Running of α: measurement and systematics

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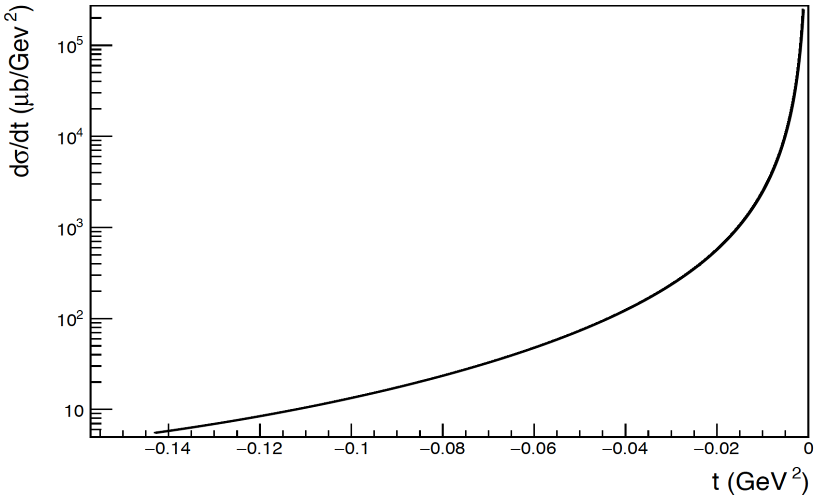
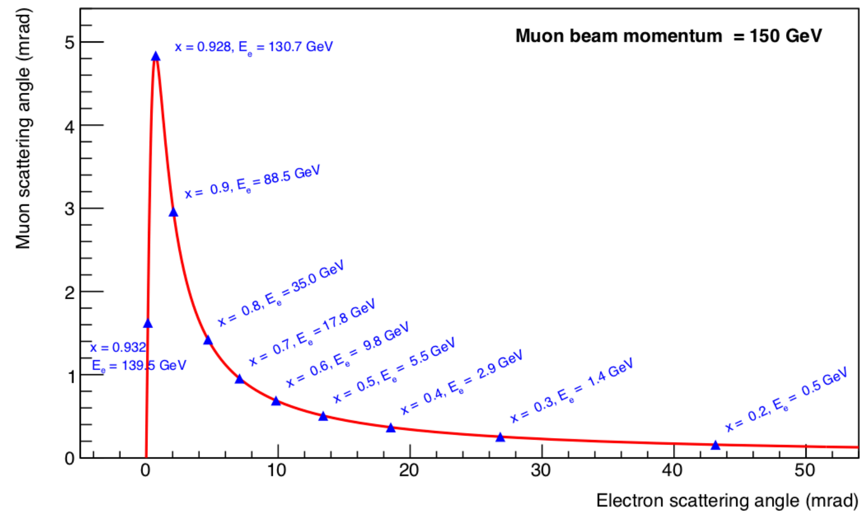
Outline

- ✧Parametrization of the hadronic running of α
- ✧Ideal statistical sensitivity in LO
- ✧NLO and signal selection
- ✧Detector smearing, Full and Fast simulation
- ✧Template fits
- ✧Conclusions / ToDos

Running α: recap for LO μ-e elastic scattering





$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_e^2, m_\mu^2)}\left[\frac{(s-m_e^2-m_\mu^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2}\right]$$

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2 \qquad \alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha(t)} \qquad \Delta\alpha(t) = \Delta\alpha_{lep}(t) + \Delta\alpha_{had}(t)$$

Simple kinematics: $t \cong -2\, m_e E_e$

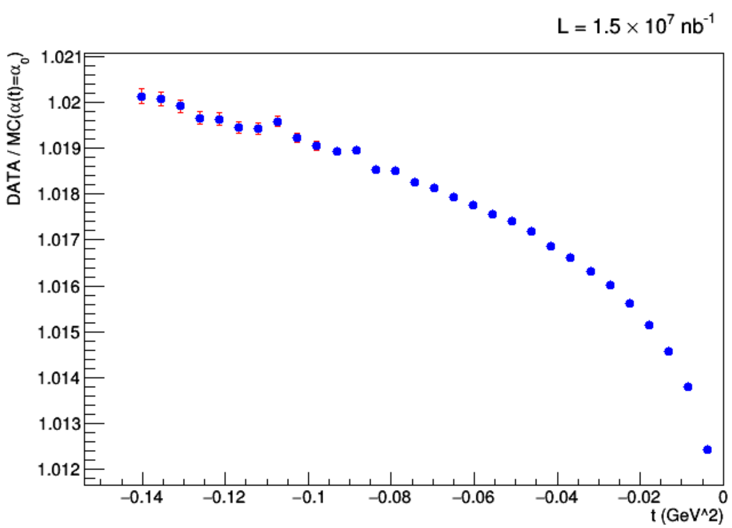
E_e can be determined from the scattering angle θ_e and the beam energy

Running α

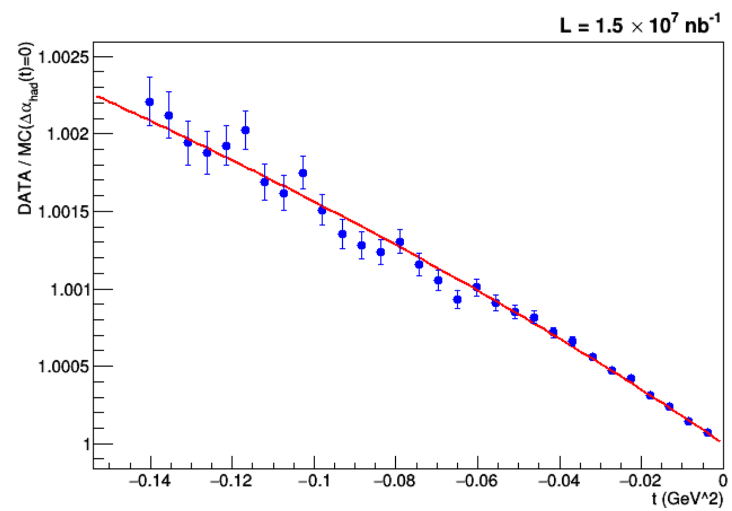
The running of α is most easily exposed by taking ratios of the observed distribution of the momentum transfer t (or equivalently the Feynman x variable, or the electron angle) and the theory expectation from MC calculated for a coupling corresponding to no running (left) or the prediction for only leptonic contribution to the running (right).

Example toy experiment shown with statistics corresponding to the nominal integrated Luminosity L = 1.5 x 10^7 nb^{-1}

TOTAL RUNNING = Lep + Had



HADRONIC RUNNING

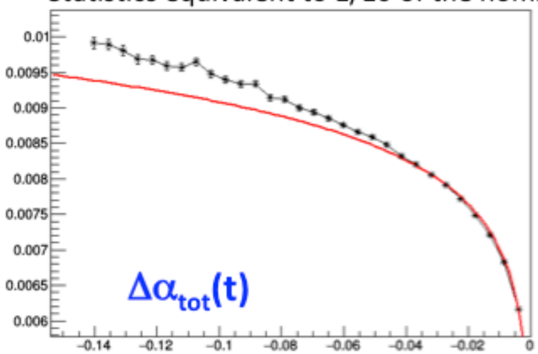


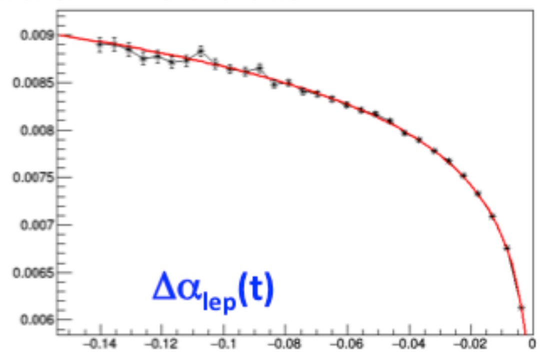
ERRATA CORRIGE

The below result shown at CERN was buggy: I had by mistake set the muon mass to 10^3 its value, so its contribution was disappearing !

Testing the total and leptonic α running

Statistics equivalent to 1/10 of the nominal one: L = 1.5 x 10^6 nb^{-1}





Fits with the Lepton-like parametrization:
TOTAL running : Does NOT work ! (there is also the hadronic component which starts rising at large |t|)
LEPTONIC running (subtracting the predicted hadronic contr.): perfect fit with
K = (2.3271 +- 0.0047) x 10^-3 (expected for leptons: α/π = 2.3228 x 10^-3)
M = m^2 = (2.649 +- 0.044) x 10^-7 m=0.5125 MeV ➔ consistent with running due to electrons (they are dominating the VP)

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CHECKS:

Leptonic running was wrong

No effect on the results of the hadronic running

Parametrization of the Hadronic running

Now using a modified Lepton-Like function (Carloni's ansatz), with 2 parameters: K' and M where K'=K/M to give the dominant low-t behaviour of the hadronic running. This change to overcome a strong (anti)correlation (~99.8%) of the original K and M observed with the template fits.

From the CERN workshop

Parametrization of the Hadronic running

POLINOMIAL 3rd order (null known-term: 3 parameters

$$\Delta\alpha_{had}(t) = c_1\,t + c_2\,t^2 + c_3\,t^3$$

PADE` approximant (3 parameters)

$$\Delta\alpha_{had}(t) = a\,t\,\frac{1+b\,t}{1+c\,t}$$

LEPTON-LIKE function (2 parameters)
M with dimension of mass squared, related to the mass of the fermion in the vacuum polarization loop
K depending on the coupling $\alpha(0)$, the electric charge and the colour charge of the fermion
Theoretically correct log(|t|) behaviour for |t|->∞; linear for t->0

$$\Delta\alpha_{had}(t) = K\left\{-\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

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Polinomial 2nd order used as a cross check.

These forms of Pol. 3rd order and Pade' are not constrained enough by our precision

Modified Lepton-Like: in the functional form replace K with K' M.
The (anti)correlation is reduced to ~-0.82, still strong but workable

IDEAL case: pure statistical sensitivity

Shown at CERN workshop for Leading Order MC

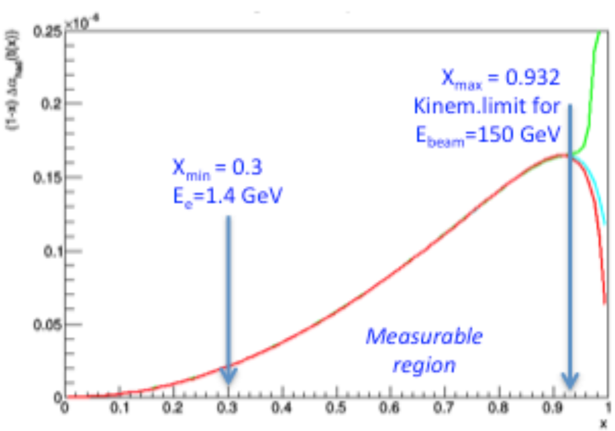
Expected significance - stat only - LO

$$t = -m_\mu^2 \frac{x^2}{1-x}$$

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

30 bins uniformly spaced in t

Fit function	χ^2 (n.d.f.)	parameter	value	meas. Integral ($\times 10^{-10}$)	extrap. a_μ^{HLO} ($\times 10^{-10}$)
polynomial 3rd-order	26.9 ± 7.3 (27)	c_1 c_2 c_3	-0.009143 ± 0.000066 -0.0179 ± 0.0024 -0.031 ± 0.017	579.3 ± 2.1	-
polynomial 2nd-order	31.3 ± 8.3 (28)	c_1 c_2	-0.009057 ± 0.000047 -0.01364 ± 0.00064	578.6 ± 2.0	-
Padé	26.9 ± 7.3 (27)	a b c	-0.009155 ± 0.000074 -0.8 ± 1.5 -2.9 ± 1.8	579.3 ± 2.1	688 ± 31
lepton-like	27.9 ± 7.4 (28)	K M	0.00720 ± 0.00037 0.0525 ± 0.0031	579.1 ± 2.1	688.9 ± 2.1



Reference used to produce the toy experiments:
$\Delta\alpha_{had}(t)$ from F.Jegerlehner's code (hadr5n12.f)
Measured integral I ($0.3<x<0.932$) = 579.2×10^{-10}
$a_\mu^{HLO} = 688.6 \times 10^{-10}$

0.3%

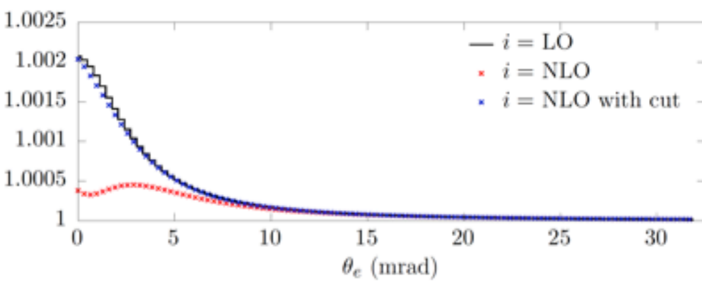
Within the acceptance all the parameterizations (but the pol-2) are equivalent.
But: difficult to constrain 3 parameters.
Lepton-Like parametrization, with only 2 parameters does an excellent job.
$M = m^2 \Rightarrow m = 0.23$ GeV which respects the physics model for a quark-like mass
$K = 0.00720 \sim f^*\alpha/\pi$, is within a factor of 2 from the expectation for quarks

NLO and selection

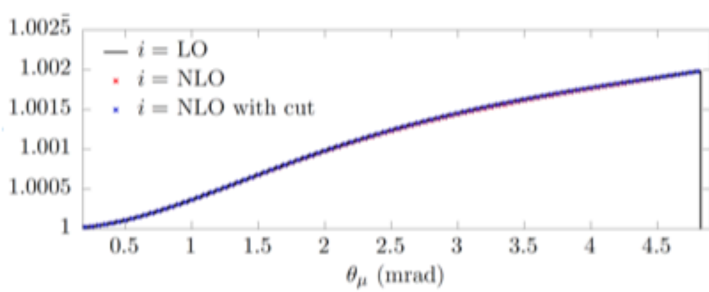
NLO

$$R_i = \frac{d\sigma(\Delta\alpha_{had}(q^2)\neq 0)}{d\sigma(\Delta\alpha_{had}(q^2)=0)}$$

