Statistical Field Theory

Studio di sistemi con molti gradi di libertà
con metodi di teoria dei campi

Cosenza (2.5), Firenze (9), Genova (4.5), Milano (8),
Pisa (5.9), Torino (6.4), Trieste (29.3) (FTE 2019)
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Research lines

- Combinatorial optimization [Caracciolo, Di Gioacchino, Erba, Fabbricatore, Gherardi, Molinari, Pastore][1]
- Statistical mechanics of data dependence in machine-learning [Gherardi, Erba][2,3]
- Rare fluctuations in spin models [Di Gioacchino, Pastore][3]
- Bosonization via Bogoliubov transformation [Caracciolo, Pastore][4]
- Applications of CFT and holography [Caracciolo, Cardella, Pastore]
- Covariant characterisation of space-times [Molinari][5]

[1] with R. Capelli (Forschungszentrum Jülich), M. D'Achille (Collège de France), E. Malatesta (Bocconi Univ.), S. Mandrà (NASA), G. Sicuro (Sapienza Univ.), A. Sportiello (CNRS)

[2] with M. Cosentino Lagomarsino (UniMi), M. Caselle and M. Osella (UniTo), R. Zecchina and C. Baldassi (Bocconi Univ.), F. Borra (Sapienza Univ.)

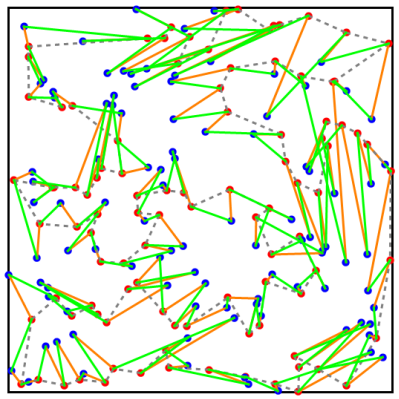
[3] with P. Rotondo (Nottingham Univ.)

[4] with F. Palumbo (INFN LNF)

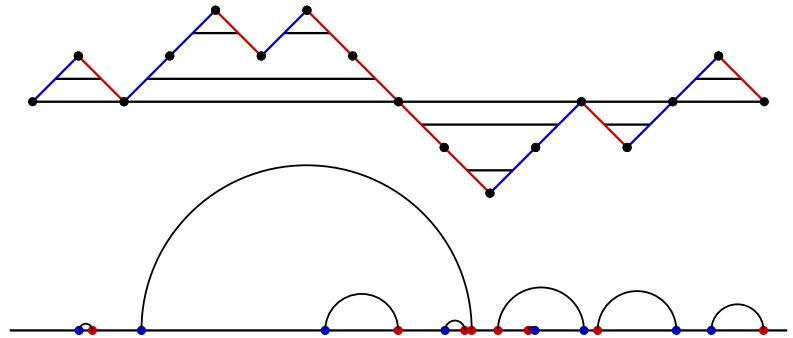
[5] with C. A. Mantica (INFN MI), S. Capozziello (INFN NA)

Combinatorial optimization

The optimal solution is obtained via a principle of minimum energy of an associated Mechanical Statistical model.

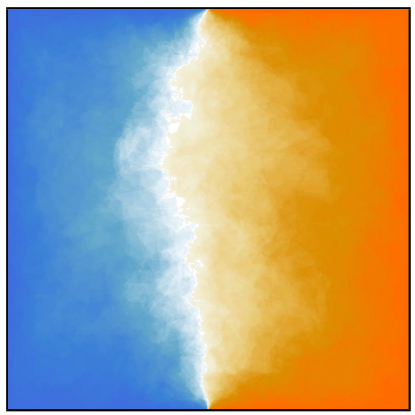


For large number N of points, in $2D$ the cost of the bipartite TSP converges to twice the cost of the assignment.

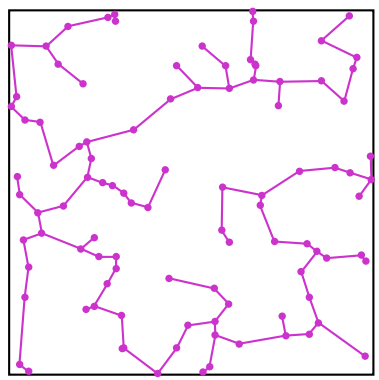


The sub-optimal, but analytical accessible, Dyck matching for a bipartite $1D$ graph with concave Euclidean weights on the links, scales with N as the optimal (intractable!) assignment.

Can the curve obtained discarding a link in a perfect $2D$ matching be described as a certain stochastic process? In figure, the left passage probability of the curve.

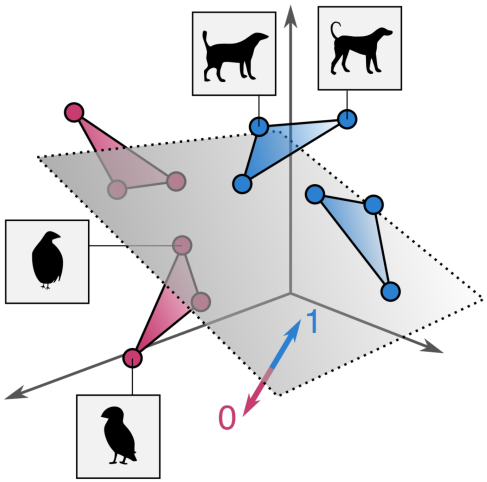


Can the length of the Euclidean Minimum Spanning Tree of a graph be obtained with Random Matrices with correlated entries?

