Sar WorS 2019 - Sardinian Workshop on Spin studies

The impact of the errors of collinear functions in describing unintegrated SIDIS data

Andrea Simonelli

SIDIS in the LARGE q_T regime

Collinear Factorization:

$$\frac{d\sigma}{dx_{Bj}\,dy\,dz_h\,dP_T^2} = \left(\frac{\alpha_S}{\pi}\right)\sum_{i\,j}\int_{x_{Bj}}^{x_{MAX}}\frac{dx}{x}\int_{z_h}^{z_{MAX}}\frac{dz}{z}\times$$

$$\times\, f_i\left(\frac{x_{Bj}}{x},Q^2\right)\left[\frac{d\hat{\sigma}_{i\,j}}{dx\,dy\,dz\,dq_T^2}\,\delta\left(z^2\,Q^2\left(\frac{P_T^2}{z_h}-\frac{1-x}{x}\,\frac{1-z}{z}\right)\right)\right]D_j\left(\frac{z_h}{z},Q^2\right)$$

In general:

MEASURED → $\mathcal{O} = H \otimes \sum_i F_i$ ← **EXTRACTED** from experimental data

↑ **COMPUTED** at FO in perturbation theory

Depending on the α_s order the collinear functions will be labeled by: LO, NLO, NNLO...

M. Anselmino, M. Boglione, A. Prokudin, and C. Türk,
"Semi Inclusive Deep Inelastic Scattering processes from small to large P_T", **Eur.Phys.J. A31 (2007) 373-381**

SIDIS in the LARGE q_T regime

Collinear Factorization:

$$\frac{d\sigma}{dx_{Bj}\,dy\,dz_h\,dP_T^2} = \left(\frac{\alpha_S}{\pi}\right)\sum_{i\,j}\int_{x_{Bj}}^{x_{MAX}}\frac{dx}{x}\int_{z_h}^{z_{MAX}}\frac{dz}{z}\times$$

$$\times\, f_i\left(\frac{x_{Bj}}{x},Q^2\right)\left[\frac{d\widehat{\sigma}_{i\,j}}{dx\,dy\,dz\,dq_T^2}\,\delta\left(z^2\,Q^2\left(\frac{P_T^2}{z_h}-\frac{1-x}{x}\frac{1-z}{z}\right)\right)\right]D_j\left(\frac{z_h}{z},Q^2\right)$$

In general:

MEASURED → $\mathcal{O} = H \otimes \sum_i F_i$ ← **EXTRACTED** from experimental data

↑ **COMPUTED** at FO in perturbation theory

Depending on the α_s order the collinear functions will be labeled by: LO, **NLO**, NNLO...

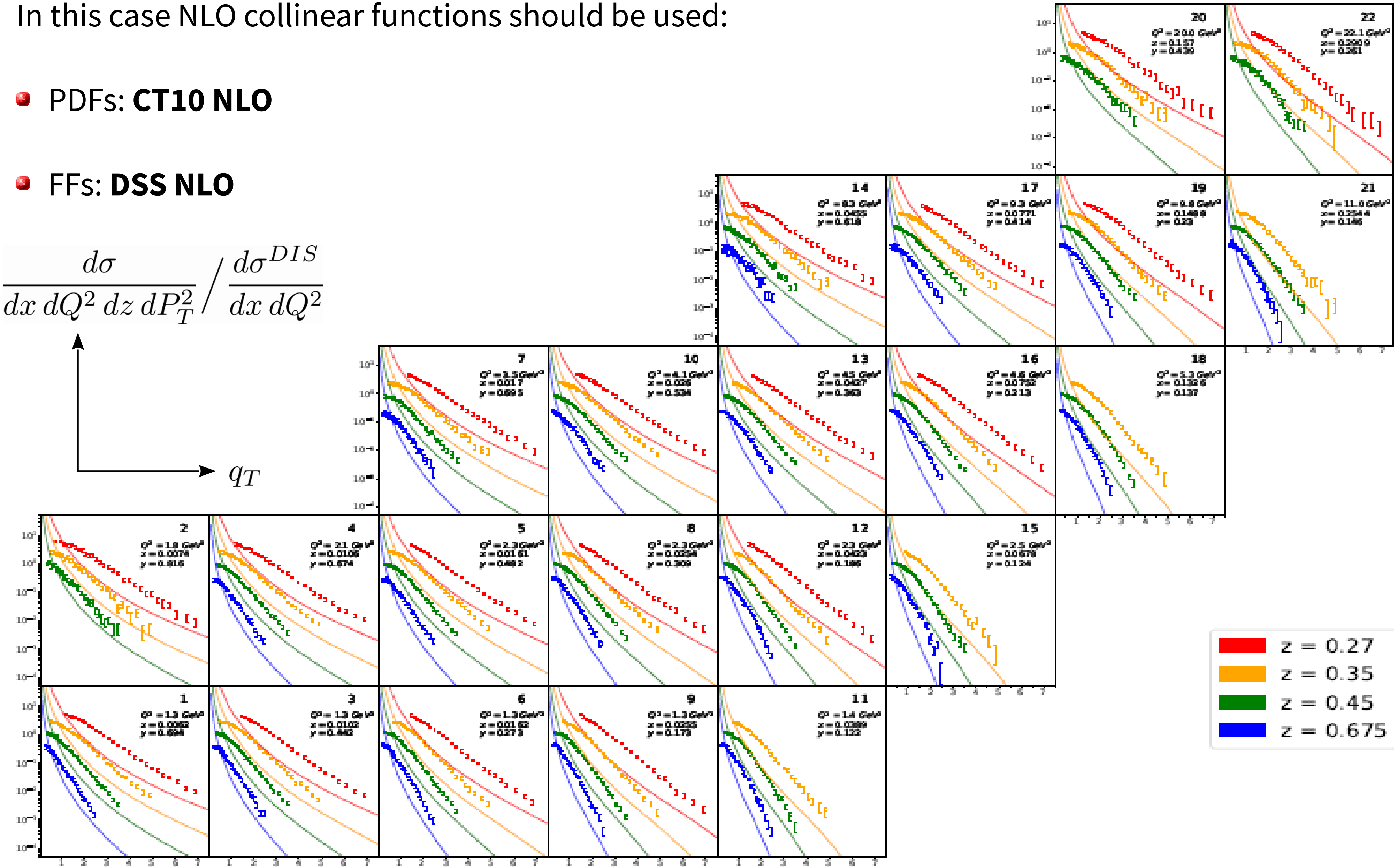
M. Anselmino, M. Boglione, A. Prokudin, and C. Türk,
“Semi Inclusive Deep Inelastic Scattering processes from small to large P_T”, **Eur.Phys.J. A31 (2007) 373-381**

Comparison with COMPASS data

In this case NLO collinear functions should be used:

- PDFs: **CT10 NLO**
- FFs: **DSS NLO**

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\,dQ^2}$$

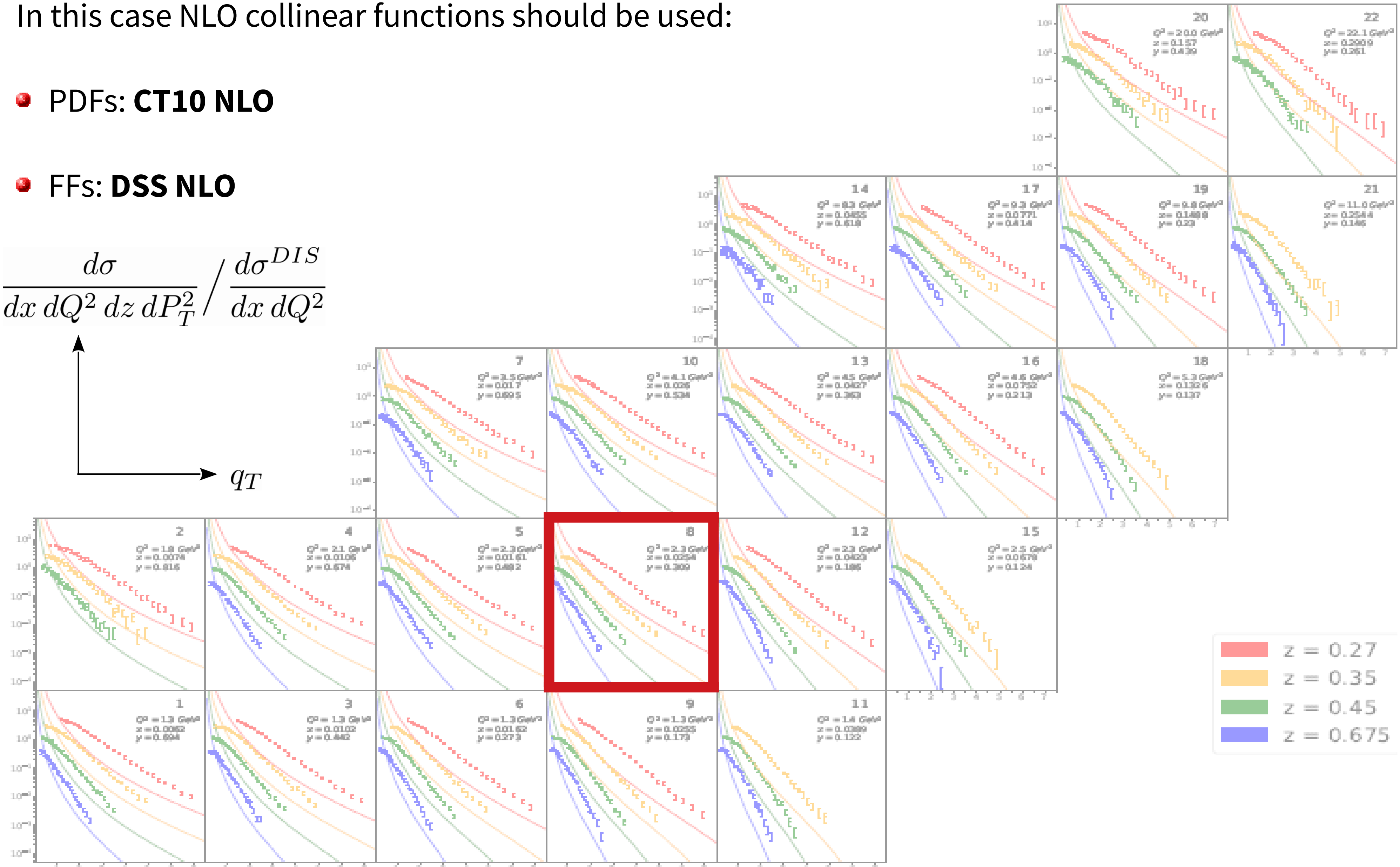


Comparison with COMPASS data

In this case NLO collinear functions should be used:

- PDFs: **CT10 NLO**
- FFs: **DSS NLO**

$$\frac{d\sigma}{dx\, dQ^2\, dz\, dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\, dQ^2}$$

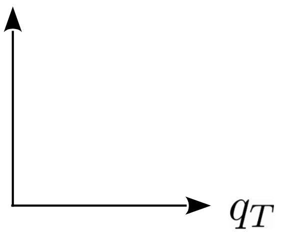


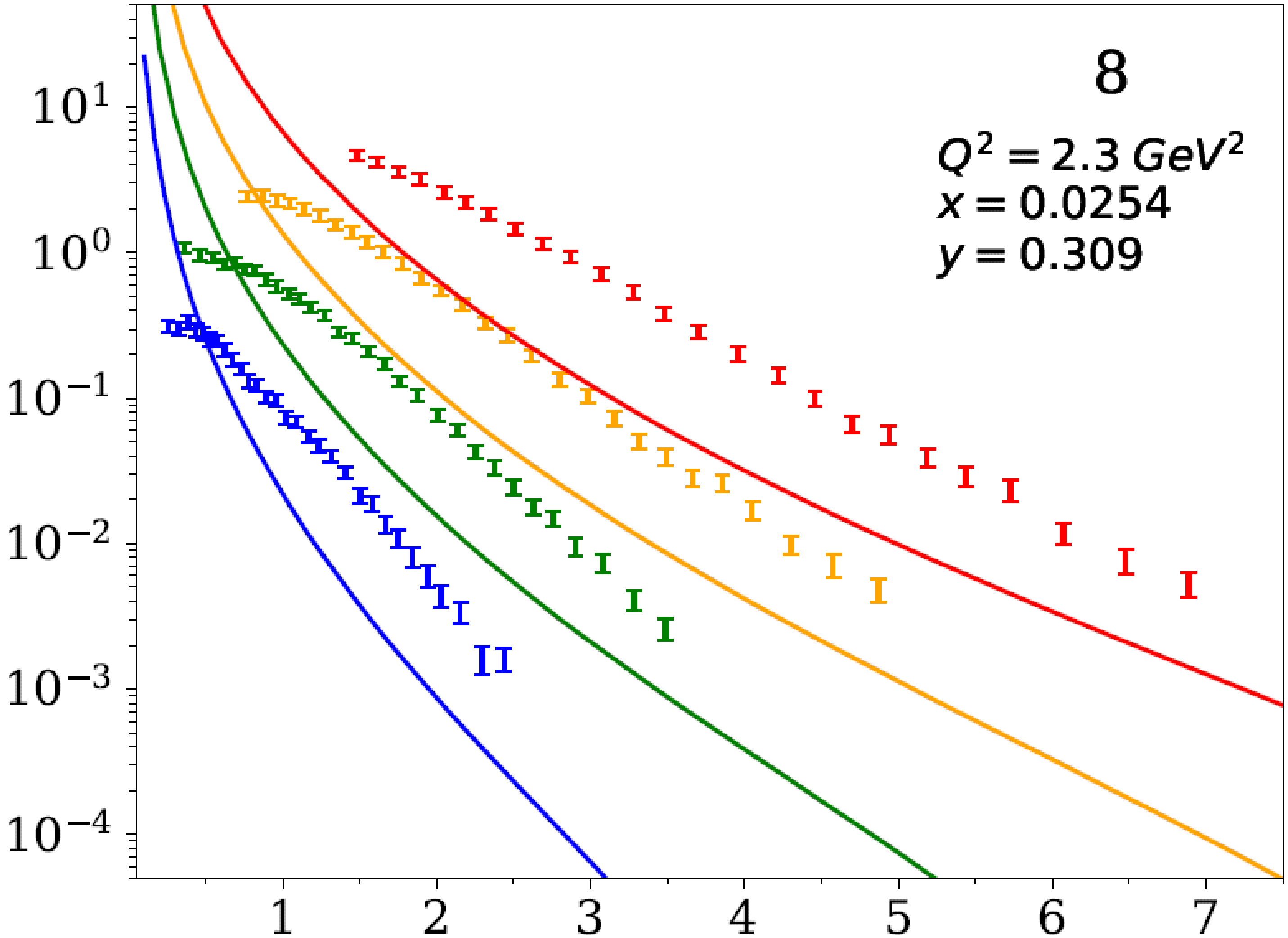
Comparison with COMPASS data

In this case NLO collinear functions should be used:

