Holographic Charge Density Waves

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June 6, 2019

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Overview

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   - What is a CDW
2. Gauge/Gravity duality
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   - The model
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Motivation

First reason: CDWs have been observed recently in experiments.[1,2]

There exists a large class of materials, showing a behavior between conventional metals and insulators. Some examples are:

- **strange metals** (unconventional scalings)
- **bad metals** (no Drude peak)

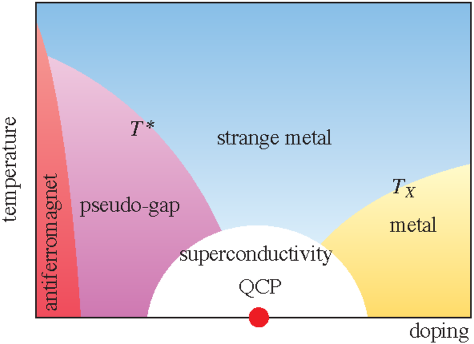
Their transport properties can not be easily explained by standard theories. A proposed explanation is given in the context of CDW.

Main goal: to find an effective field theory capable to reproduce their behavior.

[1] Kogar et al., Phys.Rev.Lett. 118, 2017
[2] Wang et al., Material Today Phys. 5, 2018

Strange/Bad metals

In the temperature/doping phase diagram, strange/bad metals are placed:



The mean free path of quasi-particles is so short that the Boltzman equation underlying Drude theory is not consistent.

These metals are characterized by a **large resistivity** even in rather **clean materials** with a long-lived momentum.

Drude-like peak

In many compounds:

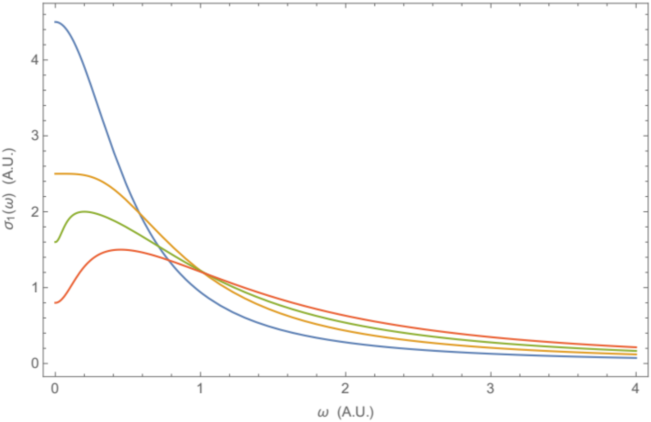


Figure: Drude peak broadens and moves away from $\omega = 0$ when the temperature is increased.

Drude-like peak

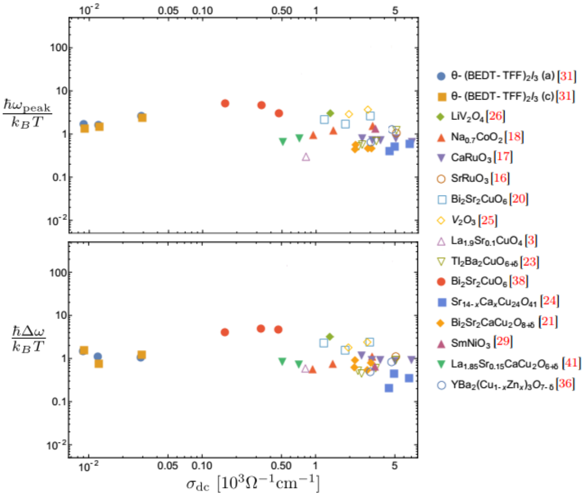


Figure: Position and width of Drude peak shown for a large variety of compounds

Scaling behavior

When turning on an external magnetic field, strange metals are characterized by an unconventional scaling behavior:

$$\rho_{xx} \sim T, \qquad \tan\theta_H = \frac{\rho_{xy}}{\rho_{xx}} \sim T^{-2}$$

In contrast with standard Fermi liquids where

$$\rho_{xx} \sim T^2$$

and in general resistivity and Hall angle should scale with opposite exponents

$$\rho_{xx} \sim T^n, \qquad \tan\theta_H \sim T^{-n}$$

Proposed explanations

In the last years, some mechanisms have been proposed that are believed to cause this behavior:

- Mott-related pseudogap
- polaron excitation
- incipient localization

Since this behavior is similar among different classes of materials, it can be useful to adopt a **less microscopic** approach.

Another possible explanation to these phenomena is believed to be the creation of **conducting Charge Density Wave** (CDW).

