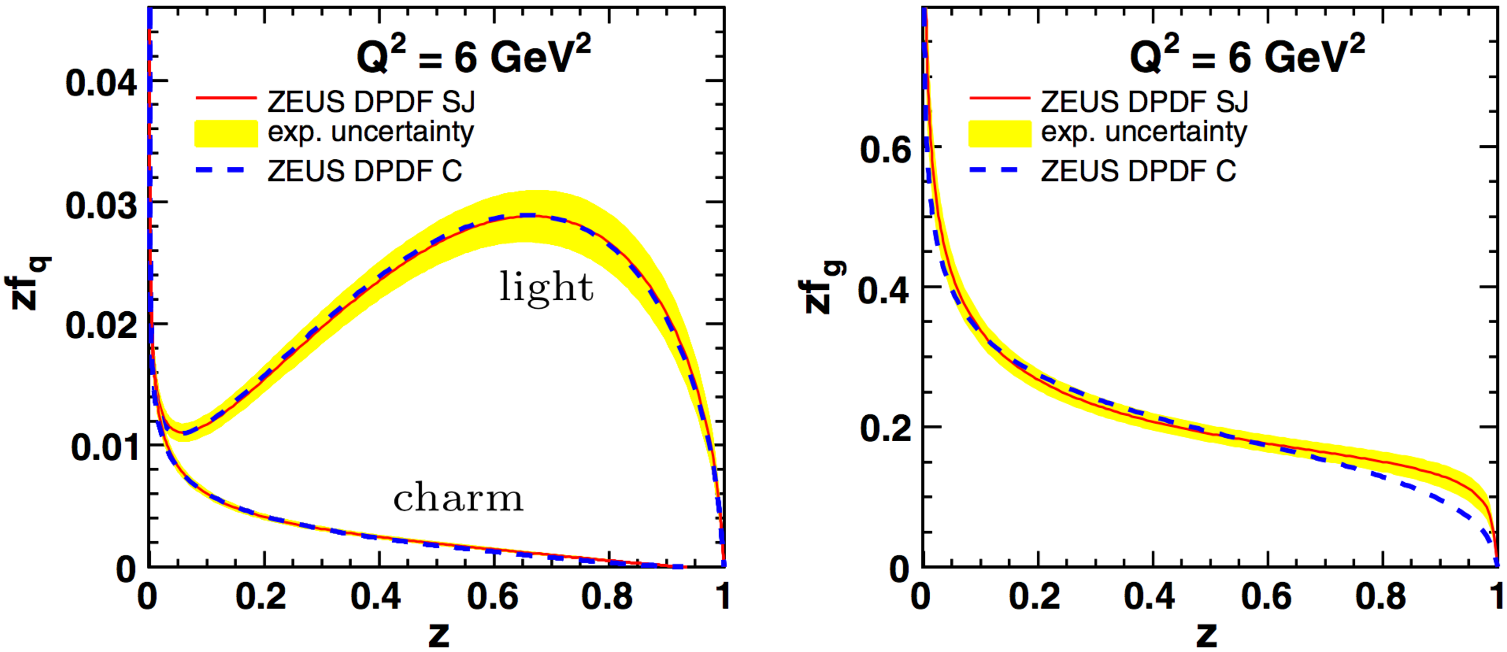
$$\mathcal{D}_q(\beta) = \beta\left(b_1(1-\beta) + b_2(1-\beta)^2\right) \qquad \mathcal{D}_g(\beta) = (a_0 + a_1\beta)(1-\beta)^2$$



- Different power behaviors for the quark and gluons
- Inputs for the diffractive PDFs at the initial scale

See also, GBW, EPJC, 2001

Compare to the Pomeron PDFs extracted from the HERA measurements



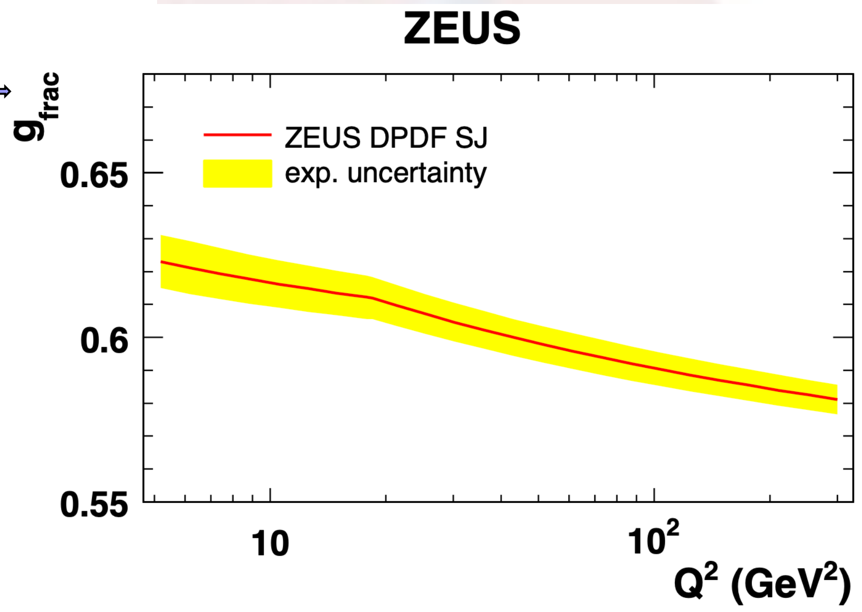
ZEUS, NPB831, 1 (2010)