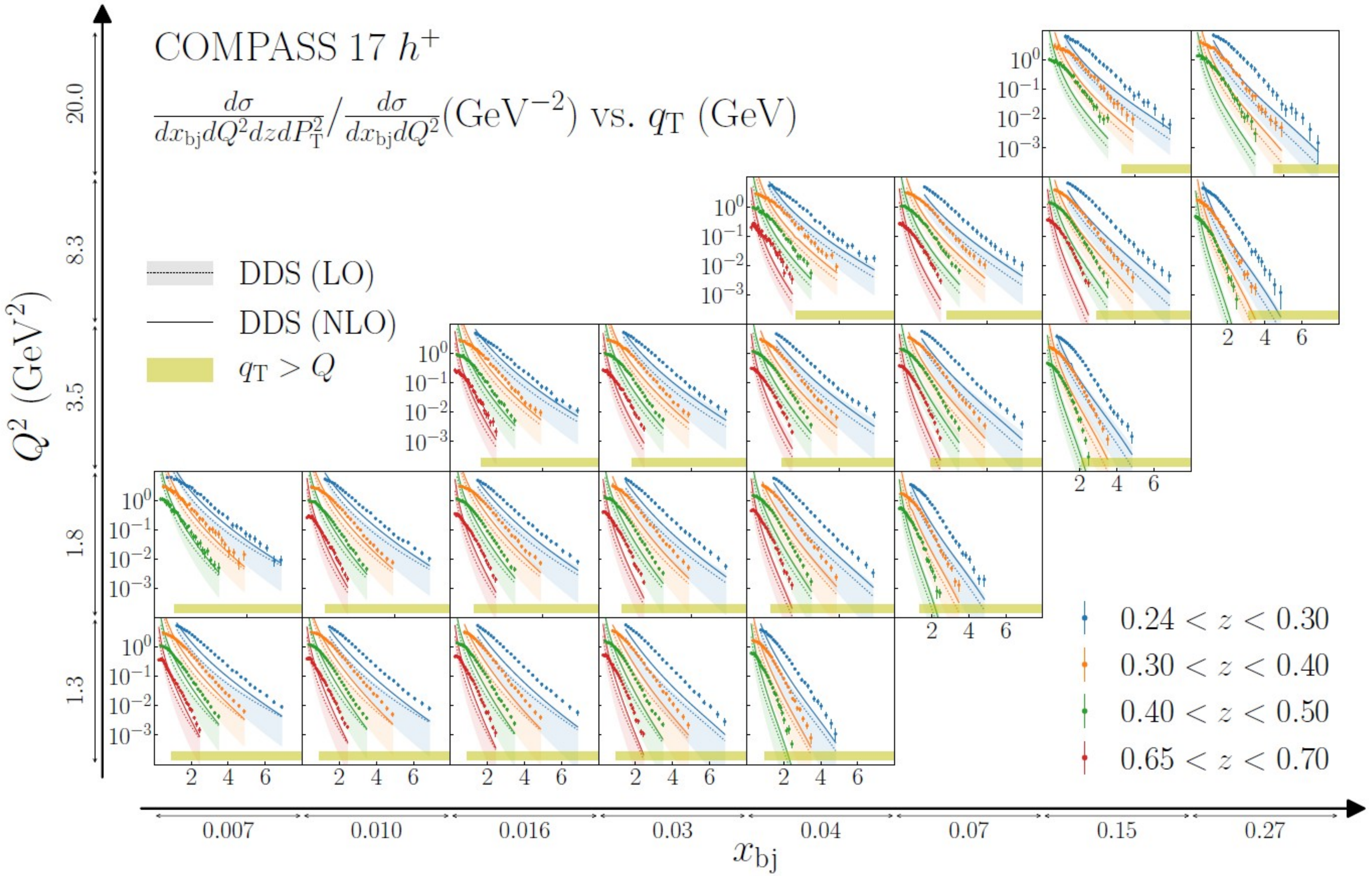
- PDFs: **CT10 NLO**
- FFs: **DSS NLO**

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\,dQ^2}$$



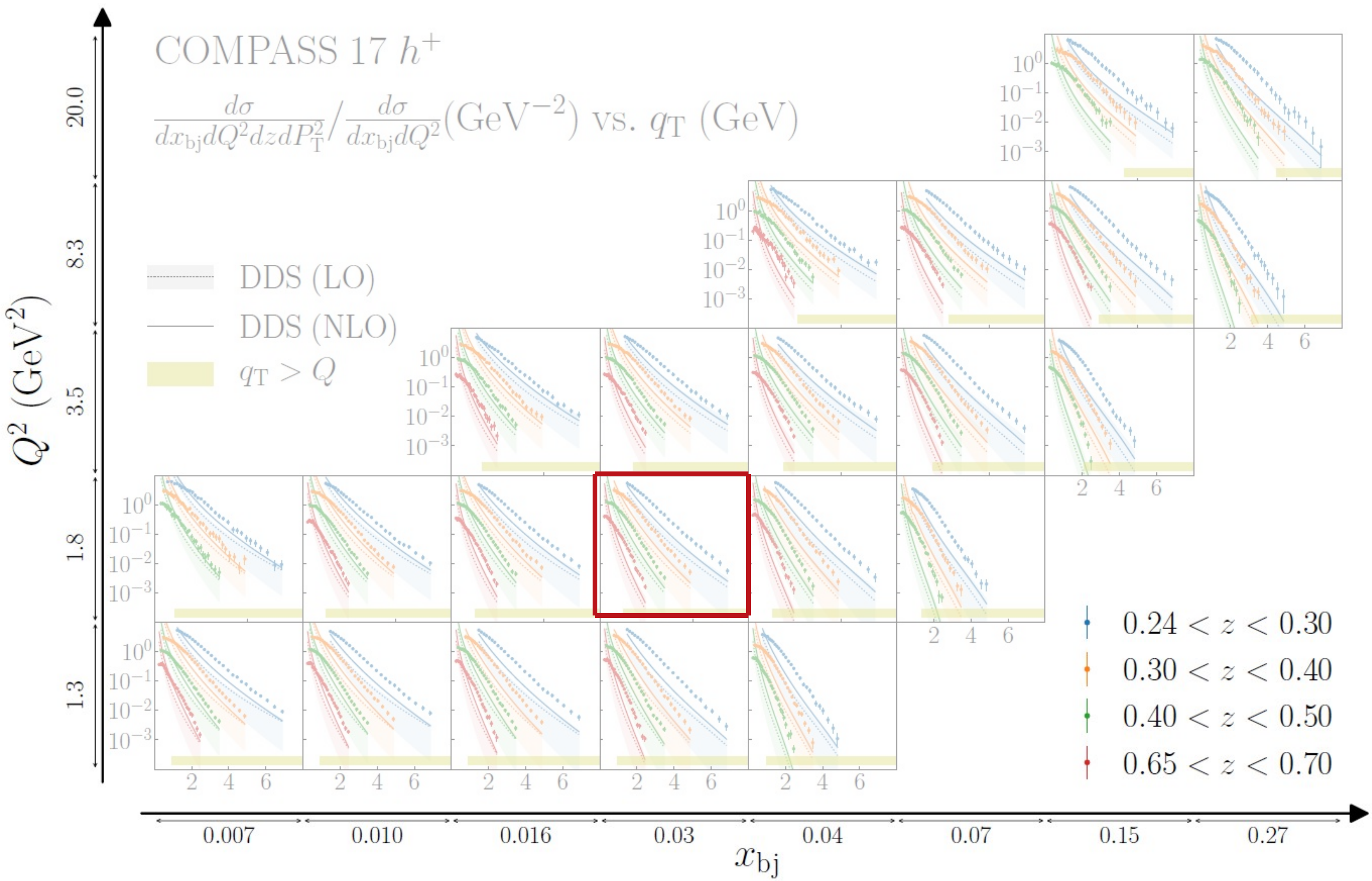


...going to $O(\alpha_s^2)$



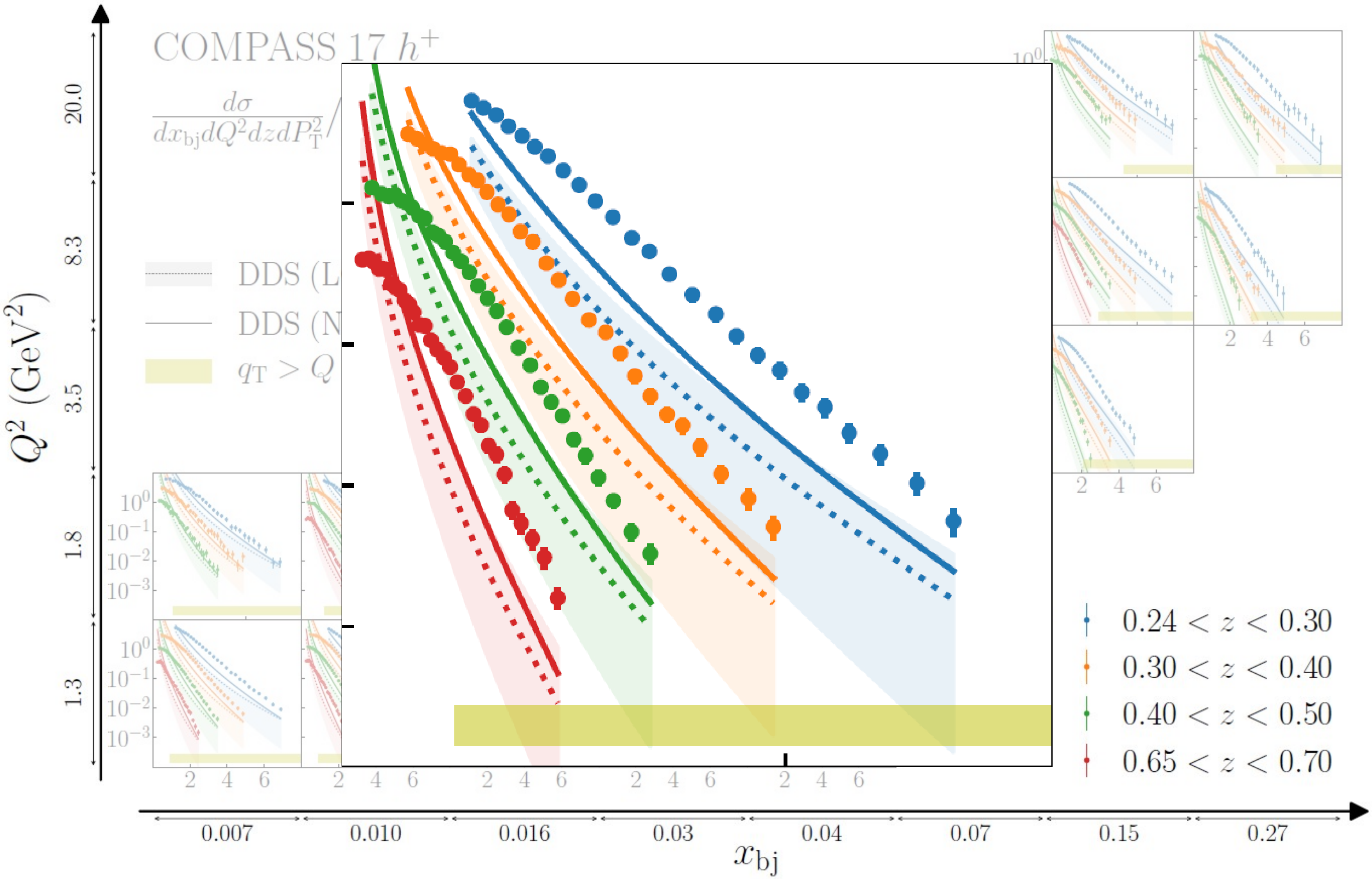
J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang,
"Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering ", **Phys.Rev. D98 (2018)**

...going to $O(\alpha_s^2)$



J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang,
"Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering ", **Phys.Rev. D98 (2018)**

