

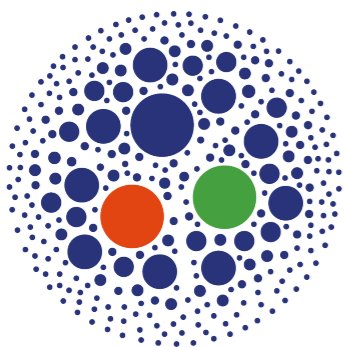
# Transverse-momentum-dependent gluon distribution in a spectator model

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in collaboration with  
**Alessandro Bacchetta, Marco Radici, Pieter Tael**



**HAS QCD**

HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS



Istituto Nazionale di Fisica Nucleare  
Sezione di Pavia

**Sar Wors 2019**  
Cagliari, 10<sup>th</sup> July 2019



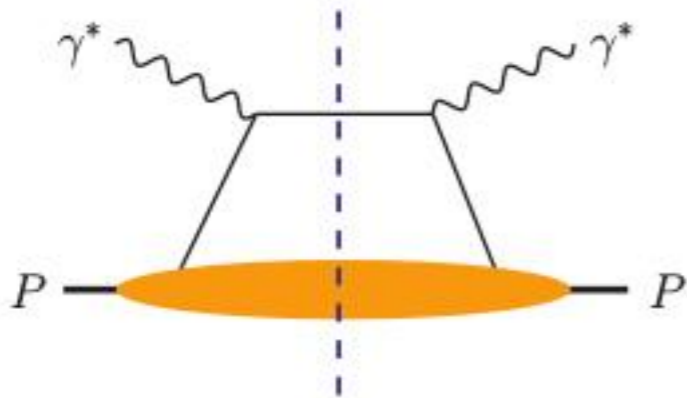
**UNIVERSITÀ  
DI PAVIA**

# Parton densities: an overview

## $p_T$ - integrated

### Collinear PDFs

- Inclusive processes
- $p_T \sim$  hardest scale



### GPDs

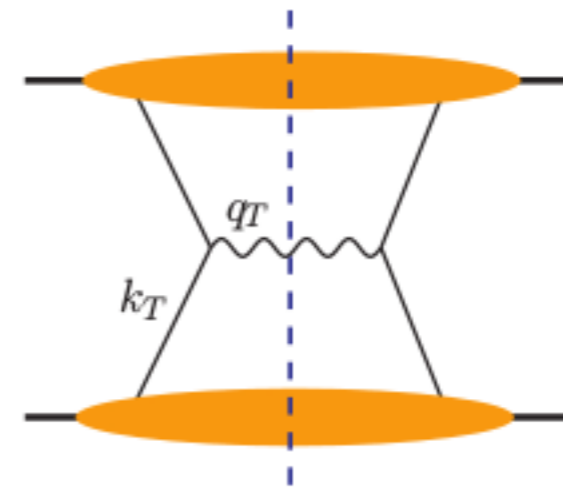
- Exclusive processes
- Skewness effects



## $p_T$ - unintegrated

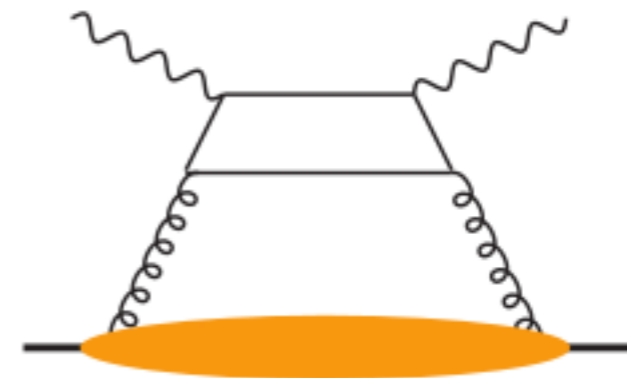
### TMDs

- (Semi-)inclusive processes
- $p_T \ll$  hardest scale



### UGDs

- High-energy factorization (**BFKL**)
- Small  $x$ , large  $p_T$



# Gluon TMDs at twist-2

gluon pol.

	U	circ.	lin.	
U	$f_1^g$		$h_1^{\perp g}$	$T$ -even
L		$g_1^g$	$h_{1L}^{\perp g}$	$T$ -odd
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$	

# Gluon TMDs at twist-2

unpolarized TMD

Boer-Mulders

gluon pol.

nucleon pol.

	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

*T*-even

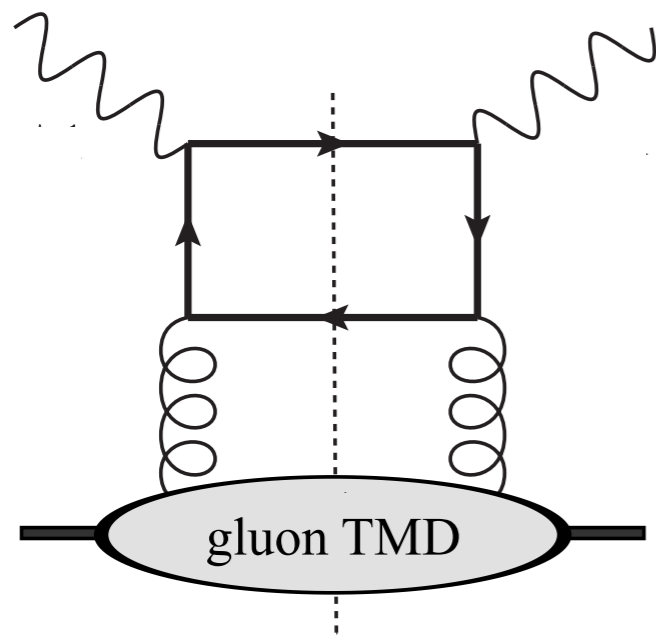
*T*-odd

helicity TMD

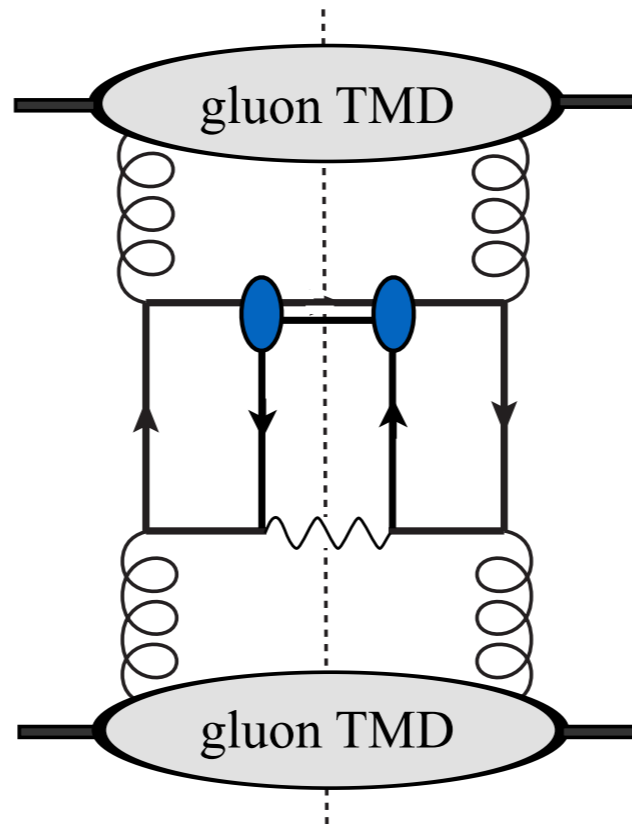
worm-gear

# Gluon TMDs: a largely unexplored territory

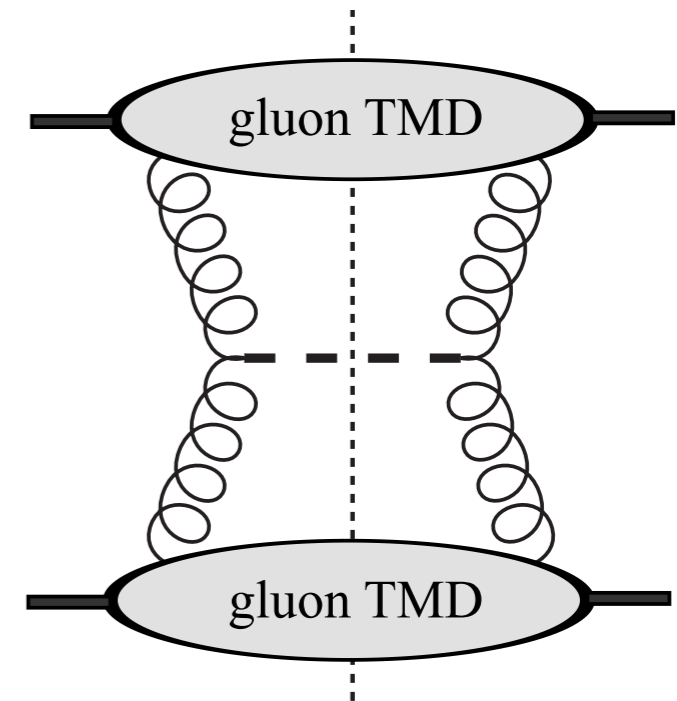
$$ep \rightarrow e + \text{jet} + \text{jet} + X$$



$$pp \rightarrow J/\Psi + \gamma + X$$



$$pp \rightarrow H(\eta_c) + X$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang, *Phys. Rev. Lett.* **108** (2012) 032002]

[W.J. den Dunnen, J.P. Lansberg, C. Pisano, M. Schlegel, *Phys. Rev. Lett.* **112** (2014) 21200]

[J.P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, *Phys. Lett. B* **784** (2018) 217]

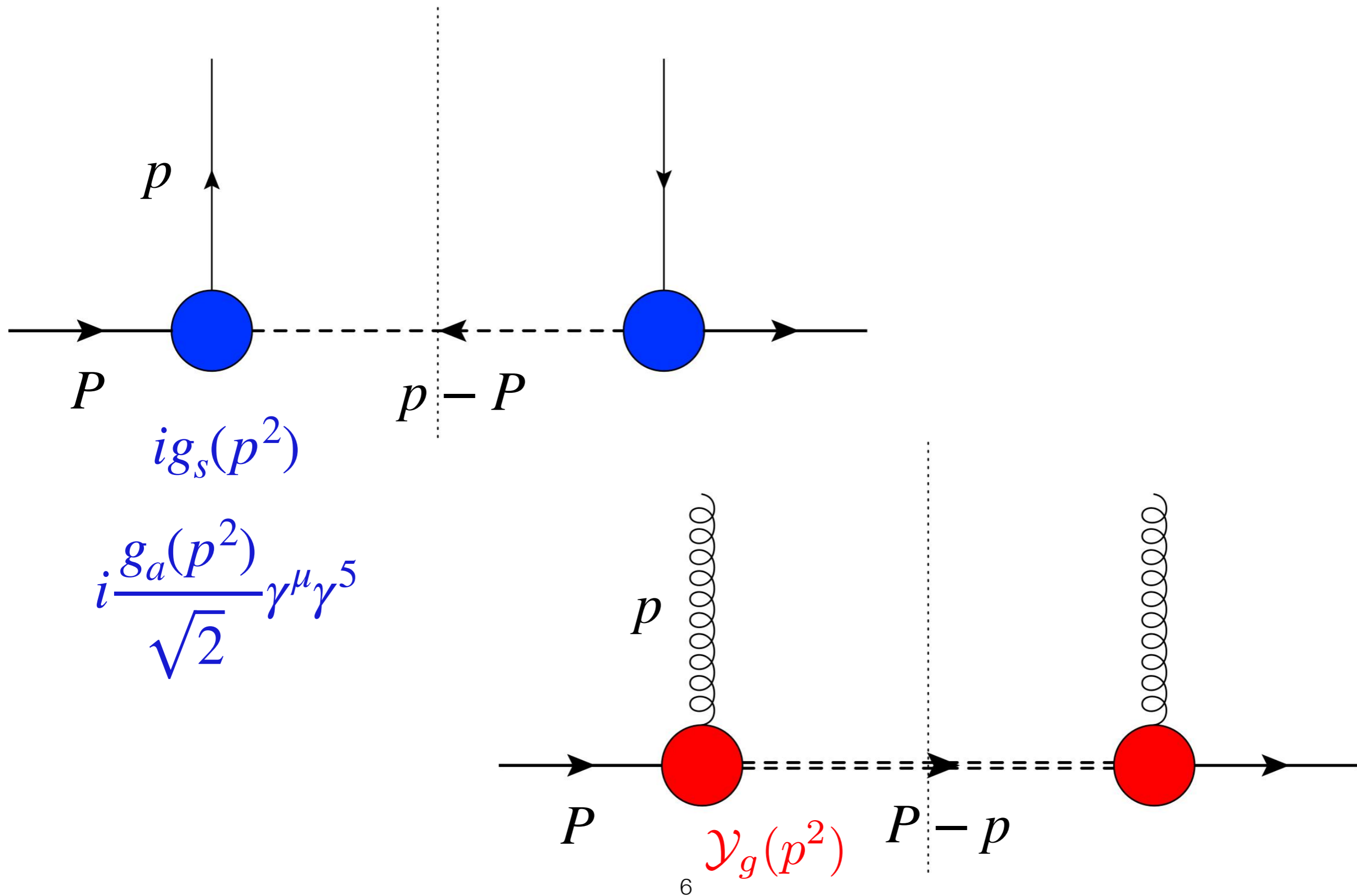
[A. Bacchetta, D. Boer, C. Pisano, P. Tael, arXiv:1809.02056 [hep-ph]]

[U. D'Alesio, C. Flore, F. Murgia, C. Pisano, P. Tael, arXiv:1811.02970 [hep-ph]]

# Motivation

- Need for a flexible model, suited to phenomenology
- Concurrent enhancement of quark-TMD description
- Consistent framework for all parton TMDs

# Effective vertices



# Quark and gluon correlators



## Scalar-diquark spectator

$$\Phi_s = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_s^2}{(p^2 - m_q^2)^2} (\not{p} + m_q) \frac{1 + \gamma^5 \not{P}}{2} (\not{P} + M_H) (\not{p} + m_q)$$



