Improving the perturbative accuracy of TMD distributions: formalism

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Introduction

- The q_{T} distribution of a generic **high-mass** (Q) system produced in hadronic collisions has two main regimes:

  - for $q_{\mathrm{T}} \gtrsim Q$ **collinear factorisation** at *fixed perturbative order* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathrm{f.o.}} = \int_0^1 dx_1 \int_0^1 dx_2 f_1(x_1, Q) f_2(x_2, Q) \frac{d\hat{\sigma}}{dq_T} + \mathcal{O}\left[\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^n\right]$$

  - for $q_{\mathrm{T}} \ll Q$ **transverse-momentum-dependent (TMD) factorisation** at *fixed logarithmic accuracy* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\mathrm{res.}} \overset{\mathrm{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2) + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

  - Collinear and TMD factorisations may eventually be **matched** to produce accurate results over the the full q_{T} spectrum.

TMD factorisation

- TMD factorisation introduces two independent *artificial* scales:
  - the **renormalisation scale μ**, originating from UV renormalisation,
  - the **rapidity scale ζ**, originating from the cancellation of the rapidity divergencies.
- The respective **evolution equations** are:

$$\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu)$$

$$\frac{\partial \ln F}{\partial \ln \mu} = \gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta}}{\mu}$$

with: $$\frac{\partial K}{\partial \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

- In addition, for small values of b_T, TMDs can be matched on coll. PDFs:

$$F(\mu,\zeta) = C(\mu,\zeta) \otimes f(\mu)$$

- The solution is:

$$F(\mu,\zeta) = \exp\left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0,\zeta_0) \otimes f(\mu_0)$$

- Anomalous dims. and matching funcs. **perturbatively** computable.

TMD factorisation

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Matching onto collinear

$$F(\mu,\zeta) = C(\mu,\zeta) \otimes f(\mu)$$

- The solution is:

Evolution (Sudakov) factor

$\mu_b = b_0 / b_T$

$$F(\mu,\zeta) = \exp\left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0,\zeta_0) \otimes f(\mu_0)$$

- Anomalous dims. and matching funcs. **perturbatively** computable.

TMD factorisation

- The single TMD distributions are then given by:

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_T; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

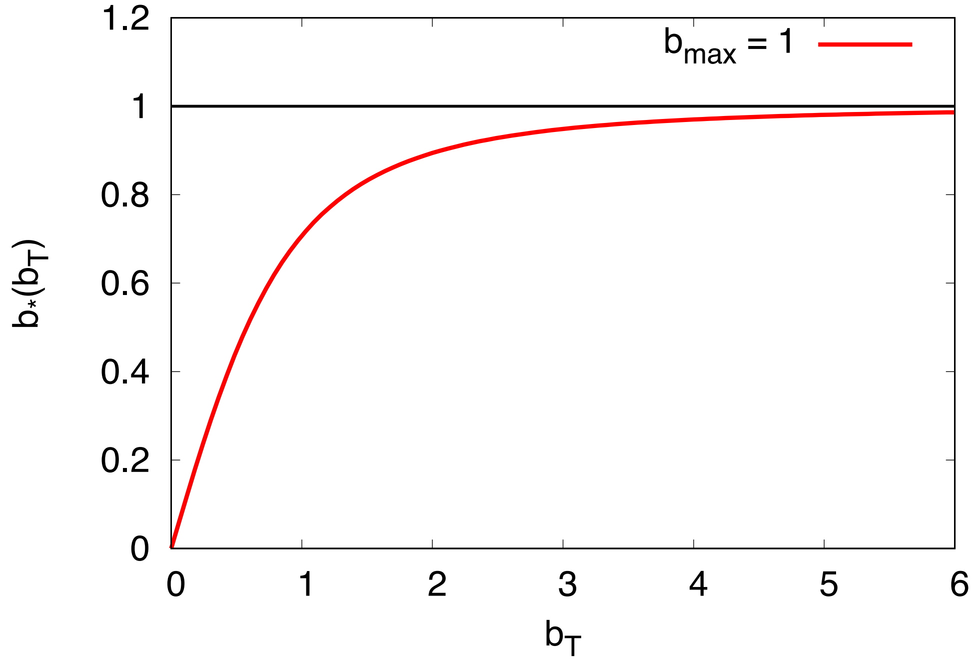
$$\times \exp\left\{ K(b_T; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \qquad : B$$

- matching to the collinear region at $b_\mathrm{T} \ll 1/\Lambda_\mathrm{QCD}$,
- factorises as *hard* (perturbative) and *longitudinal* (*i.e.* collinear, non-perturbative).

- CS and RGE evolution,
- evolution to large b_T,
- perturbative.

TMD factorisation

- When integrating over b_T, **large values of** b_T give raise to low scales in the **non-perturbative** region.



- Introduce the so-called **b*-prescription**:

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

- and rewrite:

$$F(x, b_T, \mu, \zeta) = \left[\frac{F(x, b_T, \mu, \zeta)}{F(x, b_*(b_T), \mu, \zeta)}\right] F(x, b_*(b_T), \mu, \zeta) \equiv f_{\rm NP}(x, b_T, \zeta) F(x, b_*(b_T), \mu, \zeta)$$

TMD factorisation

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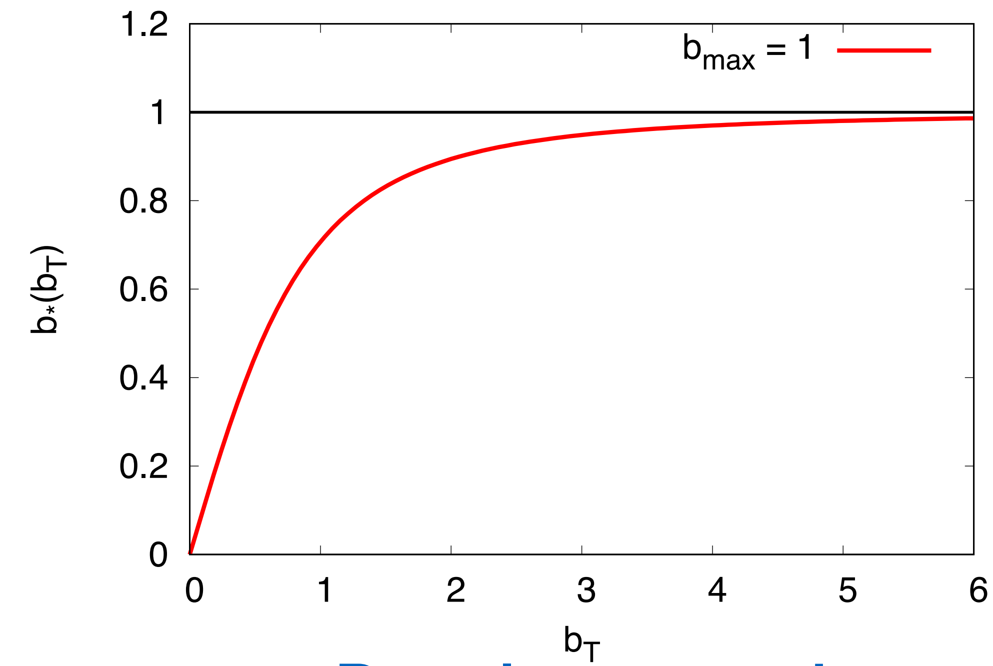

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$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

- and rewrite:

Purely perturbative

$$F(x, b_T, \mu, \zeta) = \left[\frac{F(x, b_T, \mu, \zeta)}{F(x, b_*(b_T), \mu, \zeta)}\right] F(x, b_*(b_T), \mu, \zeta) \equiv f_{\rm NP}(x, b_T, \zeta) F(x, b_*(b_T), \mu, \zeta)$$

Non-perturbative, determine from data

- Properties of $f_{\rm NP}$:
  - has to go to **one** as b_T goes to zero: reproduce the fully perturbative regime,
  - has to got to **zero** as b_T becomes large: mimic the Sudakov suppression.

- Bottom line: avoidance of the non-perturbative region upon integration in b_T implies the presence of **both** b_*-prescription and $f_{\rm NP}$.

TMD factorisation

- Final expression:

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \quad : A$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \quad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \quad : C$$

- matching to the collinear region at $b_T \ll 1/\Lambda_{QCD}$,
- factorises as *hard* (perturbative) and *longitudinal* (*i.e.* collinear, non-perturbative).