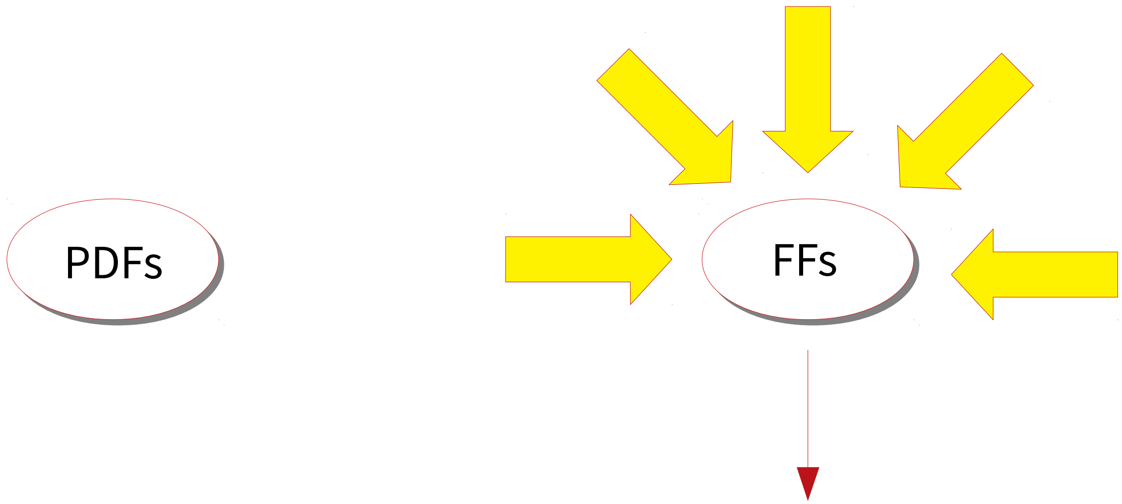
...going to $O(\alpha_s^2)$



J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang,
"Challenges with Large Transverse Momentum in Semi-Inclusive Deeply Inelastic Scattering ", **Phys.Rev. D98 (2018)**

The role of the uncertainties

- They are known to a good precision
- They are affected by rather small uncertainties

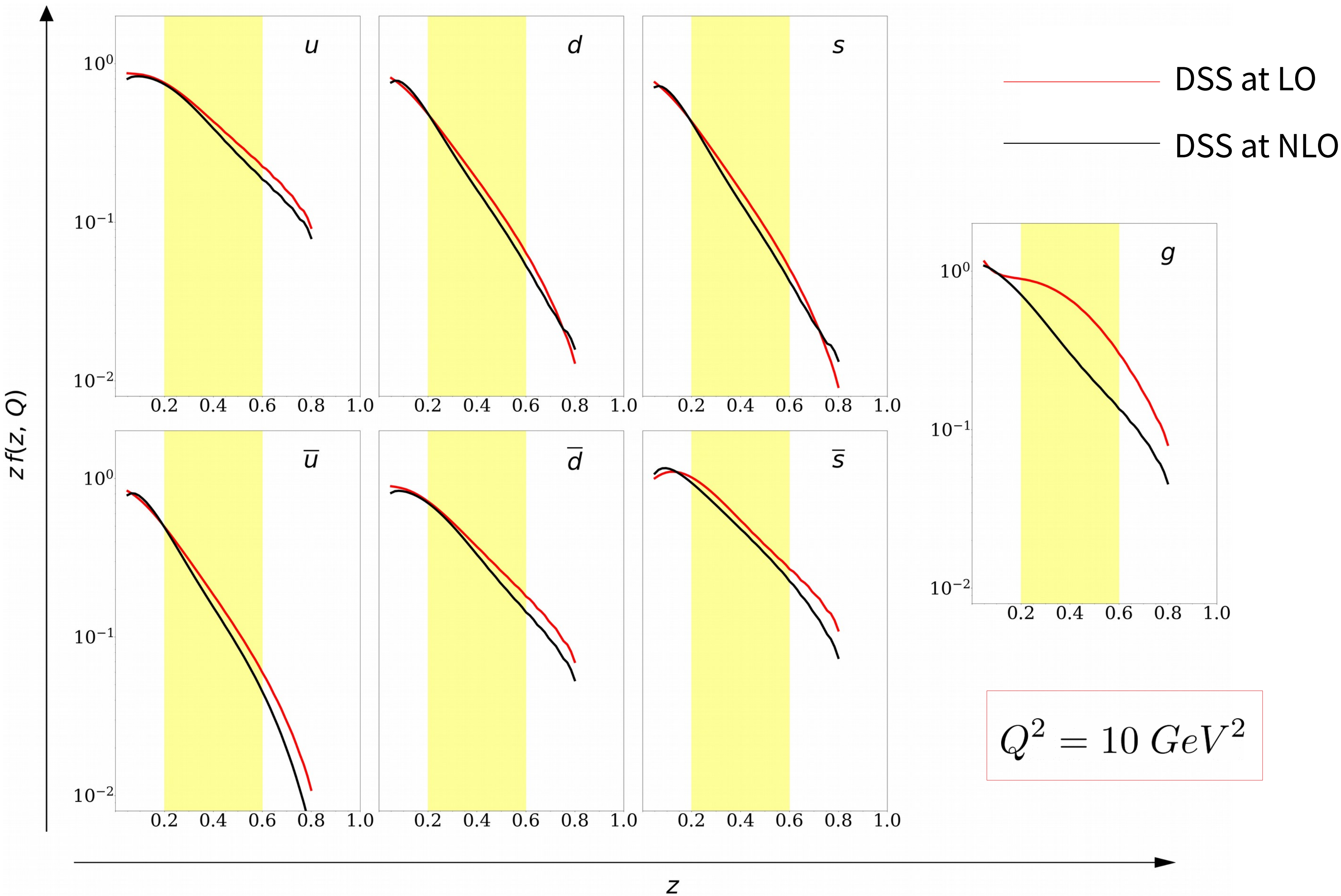


The **gluon FF** is the most difficult to determine (especially at low Q^2)

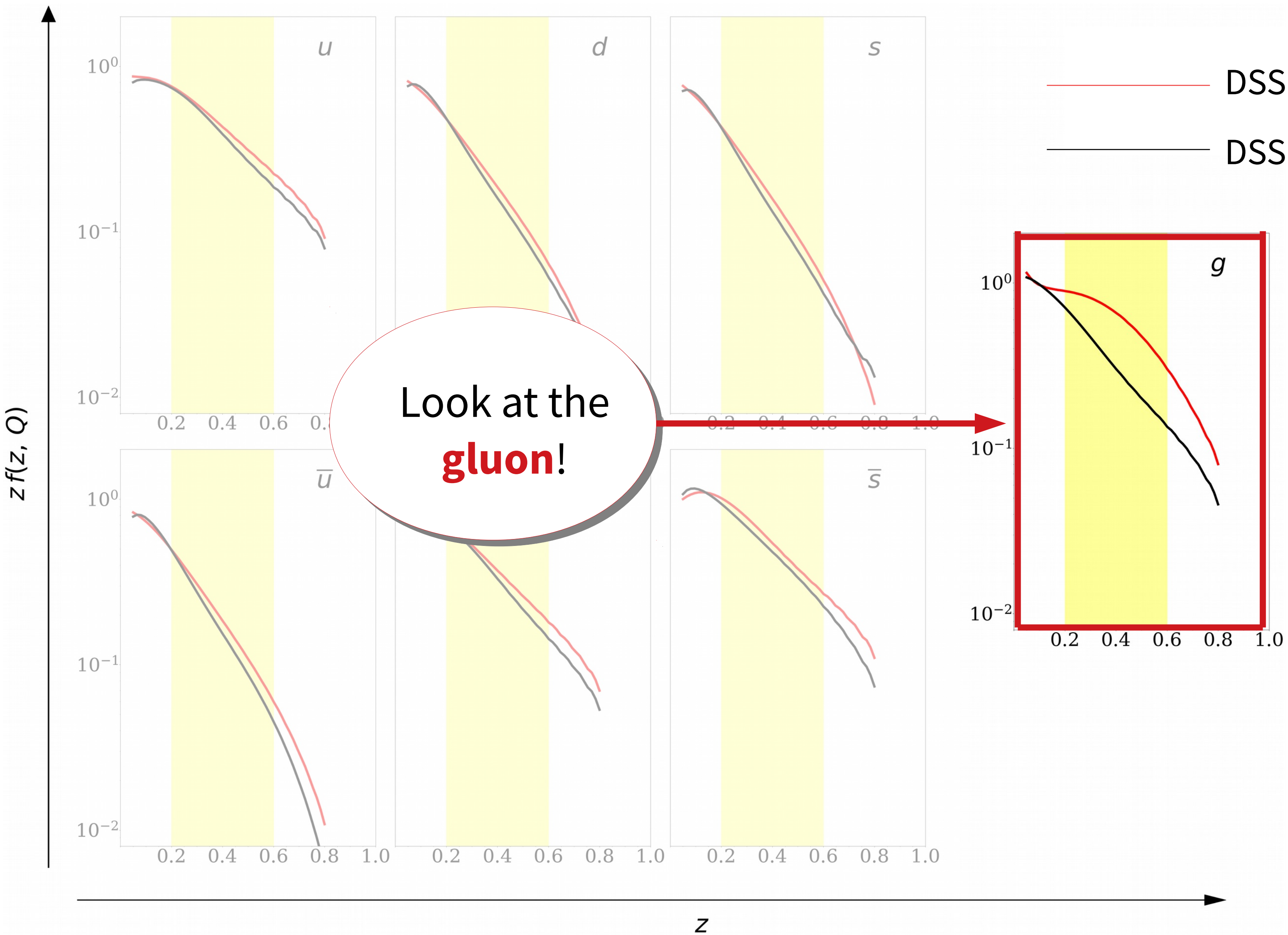
DSS are not provided with the uncertainty band

Is there a way to estimate the uncertainty of DSS set?

A different choice for FFs



A different choice for FFs



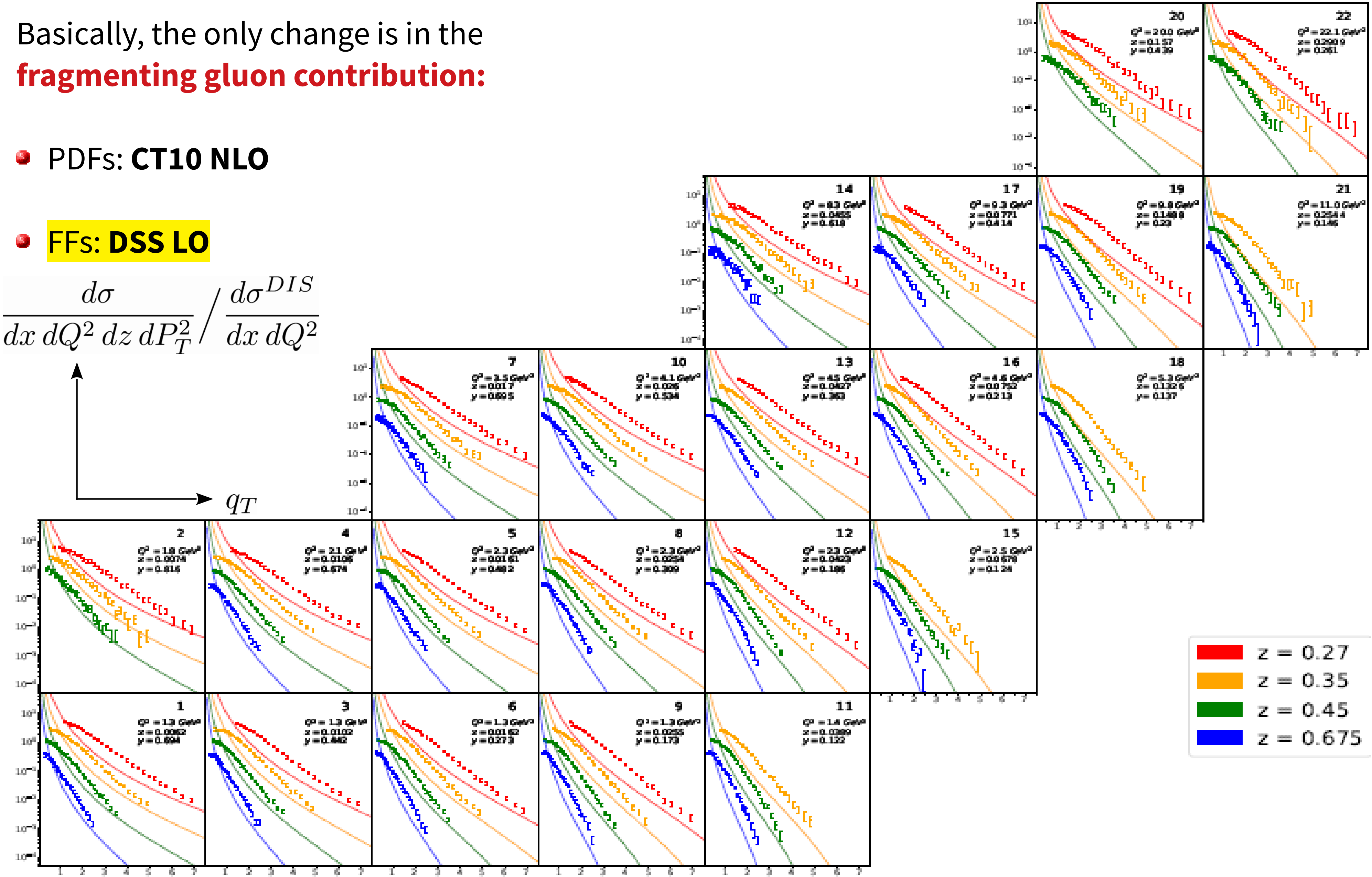
Comparison with COMPASS DATA

Basically, the only change is in the
fragmenting gluon contribution:

- PDFs: **CT10 NLO**
- FFs: **DSS LO**

$$\frac{d\sigma}{dx\, dQ^2\, dz\, dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\, dQ^2}$$

