Topological properties of high-temperature QCD and axion cosmology

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OUTLINE

- θ-dependence in QCD and axions

- θ-dependence from lattice QCD: main technical issues

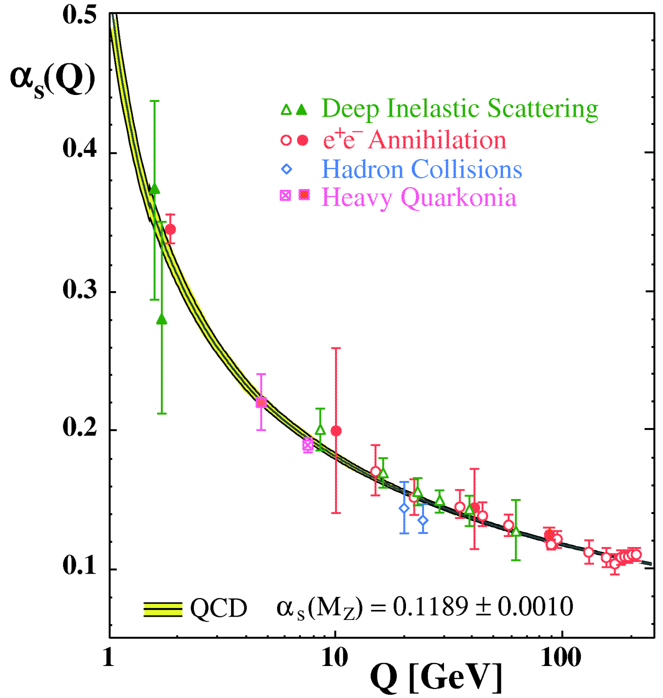
- θ-dependence in full QCD at high temperatures

Within the standard model of particle physics, strong interactions are described by Quantum Chromodynamics (QCD), the theory of quark and gluons:

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_i^f \left(D_{ij}^{\mu} \gamma_{\mu}^{E} + m_f \delta_{ij} \right) \psi_j^f + \frac{1}{4} G_{\mu\nu}^{a} G_{a}^{\mu\nu}$$

HIGH ENERGIES $\Longrightarrow$ The coupling is small, asymptotically vanishing. Perturbation theory works well.

LOW ENERGIES $\Longrightarrow$ The coupling is large, perturbation theory fails, QCD is non-perturbative.



QCD is characterized by various non-perturbative features:

confinement, chiral symmetry breaking, ...

Here we discuss those related to the classification of gauge configurations in non-trivial homotopy classes, labelled by an integer winding number $Q = \int d^4x \; q(x)$

$$q(x) = \frac{g^2}{64\pi^2} G^a_{\mu\nu}(x) \tilde{G}^a_{\mu\nu}(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu}(x) G^a_{\rho\sigma}(x)$$

$$GG \propto \vec{E}^a \cdot \vec{E}^a + \vec{B}^a \cdot \vec{B}^a \;\; ; \quad G\tilde{G} \propto \vec{E}^a \cdot \vec{B}^a$$

Homotopy group: $\pi_3(SU(3)) = \mathbb{Z}$ **(actually,** $\pi_3(SU(N_c)) = \pi_3(SU(2)) \; \forall \; N_c$**)**

$G\tilde{G}$ **is renormalizable and a possibile coupling to it is a free parameter of QCD**

$$Z(\theta) = \int [\mathcal{D}A][\mathcal{D}\bar{\psi}][\mathcal{D}\psi] \; e^{-S_{QCD}} \; e^{i\theta Q}$$

the theory at $\theta \neq 0$ **is well defined, but presents explicit breaking of** CP **symmetry**

QCD at non-zero θ

The free energy density $F(\theta) = -T \log Z/V$ is a periodic even function of θ

It is connected to the probability distribution $P(Q)$ at $\theta = 0$ via Taylor expansion:

$$F(\theta) - F(0) = \frac{1}{2} F^{(2)} \theta^2 + \frac{1}{4!} F^{(4)} \theta^4 + \ldots \quad ; \qquad F^{(2n)} = \left. \frac{d^{2n} F}{d\theta^{2n}} \right|_{\theta=0} = -(-1)^n \frac{\langle Q^{2n} \rangle_c}{V}$$

A common parametrization is the following

$$F(\theta, T) - F(0, T) = \frac{1}{2} \chi(T) \theta^2 \left[1 + b_2(T) \theta^2 + b_4(T) \theta^4 + \cdots \right]$$

$$\chi = \frac{1}{V} \langle Q^2 \rangle_0 = F^{(2)} \qquad b_2 = -\frac{\langle Q^4 \rangle_0 - 3 \langle Q^2 \rangle_0^2}{12 \langle Q^2 \rangle_0}$$

The probability distribution $P(Q)$ of the different topological sectors is a non-perturbative property of QCD

An axial $U(1)_A$ rotation of the fermion fields moves θ from the gluon to the quark sector

$$\psi_f \to e^{i\alpha\gamma_5}\psi_f \qquad \text{and} \qquad \bar{\psi}_f \to \bar{\psi}_f e^{i\alpha\gamma_5}$$
$$\Longrightarrow \quad \theta \to \theta - 2\alpha \qquad \text{and} \qquad m_f \to m_f e^{2i\alpha}$$

- **should any quark be massless (this is not the case), θ could be rotated away and θ-dependence would be trivial**

- **in the presence of light quarks (this is the case), θ-dependence can be reliably studied within the framework of chiral perturbation theory (χPT)**

Experimental bounds on the electric dipole of the moment set stringent limits to the amount of CP-violation in strong interactions.

$$|\theta| \lesssim 10^{-10}$$

So: why do we bother with θ-dependence at all?

- **θ-dependence $\longleftrightarrow P(Q)$ at $\theta = 0$ $\Longrightarrow$ it enters phenomenology anyway.**
  e.g., Witten-Veneziano mechanism: $\chi^{YM} = f_\pi^2 m_{\eta'}^2/(2N_f)$

- **Strong CP-problem: why is $\theta = 0$?** $m_f = 0$ **is ruled out.**
  A possible mechanism (Peccei-Quinn) invokes the existence of a new scalar field (axion) whose properties are largely fixed by θ-dependence

- **Axions are popular dark matter candidates, so the issue is particularly important**

The QCD axion

Main idea: add a new scalar field acquiring a VEV which breaks a $U(1)$ symmetry (Peccei-Quinn). Various high energy models exist, low energy effective lagrangian:

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a + \left(\theta + \frac{a(x)}{f_a}\right) \frac{g^2}{32\pi^2} G\tilde{G} + \ldots$$

- a is the Goldstone boson, with only derivative terms apart from a coupling to the topological charge density.
- coupling to $G\tilde{G}$ involves the decay constant f_a, supposed to be very large
- shifting $\langle a \rangle$ shifts θ by $\langle a \rangle / f_a$. However θ-dependence of QCD breaks global shift symmetry on $\theta_{eff} = \theta + \langle a \rangle / f_a$, and the system selects $\langle a \rangle$ so that $\theta_{eff} = 0$.
- Assuming f_a very large, a is quasi-static and its effective couplings (mass, interaction terms) are fixed by QCD θ-dependence. For instance

$$m_a^2(T) = \frac{\chi(T)}{f_a^2} = \frac{\langle Q^2 \rangle_{T,\theta=0}}{V f_a^2}$$

knowing $F(\theta, T)$ fixes axion parameters during the Universe evolution

Predictions about θ-dependence - I

Dilute Instanton Gas Approximation (DIGA) for high T (Gross, Pisarski, Yaffe 1981)

Classical solutions with non-trivial winding around the gauge group: instantons

characterized by various parameters: position, radiud ρ, . . .

Effective action known only perturbatively. The 1-loop one-instanton contribution is

$$\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right)$$

where $g(\rho)$ is the running coupling at the instanton scale ρ.