## Axial-vector-diquark spectator

$$\Phi_a = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_a^2}{(p^2 - m_q^2)^2} (\not{p} + m_q) \frac{\gamma^5 \gamma_\mu}{\sqrt{2}} \frac{1 + \gamma^5 \not{P}}{2} (\not{P} + M_H) \frac{\gamma^5 \gamma_\nu}{\sqrt{2}} (\not{p} + m_q) d_T^{\mu\nu} (P - p)$$



## Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[ (\not{P} + M_H) \frac{1 + \gamma^5 \not{P}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M_X) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M_H} \sigma^{\mu\nu} p_\nu$$



# Quark and gluon correlators



## Scalar-diquark spectator

$$\Phi_s = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_s^2}{(p^2 - m_q^2)^2} (\not{p} + m_q) \frac{1 + \gamma^5 \not{P}}{2} (\not{P} + M_H) (\not{p} + m_q)$$



## Axial-vector-diquark spectator

$$\Phi_a = \frac{1}{2(2\pi)^3(1-x)P^+} \frac{g_a^2}{(p^2 - m_q^2)^2} (\not{p} + m_q) \frac{\gamma^5 \gamma_\mu}{\sqrt{2}} \frac{1 + \gamma^5 \not{P}}{2} (\not{P} + M_H) \frac{\gamma^5 \gamma_\nu}{\sqrt{2}} (\not{p} + m_q) d_T^{\mu\nu} (P - p)$$



## Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr \left[ (\not{P} + M_H) \frac{1 + \gamma^5 \not{P}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M_X) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M_H} \sigma^{\mu\nu} p_\nu$$



Selection out of  
12 Dirac structures

# State of the art



First calculation of leading-twist  $T$ -even quark TMDs with scalar and axial-vector di-quarks

[R. Jakob, P. J. Mulders, and J. Rodrigues, Nucl. Phys. **A626**, 937 (1997)]



Gluon TMD PDFs and FFs

[P.J. Mulders, J. Rodrigues, Phys. Rev. **D63** (2001) 094021]  
[J. Rodrigues, PhD thesis (2001)]



Complete calculation of all the leading-twist TMDs with scalar di-quarks

[S. Meissner, A. Metz, and K. Goeke, Phys. Rev. **D76**, 034002 (2007)]



Inclusion of different axial-vector di-quark polarization states and nucleon-parton-spectator form factors

(fit to PDF parametrizations) [A. Bacchetta, F. Conti, M. Radici, Phys. Rev. **D78** (2008) 074010]

(application on azimuthal asymmetries) [A. Bacchetta, M. Radici, F. Conti, M. Guagnelli, Eur. Phys. J. **A45** (2010) 373-388]

**How to improve  
the description?**

# Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

(inspiring idea) [G.R. Goldstein, J.O.G. Hernandez, S. Liuti, Phys.Rev. **D84** (2011) 034007]

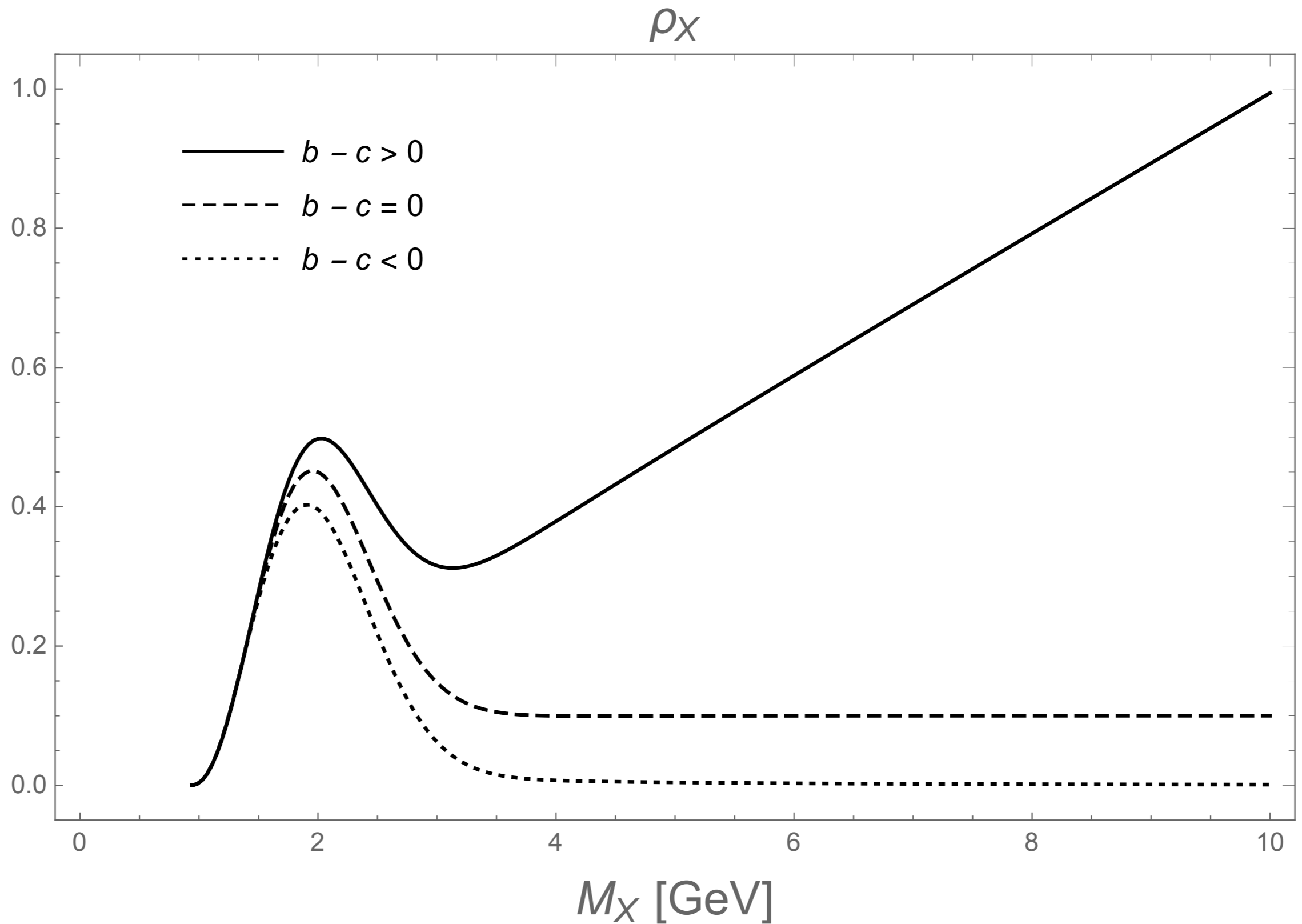
$$\rho_X \left( M_X; \{X^{(\text{pars})}\} \equiv \{A, B, b, c, C, M_D, \sigma\} \right) = \mu^{2b} \left[ \frac{A}{B + \mu^{2c}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - M_D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (b - c)$   
2 $N$ -quark contribution

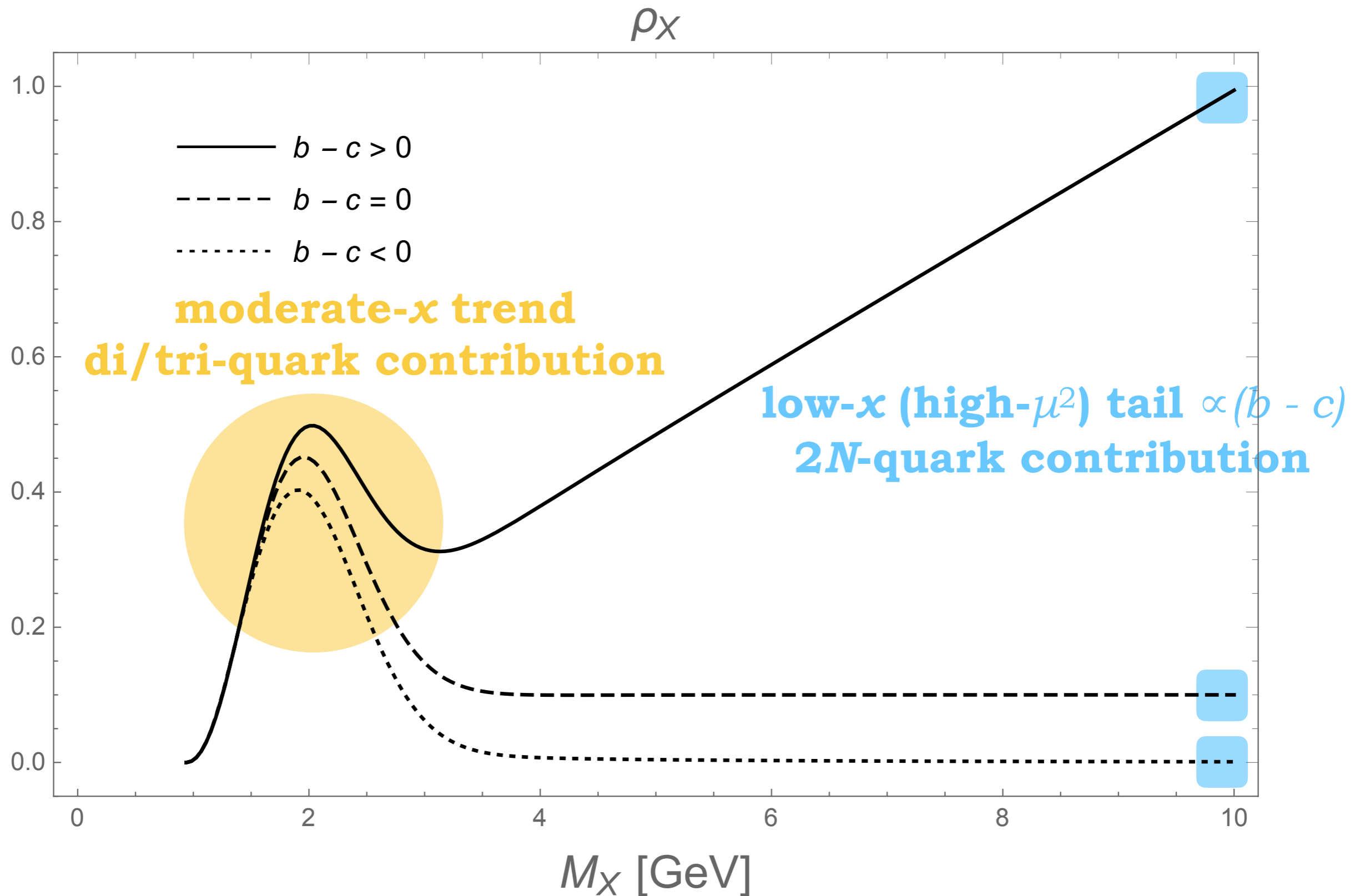
$$\mu^2 = M_X^2 - (M_H - m_{q/g})^2$$

moderate- $x$  trend  
di/tri-quark contribution

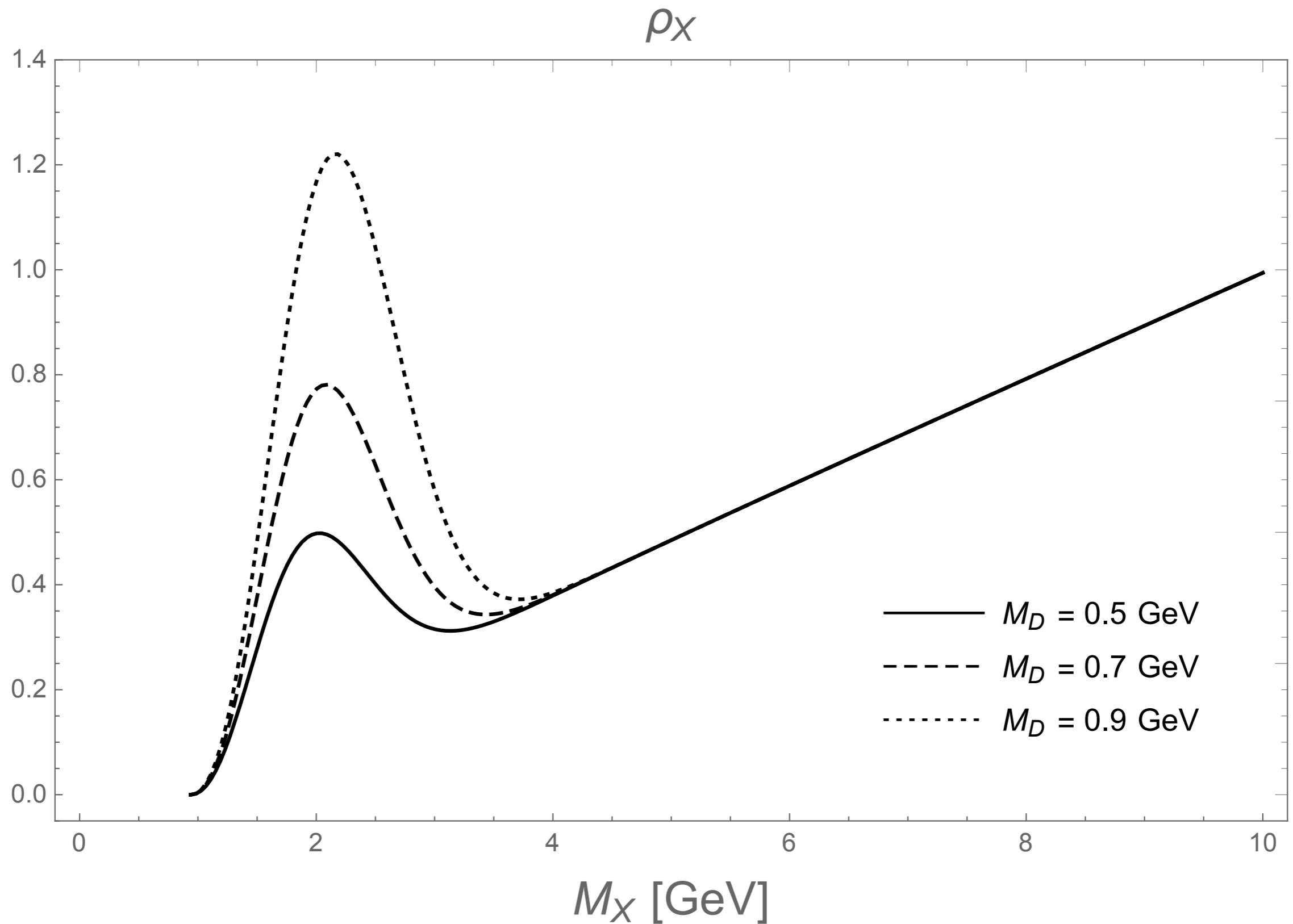
# Spectral function vs $(b - c)$







# Spectral function vs $(b - c)$



# Spectral function vs $M_D$



# Metodology

-  Calculate TMDs from parton correlators
-  Weight TMDs over  $M_X$  via spectral function
-  Integrate over parton  $p_T$  to get PDFs
-  Perform a first explorative analysis:
  - Guess-fit** of unpolarized PDFs
  - Prediction** for helicity PDFs