Momentum sum rule

$$\int d\beta x \left[\frac{2N_f df_q^D(\beta;x_{I\!P})}{dY_{I\!P}dt} + \frac{df_g^D(\beta;x_{I\!P})}{dY_{I\!P}dt}\right] \approx \frac{S_\perp^2}{(2\pi)^3} Q_s^2$$

$$\frac{2N_f \int d\beta x f_q^D(\beta;x_{I\!P})}{\int d\beta x f_g^D(\beta;x_{I\!P})} \approx 0.33$$

translate to $g_{frac} \approx 0.75$ at the saturation scale with $N_f=3$



An interesting observation

- Integrating over transverse momentum change the power behavior at large-x

$$D_q \to 4\beta^3(1-\beta)^2\left(Q_s^2/k_\perp^2\right)^2$$

$$\mathcal{D}_q(\beta) = \beta\left(b_1(1-\beta) + b_2(1-\beta)^2\right)$$

- Kt integral is dominated by lower kt region around Qs

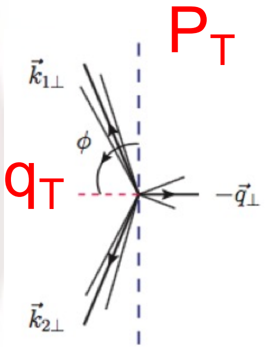
Azimuthal Angular Asymmetries from the Soft Gluon Radiation Associated with Jet

Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; arXiv: 2106.05307

Soft gluon radiations can generate an azimuthal asymmetry

Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations
  - ϕ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - <cos(ϕ)>, <cos(2ϕ)>, … divergent, $\sim 1/q_T^2$
  - Examples discussed include Vj, top quark pair production

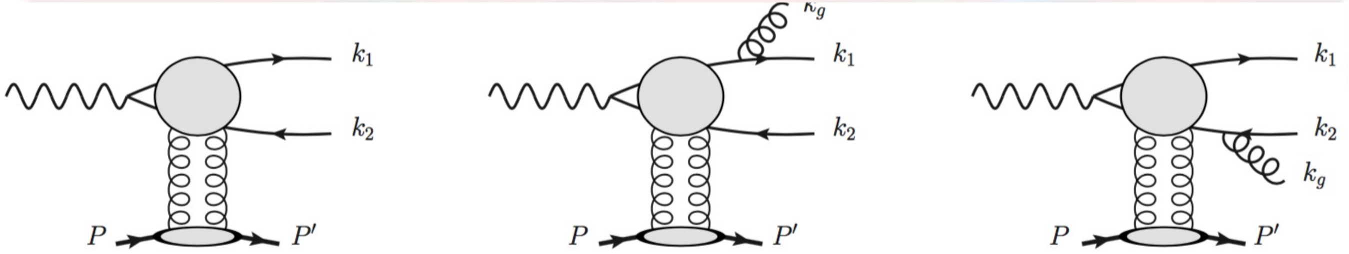


Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1\cdot k_2}{k_1\cdot k_g k_2\cdot k_g}\right)_{\vec{q}_\perp=-\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi)S_{J2}(|q_\perp|) + \cdots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774, 2106.05307
anisotropy was neglected in an earlier paper:
Hatta-Mueller-Ueda-Yuan, 1907.09491

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\,, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\,,$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\,, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\,.$$

a_0,a_2 are order 1 constants, so,

in the small-R limit, <cos(2ϕ)> goes to 1

All order resummation, in Fourier-b space

$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)}\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

EIC

Kinematics:

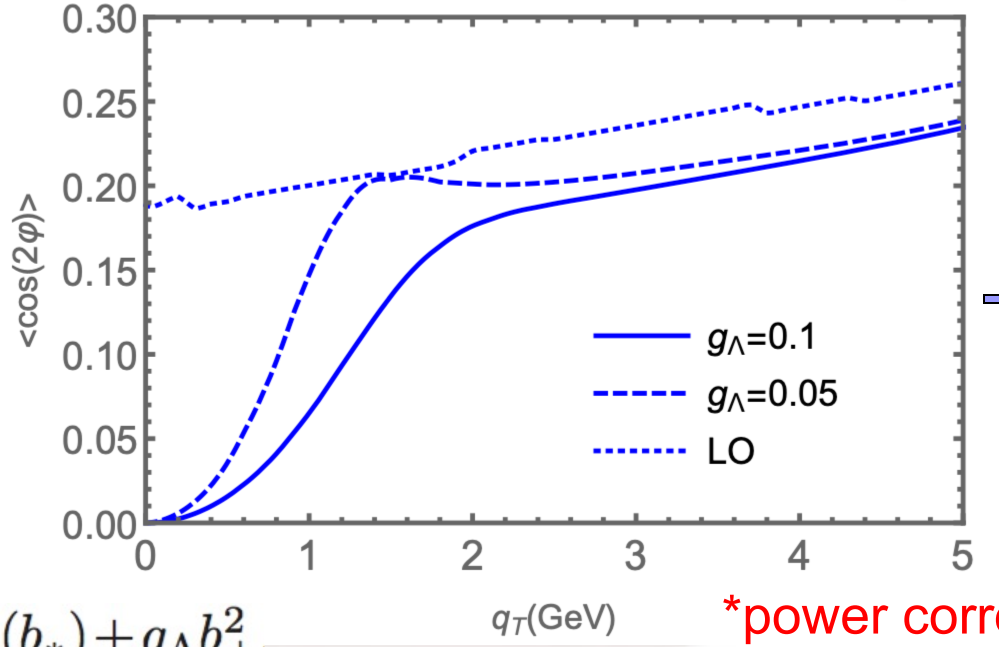
P_T~15GeV

R=0.4

y_1=y_2

Non-pert. input:

$$\Gamma_0(b_\perp) \Longrightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$$