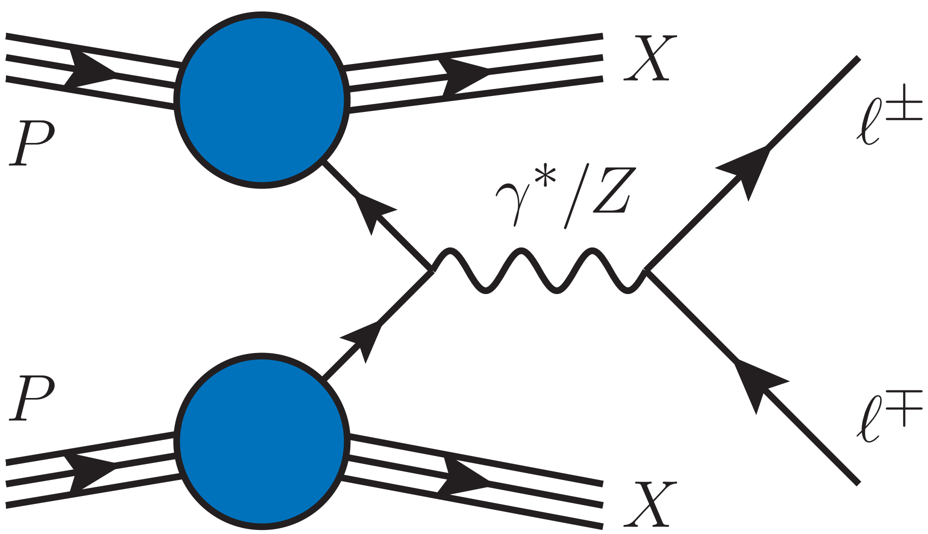
- avoid the Landau pole,
- f_{NP} accounts for the introduction of b_*,
- f_{NP} is non-perturbative thus **fit** to data.

- CS and RGE evolution,
- evolution to large b_T,
- perturbative.

TMD factorising processes

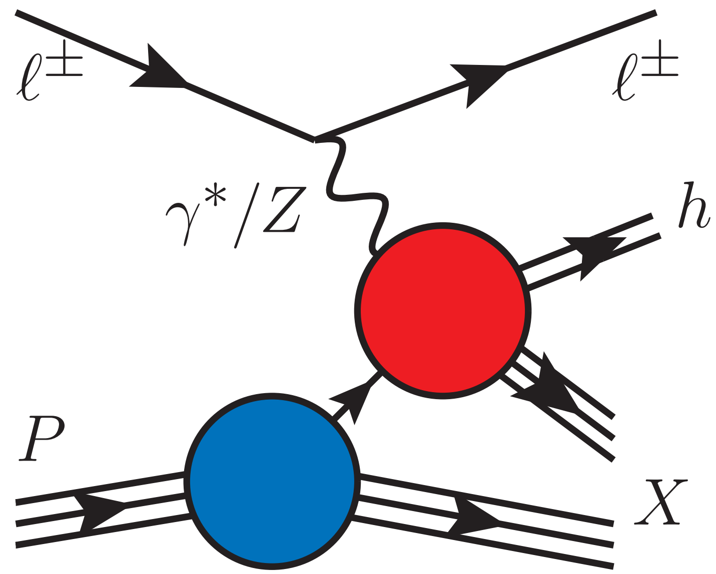
- Processes for which leading-power TMD factorisation has been **proven**:

Drell-Yan



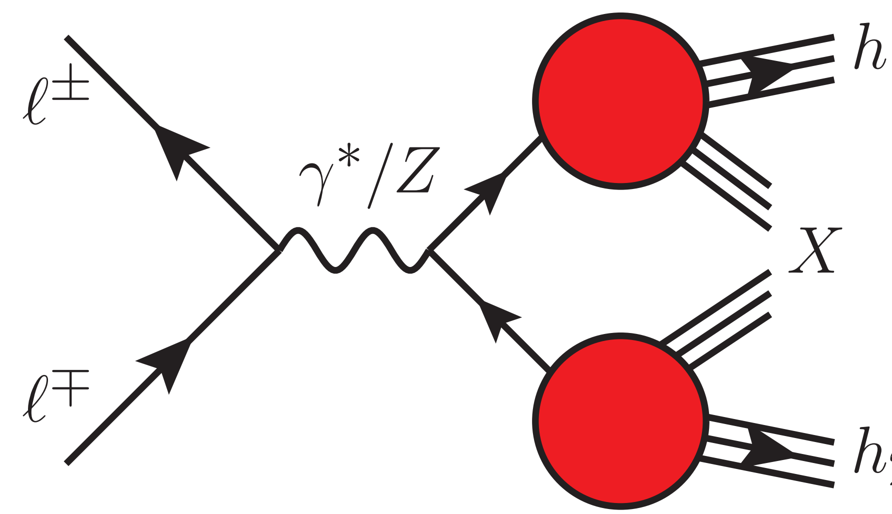
$PP \longrightarrow \ell^\pm \ell^\mp\ X$

Semi-inclusive DIS



$P\ell^\pm \longrightarrow \ell^\pm h\ X$

e^+e^- annihilation



$\ell^\pm \ell^\mp \to h_1 h_2\ X$

- **Two** TMD **PDFs**:
- Lots of data:
  - low-energy: FNAL,
  - mid-energy: RHIC,
  - high-energy: Tevatron, LHC.

- One TMD **PDF** one **FF**:
- many precise data points:
  - HERMES at DESY,
  - COMPASS at CERN.

- **Two** TMD **FFs**:
- di-hadron prod. from:
  - BELLE at KEK,
  - BABAR at SLAC.

Logarithmic counting

- TMD factorisation provides **resummation** of large logs $L = \log(q_T/Q)$:
  - implemented through the **Sudakov** form fact R.
- A **perturbative expansion** in powers of α_s of R would give:

One Sudakov for each TMD

$$R^2 = \sum_{n=0}^{\infty} a_s^n \sum_{k=1}^{2n} \widetilde{S}^{(n,k)} L^k$$

Double-log expansion

- that can be rearranged as:

$$R^2 = \sum_{m=0}^{\infty} R^2_{\mathrm{N^mLL}} \quad \text{with} \quad R^2_{\mathrm{N^mLL}} = \sum_{n=[m/2]}^{\infty} \widetilde{S}^{(n,2n-m)} a_s^n L^{2n-m}$$

Integer part of m/2

- Therefore, multiplying R by a power p of α_s gives:

$$a_s^p R^2_{\mathrm{N^mLL}} = \sum_{j=[(m+2p)/2]}^{\infty} \widetilde{S}^{(j-p,2j-(m+2p))} a_s^j L^{2j-(m+2p)} \sim R^2_{\mathrm{N^{m+2p}LL}}$$

- Bottom line: any additional power of α_s causes a shift of **two units** in the logarithmic ordering.

Logarithmic counting

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL'	α_s^2	α_s	α_s	α_s	α_s
N^2LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N^2LL'	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N^3LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \stackrel{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_1(x_1,\mathbf{b}_T,Q,Q^2) F_2(x_2,\mathbf{b}_T,Q,Q^2) + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

$$F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) = \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F) \otimes f_{j/P}(x,\mu_b)$$

$$\times \quad \exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\}$$

Matching TMD to collinear

- Accurate predictions for all q_T's by **additive matching**, order by order in perturbation theory, of collinear and TMD calculations:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{add.match.}} = \left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} + \left(\frac{d\sigma}{dq_T}\right)_{\text{f.o.}} - \left(\frac{d\sigma}{dq_T}\right)_{\text{d.c.}}$$

- In order for the match to actually take place:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \xrightarrow[\text{f.o.}]{} \left(\frac{d\sigma}{dq_T}\right)_{\text{d.c.}} \xleftarrow[q_T \ll Q]{} \left(\frac{d\sigma}{dq_T}\right)_{\text{f.o.}}$$

- Therefore, the "fixed-order" parts have to match in the relevant limits:

Log Accuracy	Minimal f.o. accuracy
NLL'	α_s (LO)
N^2LL	α_s (LO)
N^2LL'	α_s^2 (NLO)
N^3LL	α_s^2 (NLO)

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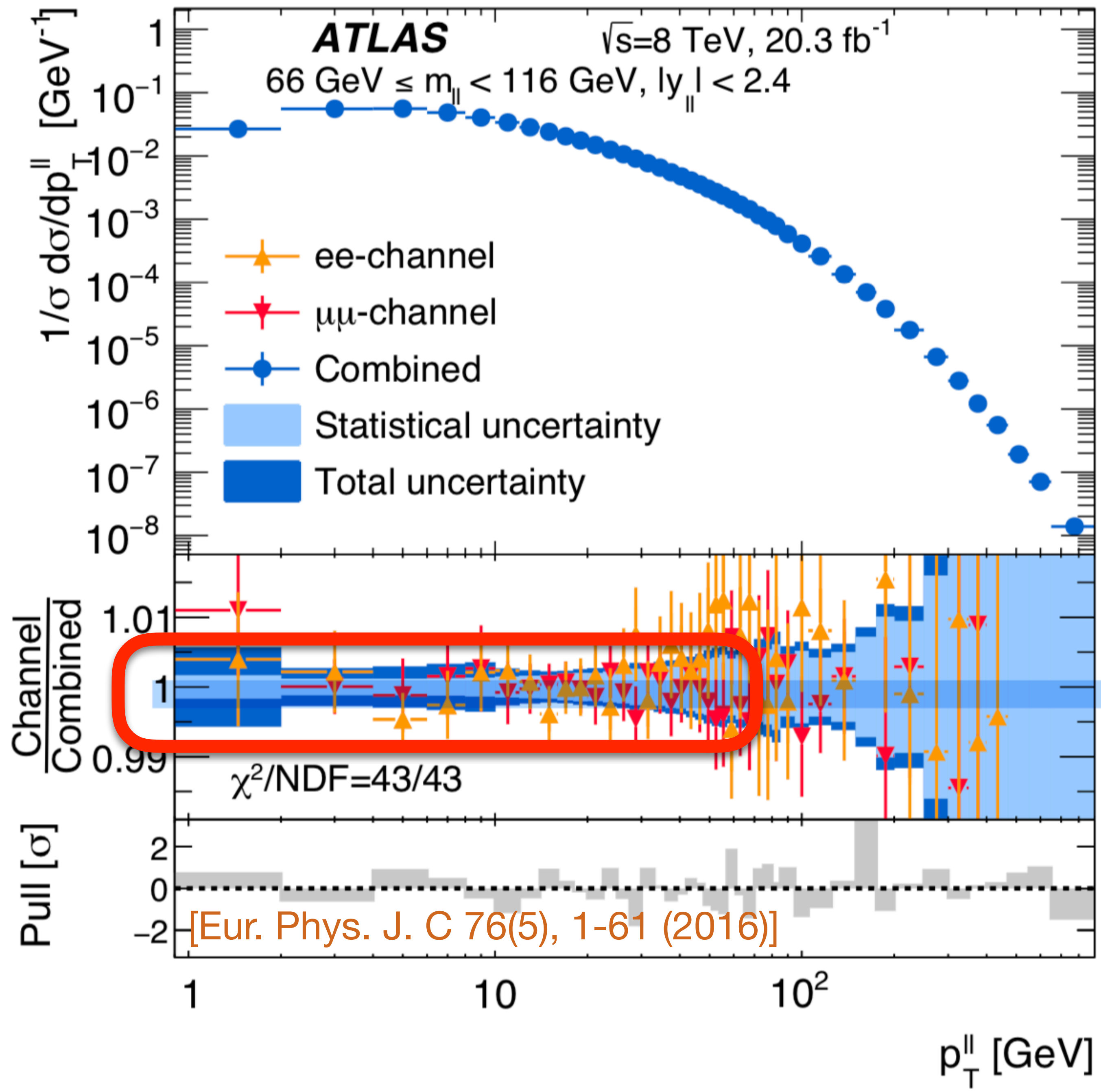
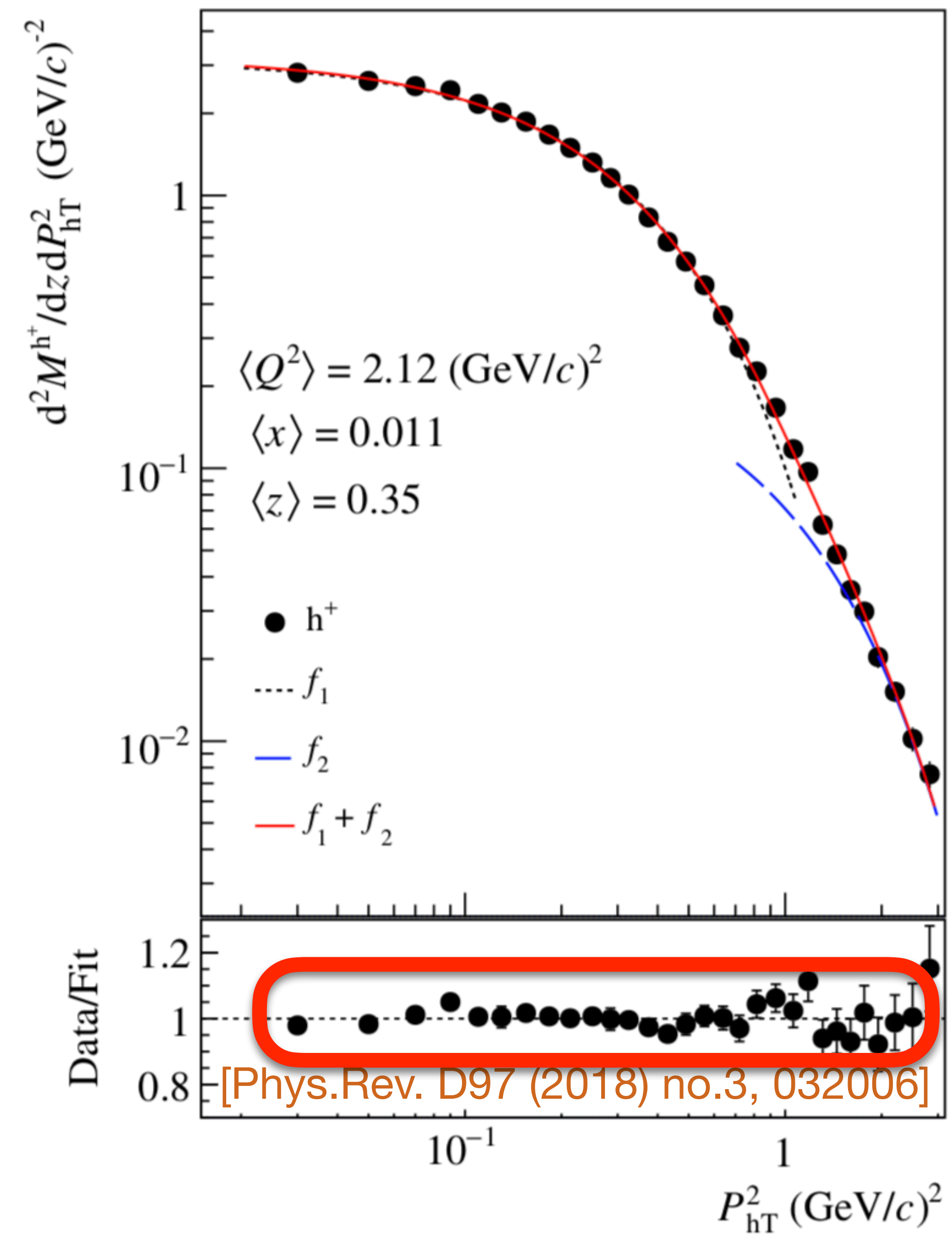
Higher-order corrections

- Measurements of q_T distributions have reached the **sub-percent level** uncs.:





- **State-of-the-art** calculations are thus necessary to hope to describe this data:
  - **higher-order** corrections and possibly **matching** between **TMD** and **collinear**.

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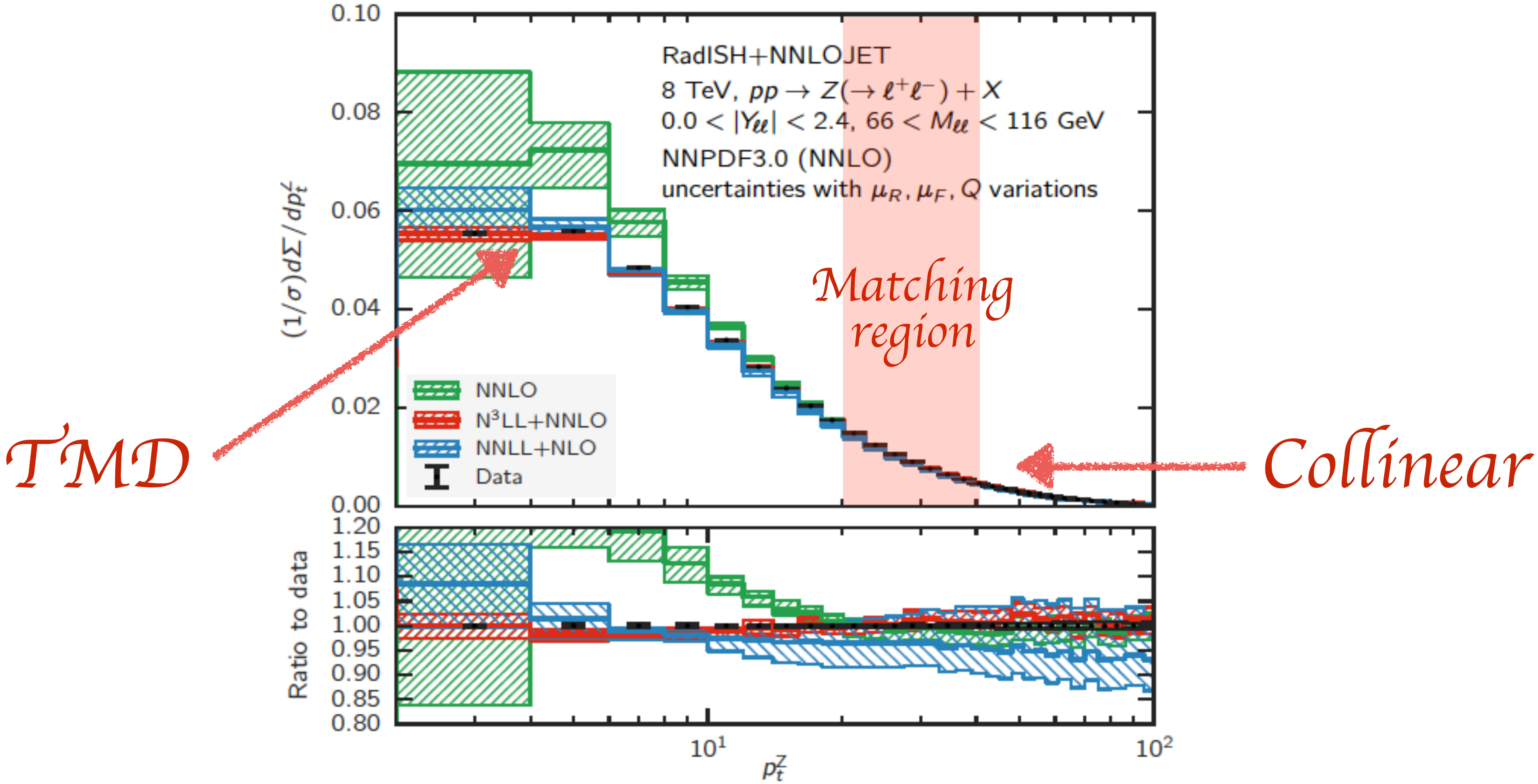
Higher-order corrections

- Current state-of-the-art: **N³LL + NNLO**:

[10.1007/JHEP12(2018)132]



- required to describe the precise ATLAS Z-production data.
- This data can be used to determine the non-pert. component.

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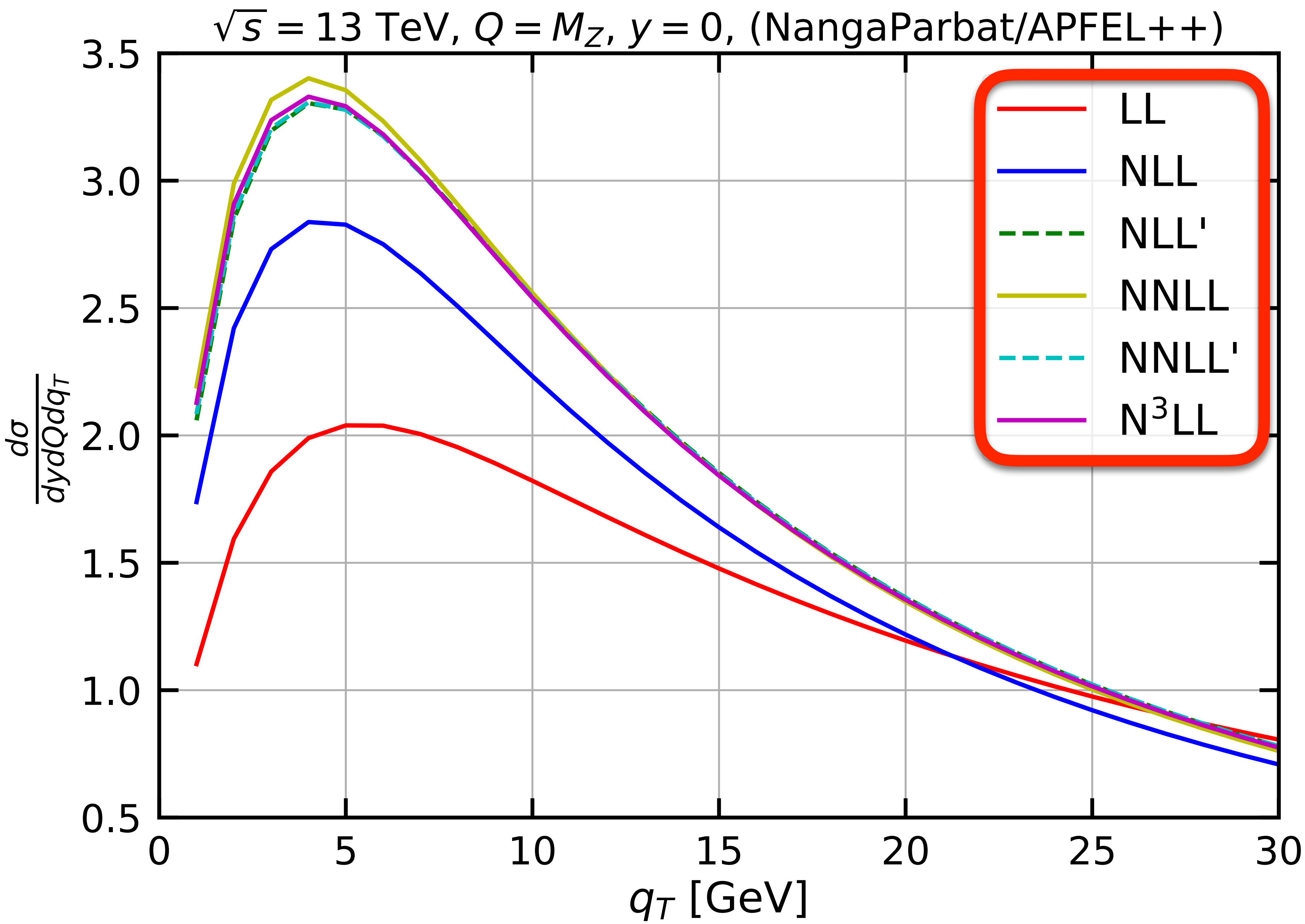
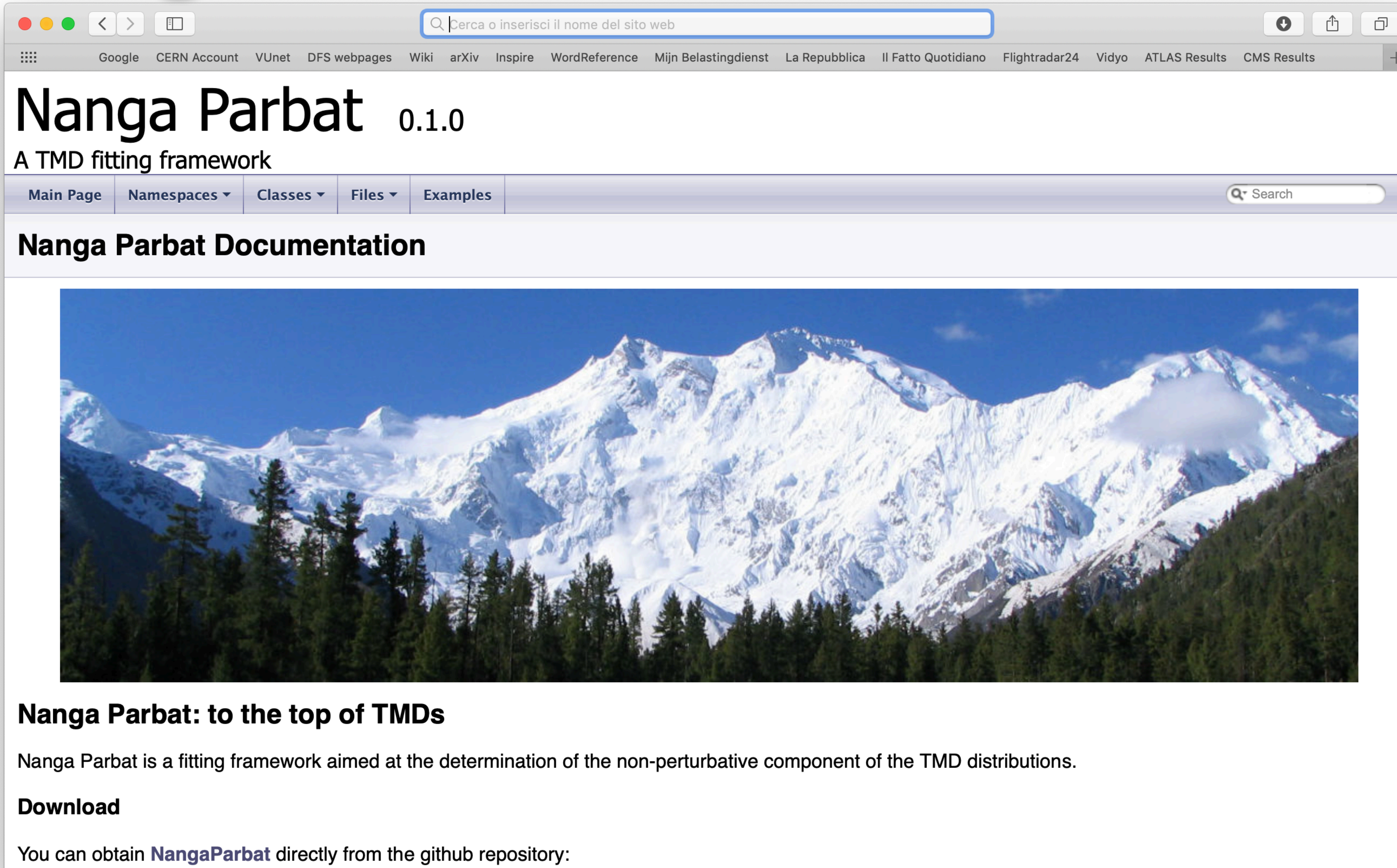
Higher-order corrections

- In Pavia, we are actively working to reach the “state-of-the-art” accuracy:
  - in fact, in the TMD region we already got there!



- A fast computation of this observable is implemented in a dedicated framework conceived to extract TMD distributions: **NangaPargat**.

NangaParbat



- The **fitting framework** based on **APFEL++** and that is currently being developed by the Pavia's group to extract TMD PDFs and FFs.

- Under development, it will eventually be **publicly** available.

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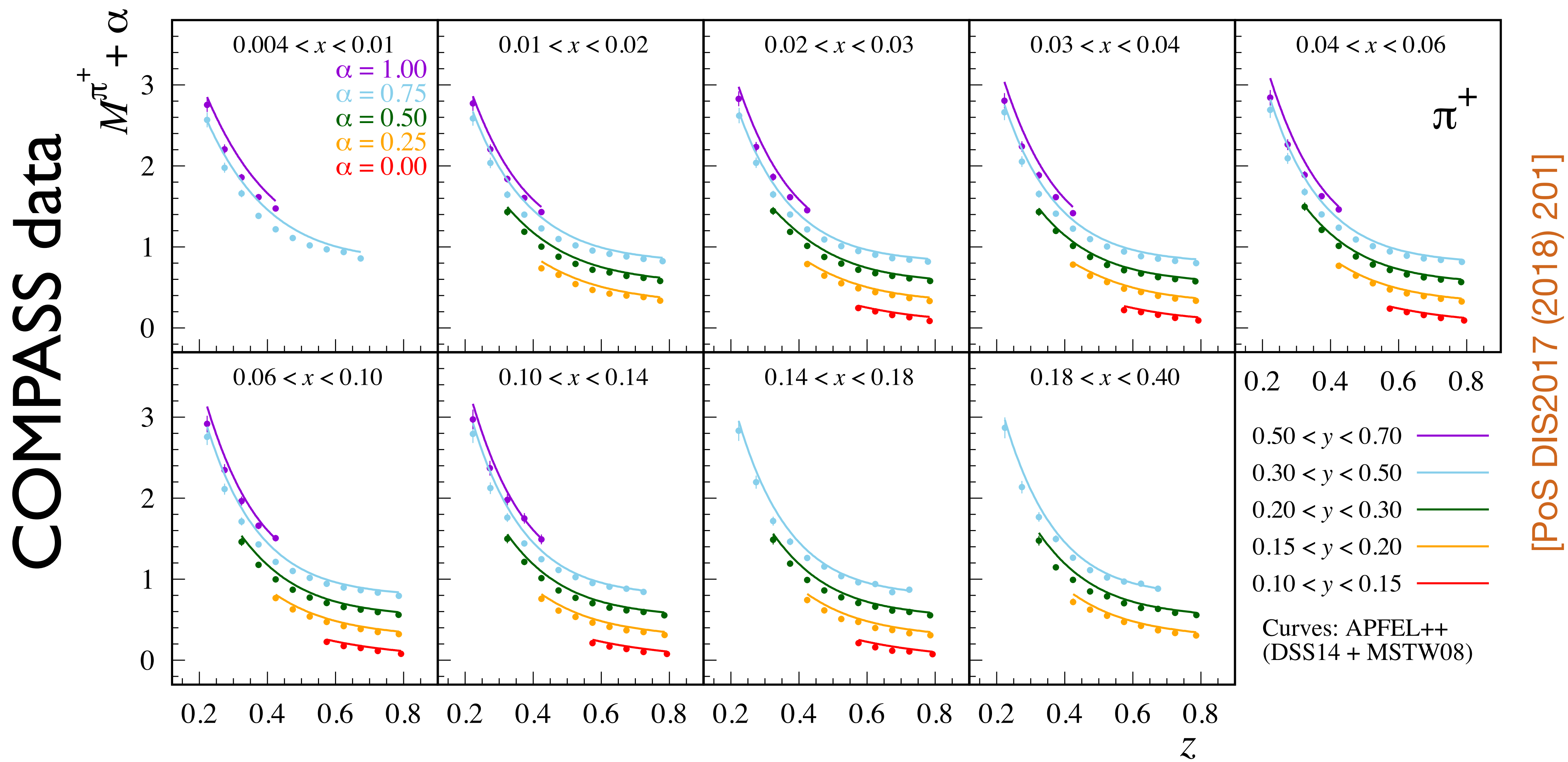
SIDIS studies: q_T-integrated multiplicities

- Let us start considering ***q_T*-integrated** SIDIS multiplicities:

$$M^h(x,z,Q^2) = \frac{d^3\sigma^h/dxdzdQ^2}{d^2\sigma/dxdQ^2}$$

  - computable in **collinear** factorisation (to $O(\alpha_s)$).



- This works pretty nicely.
- This data has actually be included in the DSS14 fit of collinear FFs.

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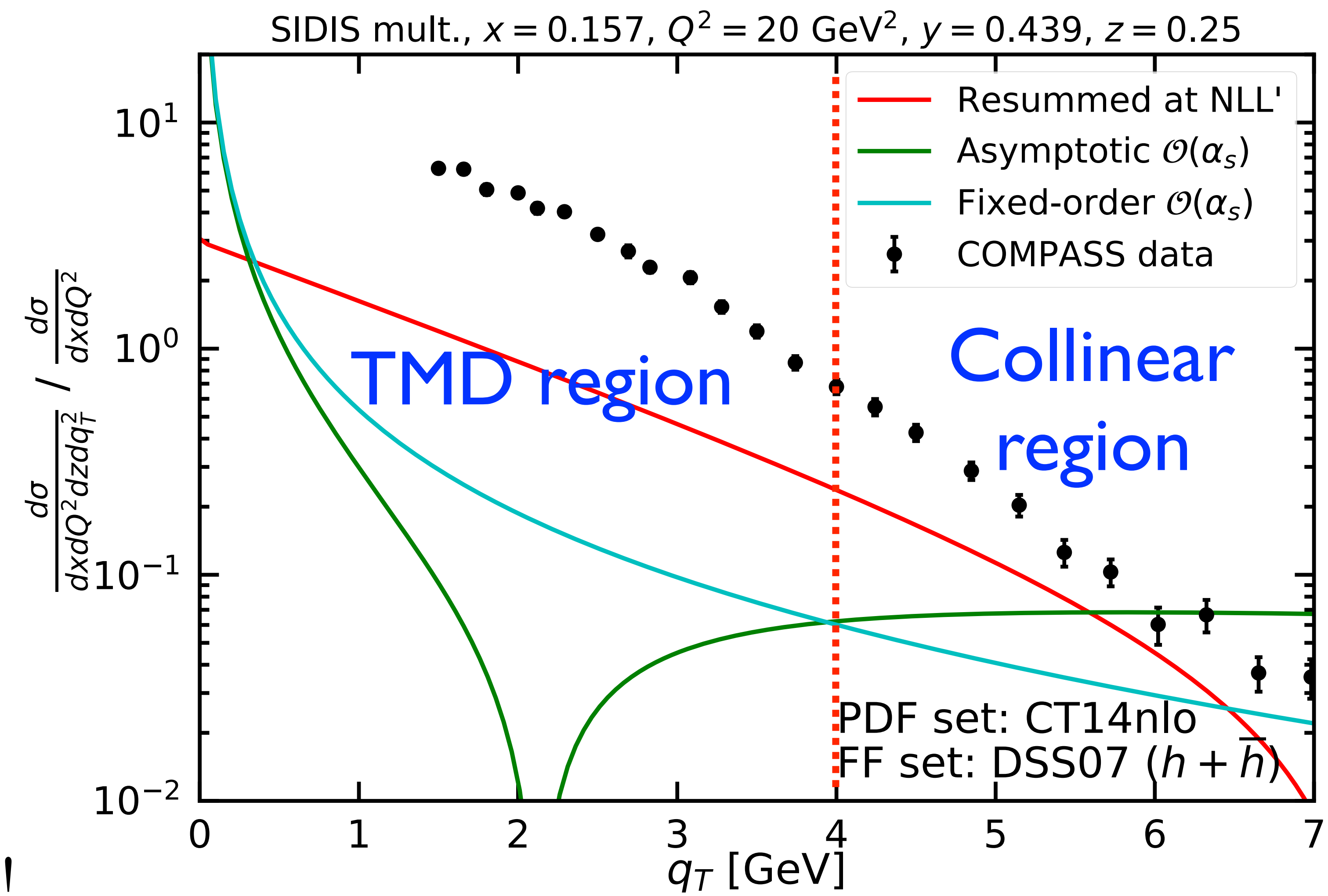
SIDIS studies: q_T-differential multiplicities

- Now, let us have a look at **q_T-differential** SIDIS multiplicities:

$$\overline{M}^h(x, z, Q^2, q_T) = \frac{d^3\sigma^h/dxdzdQ^2 dq_T^2}{d^2\sigma/dxdQ^2}$$

  - **TMD** factorisation at small q_T, **collinear** factorisation at large q_T.



- Utterly off!
- Unlikely that non-perturbative effects can accommodate such differences.
- How comes that q_T-integrated works and q_T-differential does not?

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SIDIS studies: q_T-differential multiplicities

- One may try to integrate analytically the $O(\alpha_s)$ fixed-order q_T-diff:

$$\int dq_T^2 \frac{d^3\sigma^h}{dx\,dz\,dQ^2\,dq_T^2} = \frac{d^3\sigma^h}{dx\,dz\,dQ^2}$$

- This should give the q_T-integrated cross section that we know to work.
- If one tries, one finds that this is not the case:
  - the general finding is that all terms involving **virtuals** ($q_T = 0$) are absent,
  - most noticeably, and somewhat expectedly, the $\boldsymbol{O(1)}$ contribution is not there.
- One can the try to reintroduce this terms by **expanding** the resummed cross section and retain only the terms proportional to $\boldsymbol{\delta(q_T)}$:
  - this reproduces the $O(1)$ term but at $O(\alpha_s)$ this is not enough yet,
  - **threshold-enhanced terms** are still missing from the $O(\alpha_s)$ corrections,
  - **soft-gluon** (threshold) **resummation** may help (?).

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SIDIS studies: q_T-differential multiplicities

- Bottom line:
  - the origin of the discrepancy between data and theory for q_T-differential **multiplicities** may very well be in the **denominator**.
- Are we (theorists) dividing q_T-differential cross sections by the right quantity?
- A multiplicity has to be a quantity that integrated over q_T and z should give **one**.
- Is that really the case for theoretical predictions?
- This is clearly **not a trivial question** to ask:
  - integrating over q_T requires being able to compute predictions over the **full range**.
  - This brings into play the question of **matching** TMD and collinear regimes.

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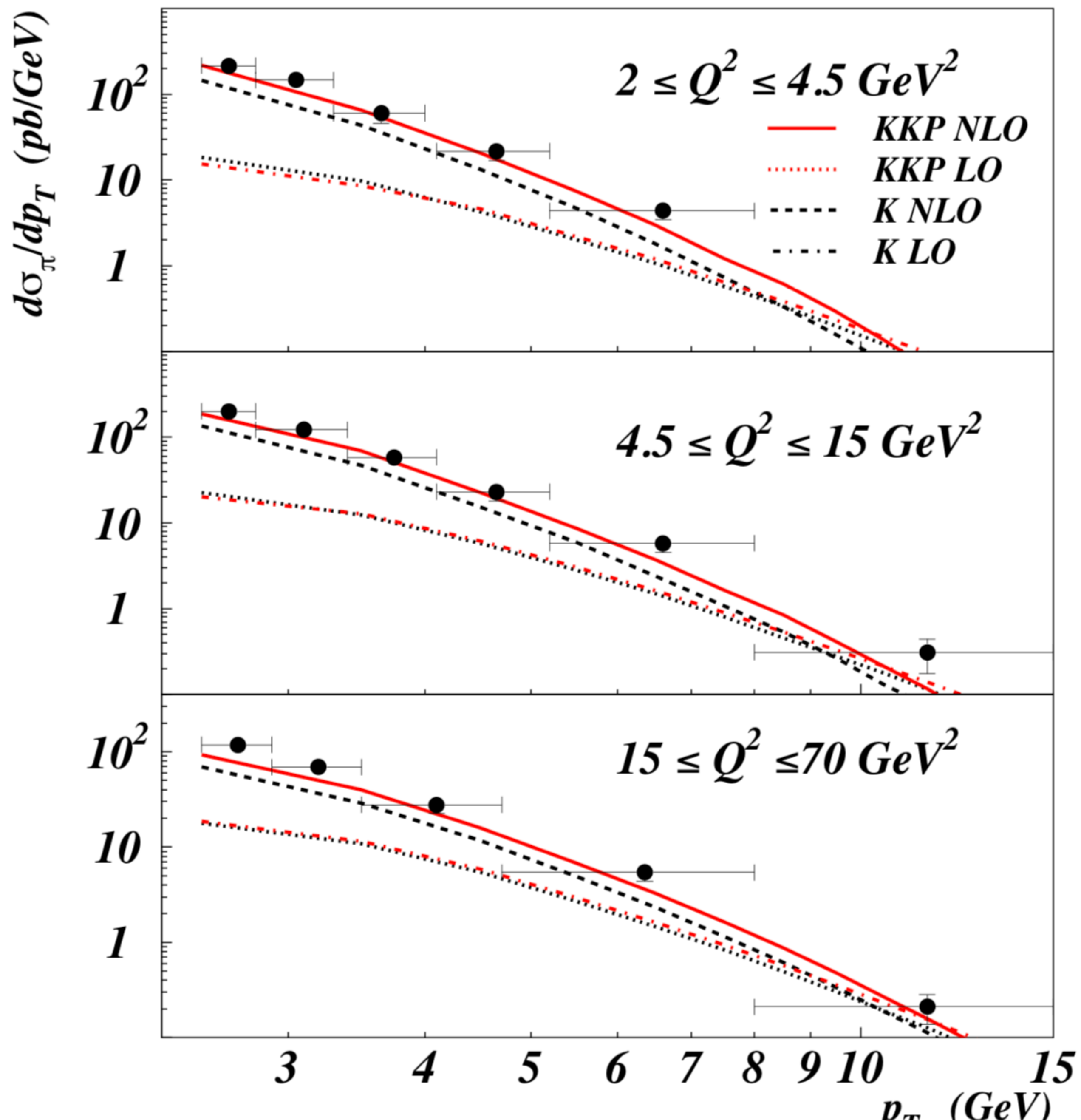
SIDIS studies: q_T-differential multiplicities

- Further indication:



[Phys.Rev. D71 (2005) 034013]

- HERA absolute cross sections are fairly described fixed-order $O(\alpha_s^2)$:
  - caveat: $\sqrt{s}$ = 300 GeV, large energy.

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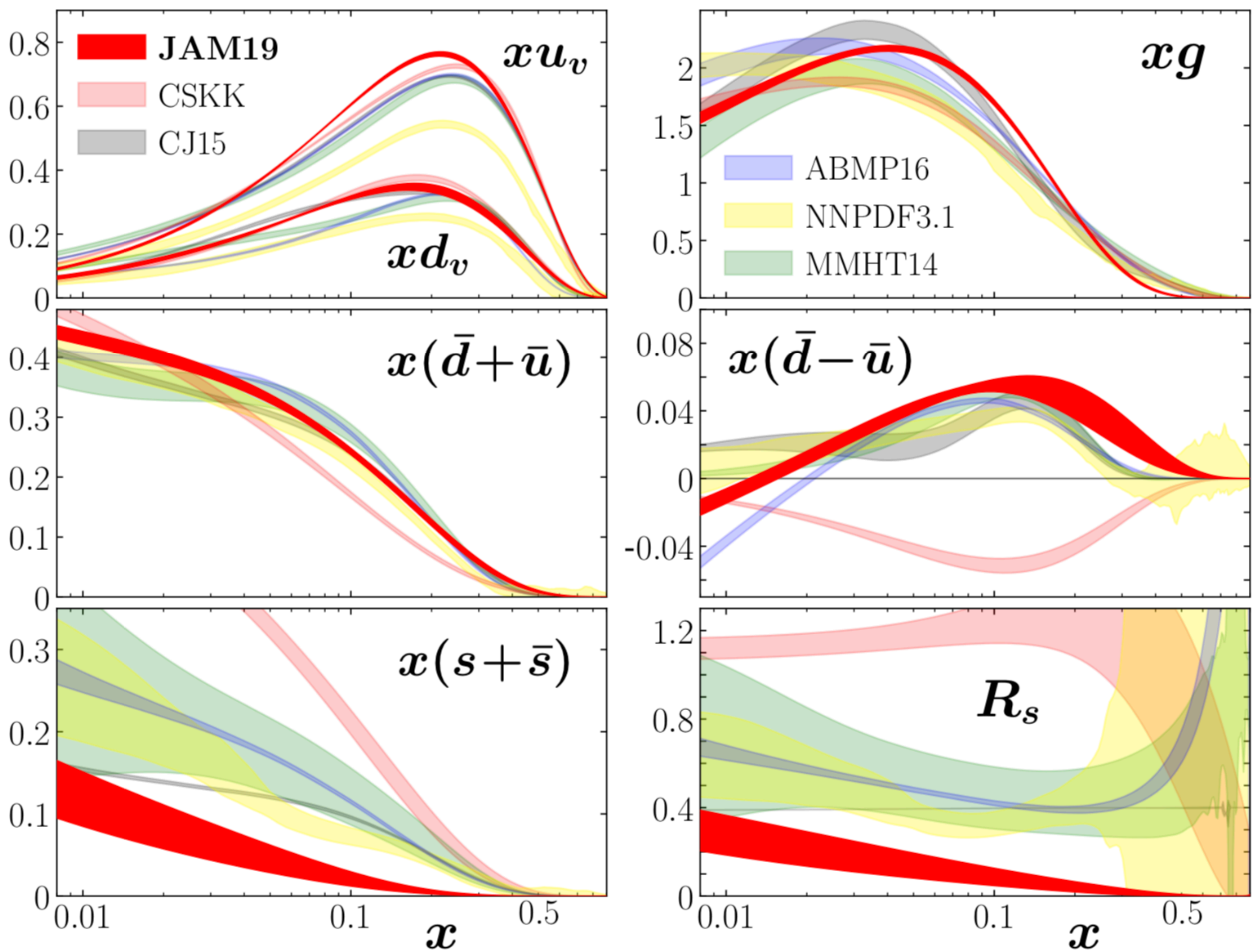
SIDIS studies: q_T-differential multiplicities

- My personal opinion: SIDIS multiplicities have to be **carefully understood** before they are included in a fit of PDFs/FFs:



[arXiv:1905.03788]

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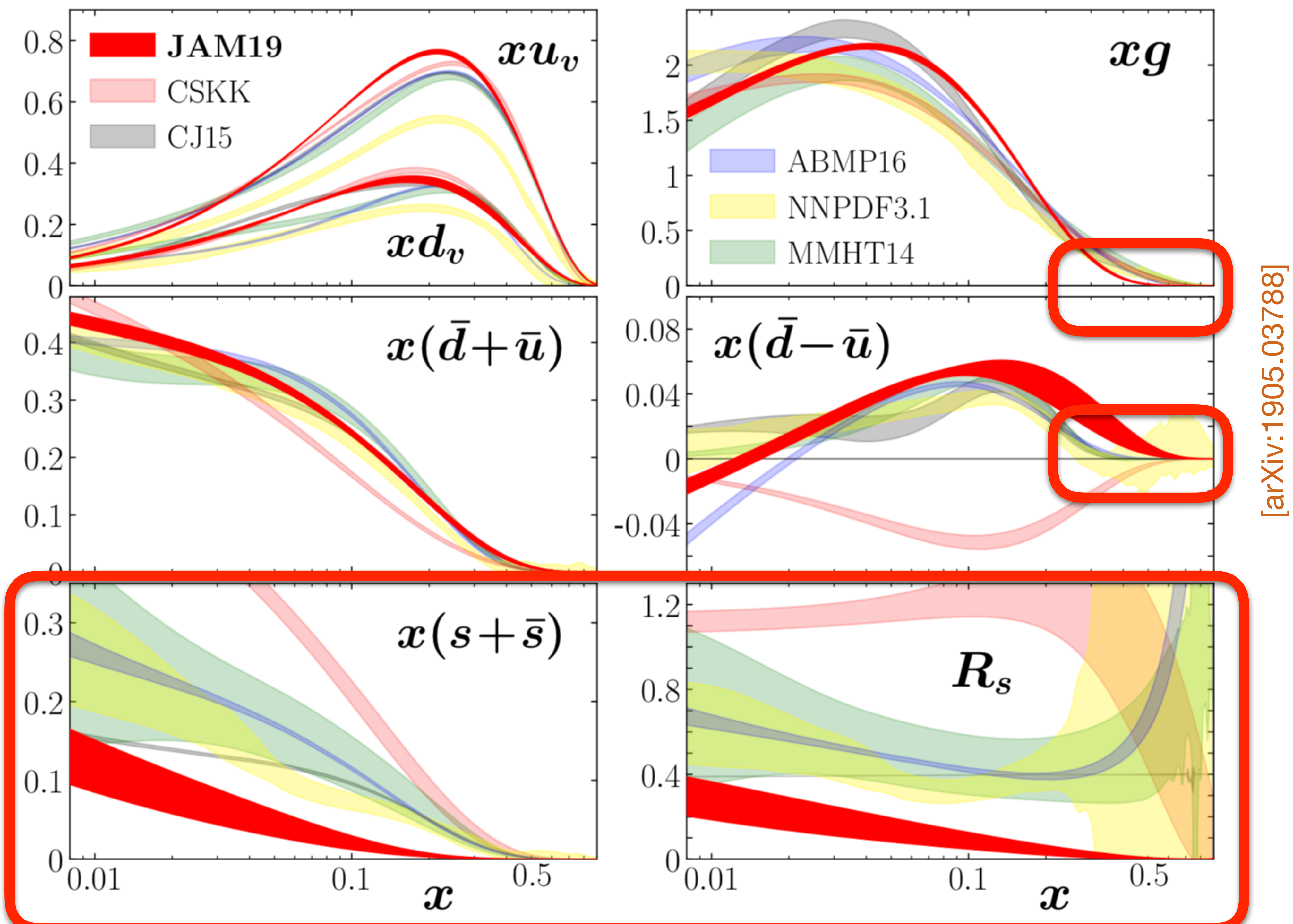

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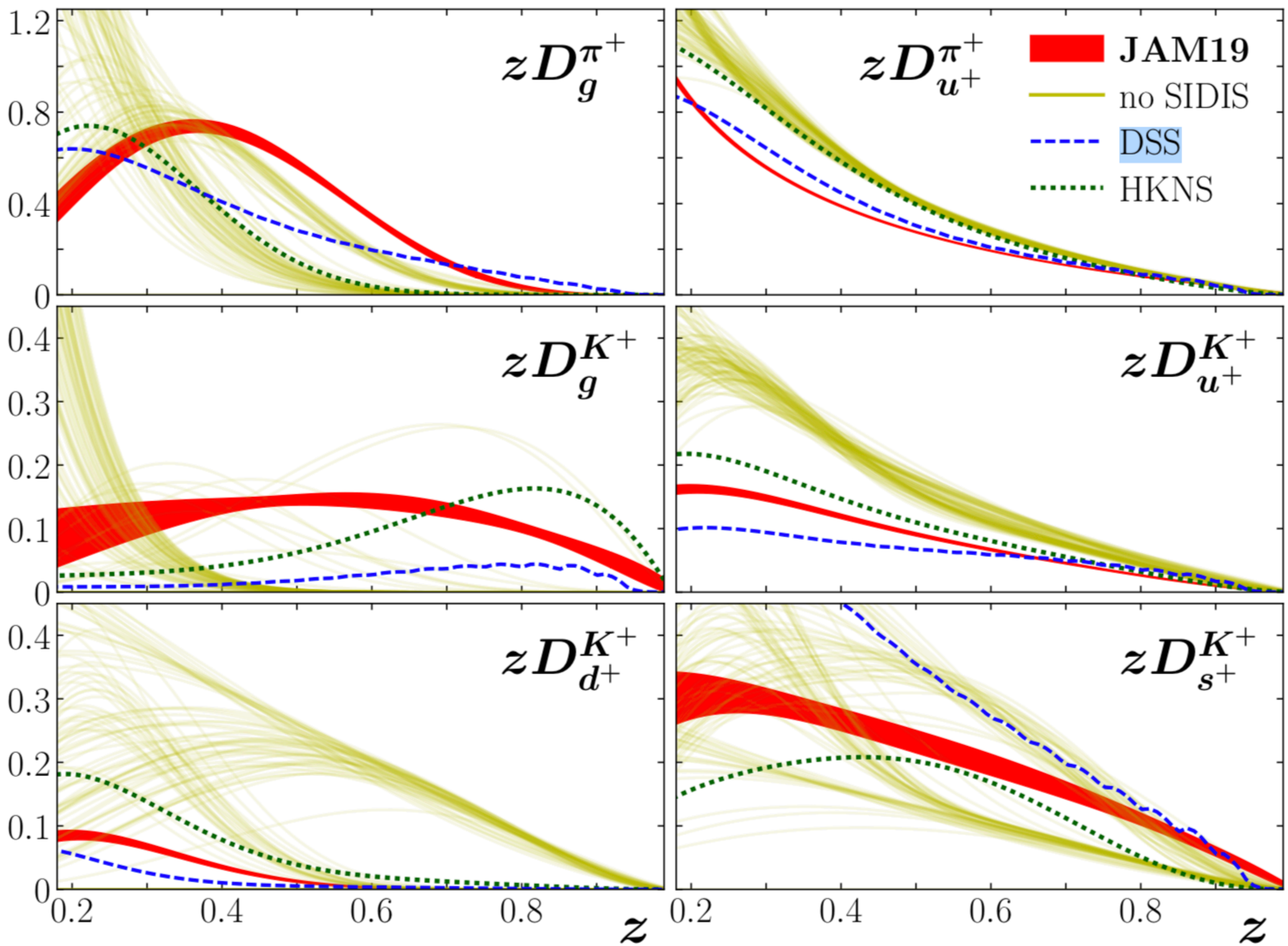

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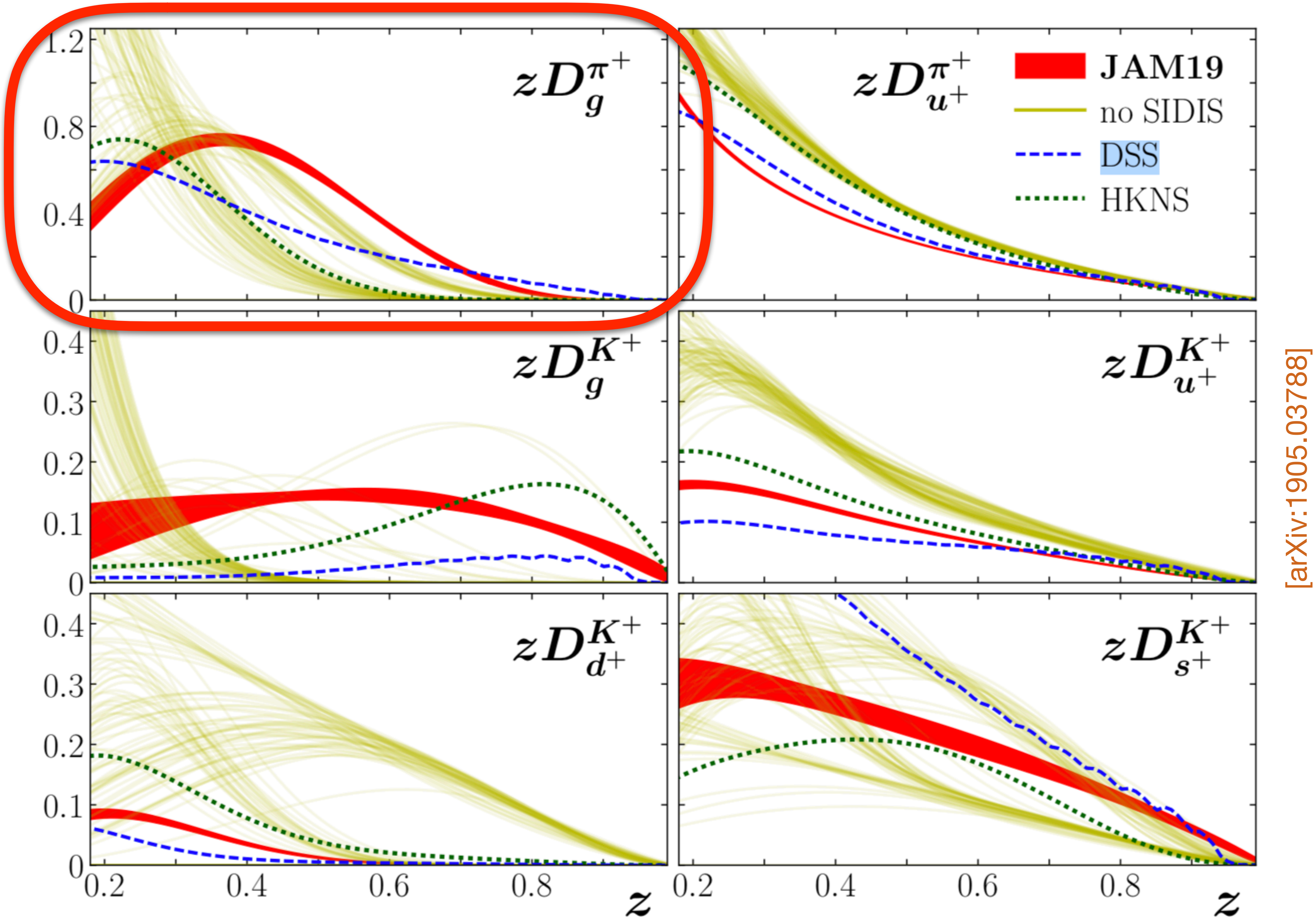


Conclusions

- **TMD factorisation** provides a valuable tool to descrive q_T distributions at small values of q_T (resummation of large logs),
- Non-perturbative component of TMDs to be determined from **data**.
- A lot of effort is being invested on the extraction of TMD PDFs and FFs:
  - wide and precise **datasets** (COMPASS, HERMES, LHC and Tevatron exps.),
  - state-of-the-art **theoretical computation** (N^3LL at small q_T),
- SIDIS multiplicities from COMPASS and HERMES are challenging:
  - **neither TMD nor collinear** factorisations seem to describe them,
  - more corrections needed (*e.g.* **soft-gluon resummation**)?
  - or just a matter of properly define the observable on the theoretical side?

Backup

Matching

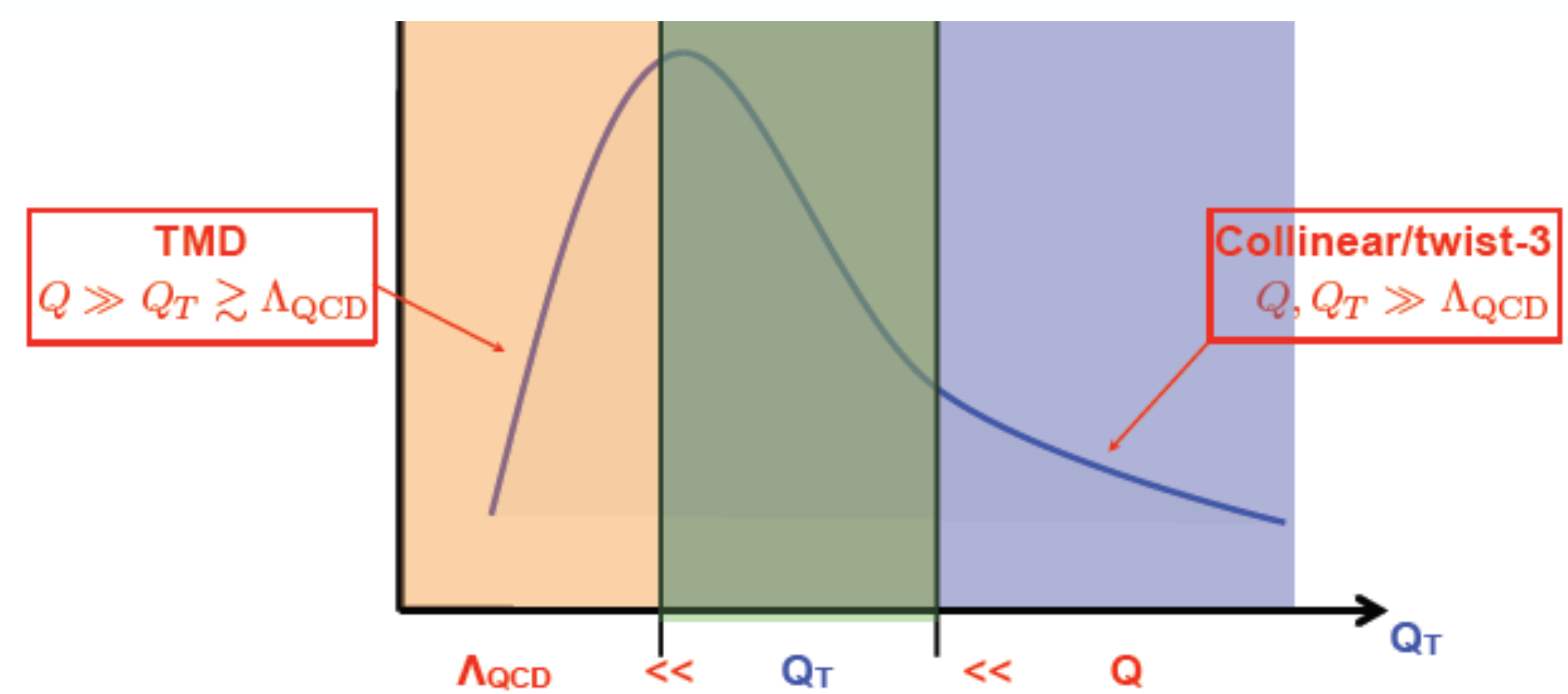
Collinear and TMD frameworks

- Referring to **colourless** final-state processes, the description of q_T dependent observables is based on two well-established frameworks:
  - the **TMD** framework for $q_T \ll Q$,
  - the **collinear** framework for $q_T \simeq Q$.
- These two regimes can be **matched** leading to *theoretically* possibly accurate predictions over the full range in q_T.
- However, assuming that collinear distributions are determined reliably, the very-low q_T (TMD) region receives **non-perturbative** contributions that need to be determined from data.
- A lot of effort has been put into the determination of the TMD non-perturbative component but still far from a general agreement:
  - different prescriptions (CSS, Parton Branching, Scimemi-Vladimirov, etc.),
  - different perturbative orders and orderings,
  - …

q_T dependence
Collinear and TMD frameworks

- At **relatively high energies**, the separation between small- and large-q_T regimes is unambiguous:
  - **safe** application of the two frameworks in the respective regions,
  - just some care required in the **transition region** $\Lambda_{QCD} \ll q_T \ll Q$ that is however well-defined (dependence on the matching prescription),
  - **limited** effect of the TMD non-perturbative contributions.

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



Matching TMD to collinear

- Multiplicative matching:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{mult.match.}} = \left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \times \left(\frac{d\sigma}{dq_T}\right)_{\text{f.o.}} \Big/ \left(\frac{d\sigma}{dq_T}\right)_{\text{d.c.}}$$