# Our parametrization

$$F^{uv}(x, \mathbf{p}_T^2) = \int dM_s^{uv} \rho_s(M_s^{uv}) \hat{F}_s^{uv}(x, \mathbf{p}_T^2) + \int dM_a^{uv} \rho_a(M_a^{uv}) \hat{F}_a^{uv}(x, \mathbf{p}_T^2)$$

$$F^{dv}(x, \mathbf{p}_T^2) = \int dM_a^{dv} \rho_a(M_a^{dv}) \hat{F}_a^{dv}(x, \mathbf{p}_T^2)$$

$$F^{\text{sea}}(x, \mathbf{p}_T^2) = \int dM_s^{\text{sea}} \rho_s(M_s^{\text{sea}}) \hat{F}_s^{\text{sea}}(x, \mathbf{p}_T^2)$$

$$F^g(x, \mathbf{p}_T^2) = \int dM^g \rho_g(M^g) \hat{F}^g(x, \mathbf{p}_T^2)$$

# Our parametrization

$$F^{u_v}(x, \mathbf{p}_T^2) = \int dM_s^{u_v} \rho_s(M_s^{u_v}) \hat{F}_s^{u_v}(x, \mathbf{p}_T^2) + \int dM_a^{u_v} \rho_a(M_a^{u_v}) \hat{F}_a^{u_v}(x, \mathbf{p}_T^2)$$

$$F^{d_v}(x, \mathbf{p}_T^2) = \int dM_a^{d_v} \rho_a(M_a^{d_v}) \hat{F}_a^{d_v}(x, \mathbf{p}_T^2)$$

$$F^{\text{sea}}(x, \mathbf{p}_T^2) = \int dM_s^{\text{sea}} \rho_s(M_s^{\text{sea}}) \hat{F}_s^{\text{sea}}(x, \mathbf{p}_T^2)$$

