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Kinematical analysis of non-collinearities

Charlotte van Hulse & Piet J Mulders

cvhuls@mail.cern.ch
p.j.g.mulders@vu.nl

European Research Council

Abstract

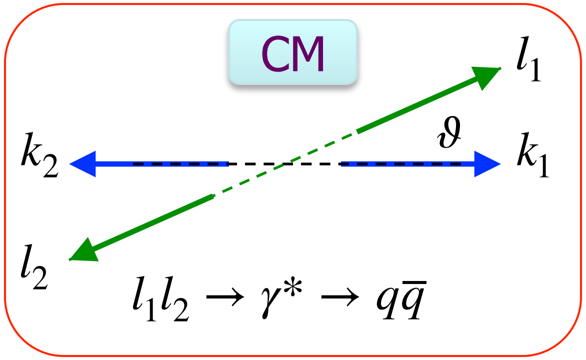
- Authors: Piet Mulders, Charlotte Van Hulse

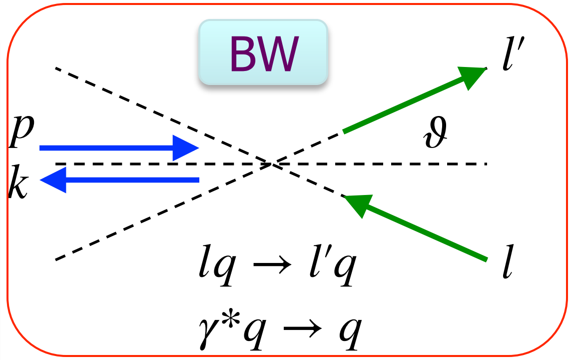
- The Drell-Yan process and electron-positron annihilation offer a natural arena for studies of non-collinearity. We [1] show how covariantly defined variables for these processes and also for semi-inclusive deep inelastic scattering are suited to get a feeling for the magnitude of intrinsic tranverse momenta.
  - [1] P.J. Mulders and C. Van Hulse ArXiv:1903.11467 [hep-ph], PRD 2019

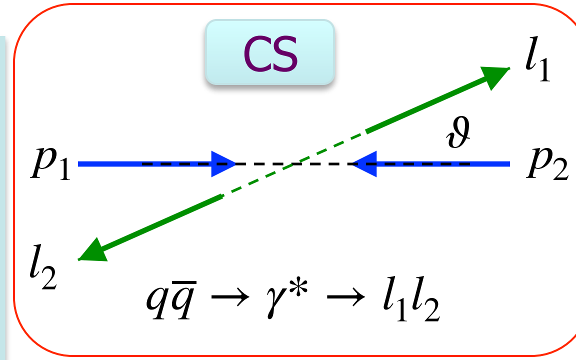
- See also:
  - PJM, hep-ph/0010199 (AIP Conf. Proc. 588 (2001) 1, p. 75-88
  - D. Boer, PJM, C. Pisano, ArXiv:0909.4652 [hep-ph], PRD 80 (2009) 094017
  - M. Boglione et al., ArXiv:1904.12882 [hep-ph]
  - Talk of Gunar Schnell

Introduction

- Importance of kinematics in multi-GeV environment of a hard process.
- Well-known for collinear situation (momentum fractions)







$$z_1 = \frac{2P_1 . q}{Q^2} \qquad z_2 = \frac{2P_2 . q}{Q^2}$$

$$\frac{P_1^+}{q^+} = \frac{2E_1^{cm}}{Q} \qquad \frac{P_2^-}{q^-} = \frac{2E_2^{cm}}{Q}$$

$$x = \frac{Q^2}{2P . q} \qquad z_h = -\frac{2P_h . q}{Q^2}$$

$$-\frac{q^+}{P^+} = \frac{Q}{2\,|\mathbf{P}^{bw}|} \qquad \frac{P_h^-}{q^-} = \frac{2\,|\mathbf{P}_h^{bw}|}{Q}$$

$$x_1 = \frac{Q^2}{2P_1 \cdot q} \qquad x_2 = \frac{Q^2}{2P_2 . q}$$

$$\frac{q^+}{P_1^+} = \frac{Q}{2E_1^{cs}} \qquad \frac{q^-}{P_2^-} = \frac{Q}{2E_2^{cs}}$$

- For two-hadron situation giving access to non-collinearity (although convoluted) and a 'second choice' of collinearity measures

$$\zeta_1^{col} = \frac{P_1 \cdot P_2}{P_2 \cdot q} \qquad \zeta_2^{col} = \frac{P_1 \cdot P_2}{P_1 \cdot q}$$

$$z_1 \zeta_2^{col} = z_2 \zeta_1^{col} = \frac{2\,P_1 \cdot P_2}{Q^2}$$

$$\xi^{col} = -\frac{P_h \cdot q}{P \cdot P_h} \qquad \zeta_h^{col} = \frac{P \cdot P_h}{P \cdot q}$$

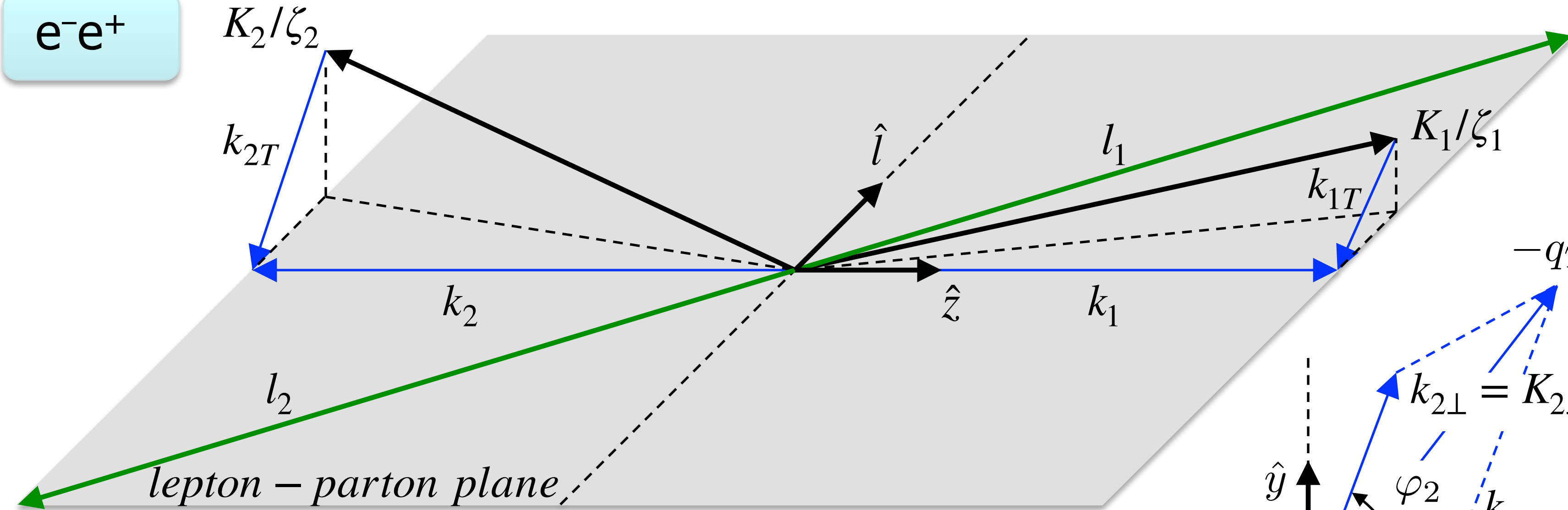
$$\frac{\zeta_h^{col}}{x} = \frac{z_h}{\xi^{col}} = \frac{2\,P \cdot P_h}{Q^2}$$

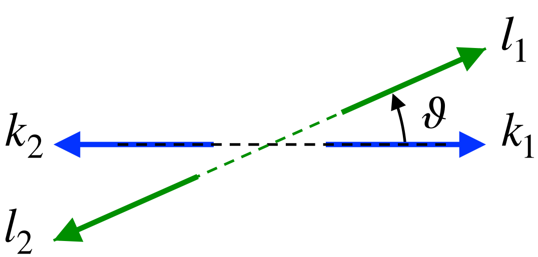
$$\xi_1^{col} = \frac{P_2 \cdot q}{P_1 \cdot P_2} \qquad \xi_2^{col} = \frac{P_1 \cdot q}{P_1 \cdot P_2}$$

$$x_1 \xi_2^{col} = x_2 \xi_1^{col} = \frac{Q^2}{2\,P_1 \cdot P_2}$$

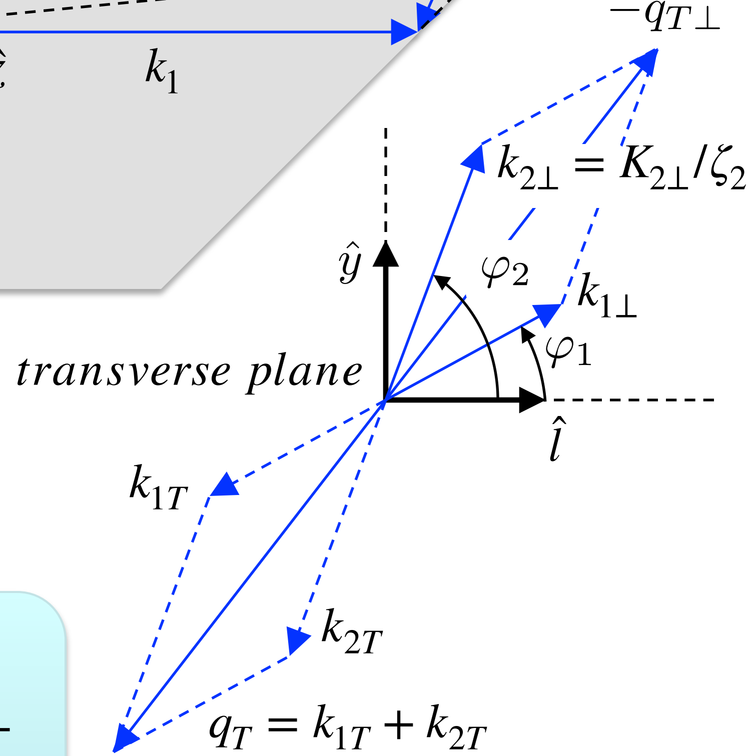
Non-collinearity in the annihilation process

e⁻e⁺





$$q_T = q - \frac{K_1}{\zeta_1} - \frac{K_2}{\zeta_2}$$



$$\hat{s} = 2l_1 \cdot l_2 = 2k_1 \cdot k_2 = Q^2$$

$$\hat{t} = -2l_1 \cdot k_1 = -Q^2 \sin^2(\theta/2)$$

$$\hat{u} = -2l_1 \cdot k_2 = -Q^2 \cos^2(\theta/2))$$

$$\cos\theta = \frac{\hat{t} - \hat{u}}{\hat{s}}$$

$$\sin\theta = \sqrt{\frac{4\,\hat{t}\hat{u}}{\hat{s}^2}}$$

Non-collinearity in annihilation process

- Non-collinearity given by q_T: $q_T = q - \frac{K_1}{\zeta_1} - \frac{K_2}{\zeta_2}$
- $K_1.q_T = K_2.q_T = 0$ → 2PI fractions

$$\frac{1}{\zeta_1} = \frac{\frac{1}{\zeta_1^{col}} - \frac{\epsilon_2}{\zeta_2^{col}}}{1-\epsilon_1\epsilon_2} \approx \frac{1}{\zeta_1^{col}} - \frac{\epsilon_2}{\zeta_2^{col}} \qquad \frac{1}{\zeta_2} = \frac{\frac{1}{\zeta_2^{col}} - \frac{\epsilon_1}{\zeta_1^{col}}}{1-\epsilon_1\epsilon_2} \approx \frac{1}{\zeta_2^{col}} - \frac{\epsilon_1}{\zeta_1^{col}}$$

$$\epsilon_1 = \frac{M_1^2}{2\,K_1\cdot K_2} \qquad \epsilon_2 = \frac{M_2^2}{2\,K_1\cdot K_2}$$