- **by asymptotic freedom, works well for small instantons, which are then exponentially suppressed, implying the validity of a dilute instanton gas approximation (DIGA)**
- **however, perturbation theory breaks down for large instantons ($1/\rho \lesssim \Lambda_{QCD}$), which become dominant, overlap with each other, and break DIGA**

Finite T acts as an infrared cut-off to the instanton radius making the 1-loop computation more and more reliable

- **instantons - antiinstantons treated as uncorrelated (non-interacting) objects Poisson distribution with an average probability density p per unit volume**

$$Z_\theta \simeq \sum \frac{1}{n_+!n_-!}(V_4 p)^{n_+ + n_-} e^{i\theta(n_+ - n_-)} = \exp\left[2V_4 p \cos\theta\right]$$

$$F(\theta,T) - F(0,T) \simeq \chi(T)(1-\cos\theta) \implies \quad b_2 = -1/12\,; \quad b_4 = 1/360\,; \ldots$$

- **Instantons of size $\rho \gg 1/T$ suppressed by thermal fluctuations, for high T instantons of effective perturbative action $8\pi/g^2(T)$ dominate. Including also leading order suppression due to light fermions and zero modes:**

$$\chi(T) \sim T^4 \left(\frac{m}{T}\right)^{N_f} e^{-8\pi^2/g^2(T)} \sim m^{N_f} T^{4-\frac{11}{3}N_c-\frac{1}{3}N_f} \propto T^{-7.66} \quad (\text{for } N_f = 2)$$

Notice: perturbative limit implies diluteness, hence DIGA, however DIGA might be good before reaching the asymptotic perturbative behavior

Predictions about θ-dependence - II

Chiral Perturbation Theory (χPT) for low T

At low T, perturbation theory breaks down, however, by $U(1)$ axial rotations, θ can be moved to the light quark masses. Then, χPT can be applied as usual.

Result for the ground state energy **(Di Vecchia, Veneziano 1980)**

$$E_0(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\theta}{2}}$$

$$\chi = \frac{z}{(1+z)^2} m_\pi^2 f_\pi^2, \quad b_2 = -\frac{1}{12} \frac{1+z^3}{(1+z)^3}, \quad z = \frac{m_u}{m_d}$$

Explicitly

$z = 0.48(3)$	$\chi^{1/4} = 75.5(5)\,\mathrm{MeV}$	$b_2 = -0.029(2)$
$z = 1$	$\chi^{1/4} = 77.8(4)\,\mathrm{MeV}$	$b_2 = -0.022(1)$

$$\Longrightarrow \quad m_a \sim 10^{-5} \left(\frac{10^{12}\mathrm{GeV}}{f_a} \right)$$

Predictions about θ-dependence - III

Large-N_c for low T $SU(N_c)$ gauge theories (Witten, 1980)

$$L_{QCD}(\theta) = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \theta \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$

$g^2 N_c = \lambda$ **is kept fixed as** $N_c \to \infty$ $\Longrightarrow$ **if any non-trivial dependence on** θ **exist in the large-**N_c **limit, the dependence must be on** $\bar{\theta} = \theta / N_c$**.**

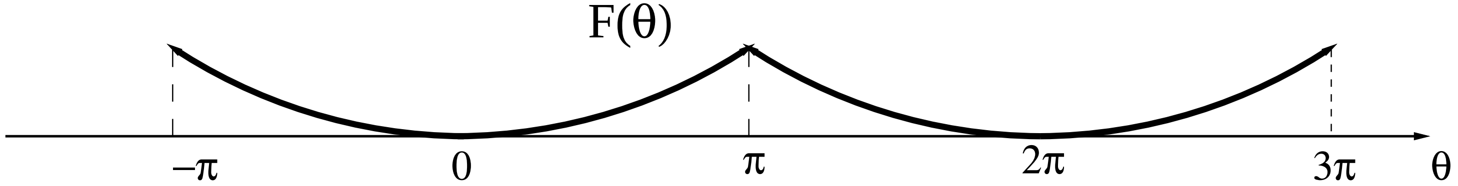
$$F(\theta, T) - F(0, T) = N_c^2 \bar{F}(\bar{\theta}, T)$$

$$\bar{F}(\bar{\theta}, T) = \frac{1}{2} \bar{\chi} \bar{\theta}^2 \left[1 + \bar{b}_2 \bar{\theta}^2 + \bar{b}_4 \bar{\theta}^4 + \cdots \right]$$

Matching powers of $\bar{\theta}$ **and** θ **we obtain**

$$\chi \sim N_c^0 \quad ; \quad b_2 \sim N_c^{-2} \quad ; \quad b_{2n} \sim N_c^{-2n}$$

$P(Q)$ **is Gaussian in the large** N_c **limit. Periodicity in** θ **enforces a multibranched structure with phase transitions at** $\theta = (2k+1)\pi$**.**



Relation between axion phenomenology and $\chi(T)$

Main source of axion relics: misalignment. Field not at the minimum after PQ symmetry breaking. Further evolution (zero mode approximation, $H =$ Hubble constant):

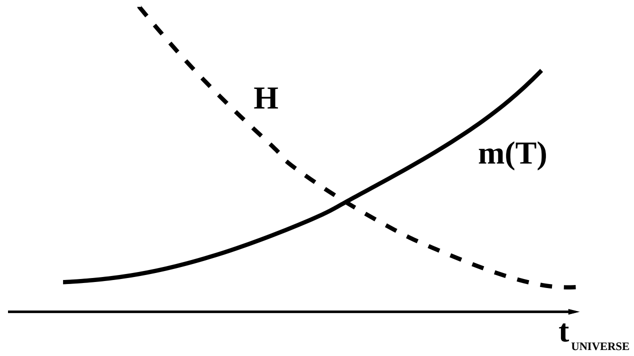
$$\ddot{a}(t) + 3H(t)\dot{a}(t) + m_a^2(T)a(t) = 0 \ ; \quad m_a^2 = \chi(T)/f_a^2$$

$T \gg \Lambda_{QCD}$ **2nd term dominates** $\Longrightarrow \quad a(t) \sim \text{const}$

$m_a \gtrsim H$ **oscillations start** $\Longrightarrow$ **adiabatic invariant**

$N_a = m_a A^2 R^3 \sim$ **number of axions (**$\sim$ **cold DM)**

$A =$ **oscill. amplitude;** $R =$ **Universe radius**



A larger $\chi(T)$ implies larger m_a and moves the oscillation time earlier (higher T, smaller Universe radius R)

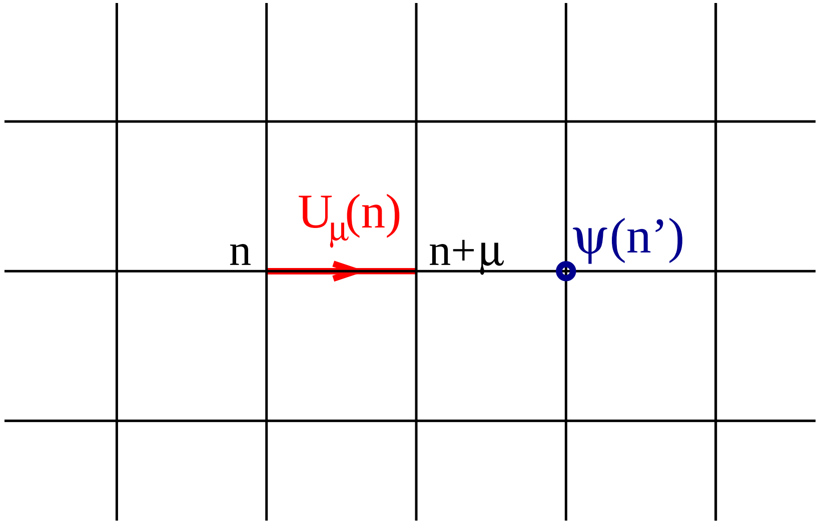
Requiring a fixed N_a ($\Omega_{axion} \sim \Omega_{DM}$)

$\chi(T)$ **grows** $\Longrightarrow$ **oscill. time anticipated** $\Longrightarrow$ **less axions** $\Longrightarrow$ **require larger f_a to maintain N_a**

On the other hand, larger f_a means smaller m_a today

θ-dependence from Lattice QCD simulations

The temperature at which DIGA sets in is not known a priori. Lattice QCD computations are the best way to obtain predictions reliable in the non-perturbative regime



