Transverse Λ polarisation in e^+e^- collisions

Sar WorS 2019 – Sardinian Workshop on Spin studies

Marco Zaccheddu - Università degli Studi di Cagliari
In collaboration with U. D’Alesio and F. Murgia

Introduction

- Transverse Lambda polarisation in high-energy inclusive processes: a long standing open issue
- Proton-proton data: large polarisation that cannot be explained within collinear factorisation in QCD at leading twist
- First phenomenological studies within a TMD scheme: Anselmino, Boer, D'Alesio, Murgia (2001)

New Datas in e^+e^- from Belle: 150 data points

Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

We report the first observation of the spontaneous polarization of Λ and $\bar{\Lambda}$ hyperons transverse to the production plane in e^+e^- annihilation, which is attributed to the effect arising from a polarizing fragmentation function. For inclusive $\Lambda/\bar{\Lambda}$ production, we also report results with subtracted feed-down contributions from Σ^0 and charm. This measurement uses a dataset of 800.4 fb^{-1} collected by the Belle experiment at or near a center-of-mass energy of 10.58 GeV. We observe a significant polarization that rises with the fractional energy carried by the $\Lambda/\bar{\Lambda}$ hyperon.

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Benefits:

- No PDFs
- Cleaner process
- TMD Factorization proven

Contents

- TMDs FF with Helicity Formalism
- $e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$
- $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- Phenomenolgy
- Preliminary Fits

Helicity Formalism: TMD Fragmentation Functions for quarks

Quark Polarisation States determine the Hadron ones

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{k}_{\perp h}) = \sum_{\lambda_q,\lambda'_q} \rho^{q,s_q}_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{k}_{\perp h})$$

$$P^h_j\,\hat{D}_{h/q,s_q} = \hat{D}^h_{S_j/q,s_q} - \hat{D}^h_{-S_j/q,s_q} = \Delta\hat{D}^h_{S_j/s_q}.$$

Helicity density matrix

$$\rho^{i,s_i}_{\lambda_i,\lambda'_i} = \frac{1}{2}\begin{pmatrix} 1+P^i_z & P^i_x - iP^i_y \\ P^i_x + iP^i_y & 1-P^i_z \end{pmatrix}$$

8 independent Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta\hat{D}^h_{S_Y/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

Amsterdam notation:

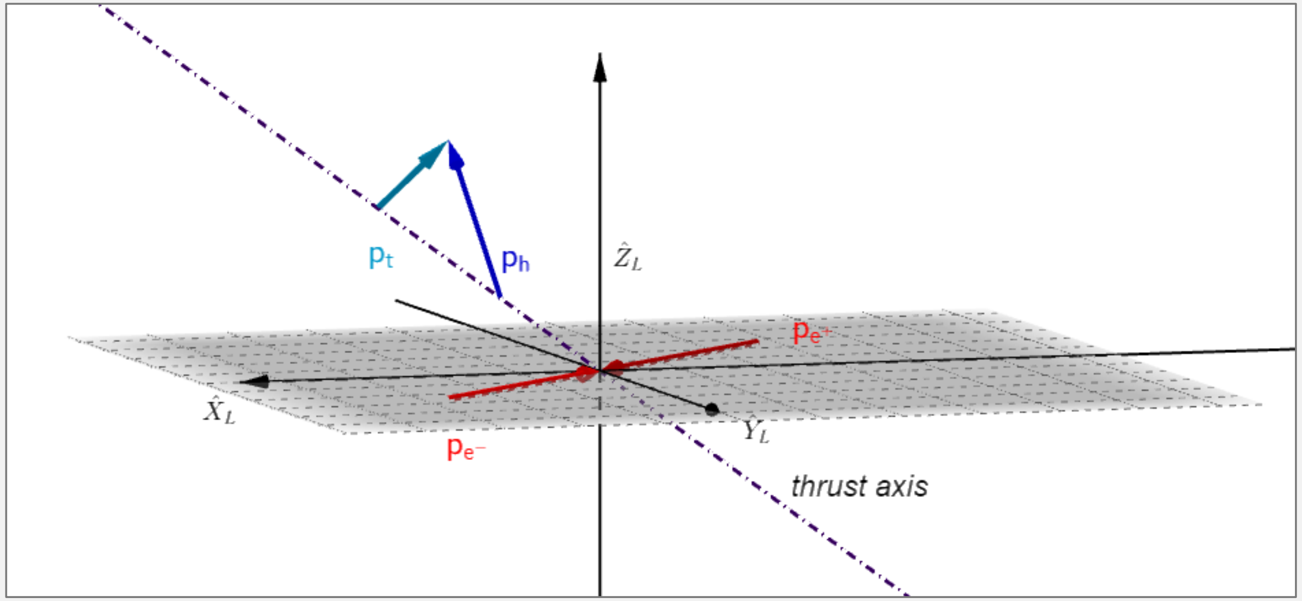
Collins $\Delta^N D_{h/q^\uparrow} \longleftrightarrow H_1^\perp$

Polarizing $\Delta D^h_{S_Y/q} \longleftrightarrow D_{1T}^\perp$

$$\begin{cases}\Delta^- D^{h_1}_{S_Y/s_T} \\ \Delta D^{h_1}_{S_X/s_T}\end{cases} \leftrightarrow \begin{cases}H_1 \\ H_{1T}^\perp\end{cases}$$

Polarisation: $e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$

$$\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}} \frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c}\sum_{\{\lambda\}} \frac{1}{32\pi s}\frac{1}{4}\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,p_{\perp 1})$$



Polarisation given along the hadron helicity axes:

$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

Polarisation measured along: $\hat{n} = \hat{q}_1 \times \hat{P}_{h1}$

It coincides with: $\hat{n} = \hat{Y}_{h1}$

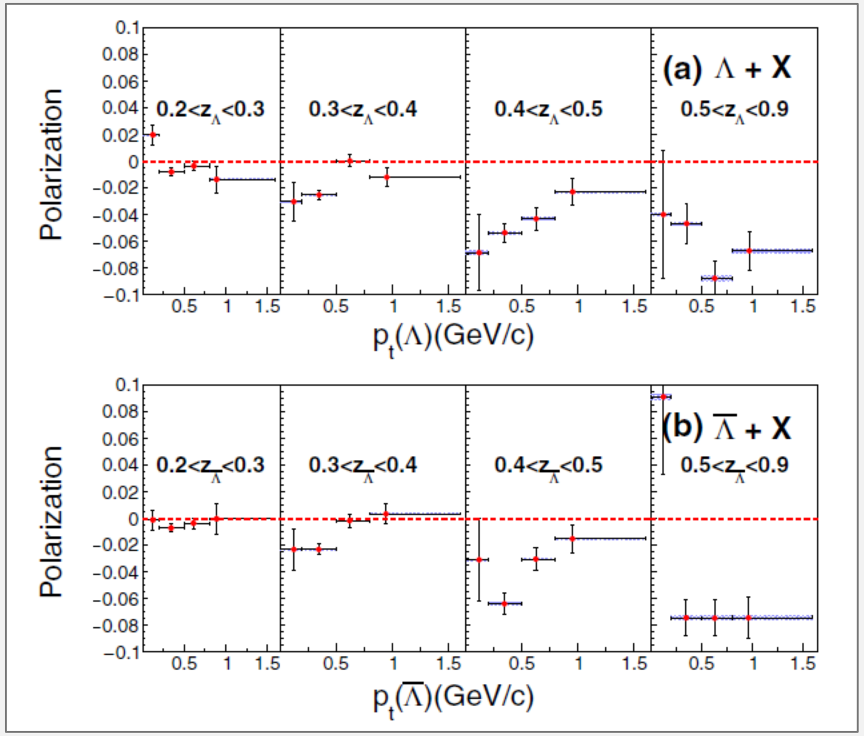
Polarisation: $e^+e^- \to h_1(\text{jet})X$