- Some special frames:
  - Hadrons collinear: $K_{1T} = K_{2T} = 0$
  - γ^* collinear with one of the hadrons:
    - $q_\perp = K_{1\perp} = 0 \;\Rightarrow\; q_T = -K_{2\perp(qK_1)}/\zeta_2$
    - $q_\perp = K_{2\perp} = 0 \;\Rightarrow\; q_T = -K_{1\perp(qK_2)}/\zeta_1$
  - γ^* collinear with jet (cm frame)
    - $q_T = -K_{1\perp}/\zeta_1 - K_{2\perp}/\zeta_2 \equiv k_{1T} + k_{2T}$ (there are small components along jet)

LT leaving k^- invariant

$$[K^-, \frac{M^2}{2K^-}, \mathbf{0}] \leftrightarrow [K^-, \frac{\mathbf{M^2} + \mathbf{K}_\perp^2}{2K^-}, -\mathbf{K}_\perp]$$

$$[k^-, k^+, \mathbf{0}] \leftrightarrow [k^-, k^+ + \frac{\mathbf{k}_\perp^2}{2k^-}, -\mathbf{k}_\perp]$$

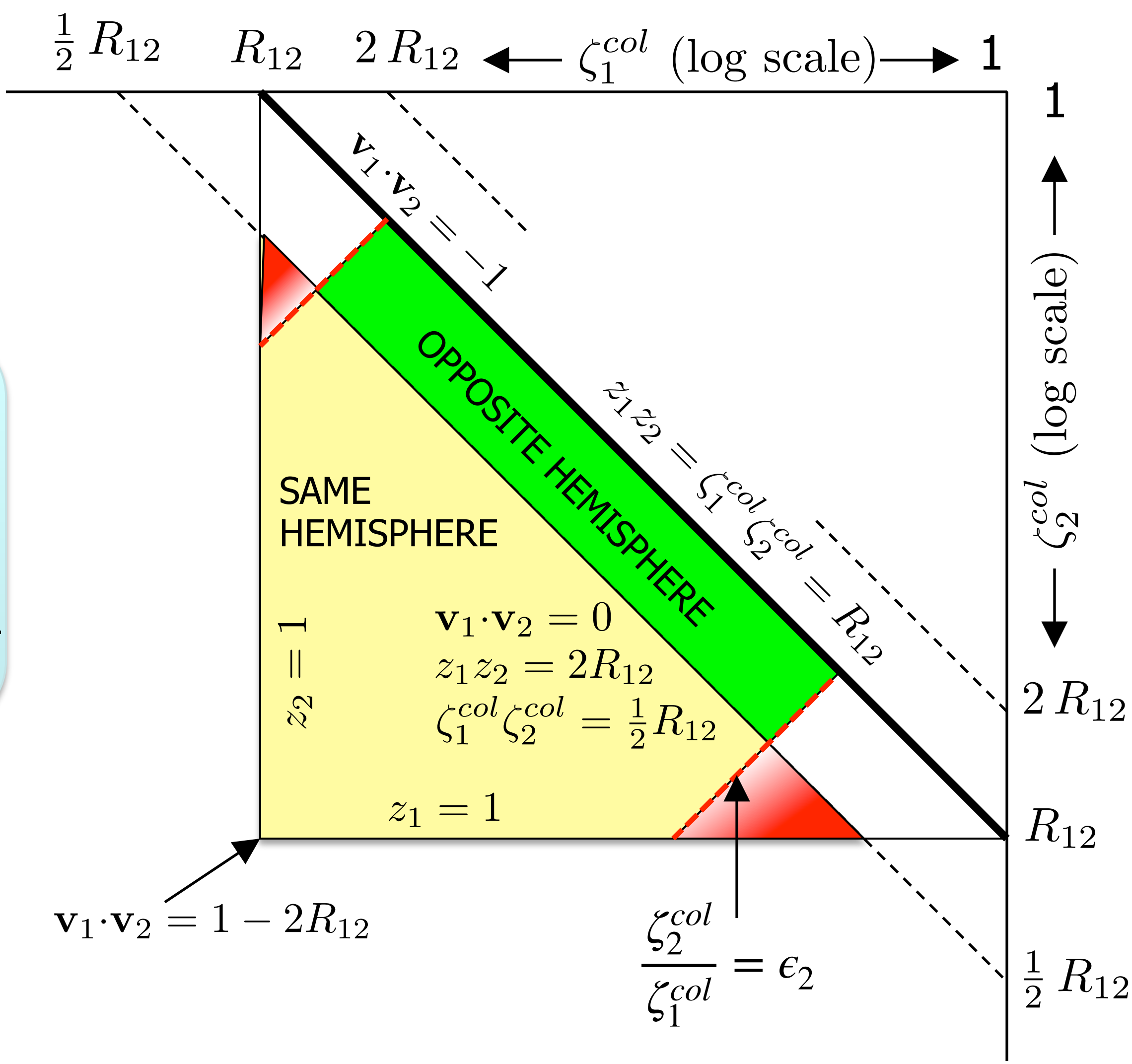
- Measures of non-collinearity (no theoretical bias!)
  - $q_T^2 = Q^2\left(1 - \frac{K_1\cdot q}{\zeta_1\,Q^2} - \frac{K_2\cdot q}{\zeta_2\,Q^2}\right) = \frac{Q^2}{2}\left(2 - \frac{z_1}{\zeta_1} - \frac{z_2}{\zeta_2}\right)$
  - $D_T \equiv -\frac{4\,\epsilon_{\mu\nu\rho\sigma} l_1^\mu l_2^\nu K_1^\rho K_2^\sigma}{\zeta_1\zeta_2\,Q^3} \approx q_T^y \sin\theta + \frac{k_{1T}^{cm}\times k_{2T}^{cm}}{Q}\cos\theta$

- Allowed regions for given hadron pairs:

$$R_{12}^{ann} = \frac{2\,K_1 \cdot K_2}{Q^2} \approx \frac{s_{12}}{Q^2}$$

$$z_1 \zeta_2^{col} = z_2 \zeta_1^{col} = R_{12}^{ann}$$

$$\frac{\zeta_1^{col} \zeta_2^{col}}{R_{12}} = \frac{1 - \mathbf{v}_1^{cm} \cdot \mathbf{v}_2^{cm}}{2}$$



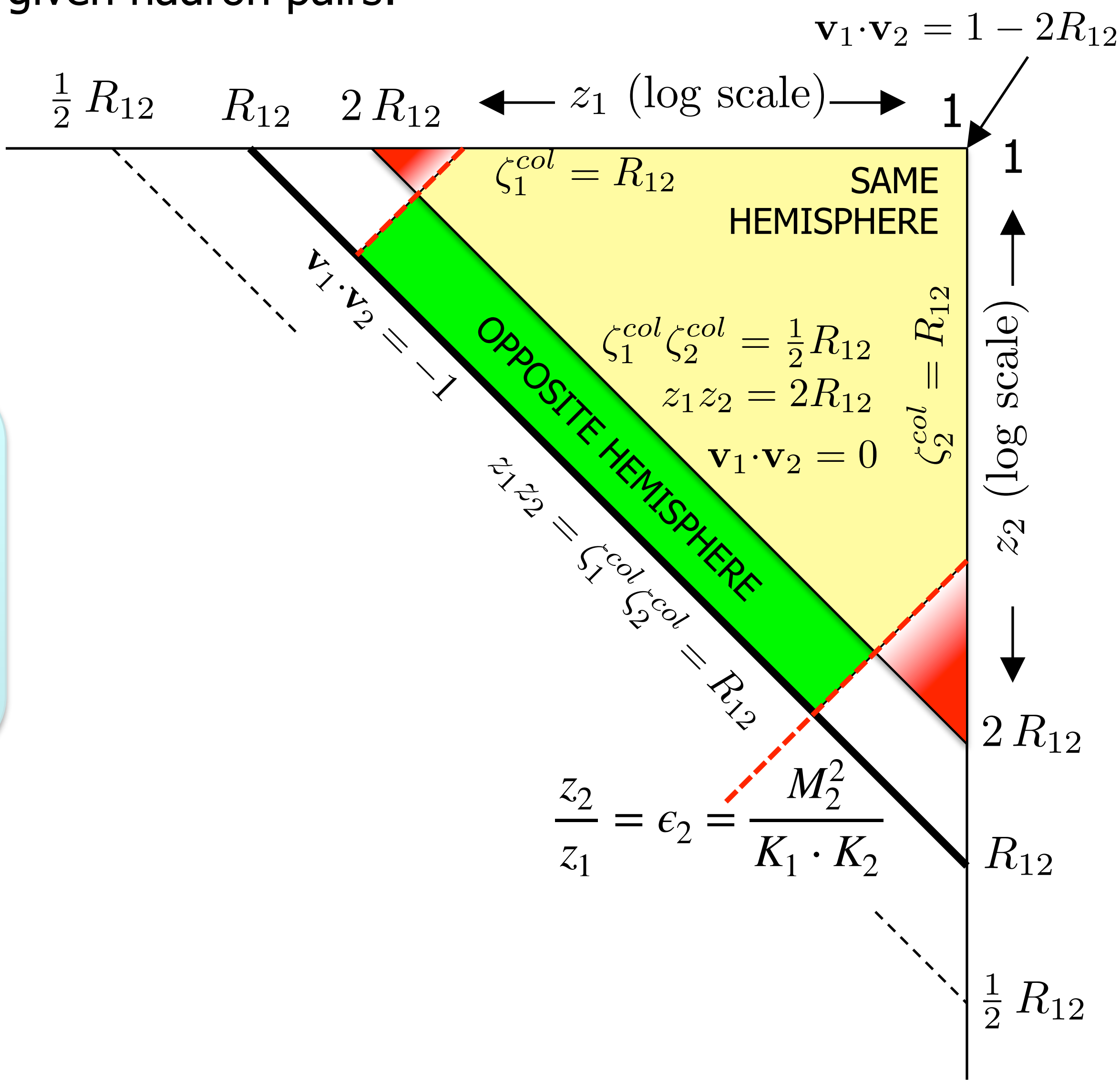
The annihilation process (fixing R_{12})

- Allowed regions for given hadron pairs:

$$R_{12}^{ann} = \frac{2\,K_1 \cdot K_2}{Q^2} \approx \frac{s_{12}}{Q^2}$$

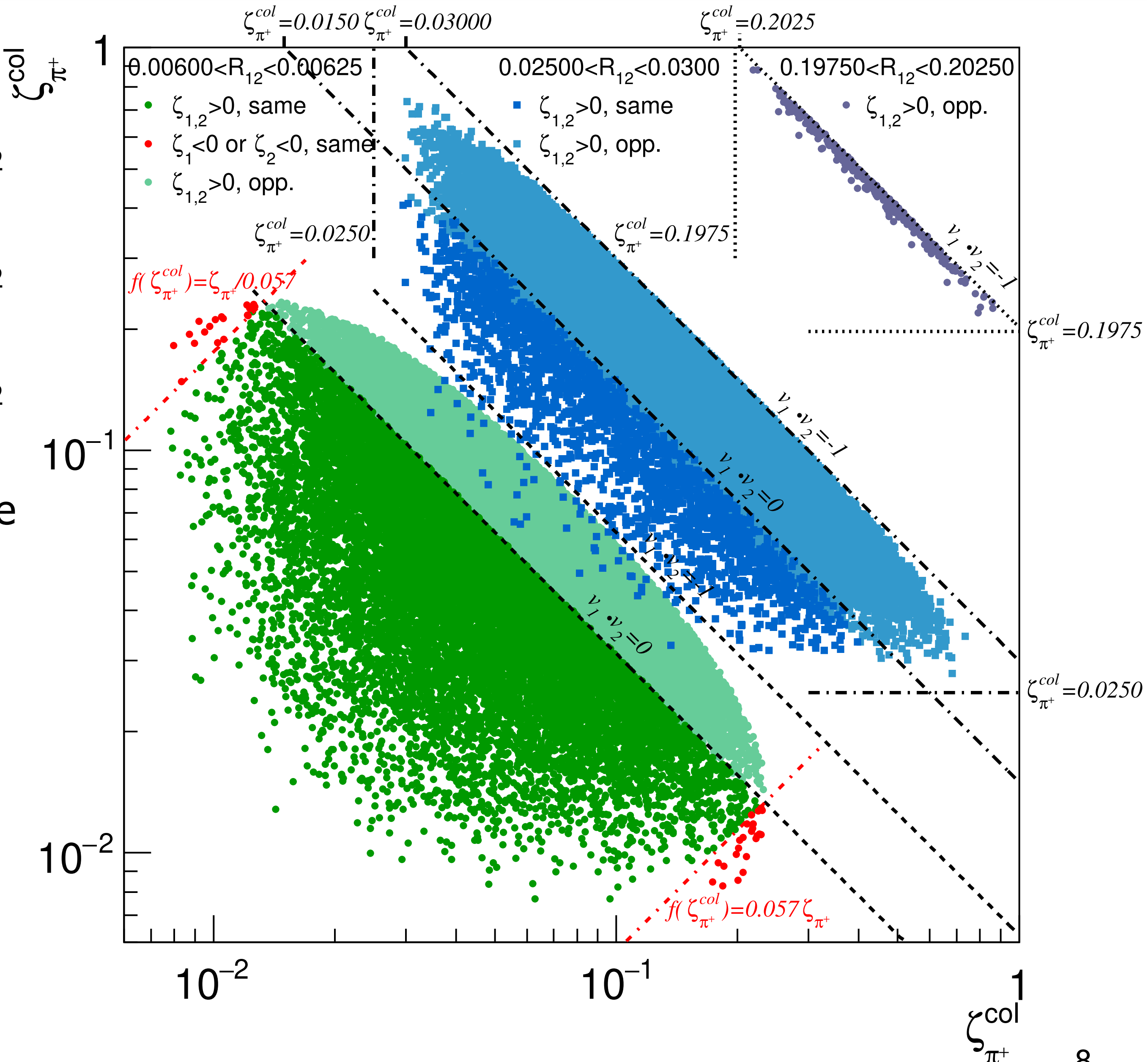
$$z_1 \zeta_2^{col} = z_2 \zeta_1^{col} = R_{12}^{ann}$$

