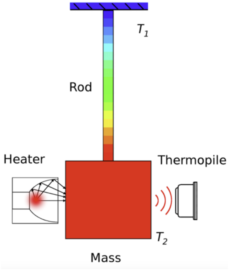
Thermoelastic fluctuations around nonequilibrium states

Gianmaria Falasco, University of Luxembourg

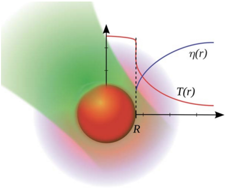
Giorgio Mentasti and Livia Conti, UNIPD/INFN

GRASS 2019, Padova

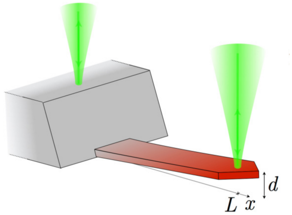
Temperature gradients in applications



macroresonators (L. Conti's lab, JSP 13)



nanoparticle tracking/trapping
(F. Chicos's lab, PRL 09)



AFM cantilivers (L. Bellon's lab, PRE 17)

Out of equilibrium: 1 d.o.f

Currents due to inhomogeneous energy input

Out of equilibrium: 1 d.o.f

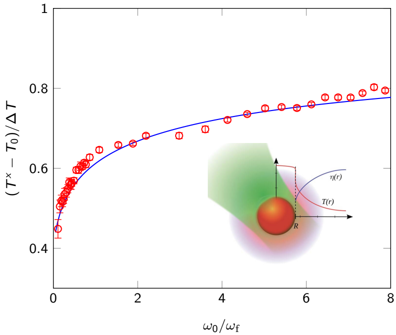
Currents due to inhomogeneous energy input

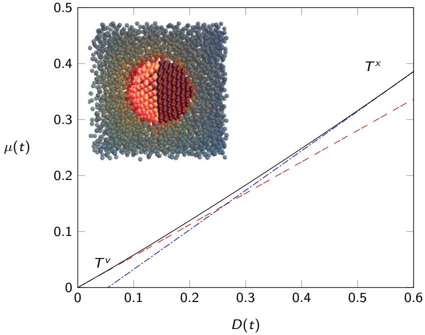
Insights from paradigm: heated Brownian nanoparticle
(Falasco et al. PRE '14)

Out of equilibrium: 1 d.o.f

Currents due to inhomogeneous energy input

Insights from paradigm: heated Brownian nanoparticle
(Falasco et al. PRE '14)





No equipartition ✘
energy $m\omega_0^2\langle x^2\rangle/2 \neq k_B T/2$

No fluctuation-dissipation relation ✘
μ response $\neq$ D fluctuations

Out of equilibrium: many d.o.f.'s

Insights from simple model: harmonic oscillators in temperature gradient

Out of equilibrium: many d.o.f.'s

Insights from simple model: harmonic oscillators in temperature gradient

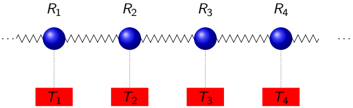
$$\ddot{R}_i = -\sum_j A_{ij} R_j - \gamma \dot{R}_i + f_i \qquad \langle f_i f_j \rangle = 2\gamma k_\mathrm{B} T_i \delta_{ij}$$

Out of equilibrium: many d.o.f.'s

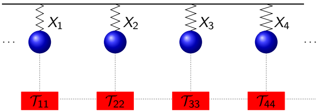
Insights from simple model: harmonic oscillators in temperature gradient

$$\ddot{R}_i = -\sum_j A_{ij} R_j - \gamma \dot{R}_i + f_i \qquad \langle f_i f_j \rangle = 2\gamma k_B T_i \delta_{ij}$$

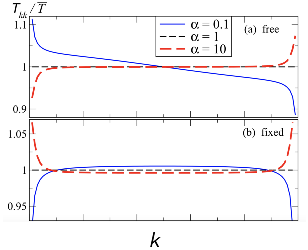
Oscillators



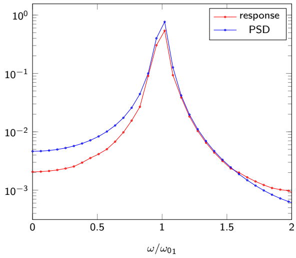
Correlated modes



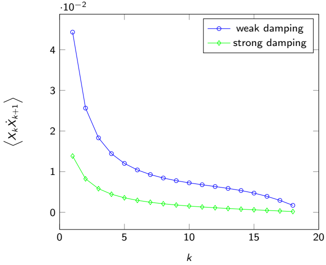
Out of equilibrium: many d.o.f.'s (cont.)



no equipartition among modes ✘



no fluctuation-dissipation relation ✘



non-zero mode correlations $\sim$ heat fluxes✘

Catch

Kinetics aspects become relevant:
boundary conditions, transport coefficients...

Dynamics cannot be leapfrogged!