M.Alacevich et al, JHEP02(2019)155



Without any selection the signal sensitivity of the electron angle is destroyed -> necessary to implement an "elastic" selection



The muon angle instead is a robust observable, stable w.r.t. radiative corrections -> it can be used with an inclusive selection (theoretically advantageous)

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In addition to the inclusive selection, which is good for the muon angular distribution, I tested a selection with:

$\theta_\mu > 0.2$ mrad

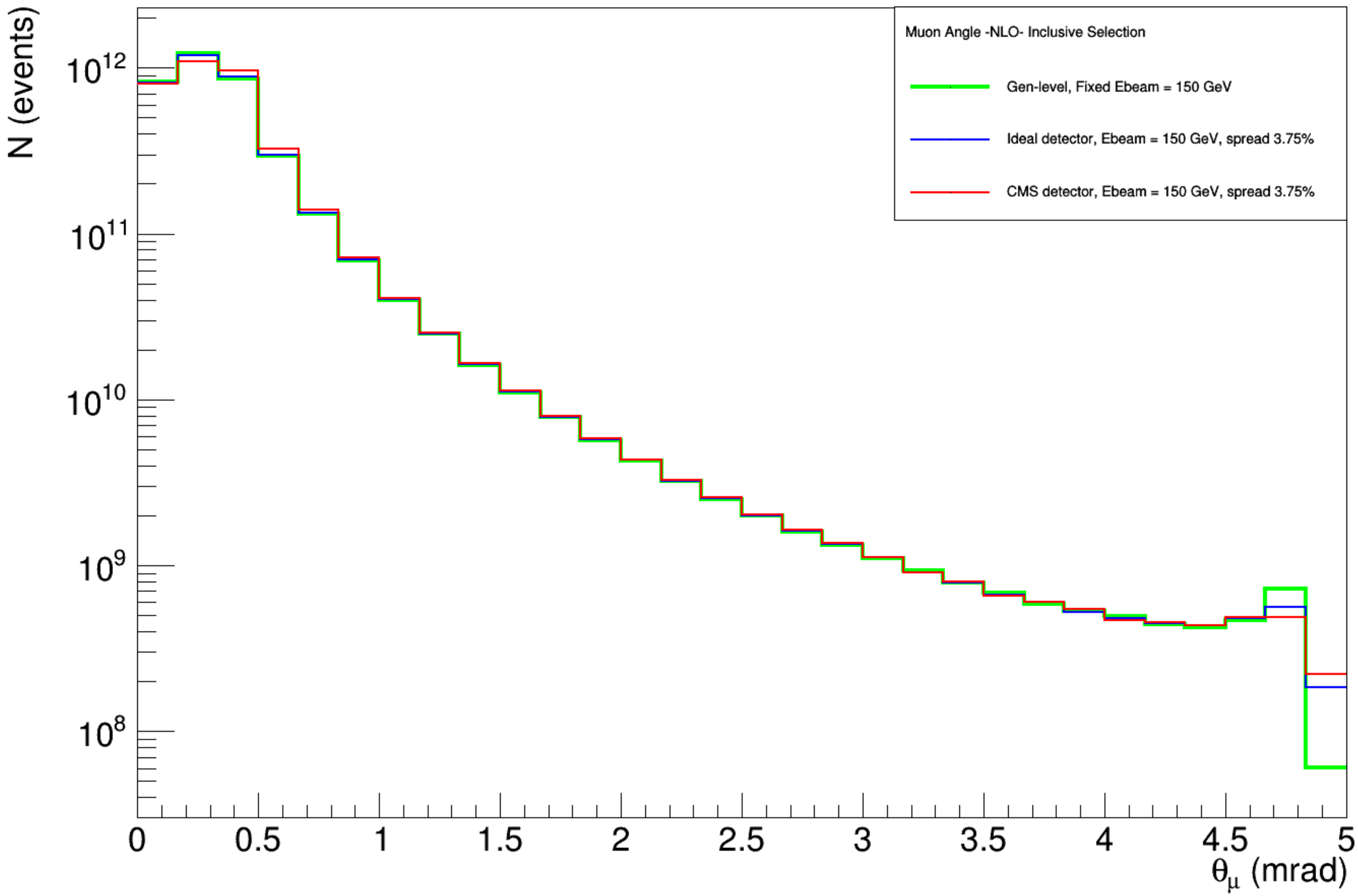
which (at least for pure signal at NLO) is useful for the electron angular distribution

Also: tested sensitivity-based selections, cutting around the elastic band, mostly as a check by now

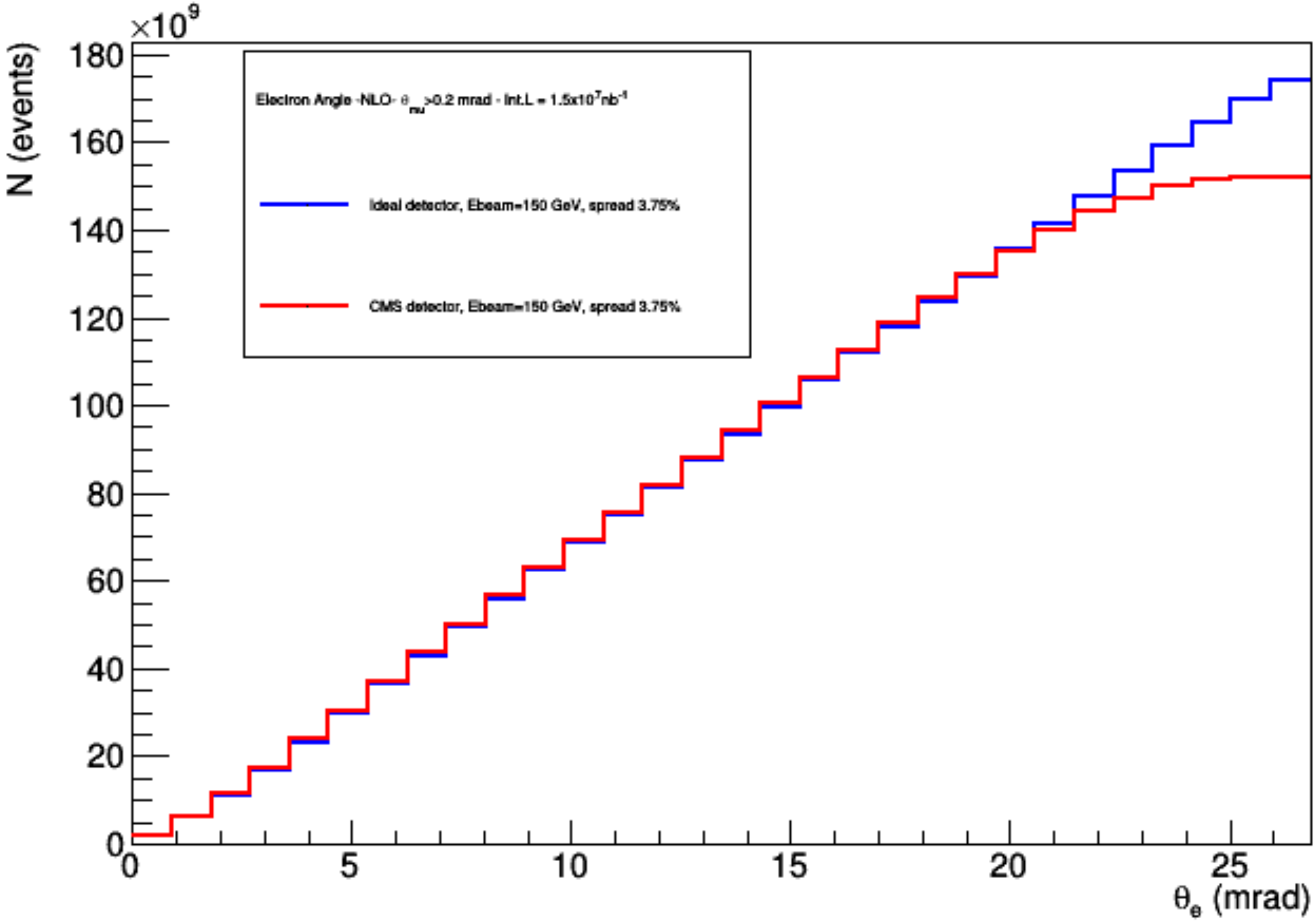
Experimental Smearing sources

Studied with a Fast-MC approach:

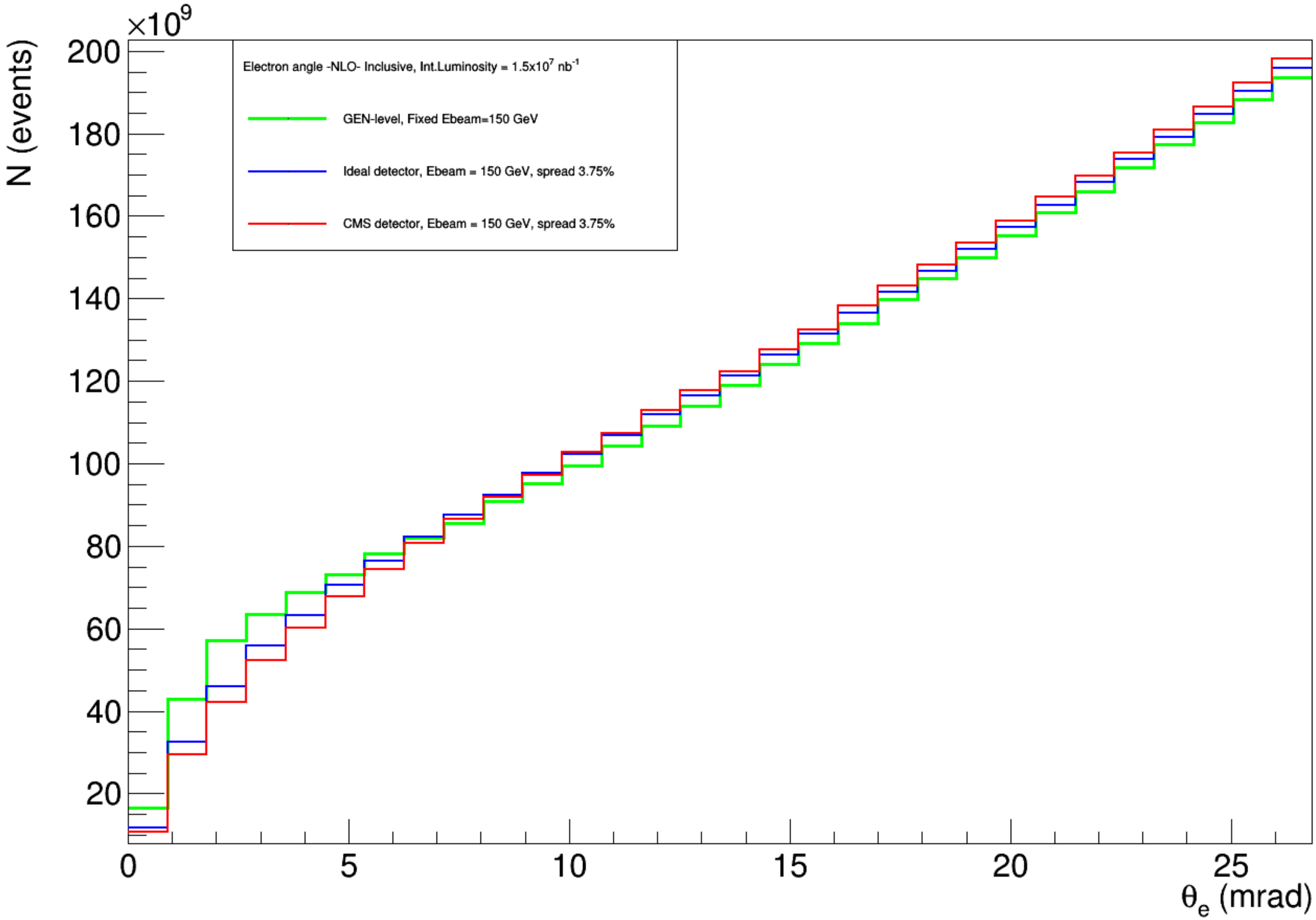
- CMS-like parametrization (by Antonio) of the angular resolution as a function of momentum, considering 1cm Be target and a tracking station made of 3 x and 3 y CMS modules (each with double layer, in total ~12 300um Si layers, i.e. 3.6 mm Si). This parametrization includes both the intrinsic resolution and the material effect of the detector.
- Ideal detector parametrization, consisting of 1cm Be target and a baseline 0.02 mrad angular resolution (as in our EPJC paper).
- Beam Energy smearing: assumed E=150 GeV but studied the effect of a smeared beam with 3.75% energy spread (special MC samples by Carlo)
- (not yet) Bremsstrahlung effect on the scattered electron. Antonio has a good parametrization for this effect, but it should not have a big impact.