Classical theory of CDW

Firstly proposed by Peierls in 1955, the resulting charge density, in presence of electron-phonon interaction is[3]:

$$\rho(\mathbf{r}) = \rho_0 + \rho_1 \cos(2\mathbf{k_F} \cdot \mathbf{r} + \varphi)$$

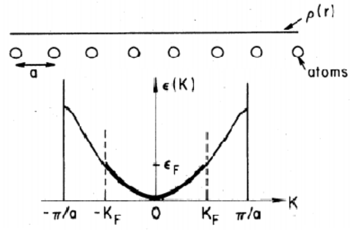


Figure: Distortion of the ionic lattice modulates the charge density and opens a gap at the Fermi energy.

[3] Grüner, Rev.Mod.Phys. 60. 1988

Hydrodynamic result

In hydrodynamics a well-known result for the conductivity is[4]:

$$\sigma(\omega) = \sigma_0 + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Gamma - i\omega)(\Omega - i\omega) + \omega_0^2}$$

that in large Ω limit gives the Drude-peak behavior:

$$\sigma(\omega) = \sigma_0 + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma - i\omega} + O(\Omega^{-1})$$

Also the strange scaling of the Hall angle can be explained within this context:

$$\tan\theta_H \sim B \frac{\rho^2}{\chi_{PP}\Gamma} f(\Omega, \Gamma, \sigma_0)$$

[4] Delacrtaz et al., SciPost Phys. 7, 2017

Why Holography?

Problem:
classical theories (weakly coupled) of CDW lead to **insulating systems**. Even if the CDW is incommensurate w.r.t. the underlying lattice, the impurities pin the CDW, making it an insulator.

Then why **holography** may help?

- Some holographic models lead to **conducting CDW**.
- Relatively easy to access the strong coupled regime.
- Intermediate approach between existing field theory approaches working only in certain limits.

Historical introduction

Main idea: connect a Conformal Field Theory to a Classical Theory of Gravity in one more dimension.

- The first example found by Maldacena in 1997 connecting type IIB superstring on $AdS_5 \times S_5$ and the $\mathcal{N} = 4$ SYM in the large-N limit at strong coupling.
- Many other examples in different number of dimensions.
- Generalizations to QCD and **condensed matter systems**.

Remark: string theory can be used as a bridge between quantum field theory and gravity in order to prove the duality, but it is not necessary to work with the duality.

The GKPW formula

The main ingredient to write the *holographic dictionary* is the GKPW formula[5]:

$$Z_{CFT}[\{h(x)\}] = Z_{AdS}[\{h(x_\mu, r)\}]$$

It brings some consequences:

- the boundary value of a bulk field is the source of an operator in the CFT
- the first free term in the boundary expansion of fields is the VEV of the corresponding operator

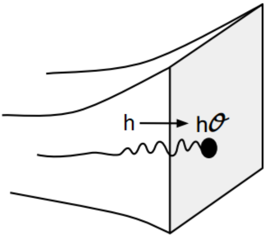


Figure: Boundary value of the fields

[5] Hartnoll at al., arXiv:1612.07324[hep-th], 2016

The GKPW formula

- the extra radial coordinate geometrizes the renormalization group
- gauge symmetries in the bulk correspond to global symmetries in the CFT

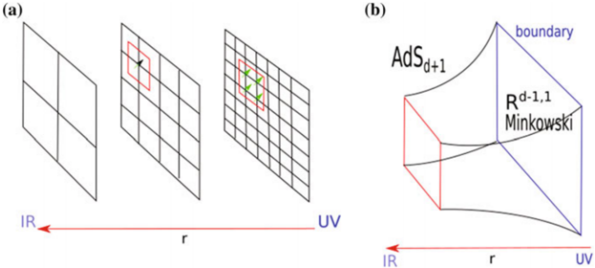


Figure: Relation between the renormalization group flow and the extra radial coordinate in the bulk

Finite temperature

Field theory side:

- compactify the temporal direction
- the temperature is: $T = 1/L_T$

Gravity side:

- Black-hole solution: $ds^2 = \frac{L^2}{r^2}(-f(r)dt^2 + \frac{1}{f(r)}dr^2 + d\vec{x}^2)$
- the temperature is: $T = \frac{|f'(rh)|}{4\pi}$

Rule: a quantum field theory at finite temperature is dual to a black-hole solution in the gravity side, and the black-hole temperature is exactly the temperature of the field theory.

