DYNamical systems and non equilibrium states of complex SYStems : MATHematical methods and physical concepts

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### Abstract

Our project has a twofold purpose: firstly it aims at a better understanding of fundamental properties of classical and quantum dynamical systems, with a particular prominence on unconventional transport and dynamical features (fractal spectra, weak chaos, infinite ergodic theory, almost resonant quantum systems). Secondly it addresses features that characterize the behavior of many particle systems, with a methodology that takes into account the former dynamical perspective: examples include normal and anomalous heat conduction in long chains, equilibrium and non equilibrium aspects of systems with long range interactions - with an emphasis on self-gravitating systems-, coherent transport in light harvesting systems, and statistical analysis of social and economic systems.

#### NETWORK



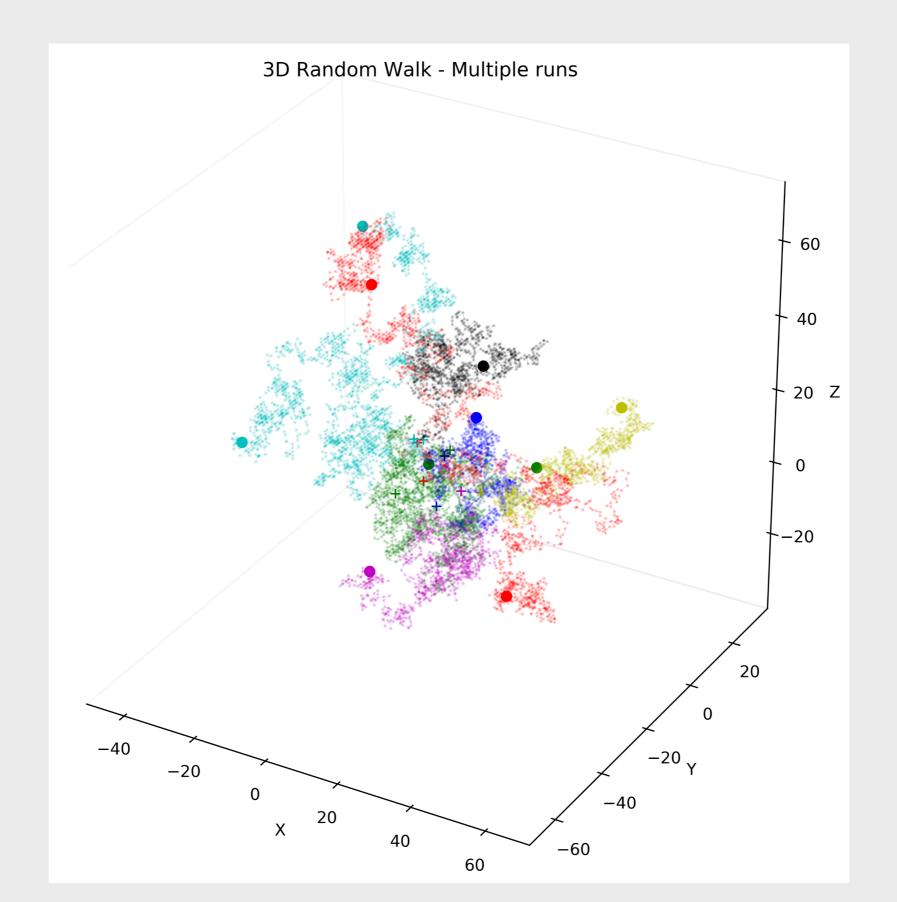


# DYNSYSMATH

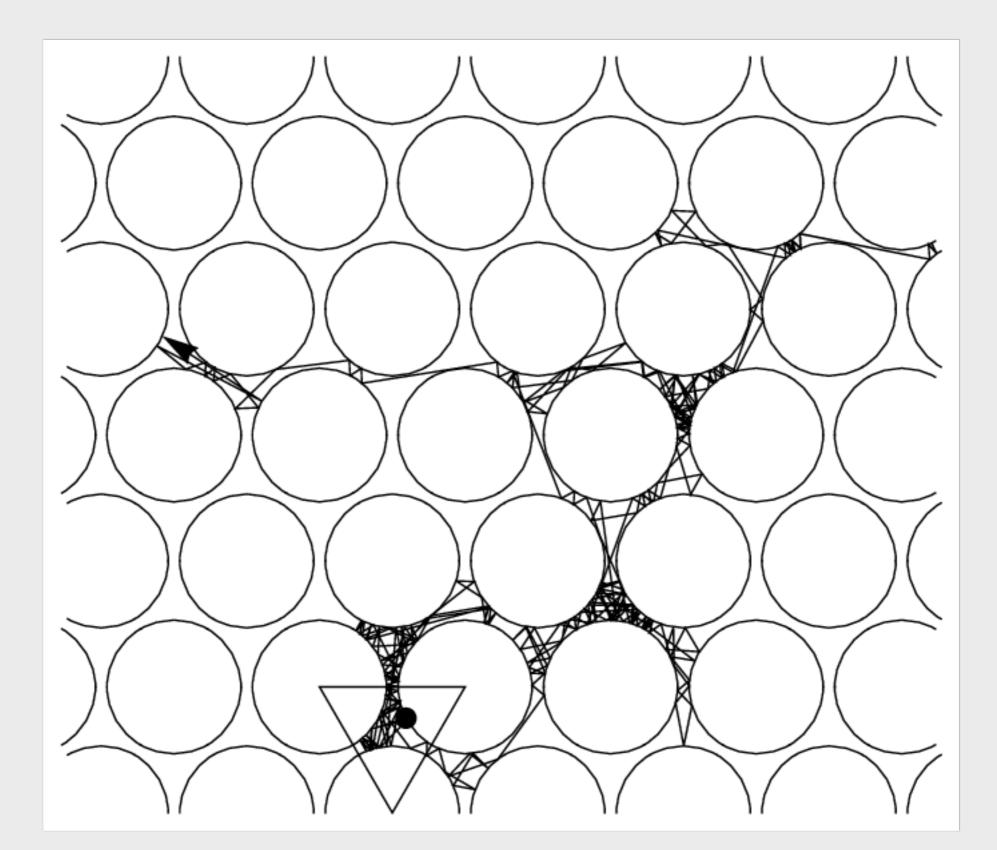
## Temi generali dell'iniziativa specifica