N (events)
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
θ_{μ} (mrad)
Muon Angle -NLO- Inclusive Selection
Gen-level, Fixed Ebeam = 150 GeV
Ideal detector, Ebeam = 150 GeV, spread 3.75%
CMS detector, Ebeam = 150 GeV, spread 3.75%



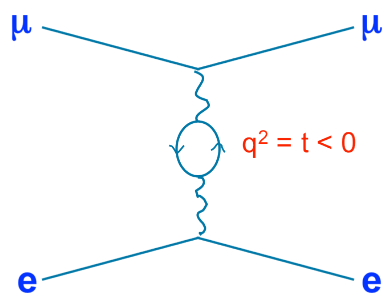
×10^9
N (events)
Electron Angle -NLO- θ_mu>0.2 mrad - Int.L = 1.5x10^7nb^-1
Ideal detector, Ebeam=150 GeV, spread 3.75%
CMS detector, Ebeam=150 GeV, spread 3.75%
θ_e (mrad)
0
5
10
15
20
25
20
40
60
80
100
120
140
160
180

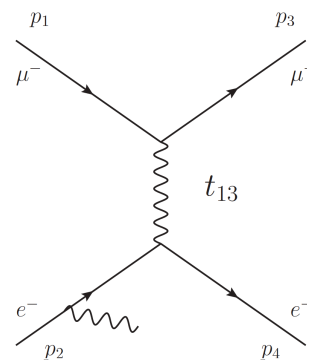


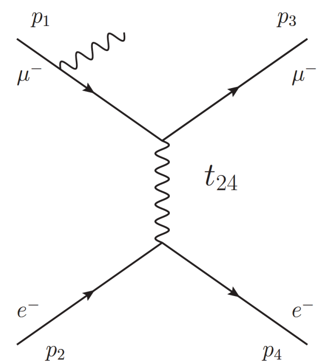
N (events)
×10^9
200
180
160
140
120
100
80
60
40
20
0
5
10
15
20
25
θe (mrad)
Electron angle -NLO- Inclusive, Int.Luminosity = 1.5x10^7 nb^-1
GEN-level, Fixed Ebeam=150 GeV
Ideal detector, Ebeam = 150 GeV, spread 3.75%
CMS detector, Ebeam = 150 GeV, spread 3.75%

NLO template fit of α(t)

At NLO a simple fit with an analytical function is not possible anymore







There is no more a unique t variable related to the running α(t) in any given event
The relevant t depends on the amplitude and is the one calculated from the fermion line without the radiated photon.

The NLO matrix element for real emission is the sum of three contributions related to the emission of a photon from one or the other fermion line and their interference.
The full matrix element can be reweighted according to this splitting in 3 pieces, using a parametric function for the $\Delta\alpha_{had}$. (reweighting core code by Carlo)

Extraction of the running alpha has to use a template method

Template fits

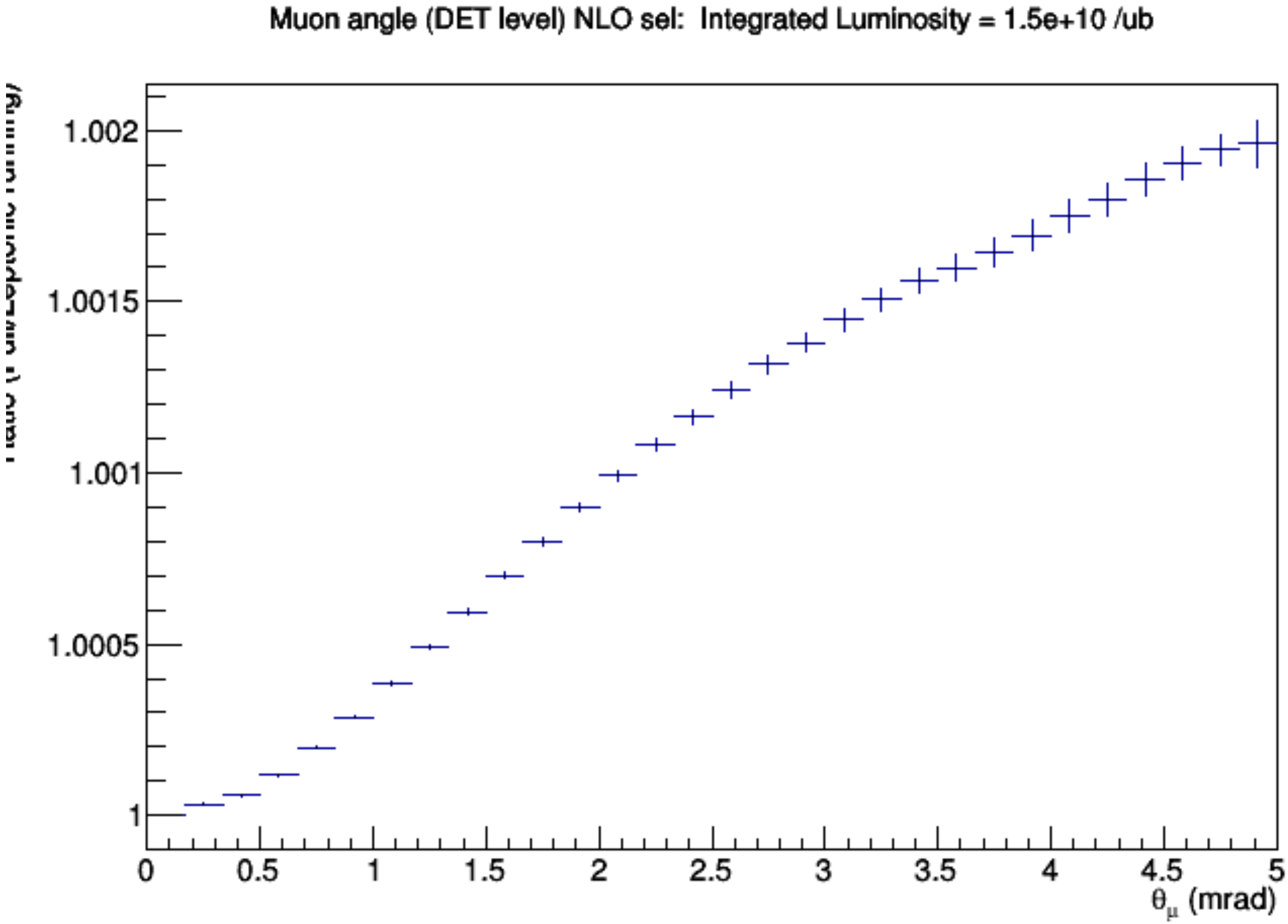
Implemented and validated with the following sequence:

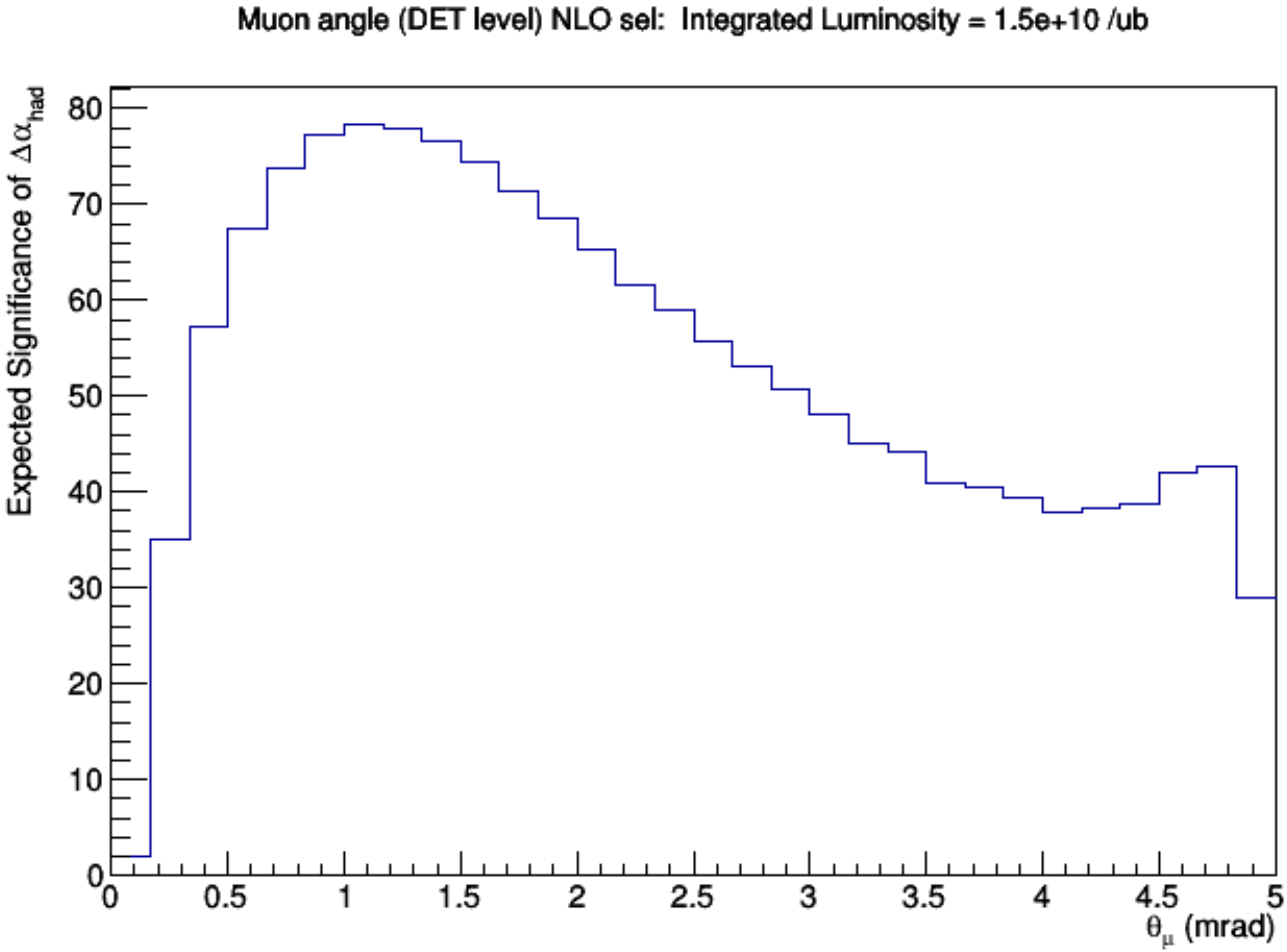
- ✓ Template fit of LO electron angle distribution, to be compared with the simple analytical fit obtained for the CERN workshop
- ✓ Template fit of LO muon angle distribution (inclusive), to be compared with the electron one
- ✓ Template fit of NLO muon angle distribution, to be compared with LO muon
- ✓ Template fit of NLO electron angle distribution with a selection $\theta_\mu > 0.2$ mrad
- ✓ Template fit of NLO electron angle distribution (inclusive !)
- ✓ Template fit of NLO 2D distribution of electron and muon angles, both inclusive and with different kind of selections
- ✓ All cases have been checked both for the ideal (generator level) distributions and for the experimental (detector level) distributions, using the given models
- ✓ Effect of the beam energy spread corresponding to the M2 muon beam studied as another experimental effect

Technical details (important contribution by Paolo)

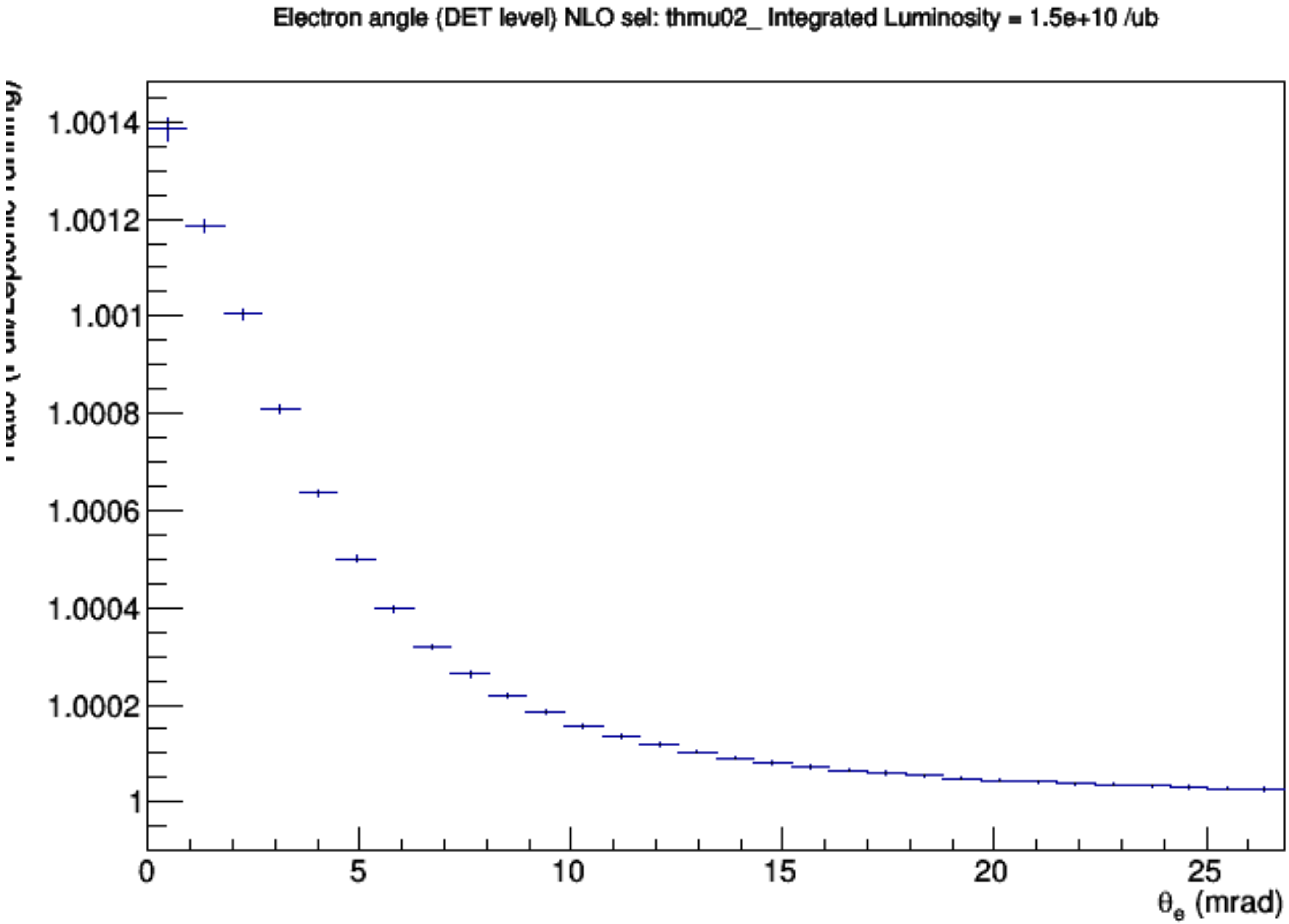
strong correlation between the two fit parameters, overcome with a modified parametrization