Only two possibile cross sections

$$\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\hat{D}_{h/q}(z_1, p_{1\perp h_1})$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1, p_{1\perp h_1})$$

$$P_Y^{h_1} = \frac{\sum_q e_q^2 \Delta D^{h_1}_{S_Y/q}(z_1, p_{\perp h_1})}{\sum_q e_q^2 D_{h/q}(z_1, p_{\perp h_1})}$$



It is possible to extract directly

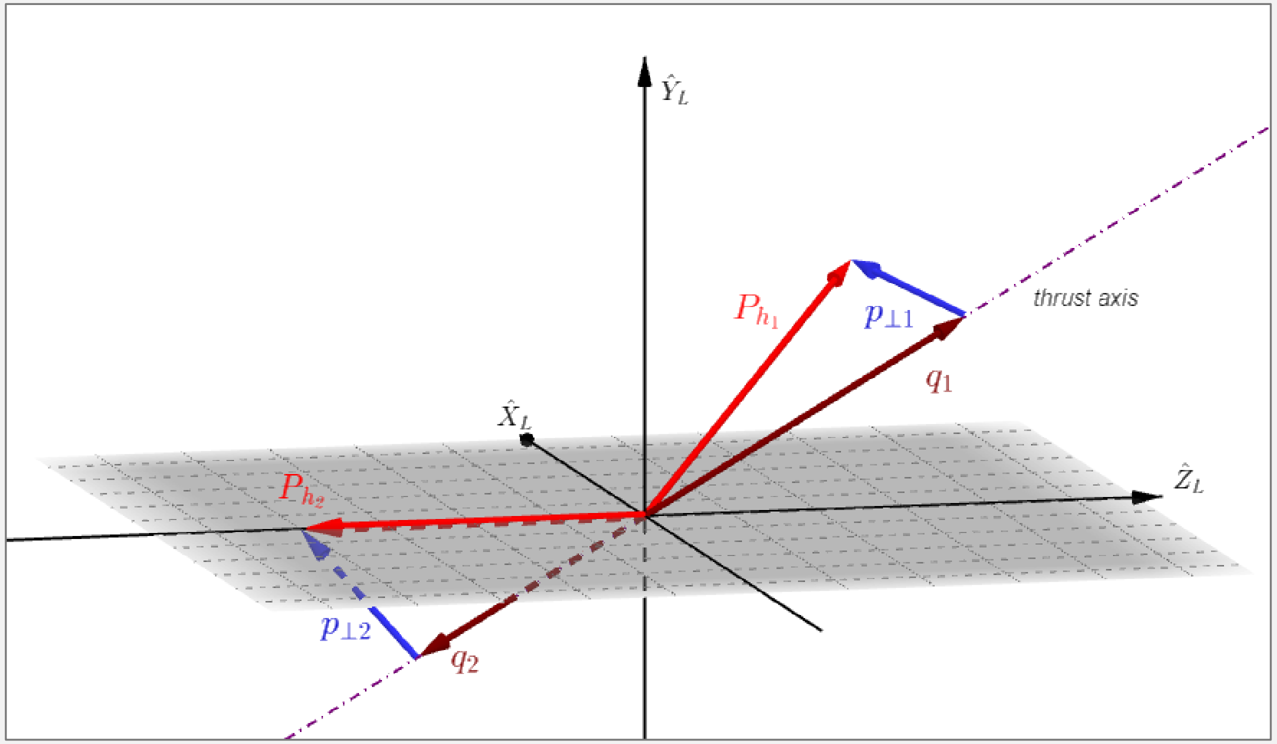
Lambda Polarising FF

$$\Delta\hat{D}^h_{S_Y/q}$$

Polarisation: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$

$$\rho^{h_1,S_{h_1}}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2,S_{h_2}}_{\lambda_{h_2},\lambda'_{h_2}}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp h_1} dz_2 d^2\mathbf{p}_{\perp h_2}}$$

$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{3}{32\pi s}\frac{1}{4}M_{\lambda_c\lambda_d,\lambda_a\lambda_b}M^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp h_1})\hat{O}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp h_2})$$



Polarisation given along the hadron helicity axes:

$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

$$\mathcal{P}^{h_1} = P_x^{h_1}\hat{X}_1 + P_y^{h_1}\hat{Y}_1 + P_z^{h_1}\hat{Z}_1$$

Kinematics

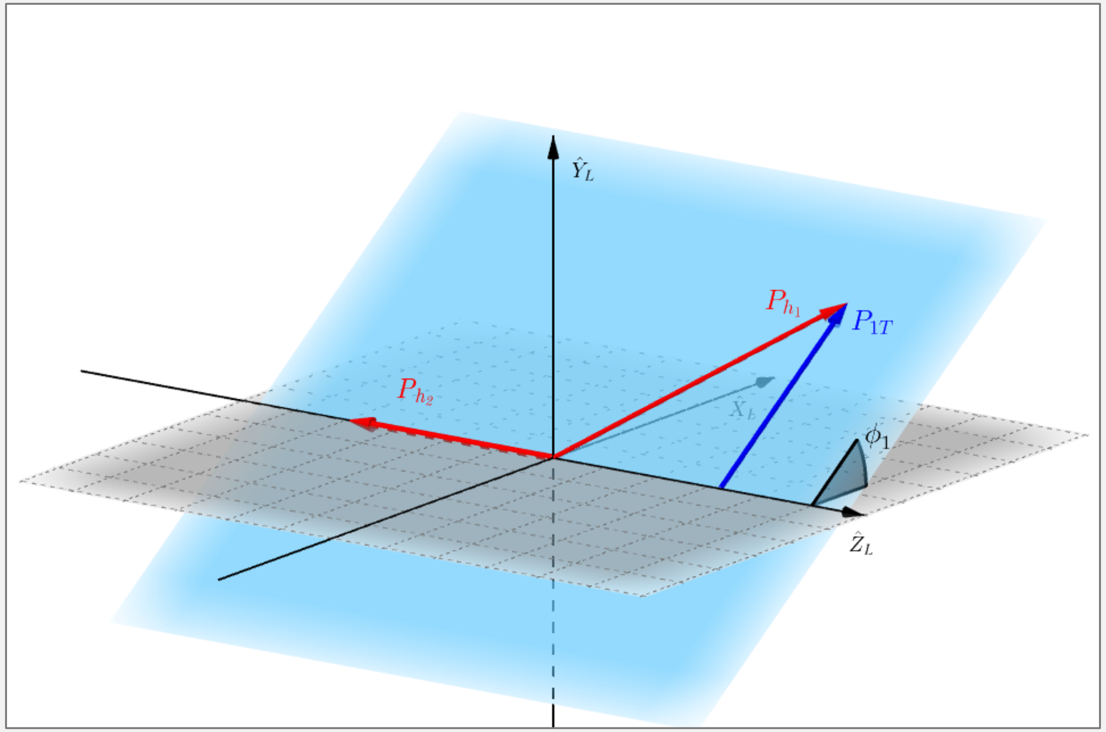
$$P_1 = z_{p_1}q_1 + p_{\perp 1}$$

$$P_2 = z_{p_2}q_2 + p_{\perp 2}$$

The polarisation is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

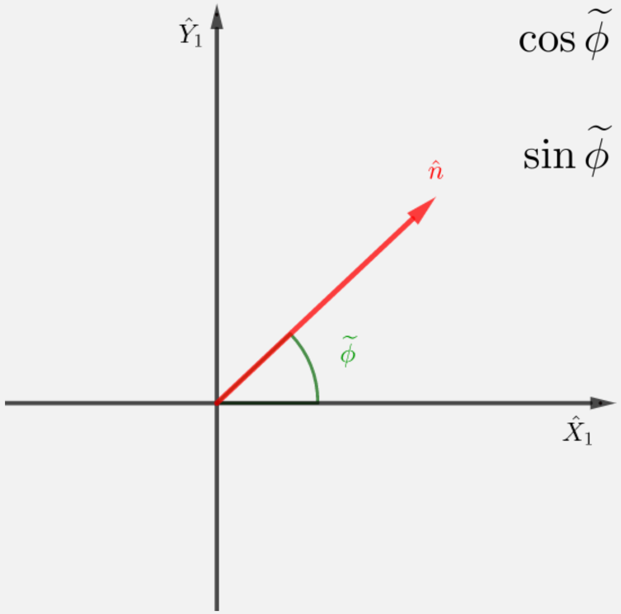
Exp. Datas depend only on energy fraction z_1, z_2