Gauge fields are 3×3 unitary complex matrixes living on lattice links (link variables)

$$U_\mu(n) \simeq \mathcal{P} \exp\left(ig \int_n^{n+\mu} A_\mu dx_\mu \right)$$

Fermion fields live on lattice sites

Pure Gauge Term:

$\int d^4x G^a_{\mu\nu} G_a^{\mu\nu} \Rightarrow S_G =$ **sum of traces of path-order loop products of link variables**

Fermion Action Discretization (in brief):

$\int d^4x \bar{\psi}^f_i \left(D^\mu_{ij} \gamma^E_\mu + m_f \delta_{ij} \right) \psi^f_j \Rightarrow S_F = \bar{\psi}_n M[U]_{n,m} \psi_m$ **(M $\equiv$ fermion matrix)**

The thermal QCD partition function is rewritten in terms of an Euclidean path integral

$$Z(V,T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}}}{T}}\right) \Rightarrow \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-(S_G[U]+\bar{\psi}M[U]\psi)} = \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

$$T = \frac{1}{\tau} = \frac{1}{N_t a(\beta, m)}$$

τ **is the extension of the compactified time**

$g_0 \rightarrow 0$ **(**$\beta \rightarrow \infty$**)** $\Longrightarrow$ $a \rightarrow 0$

Dynamical fermion contributions are encoded in the fermion determinant $\det M[U]$

As long as $\mathcal{D}U e^{-S_G} \det M[U]$ is positive, it can be interpreted as a probability distribution $\mathcal{D}U\mathcal{P}[U]$ over gauge link configurations.

The path integral is then sampled by Monte-Carlo methods

Numerical Problems in Lattice QCD simulations

main technical issues that one has to face

- **topological charge renormalizes, naive lattice discretizations are non-integer valued. Various methods devised leading to consistent results**
  - **field theoretic compute renormalization constants and subtract**
  - **fermionic definitions use the index theorem to deduce Q from fermionic zero modes**
  - **smoothing methods use various techniques to smooth gauge fields and recover integer Q**

- **Sign problem at $\theta \neq 0$ $\Longrightarrow$ Taylor expansion from cumulants at $\theta = 0$**

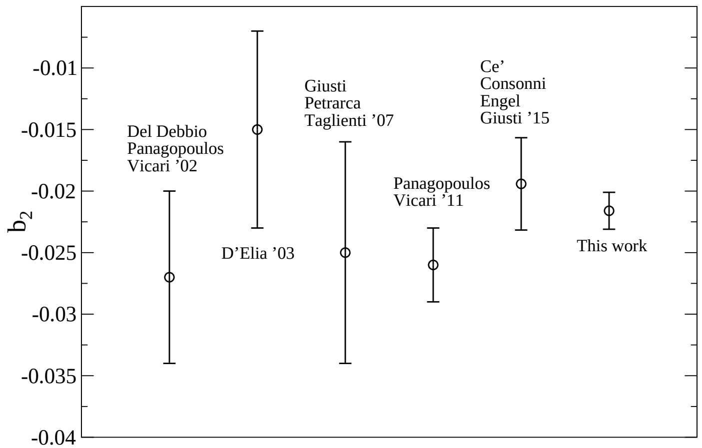
- **Freezing of topological modes in the continuum (known algorithms become non-ergodic)**

- **Approach to the continuum limit quite rough in presence of dynamical fermions**

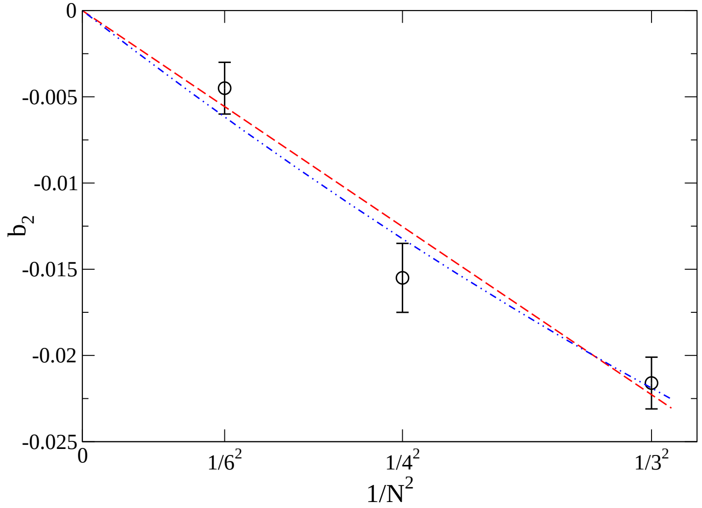
- **On a finite volume, one may have $\langle Q^2 \rangle = \chi V \ll 1$ at very high T Need to correctly sample very rare events at high T**

Pure gauge results: $T = 0$ (Yang-Mills vacuum)

Topological susceptibility well known, with increasing refinement, since many years, and compatible with the Witten-Veneziano mechanism for $m_{\eta'}$, $\chi^{1/4} \sim 180$ MeV



Determination of b_2 more difficult. Most recent determination for $SU(3)$ (Bonati, MD, Scapellato, 1512.01544) obtained by introducing an external imaginary θ source to improve signal/noise.



Clear evidence for the predicted large-N_c scaling of b_2:

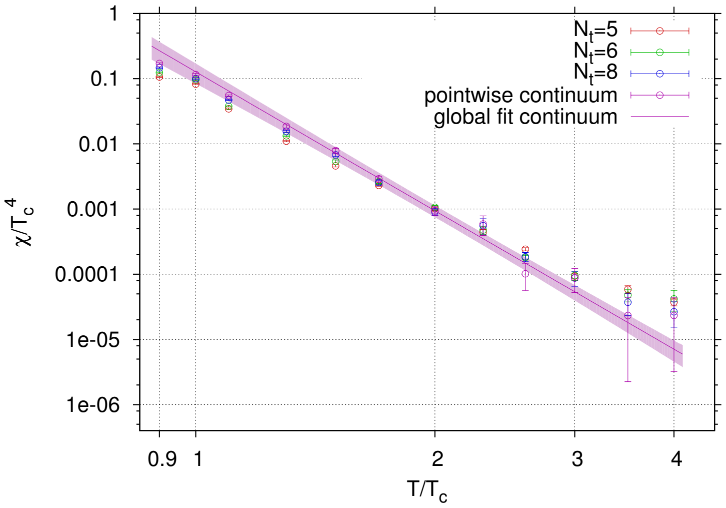
$$b_2 \simeq \frac{\bar{b}_2}{N^2}$$

with $\bar{b}_2 = -0.20(2)$

(Bonati, MD, Rossi, Vicari, 1607.06360)

Pure gauge results: Finite T, across and above T_c

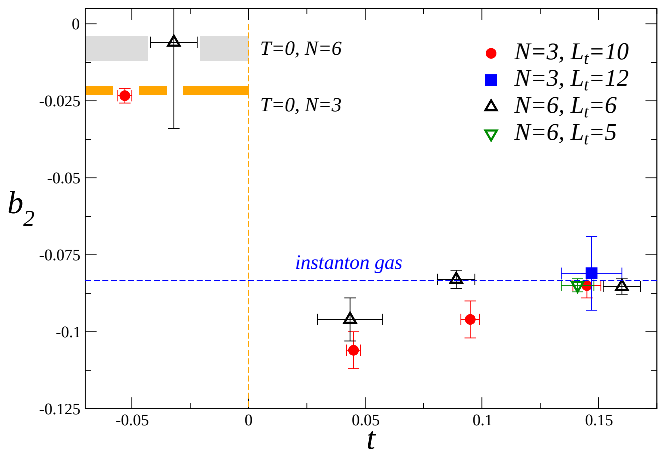
Topological activity stays almost unchanged till T_c and then χ drops suddenly. Well known since many years



from S. Borsanyi et al. 1508.06917

The perturbative power law behavior predicted for χ at high T has been verified

$\chi(T) \propto 1/T^b$, where $b = 7.1(4)(2)$ (perturbative prediction $b = 7$)



from Bonati, MD, Panagopoulos, Vicari 1301.7640

DIGA values for higher cumulants reached quite soon, already for $T \gtrsim 1.1\,T_c$.