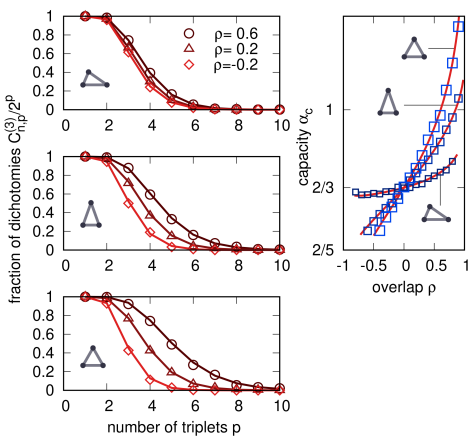


Statistical mechanics of machine-learning

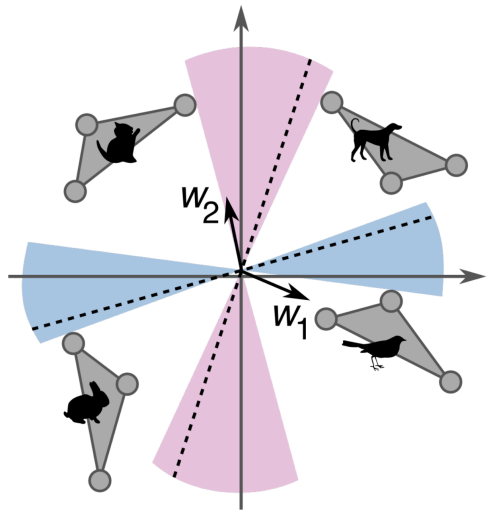
Artificial intelligence and neural networks have growing success in applications, but theoretical comprehension lacks. Important advancements were achieved by statistical physics methods for unstructured random data.



A *dichotomy* is a paradigmatic task for a learning algorithm: distinguish two different labels in a dataset (red/blue, bird/mammal). The simplest way is by linear separation (hyperplanes).



Numerical and theoretical results for triangles: admissible dichotomies and capacity.



But the data have a structure (geometrical multiplets, species)! Not all the planes are admissible.

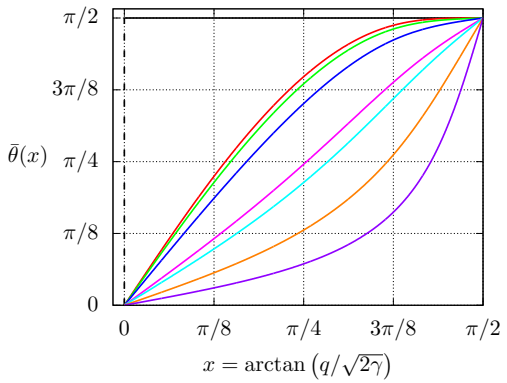
An open question

How can one quantify and predict the influence of data structure on the performances of deep neural networks?

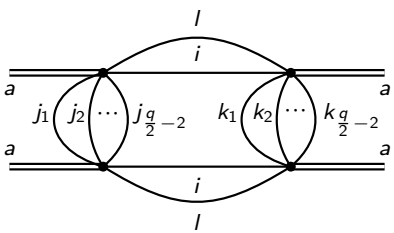
Non-perturbative Field Theory and Gravity

Bosonization via Bogoliubov transformation

Can the chiral symmetry breaking mechanism be described in a way similar to BCS superconductivity phase? We use a quantum many-body, Bogoliubov approach to study relativistic fermion models.
In figure, the chiral angle in the 't Hooft model, for different value of quark mass and gauge coupling.



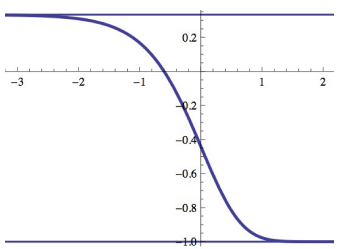
Sachdev-Ye-Kitaev model and holography



We continue the study of the SYK model by techniques of spin-glass systems, and its possible relation via holographic duality to black hole physics.

Covariant characterisation of space-times

In the context of $f(R)$ gravity, we investigate the behaviour of perfect fluids in Generalized Robertson-Walker space-times, with some amusing implications for a toy model of cosmology.
In figure, the ratio p/ρ as a function of time.



Rare fluctuations in spin models

Large deviations

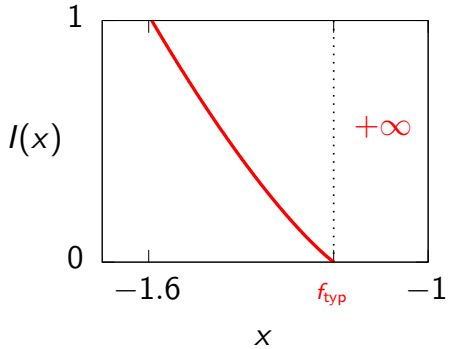
The probability to find a random variable f_N in an interval $[x, x + \mathrm{d}x]$ satisfies a *large deviation principle* if

$$P(f_N \in [x, x + \mathrm{d}x]) \sim e^{-NI(x)}\,\mathrm{d}x$$

with the so-called *rate function* $I(x)$.

Disordered spin systems

In a disordered system the free energy is a random variable. We evaluated the corresponding rate function for a model of spherical spins.



The open problem: very large deviations

An infinite rate function means a *different scaling* with N at the exponent of the large deviation principle! How to find it?

Last two years... (since 2017/07)

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