Previous approaches

Past approaches rely on global equilibrium

⋆ Mode decomposition (A. Gillespie et al.. PRD '94... K. Komori et al. PRD '18) ✘
modes are correlated!

⋆ Fluctuation-dissipation relation (Y. Levin PRD '97...) ✘
response $\neq$ correlations

Set up fluctuating dynamics using only local equilibrium

Fluctuating hydrodynamics

displacement $d(r,t)$

$$\rho \partial_t d = \nabla \cdot \sigma$$
$$\sigma \sim \lambda \nabla^2 d + \alpha \nabla T$$

temperature $T(r,t)$

$$\partial_t T = \nabla \cdot J$$
$$J \sim \kappa \nabla T + \alpha \nabla T$$

Fluctuating hydrodynamics

displacement $d(r,t)$

$$\rho \partial_t d = \nabla \cdot \sigma$$
$$\sigma \sim \lambda \nabla^2 d + \alpha \nabla T$$

temperature $T(r,t)$

$$\partial_t T = \nabla \cdot J$$
$$J \sim \kappa \nabla T + \alpha \nabla T$$

Introduce fluctuations

$$d \rightarrow \langle d \rangle + \delta d \qquad T \rightarrow \langle T \rangle + \delta T$$

Fluctuating hydrodynamics

displacement $d(r,t)$

$$\rho\partial_t d = \nabla\cdot\sigma$$
$$\sigma \sim \lambda\nabla^2 d + \alpha\nabla T$$

temperature $T(r,t)$

$$\partial_t T = \nabla\cdot J$$
$$J \sim \kappa\nabla T + \alpha\nabla T$$

Introduce fluctuations

$$d \to \langle d\rangle + \delta d \qquad T \to \langle T\rangle + \delta T$$

due to stochastic forces accounting for microscopic d.o.f.'s

$$\sigma \to \langle\sigma\rangle + \tau \qquad J \to \langle J\rangle + Q$$

Fluctuating hydrodynamics

displacement $d(r,t)$

$$\rho\partial_t d = \nabla\cdot\sigma$$
$$\sigma \sim \lambda\nabla^2 d + \alpha\nabla T$$

temperature $T(r,t)$

$$\partial_t T = \nabla\cdot J$$
$$J \sim \kappa\nabla T + \alpha\nabla T$$

Introduce fluctuations

$$d \to \langle d\rangle + \delta d \qquad T \to \langle T\rangle + \delta T$$

due to stochastic forces accounting for microscopic d.o.f.'s

$$\sigma \to \langle\sigma\rangle + \tau \qquad J \to \langle J\rangle + Q$$

assumed in local equilibrium

$$\big\langle \tau(r,t)\tau(r',t')\big\rangle \sim k_\mathrm{B}\langle T(r)\rangle\delta(t-t')\delta(r-r') \times \mathrm{Im}\lambda(t)$$
$$\big\langle Q(r,t)Q(r',t')\big\rangle \sim k_\mathrm{B}\langle T(r)\rangle^2\delta(t-t')\delta(r-r') \times \kappa$$

Easy: const. coefficients, no geometry

Explicit solution: $(\delta d \; \delta T)(k,\omega) = R(k,\omega) \cdot (\tau \; Q)(k,\omega)$

- $\star$ Equilibrium: $\langle \delta d^2 \rangle \sim k_{\mathrm{B}} T / E \, V$ ✓
- $\star$ Nonequilibrium: fluctuation-dissipation only neglecting correlations in τ and Q

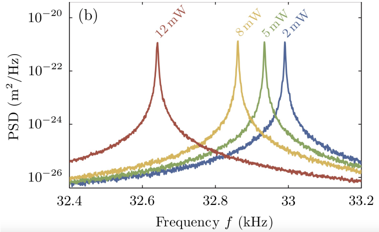
$$\mathrm{PSD}(\omega) \sim k_{\mathrm{B}} \int drT(r) \times R''(r,\omega)$$

- $\star$ $\Delta T / T \lesssim 10\%$: negligible NE correction

T-induced inhomogeneities

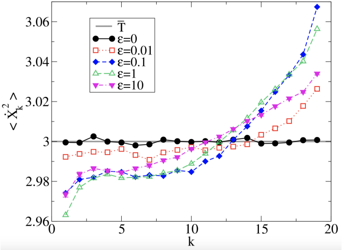
Nonlinearities are expected to matter

Cantilever



softening with heating
(Geitner et al. PRE '17)

Nonharmonic oscillators



energy redistribution
(Falasco et al. NJP '16)

Wrap-up

- ⋆ Use what is know about nonequilibrium Stat Mech
- ⋆ Material properties & B.C.'s influence energy partition and PSD
- ⋆ Fluctuating dynamics with local equilibrium
- ⋆ Identify conditions for phenomenological approaches to work
- ⋆ Work in progress: readout PSD under realistic (nonlinear) conditions