$$\frac{R_{12}}{z_1 z_2} = \frac{1 - \mathbf{v}_1^{cm} \cdot \mathbf{v}_2^{cm}}{2}$$



Monte Carlo simulation (Pythia, $e^+e^- \to \pi\pi$ at 10.58 GeV)

- Two pions
  - $R_{12} \sim 0.2$
    $s_{12} \sim (4.7\ \text{GeV})^2$
  - $R_{12} \sim 0.03$
    $s_{12} \sim (1.8\ \text{GeV})^2$
  - $R_{12} \sim 0.006$
    $s_{12} \sim (0.8\ \text{GeV})^2$
- For large R_{12} only opposite-hemisphere
- Regions with ζ_π negative (red) for same-hemisphere



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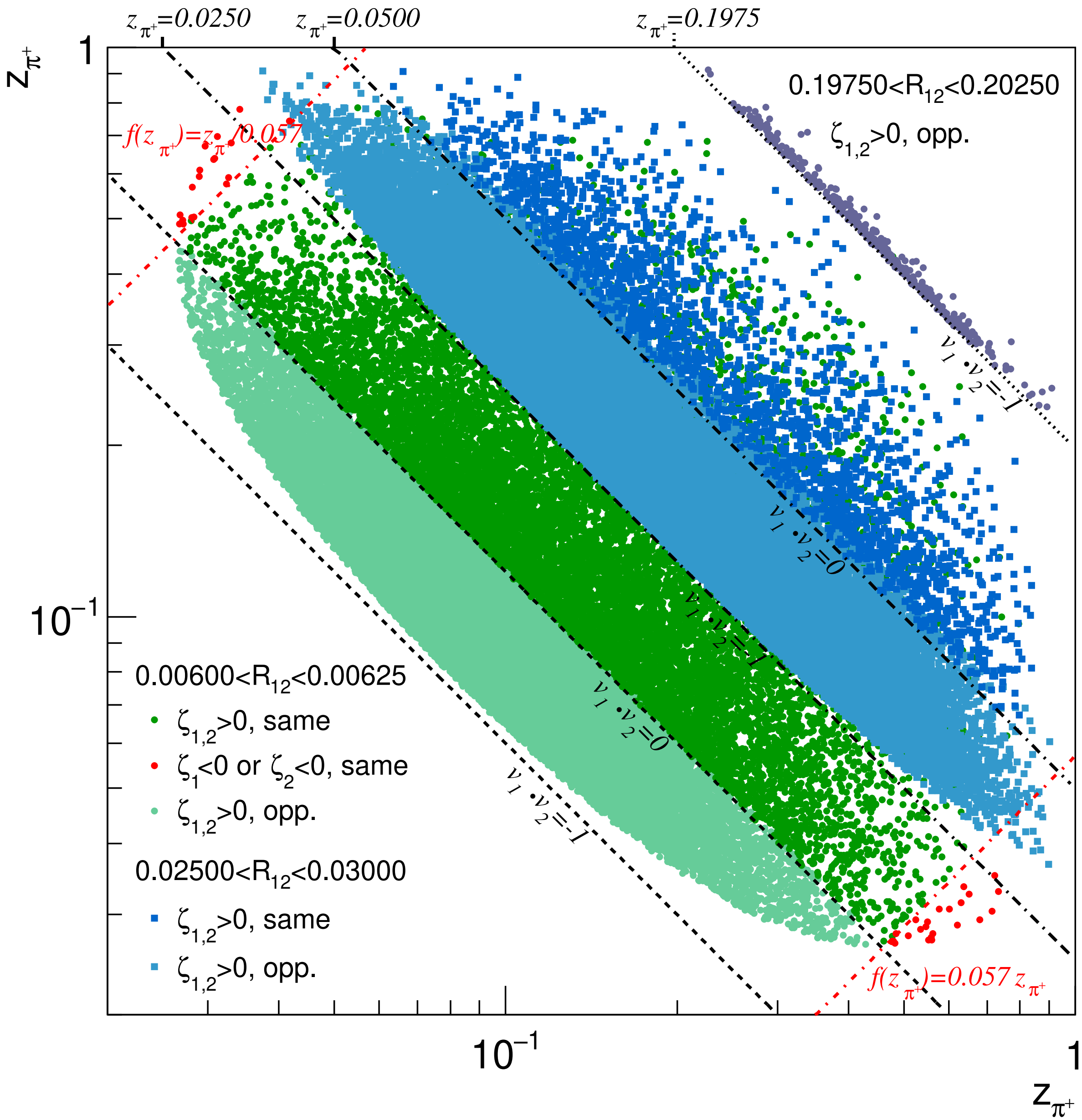
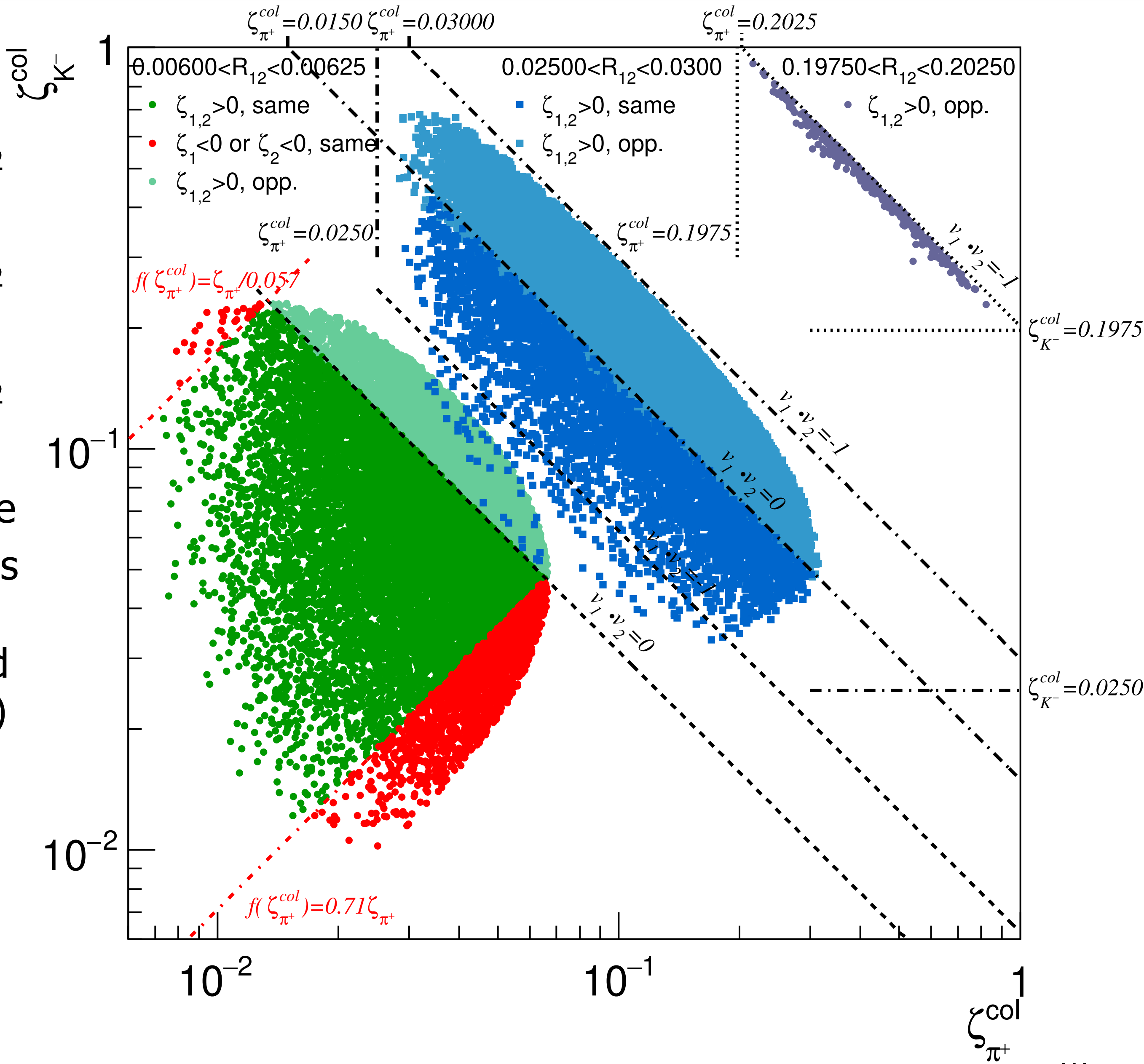


Monte Carlo simulation (Pythia, $e^+e^- \rightarrow \pi K$ at 10.58 GeV)

- Pion-kaon pair
  - $R_{12} \sim 0.2$
    $s_{12} \sim (4.7\text{ GeV})^2$
  - $R_{12} \sim 0.03$
    $s_{12} \sim (1.8\text{ GeV})^2$
  - $R_{12} \sim 0.006$
    $s_{12} \sim (0.8\text{ GeV})^2$
- For large R_{12} only opposite-hemisphere
- For small R_{12} there is a small region (red) with ζ_π negative and a larger region (red) with ζ_K negative for same-hemisphere pairs