$$F^g(x, \mathbf{p}_T^2) = \int dM^g \rho_g(M^g) \hat{F}^g(x, \mathbf{p}_T^2)$$

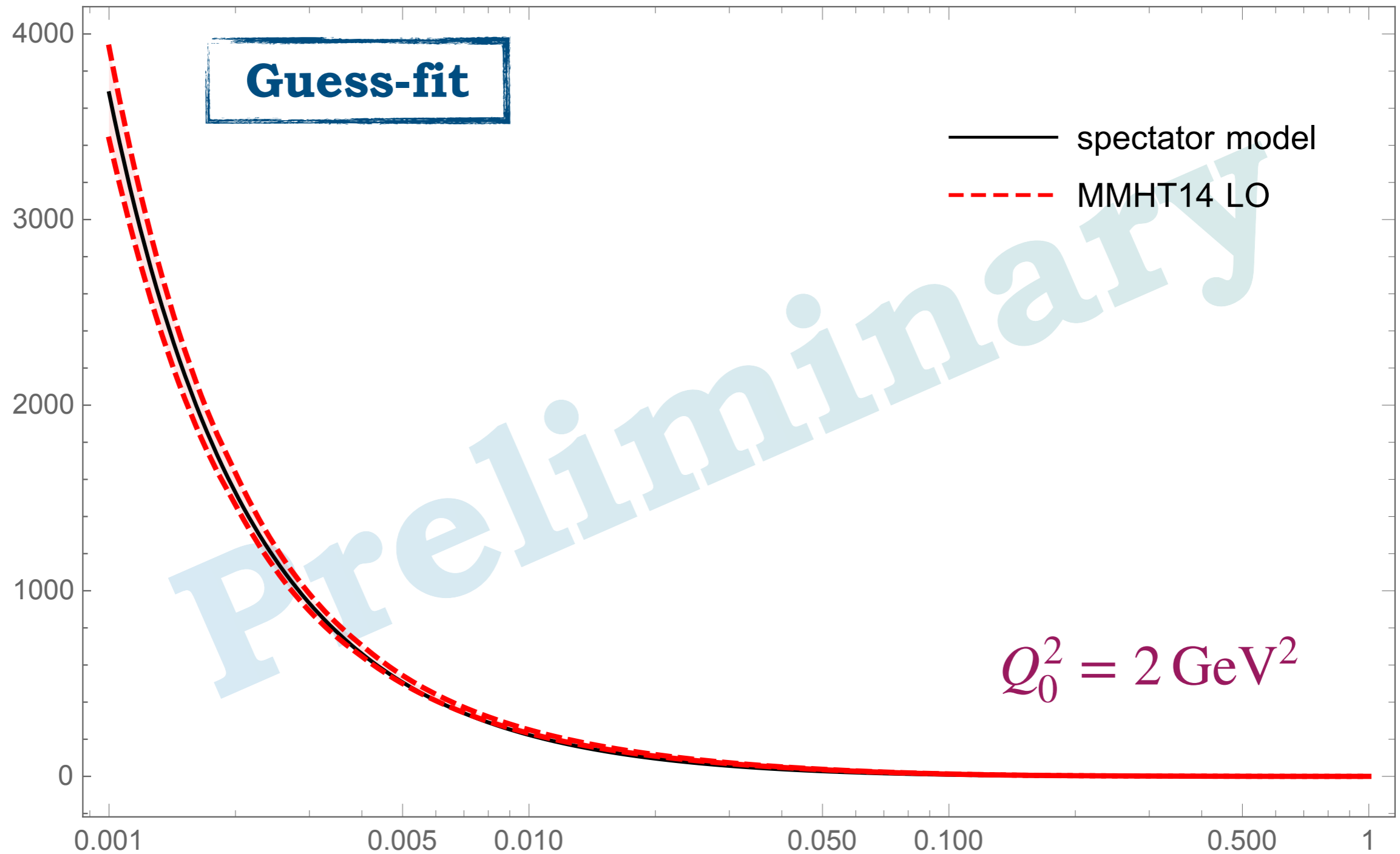
**scalar di-quark**

**axial-vector di-quark**

**spin-1/2**

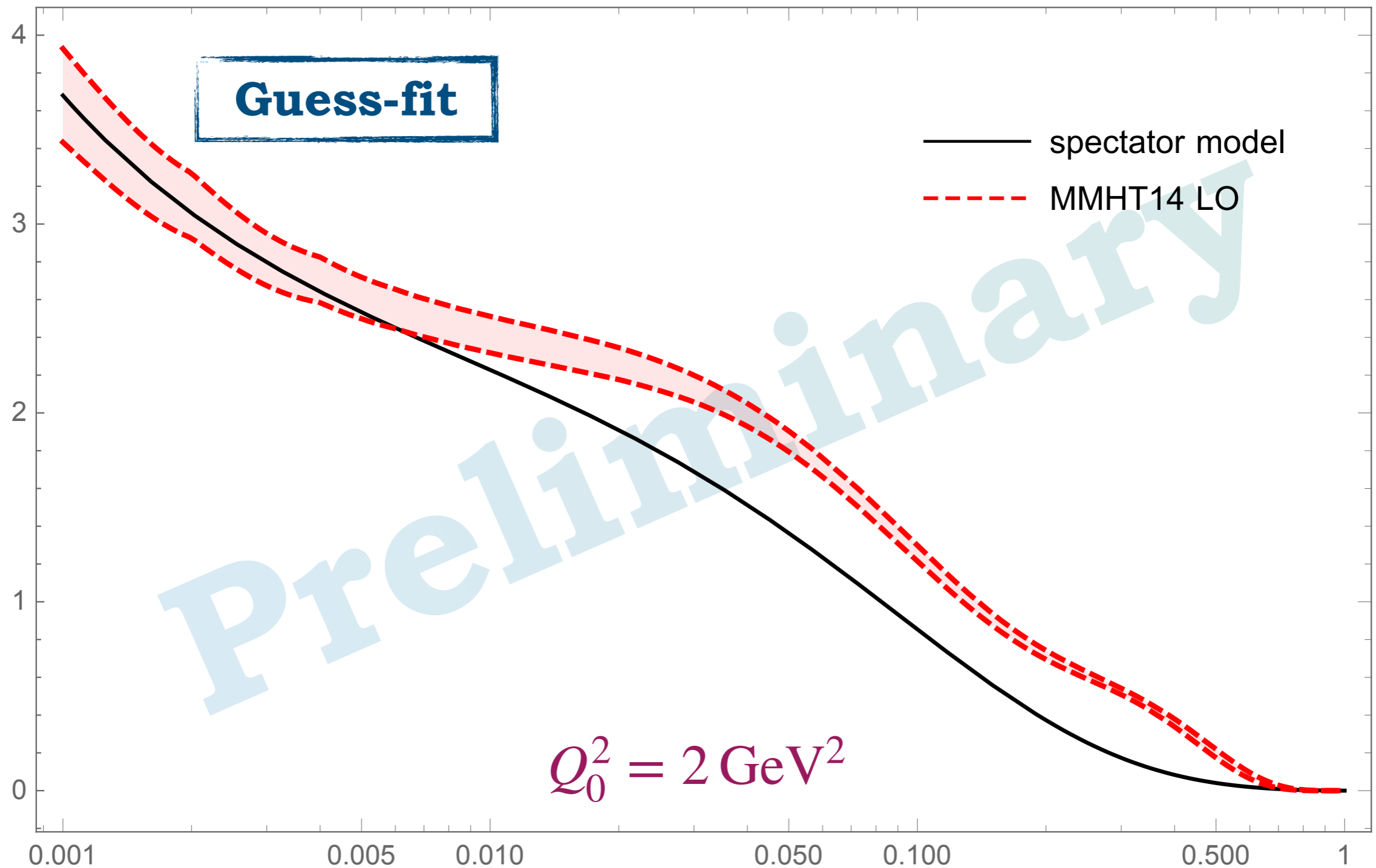
# Unpolarized gluon PDF

$$f_1^g(x)$$



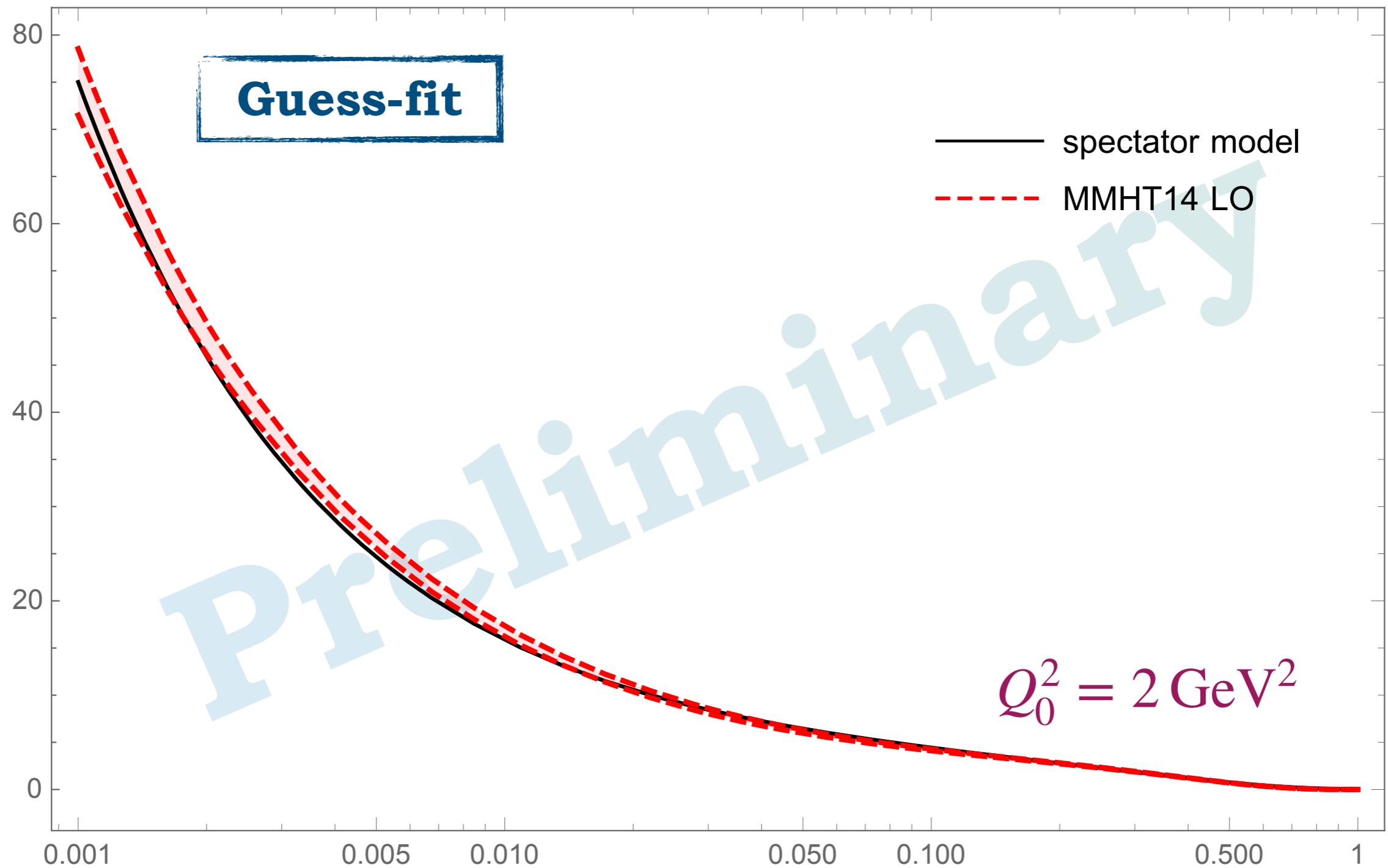
# Unpolarized gluon PDF

$$xf_1^g(x)$$



# Unpolarized valence-up PDF

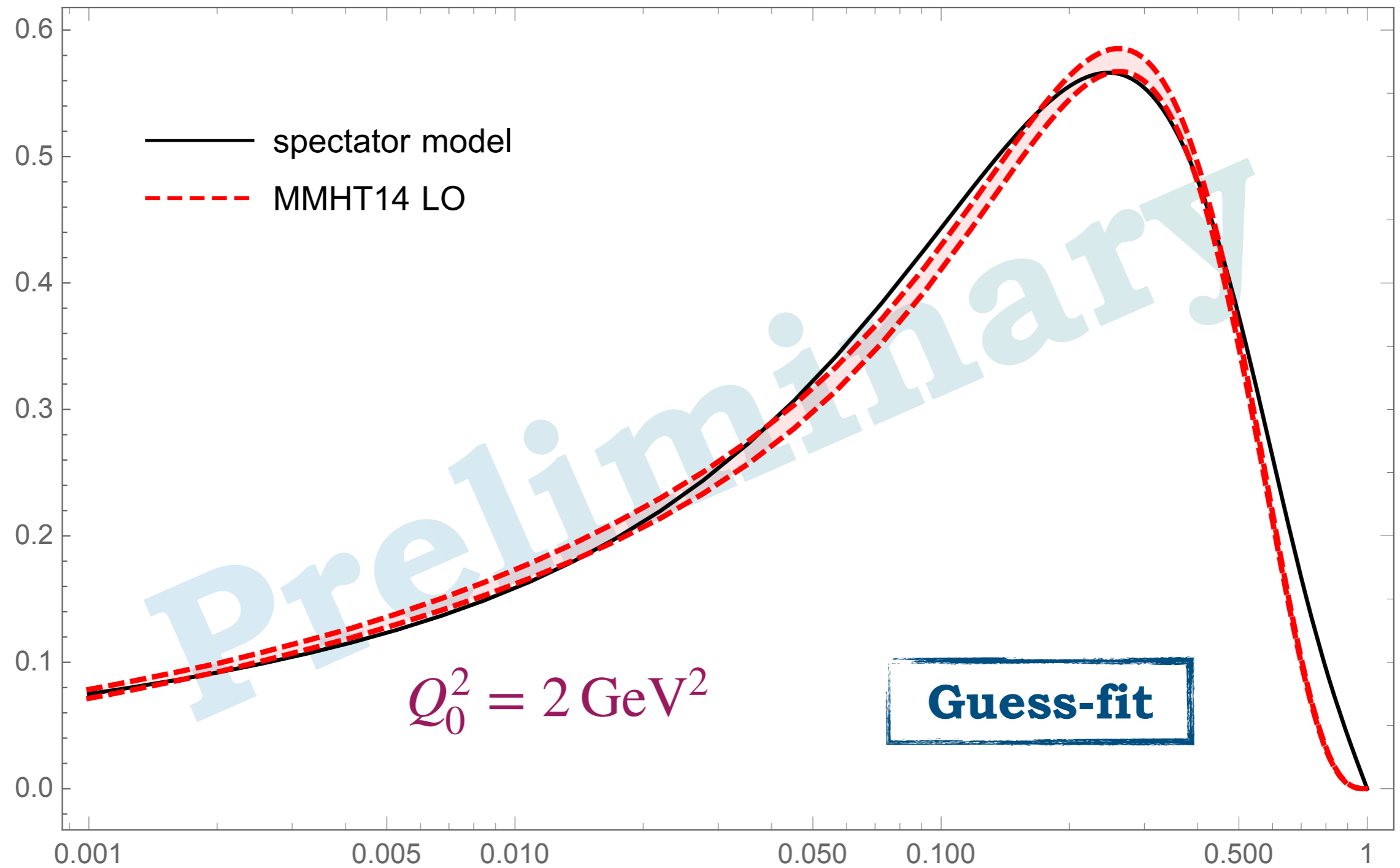
$$f_1^{u_v}(x)$$



x

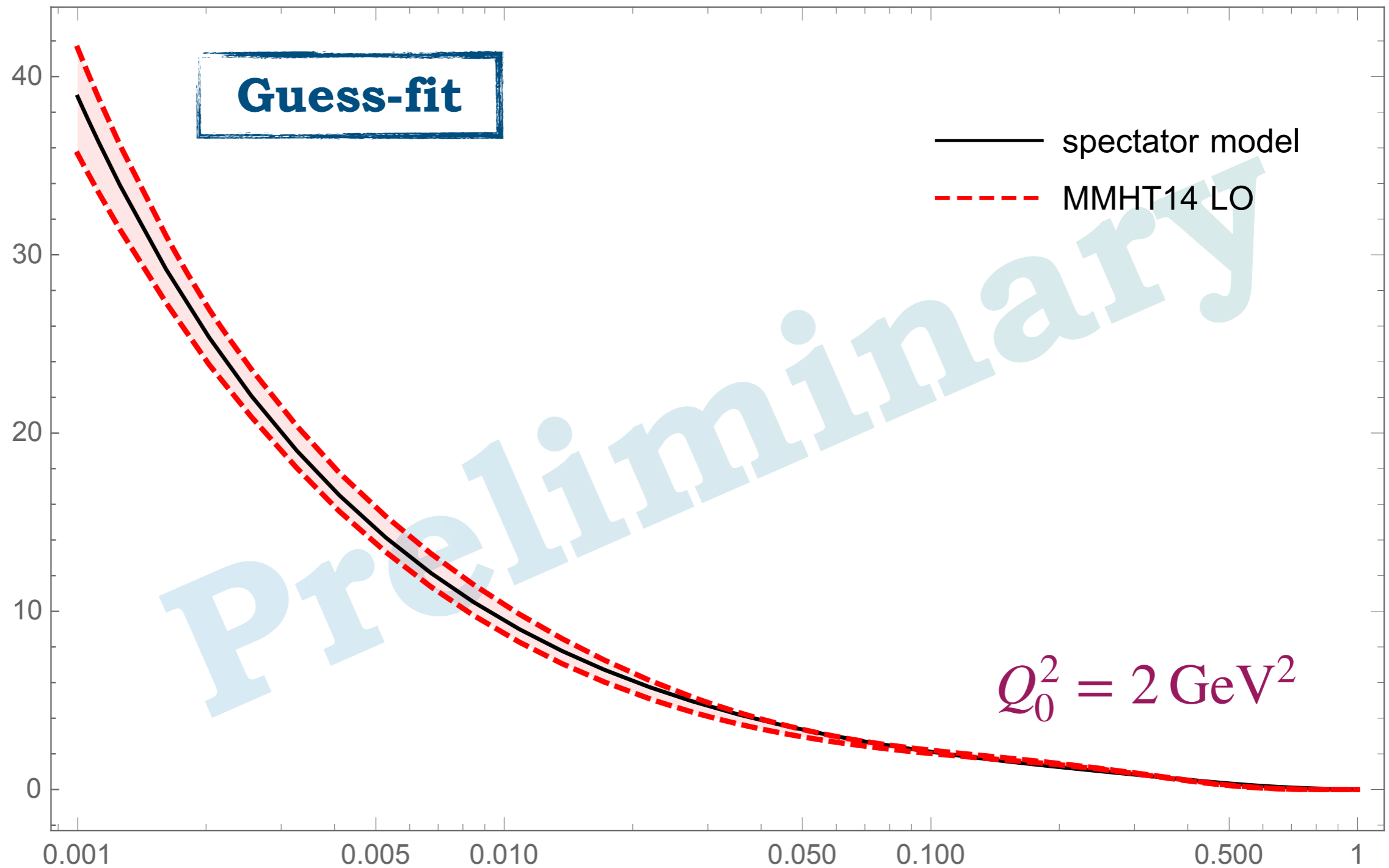
# Unpolarized valence-up PDF

$$xf_1^{u_v}(x)$$



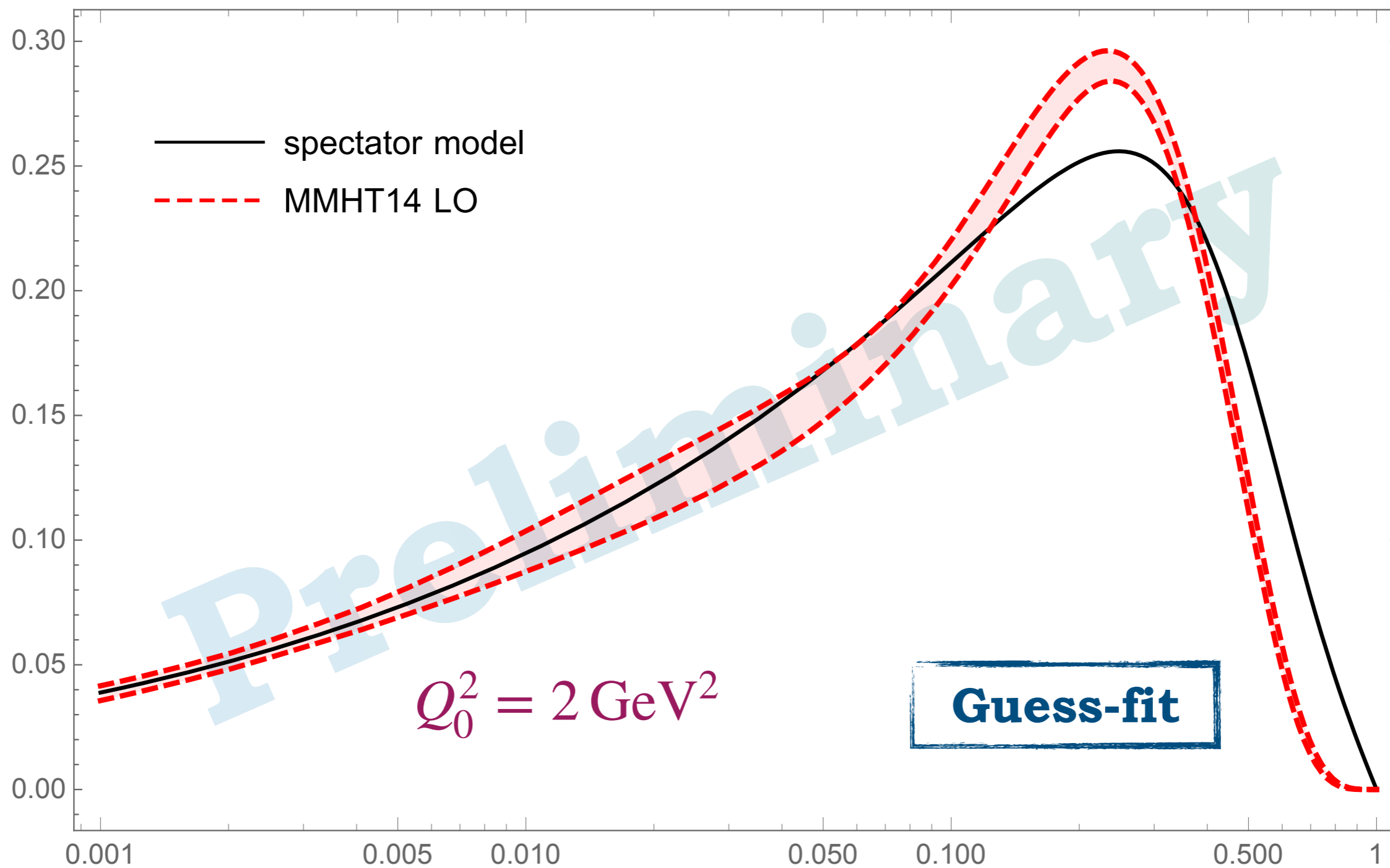
# Unpolarized valence-down PDF

$$f_1^{d_v}(x)$$



# Unpolarized valence-down PDF

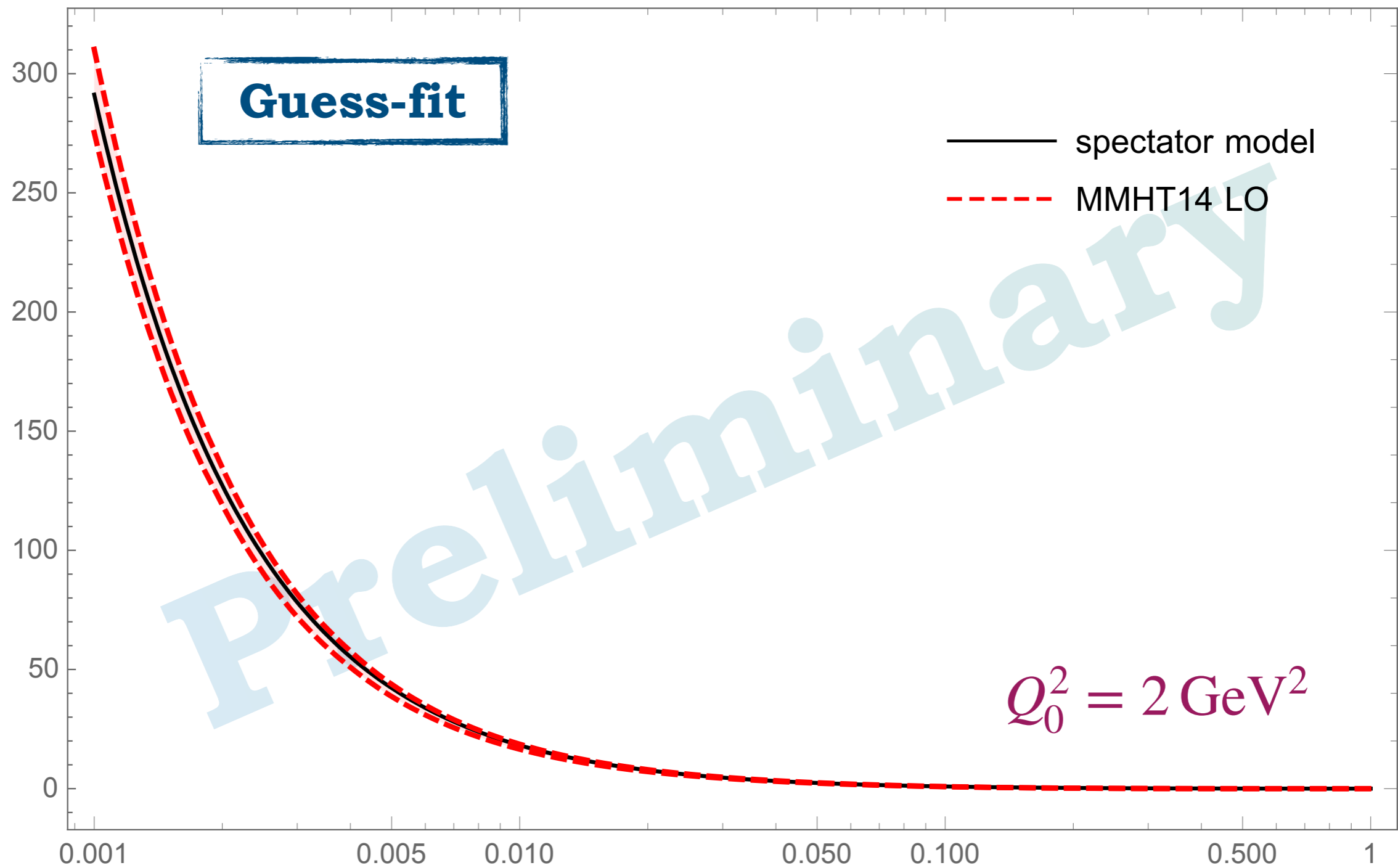
$$xf_1^{d_v}(x)$$





# Unpolarized sea PDF

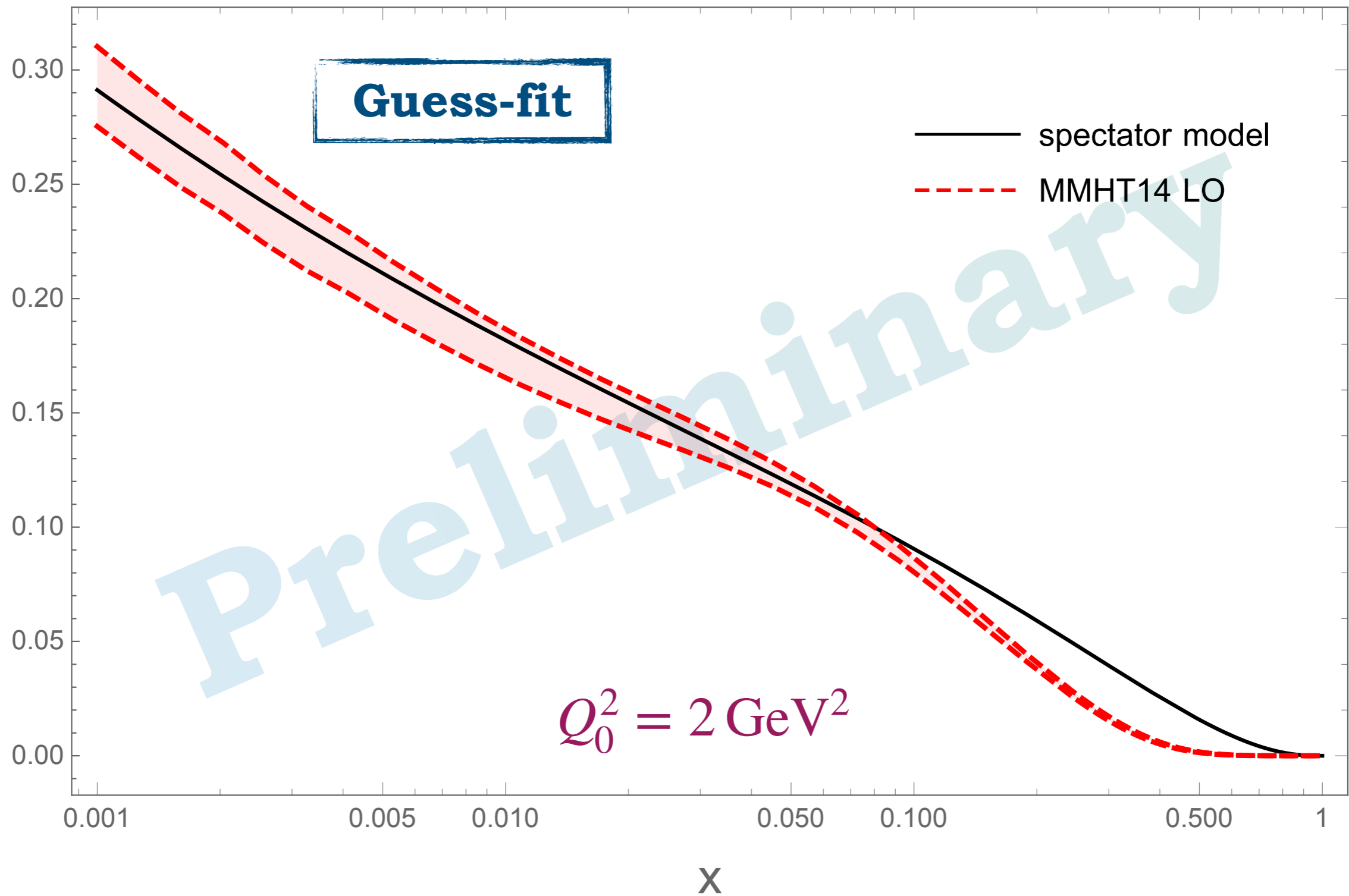
$$f_1^{\text{sea}}(x)$$



x

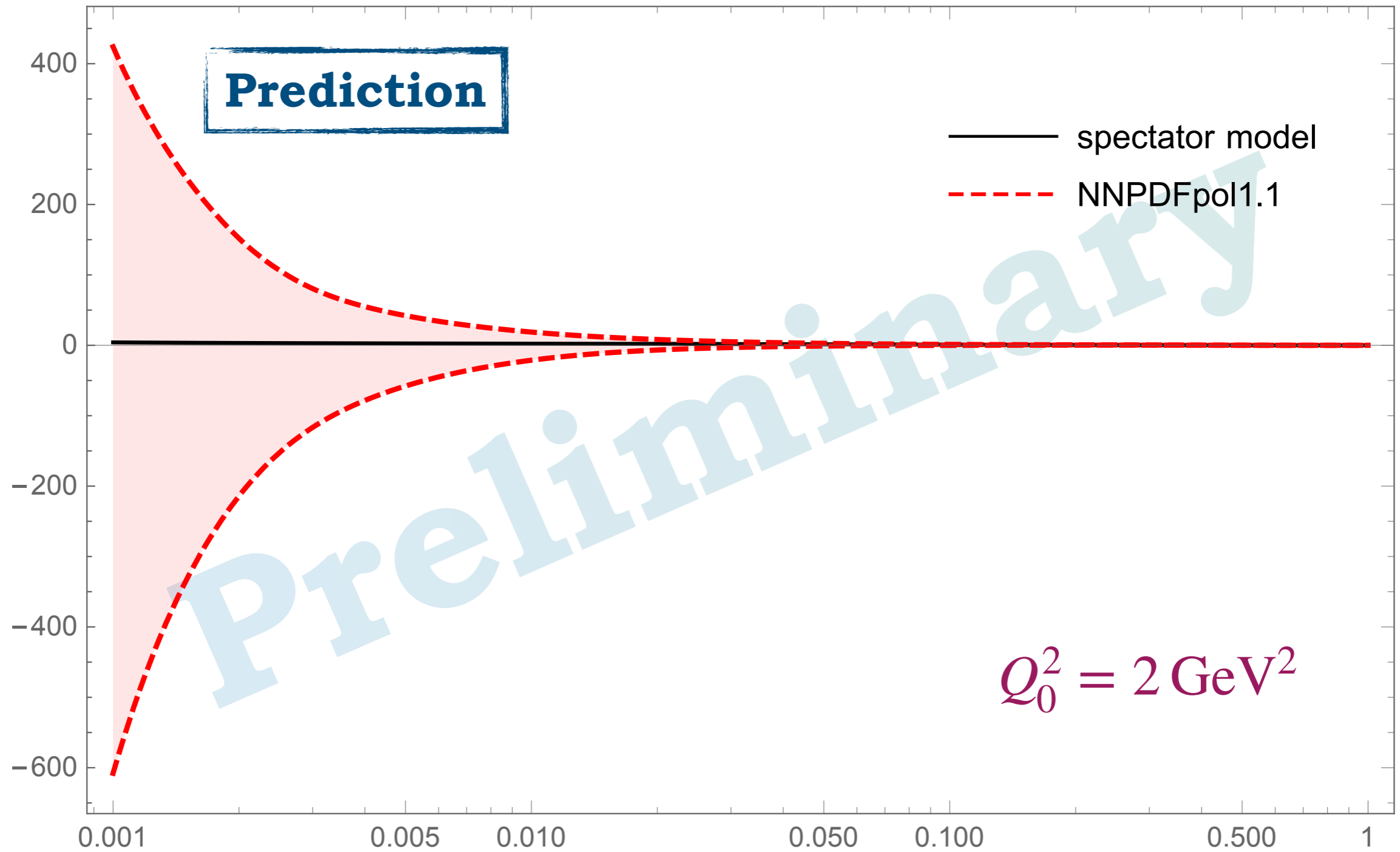
# Unpolarized sea PDF

$$xf_1^{\text{sea}}(x)$$



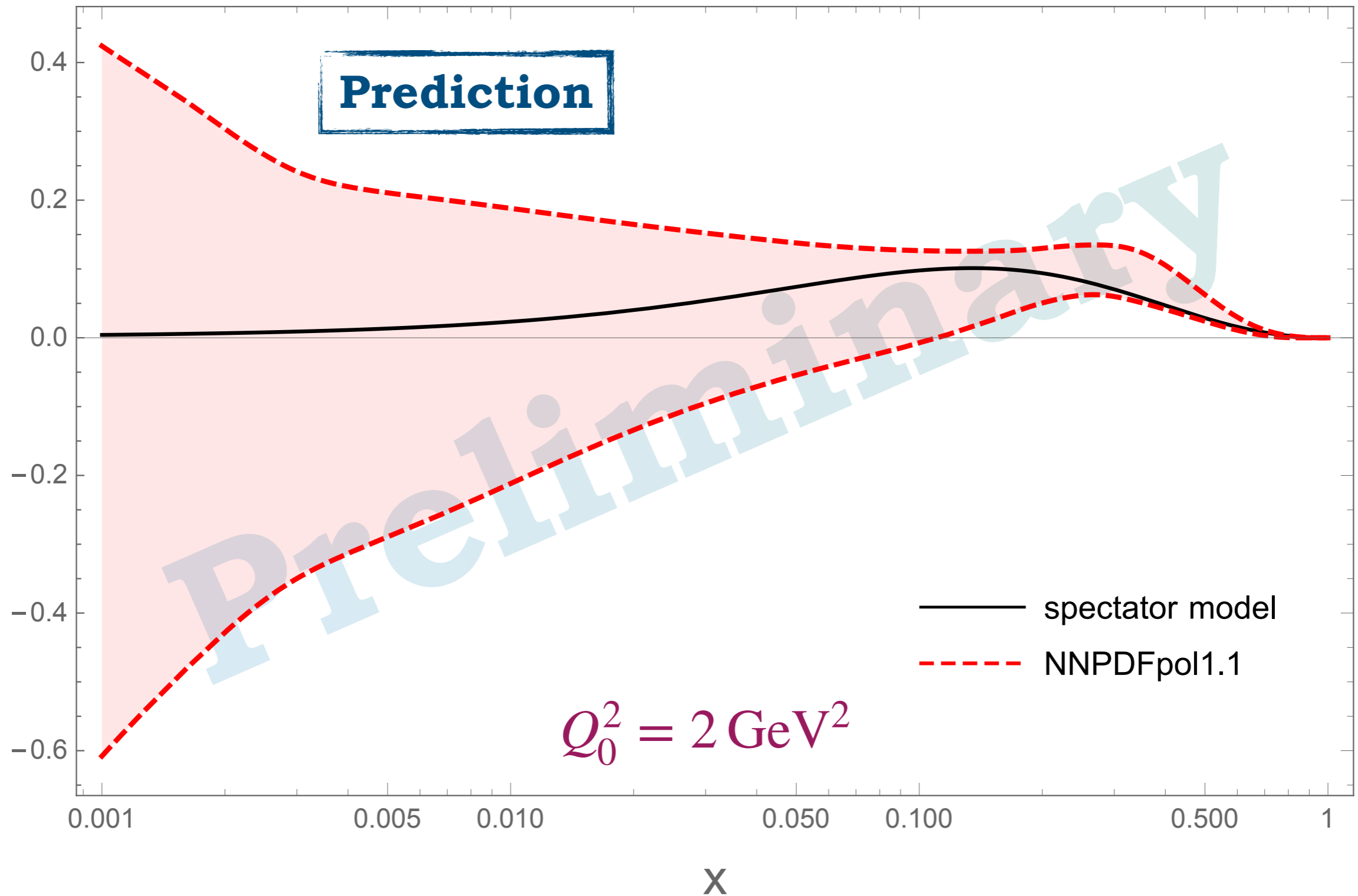
# Helicity gluon PDF

$$g_1^g(x)$$



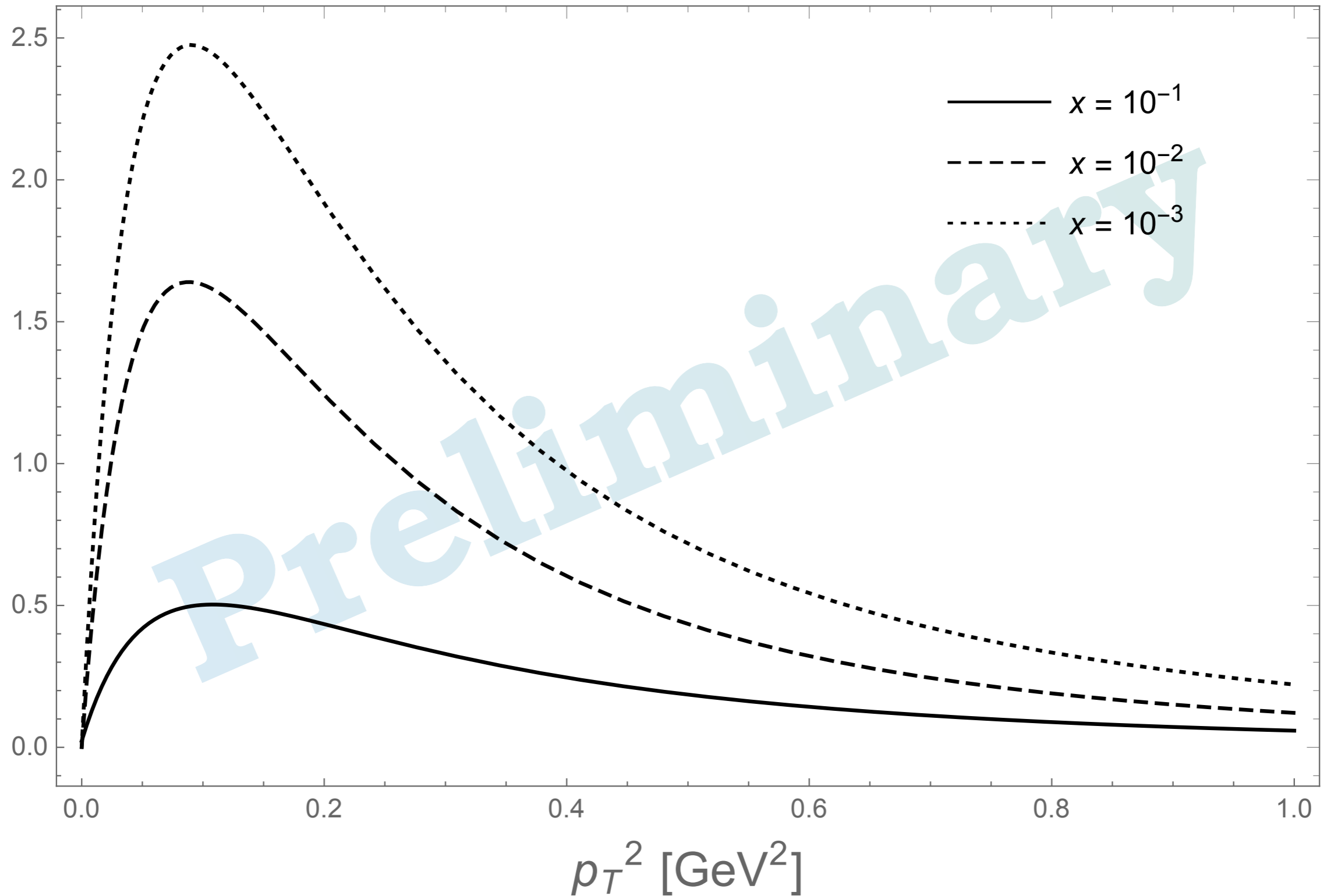
# Helicity gluon PDF

$$xg_1^g(x)$$



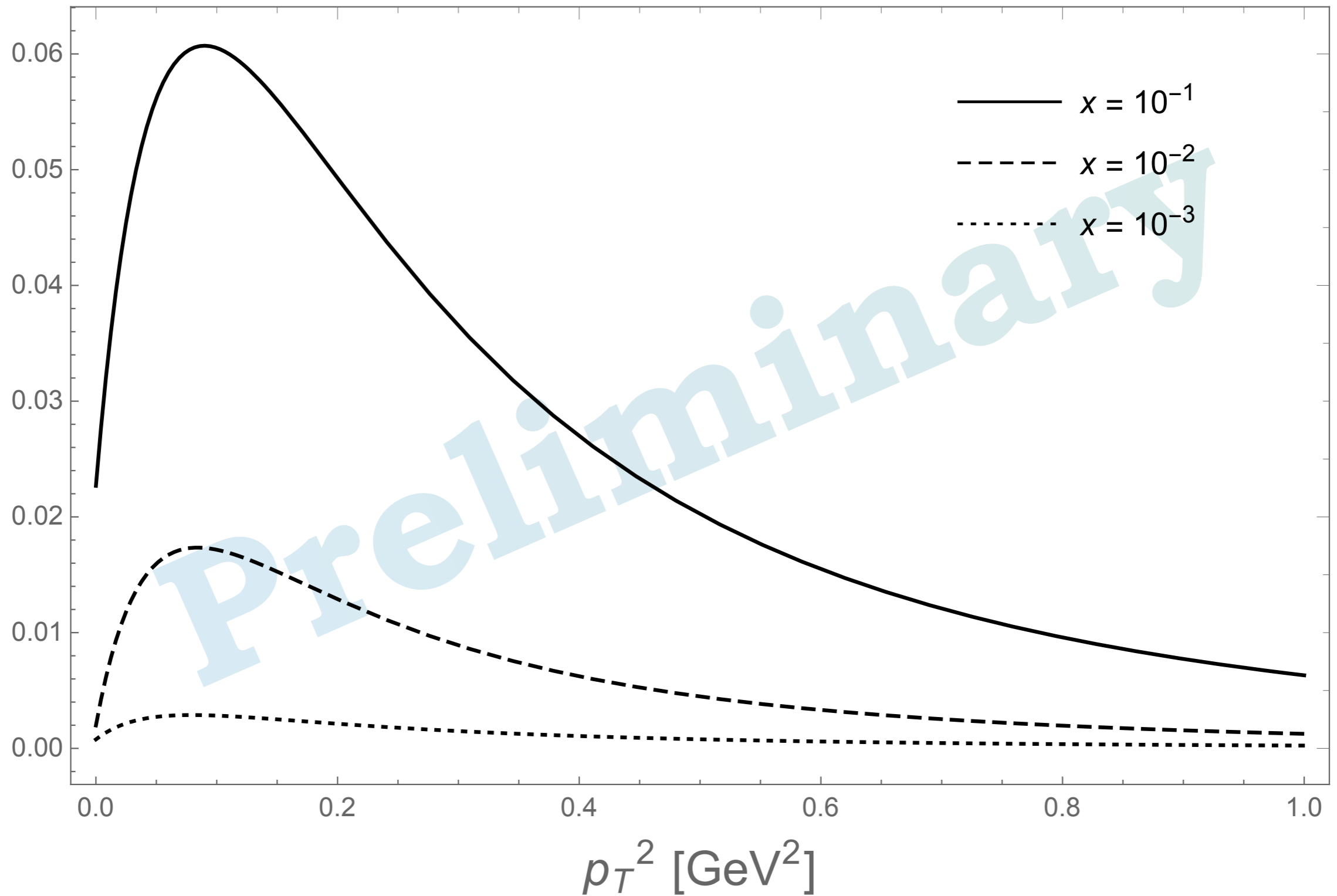
# Unpolarized gluon TMD

$$xf_1^g(x, p_T^2)$$



# Helicity gluon TMD

$$xg_{1L}^g(x, p_T^2)$$



# Conclusions & Outlook

- Systematic calculation of all twist-2  $T$ -even parton TMDs
- Spectral mass to catch small- and moderate- $x$  effects
- Encouraging results from **guess-fit** of  $f_1$  + **prediction** for  $g_1$
- Simultaneous fit** of  $f_1$  and  $g_1$  with **BFKL** input on small- $x$  tail
- Effect of collinear evolution to be gauged
- $T$ -odd TMDs with one-gluon exchange in eikonal approximation
- Extraction of gluon TMDs from *golden channels*