Small deviations compatible with repulsive instanton-instanton interactions

Pure gauge topology well under control, both at zero and finite T, since a few years

WHY ARE FERMIONS MUCH MORE DIFFICULT?

$$Z(V,T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}}}{T}}\right) \Rightarrow \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

$$M \sim D_\mu \gamma_u + m_q$$

Dynamical fermions are notoriously difficult, but for topology the situation is worse.

The continuum Dirac operator $D_\mu \gamma_\mu$ has exact zero modes for $Q \neq 0$, with well defined chirality

Bccause of the vanishing fermion determinant, that suppresses non-zero topological sectors and make θ-dependence vanish as $m_q \to 0$.

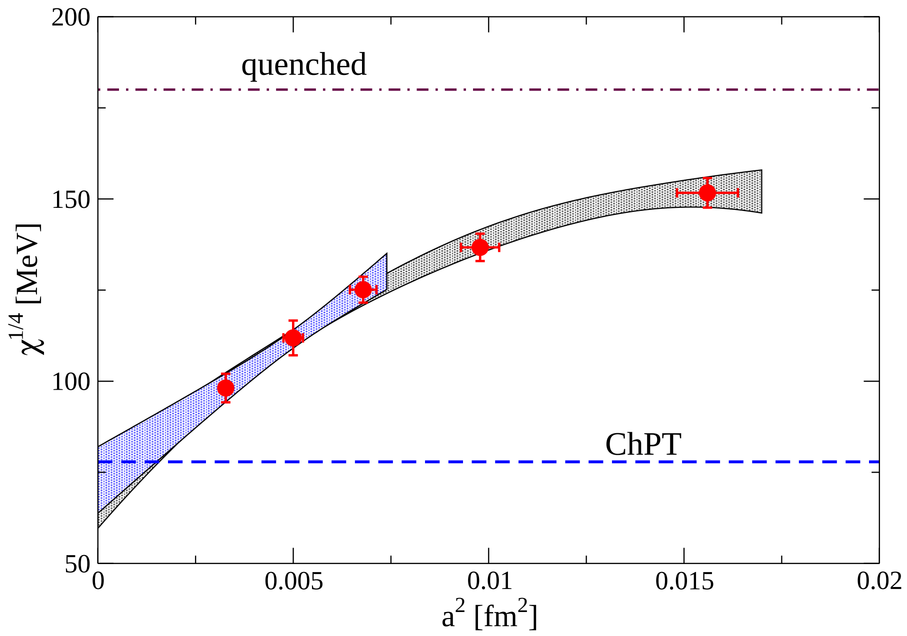
If the lattice discretization of M is far from continuum and has poor chiral properties, $\det M$ will fail its task and let many more $Q \neq 0$ configurations in than it should.

Full QCD results

I will show some results obtained for $N_f = 2 + 1$ QCD with physical quark masses

C. Bonati et al., JHEP 1603 (2016) 155 [arXiv:1512.06746]

stout improved staggered fermions, a tree-level Symanzik gauge action

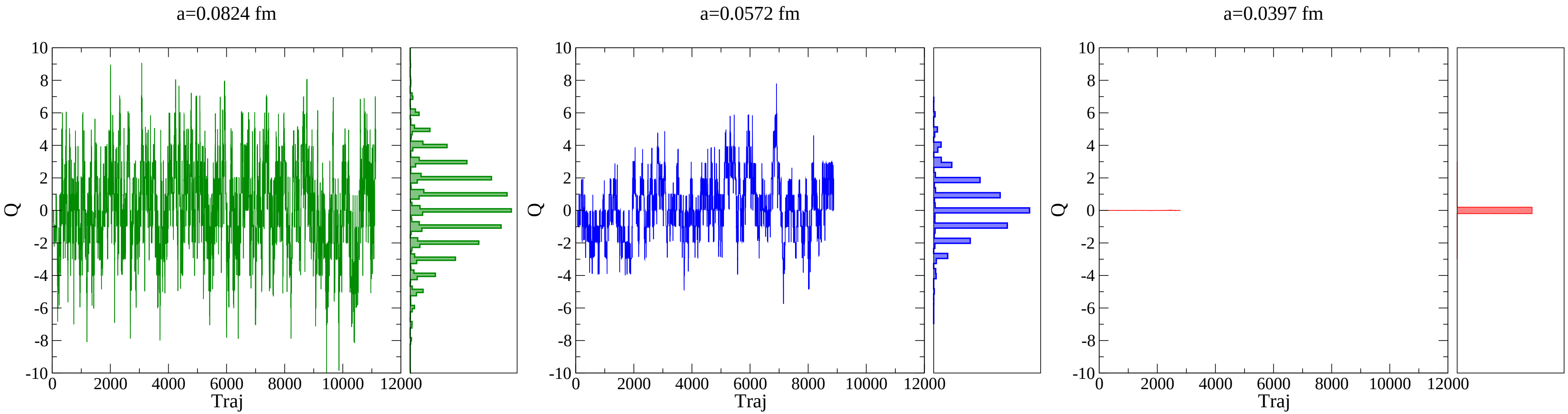


The approach to the continuum limit is quite slow and lattice spacing well below 0.1 fm are needed

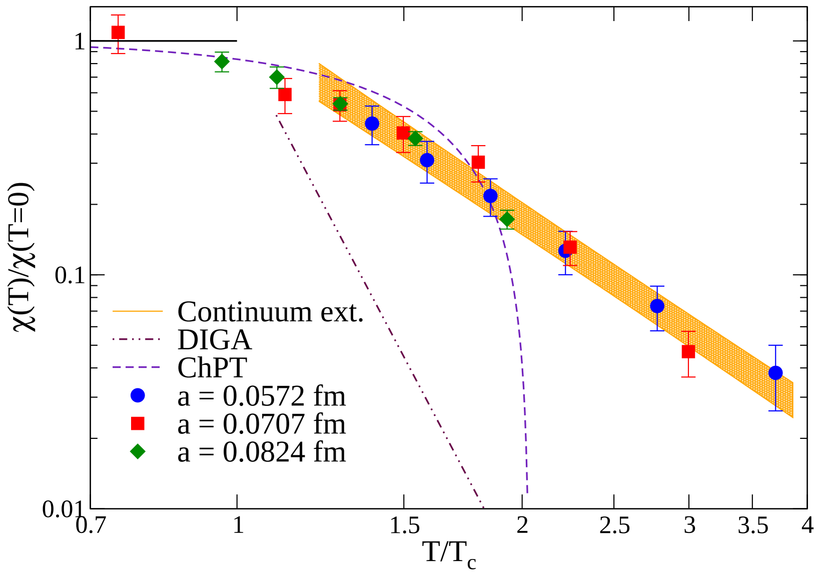
continuum limit compatible with ChPT ($73(9)\mathrm{MeV}$ against $77.8(4)\mathrm{MeV}$)

slow convergence to the continuum is strictly related to the slow approach to the correct chiral properties of fermion fields

On the other hand, the need for quite small lattice spacings, in order to correctly extrapolate to the continuum limit, brings us close to a completely frozen topology



Finite T results from the same paper



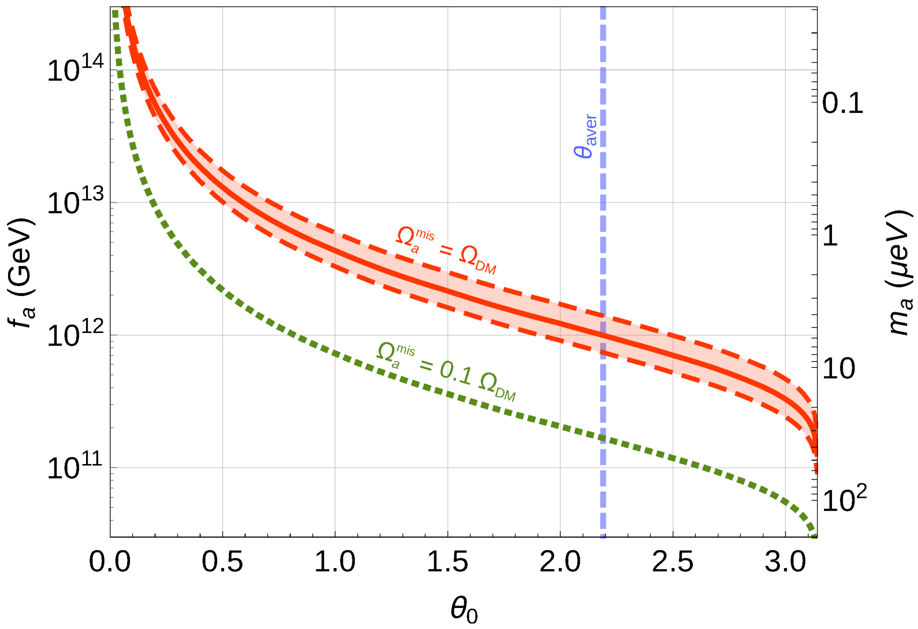
Cut-off effects seemed strongly reduced after taking the ratio $\chi(T)/\chi(T = 0)$, giving us the false illusion of a smooth continuum extrapolation