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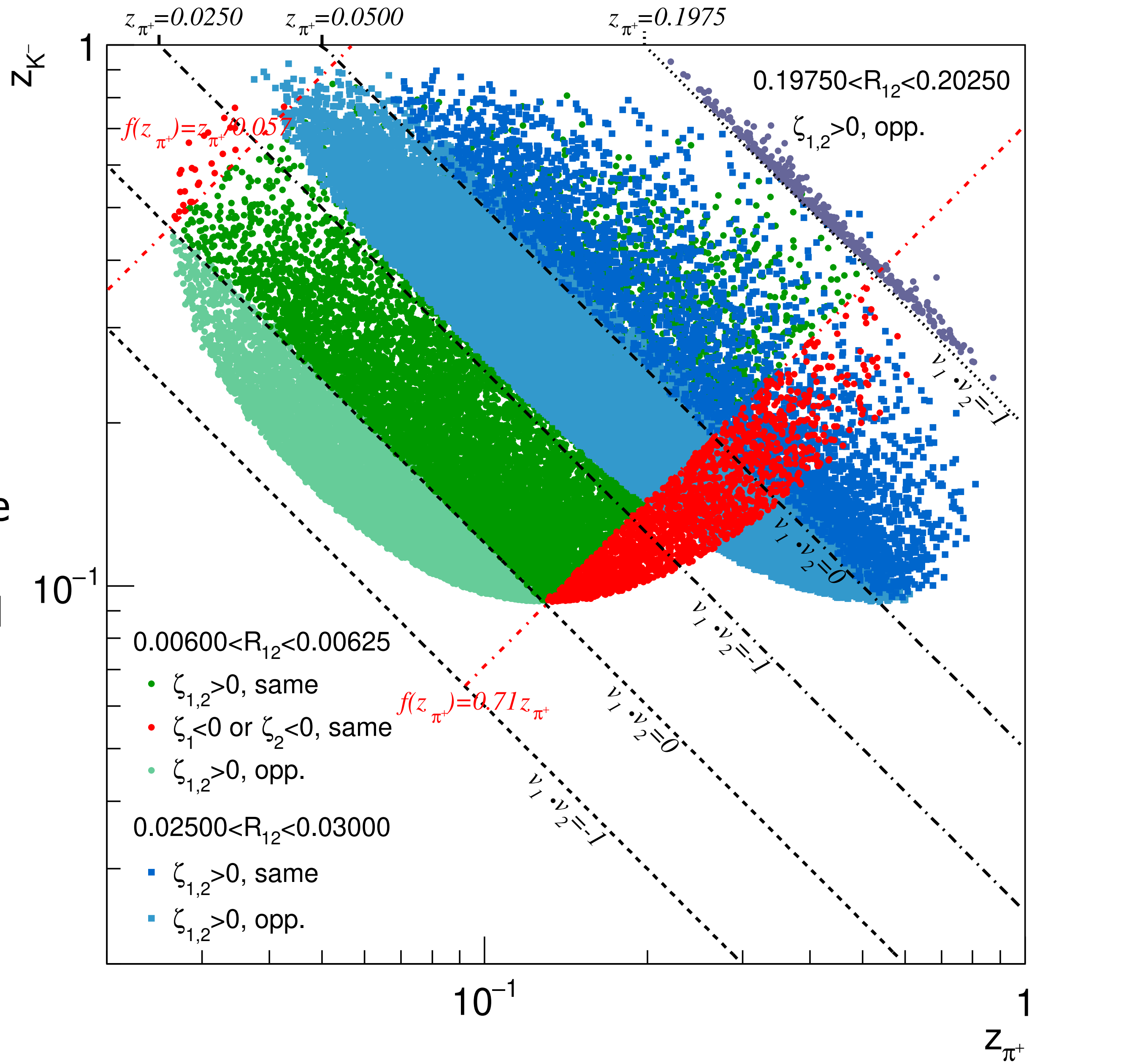
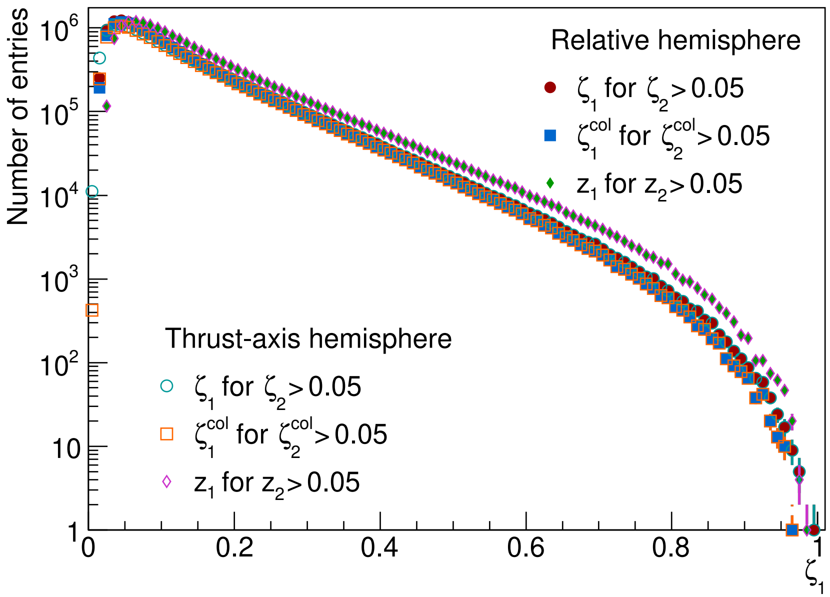
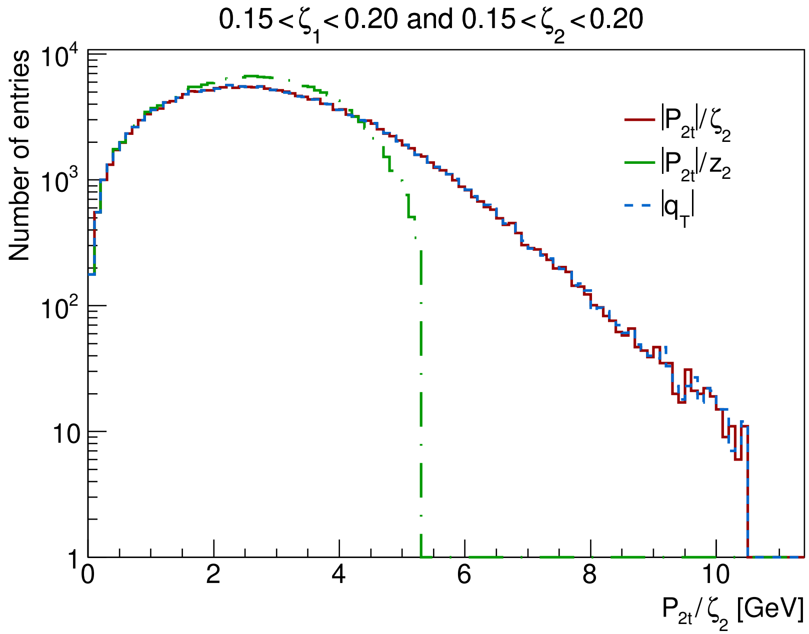
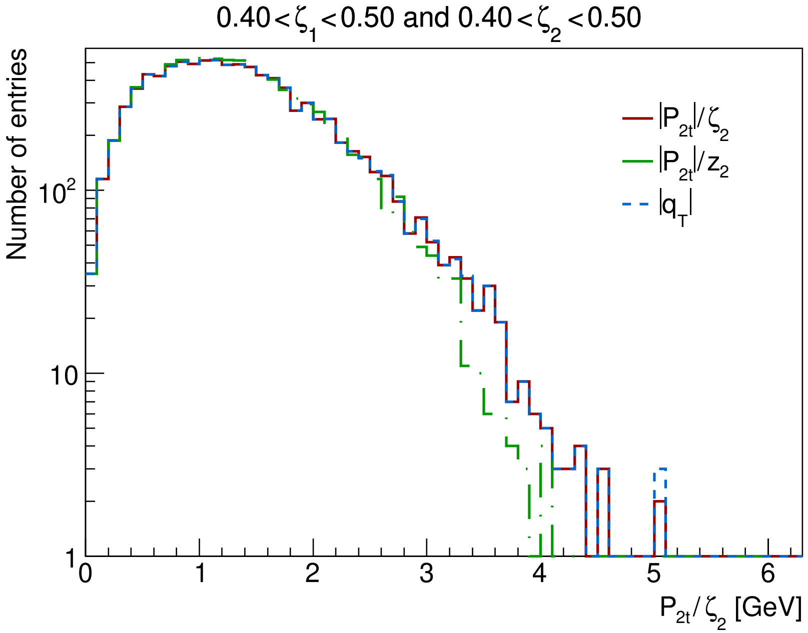


Use of different fractions

- Overall z, ζ and ζ^{col} not very different
- For small z and ζ^{col} one does not find the correct q_T. It requires ζ !

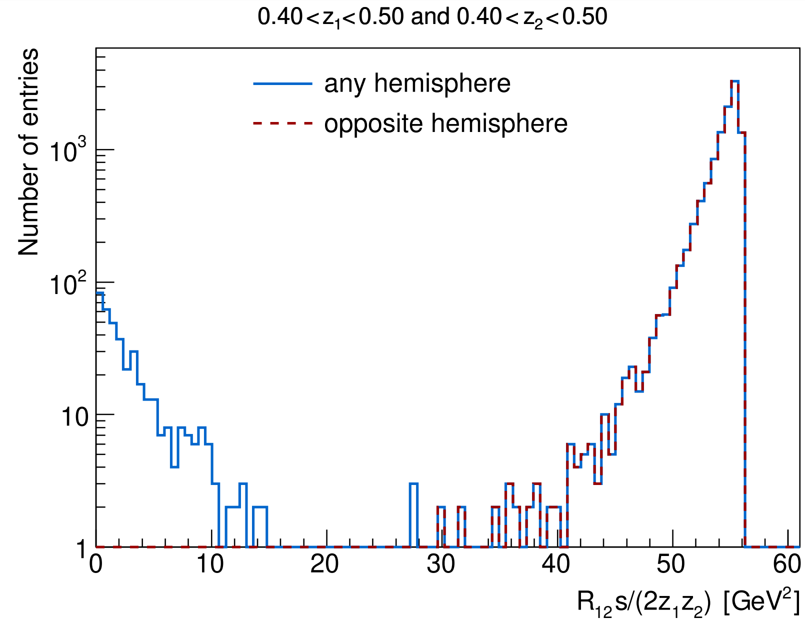


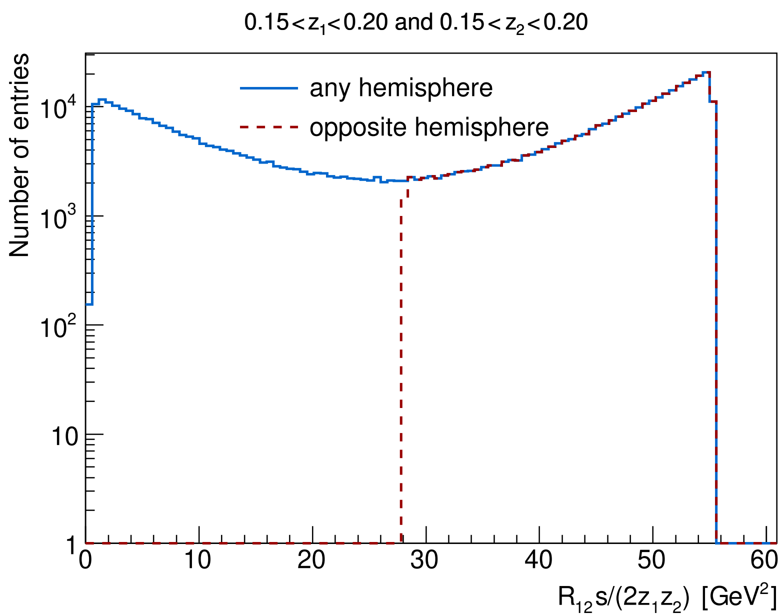


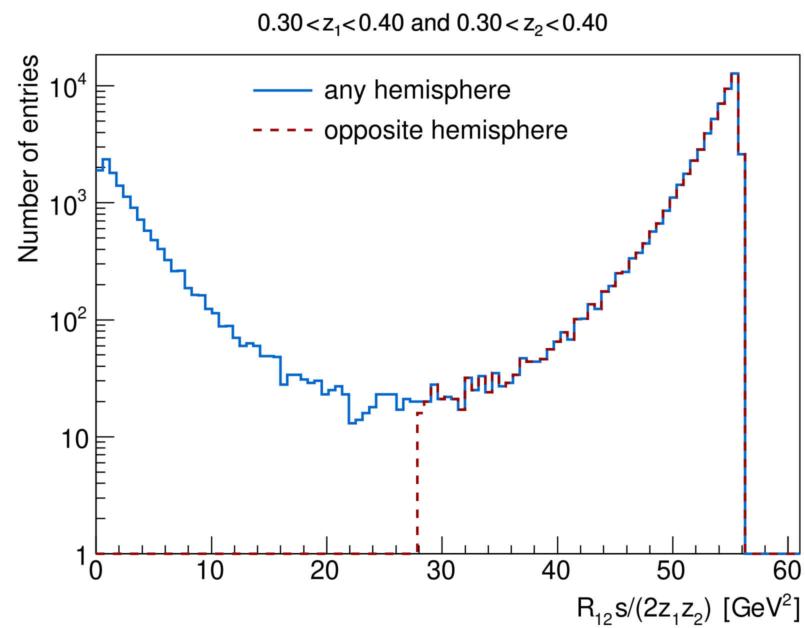


Separation of hemispheres

- Fine at large fractions
- Impossible at small fractions, but formally possible via $v_1.v_2$ for given pairs obtained from R_{12} and ζ's