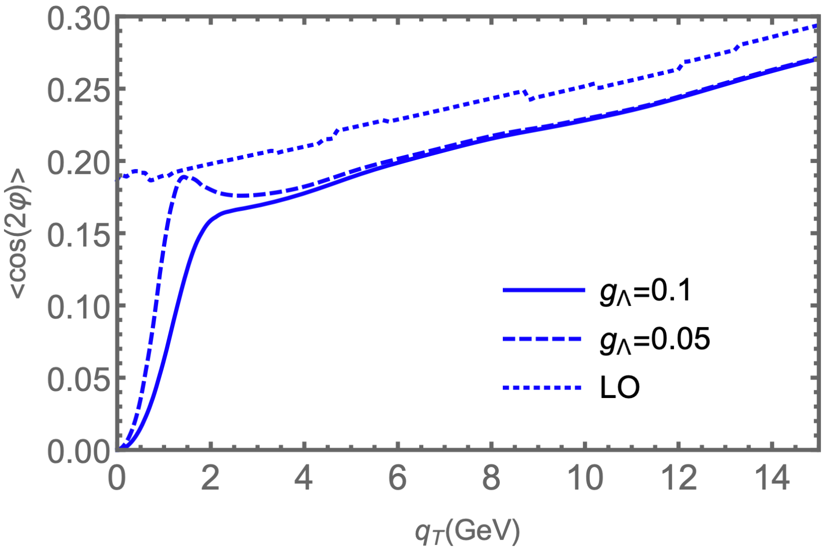


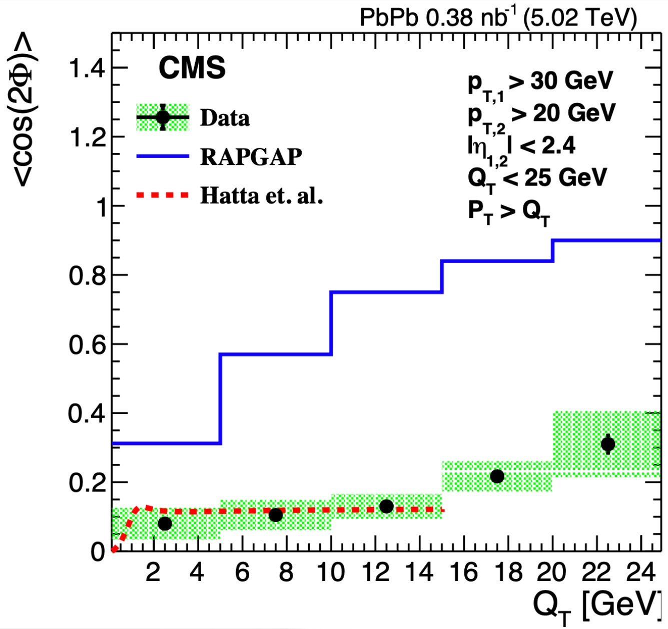
$\alpha_2/\alpha_0 \approx 0.14$

*power corrections included

Compare to recent CMS measurement

P_T~35GeV, R=0.4 ,$y_1=y_2$

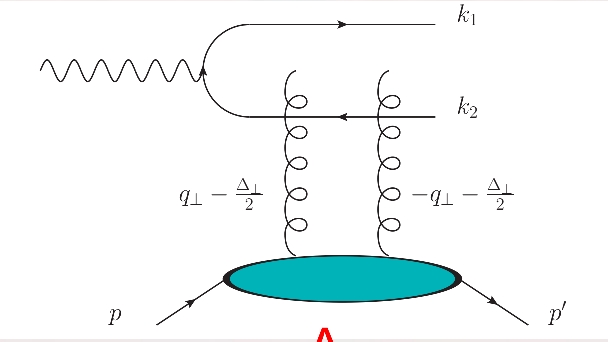




Michael Murray DIS 2022

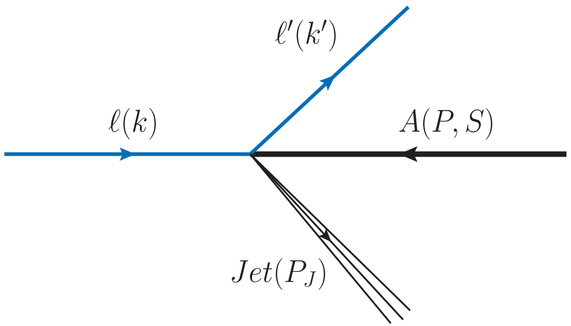
Comments

- To avoid the soft gluon radiation contribution, we need to reconstruct nucleon/nucleus recoil momentum to study the tomography