$\uparrow$ $\rightarrow$ q_T

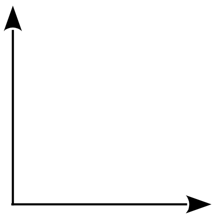


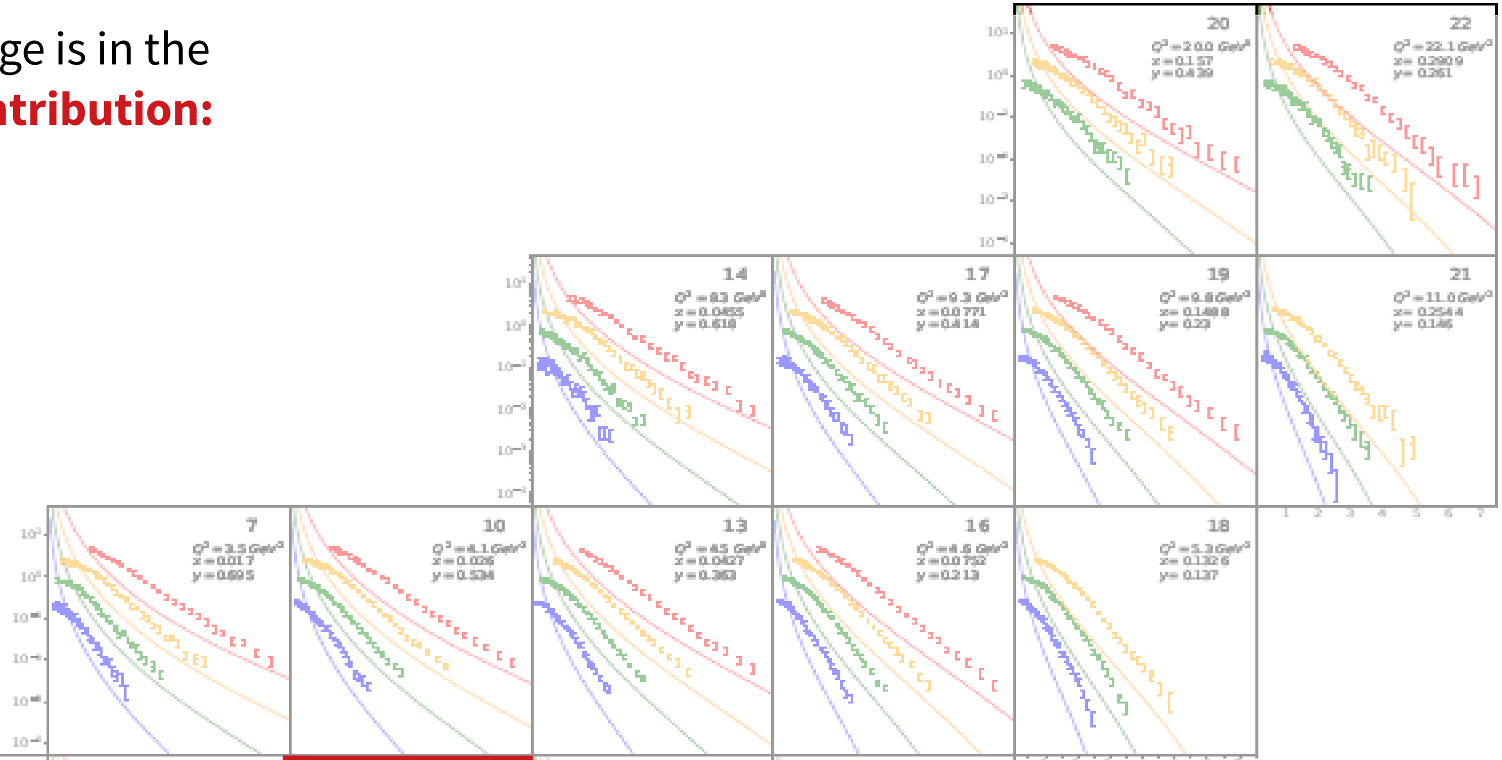
Comparison with COMPASS DATA

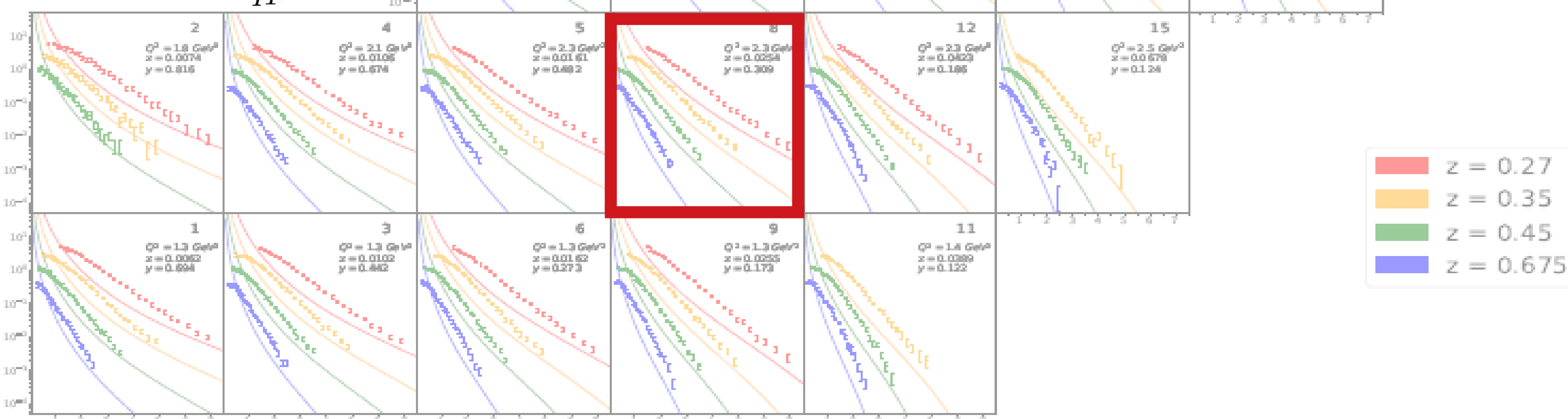
Basically, the only change is in the **fragmenting gluon contribution:**

- PDFs: **CT10 NLO**
- FFs: **DSS LO**

$$\frac{d\sigma}{dx\, dQ^2\, dz\, dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\, dQ^2}$$





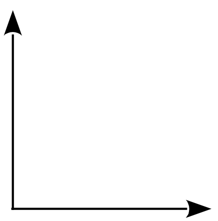


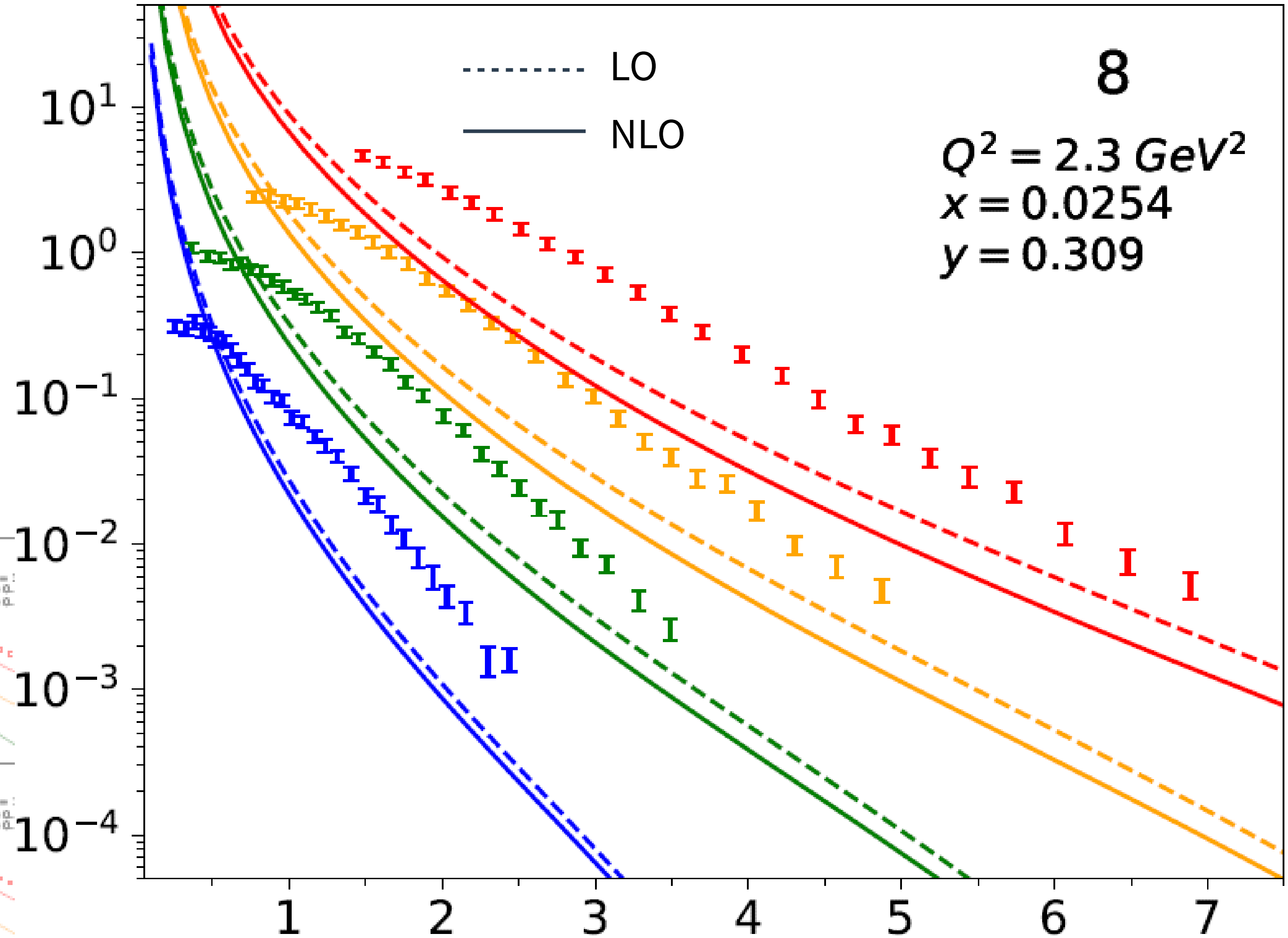
Comparison with COMPASS DATA

Basically, the only change is in the
fragmenting gluon contribution:

- PDFs: **CT10 NLO**
- FFs: **DSS LO**

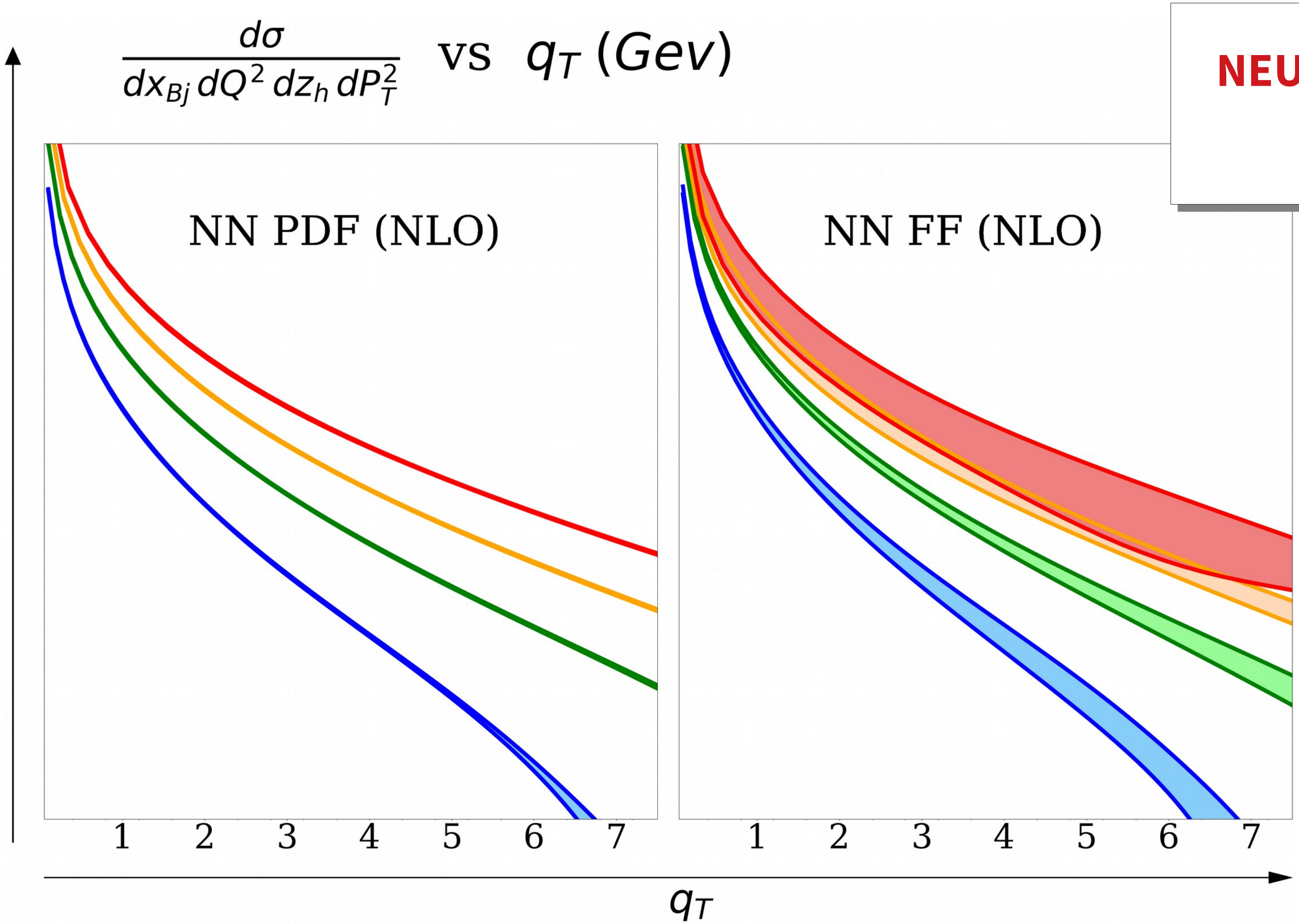
$$\frac{d\sigma}{dx\, dQ^2\, dz\, dP_T^2} \Big/ \frac{d\sigma^{DIS}}{dx\, dQ^2}$$





Errors and Collinear Functions

How much does the **error associated to the extraction of the collinear functions** affect the SIDIS cross section at large q_T?