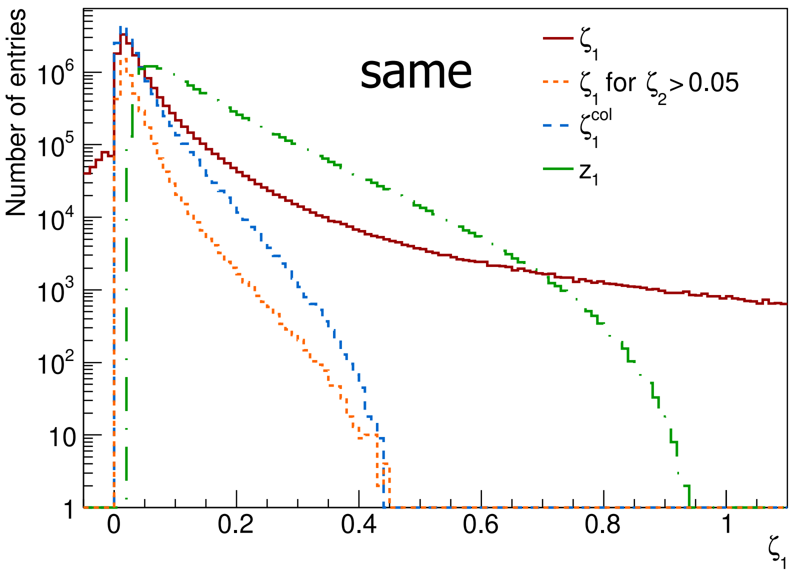


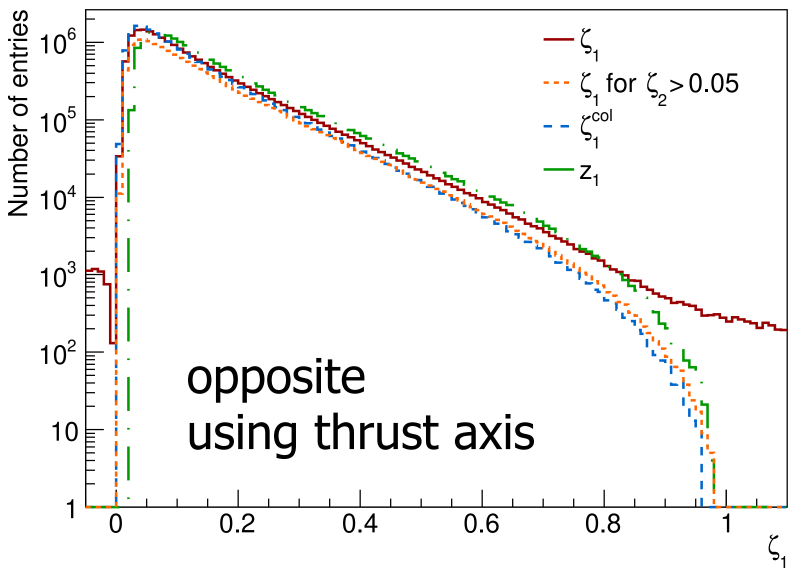


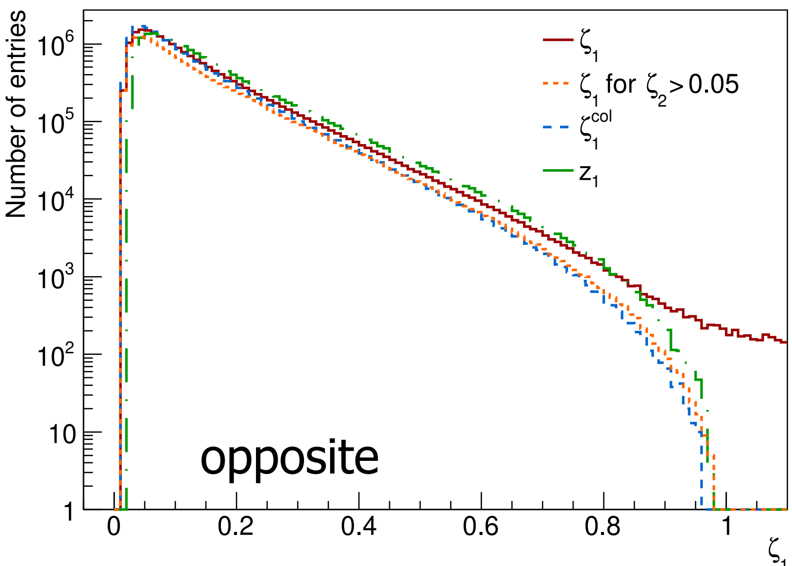


Range of 2PI variable ζ

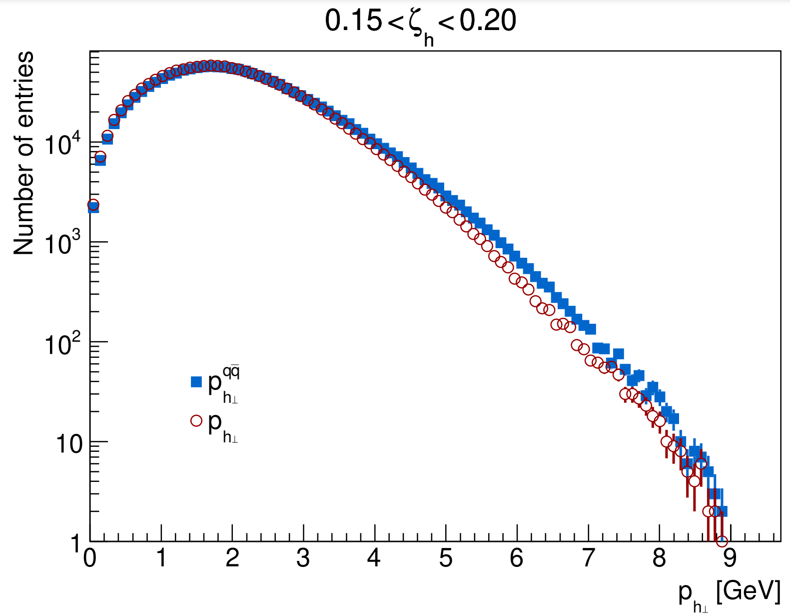
- ζ can be negative for same hemisphere hadrons
- Negative values even occur (although suppressed) when using the thrust axis (replacing second hadron)

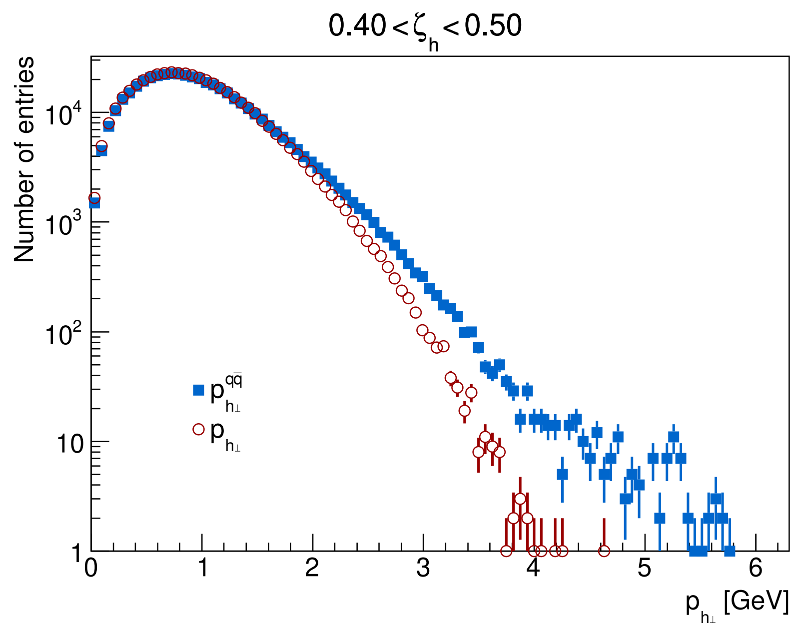






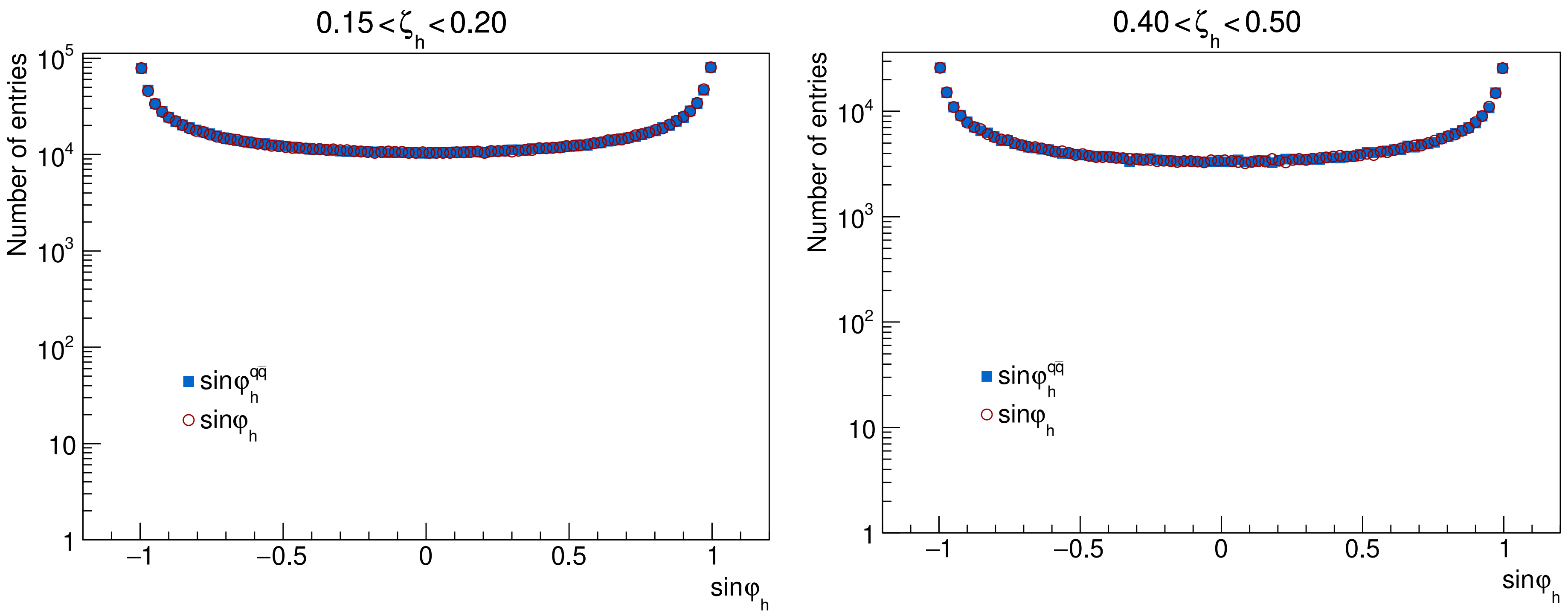
- Experimental acces to qT using one hadron and thrust axis (red) compares well with full two-hadron analysis using MC qqbar axis (blue).





Using the determinant

- $$D_T \equiv -\frac{4\,\epsilon_{\mu\nu\rho\sigma} l_1^\mu l_2^\nu K_1^\rho K_2^\sigma}{\zeta_1 \zeta_2 Q^3} \approx q_T^y \sin\theta + \frac{k_{1T}^{cm} \times k_{2T}^{cm}}{Q} \cos\theta$$
- Data (extracted from $D_T/(|q_T| \sin\theta)$) show a full range of events as a function of $\sin\phi_h$. Experimental acces using one hadron and thrust axis (red) compares well with full two-hadron analysis using MC qqbar axis (blue).