Observed drop of the chiral susceptibility much smoother than perturbative estimate:

$\chi(T) \propto 1/T^b$ **with** $b = 2.90(65)$ **(DIGA prediction:** $b = 7.66 \div 8$**)**

A slow decaying of χ means an earlier oscillation time during the Universe evolution (i.e. higher T)

$\Longrightarrow$ **larger f_a to account for dark matter** $\Longrightarrow$ **smaller value of m_a today**



An unknown variable is the initial misalignment θ_0. Moreover, if PQ symmetry breaks before inflation the initial value is constant, otherwise an average over the initial value has to be performed.

Order of magnitude prediction for present axion mass: $m_a \sim 10\ \mu\text{eV}$

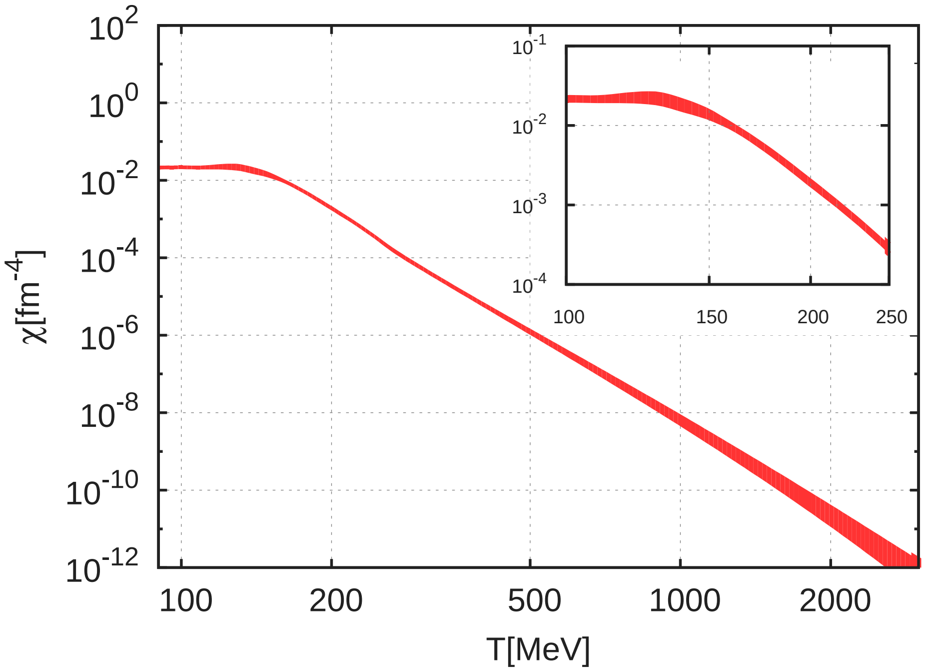
The relatively slow drop of χ with T, compare to DIGA, has not been confirmed by later lattice studies, obtaining results more in line with DIGA predictions

P. Petreczky, H. P. Schadler and S. Sharma, arXiv:1606.03145

S. Borsanyi *et al.*, **arXiv:1606.07494**

Y. Taniguchi, K. Kanaya, H. Suzuki and T. Umeda, arXiv:1611.02411

F. Burger *et al*, arXiv:1705.01847



Most precise and extended (in T) results reported in S. Borsanyi *et al.*, **arXiv:1606.07494**

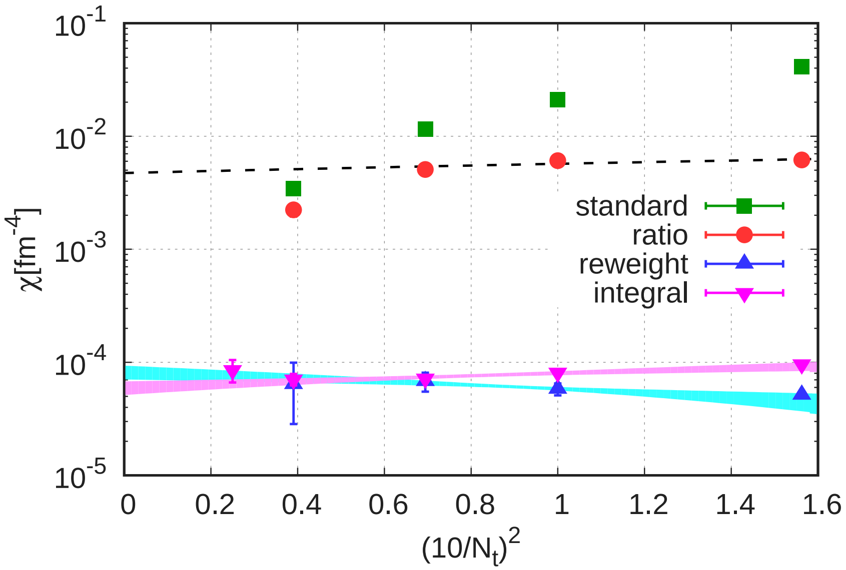
Axion mass moves to around or above $100\ \mu\mathrm{eV}$. That can be a dramatic difference for experimental detection

Discussion: **Improved results reported in S. Borsanyi *et al.*, arXiv:1606.07494 are based on two main strategies/assumptions:**

- **In order to account for discretization artifacts of the fermion determinant, gauge configurations with non-zero Q are reweighted a posteriori by a factor**

$$\frac{m_f}{m_f + i\lambda}$$

for each unit of Q, where λ is one of the first Q eigenvalues that should be zero.
that induces non-local modifications of the simulated theory which is difficult to predict



Results from S. Borsanyi *et al.*, arXiv:1606.07494

- **To avoid sampling problems (freezing and rare events), the determination of $\langle Q^2 \rangle$ is based on simulations at fixed topological sector $Q = 0$ and $Q = 1$ and on the determination of the temperature dependence of Z_1/Z_0, then assuming that**

$$\langle Q^2 \rangle \simeq \frac{Z_1}{Z_0}$$

the assumption is certainly true when $\langle Q^2 \rangle \ll 1$, however on large enough volumes $\langle Q^2 \rangle = \chi V$ will not be small. Assuming one can infer the large volume limit from simulations where $\chi V \ll 1$ requires to assume DIGA a priori, i.e. a gas of non-interacting instantons

The only way to directly check DIGA is to compare Z_Q/Z_0 for larger values of Q.