$$\begin{cases} P_1 = \left(P_{1T}\cos\phi_1, P_{1T}\sin\phi_1, P_{1L}\right) \\ p_{\perp 1}^2 \simeq P_{1T}^2 + \left(\frac{z_1}{z_2}\right)^2 p_{\perp 2}^2 - 2\frac{z_1}{z_2}P_{1T}p_{\perp 2}\cos(\phi_1 - \varphi_2) \end{cases}$$

The polarisation projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1}\cos\widetilde{\phi} + P_y^{h_1}\sin\widetilde{\phi}$$



$$\cos\widetilde{\phi} \simeq \frac{z_{h_1}}{z_{h_2}}\frac{p_{\perp 2}}{p_{\perp 1}}\sin(\phi_1 - \varphi_2)$$

$$\sin\widetilde{\phi} \simeq \frac{P_{1T}}{p_{\perp 1}} - \frac{z_{h_1}}{z_{h_2}}\frac{p_{\perp 2}}{p_{\perp 1}}\cos(\phi_1 - \varphi_2)$$

Polarisation: $e^+e^- \to h^\uparrow{}_1 h_2 X$

$d\sigma^{unpol}$

$$D_{h_1/q} D_{h_2/\bar{q}}$$

$$\Delta^N D_{h_1/q^\uparrow} \Delta^N D_{h_2/\bar{q}^\uparrow} \cos(2\varphi_2 + \phi_1^{h_1})$$

$d\sigma^{\uparrow_Y} - d\sigma^{\downarrow_Y}$

$$\Delta D^{h_1}_{S_Y/q} D_{h_2/\bar{q}}$$

$$\Delta^- D^{h_1}_{S_Y/s_T} \Delta^N D_{h_2/\bar{q}^\uparrow} \cos(2\varphi_2 + \phi_1^{h_1})$$

$d\sigma^{\uparrow_X} - d\sigma^{\downarrow_X}$

$$\Delta D^{h_1}_{S_X/s_T} \Delta^N D_{h_2/\bar{q}^\uparrow} \sin(2\varphi_2 + \phi_1^{h_1})$$

In order to obtain the polarisation we firstly calculate the polarisation along the Y and X helicity axes:

$$\langle P_Y^{h_1} \rangle = \frac{\langle d\sigma^{\uparrow_Y} - d\sigma^{\downarrow_Y} \rangle}{\langle d\sigma \rangle}$$

$$\langle P_X^{h_1} \rangle = \frac{\langle d\sigma^{\uparrow_X} - d\sigma^{\downarrow_X} \rangle}{\langle d\sigma \rangle}$$

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp h_1} dz_2 d^2\mathbf{p}_{\perp h_2}}$$

$$d^2 p_{\perp 1} \longrightarrow dP_{1T} d\phi_1$$

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 dP_{1T} d\phi_1 dz_2 d^2\mathbf{p}_{\perp h_2}}$$

We use for the FF the following parametrisations:

$$\Delta D^h_{S_Y/q}(z,p_\perp) \quad = \Delta D^h_{S_Y/q}(z)\sqrt{2e}\,\frac{p_\perp}{M_p}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_p}}{\pi\langle p_\perp^2\rangle_h}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

$$\frac{1}{\langle p_\perp^2\rangle_p} = \frac{1}{M_p^2} + \frac{1}{\langle p_\perp^2\rangle_h}$$

$$\int dP_{1T} d\phi_1 dp_{\perp 2} d\varphi_2$$

We find an expression for the polarisation that depends only on the energy fractions z_1 *and* z_2

$$P^{h_1}\cdot\hat{n} :: \quad \frac{\sum_q \Delta D^h_{S_Y/q}(z) D_{h_2/\bar{q}}(z_2)}{\sum_q D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

$$\times \quad \frac{\sqrt{2e\pi}}{2M_p}\,\frac{\langle p_\perp^2\rangle_p^2}{\langle p_\perp^2\rangle_1}\,\frac{z_2}{\sqrt{z_1^2\langle p_\perp^2\rangle_2 + z_2^2\langle p_\perp^2\rangle_p}}$$

Phenomenology

From datas we can extract different information, particularly:

- $\Lambda(jet)X$: Lambda polarising width $\langle p_\perp^2 \rangle_p$
- $\Lambda\pi X$: Polarising FF (u,d)
- $\Lambda k X$: Polarising FF (u,s)

Fitted parameters:

Flav.	$\mathcal{N}_q^p$	α_q	β_q	$\langle p_\perp^2 \rangle_p$
u	$\mathcal{N}_u^p$			
d	$\mathcal{N}_d^p$			
s	$\mathcal{N}_s^p$	α_s		$\langle p_\perp^2 \rangle_p$
sea	$\mathcal{N}_{sea}^p$		β_{sea}	

Polarising parametrization:

$$\Delta D_{S_Y/q}^h(z) = \mathcal{N}_q^p(z) D_{h/q}(z)$$

$$\mathcal{N}_q^p(z) = \mathcal{N}_q^p z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

Since we did not have a set of FF that separates the Λ and $\bar{\Lambda}$:

$$D_q^\Lambda = D_q^{\Lambda^0} + D_q^{\bar{\Lambda}^0} \longrightarrow D_q^\Lambda = D_q^{\Lambda^0} + D_{\bar{q}}^{\Lambda^0}$$

$$D_{\bar{q}}^{\Lambda^0} = (1-z)^\alpha D_q^{\Lambda^0}$$

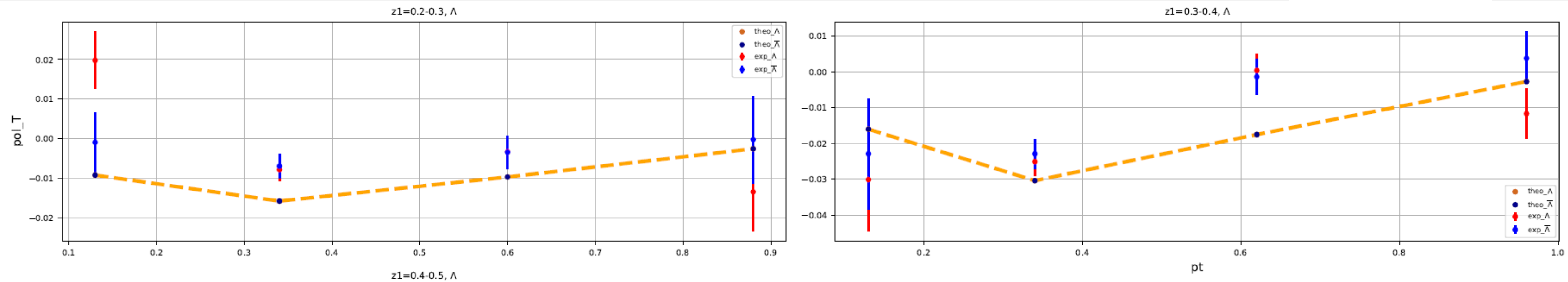
$$D_q^{\Lambda^0} = \frac{1}{1+(1-z)^\alpha} D_q^\Lambda \qquad \alpha = 1, 2$$