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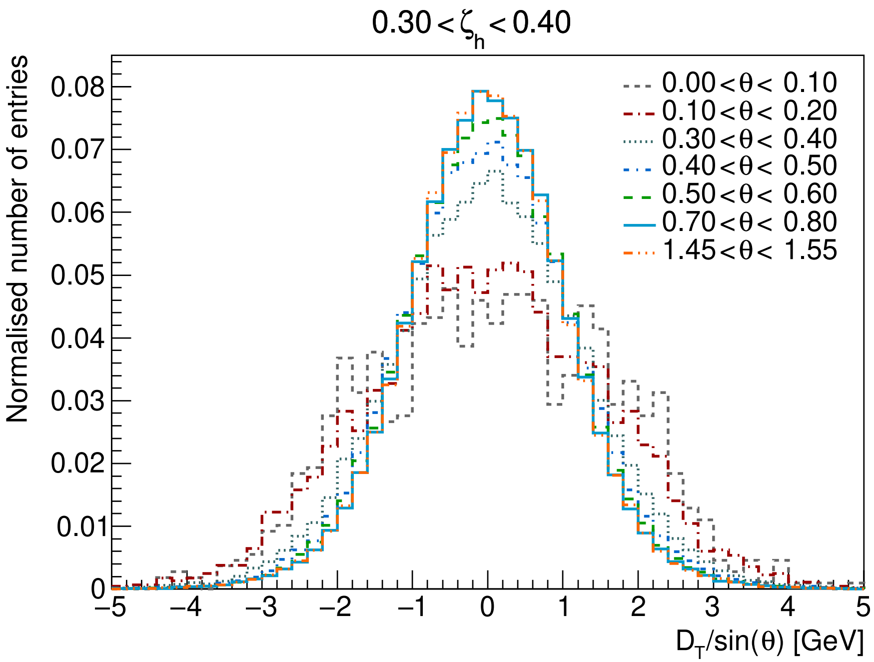
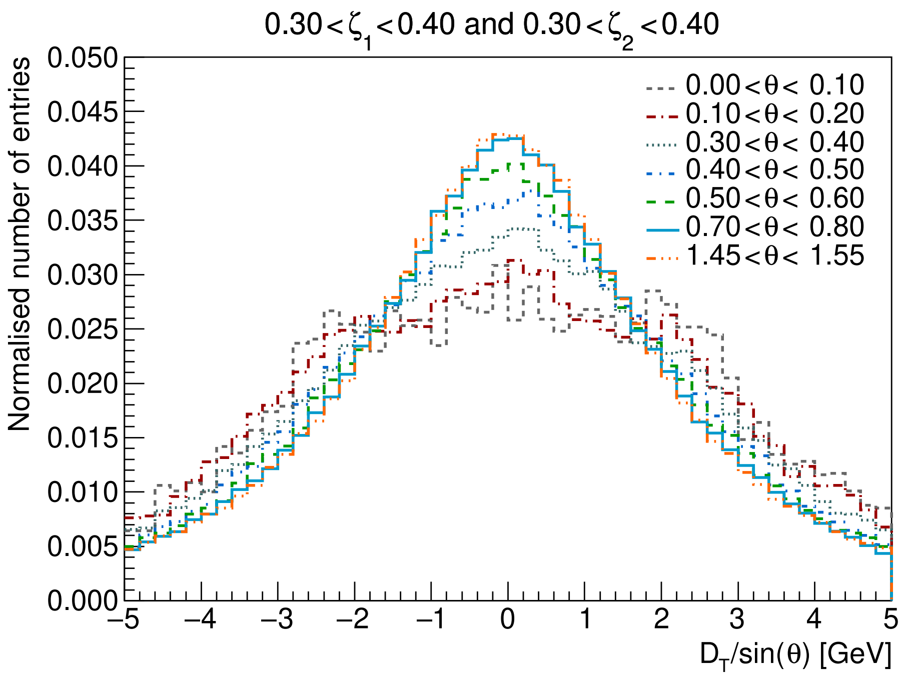
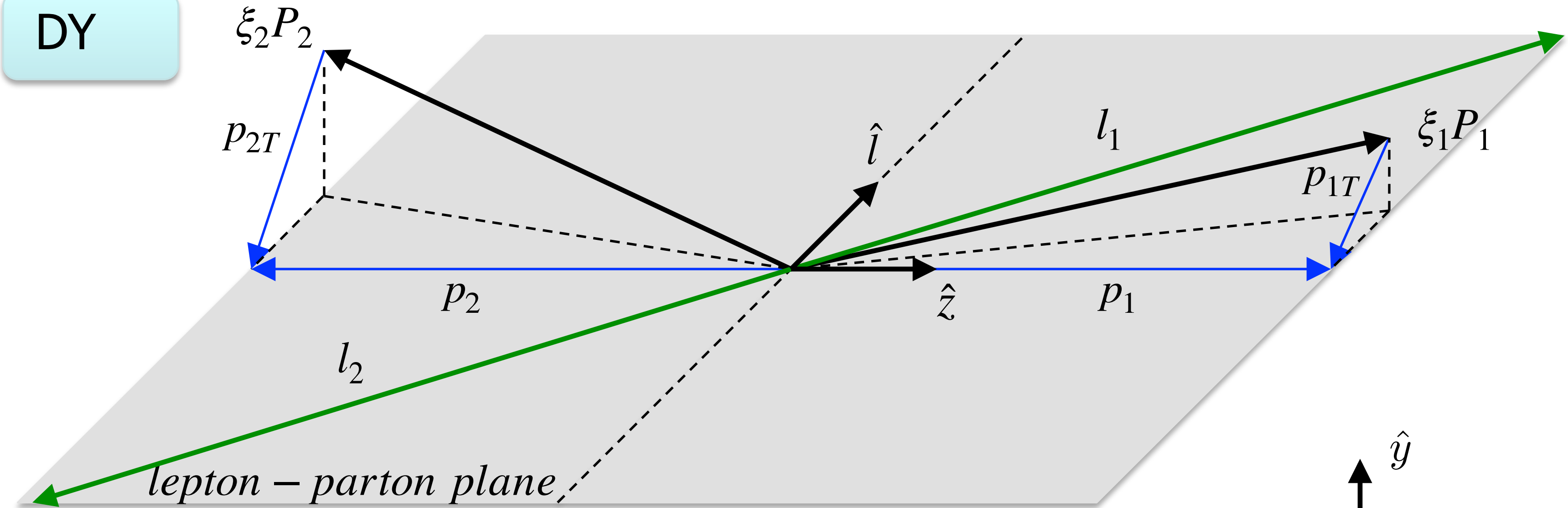


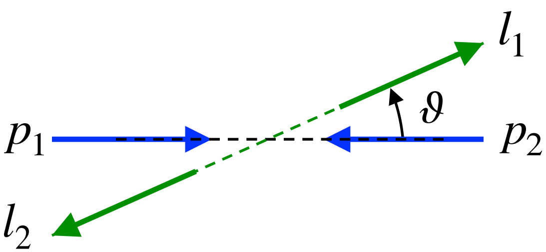


- As expected about a factor square root two between widths for the two-hadron (right) and hadron-thrust axis analysis (left)

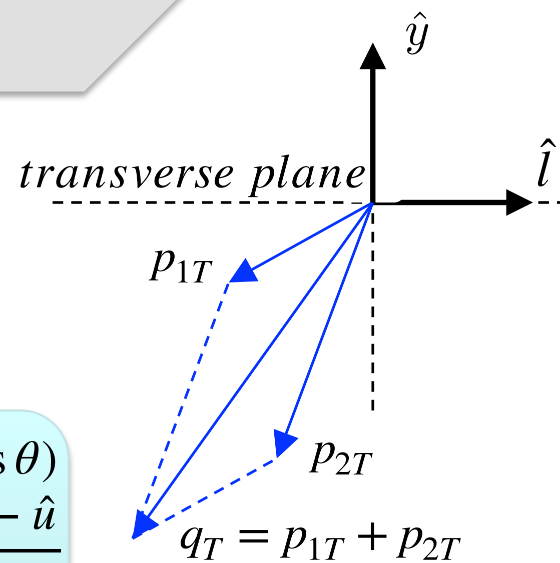
Non-collinearity in the Drell-Yan process

DY





$$q_T = q - \xi_1 P_1 - \xi_2 P_2$$



$$\hat{s} = 2l_1 \cdot l_2 = 2p_1 \cdot p_2 = Q^2 \qquad y = \frac{1}{2}(1 - \cos\theta)$$

$$\hat{t} = -2l_1 \cdot p_1 = -Q^2 \sin^2(\theta/2) = -y\,Q^2 \qquad \cos\theta = \frac{\hat{t} - \hat{u}}{\hat{s}}$$

$$\hat{u} = -2l_1 \cdot p_2 = -Q^2 \cos^2(\theta/2)) = -(1-y)Q^2$$

Non-collinearity in Drell-Yan process

- Non-collinearity given by q_T: $q_T = q - \xi_1 P_1 - \xi_2 P_2$
- $P_1.q_T = P_2.q_T = 0$ → 2PI fractions

$$\xi_1 = \frac{\xi_1^{col} - \epsilon_2 \xi_2^{col}}{1 - \epsilon_1 \epsilon_2} \approx \xi_1^{col} - \epsilon_2 \xi_2^{col} \qquad \xi_2 = \frac{\xi_2^{col} - \epsilon_1 \xi_1^{col}}{1 - \epsilon_1 \epsilon_2} \approx \xi_2^{col} - \epsilon_1 \xi_1^{col}$$

$$\epsilon_1 = \frac{M_1^2}{P_1 \cdot P_2}$$

$$\epsilon_2 = \frac{M_2^2}{P_1 \cdot P_2}$$

- Some special frames:
  - Hadrons collinear: $P_{1T} = P_{2T} = 0$
  - γ^* collinear with one of the hadrons:
    - $q_\perp = P_{1\perp} = 0 \;\Rightarrow\; q_T = -\,\xi_2\, P_{2\perp(qP_1)}$
    - $q_\perp = P_{2\perp} = 0 \;\Rightarrow\; q_T = -\,\xi_1\, P_{1\perp(qP_2)}$
  - γ^* collinear with jet (Collins-Soper frame)
    - $q_T = -\,\xi_1\, P_{1\perp} - \xi_2\, P_{2\perp} \equiv p_{1T} + p_{2T}$ (there are small component along jet)

LT leaving p^+ invariant

$$[\frac{M^2}{2P^+}, P^+, \mathbf{0}] \;\leftrightarrow\; [\frac{\mathbf{M}^2 + \mathbf{P}_\perp^2}{2P^+}, P^+, -\mathbf{P}_\perp]$$

$$[p^-, p^+, \mathbf{0}] \;\leftrightarrow\; [p^- + \frac{\mathbf{p}_\perp^2}{2p^+}, p^+, -\mathbf{p}_\perp]$$

- Measures of non-collinearity (no theoretical bias!)
  - $$q_T^2 = Q^2\left(1 - \xi_1 \frac{P_1 \cdot q}{Q^2} - \xi_2 \frac{P_2 \cdot q}{Q^2}\right) = \frac{Q^2}{2}\left(2 - \frac{\xi_1}{x_1} - \frac{\xi_2}{x_2}\right)$$
  - $$D_T \equiv -\frac{4\xi_1 \xi_2\, \epsilon_{\mu\nu\rho\sigma} l_1^\mu l_2^\nu P_1^\rho P_2^\sigma}{Q^3}$$