Independent checks to assess the impact of such assumptions are surely due

Defeating the rare event problem by a multicanonical approach

C. Bonati, MD, G. Martinelli, F. Negro, F. Sanfilippo and A. Todaro, arXiv:1807.07954

The idea is to modify the probability distribution, by adding a Q dependent potential to the action and then reweight

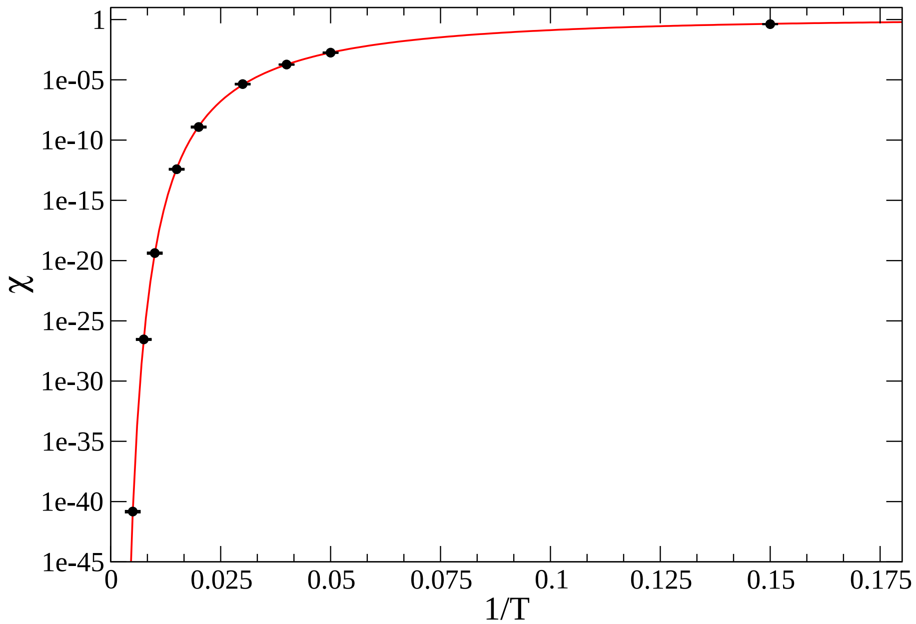
$$\langle Q^2 \rangle = \frac{\int \mathcal{D}U e^{-S_{QCD}} \, Q^2}{\int \mathcal{D}U e^{-S_{QCD}}} = \frac{\int \mathcal{D}U e^{-S_{QCD}-V(Q)} \, Q^2 e^{V(Q)}}{\int \mathcal{D}U e^{-S_{QCD}}} = \frac{\langle Q^2 e^{V(Q)} \rangle_V}{\langle e^{V(Q)} \rangle_V}$$

If $V(Q)$ is chosen so as to enhance high Q configurations, the rare events will be sampled more frequently and then correctly reweighted. The improvement in the statistical error can be impressive.

A similar strategy is adopted in metadynamics, where $V(Q)$ is made dynamical

(Laio, Martinelli, Sanfilippo, arXiv:1508.07270)

This strategy, applied to the case of the path integral on a circle, works impressively: we have been able to determine $\langle Q^2 \rangle$ over 40 order of magnitudes



From C. Bonati, MD, "Topological critical slowing down: seven variations on a toy model", arXiv:1709.10034

Similar strategies have been proposed in the pure gauge case

(P. T. Jahn, G. D. Moore and D. Robaina, arXiv:1806.01162)

We have applied this strategy with the same discretization as before

$N_f = 2+1$ QCD, stout improved staggered fermions, a tree-level Symanzik gauge action, physical quark masses

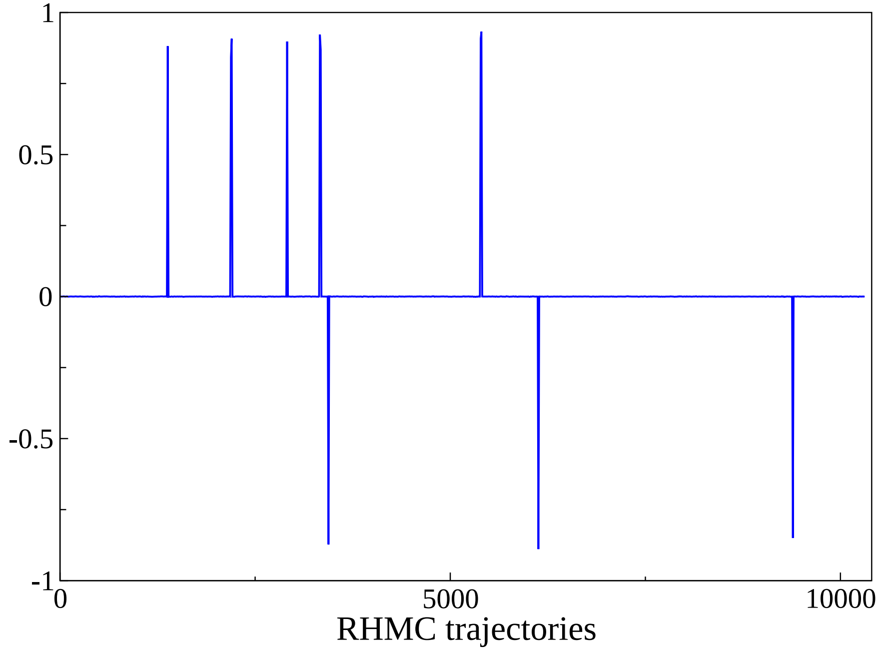
Main technical issues of the numerical implementation:

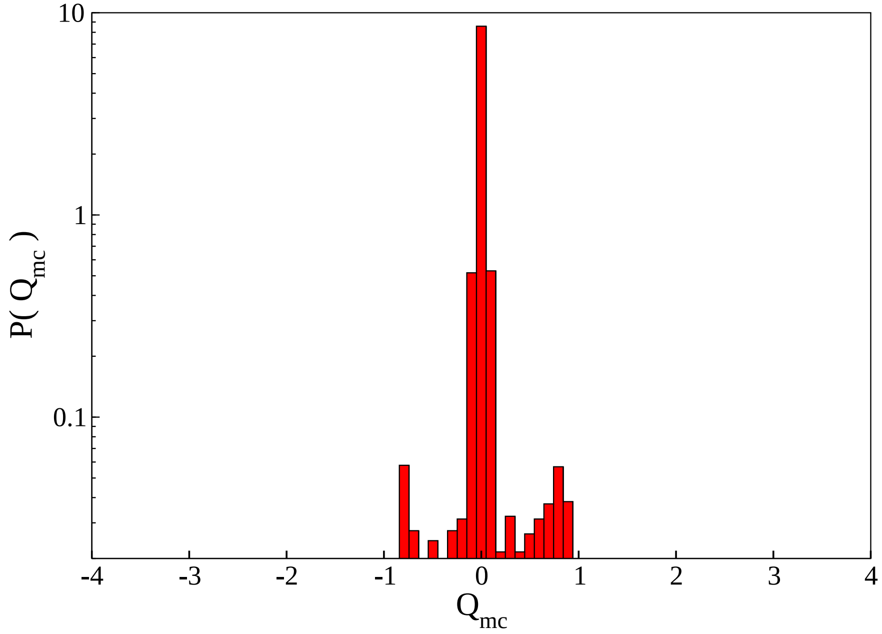
- the charge Q_{mc} entering the bias potential must be simple enough to permit integration of MD equations with reasonable overhead, have a good overlap with the true topological background Q

  best choice: field theoretic definition after 10-20 steps of stout smearing (30-60% overhead)

- Reweighting is usually plagued by bad overlap between target and the actual distribution.

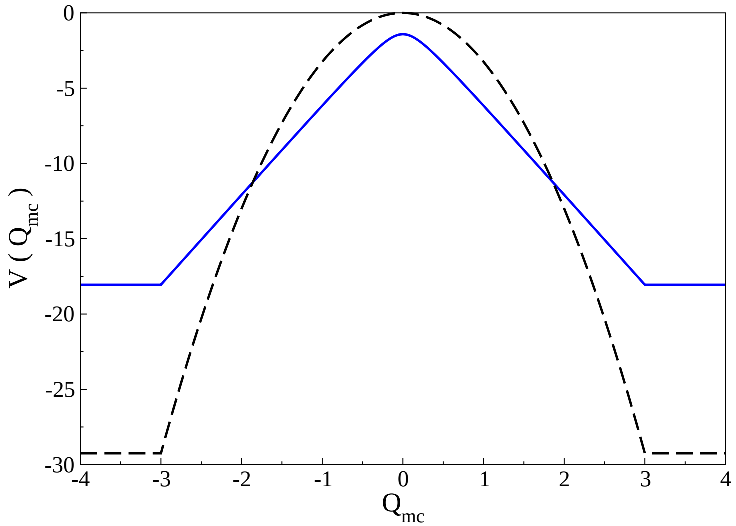
  This can happen (and can be avoided) also in our case