**...grazie!** 🙄



# **Backup slides**

# Spectator-model gluon TMDs (1)

$$\begin{aligned}
 \hat{f}_1^g(x, \mathbf{p}_T^2) = & \left[ 4g_1^2 M_H^2 (x^2 (M_X - M_H(1-x))^2 + \mathbf{p}_T^2 ((x-2)x + 2)) \right. \\
 & - 4g_1 g_2 M_H x^2 (M_H + M_X) ((M_X - M_H(1-x))^2 + \mathbf{p}_T^2) \\
 & + g_2^2 (\mathbf{p}_T^2 x (M_H^2 (3x-2) + 2M_H M_X x + M_X^2 (x+2)) \\
 & \left. + x^2 (M_H + M_X)^2 (M_X - M_H(1-x))^2 + 2(\mathbf{p}_T^2)^2 \right] \\
 & / 4(2\pi)^3 M_H^2 x (L_g^2(m_g^2) + \mathbf{p}_T^2)^2
 \end{aligned}$$

$$\begin{aligned}
 \hat{g}_{1L}^g(x, \mathbf{p}_T^2) = & \left[ (2g_1 M_H - g_2 (M_H + M_X)) (2g_1 M_H (x (M_X - M_H(1-x))^2 - \mathbf{p}_T^2 (x-2)) \right. \\
 & + g_2 (-M_H^2 x^3 (M_H + M_X) + \mathbf{p}_T^2 x (M_X - 3M_H) + 2\mathbf{p}_T^2 (M_H - M_X) \\
 & \left. + 2M_H x^2 (M_H - M_X) (M_H + M_X) - x (M_H - M_X)^2 (M_H + M_X)) \right] \\
 & / 4(2\pi)^3 M_H^2 (L_g^2(m_g^2) + \mathbf{p}_T^2)^2
 \end{aligned}$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

# Spectator-model gluon TMDs (2)

$$g_{1T}^g(x, \mathbf{p}_T^2) = \left[ \left( (2g_1 M_H - g_2(M_H + M_X))((M_X - M_H(1-x)) \right. \right. \\ \left. \left. (-2g_1 M_H(1-x) - g_2 M_X x) - g_2 \mathbf{p}_T^2) \right) \right] \\ / (2(2\pi)^3 M_H (L_X^2(m_g^2) + \mathbf{p}_T^2)^2)$$

$$h_1^{\perp g}(x, \mathbf{p}_T^2) = \left[ (4g_1^2 M_H^2(1-x) + g_2^2(x(M_X^2 - M_H^2(1-x)) + \mathbf{p}_T^2)) \right] \\ / ((2\pi)^3 x (L_X^2(m_g^2) + \mathbf{p}_T^2)^2)$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

# Spectator-model quark TMDs scalar di-quark (1)

$$f_1^{q(s)}(x, \mathbf{p}_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{[(m + xM)^2 + \mathbf{p}_T^2] (1 - x)}{2 [\mathbf{p}_T^2 + L_s^2(m^2)]^2}$$