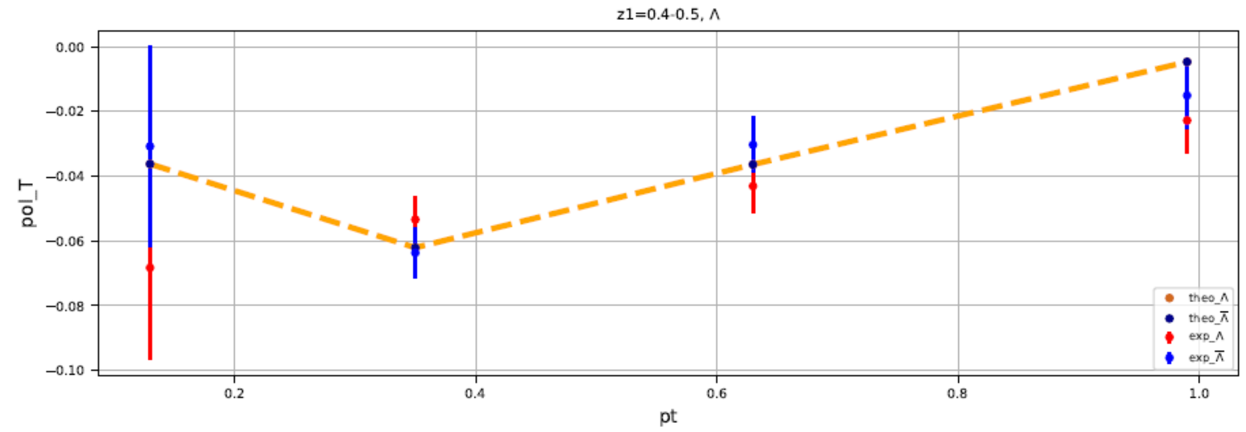
$$D_{\bar{q}}^{\Lambda^0} = \frac{(1-z)^\alpha}{1+(1-z)^\alpha} D_q^\Lambda$$

Global Fit

Polarisation: $\Lambda(jet)$

$\chi^2/dof = 2.5683$





Global Fit	
Nu	0.21 ± 0.01
Nd	-0.44 ± 0.03
Ns	-0.37 ± 0.08
Nsea	-0.26 ± 0.03
α	1.7 ± 0.3
β	2.4 ± 0.4
$p_\perp$	0.11 ± 0.06

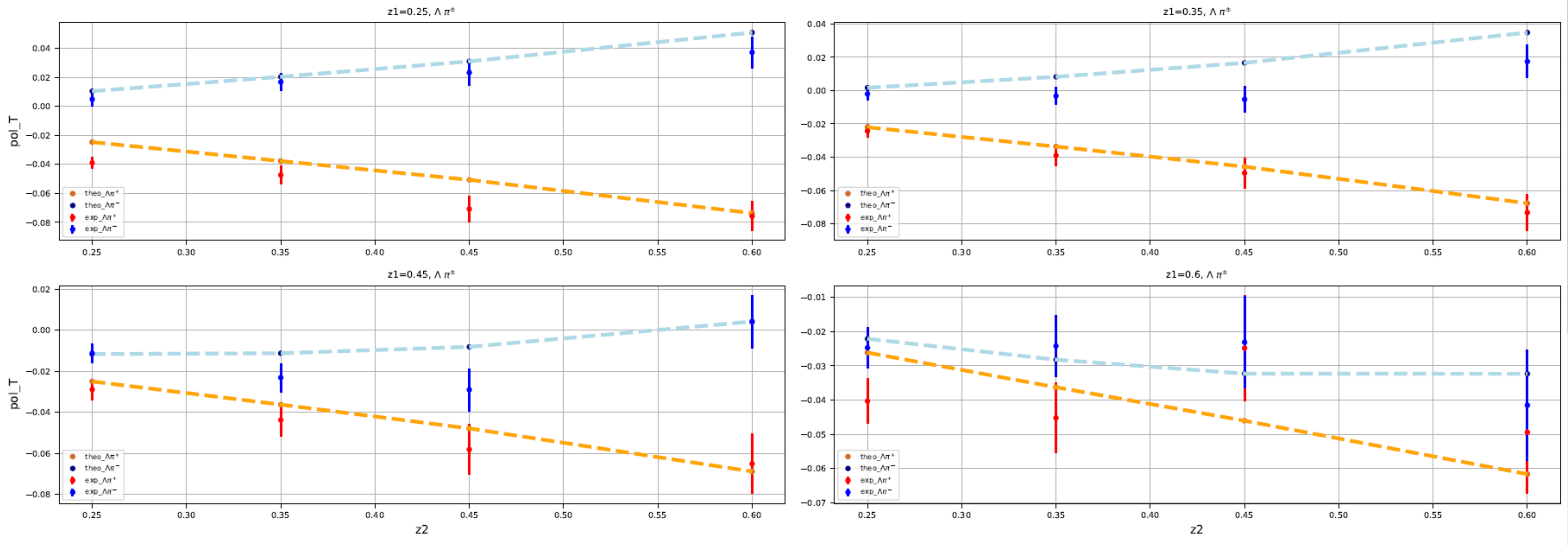
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Global Fit

Polarisation: $\Lambda\pi$

$\chi^2/dof = 2.5683$



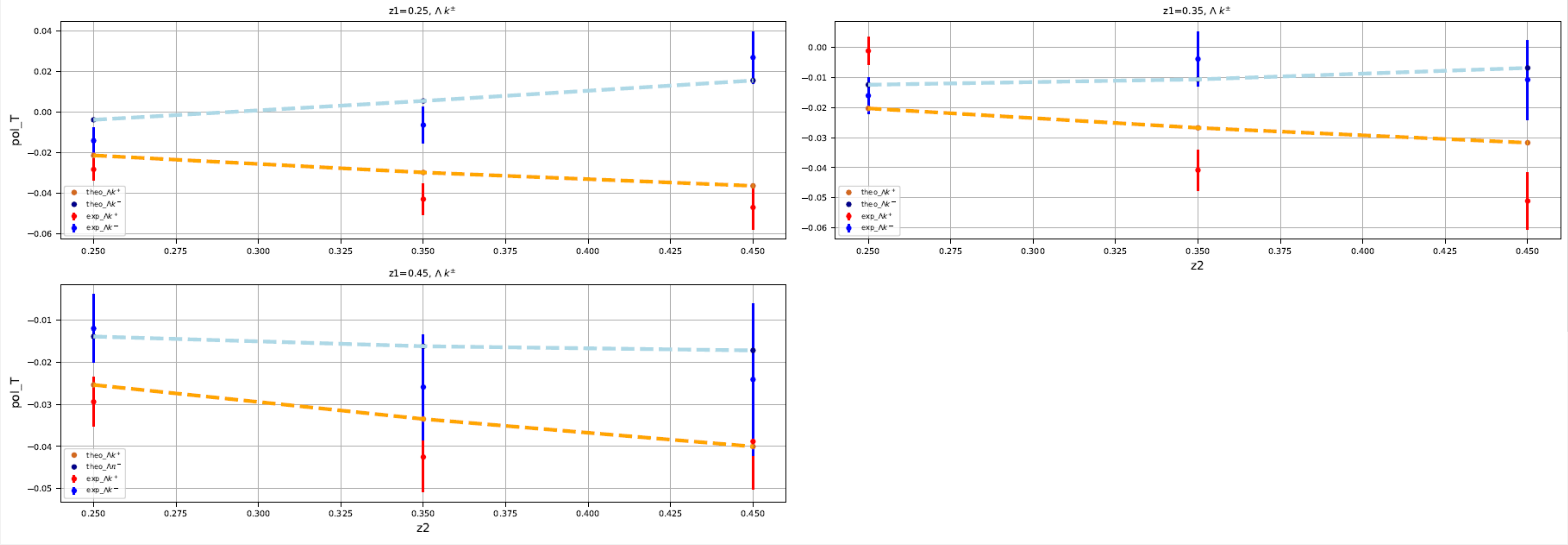
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Global Fit

Polarisation: Λk

$\chi^2/dof = 2.5683$



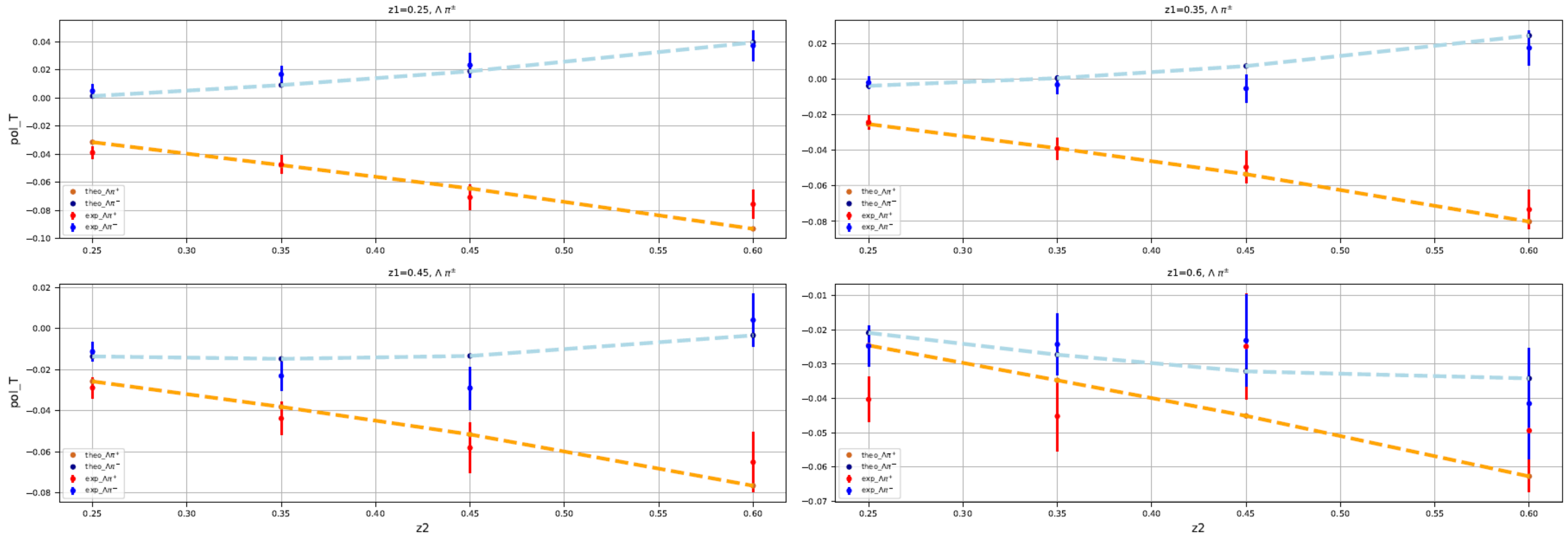
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Fit: associated production only

Polarisation: $\Lambda\pi$

$\chi^2/dof = 1.499$



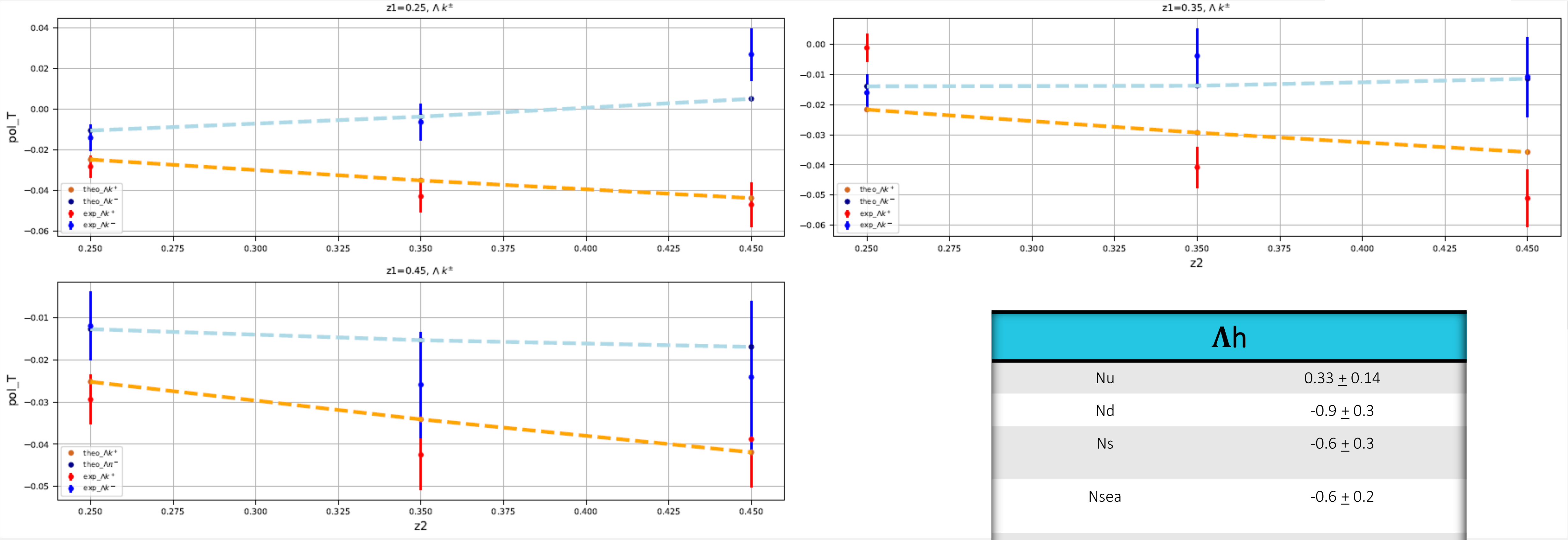
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Fit: associated production only

Polarisation: Λk

$\chi^2/dof = 1.499$



Λh	
Nu	0.33 ± 0.14
Nd	-0.9 ± 0.3
Ns	-0.6 ± 0.3
Nsea	-0.6 ± 0.2
α	1.5 ± 0.3
β	2.7 ± 0.6
$p_\perp$	0.06 ± 0.02

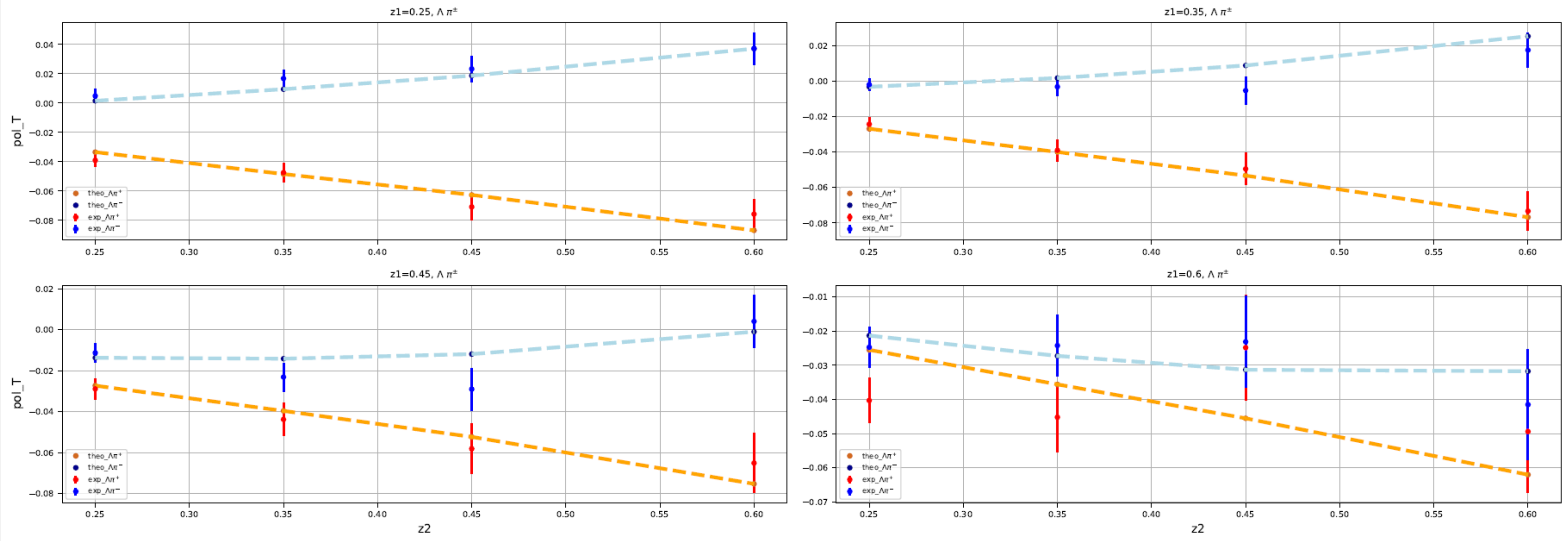
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Fit: associated production only (fixed gaussian width)

Polarization: $\Lambda\pi$

$\chi^2/dof = 1.5178$



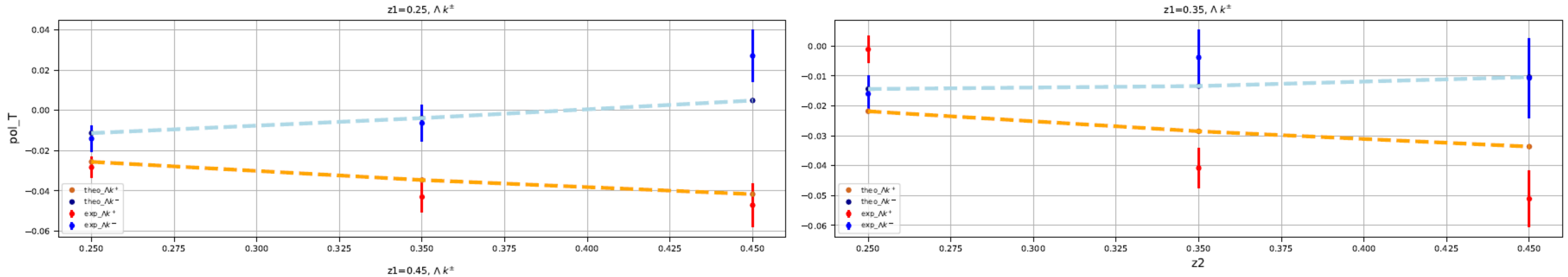
Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

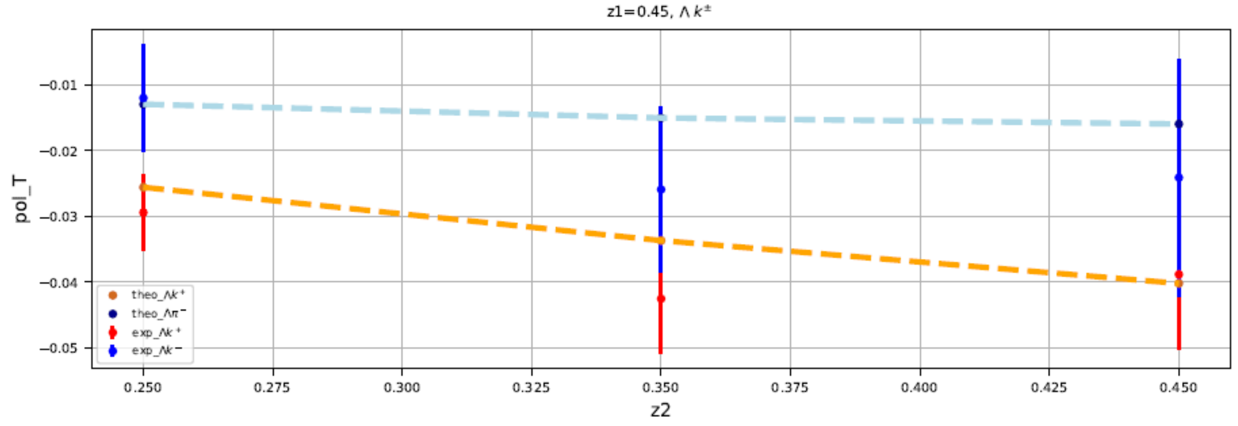
Preliminary

Fit: associated production only (fixed gaussian width)

Polarisation: Λk

$\chi^2/dof = 1.5178$





Λh (fixed gaussian width)	
Nu	0.18 ± 0.01
Nd	-0.49 ± 0.03
Ns	-0.31 ± 0.07
Nsea	-0.34 ± 0.06
α	1.5 ± 0.3
β	2.9 ± 0.5
$p_\perp$	0.11

Datas cut for $z > 0.5$: $\Lambda(jet), \Lambda k$

Preliminary

Conclusions

- Global fit : $\chi^2/dof = 2.5683$
- Associated fit with no Lambda jet: $\chi^2/dof = 1.499$
- Associated fit with no Lambda jet, fixing width: $\chi^2/dof = 1.5178$

Lead to different width $\langle p_\perp^2 \rangle_p$

What's next?

- Different choices of the Gaussian width for the unpolarised and/or the polarising FF: z dependence, flavour dependence.
- Functional form of the polarising FF.
- Predictions for proton-proton and comparison with existing data and previous extractions

Grazie

Marco Zaccheddu

Marco.zaccheddu@ca.infn.it

Università degli Studi di Cagliari

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$
$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{D_{h_1/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$
$$+\frac{1}{4}\sin^2\theta\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$
$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2})\Big(1+\cos^2\theta\Big)$$
$$+\frac{1}{2}\sin^2\theta\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$
$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_X/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2 p_{\perp 2}}$$
$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_Z/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

TMD Formalism and Azimuthal Asymmetries

Generalization of PDF and FF including a Transverse Momentum Dependence.

Correlation Azimuthal distribution - Spin

PDF	$\Delta^N f_{q/p^\uparrow}$	Sivers 1991
FF	$\Delta^N D_{h/q^\uparrow}$	Collins 1993

Spin ½ Hadron:

Collinear pQCD → 3 PDF, 3 FF

TMD Formalism (Correlation Spin - $k_\perp$) → 8 TMD- PDF, 8 TMD- FF

Spin and TMD Effects

Single spin Asymmetry (SSA)

$$a^{\uparrow}b \to cd$$

$$a_N = \frac{d\sigma^{a^{\uparrow}b\to cd} - d\sigma^{a^{\downarrow}b\to cd}}{d\sigma^{a^{\uparrow}b\to cd} + d\sigma^{a^{\downarrow}b\to cd}}$$

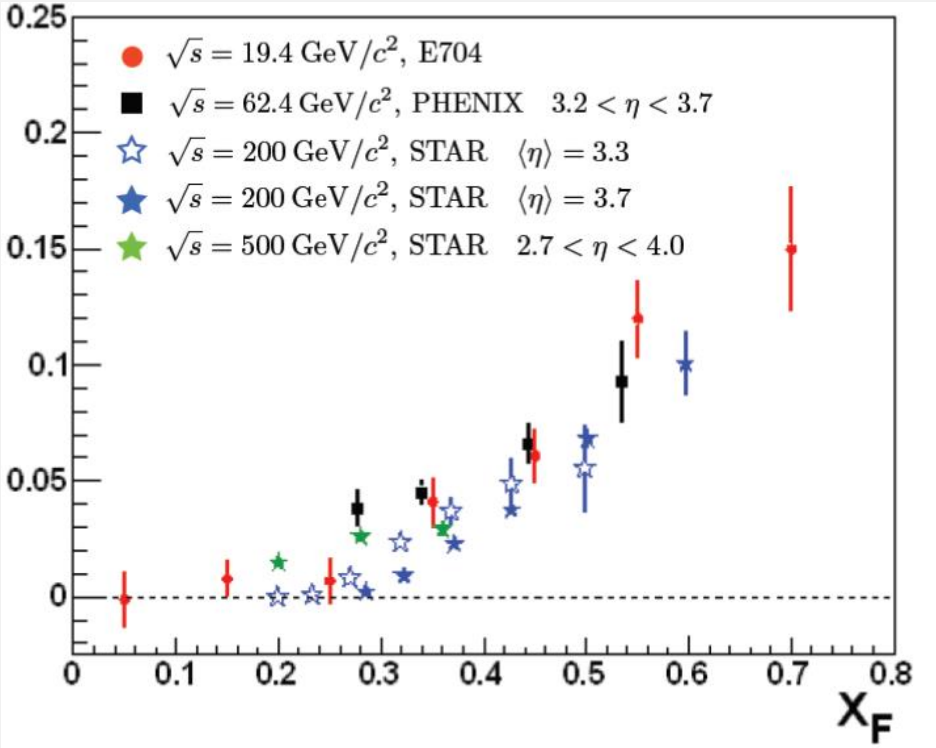
$$a_N \propto \alpha_s \frac{m}{\sqrt{s}} \simeq \alpha_s \frac{m}{p_\perp}$$ [Kane, Pumplin, Repko 1978]

Collinear pQCD

$$p^{\uparrow}p \to \pi X$$

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$$A_N \leq 1-2\%$$



Spin and transverse momentum effects were considered negligible.

But Experimental Data show $A_N \simeq 20\%$

Cross Sections and Polarization States for $e^+e^- \to h^\uparrow{}_1 h_2 X$

$$\rho^{h_1,S_{h_1}}_{\lambda_{h_1},\lambda'_{h_1}} \rho^{h_2,S_{h_2}}_{\lambda_{h_2},\lambda'_{h_2}} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta\, dz_1 d^2\mathbf{k}_{\perp h_1} dz_2 d^2\mathbf{k}_{\perp h_2}}$$

$$= \sum_{q_c} \sum_{\{\lambda\}} \frac{3}{32\pi s} \frac{1}{4} M_{\lambda_c\lambda_d,\lambda_a\lambda_b} M^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b} \hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{k}_{\perp h_1}) \hat{O}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{k}_{\perp h_2})$$

$d\sigma^{nonpol.}$

Cross Sections and Polarization States for $e^+e^- \to h^\uparrow{}_1 h_2 X$

$$\Delta^N D_{h_1/q^\uparrow} \Delta^N D_{h_2/\bar{q}^\uparrow} \cos(\varphi_{h_1} + \varphi_{h_2})$$

$d\sigma^{\uparrow_Y} - d\sigma^{\downarrow_Y}$

$$\Delta D^{h_1}_{S_Y/q} D_{h_2/\bar{q}}$$

$$\Delta^- D^{h_1/q}_{S_Y/s_T} \Delta^N D_{h_2/\bar{q}^\uparrow} \cos(\varphi_{h_1} + \varphi_{h_2})$$

$$\langle P^{h_1}_Y \rangle = \frac{\langle d\sigma^{\uparrow_Y} - d\sigma^{\downarrow_Y} \rangle}{\langle d\sigma \rangle}$$

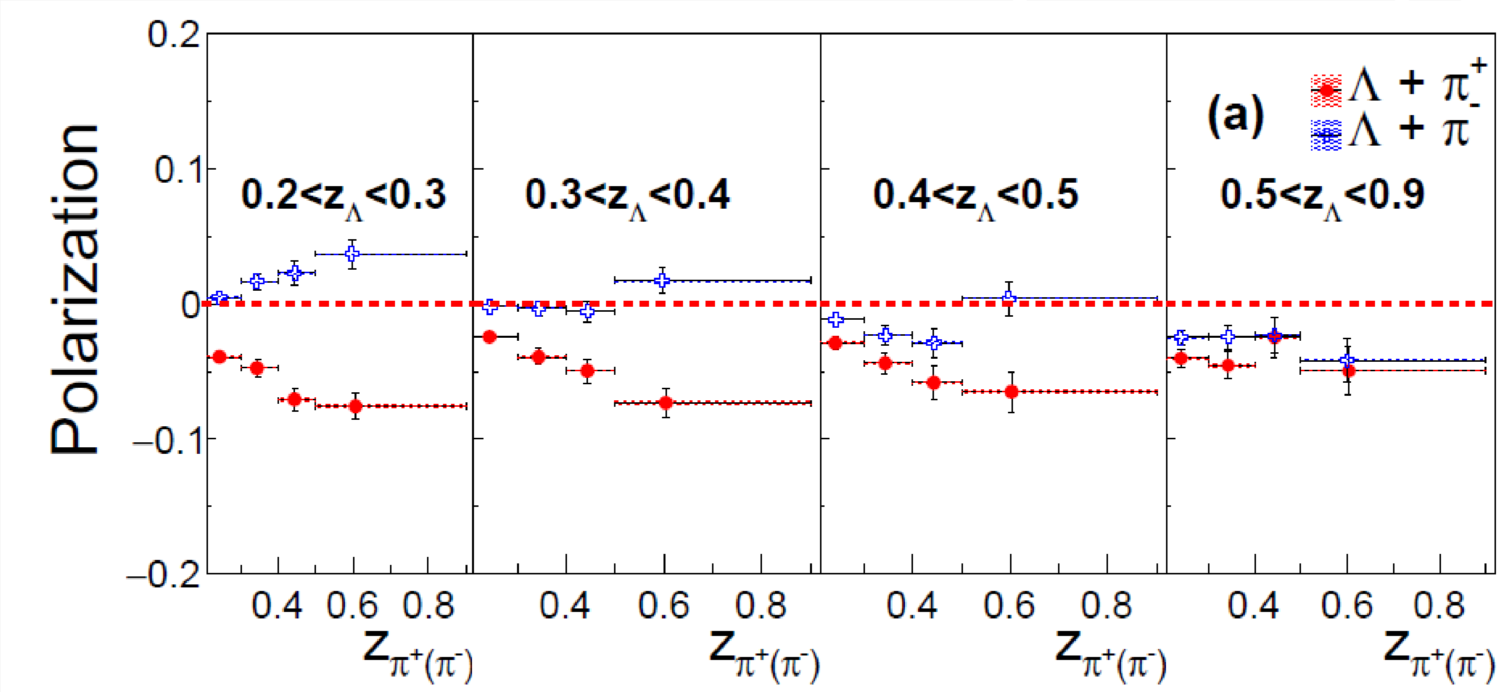
$$\Delta \hat{D}^h_{S_Y/q}$$

Polarizing FF

Observation of Transverse Λ Hyperon Polarization in e^+e^- Annihilation at Belle (2018)