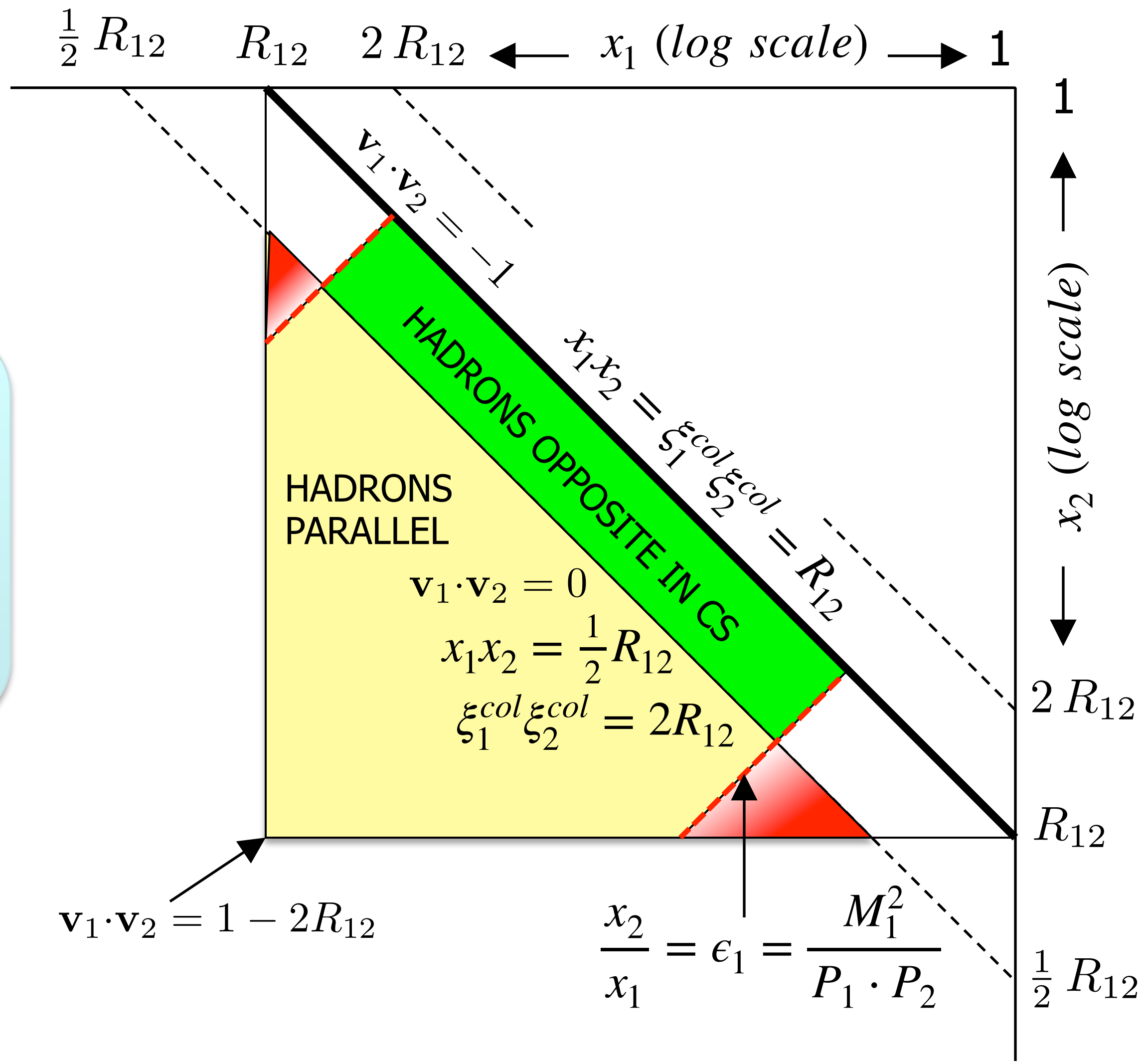
The Drell-Yan process (fixing R_{12})

- Allowed regions for given hadron pairs:

$$R_{12}^{DY} = \frac{Q^2}{2\,P_1 \cdot P_2} \approx \frac{Q^2}{s}$$

$$x_1 \xi_2^{col} = x_2 \xi_1^{col} = R_{12}^{DY}$$

$$\frac{x_1 x_2}{R_{12}} = \frac{1 - \mathbf{v}_1^{cs} \cdot \mathbf{v}_2^{cs}}{2}$$



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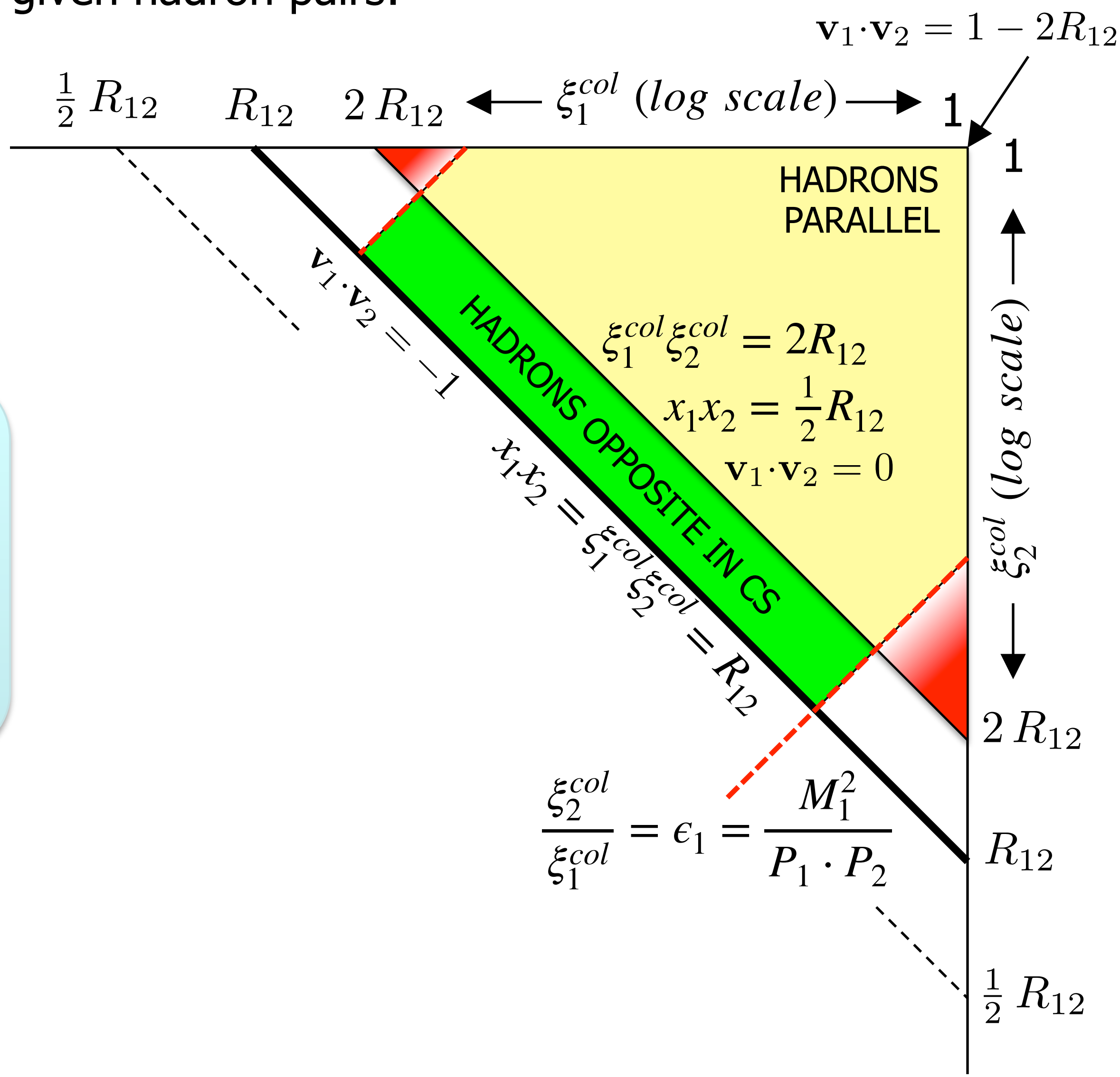
$$R_{12}^{DY} = \frac{Q^2}{2\,P_1 \cdot P_2} \approx \frac{Q^2}{s}$$

$$x_1 \xi_2^{col} = x_2 \xi_1^{col} = R_{12}^{DY}$$

$$\frac{R_{12}}{\xi_1^{col}\xi_2^{col}} = \frac{1 - \mathbf{v}_1^{cs} \cdot \mathbf{v}_2^{cs}}{2}$$

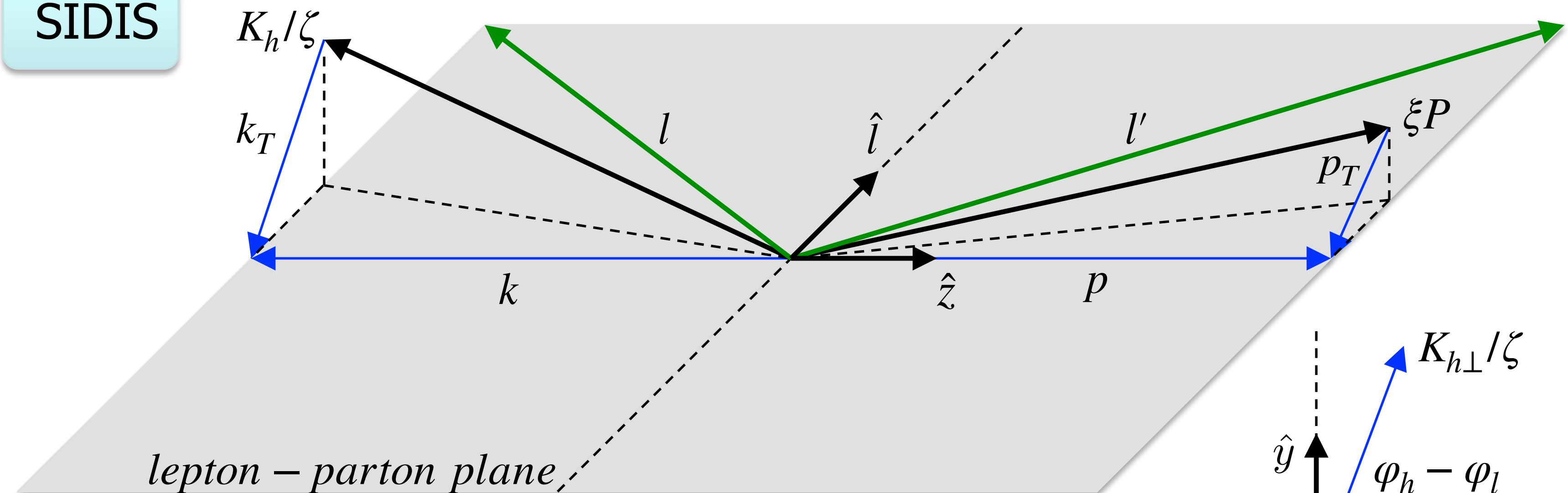


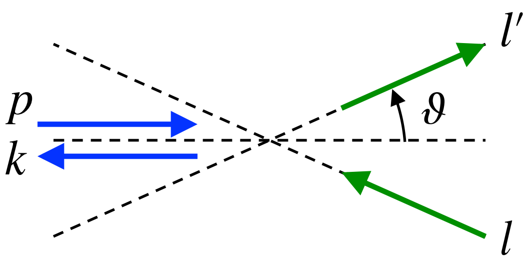
- Combine these two cases (annihilation and DY) in hadron induced hadroproduction with underlying partonic hard process $p_1 + p_2 \rightarrow k_1 + k_2$

- Dijet imbalance in hadronic collisions
Daniel Boer (Vrije U., Amsterdam & Groningen, KVI), Piet J. Mulders (Vrije U., Amsterdam), Cristian Pisano (Vrije U., Amsterdam & Cagliari U. & INFN, Cagliari). Sep 2009. 14 pp.
Published in **Phys.Rev. D80 (2009) 094017**
DOI: 10.1103/PhysRevD.80.094017
e-Print: **arXiv:0909.4652 [hep-ph] | PDF**

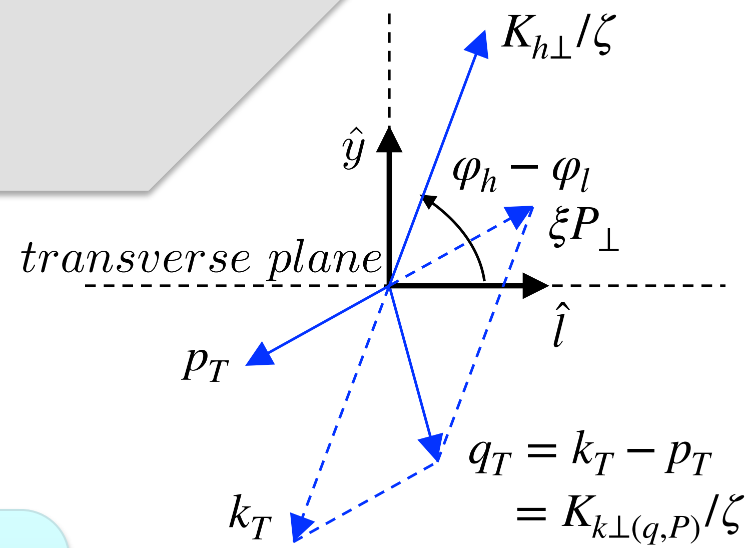
Non-collinearity in semi-inclusive deep inelastic scattering