MC history of Q and probability distribution of Q_{mc} on a sample run

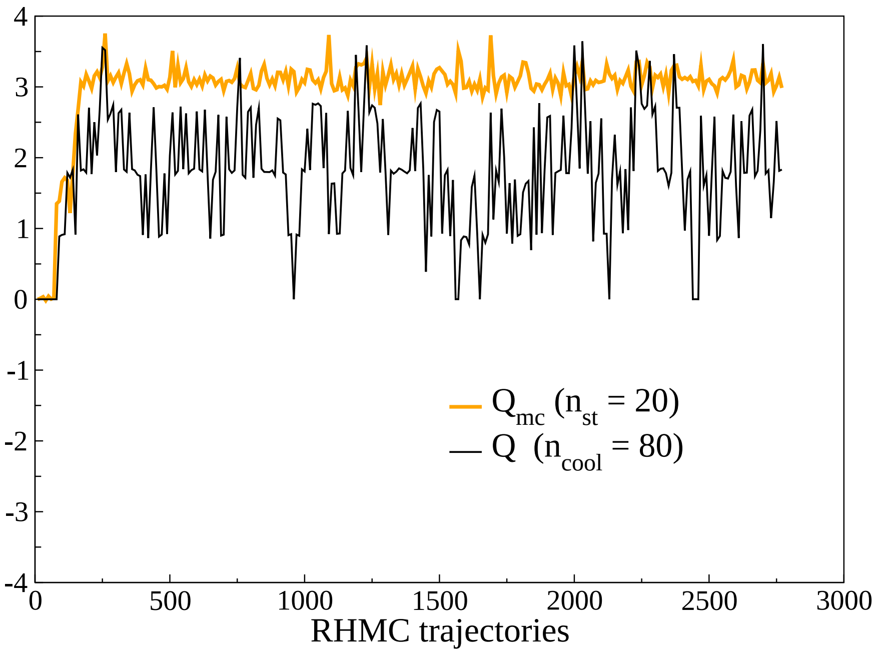
$32^3 \times 8$ **lattice,** $a = 0.0572$ **fm,** $T \simeq 430$ **MeV**

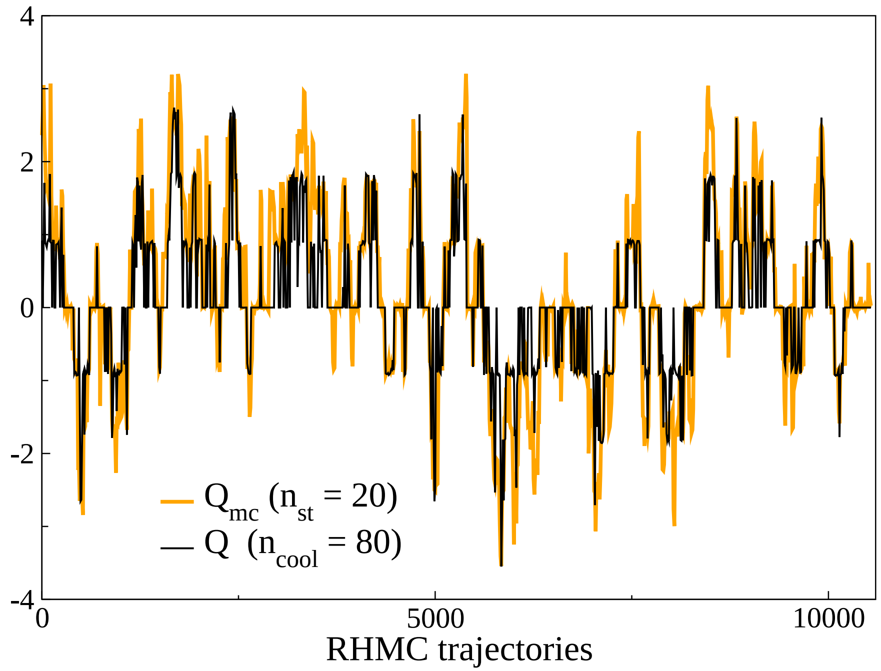


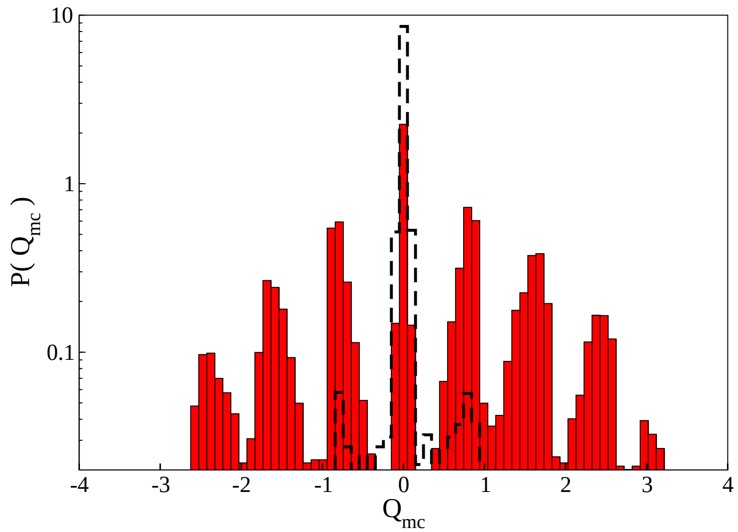
Two possibile choices of the bias potential and the corresponding MC histories:

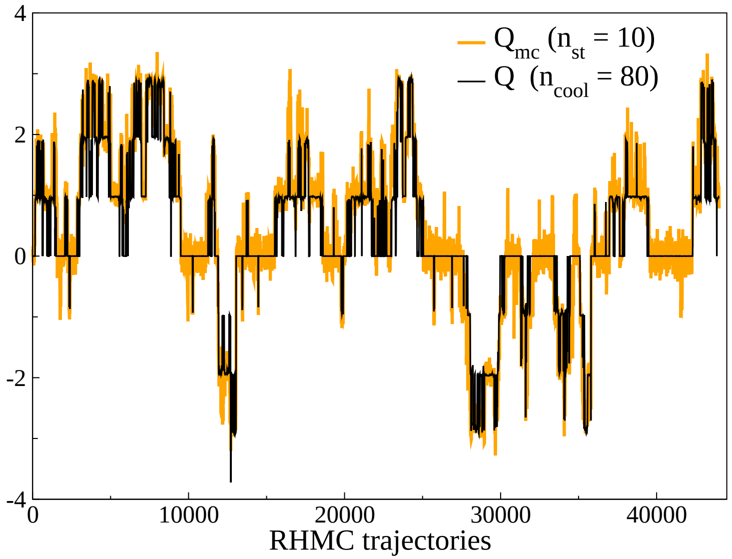
$$V(Q_{mc}) = -a_q\, Q_{mc}^2\,; \quad a_q = 3.25$$

$$V(Q_{mc}) = -\sqrt{(B\, Q_{mc})^2 + C}\,; \;\; B = 2,\; C = 6$$









Modified probability distribution of Q_{mc} from the “good” run

reweighted result $a^4\chi = (6.1 \pm 1.1) \times 10^{-8}$

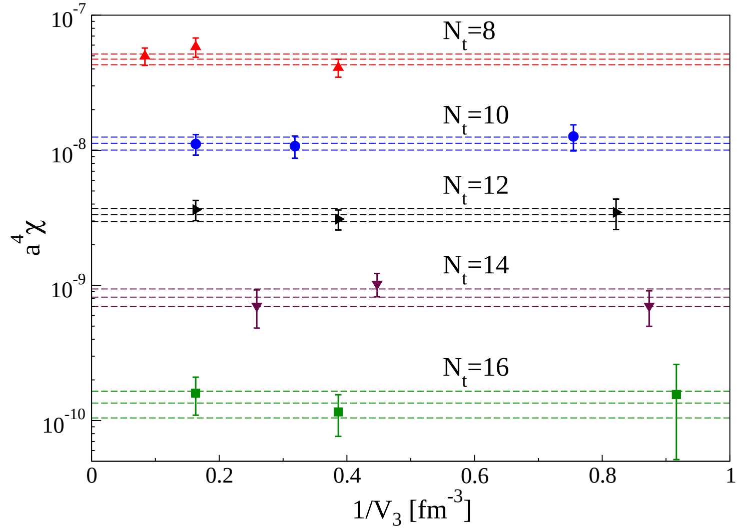
standard method $a^4\chi = (4.1 \pm 1.6) \times 10^{-8}$