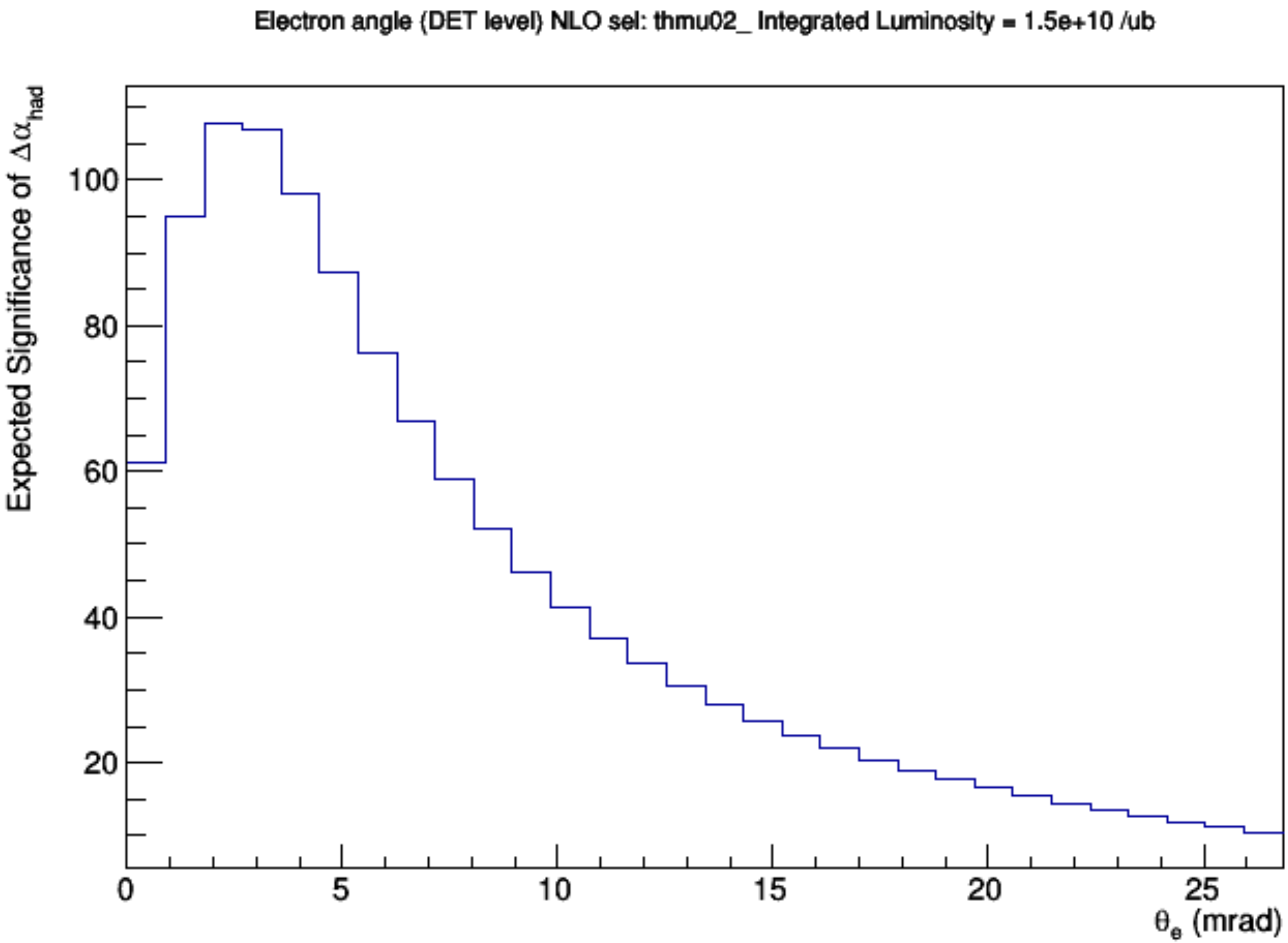
Fit stability tested with two different strategies: exact algebraic chi2-minimization and numerical minimization using ROOT. Both methods give the EXACT SAME results except for extremely pathological cases.

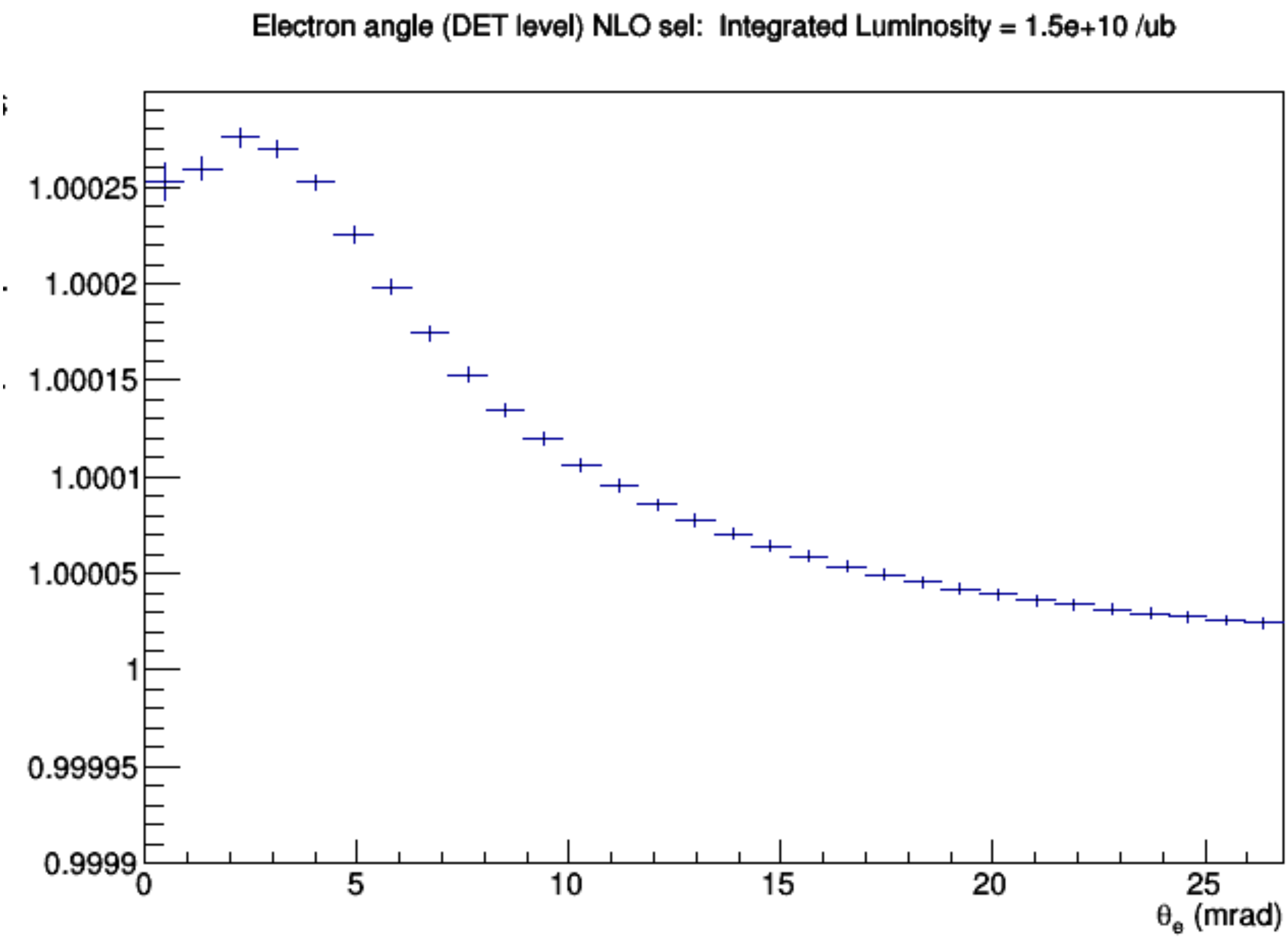

Muon angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub
1.002
1.0015
1.001
1.0005
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
θ_μ (mrad)



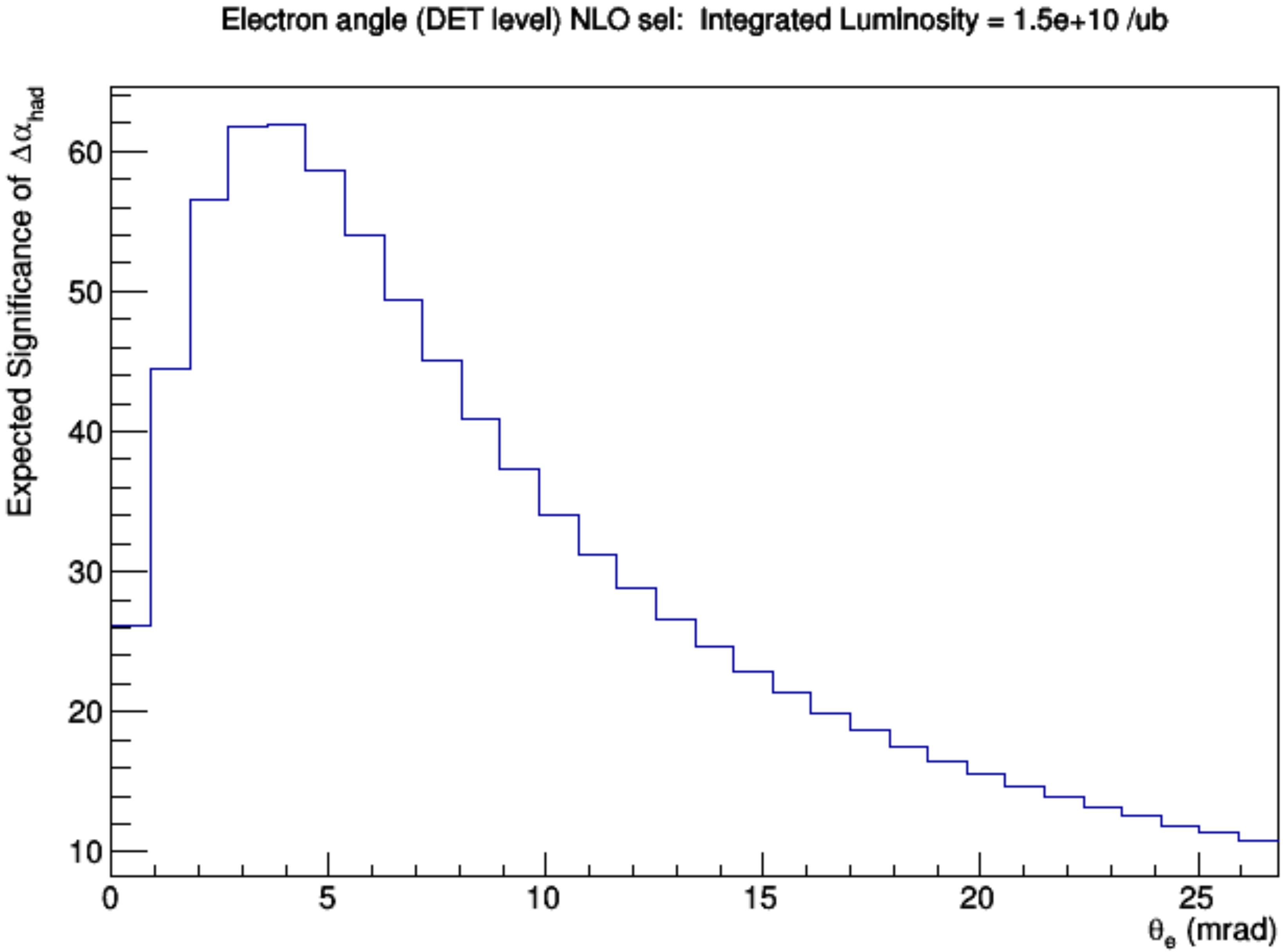
Muon angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub
Expected Significance of $\Delta\alpha_{had}$
0
10
20
30
40
50
60
70
80
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
θ_{μ} (mrad)


Electron angle (DET level) NLO sel: thmu02_ Integrated Luminosity = 1.5e+10 /ub
1.0014
1.0012
1.001
1.0008
1.0006
1.0004
1.0002
1
0
5
10
15
20
25
θe (mrad)