Δ_T

Lepton-jet correlation in DIS



Quark distribution⊗soft factor

$$\frac{d^5\sigma(\ell p \to \ell' J)}{dy_\ell d^2 k_{\ell\perp} d^2 q_\perp} = \sigma_0 \int d^2 k_\perp d^2 \lambda_\perp x f_q(x, k_\perp, \zeta_c, \mu_F) \times H_{\rm TMD}(Q, \mu_F) S_J(\lambda_\perp, \mu_F)\, \delta^{(2)}(q_\perp - k_\perp - \lambda_\perp)\ .$$

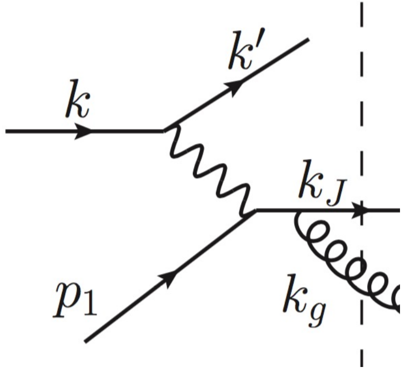
Liu-Ringer-Vogelsang-Yuan 1812.08077, 2007.12866

(Lab frame)

Total transverse momentum of the lepton+jet probes the TMD quark distribution

Soft gluon radiation

$$g^2 \int \frac{d^3k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F S_g(k_J, p_1)$$
$$= \frac{\alpha_s}{2\pi^2} \frac{1}{q_\perp^2} \left[\ln \frac{Q^2}{q_\perp^2} + \ln \frac{Q^2}{k_{\ell\perp}^2} \right.$$
$$\left. + c_0 + 2c_1 \cos(\phi) + 2c_2 \cos(2\phi) + \cdots \right],$$



$$S_g(k_J, p_1) = \frac{2k_J \cdot p_1}{k_J \cdot k_g p_1 \cdot k_g}$$

- Small-R limit,

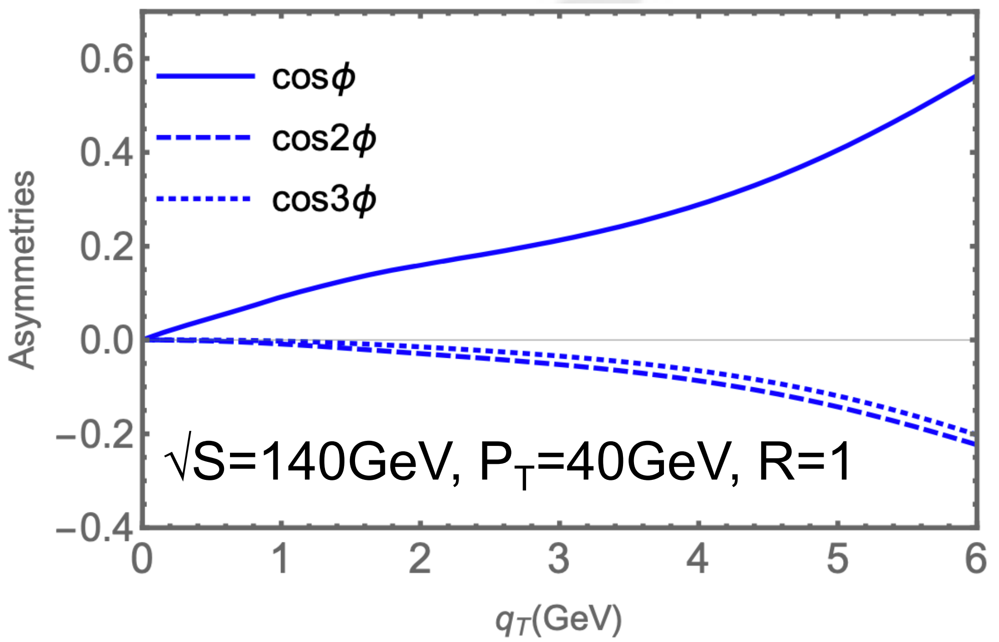
$$\ln \frac{1}{R^2} + 2\cos(\phi)\left(\ln \frac{1}{R^2} + 2\ln(4) - 2\right) + 2\cos(2\phi)\left(\ln \frac{1}{R^2} - 1\right)$$

Final result depends on the quark TMD

$$\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp}d^2q_\perp} = \sum_{n=1} 2\cos(n\phi)\int \frac{b_\perp db_\perp}{(2\pi)} J_n(|q_\perp||b_\perp|)$$
$$\times e^{-\,\mathrm{Sud}} \sum_q \sigma_0 x_q f_q(x_q, \mu_b) \qquad (15)$$
$$\times \int d|q'_\perp| J_n(|b_\perp||q'_\perp|) \frac{C_F\alpha_s c_n(q'^2_\perp)}{|q'_\perp|\pi}.$$

$$\mathrm{Sud}(\mu_b^2, P_\perp^2, R) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left\{\frac{\alpha_s(\mu)C_F}{\pi}\left[\ln\frac{Q^2}{\mu^2} + \ln\frac{Q^2}{P_\perp^2}\right.\right.$$
$$\left.\left. -\frac{3}{2} + c_0(R)\right]\right\}, \qquad (14)$$