Measurement for Λ polarization in:

- $e^+e^- \to \Lambda\,(\text{jet})X$
- $e^+e^- \to \Lambda\,\pi X$



Benefits:

- TMD factorization is valid;
- NO PDF
- Extraction Polarizing FF

Transverse Polarization predicted in the Helicity Frame of the Hadron

$$P_Y^{\Lambda}$$

Measured along:

$$\hat{n} = \frac{\vec{P}_\Lambda \times \vec{P}_\pi}{|\vec{P}_\Lambda \times \vec{P}_\pi|}$$

FF properties

$$\hat{D}_{h/q}(z,\mathbf{k}_{\perp,h}) = D_{h/q} = (D^{++}_{++} + D^{++}_{--})$$
$$\hat{D}_{h/q,s_T}(z,\mathbf{k}_{\perp,h}) = \hat{D}_{h/q} + \frac{1}{2}\Delta\hat{D}_{h/q,s_T}$$
$$\Delta\hat{D}_{h/q,s_T}(z,\mathbf{k}_{\perp,h}) = \Delta^N D_{h/q^\uparrow}\sin(\phi_{s_q}-\phi_h) = 4ImD^{++}_{+-}\sin(\phi_{s_q}-\phi_h) \quad [Collins]$$
$$\Delta\hat{D}^{h/q}_{S_Z/s_L}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_Z/s_L} = (D^{++}_{++} - D^{++}_{--})$$
$$\Delta\hat{D}^{h/q}_{S_Z/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_Z/s_T}\cos(\phi_{s_q}-\phi_h) = 2ReD^{++}_{+-}\cos(\phi_{s_q}-\phi_h)$$
$$\Delta\hat{D}^{h/q}_{S_X/s_L}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_X/s_L} = 2ReD^{+-}_{++}$$
$$\Delta\hat{D}^{h/q}_{S_X/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h/q}_{S_X/s_T}\cos(\phi_{s_q}-\phi_h) = (D^{+-}_{+-} + D^{+-}_{-+})\cos(\phi_{s_q}-\phi_h)$$
$$\Delta\hat{D}^{h}_{S_Y/q}(z,\mathbf{k}_{\perp,h}) = \Delta D^{h}_{S_Y/q} = -2ImD^{+-}_{++} \quad [Polarizing]$$
$$\Delta\hat{D}^{h/q}_{S_Y/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta\hat{D}^{h/q}_{S_Y/c} + \Delta^-\hat{D}^{h/q}_{S_Y/s_T}$$
$$\Delta^-\hat{D}^{h/q}_{S_Y/s_T}(z,\mathbf{k}_{\perp,h}) = \Delta^- D^{h/q}_{S_Y/s_T}\sin(\phi_{s_q}-\phi_h) = (D^{+-}_{+-} - D^{+-}_{-+})\sin(\phi_{s_q}-\phi_h)$$

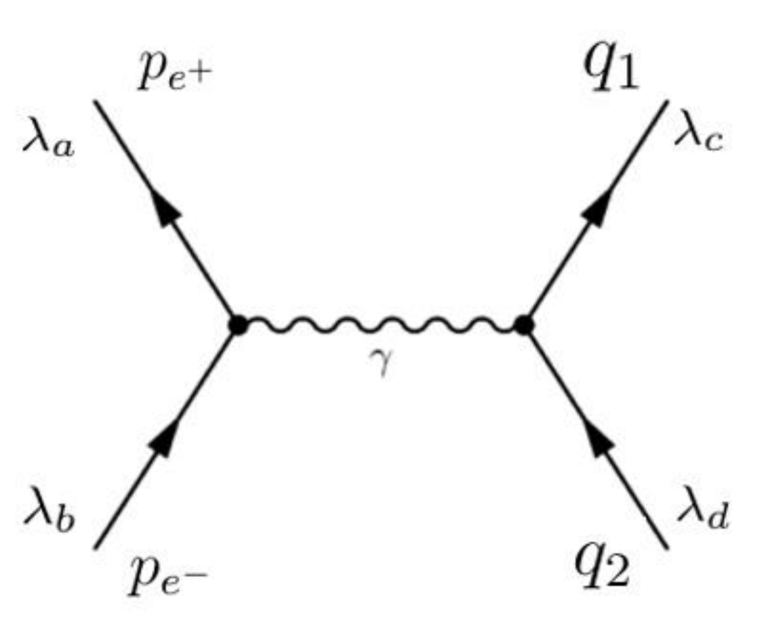
$$D^{++}_{++} = D^{--}_{--}$$
$$D^{++}_{--} = D^{--}_{++}$$
$$D^{++}_{+-} = -D^{--}_{-+}$$
$$D^{++}_{-+} = -D^{--}_{+-}$$
$$D^{+-}_{++} = -D^{-+}_{--}$$
$$D^{-+}_{++} = -D^{+-}_{--}$$
$$D^{+-}_{+-} = D^{-+}_{-+}$$
$$D^{+-}_{-+} = D^{-+}_{+-}$$
$$D^{++}_{+-} = (D^{++}_{-+})^*$$
$$D^{+-}_{++} = (D^{-+}_{++})^*$$

$$P_T \cdot \hat{n} = P^{h_1}_X \cos\widetilde{\phi} + P^{h_1}_Y \sin\widetilde{\phi}$$

Helicity Scattering Amplitudes

$$M^0_{\lambda_c\lambda_d,\lambda_a\lambda_b}$$

$$q_1 = E(1,0,0,1) \qquad p_{e^+} = E(1,\sin\theta,0,\cos\theta)$$
$$q_2 = E(1,0,0,-1) \qquad p_{e^-} = E(1,-\sin\theta,0,-\cos\theta).$$



$$M^0_{+-,+-} = M^0_{-+,-+} = M^0_2$$
$$M^0_{+-,-+} = M^0_{-+,+-} = M^0_3$$

$$M^0_2 = -ie^2 e_q(1-\cos\theta);$$
$$M^0_3 = -ie^2 e_q(1+\cos\theta);$$

Cross Sections

$$d\sigma^{unpol.}$$
$$=\frac{3\pi\alpha^2 e_q^2}{2s}\left\{(1+\cos^2\theta)D_{h_1/q}D_{h_2/\bar{q}}+\frac{1}{4}\sin^2\theta\,\Delta^N D_{h_1/q^\uparrow}\Delta^N D_{h_2/\bar{q}^\uparrow}\cos(\varphi_{h_1}+\varphi_{h_2})\right\}$$

$$d\sigma^{\uparrow_Y}-d\sigma^{\downarrow_Y}$$
$$=\frac{3\pi\alpha^2 e_q^2}{2s}\left\{(1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}D_{h_2/\bar{q}}+\frac{1}{2}\sin^2\theta\,\Delta^- D^{h_1/q}_{S_Y/s_T}\Delta^N D_{h_2/\bar{q}^\uparrow}\cos(\varphi_{h_1}+\varphi_{h_2})\right\}$$