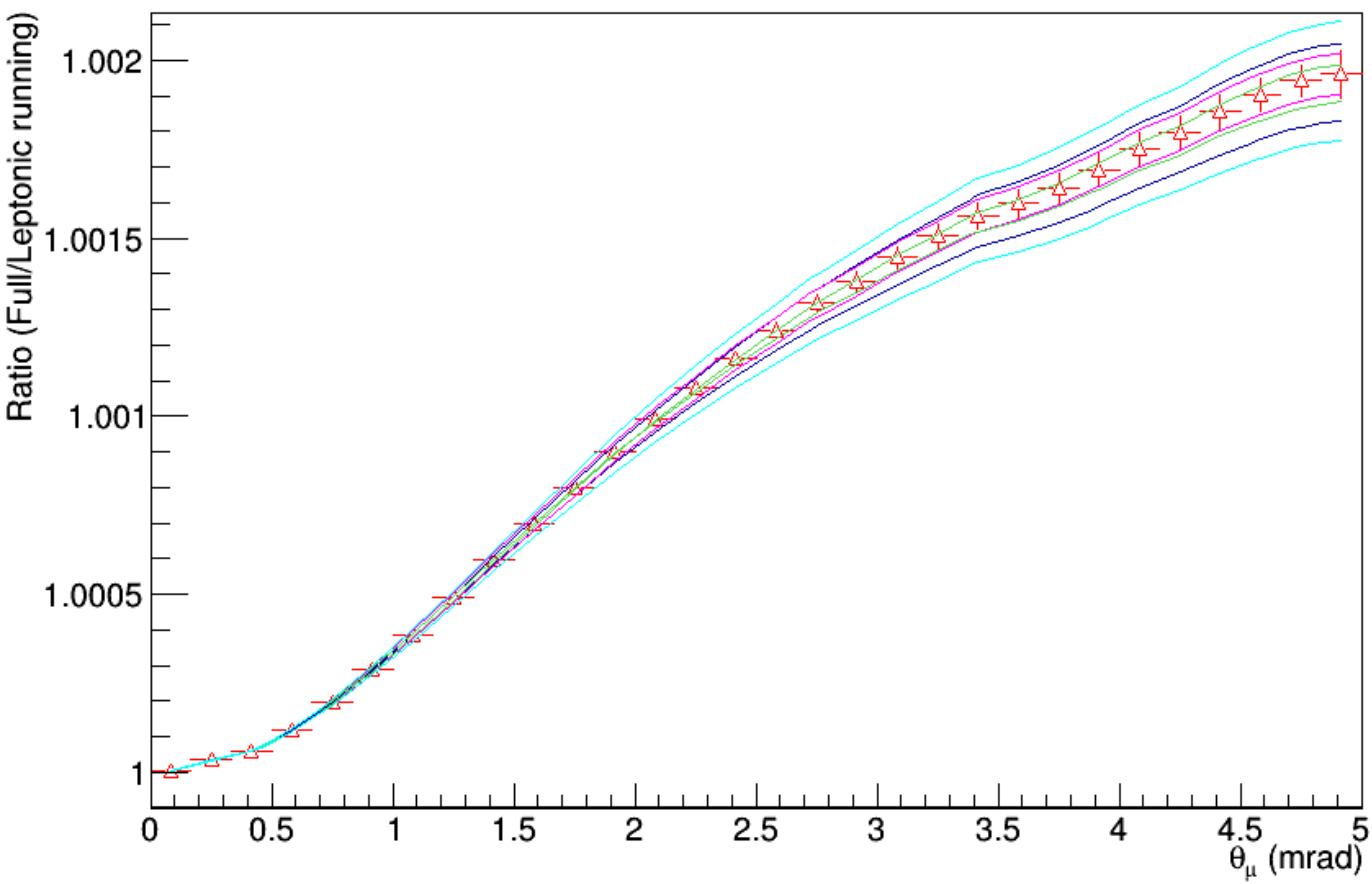

Electron angle (DET level) NLO sel: thmu02_ Integrated Luminosity = 1.5e+10 /ub
Expected Significance of $\Delta\alpha_{had}$
100
80
60
40
20
0
5
10
15
20
25
θ_e (mrad)

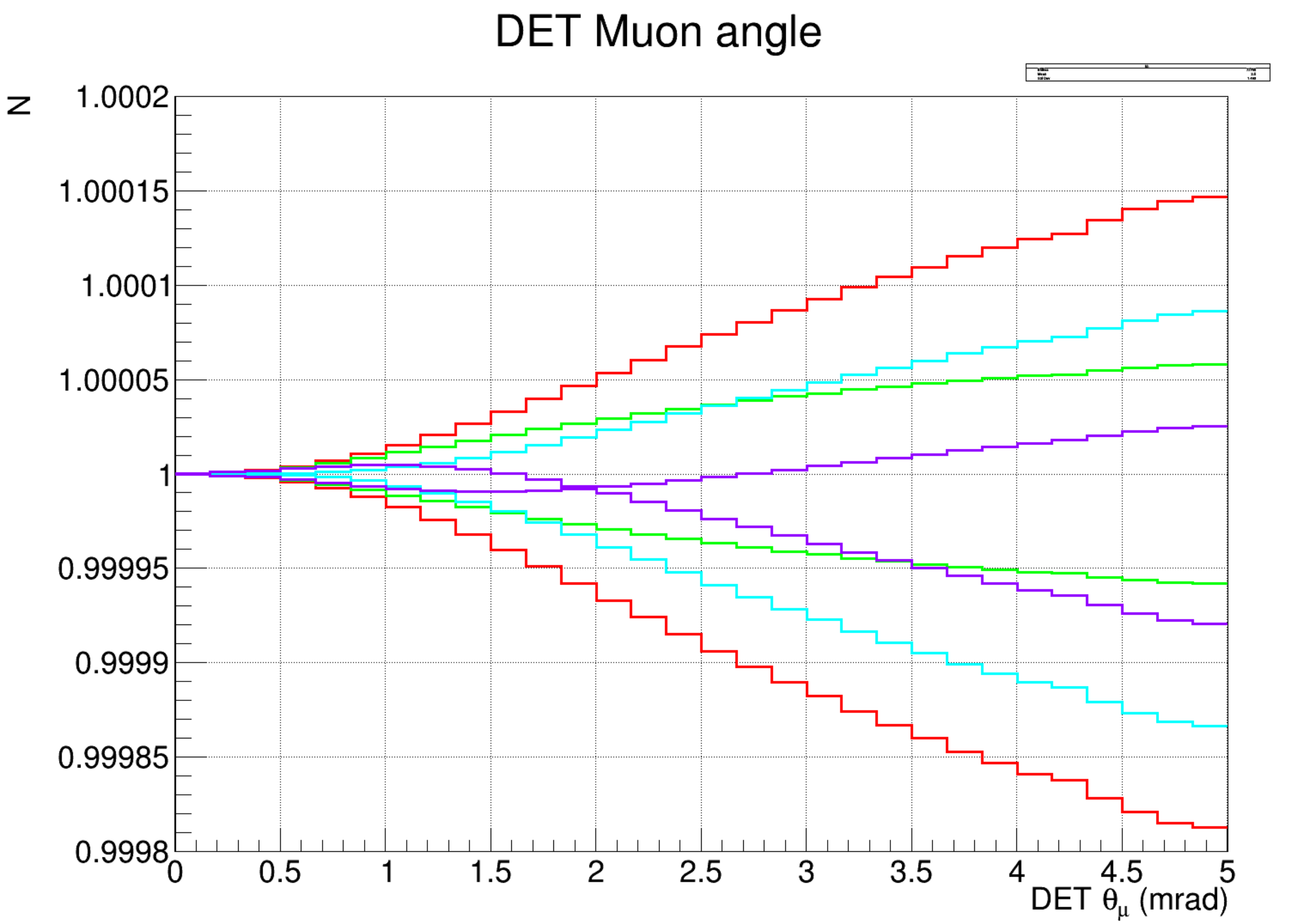


Electron angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub
1.00025
1.0002
1.00015
1.0001
1.00005
1
0.99995
0.9999
0
5
10
15
20
25
θ_e (mrad)

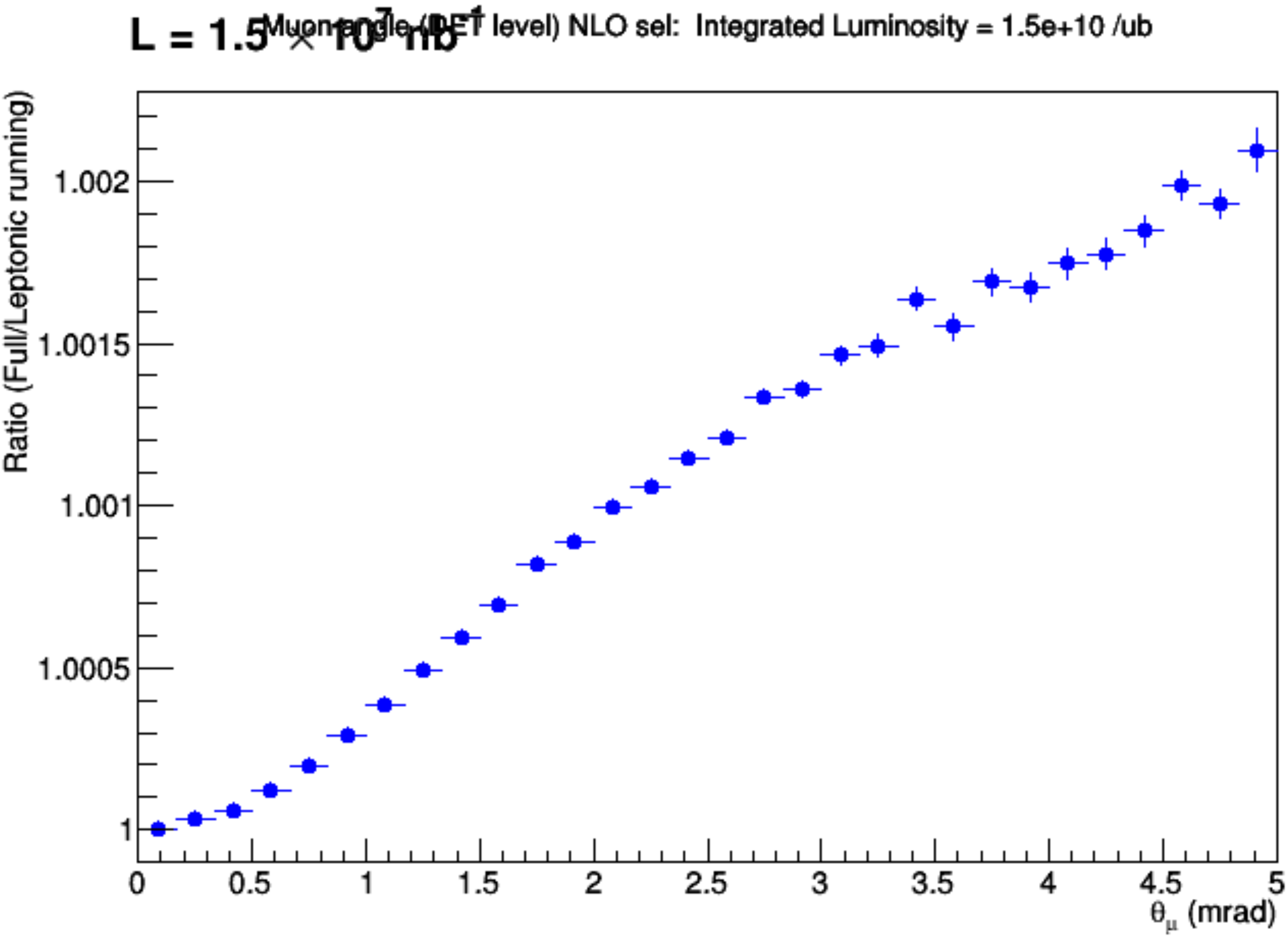

Electron angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub
Expected Significance of $\Delta\alpha_{had}$
10
20
30
40
50
60
0
5
10
15
20
25
θ_e (mrad)

Muon angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub

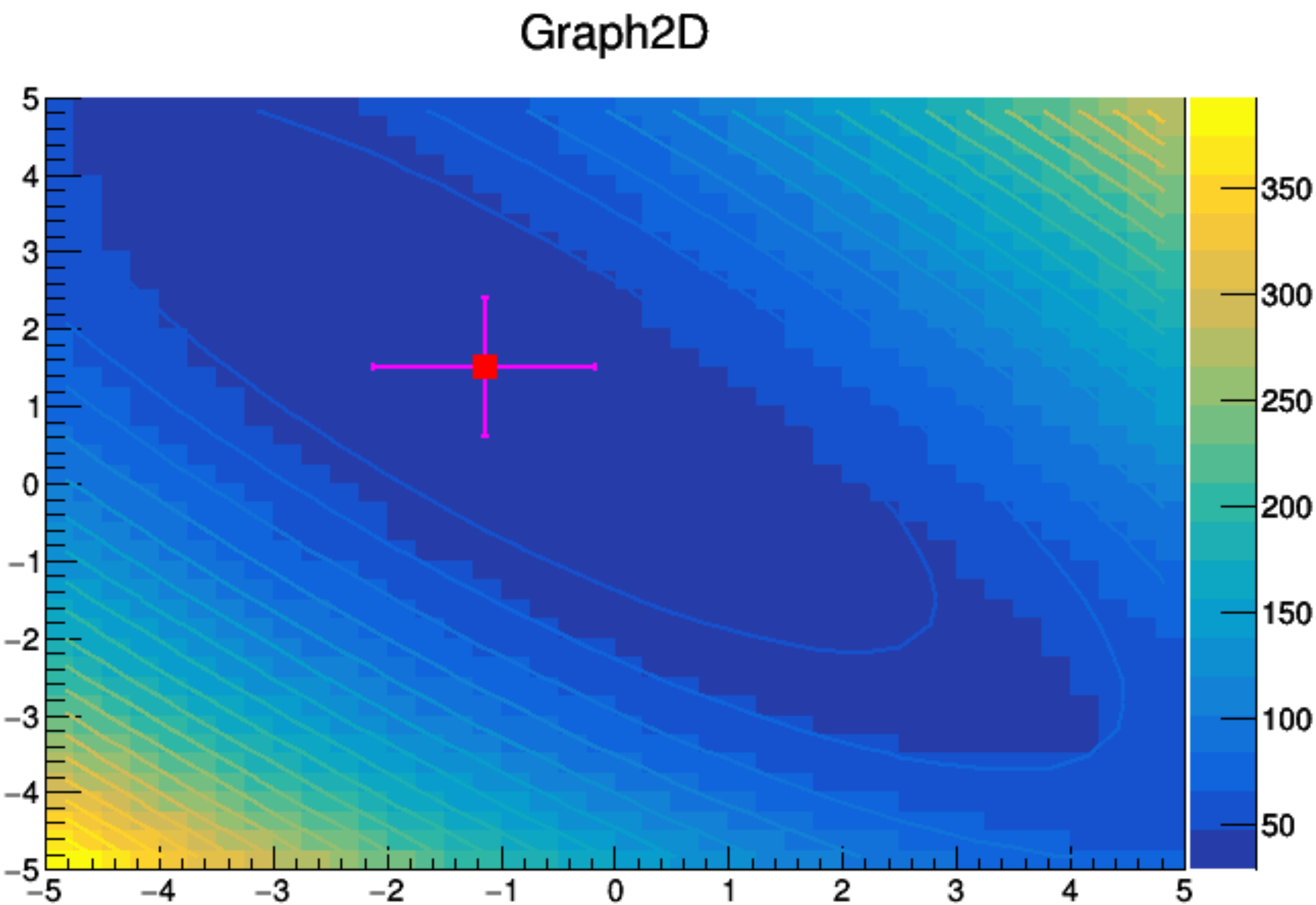




DET Muon angle
N
1.0002
1.00015
1.0001
1.00005
1
0.99995
0.9999
0.99985
0.9998
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
DET θ_μ (mrad)

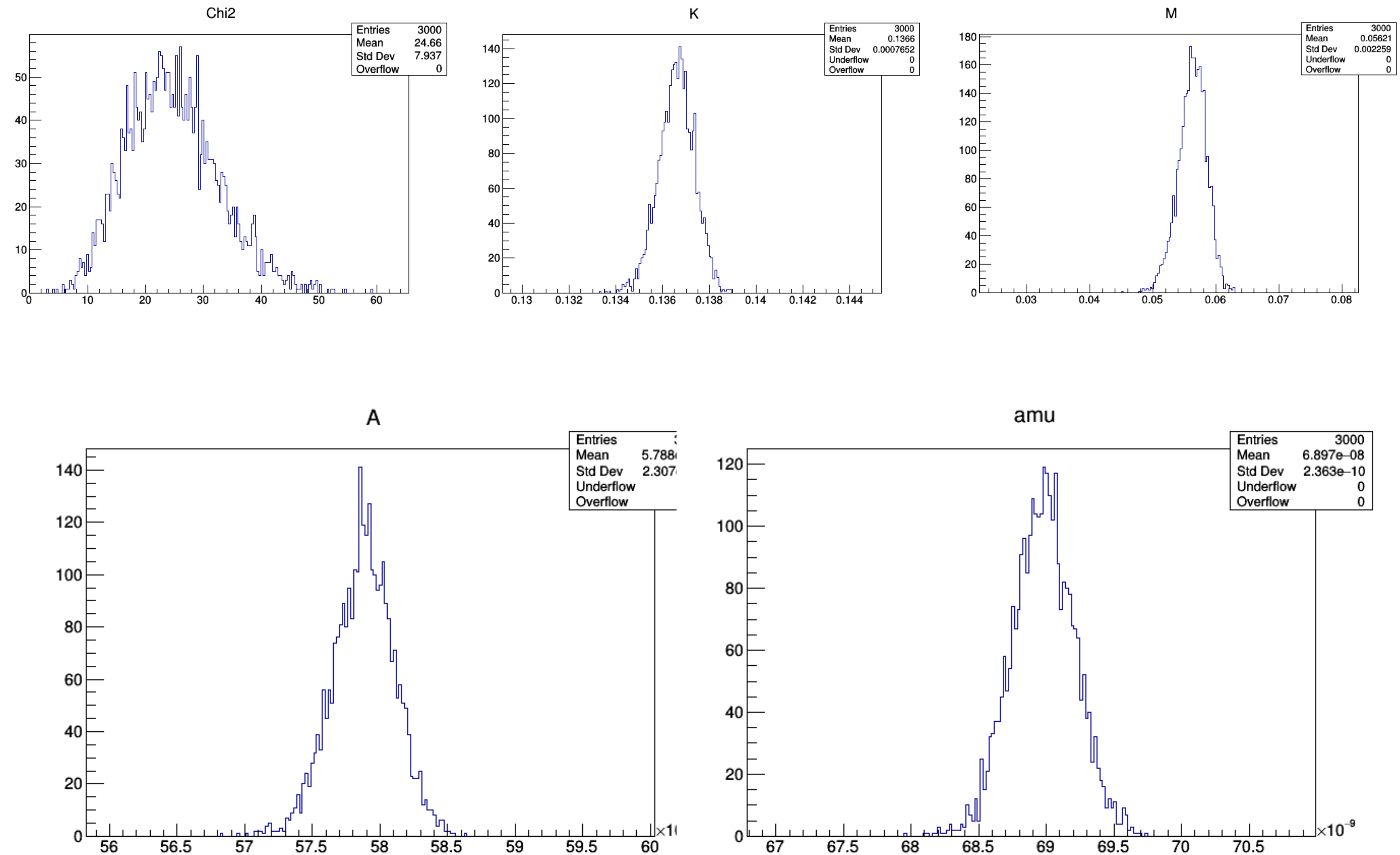


$L = 1.5 \times 10^7\ nb^{-1}$
Muon angle (DET level) NLO sel: Integrated Luminosity = 1.5e+10 /ub
Ratio (Full/Leptonic running)
1.002
1.0015
1.001
1.0005
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
θ_μ (mrad)

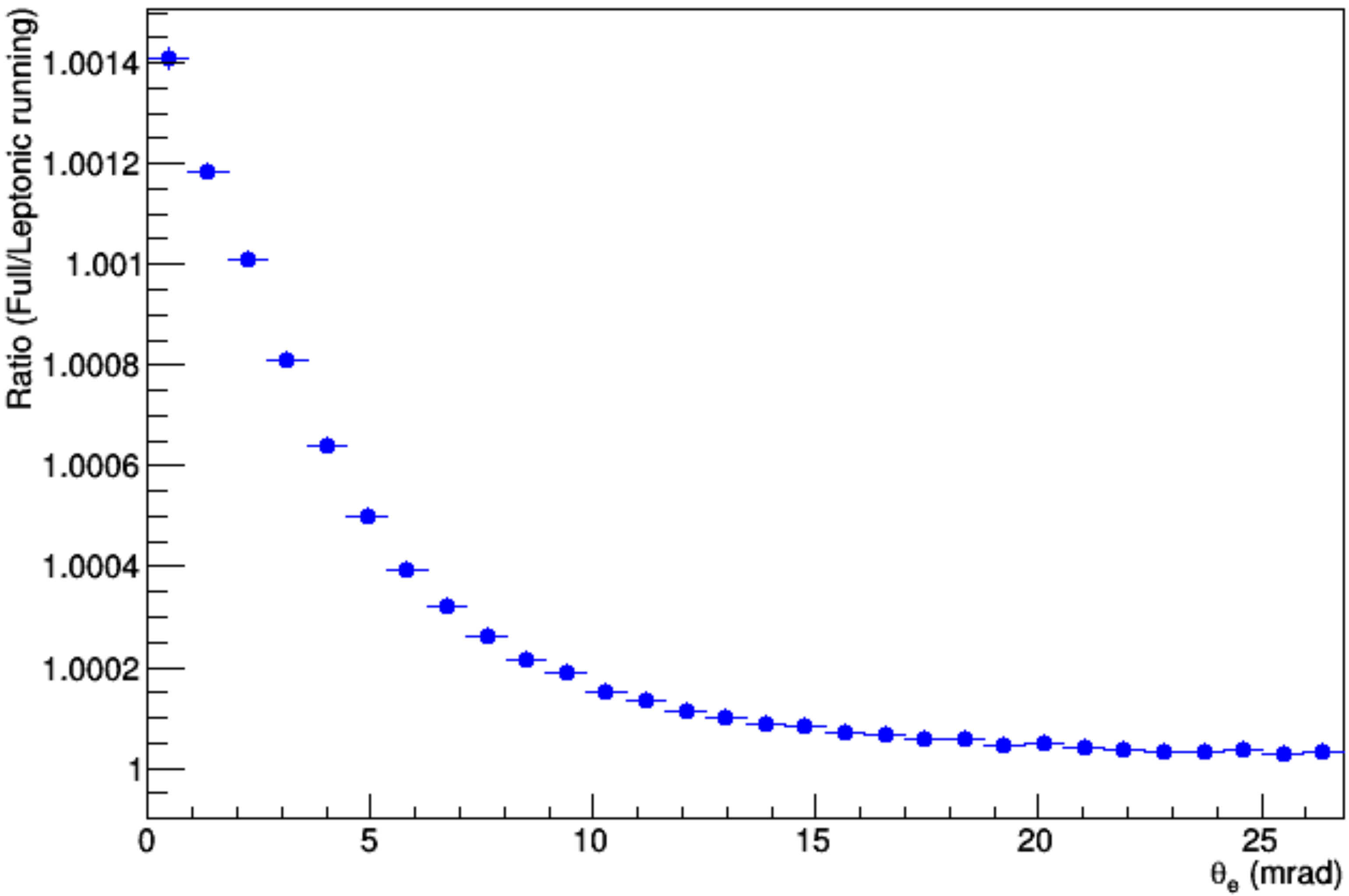


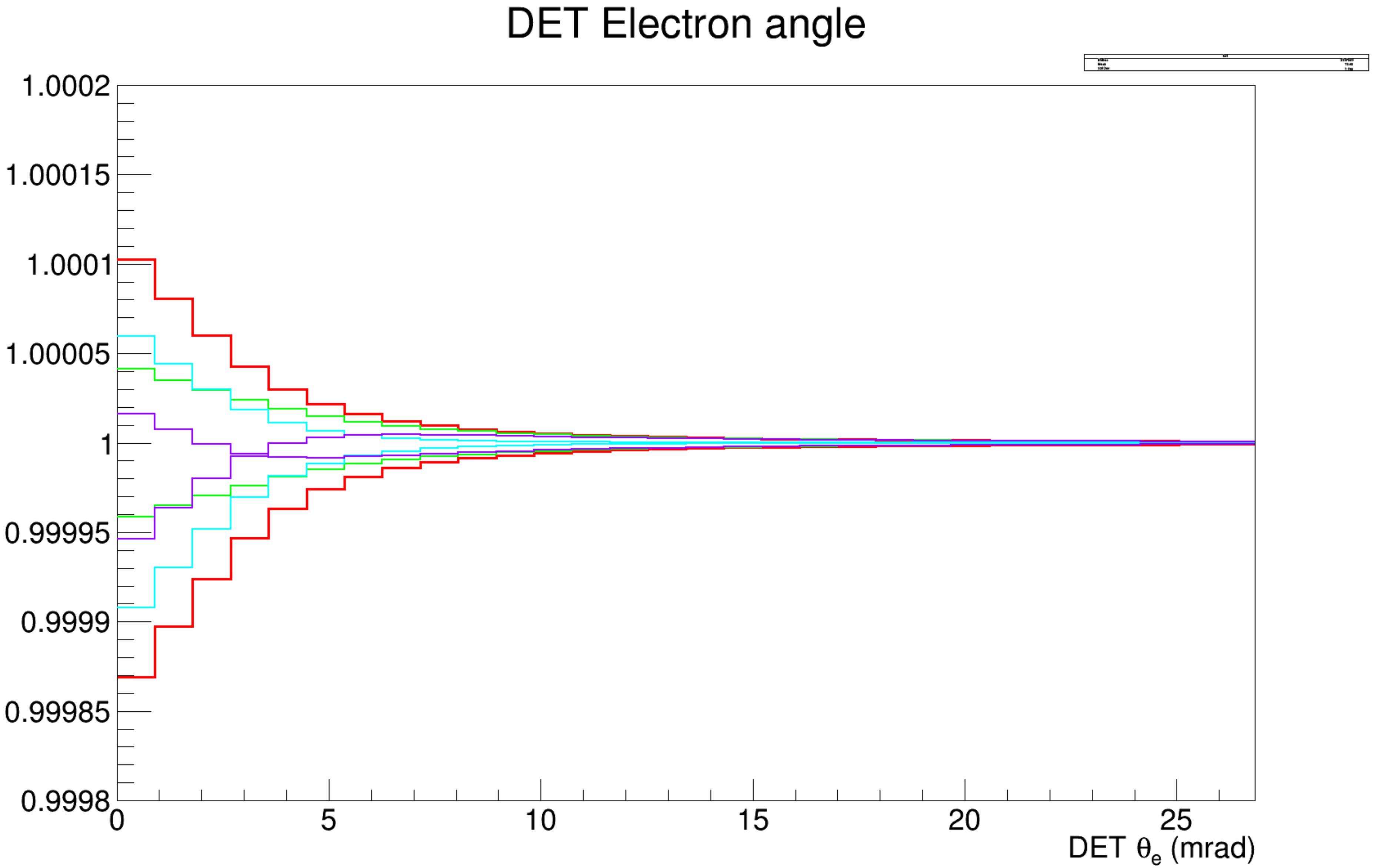
Graph2D
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5
350
300
250
200
150
100
50

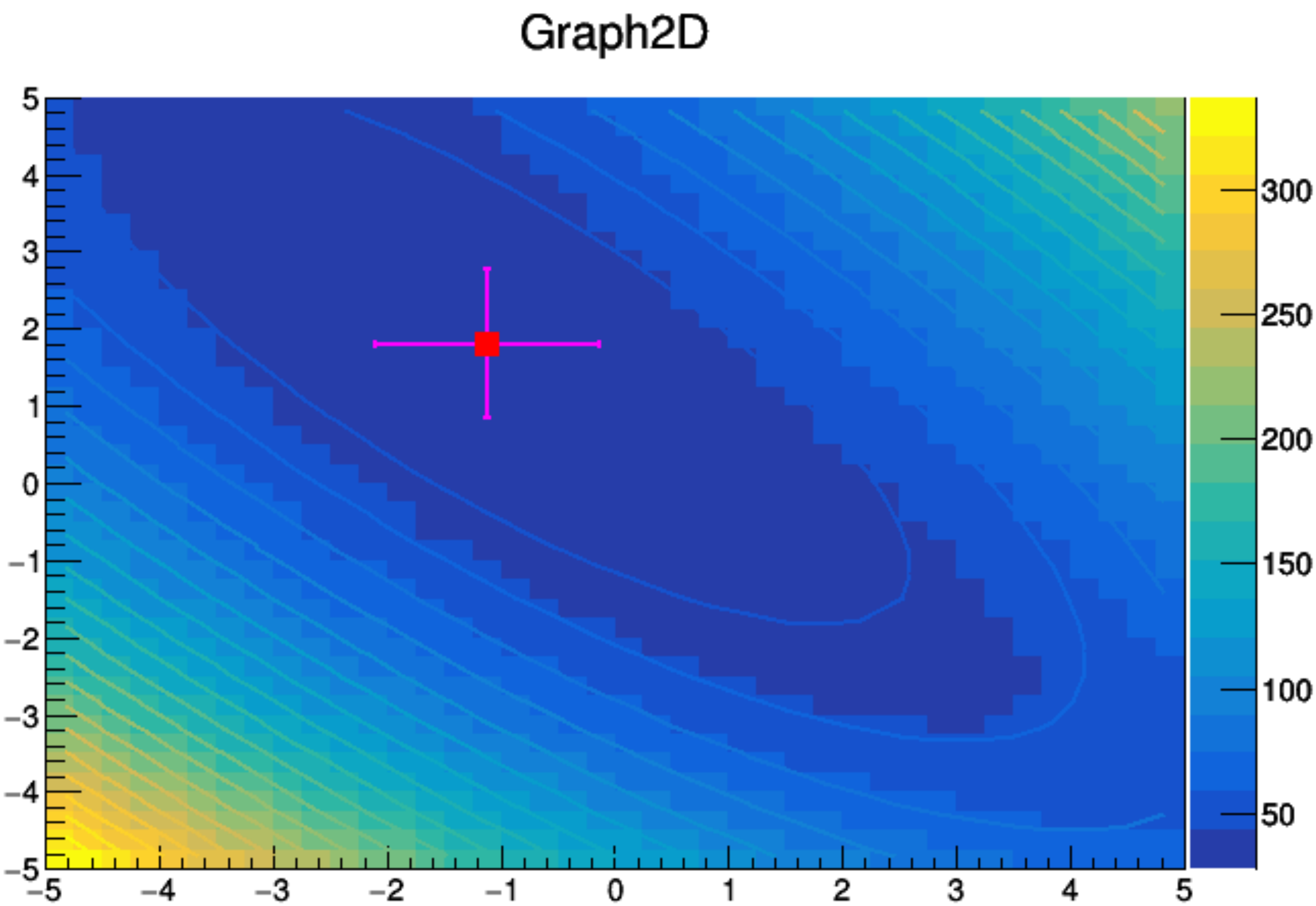
Template fit of muon angle (NLO inclusive, with detector resolution and beam E spread)



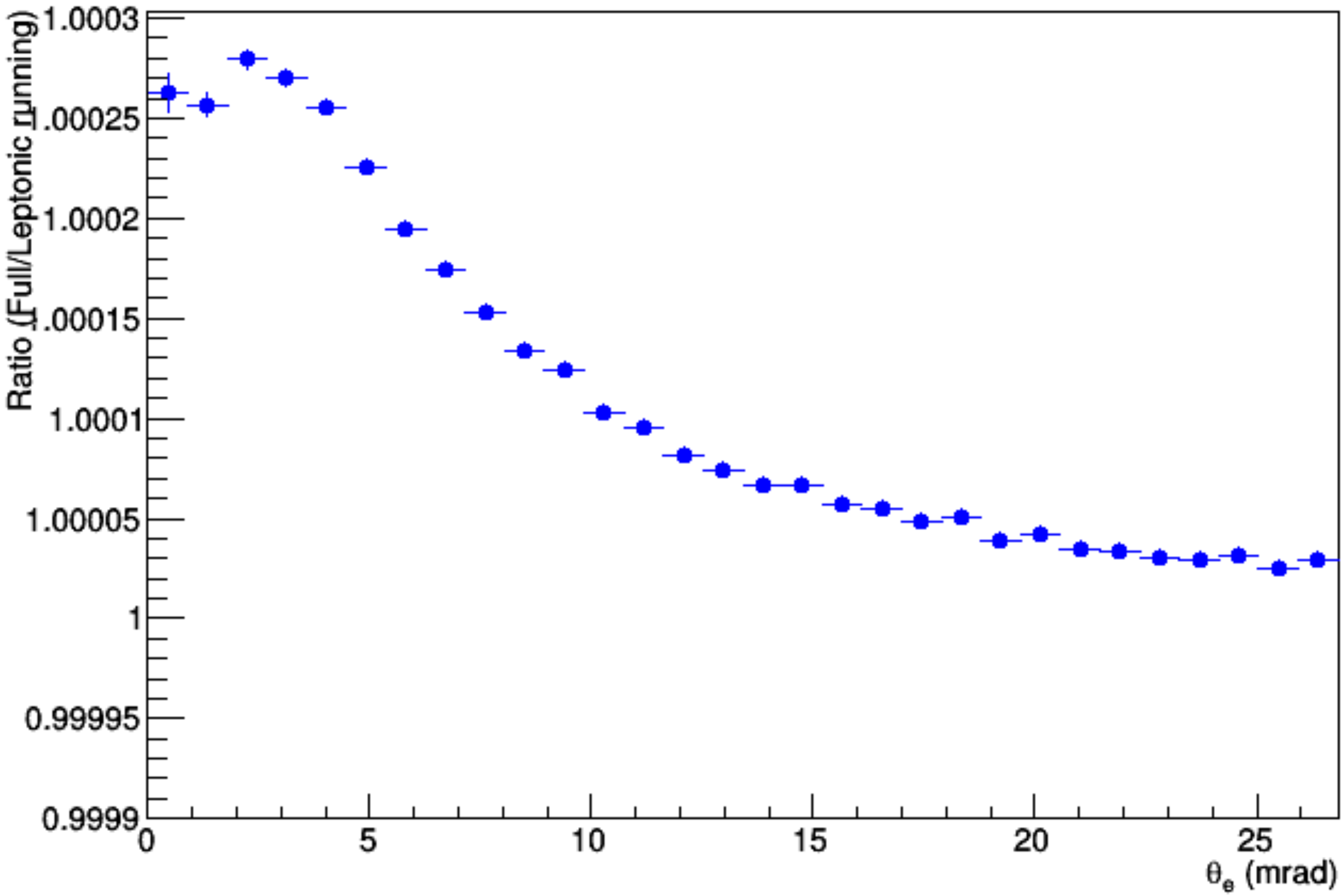
Electron angle (NLO, θ_μ>0.2 mrad, detector resolution and beam E spread)

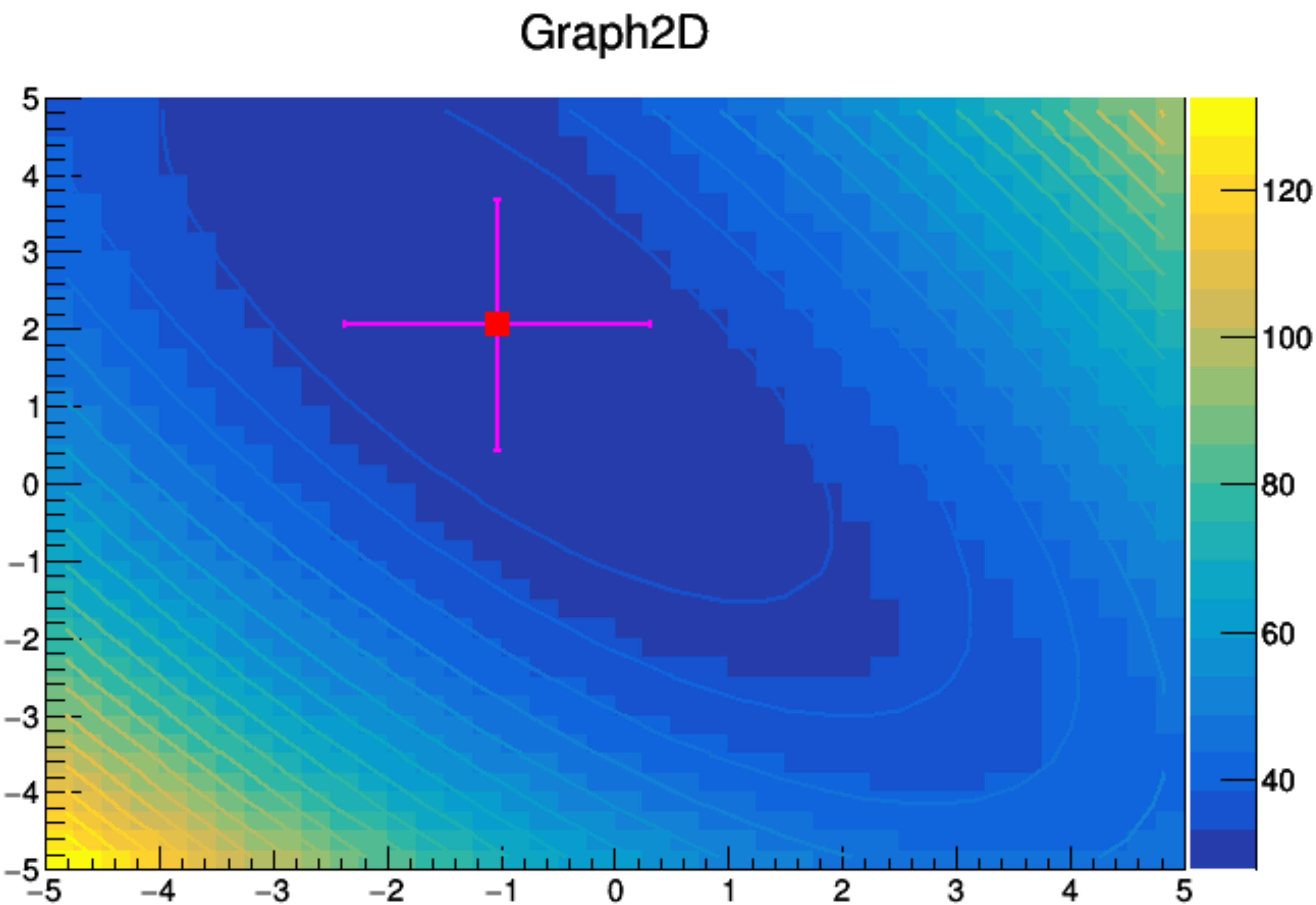



DET Electron angle
1.0002
1.00015
1.0001
1.00005
1
0.99995
0.9999
0.99985
0.9998
0
5
10
15
20
25
DET θ_e (mrad)


Graph2D
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5
300
250
200
150
100
50

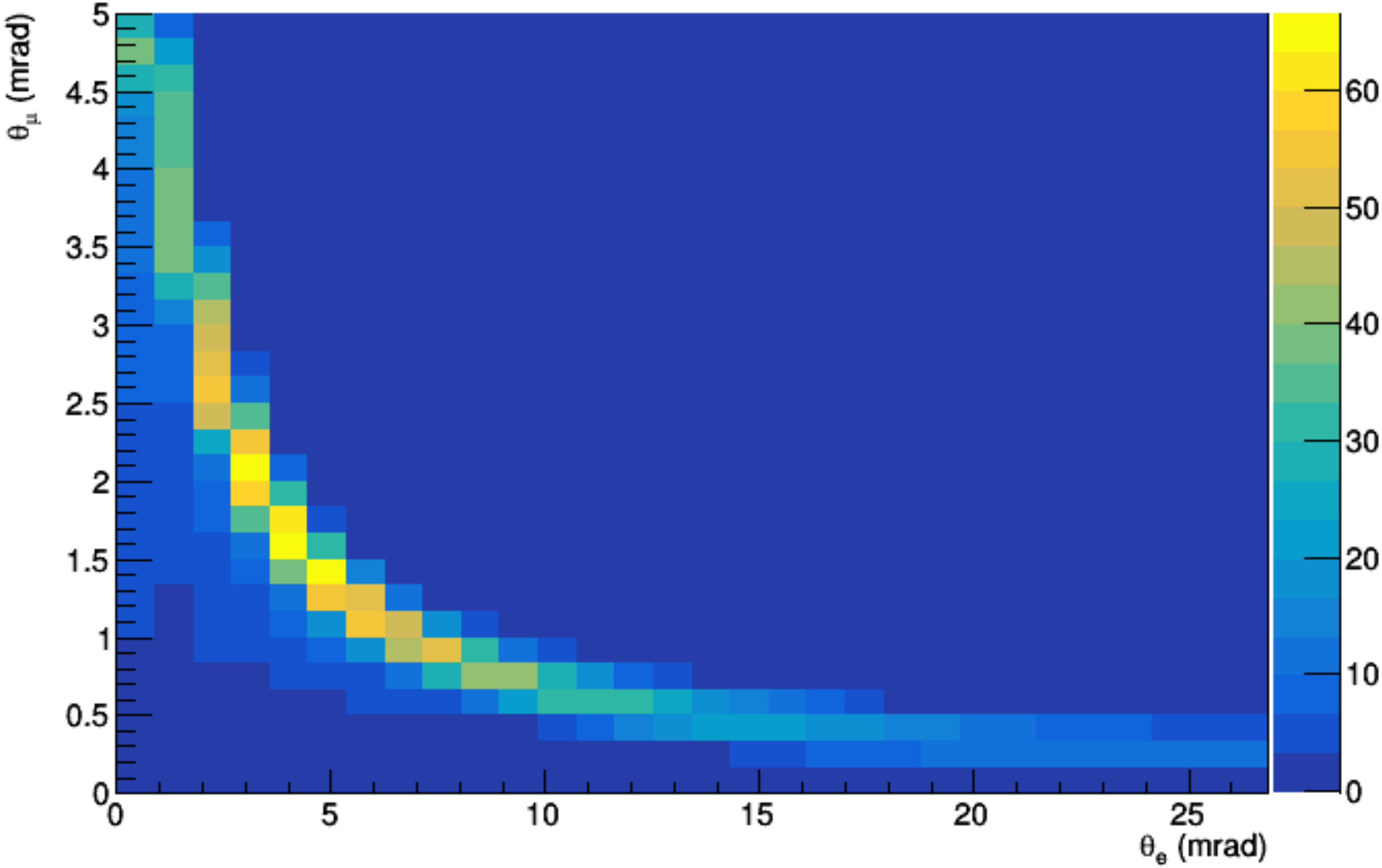
Electron angle (NLO, inclusive, detector resolution and beam E spread)

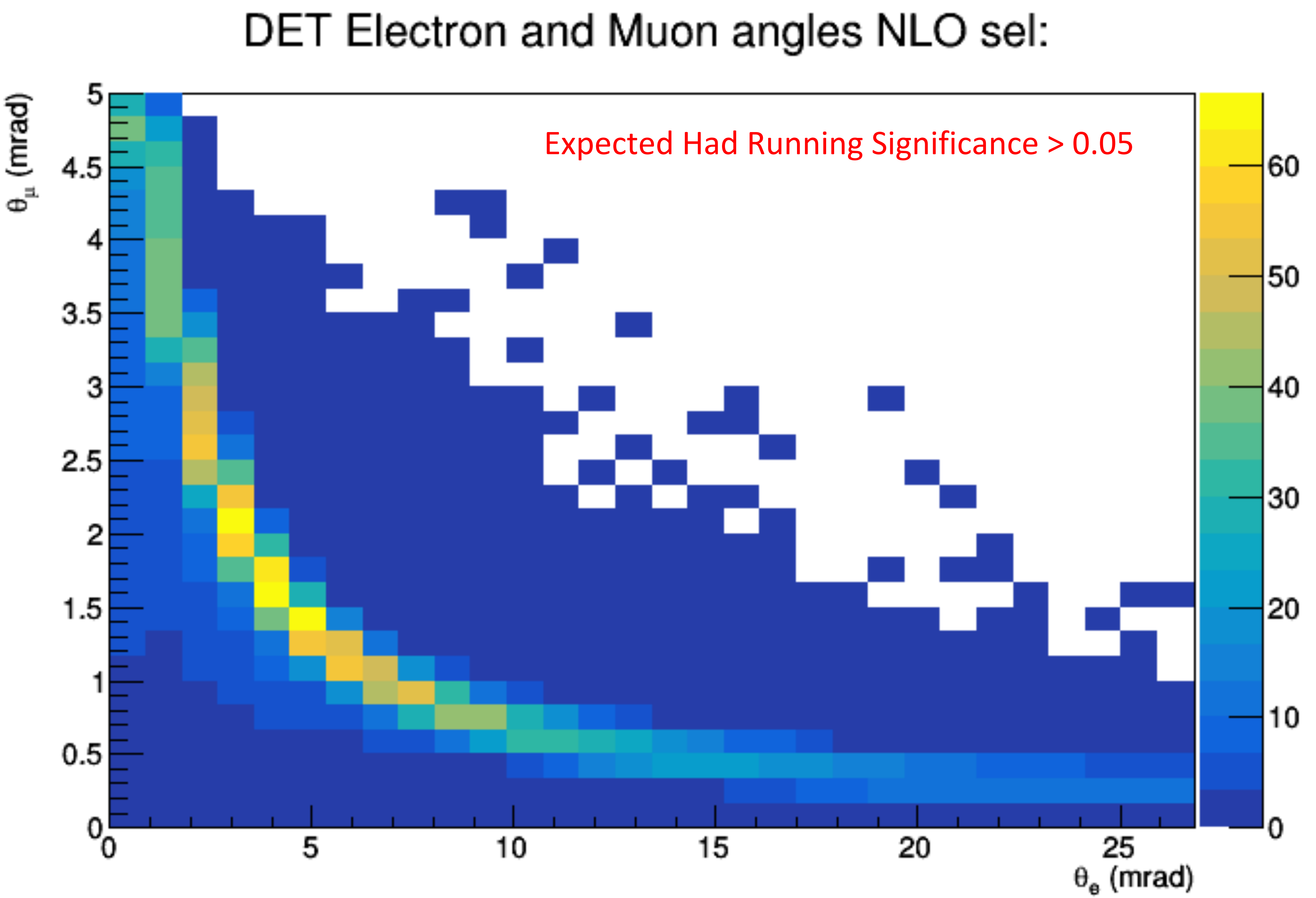


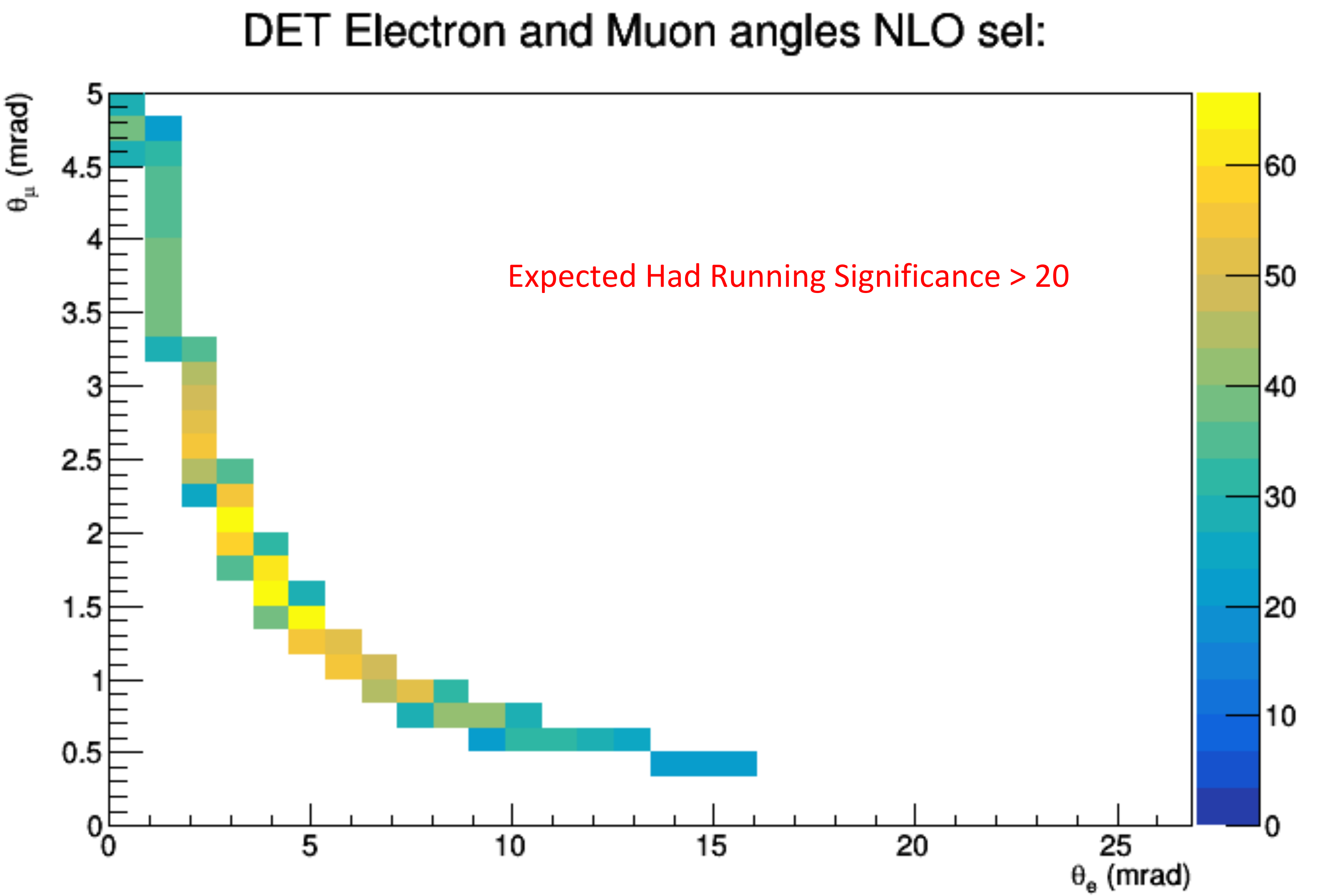

Graph2D
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5
120
100
80
60
40

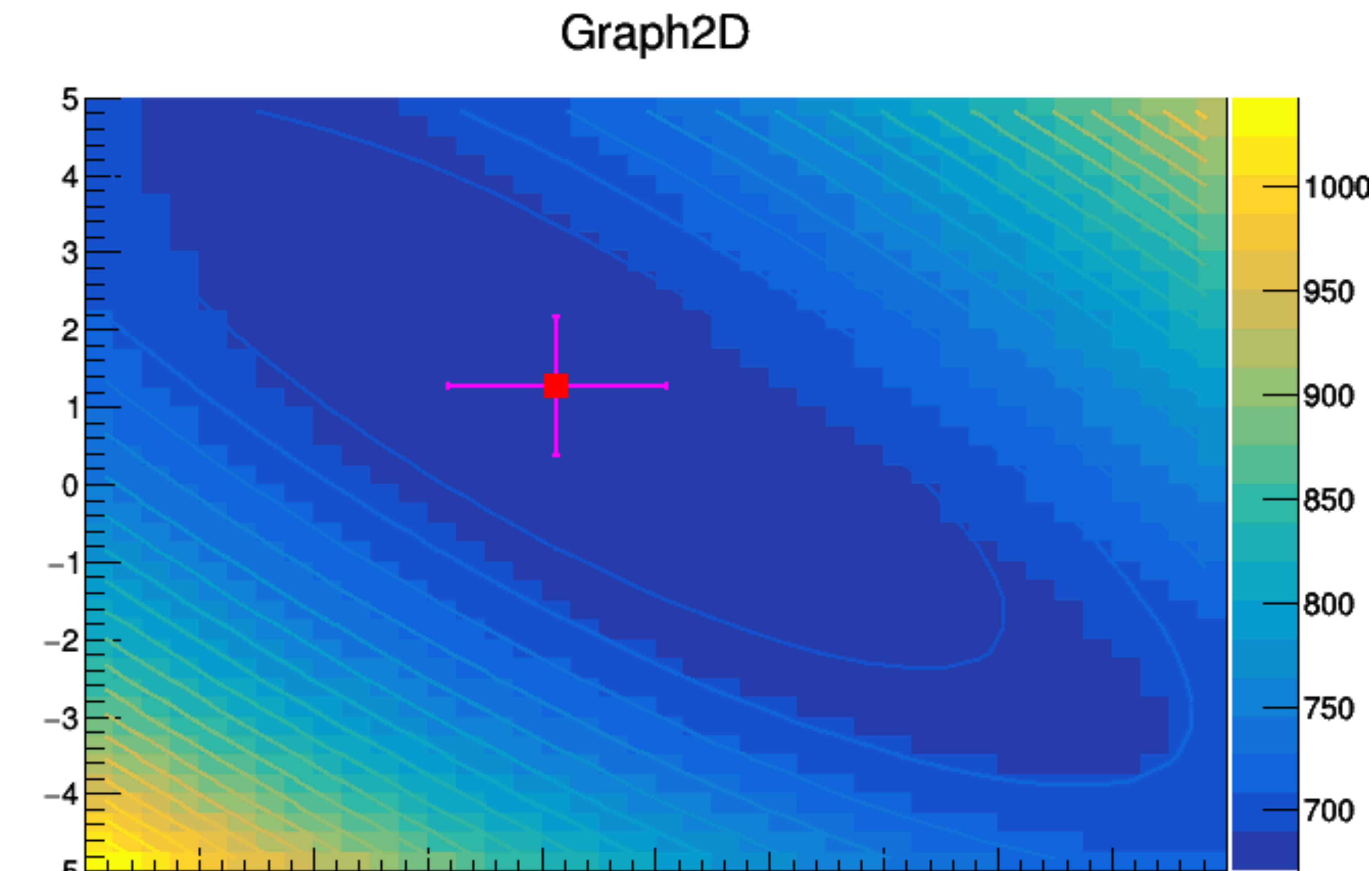
2D fit of electron-muon angles

DET Electron and Muon angles NLO sel:




DET Electron and Muon angles NLO sel:
Expected Had Running Significance > 0.05
θμ (mrad)
θe (mrad)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
5
10
15
20
25
0
10
20
30
40
50
60


DET Electron and Muon angles NLO sel:
Expected Had Running Significance > 20
θ_μ (mrad)
θ_e (mrad)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
5
10
15
20
25
0
10
20
30
40
50
60


Graph2D
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5
1000
950
900
850
800
750
700

Conclusions

- Measurement of α needs smart approaches to overcome the technical difficulty of simulating ~10^{13} events, which have to be processed quickly with variable selections and detector parameters to study the experimental systematics.
  - The analysis workflow has to use a Fast simulation approach
    - Detector response has to be parameterized on Full GEANT4 simulation, and in the real experiment it will have to be determined from real data, as the GEANT4 description of basic processes (e.g.multiple scattering) will not be sufficient to meet the needed precision.
  - Radiative corrections: NLO QED effects have to be taken into account, as they impact the measurement as much as the detector effects (or in some cases more).
  - We think to have figured out a strategy -> DONE and Validated

- On the path discussed at the CERN workshop: complete fit of the signal including detector resolution effects and beam energy spread is done.

- To-Do: systematic tests with real systematic errors.
  - Beam energy calibration to be tested first: Carlo already prepared MC samples with energy shifted by +/- 1 − 5 MeV, according to our expected accuracy.
  - Other systematic tests will follow

- Priority: write section on the LoI (on-going)

- There is still much to be done …