Estimate for EIC kinematics



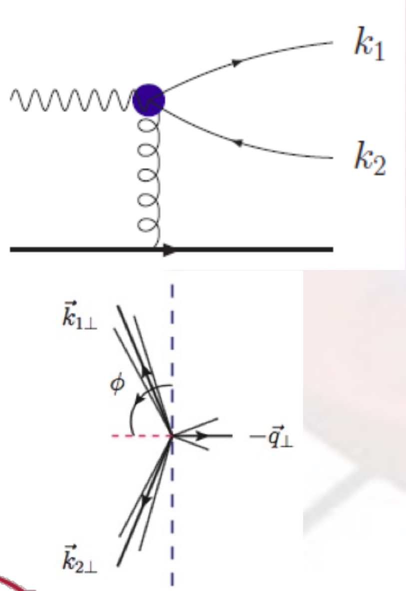
- TMD quark follows SIYY parameterization
  - 1406.3073

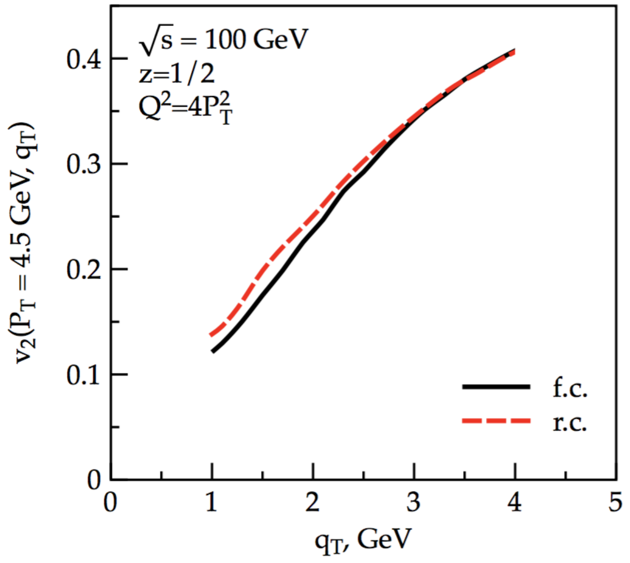
$$\mathrm{Sud}(b_\perp) \to \mathrm{Sud}(b_*) + \mathrm{Sud}^{q}_{\mathrm{NP}}(b_\perp) + \mathrm{Sud}^{\mathrm{jet}}_{\mathrm{NP}}(b_\perp)$$

$$\mathrm{Sud}^{q}_{\mathrm{NP}}(b_\perp) = 0.106\, b_\perp^2 + 0.42 \ln(Q/Q_0) \ln(b_\perp/b_*) \qquad \mathrm{Sud}^{\mathrm{jet}}_{\mathrm{NP}}(b_\perp) = g_\Lambda b_\perp^2$$

Inclusive Dijet in DIS

- Cos(2ϕ) anisotropy was proposed to study the linearly polarized gluon distribution





CGC calculation: Dumitru-Lappi-Skokov, 1508.04438

see also,
Boer-Brodsky-Mulders-Pisano 1011.4225
Metz-Zhou, 1105.1991
Boer et al., 1702.08195, 1605.07934
Mantysaari et al., 1902.05087, 1912.05586

Three contributions to cos(2ϕ) asymmetry

$$\frac{d^4\sigma}{d\Omega} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \left[\widetilde{W}_0^{\gamma^* p}(|b_\perp|) - 2\cos(2\phi_b)\widetilde{W}_2^{\gamma^* p}(|b_\perp|)\right]$$

$$\widetilde{W}_0^{\gamma^* p}(b_\perp) = x_g f_g(x_g, \mu_b) e^{-\mathrm{Sud}_{\mathrm{pert}}^{\gamma^* p}(b_*) - \mathrm{Sud}_{\mathrm{NP}}^{\gamma^* p}(b_\perp)}$$

$$\widetilde{W}_2^{\gamma^* p}(b_\perp) = e^{-\mathrm{Sud}_{\mathrm{pert}}^{\gamma^* p}(b_*) - \mathrm{Sud}_{\mathrm{NP}}^{\gamma^* p}(b_\perp)} \times \left[x_g f_g(x_g, \mu_b)\left(\boxed{\alpha_2^{\gamma g}} + \boxed{\frac{\sigma_2}{\sigma_0} g_h(b_\perp)}\right) + \boxed{\frac{\sigma_2}{\sigma_0}\int \frac{dx'}{x'} x_g f_i(x', \mu) C_{h/i}^{(1)}\left(\frac{x_g}{x'}\right)}\right]$$

Soft gluon from jet

Intrinsic linearly polarized gluon

Collinear splitting contribution

- Two loop calculation: Gutierrez-Reyes, et al., 1907.03780

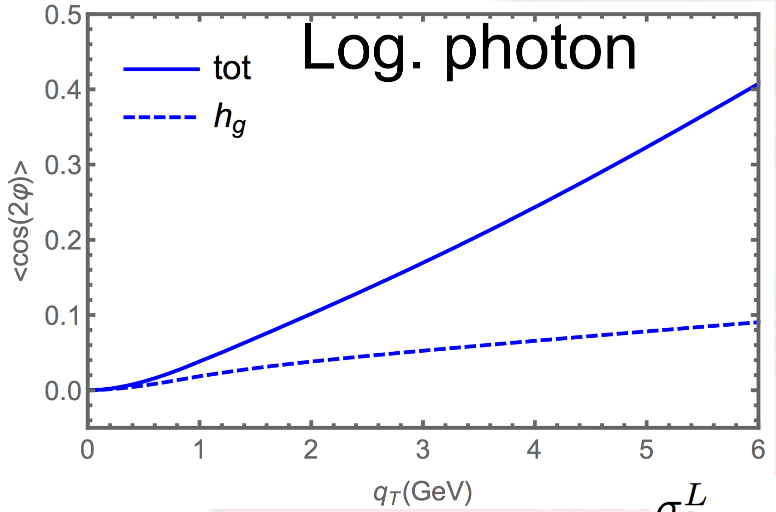
- Numerically, contribution from soft gluon with jet is sizable
  - This can also be studied in real photon scattering process, where there is no linearly polarized gluon contribution
- The difference between the transverse and longitudinal photons purely comes from the linearly polarized gluon distribution

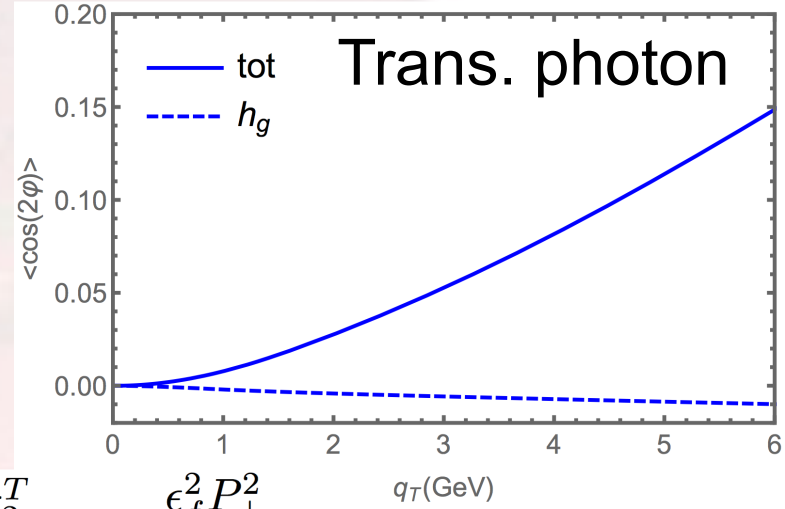
$$\frac{\sigma_2^L}{\sigma_0^L} = \frac{1}{2} \qquad \frac{\sigma_2^T}{\sigma_0^T} = -\frac{\epsilon_f^2 P_\perp^2}{\epsilon_f^4 + P_\perp^4}$$

Conclusion

- Small-x physics provides a unique opportunity to explore nucleon tomography through parton Wigner distributions
  - Unified description with dipole amplitude starts to emerge
- Further developments are needed to explore the full potential of the future electron-ion collider
  - More processes to probe the dipole amplitudes, including its spin-dependence
  - Soft gluon radiation can generate a sizable azimuthal asymmetry, offering challenges and opportunities

Numerically, contribution from soft gluon with jet is sizable, Q=10GeV, P_T=15GeV, R=0.4





$$\frac{\sigma_2^L}{\sigma_0^L} = \frac{1}{2} \qquad \frac{\sigma_2^T}{\sigma_0^T} = -\frac{\epsilon_f^2 P_\perp^2}{\epsilon_f^4 + P_\perp^4}$$

- The difference between the above two purely comes from the linearly polarized gluon distribution

All order resummation, in Fourier-b space (no power corrections)

$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)}\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

CMS

Kinematics:

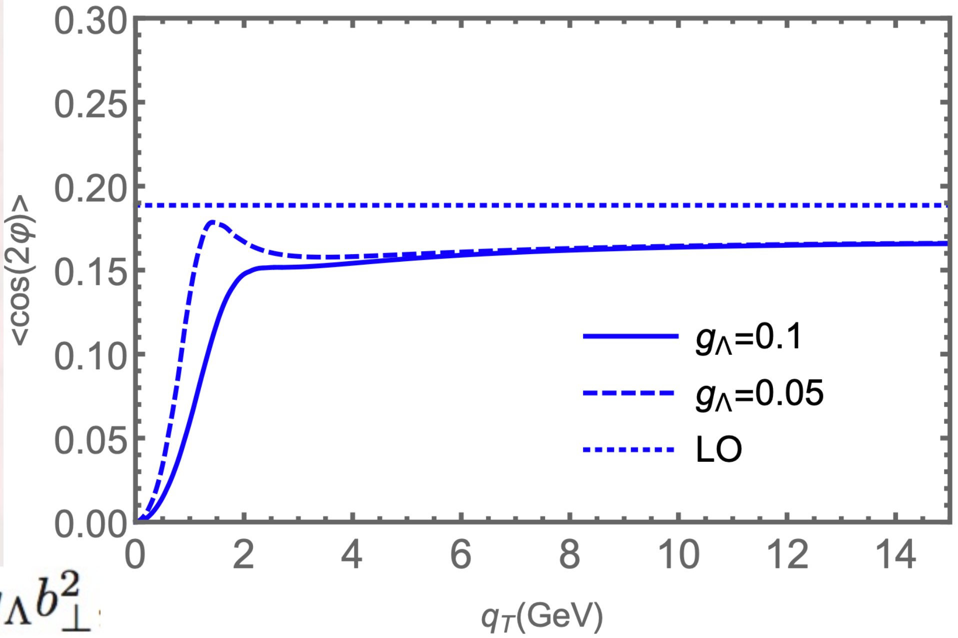
P_T~35GeV

R=0.4

$y_1=y_2$

Non-pert. input:

$$\Gamma_0(b_\perp) \Longrightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$$