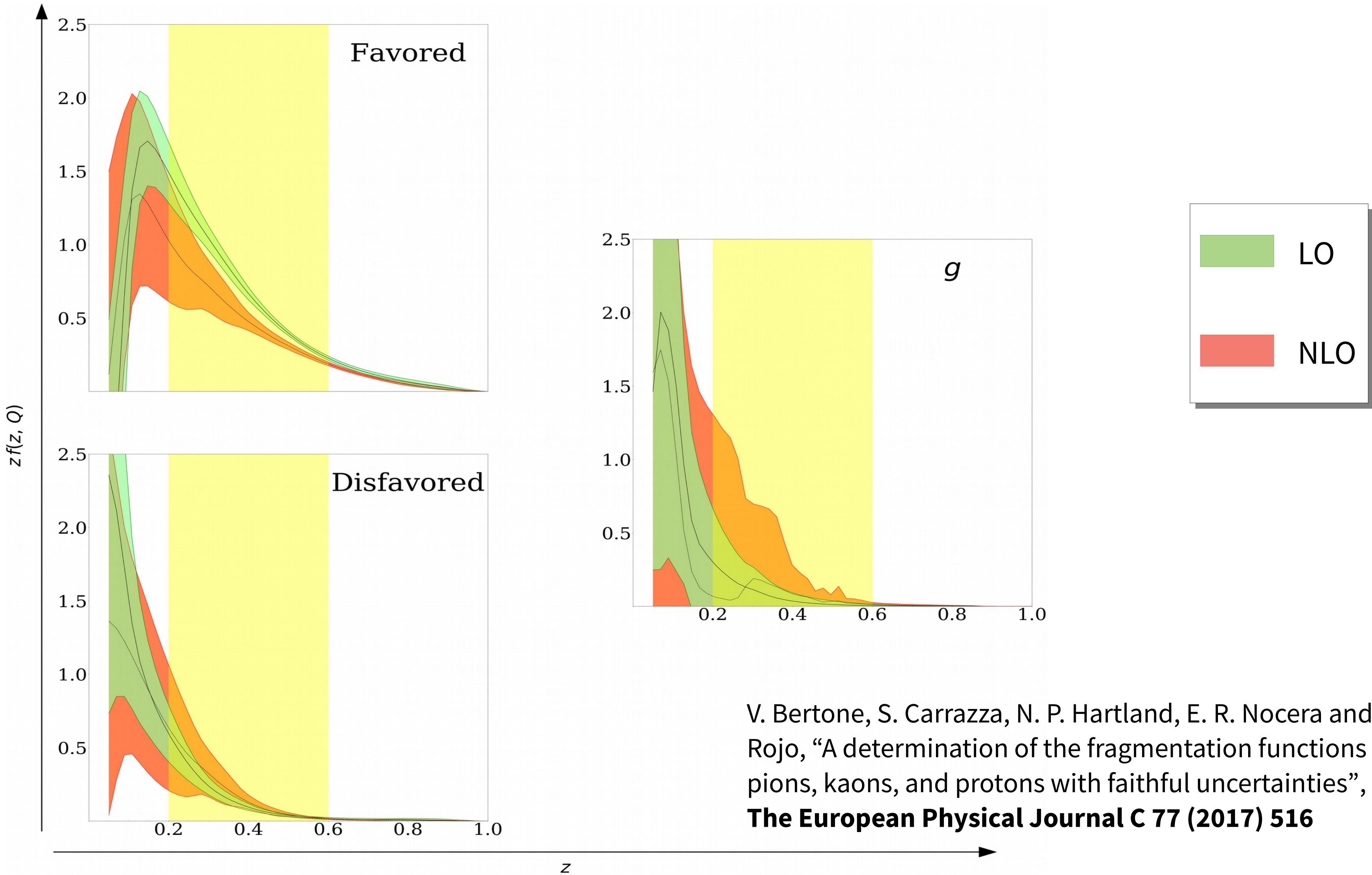
NEURAL NETWORK SETS

$Q^2 = 2.1\ GeV^2$

$x = 0.0106$

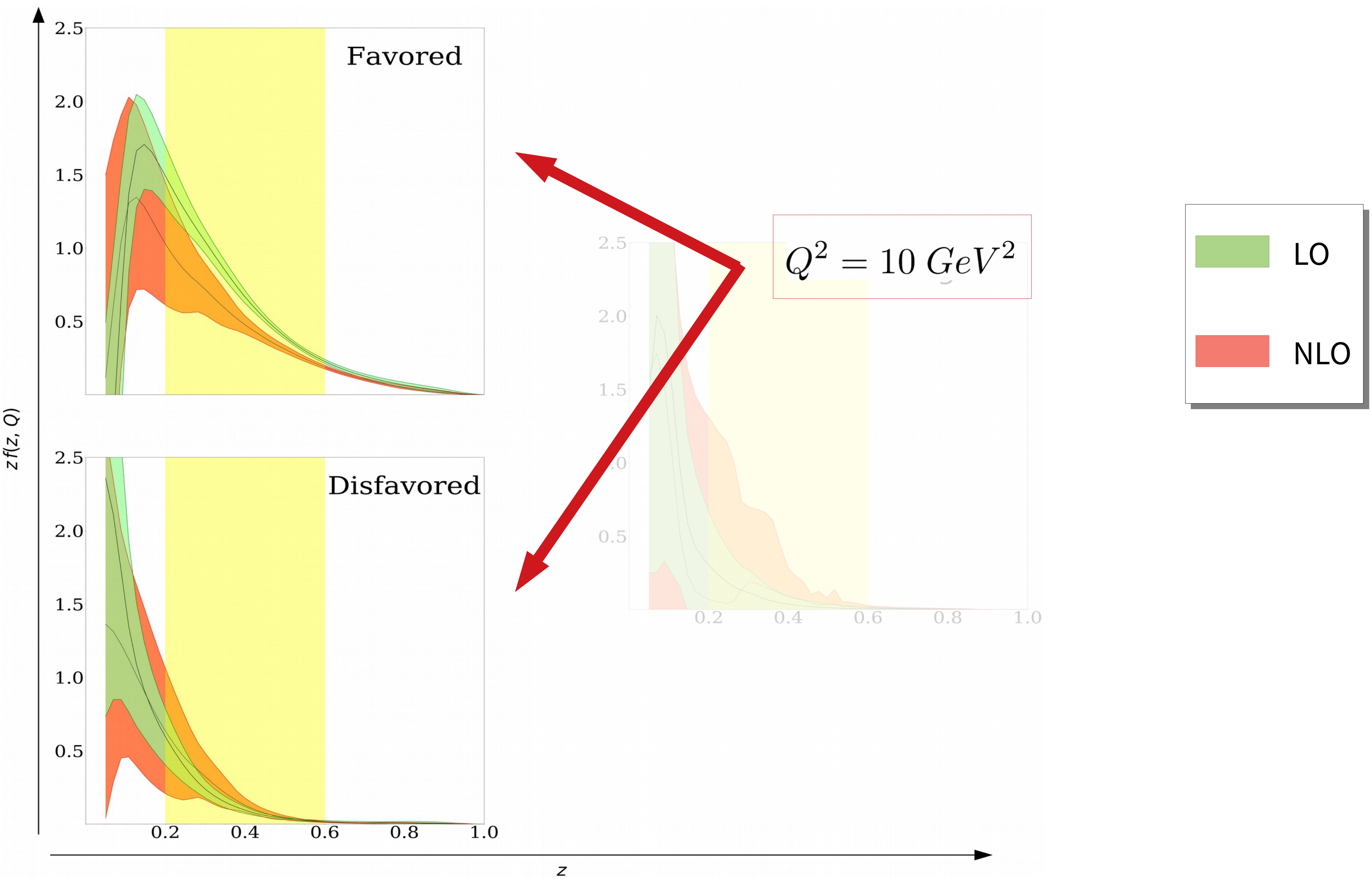
$y = 0.674$

Neural Network FFs



V. Bertone, S. Carrazza, N. P. Hartland, E. R. Nocera and J. Rojo, “A determination of the fragmentation functions of pions, kaons, and protons with faithful uncertainties”, **The European Physical Journal C 77 (2017) 516**