taking into account computational overhead (60%), the gain is around 2.5

As the lattice spacing decreases χ drops down and the gain increases

$48^3 \times 16$ **lattice,** $a = 0.0286$**,** $T = 430$ **MeV**

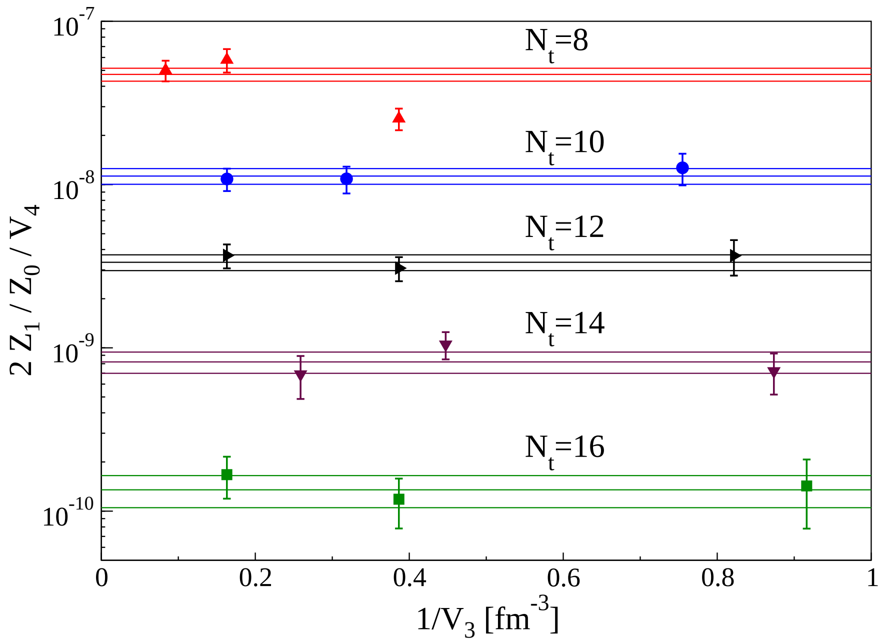
In this case $\langle Q^2 \rangle = 2.1(7) \times 10^{-4}$ **and the estimated gain is** $O(10^3)$**.**

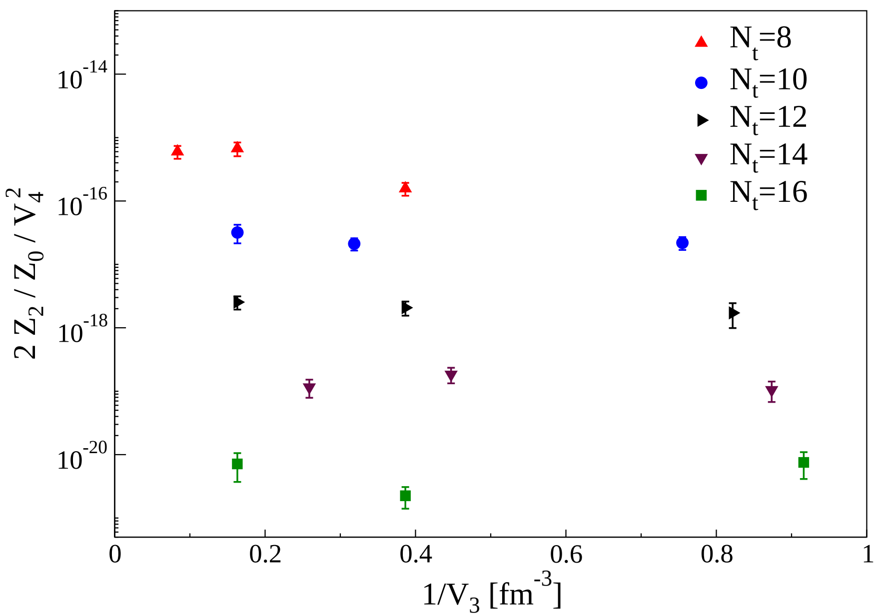


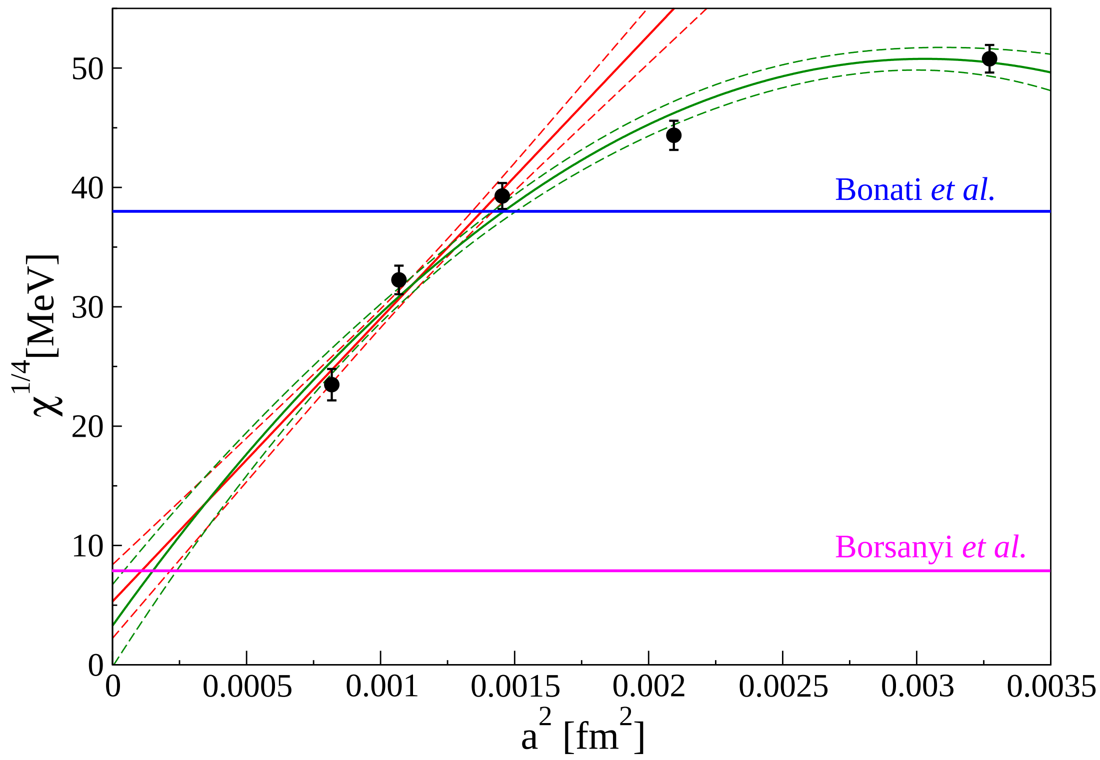
$a^4\chi$ **for various volumes and lattice spacings at** $T = 430$ **MeV**

most signal from Z_1/Z_0

however we estimate Z_2/Z_0 **as well: it scales according to DIGA**







Despite the good results obtained at finite lattice spacing, the continuum extrapolation is rough

$$\chi_{cont}^{1/4} = (3 \pm 3 \pm 2)\,\mathrm{MeV}$$

Lattice artifacts of our Dirac operator are still too large

Result in agreement with Borsanyi et al.

CONCLUSIONS

- Obtaining results for the θ-dependence of full QCD at $T \gg T_c$ is a hard task

- Using a multicanonical approach, we managed to confirm results obtained under reasonable assumptions in S. Borsanyi *et al.*, arXiv:1606.07494.

- However, we are still plagued by large lattice artifacts and a rough continuum extrapolation, leading to large final uncertainties. Any alternative to the eigenvalue reweighting adopted in S. Borsanyi *et al.*?
  - go to smaller lattice spacings (need good strategies to defeat topological freezing, e.g., metadynamics)
  - adopt a different Dirac operator for MC sampling (e.g., overlap, but computationally expensive ...)
  - adopt a different definition of Q, e.g. that based on spectral projectors as in C. Alexandrou, arXiv:1709.06596, which seems to have strongly reduced lattice artifacts for full QCD simulations