$$g_{1L}^{q(s)}(x, \mathbf{p}_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{[(m + xM)^2 - \mathbf{p}_T^2] (1 - x)}{2 [\mathbf{p}_T^2 + L_s^2(m^2)]^2}$$

$$g_{1T}^{q(s)}(x, \mathbf{p}_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{M (m + xM) (1 - x)}{[\mathbf{p}_T^2 + L_s^2(m^2)]^2}$$

$$h_{1L}^{\perp q(s)}(x, \mathbf{p}_T^2) = -g_{1T}^{q(s)}(x, \mathbf{p}_T^2)$$

$$L_X^2(m^2) = xM_X^2 + (1 - x)m^2 - x(1 - x)M_H^2$$

# Spectator-model quark TMDs

## scalar di-quark (2)

$$h_{1T}^{q(s)}(x, \mathbf{p}_T^2) = f_1^{q(s)}(x, \mathbf{p}_T^2)$$

$$h_{1T}^{\perp q(s)}(x, \mathbf{p}_T^2) = -\frac{g_s^2}{(2\pi)^3} \frac{M^2 (1-x)}{[\mathbf{p}_T^2 + L_s^2(m^2)]^2}$$

$$\begin{aligned} h_1^{q(s)}(x, \mathbf{p}_T^2) &= h_{1T}^{q(s)}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T^2}{2M^2} h_{1T}^{\perp q(s)}(x, \mathbf{p}_T^2) \\ &= \frac{g_s^2}{(2\pi)^3} \frac{(m + xM)^2 (1-x)}{2[\mathbf{p}_T^2 + L_s^2(m^2)]^2} = \frac{1}{2} \left( f_1^{q(s)}(x, \mathbf{p}_T^2) + g_1^{q(s)}(x, \mathbf{p}_T^2) \right) \end{aligned}$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

# Spectator-model quark TMDs axial-vector di-quark (1)

$$f_1^{q(a)}(x, \mathbf{p}_T^2) = \frac{g_a^2}{(2\pi)^3} \frac{\mathbf{p}_T^2 (1+x^2) + (m+xM)^2 (1-x)^2}{2 [\mathbf{p}_T^2 + L_a^2(m^2)]^2 (1-x)}$$

$$g_{1L}^{q(a)}(x, \mathbf{p}_T^2) = \frac{g_a^2}{(2\pi)^3} \frac{\mathbf{p}_T^2 (1+x^2) - (m+xM)^2 (1-x)^2}{2 [\mathbf{p}_T^2 + L_a^2(m^2)]^2 (1-x)}$$

$$g_{1T}^{q(a)}(x, \mathbf{p}_T^2) = \frac{g_a^2}{(2\pi)^3} \frac{M x (m+xM)}{[\mathbf{p}_T^2 + L_a^2(m^2)]^2}$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

# Spectator-model quark TMDs axial-vector di-quark (2)

$$h_{1L}^{\perp q(a)}(x, \mathbf{p}_T^2) = g_{1T}^{q(a)}(x, \mathbf{p}_T^2)/x ,$$

$$h_{1T}^{q(a)}(x, \mathbf{p}_T^2) = -\frac{g_a^2}{(2\pi)^3} \frac{x \mathbf{p}_T^2}{[\mathbf{p}_T^2 + L_a^2(m^2)]^2 (1-x)}$$

$$h_{1T}^{\perp q(a)}(x, \mathbf{p}_T^2) = 0$$

$$h_1^{q(a)}(x, \mathbf{p}_T^2) = h_{1T}^{q(a)}(x, \mathbf{p}_T^2)$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M_H^2$$

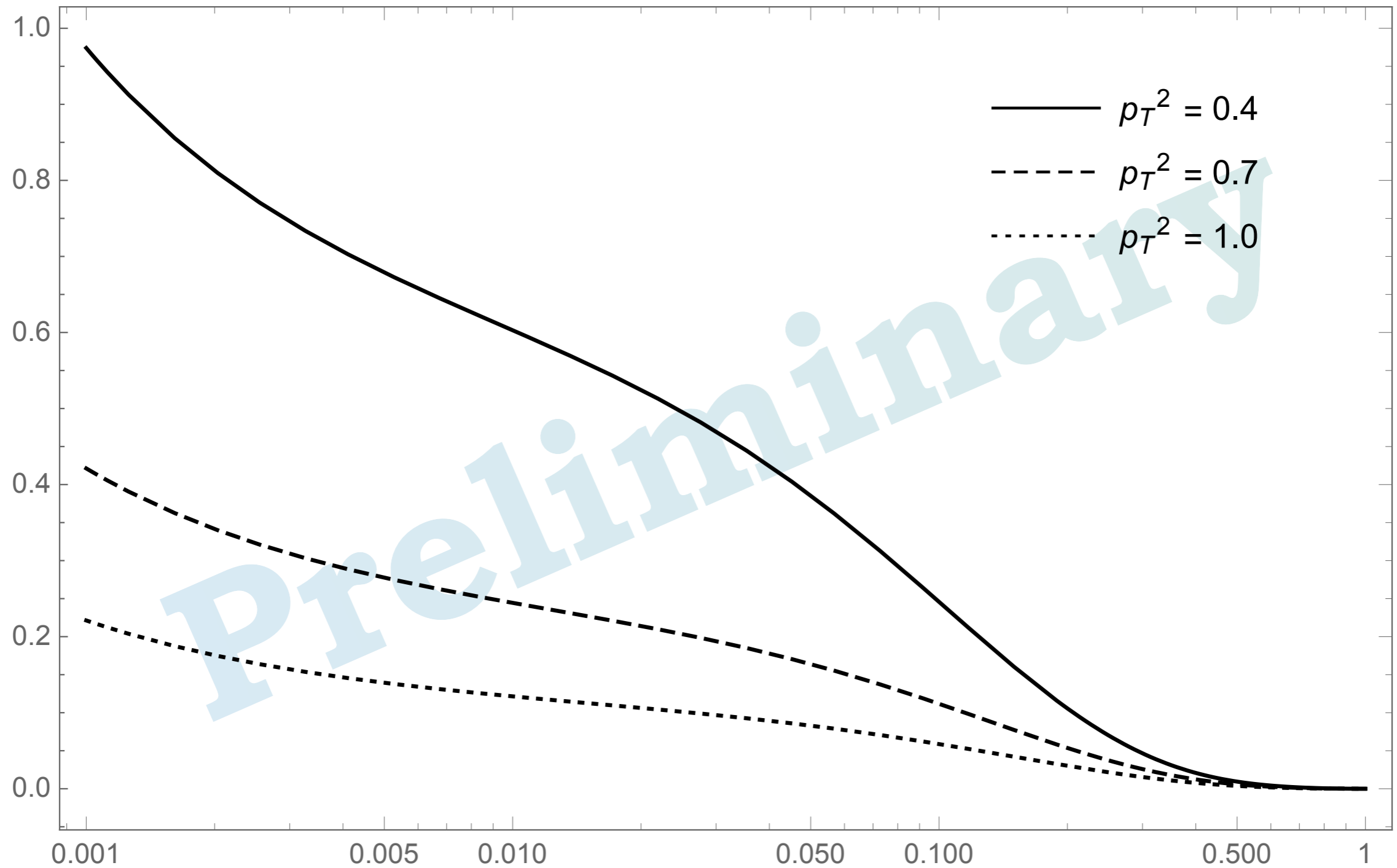
# Nucleon-parton-spectator effective vertex

$$g_X(p^2) = \begin{cases} g_X^{p.l.} & \text{pointlike} \\ g_X^{dip} \frac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2} & \text{dipolar} \\ g_X^{exp} e^{(p^2 - m^2)/\Lambda_X^2} & \text{exponential} \end{cases}$$



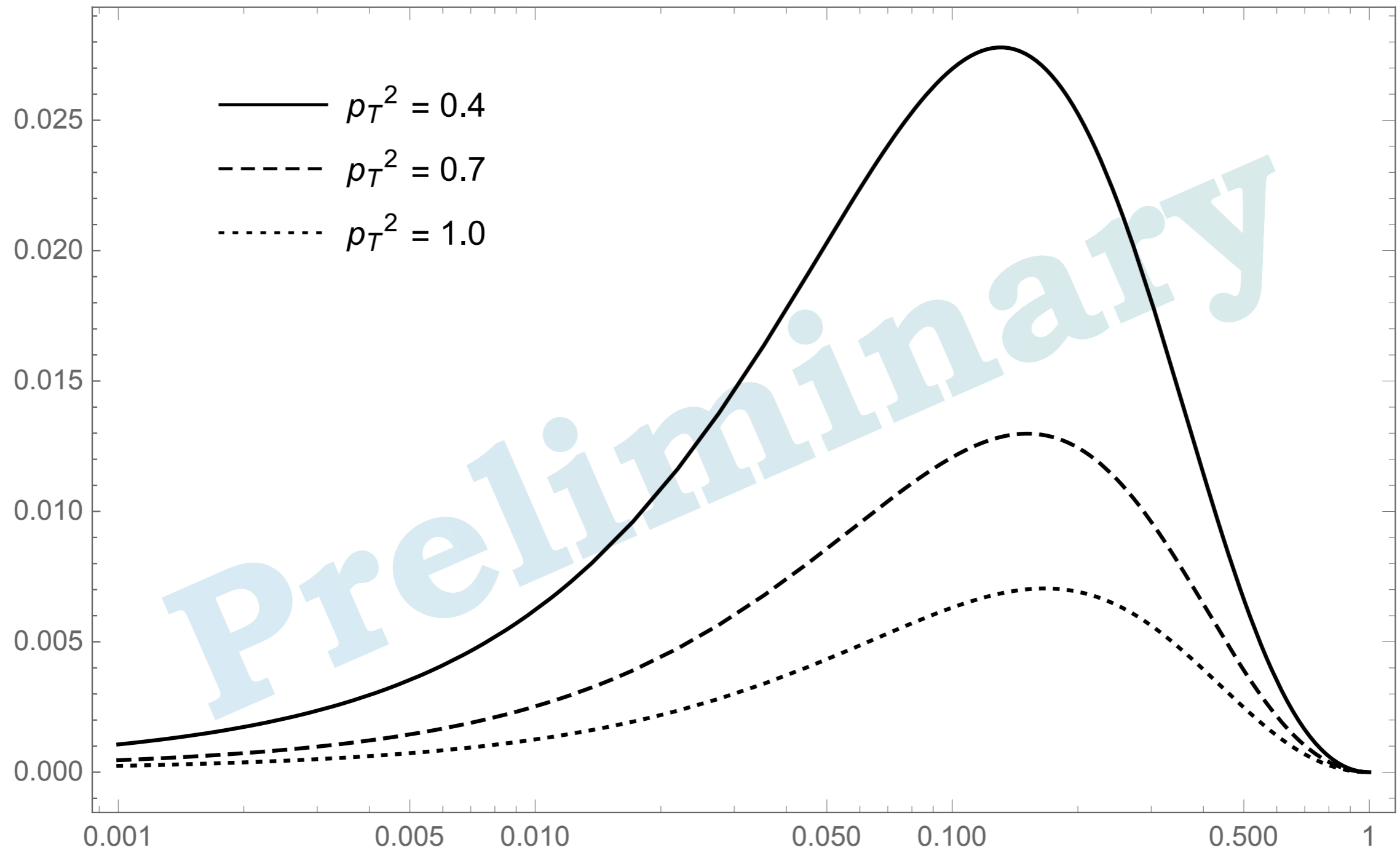
# Unpolarized gluon TMD

$$xf_1^g(x, p_T^2)$$



# Helicity gluon TMD

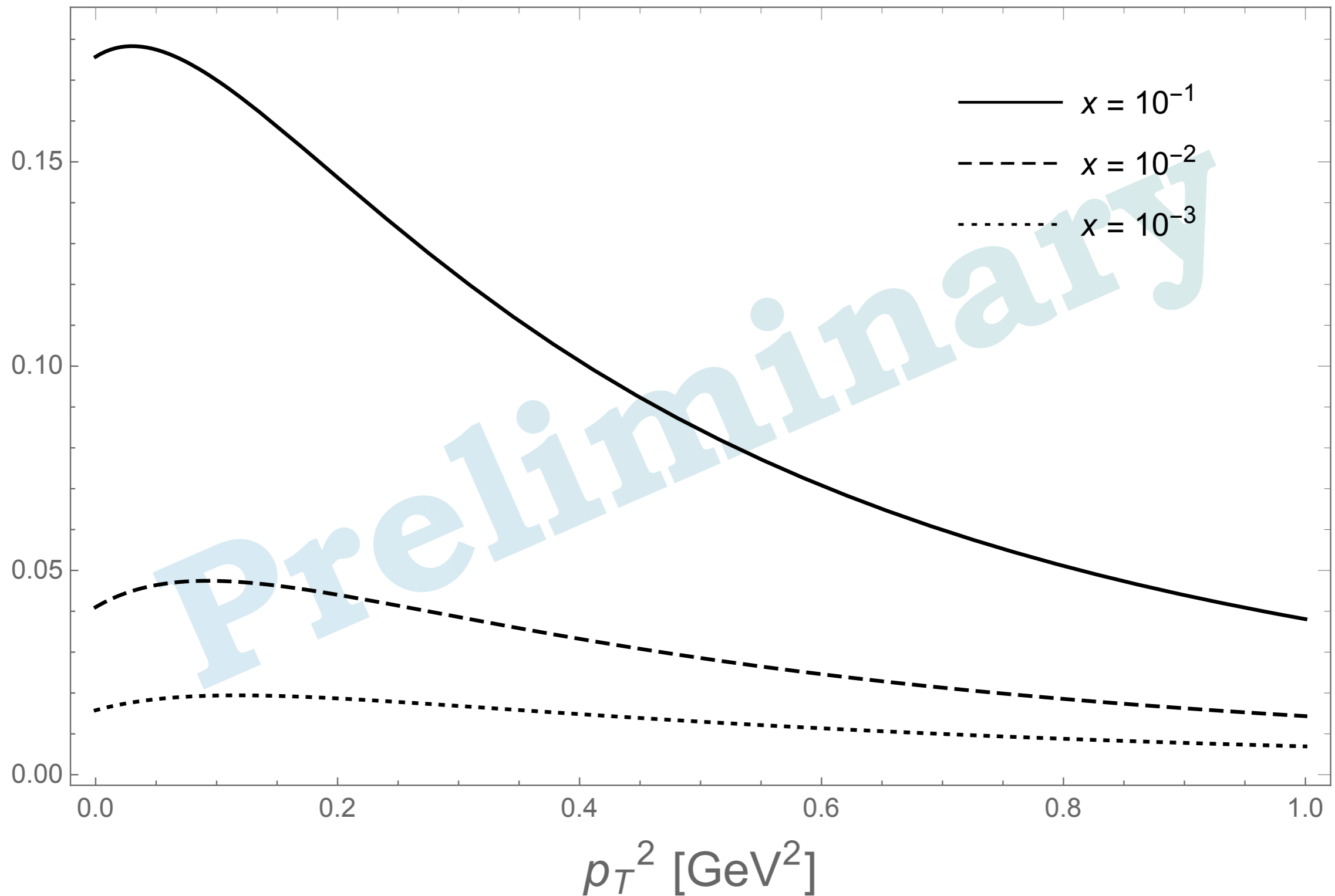
$$xg_1^g(x, p_T^2)$$



Preliminary

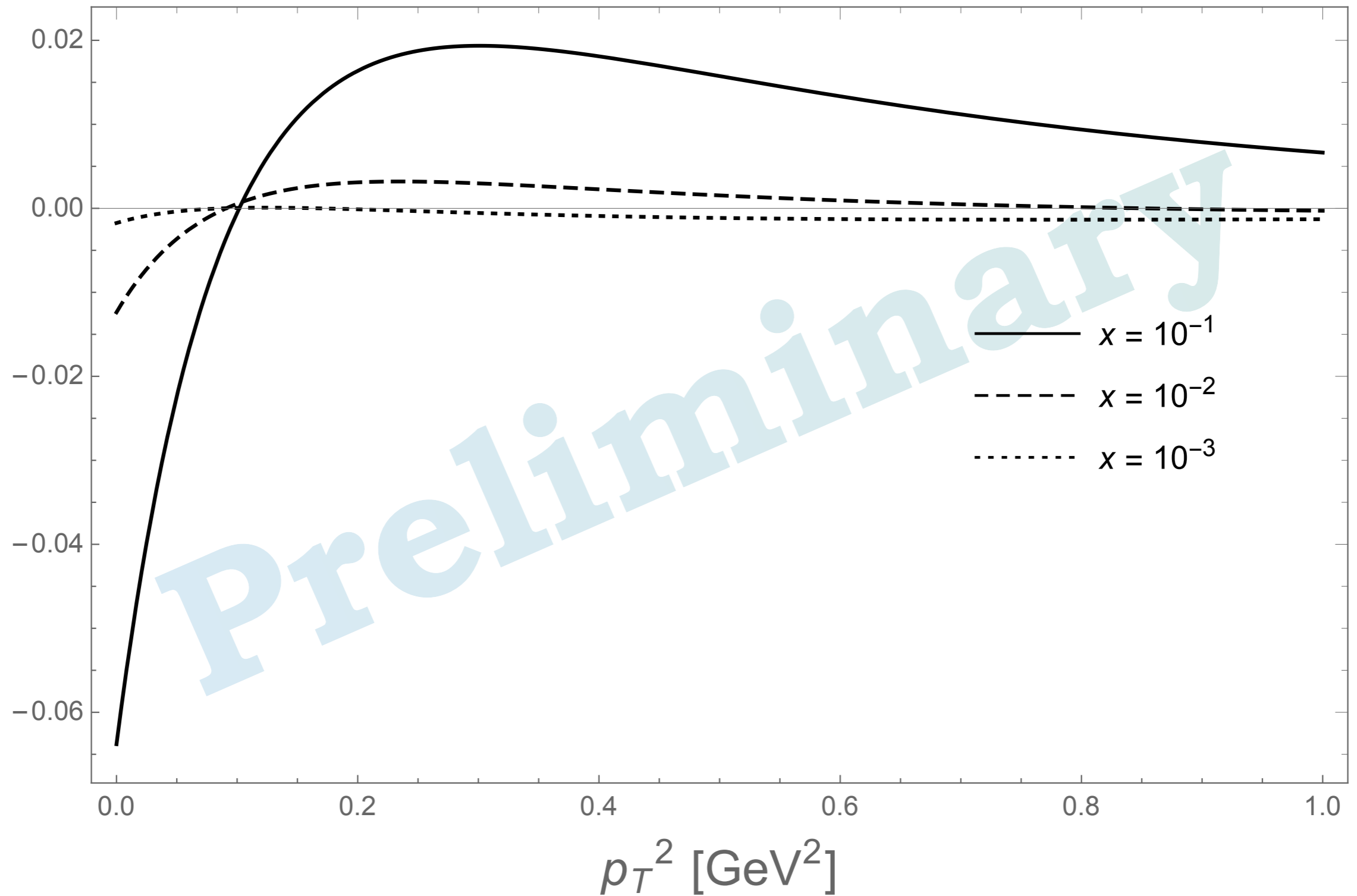
# Unpolarized valence-up TMD

$$xf_1^{u_v}(x, p_T^2)$$



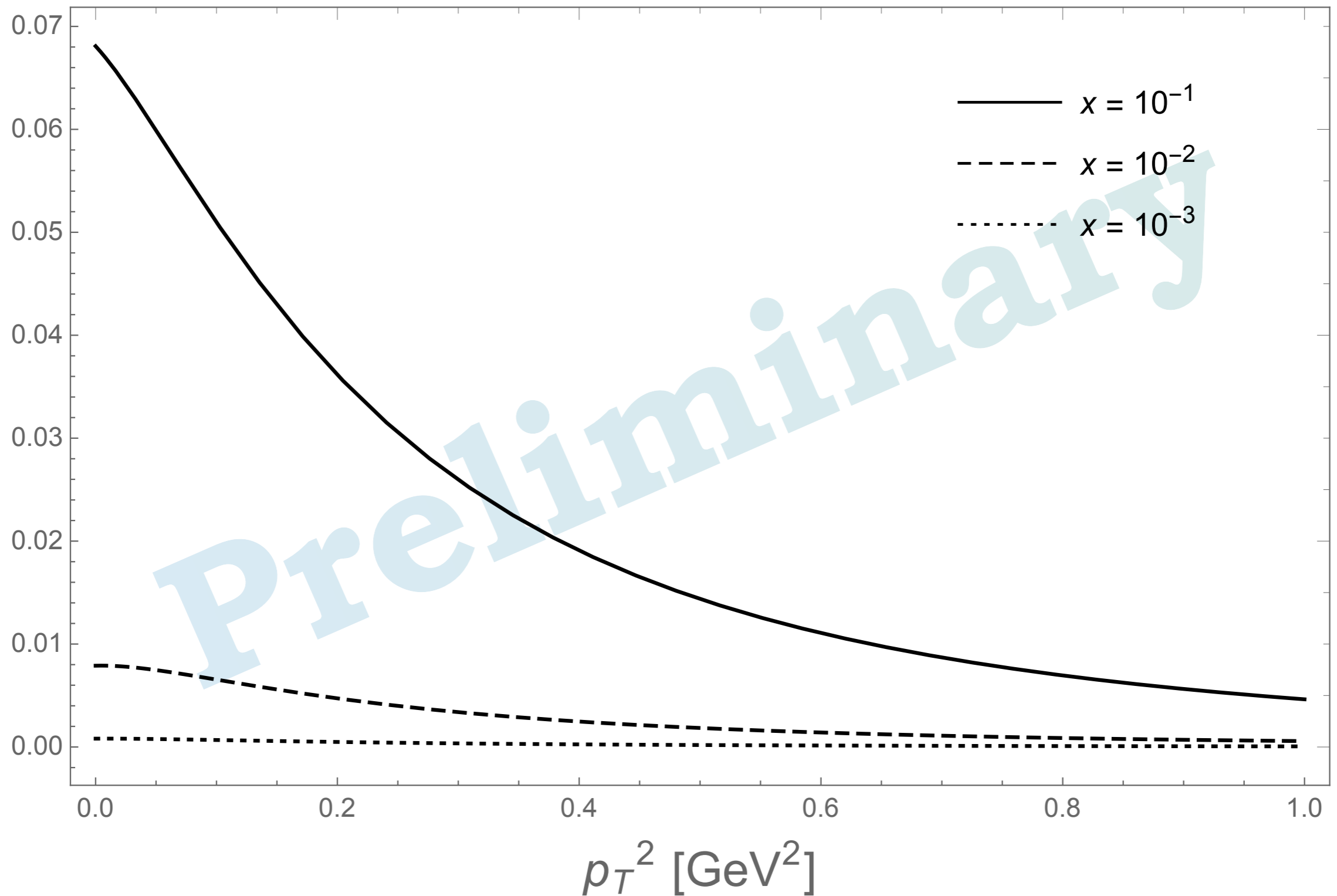
# Helicity valence-up TMD

$$xg_{1L}^{u_v}(x, p_T^2)$$



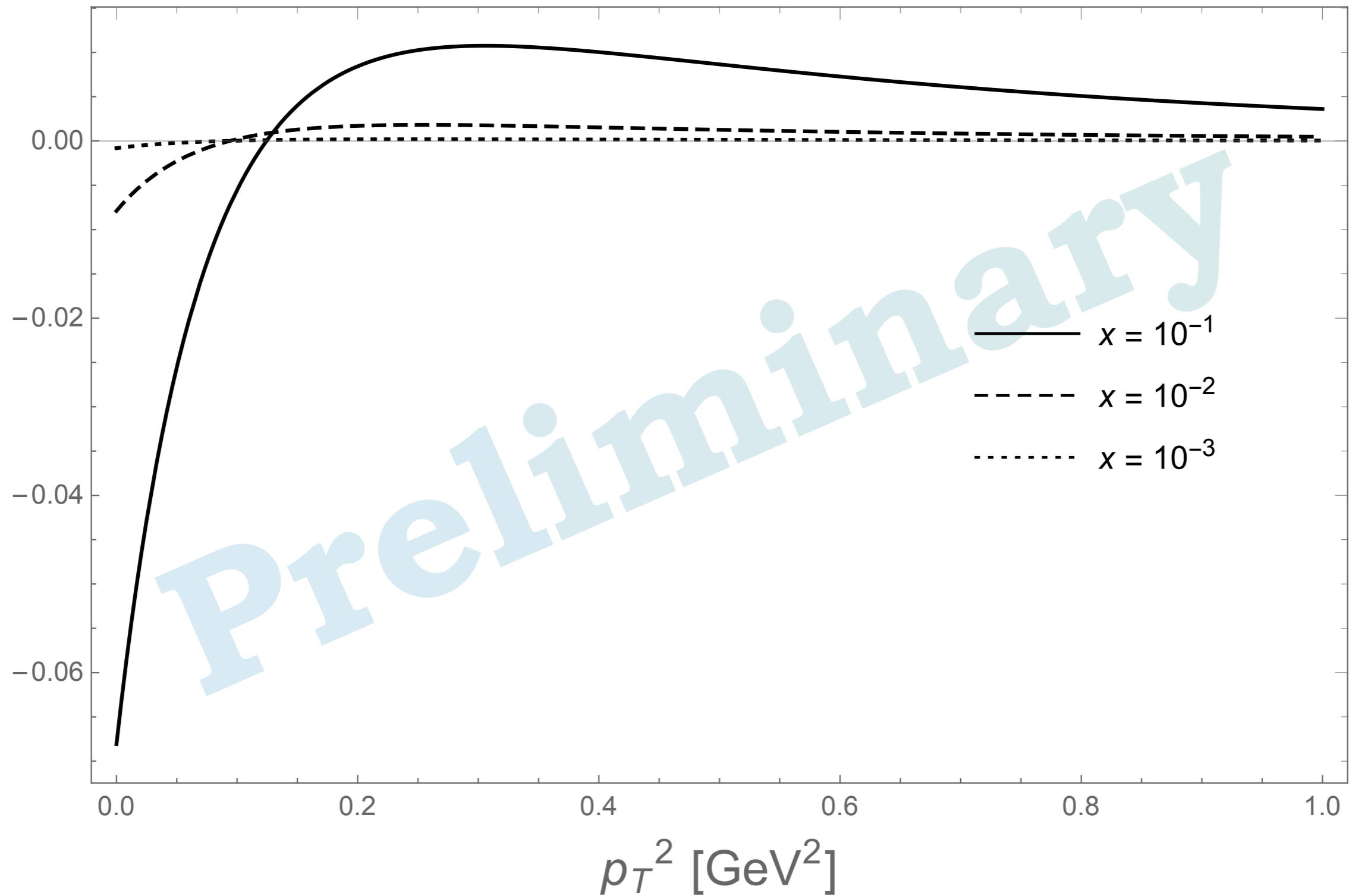
# Unpolarized valence-down TMD

$$xf_1^{d_v}(x, p_T^2)$$



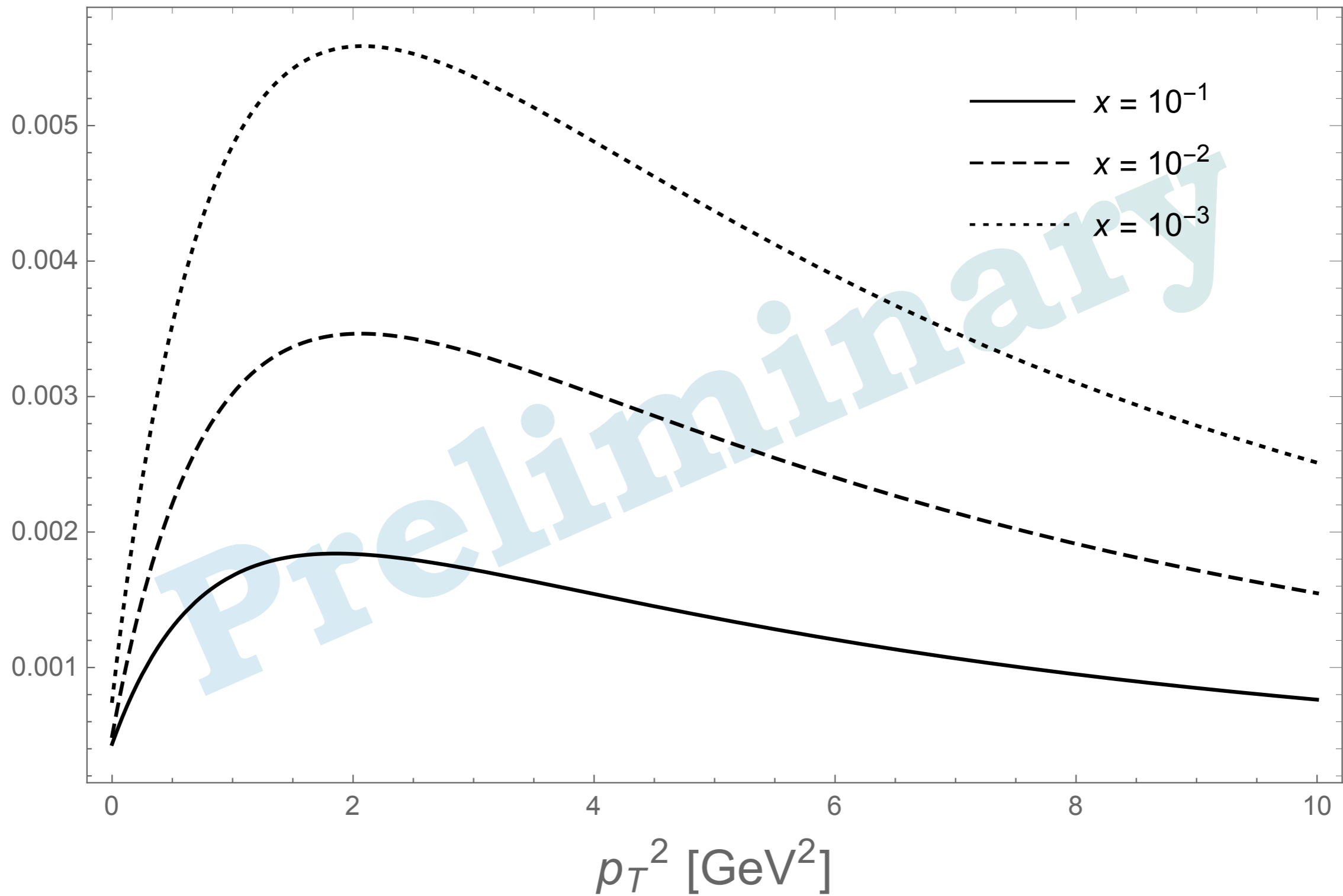
# Helicity valence-down TMD

$$xg_{1L}^{d_v}(x, p_T^2)$$



# Unpolarized sea TMD

$$xf_1^{sea}(x, p_T^2)$$



# Helicity sea TMD

$$xg_{1L}^{sea}(x, p_T^2)$$