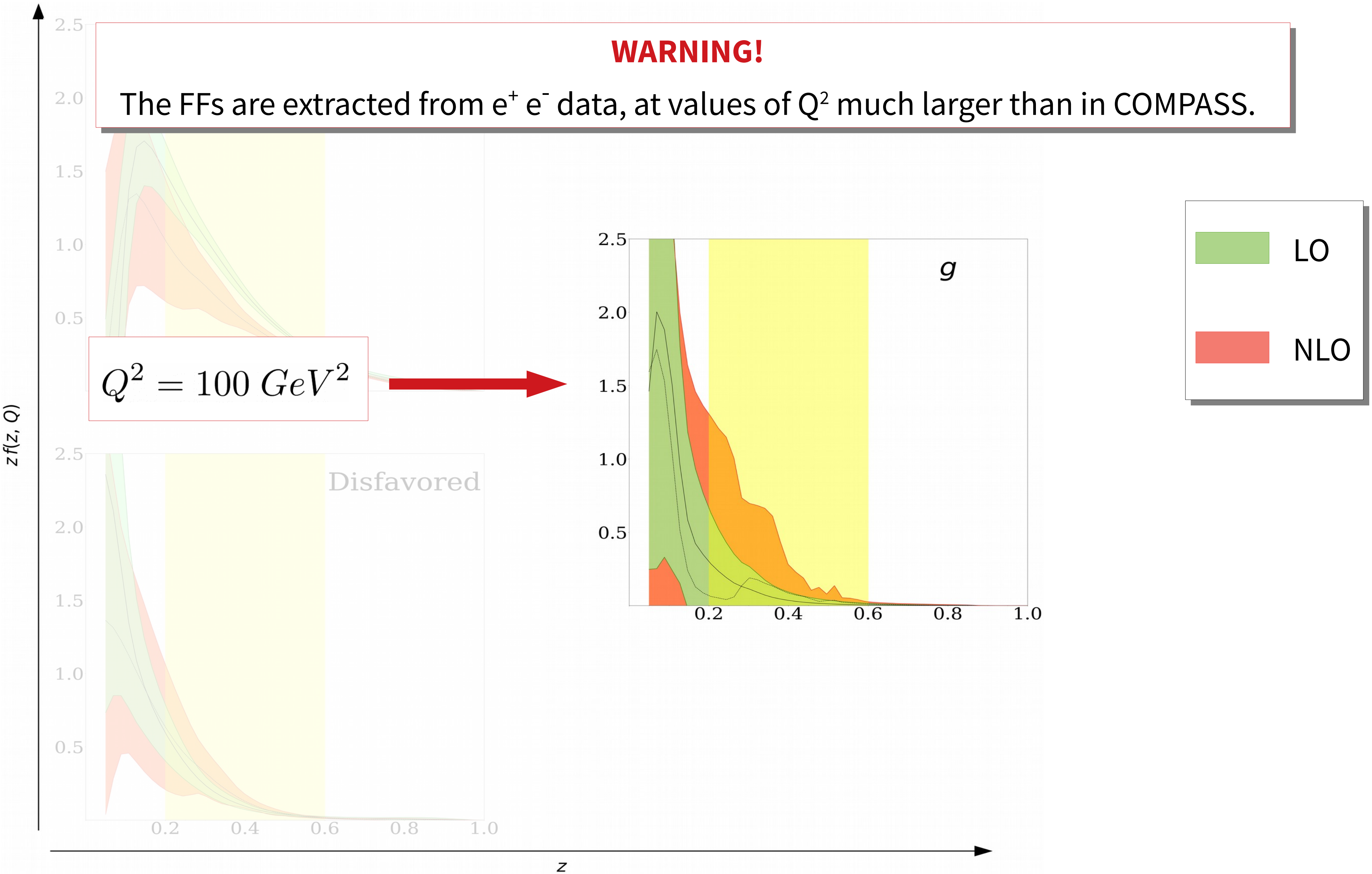
Neural Network FFs



Neural Network FFs

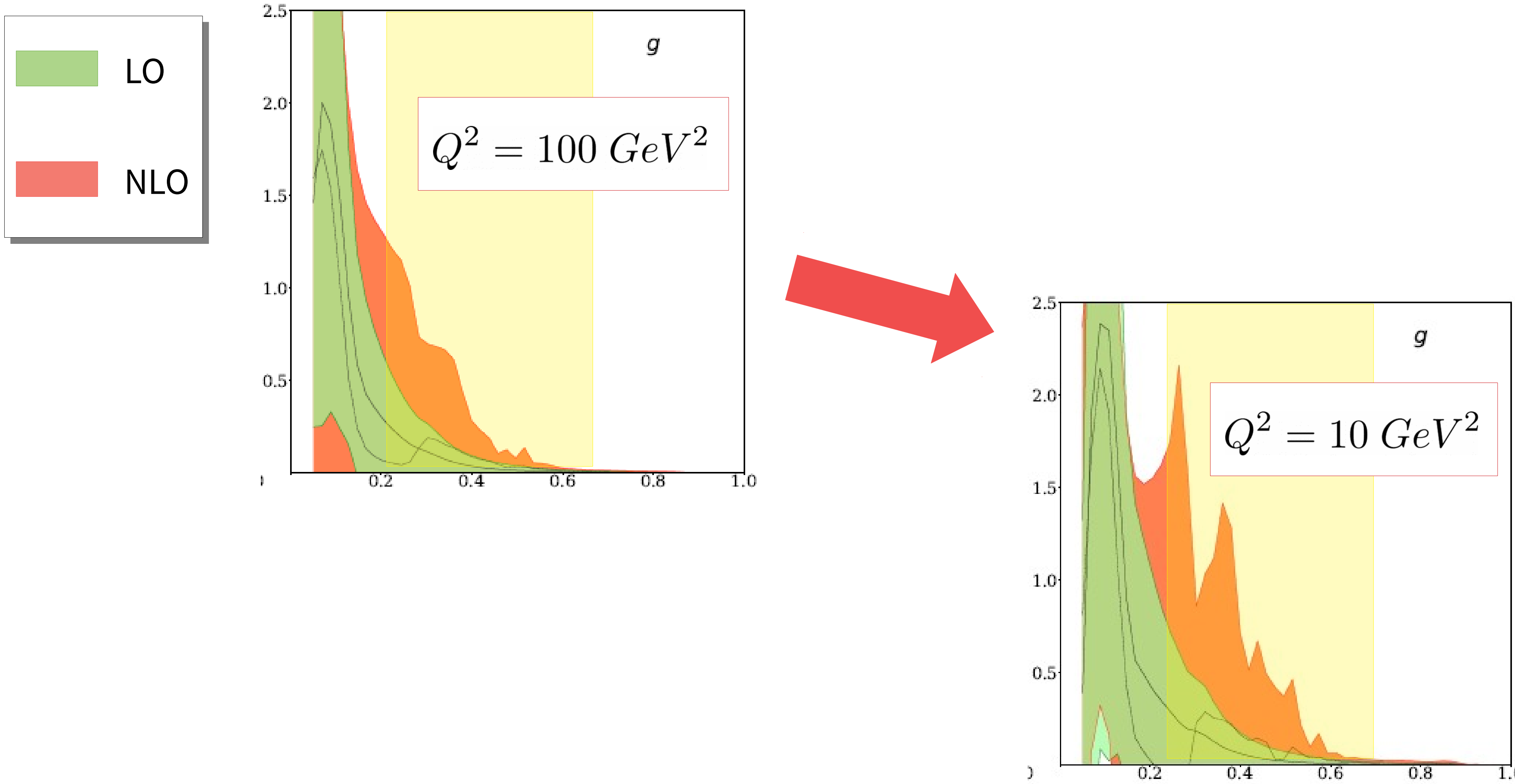
WARNING!

The FFs are extracted from $e^+ e^-$ data, at values of Q^2 much larger than in COMPASS.



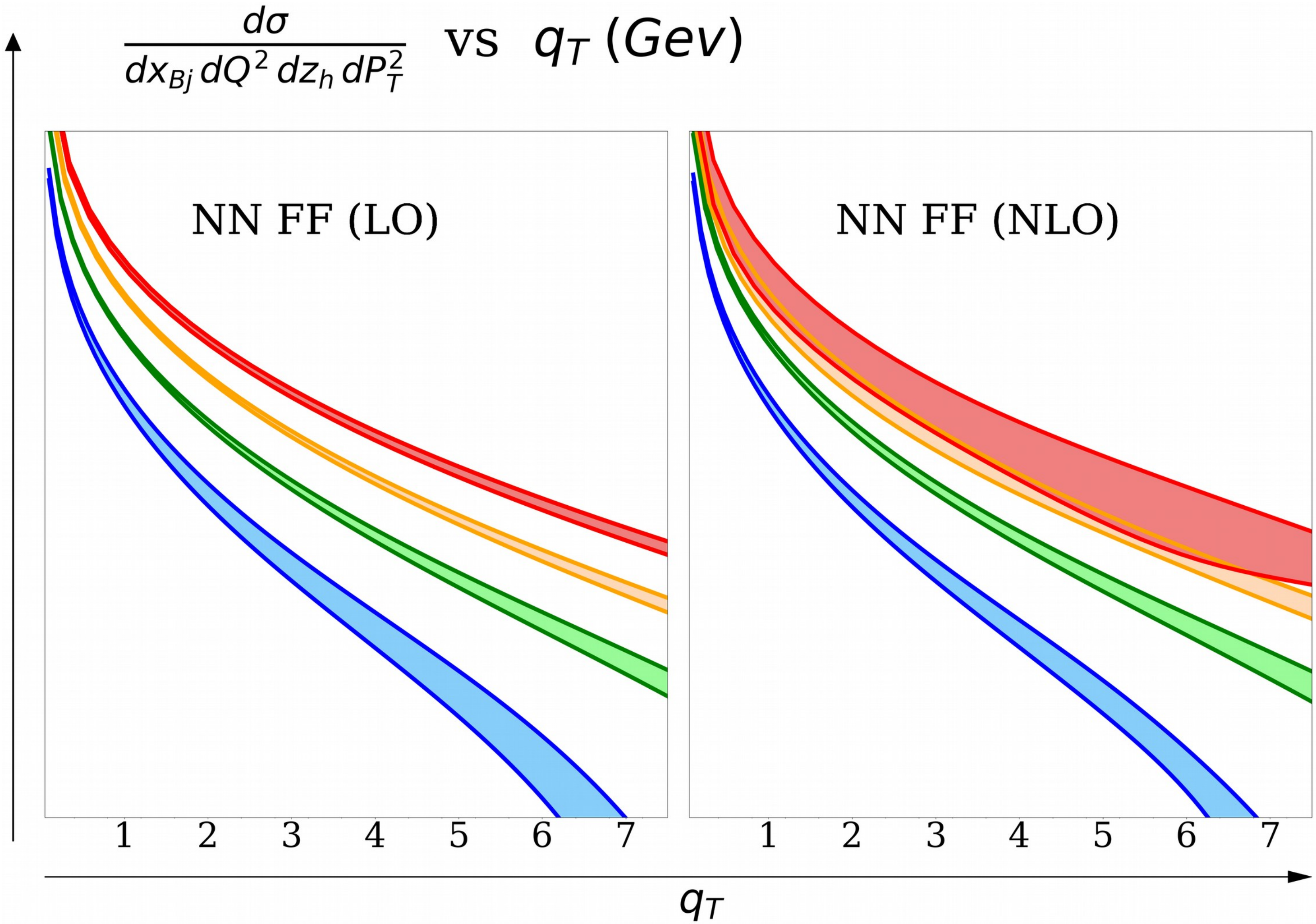
NN FF at NLO vs NN FFs at LO

...This is what happens using a COMPASS-like Q^2:



NN FFs: Comparison in the SIDIS cross section

The error bars associated to NLO FFs are on average **larger**:



$Q^2 = 2.1\ GeV^2$
$x = 0.0106$
$y = 0.674$

SIDIS in the LOW q_T regime

TMD Factorization:

$$\frac{d\sigma}{dx_{Bj}\, dy\, dz_h\, dq_T^2} = \pi\, z^2\, H\,(Q;\,\mu) \int \frac{d^2\vec{b_T}}{(2\pi)^2}\, e^{i\,\vec{q_T}\cdot\vec{b_T}} \sum_j e_j^2\, \widetilde{F}\,(x,\, b_T,\mu,\zeta_F)\; \widetilde{D}\,(z,\, b_T,\mu,\zeta_D)$$

TMD FUNCTIONS

Same collinear functions used in collinear factorization!

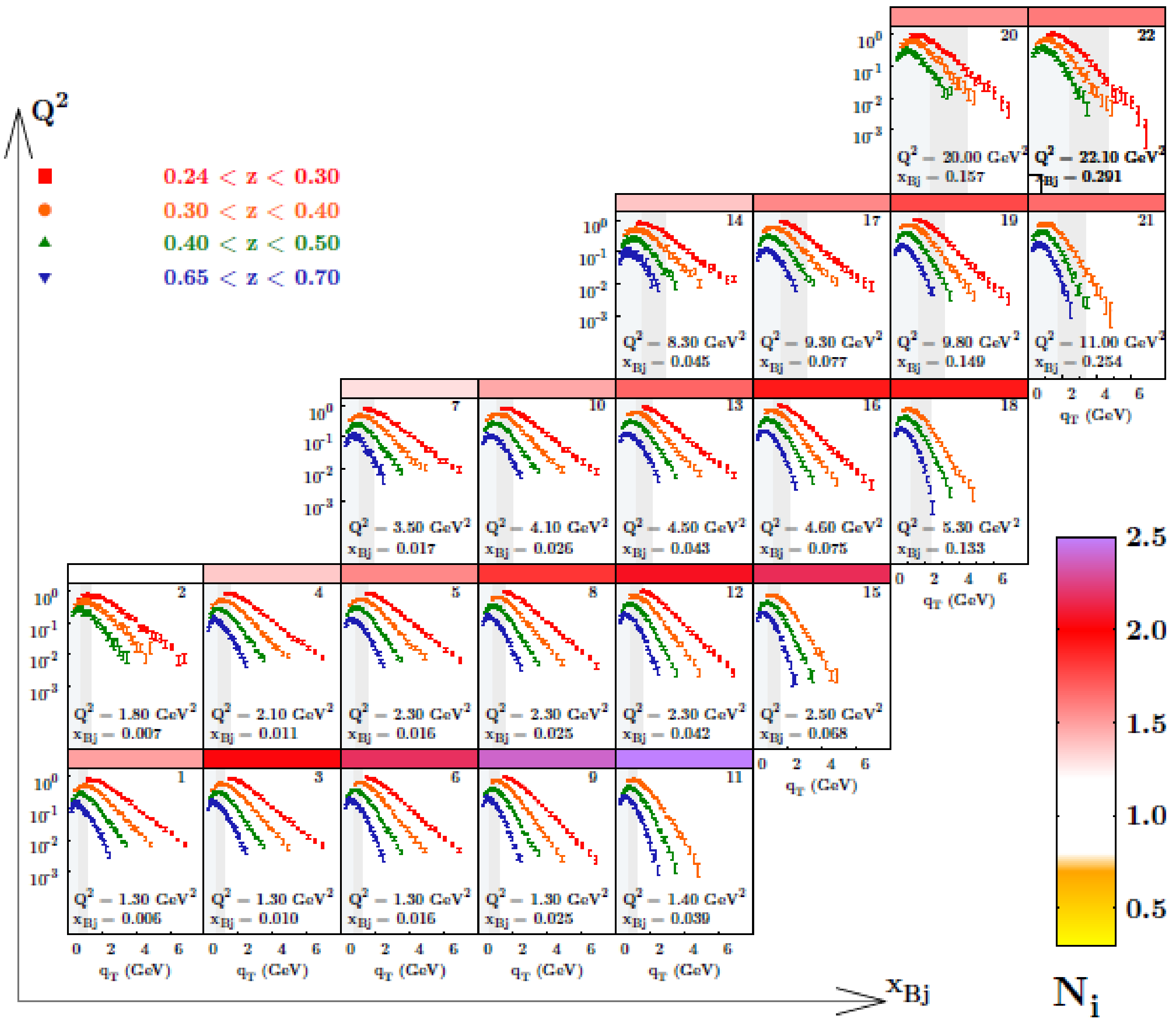
The **TMD PDF** is a complex object...

$$\widetilde{F}\,(x,\, b_T,\mu,\zeta_F) = \sum_j \int_x^1 \frac{d\widehat{x}}{\widehat{x}}\, C_{f\,j}\left(\frac{x}{\widehat{x}}\right) f_j\,(\widehat{x},\,\mu_b) \times$$

$$\times\; exp\left\{\frac{1}{2}\, log\left(\frac{\zeta_F}{\mu^2}\right) \widetilde{K}\,(b_*\,\mu_b) + \int_{mu_b}^{Q} \frac{d\mu}{\mu}\gamma_F\left(\mu,\,\frac{\zeta_F}{\mu^2}\right)\right\} \times M_F\,(x,\, b_T)$$

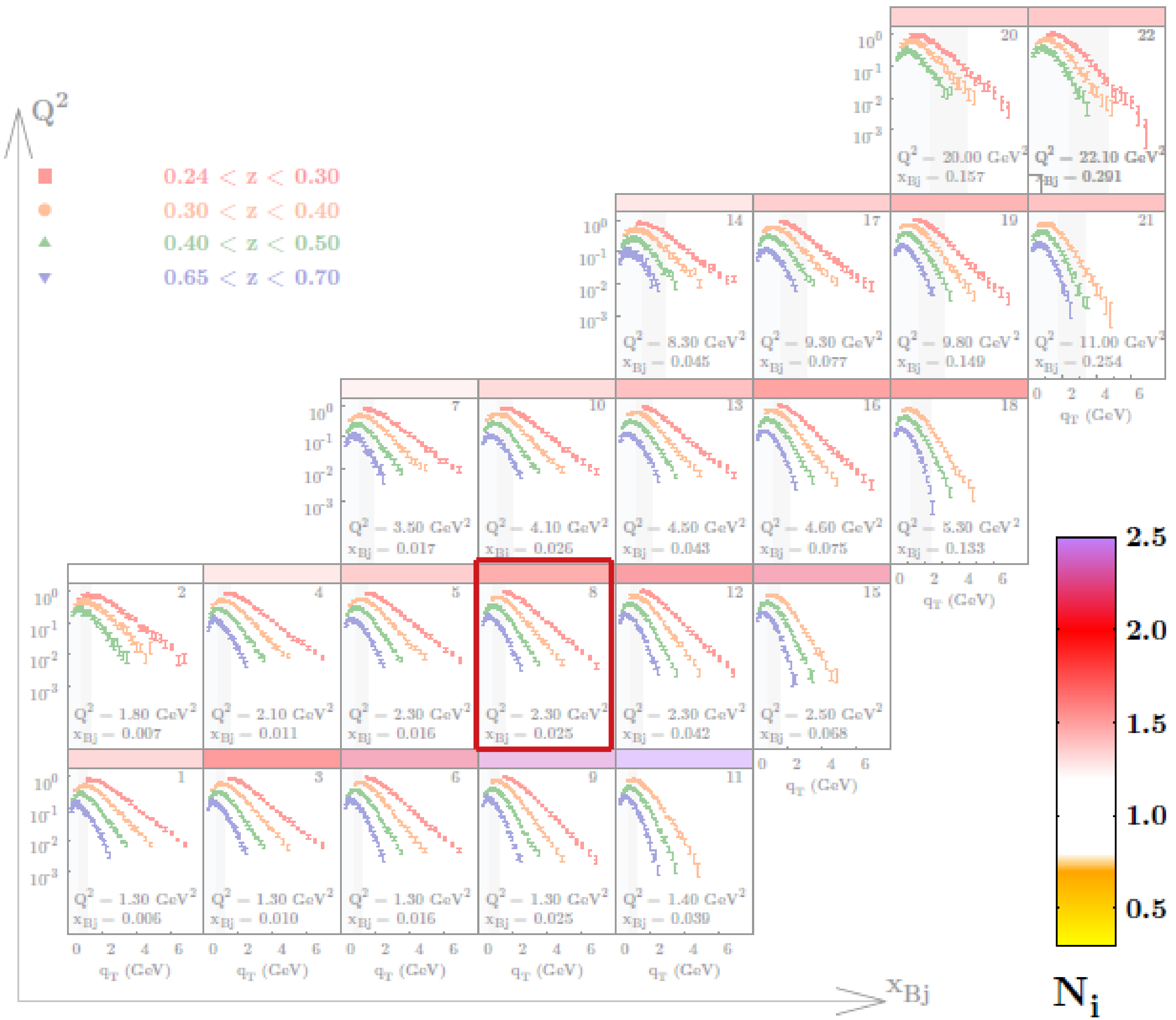
M. Boglione, J.O. Gonzalez Hernandez, S. Melis, and A. Prokudin
“A study on the interplay between perturbative QCD and CSS/TMD formalism in SIDIS processes”, **JHEP 1502 (2015) 095**