Holographic dictionary

Boundary:

- Partition function
- Source/vev of operators
- Two point functions
- Global symmetries
- Renormalization group flow
- Finite temperature

Gravity:

- Partition function
- Leading/subleading boundary values of fields
- Ratio SL/L boundary values
- Gauge symmetries
- Evolution in the radial direction
- Black-hole solution

Holographic model

We start by considering the following Einstein-Maxwell-Dilanton action plus an **axion** term[6]:

$$S = \int d^{3+1}x\sqrt{-g}\left(R - V(\phi) - \frac{1}{2}(\partial\phi)^2 - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}Y(\phi)\sum_{i=1,2}(\partial\psi_i)^2\right)$$

The real couplings V, Z and Y near the boundary $(r \to 0)$ behave like:

$$V_{UV} = -6 + \frac{1}{2}m\phi^2 + \ldots, \quad Z_{UV} = 1 + z_1\phi + \ldots, \quad Y_{UV} = y_2\phi^2 + \ldots$$

in order to guarantee an asymptotically AdS_4 space-time and the spontaneous breaking of translations.

[6] Amoretti et al., Phys.Rev.Lett. 120, 2018

Background ansatz

The full background ansatz is:

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}^2, \quad A = a(r)dt, \quad \psi_i = kx_i$$

By using the eom, these functions are constrained to be (near the boundary):

$$D(r) = \frac{1}{r^2}(1 + d_3 r^3 + ...), \quad B(r) = \frac{1}{r^2}, \quad C(r) = \frac{1}{r^2}(1 - \frac{d_3}{2} r^3 + ...)$$

$$a(r) = \mu + \rho r + ...$$

The axion is linear in k in order to reproduce the CDW behavior and break translations.

Spontaneous Symmetry Breaking

The real scalar has the following boundary expansion:

$$\phi(r) = \phi_{(s)} r^{3-\Delta} + \phi_{(v)} r^{\Delta} + ..., \qquad r \to 0$$

We set:

$$\Delta = 2 \qquad \text{Scaling dimension of the scalar operator.}$$
$$m^2 = \Delta(\Delta - 3) = -2 \qquad \text{Squared mass of the scalar field.}$$

Translations are **spontaneously** broken if we set:

$$\phi_{(s)} = 0$$

Keeping $\phi_{(s)} \neq 0$ would lead to explicit breaking of the symmetry.

Renormalized on-shell action

The on-shell action is divergent, so it must be renormalized by subtracting appropriate counter-terms.:

$$S_{c.t.} = \int_{r=\epsilon} d^3x\sqrt{-\gamma}\Big(2\mathcal{K} + 4 + \frac{1}{2}\phi^2 + \mathrm{R}[\gamma] - \frac{1}{2}Y(\phi)\sum_{i=1,2}(\psi_i - kx_i)^2\Big)$$

Using the EOM, we can fix the boundary expansions of the fields and finally write the renormalized Euclidean on-shell action:

$$I_{ren} = \beta V_{(2)}\left(\frac{3d_3}{2} + k^2 I_Y(0)\right)$$

Where $I_Y(0)$ is a surviving bulk term:

$$I_Y(r) = \int_{r_h}^{r} dr'\sqrt{B(r')D(r')}\,Y(\phi)$$

Thermodynamics

Assuming the existence of a regular horizon, we get:

$$s = 4\pi C(r_h), \qquad T = \frac{1}{4\pi}\sqrt{-\frac{B'(r)D'(r)}{B^2(r)}}'\Bigg|_{r=r_h}$$

while the pressure is given by:

$$p = -\frac{I_{ren}}{\beta V_{(2)}} = -\frac{3d_3}{2} - k^2 I_Y(0)$$

In order to evaluate the one-point-functions we expand the fields at linear order in fluctuations:

$$g_{\mu\nu} = g^b_{\mu\nu}(r) + h_{\mu\nu}(x_M), \qquad A_\mu = A^b_\mu(x_M) + \delta A_\mu(x_M),$$

$$\phi = \phi^b(r) + \delta\phi(x_M), \qquad \psi_i = \psi^b_i(r) + \delta\psi_I(x_M).$$

One-point-function and Ward Identities

Then the renormalized action reads:

$$S_{ren}^{(1)} = \int d^3x \left[\frac{3}{2} d_3 h_{tt}^{(0)} + \frac{3}{4} d_3 h_{xx}^{(0)} + \frac{3}{4} d_3 h_{yy}^{(0)} - \rho \delta A_t^{(0)} - \phi_{(v)} \delta \phi_{(s)} \right]$$

From where we can extract:

$$\langle T^{tt} \rangle = \epsilon = -3d_3, \qquad \langle T^{xx} \rangle = \langle T^{yy} \rangle = -\frac{3}{2} d_3 = p + k^2 I_Y(0)$$

$$\langle J_t \rangle = \rho, \qquad \langle O_\phi \rangle = \phi_{(v)}$$

The Ward Identities are satisfied

$$\langle T^\mu_\mu \rangle = 0, \quad \partial_\mu \langle T^{\mu\nu} \rangle = 0, \quad \partial_\mu \langle J^\mu \rangle = 0$$

as it should be in case of SSB.

DC transport coefficients

Transport coefficients are defined by the relation:

$$\begin{pmatrix} \vec{J} \\ \vec{J_Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T / T \end{pmatrix}$$

For simplicity we consider in details the electric conductivity, that in case of spontaneous symmetry breaking reads:

$$\sigma(\omega) = \frac{i}{\omega} G^R_{JJ}(\omega, q = 0) \xrightarrow[\omega \to 0]{} \sigma_0 + \frac{\chi^2_{JP}}{\chi_{PP}} \frac{i}{\omega}$$

Where σ_0 is the incoherent conductivity (i.e. the momentum-independent component of σ) while the imaginary pole is due to a $\delta(\omega)$ contribution.

Incoherent conductivity

We have introduced the static susceptibilities:

$$\chi_{PP} = \frac{\delta \langle T^{tx} \rangle}{\delta v^x} = -\frac{9}{2} d_3, \qquad \chi_{JP} = \frac{\delta \langle J^x \rangle}{\delta v^x} = \rho$$

The incoherent conductivity is defined by the formula:

$$\sigma_0 = \frac{1}{\chi_{PP}^2} \lim_{\omega \to 0} G^R_{J_{inc} J_{inc}}(\omega, q = 0)$$

J_{inc} is the incoherent current orthogonal to momentum P:

$$J_{inc} = \chi_{PP} J - \chi_{JP} P, \quad \text{s.t.} \quad \langle J_{inc} P \rangle = 0$$

How to proceed

As usual in holography we will follow this recipe:

- Find some proper **conserved quantity**.
- Check that they asymptote the right physical quantities at the boundary.
- Evaluate them at the horizon imposing **regularity** of the fields.

The last point requires that in the Eddington-Finklestein coordinates:

$$v = t + \frac{1}{4\pi T} \log(r_h - r) + O(r_h - r)$$

the fields must be regular at the horizon, since $r = r_h$ is just an apparent singularity, and not a true space-time singularity.

Boundary conditions and conserved quantities

Since we are interested in DC coefficients, we turn on the following linear in time perturbations:

$$\delta a_x(r,t) = a_x(r) - p_1(r)t, \quad \delta h_{tx}(r,t) = h_{tx}(r) - p_2(r)t,$$

$$\delta h_{rx}(r,t) = h_{rx}(r), \quad \delta\psi_i(r) = \chi_i(r)$$

The conserved quantities can be found by considering

$$\sqrt{-g}Z(\phi)F^{rx}, \qquad G^{\mu\nu}(k^\mu)$$

which lead to the electric and heat current at the boundary respectively.

Result

Evaluating these quantities at the horizon, and dividing by the source:

$$\sigma_0 = \frac{Z_h(k^2 I_Y + sT)^2}{(k^2 I_Y + \mu\rho + sT)^2} + \frac{4\pi\rho^2 k^2 I_Y^2}{sY_h(k^2 I_Y + \mu\rho + sT)^2}$$

A few comments:

- Free energy is minimized for $k = 0$ (no symmetry breaking at all)
- Recover known solution in $k \to 0$ limit
- Stable solution with $k \neq 0$ is given by high-derivatives models

Scaling behavior

In order to study the temperature dependence of the conductivity, we assume the following behavior for the scalar couplings:

$$V_{IR} = V_0 e^{-\delta\phi}, \quad Y_{IR} = Y_0 e^{\nu\phi}, \quad Z_{IR} = Z_0 e^{\gamma\phi}$$

We are also assuming **hyperscaling** violation, then the metric reads:

$$ds^2 = \xi^\theta \Big[-f(\xi)\frac{dt^2}{\xi^{2z}} + \frac{L^2 d\xi^2}{\xi^2 f(\xi)} + \frac{d\vec{x}^2}{\xi^2} \Big], \quad f(\xi) = 1 - \left(\frac{\xi}{\xi_h}\right)^{2+z-\theta}$$

Setting $\phi = \kappa \log(\xi)$ and $A = A_0 \xi^{\zeta - z} dt$, the thermodynamics quantities scale as:

$$T \sim \xi_h^{-z}, \qquad S \sim T^{\frac{2-\theta}{z}}$$

Consistency relations

In order for the theory to be consistent, we should require some constraints:

- Null Energy Condition:

$$(2-\theta)(2z-2-\theta) \geqslant 0, \quad (z-1)(2+z-\theta) \geqslant 0$$

- Positivity of specifc heat:

$$\frac{2-\theta}{z} \geqslant 0$$

- Marginal or irrelevant deformation sourced by ψ_i:

$$-\frac{2+\kappa\nu}{z} \geqslant 0$$

- Marginal or irrelevant deformation sourced by A^t:

$$-\frac{\zeta+2-\theta}{z} \geqslant 0$$

Temperature dependence

We can now look at the incoherent conductivity, which factorizing the bulk integral becomes:

$$\sigma_0 = \frac{k^4 I_Y^2}{(k^2 I_Y + \mu\rho)^2}\left(Z_h + \frac{4\pi\rho^2}{sY_h k^2}\right)$$

The prefactor is dominated by the *UV* behavior and it approaches a constant, so all the temperature dependence is encoded in the *IR* expansion of the fields, leading to:

$$\sigma_0 \sim \frac{k^4 I_Y^2}{(k^2 I_Y + \mu\rho)^2} T^{\frac{2z-\theta-2\tilde{\Delta}}{z}}$$

Working with the parameters θ and z, we see that within the consistency relations, there is room enough to get **conducting behavior** $d\sigma_0/dT < 0$

Adding a Magnetic Field

Almost all experiments with bad (or strange) metals are performed with an external **magnetic field**.
It is important to study the effects of this new field on the transport coefficients.

In order to add an external magnetic field, we slightly modify the ansatz for the gauge field:

$$A = A(r)dt - Bydx$$

Generating a constant magnetic field orthogonal to the two spatial dimensions.

Magnetization currents

The main effect of adding an external magnetic field is to generate an extra-term to the action

$$S = S_{B=0} + \int_{r=\epsilon} d^3x BM(0), \text{ with } M(r) = \int_{r_h}^{r} dr' \frac{B\sqrt{B(r')D(r')}Z(\phi)}{C(r')}$$

proportional to the magnetization density.

Also the currents are modified, e.g.:

$$\mathcal{J}^x(r) = J^x(r), \quad \mathcal{J}^y(r) = J^y(r) - \xi M(r)$$

such that the new currents are radially conserved

$$\partial_r \mathcal{J}^x(r) = 0, \quad \partial_r \mathcal{J}^y(r) = 0$$

Transport coefficients

With the introduction of the magnetic field, transport coefficients become matrices, e.g.:

$$\sigma \to \hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

where in case of isotropic systems:

$$\sigma_{xx} = \sigma_{yy}, \qquad \sigma_{yx} = -\sigma_{xy}$$

The same holds for the others coefficients, so we will have to evaluate 6 coefficients (instead of 3).

Results

Incoherent conductivities:
The complete expressions of the conductivities are:

$$\sigma_{0,xx} = \frac{\rho^2}{\rho^2 + B^2 Z_h^2} \frac{(I_Y k^2 + sT)^2 Z_h}{\chi_{PP}^2} + \frac{4\pi I_Y^2 k^2}{s Y_h \chi_{PP}^2}$$

$$\sigma_{0,xy} = \frac{B\rho}{(\rho^2 + B^2 Z_h^2)\chi_{PP}^2}\Big(M^2\rho^2 - Z_h^2\big[(I_Y k^2 + sT)^2 - B^2 M^2\big]\Big)$$

It is interesting noting that in the $B \to 0$ limit $\sigma_{0,xx}$ recovers the previous result, while $\sigma_{0,xy}$ vanishes

$$\sigma_{0,xx}\big|_{B=0} = \sigma_0, \qquad \sigma_{0,xy}\big|_{B=0} = 0$$

as it should be.

Conclusion

To resume:

- There exists a class of holographic models which can be used as Effective Field Theories to describe conducting CDW
- We showed how to implement SSB and evaluated some transport coefficients in this scenario
- Conducting behavior is allowed
- We added an external magnetic field and discussed about some consequences

Outlook:

- Evaluate other coefficients with $B \neq 0$
- Study the scaling behavior in a magnetic field
- Check the DC limit of transport coefficients with the numerics

Tanks for the attention!