All order resummation, in Fourier-b space (with power corrections)

$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)}\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

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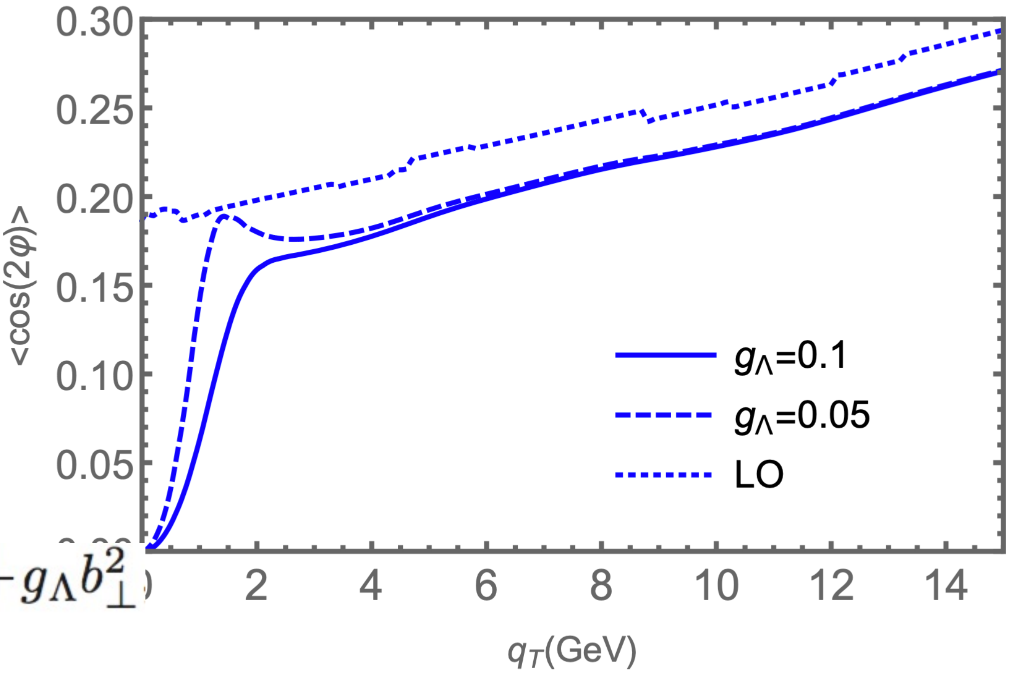
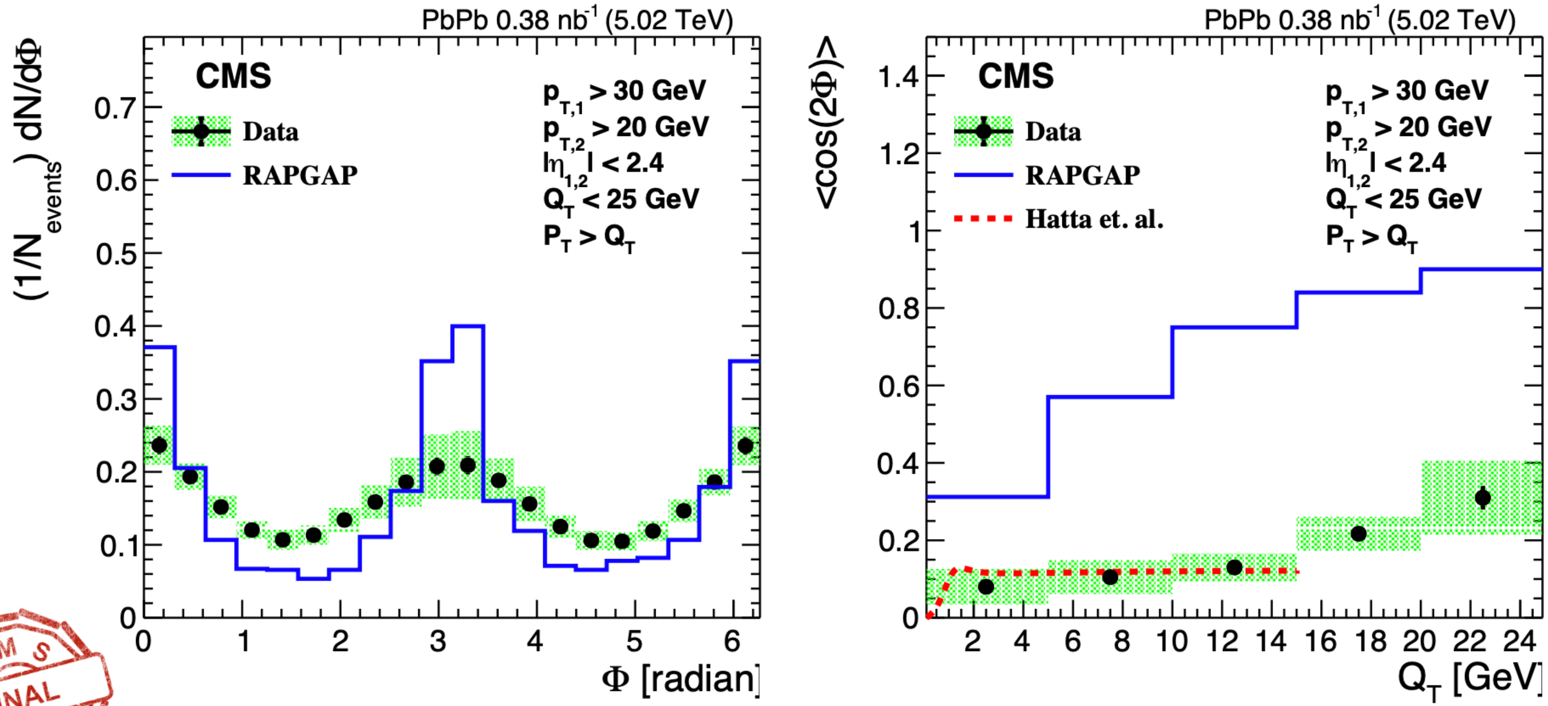


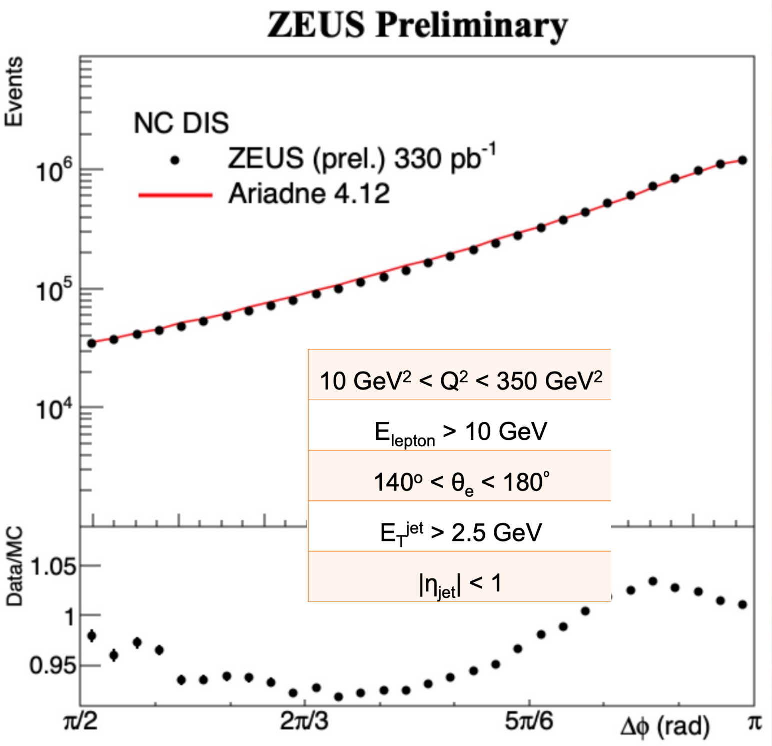


CMS FINAL DIS22

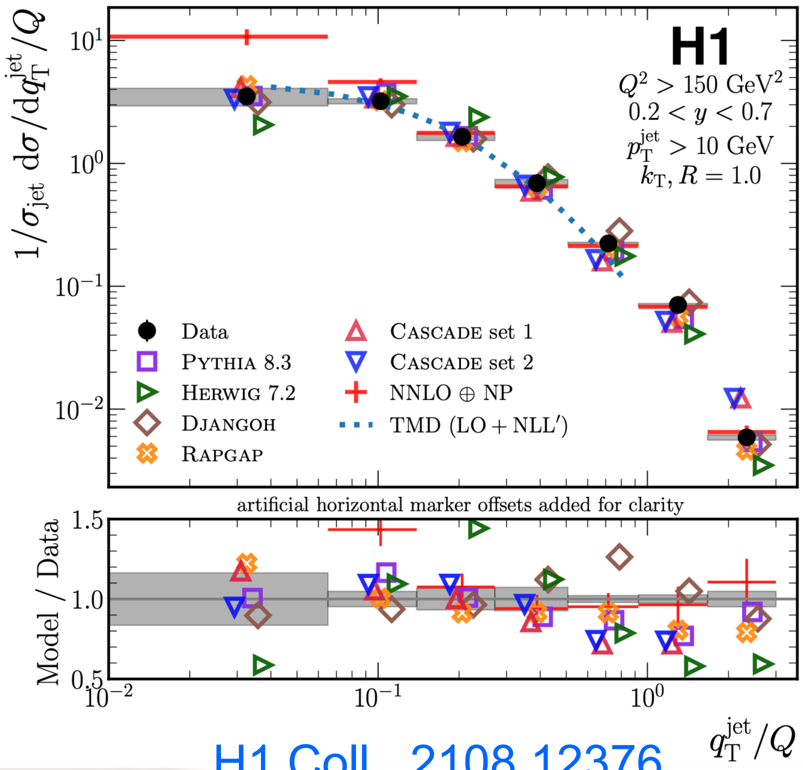
$\langle\cos(2\phi)\rangle$ in data is below back-to-back expectation and RAPGAP prediction

$\langle\cos(2\phi)\rangle$ constant for $Q_T > 2$ GeV in the Hatta calculations which just include soft radiation and no effect from elliptic gluons.

Re-analysis of HERA Data



Amilkar Quintero, 2019
EIC-meeting, Paris



H1 Coll., 2108.12376
M. Arratia, DIS 2021