Normalization problem at low-q_T



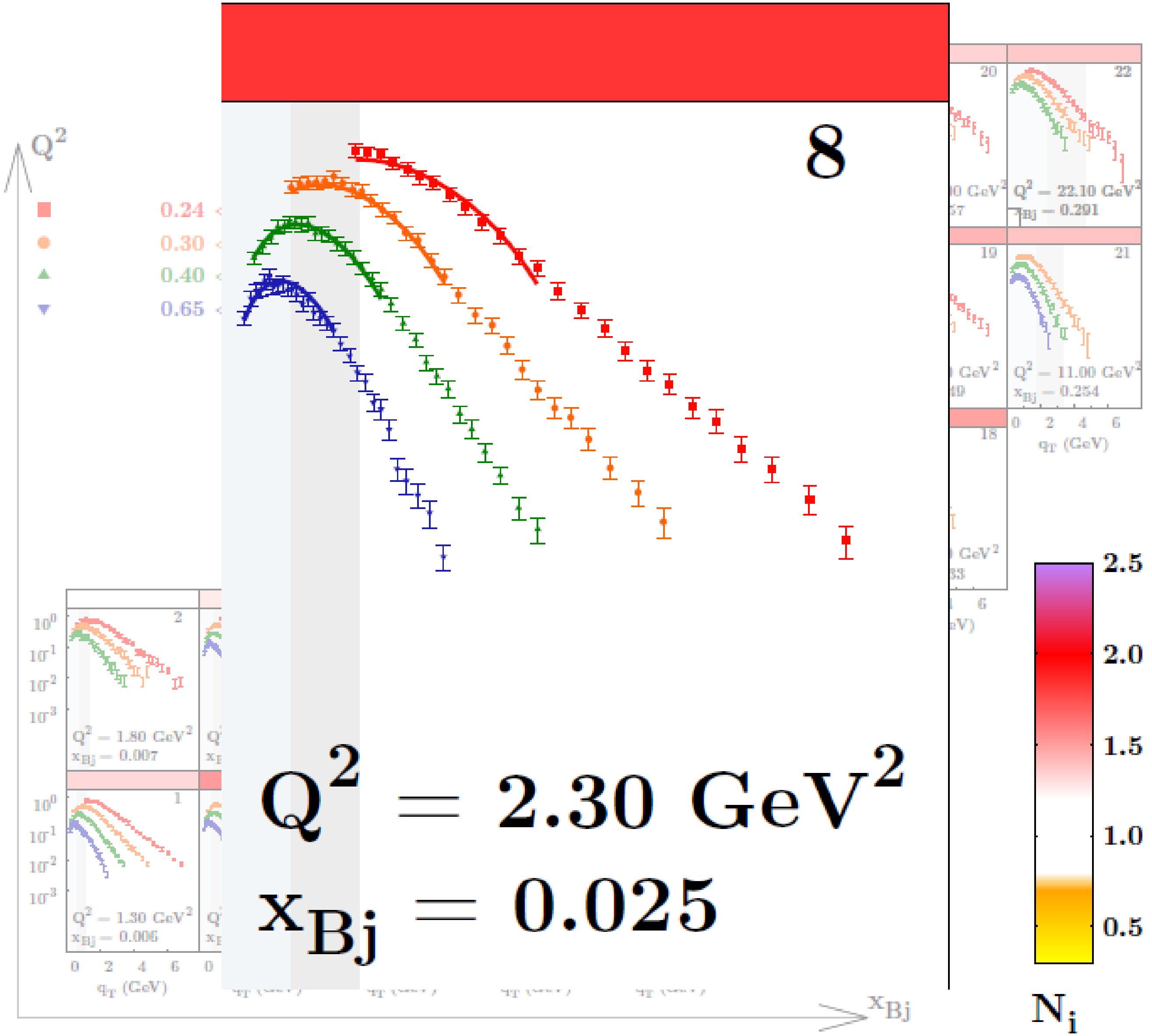
Low-q_T cross sections evaluated using **DSS at NLO**

Normalization problem at low-q_T



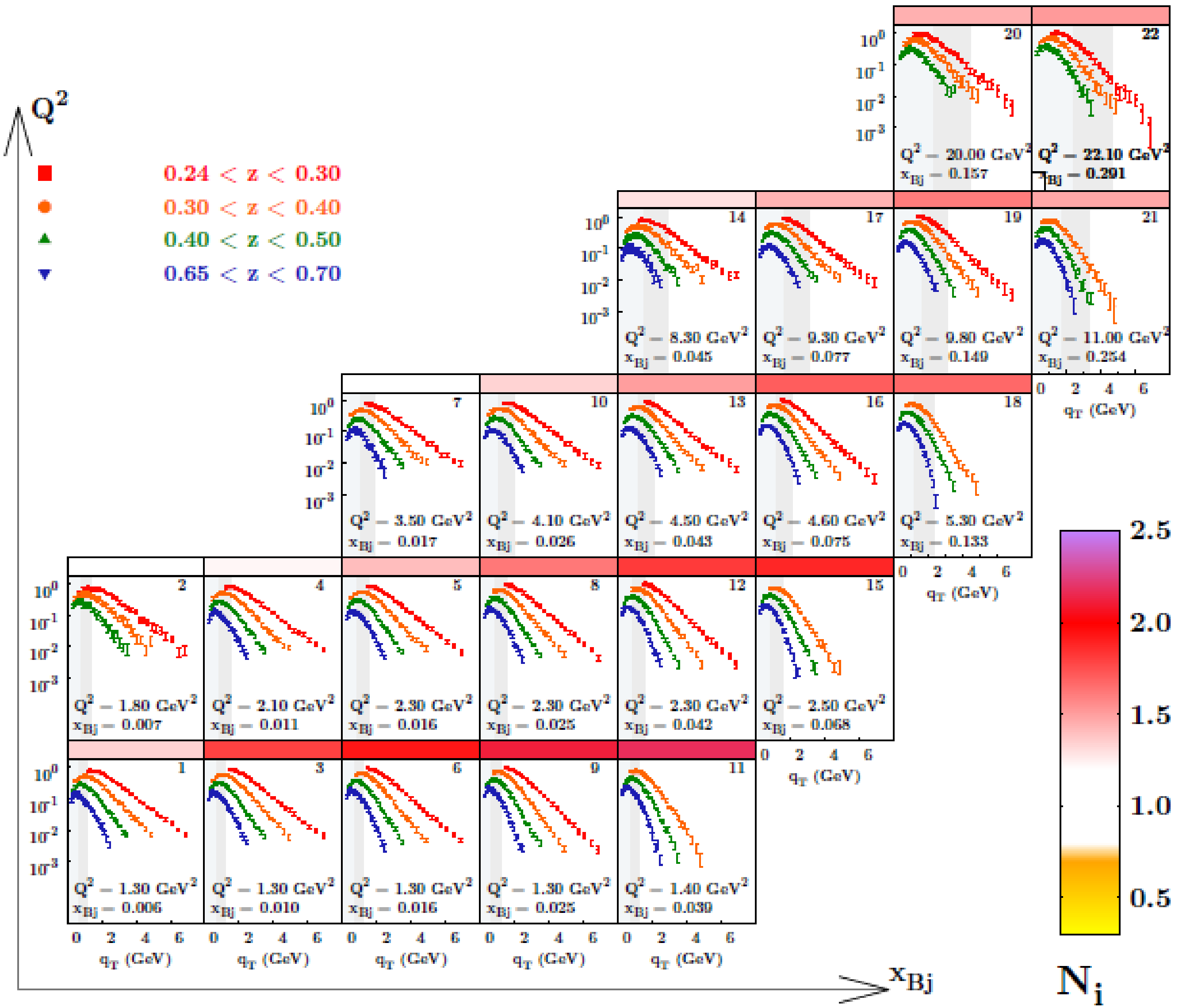
Low-q_T cross sections evaluated using **DSS at NLO**

Normalization problem at low-q_T



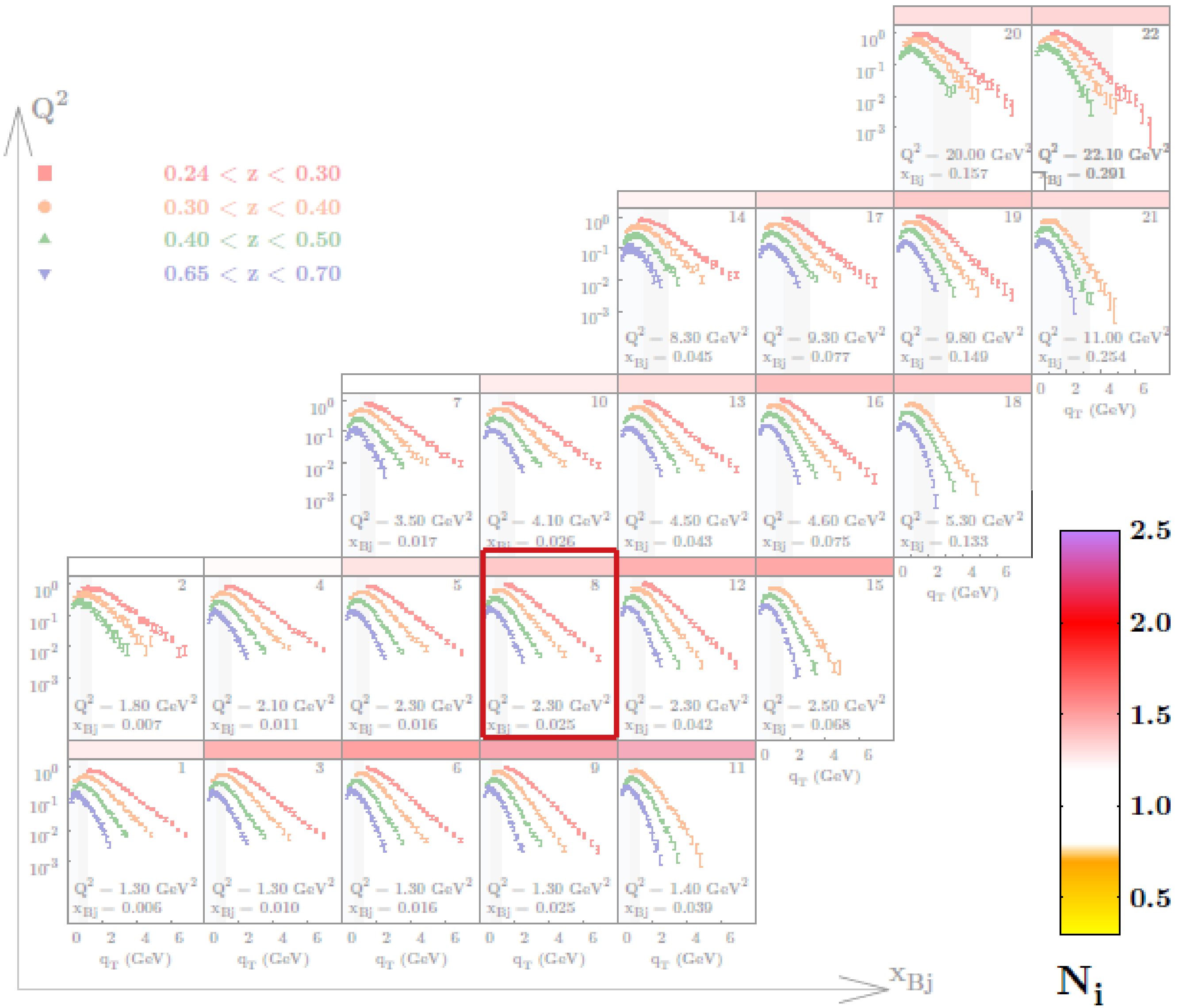
Low-q_T cross sections evaluated using **DSS at NLO**