SIDIS





$$q_T = q + \xi P - \frac{P_h}{\zeta_h}$$



$$\hat{t} = -\,2\,l \cdot l' = -\,2\,p \cdot k = -\,Q^2$$

$$\hat{s} = 2\,l \cdot p = \frac{1}{y}\,Q^2$$

$$\hat{u} = -\,2\,l \cdot k = -\frac{1-y}{y}\,Q^2$$

$$\cos\theta = \frac{-\,\hat{t}}{\hat{s} - \hat{u}}$$

Non-collinearity in SIDIS

- Non-collinearity given by q_T: $q_T = q + \xi P - \frac{K_h}{\zeta_h}$
- $P.q_T = K_h.q_T = 0$ → 2PI fractions

$$\xi = \frac{\xi^{col} + \frac{\epsilon_h}{\zeta^{col}}}{1 - \epsilon\epsilon_h} \approx \xi^{col} + \frac{\epsilon_h}{\zeta^{col}} \qquad \frac{1}{\zeta_h} = \frac{\frac{1}{\zeta_h^{col}} + \epsilon\xi^{col}}{1 - \epsilon\epsilon_h} \approx \frac{1}{\zeta_h^{col}} + \epsilon\xi^{col}$$

$$\epsilon = \frac{M^2}{P \cdot K_h}$$

$$\epsilon_h = \frac{M_h^2}{P \cdot K_h}$$

- Some special frames:
  - Hadrons collinear: $P_T = K_{hT} = 0$
  - γ^* collinear with one of the hadrons:
    - $q_\perp = P_\perp = 0 \;\Rightarrow\; q_T = -P_{h\perp(qP)}/\zeta_h$
  - γ^* collinear with partons (Brick-Wall frame)
    - $q_T = \xi P_\perp - P_{h\perp}\zeta_h \equiv k_T - p_T$ (there are small component along jet)

- Measures of non-collinearity (no theoretical bias!)
  - $q_T^2 = -Q^2\left(1 - \xi\frac{P \cdot q}{Q^2} + \frac{K_h \cdot q}{\zeta_h Q^2}\right) = -\frac{Q^2}{2}\left(2 - \frac{\xi}{x} - \frac{z_h}{\zeta_h}\right)$
  - $D_T \equiv -\frac{4\xi\,\epsilon_{\mu\nu\rho\sigma} l_1^\mu l_2^\nu P^\rho K_h^\sigma}{\zeta Q^3}$

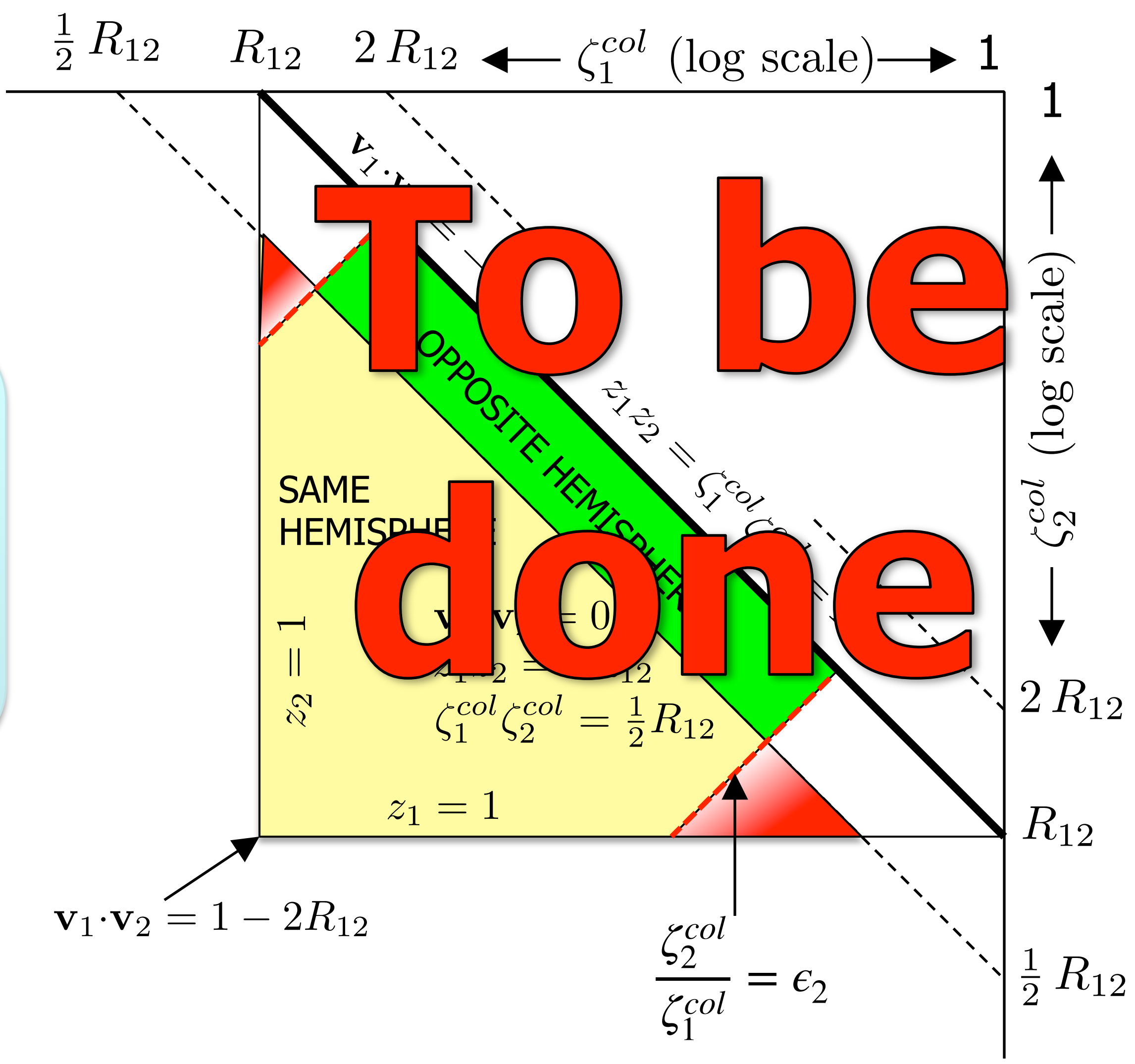
SIDIS (fixing R_{12})

- Allowed regions for given hadron pairs (target and current fragmentation)

$$R_{12}^{SIDIS} = \frac{2\,P\cdot P_h}{Q^2}$$

$$\frac{\zeta_h^{col}}{x} = \frac{z_h}{\xi^{col}} = R_{12}^{SIDIS}$$

$$\frac{\zeta_h}{\xi\,R_{12}} = \frac{1-\mathbf{v}^{bw}\cdot\mathbf{v}_h^{bw}}{2\,|\mathbf{v}^{bw}|\,|\mathbf{v}_h^{bw}|}$$



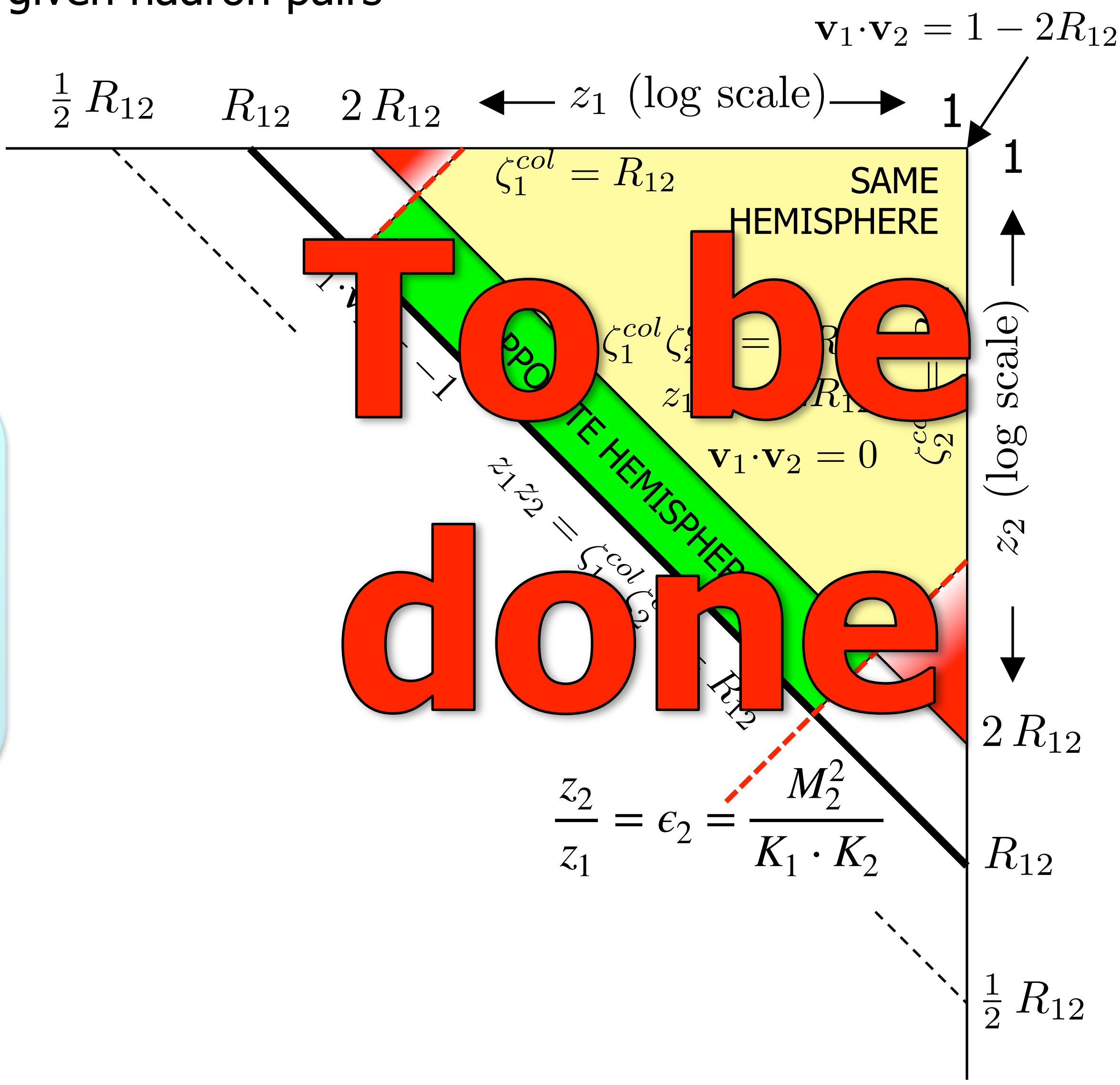
SIDIS (fixing R_{12})

- Allowed regions for given hadron pairs

$$R_{12}^{SIDIS} = \frac{2\,P\cdot P_h}{Q^2}$$

$$\frac{\zeta_h^{col}}{x} = \frac{z_h}{\xi^{col}} = R_{12}^{SIDIS}$$

$$\frac{x\,R_{12}}{z_h} = \frac{1 - \mathbf{v}^{bw}\cdot\mathbf{v}_h^{bw}}{2\,|\mathbf{v}^{bw}|\,|\mathbf{v}_h^{bw}|}$$



Concluding remarks

- Collinear fractions are key ingredients in the hadron – parton transition (in processes such as DIS and single hadron production in annihilation
- Difference between collinear fractions contain information on (convoluted) 3D structure in processes like SIDIS, two-hadron inclusive annihilation and Drell-Yan.
- Inclusion of lepton plane provides additional (also convoluted) information on individual transverse momenta (azimuthal structure in transverse plane).