- Metodi statistici in caos classico
- Caos quantistico e applicazioni
- Trasporto anomalo deterministico e stocastico
- Sistemi con interazioni a lungo raggio
- Applicazioni a fenomeni economici e sociali

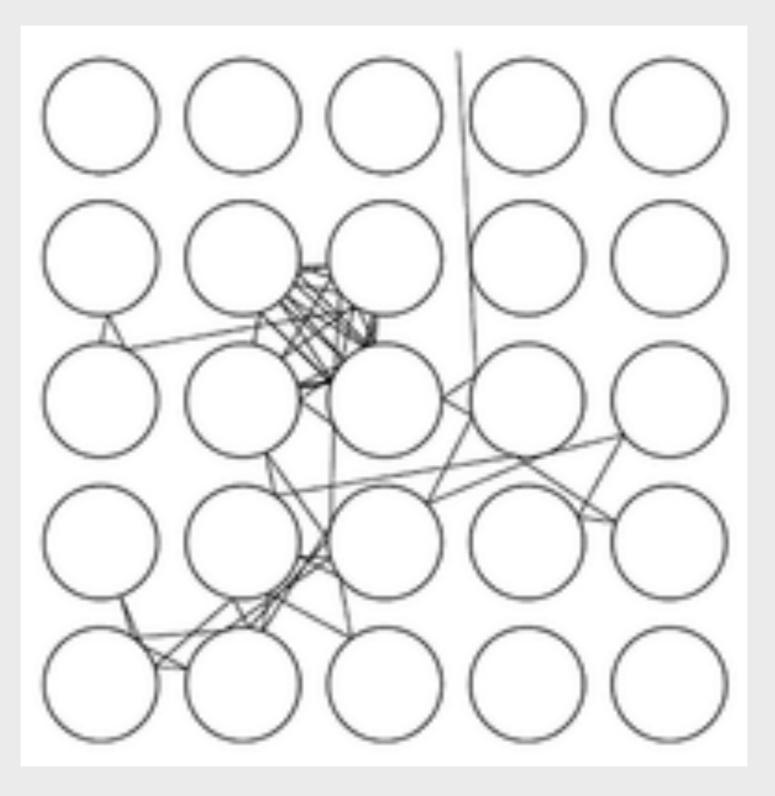
## Trasporto stocastico normale: random walk



## Trasporto deterministico normale: gas di Lorentz a orizzonte infinito



Trasporto deterministico anomalo: gas di Lorentz a orizzonte infinito

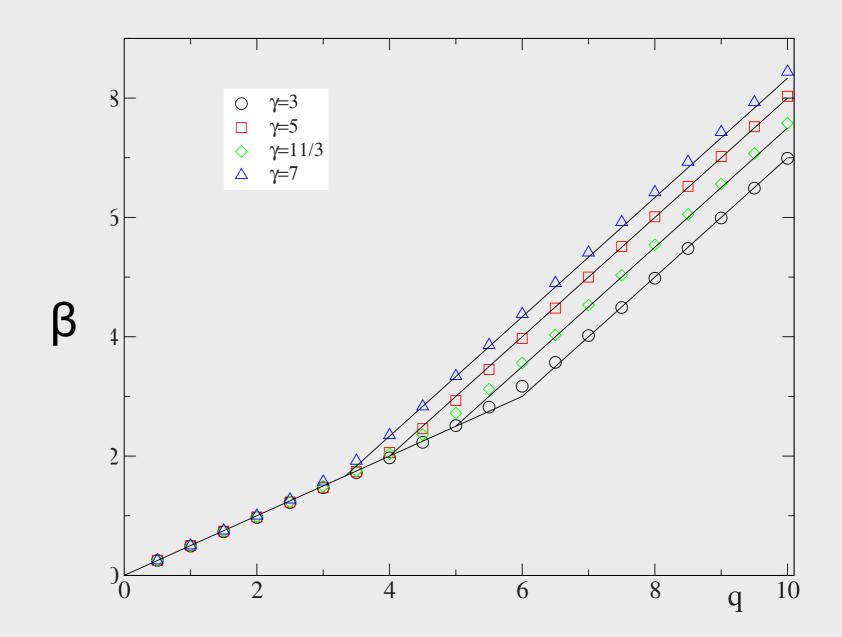


## Trasporto stocastico anomalo: gas di Lorentz a orizzonte infinito



## Normale contro Anomalo

$$\langle |x_t - x_0|^q \rangle \sim t^{q/2} \qquad \quad \langle |x_t - x_0|^q \rangle \sim t^{\beta(q)}$$



.. oltre il trasporto (parole chiave) ..

- Universalità di Sparre Andersen
- Problemi di primo passaggio
- Statistiche dei tempi di residenza (Darling-Kac, Lamperti)
- Statistiche dei records

## Una complicazione supplementare..

Le tecniche standard (Montroll-Weiss, renewal, espansioni in orbite periodiche ...) **non** funzionano per mezzi eterogenei, o sistemi con persistenza

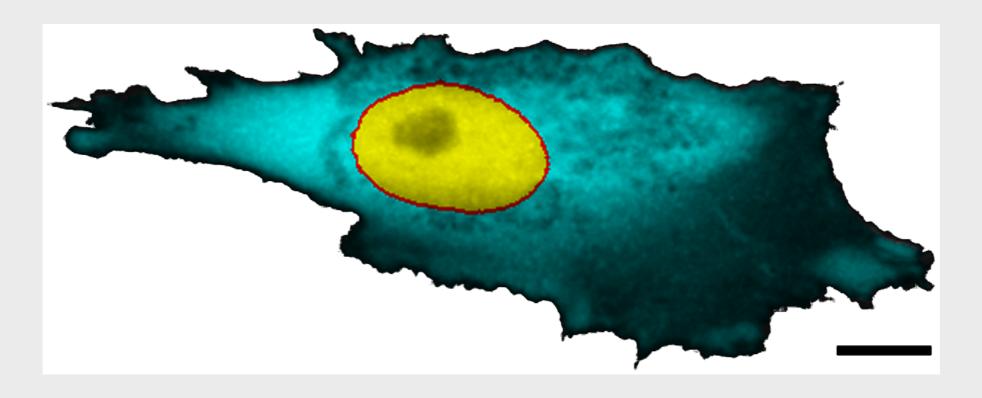
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PLos one

## Protein Diffusion in Mammalian Cell Cytoplasm

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## Un esempio (ingannevolmente) semplice

### CENTRALLY BIASED DISCRETE RANDOM WALK

By J. GILLIS (Rehovoth, Israel)

[Received 28 February 1956]

#### 1. Introduction

WE denote by **m** the general lattice point  $(m_1, m_1, ..., m_d)$  in a *d*-dimensional lattice and by  $\mathbf{E}_i$  the unit vector parallel to the positive direction of the *i*th axis. We now consider a random walk starting at the origin and such that the only steps permitted are of the type  $\mathbf{m} \to \mathbf{m} \pm \mathbf{E}_i$ , with respective probabilities  $P_i(\mathbf{m})$ ,  $Q_i(\mathbf{m})$  (i = 1, 2, ..., d). A recurrent point is defined as one through which the walk will, with probability 1, pass an infinite number of times. The main purpose of this paper is to prove Theorem 3 below. However, the proof will require two preliminary results which it will be convenient to state separately as Theorems 1 and 2.

THEOREM 1. In a discrete random walk on a one-dimensional lattice let  $p_{i,j}$  denote the probability of a step from lattice point i to j. Suppose further that  $p_{i,j} = p_{i,j} = \frac{1}{2}$  (1.1)

$$p_{0,1} = p_{0,-1} = \frac{1}{2},$$
 (1.1)

$$p_{i,i+1} = \frac{1}{2}(1-\epsilon/i)$$
 (i = +1, +2,...), (1.2)

$$p_{i,i-1} = \frac{1}{2}(1+\epsilon/i)$$
 (1.3)

$$p_{i,j} = 0$$
 when  $|i-j| \neq 1$ , (1.4)

where  $|\epsilon| < 1$ .

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PAPER: Classical statistical mechanics, equilibrium and non-equilibrium

# Non-homogeneous persistent random walks and Lévy–Lorentz gas

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### Passaggio al continuo, equazioni di Fokker Planck con correnti o diffusione non omogenea

## Generalizzazione di argomenti alla Darling-Kac