Normalization problem at low-q_T



Low-q_T cross sections evaluated using **DSS at LO**

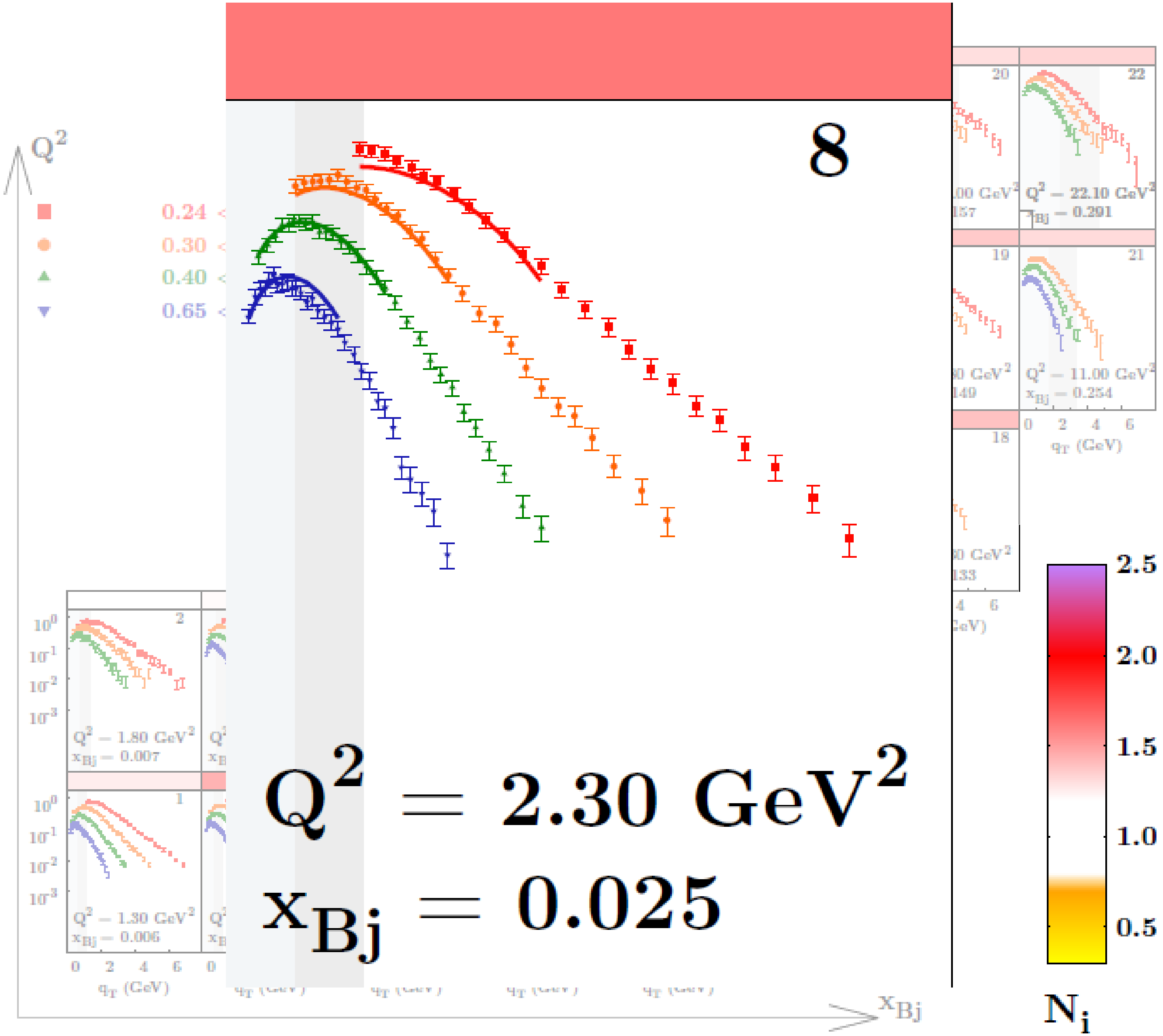
Normalization problem at low-q_T



Low-q_T cross sections evaluated using **DSS at LO**

Andrea Simonelli

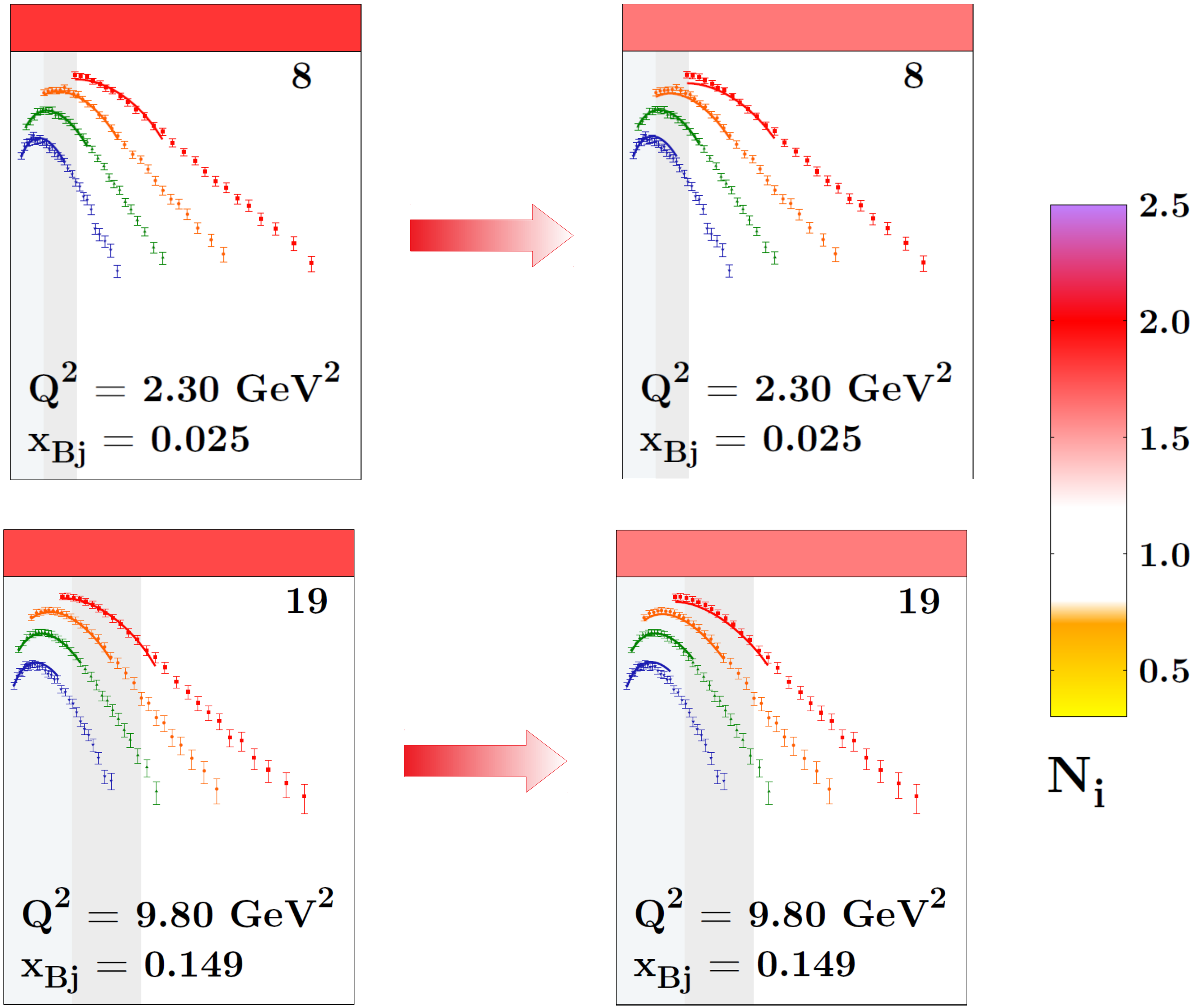
Normalization problem at low-q_T



Low-q_T cross sections evaluated using **DSS at LO**

Andrea Simonelli

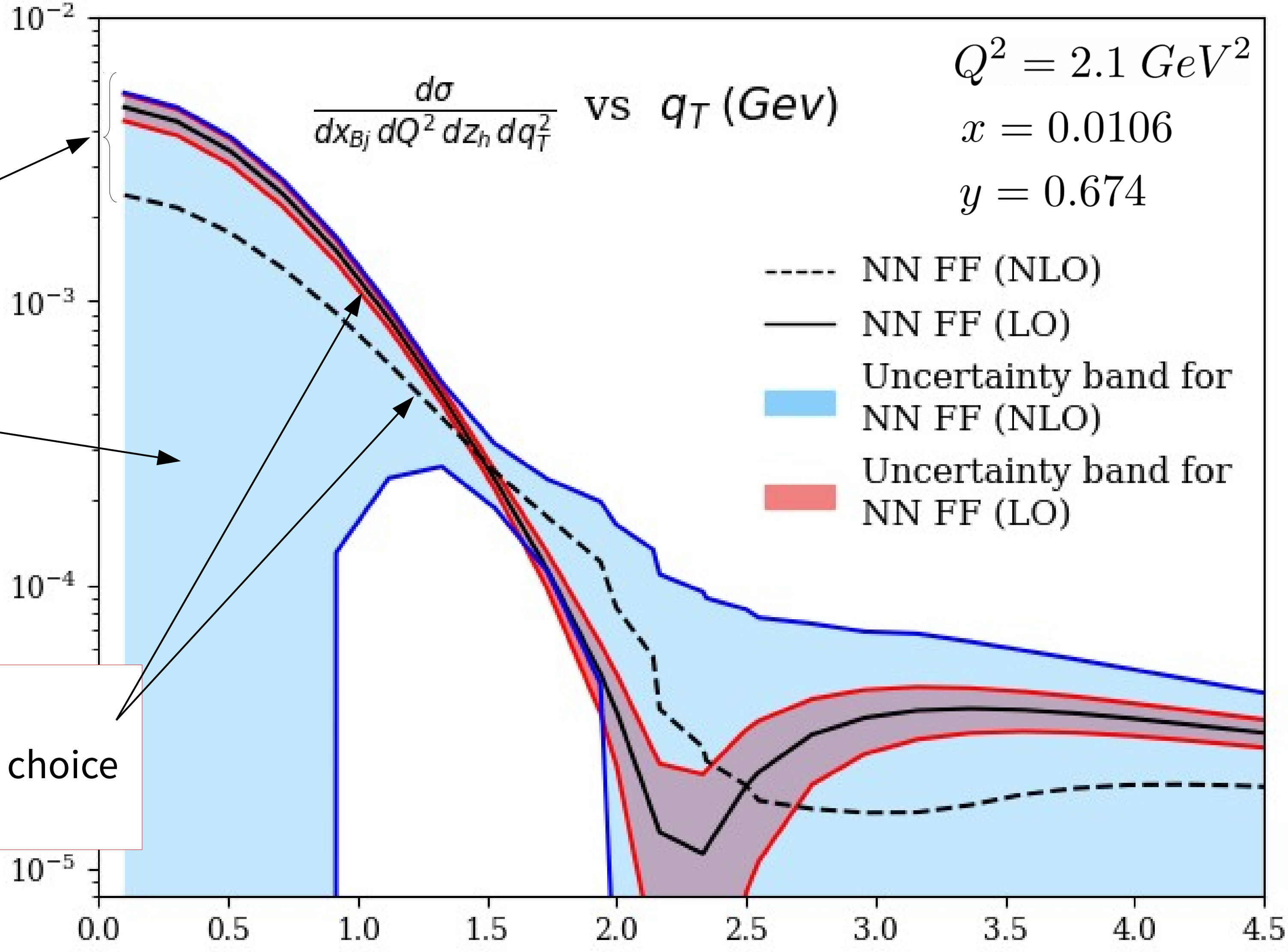
Normalization problem at low-q_T



Using DSS at LO the problem is less severe

Collinear Functions in the low-q_T cross section

ERRORS:
- Factor of ~ 5
- Very large band in low q_T region for the right FFs.



CENTRAL LINES:
Effect of a different choice of the FF set.

THANK YOU FOR YOUR ATTENTION!

BACK UP SLIDES

A different choice for FFs

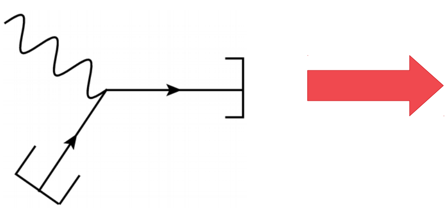
Let's consider again the collinear factorization theorem:

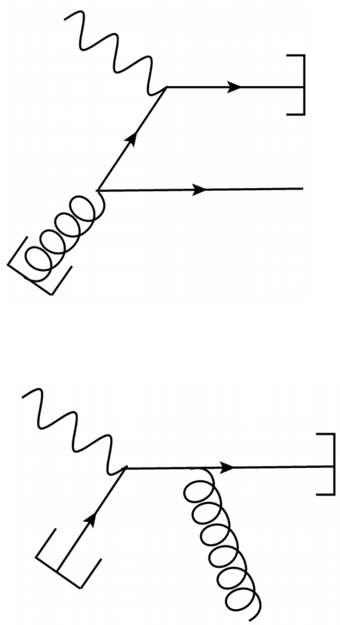
$$\mathcal{O} = H \otimes \sum_i F_i$$

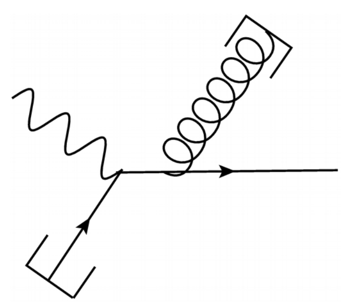
As the order of α_s increases, the **HARD** part grows since the **phase space enlarges** more and more.

As a consequence the **COLLINEAR FUNCTIONS** contribution decreases.

For SIDIS:







NN FFs: Comparison in the SIDIS cross section

Central lines for NN FFs: