

# Describing Unpolarized SIDIS data at order $\alpha_s$



UNIVERSITÀ  
DEGLI STUDI  
DI TORINO

J Osvaldo Gonzalez-Hernandez  
University of Turin

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# Outlook

- **Importance of unpolarized functions**
- **Some challenges extracting unpolarized functions from SIDIS**
- **Signals of non-perturbative dominance**
- **Testing the kinematics where factorization theorems hold.**

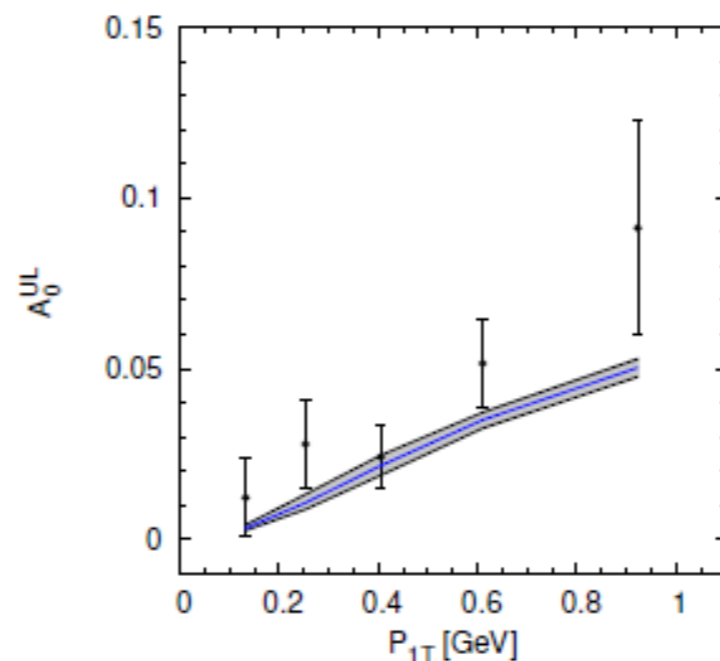
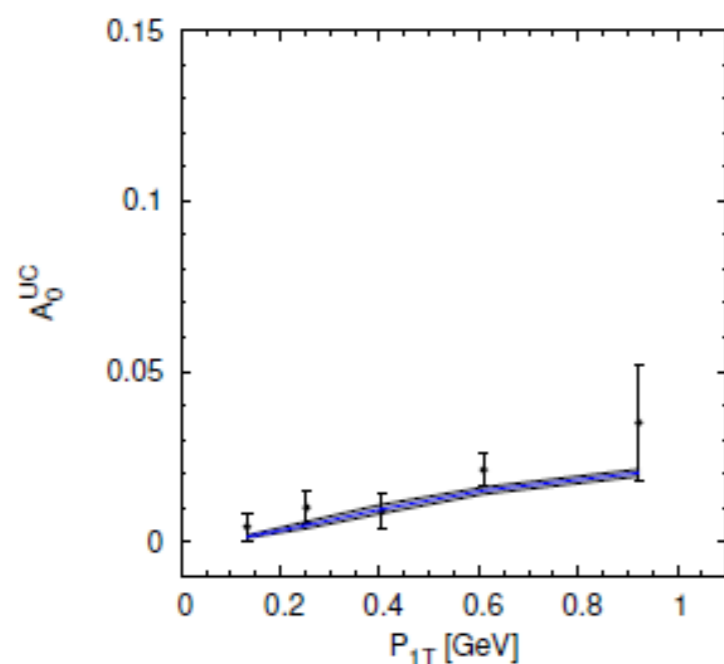
**TMD physics, rich phenomenology.**

**Gateway to 3D structure of hadrons**

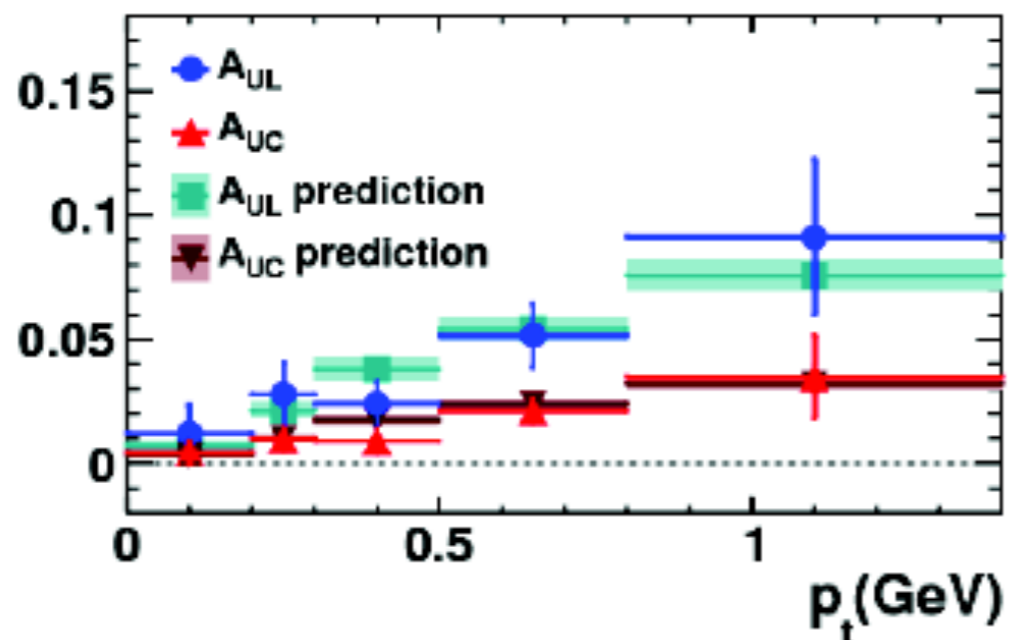
# $e^+e^-$ double ratios (Collins function)

$Q^2 = 13 \text{ GeV}^2$

Predictions for BES III



Simple gaussian picture



Picture within QCD-factorization

Kang, Prokudin, Sun, Yuan

Phys.Rev. D93 (2016) no.1, 014009

arXiv:1505.05589 [hep-ph] JLAB-THY-15-2044

# Input needed : Unpolarized functions

$e^+e^- \rightarrow \pi\pi X$

**SIDIS**

**Unpolarized TMDFF**

**Collins TMDFF**

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2 \mathbf{P}_{1T} d \cos \theta_2} = \frac{3\pi\alpha^2}{2s} \left\{ D_{h_1 h_2} + N_{h_1 h_2} \cos 2\phi_1 \right\}$$

$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

**Ratio**

$$D^U = D_{\pi^+ \pi^-} + D_{\pi^- \pi^+}$$

$$N^U = N_{\pi^+ \pi^-} + N_{\pi^- \pi^+}$$

$$D^L = D_{\pi^+ \pi^+} + D_{\pi^- \pi^-}$$

$$N^L = N_{\pi^+ \pi^+} + N_{\pi^- \pi^-}$$

$$D^C = D^U + D^L$$

$$N^C = N^U + N^L,$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) \boxed{A_0^{UL(C)} \text{ Double Ratio}}$$

$$\frac{d\sigma^{\ell(S_\ell)+p(S) \rightarrow \ell' h X}}{dx_B dQ^2 dz_h d^2 \mathbf{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + \dots \right.$$

$$\left. + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}$$

$$A_{UT}^{\sin(\phi_h + \phi_S)}$$

**Ratio**

$$F_{UT}^{\sin(\phi_h + \phi_S)}$$

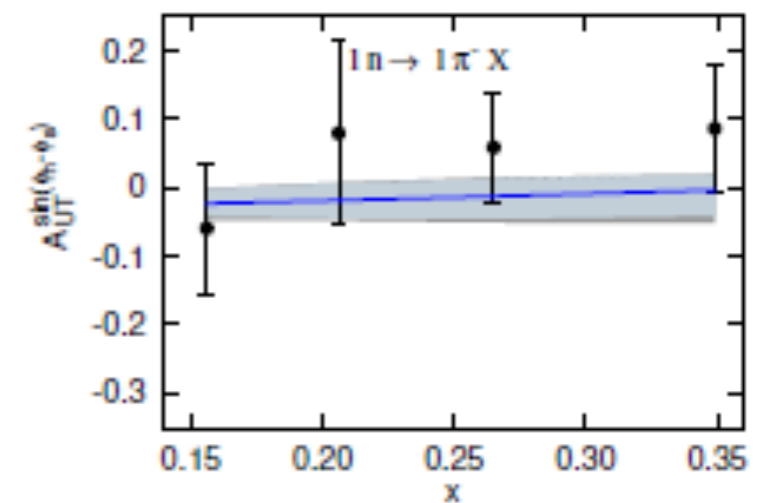
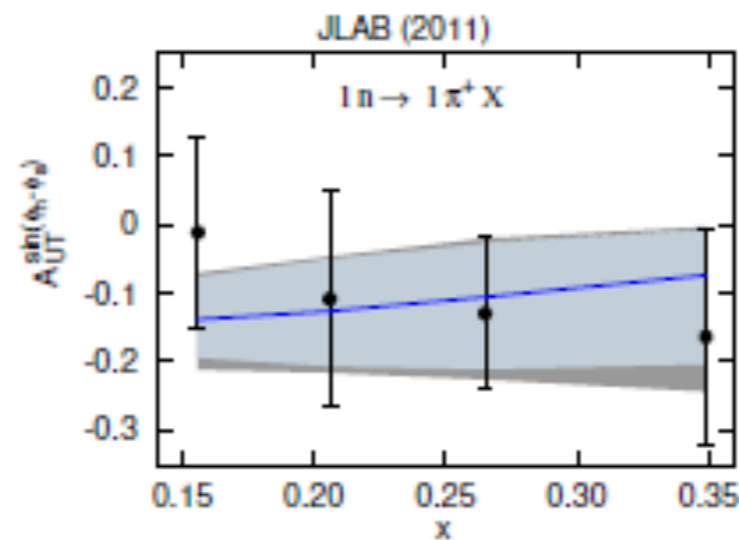
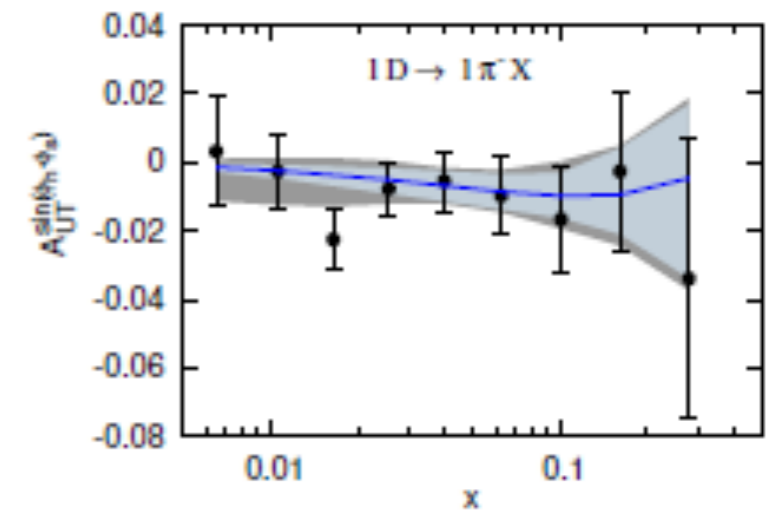
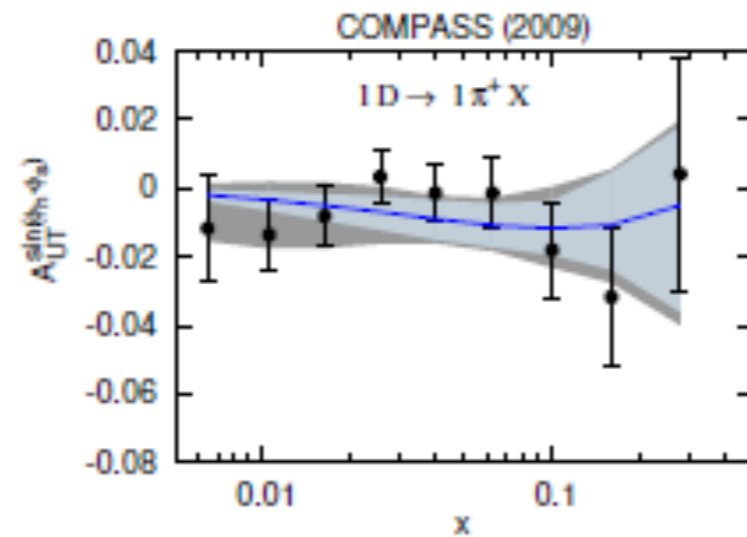
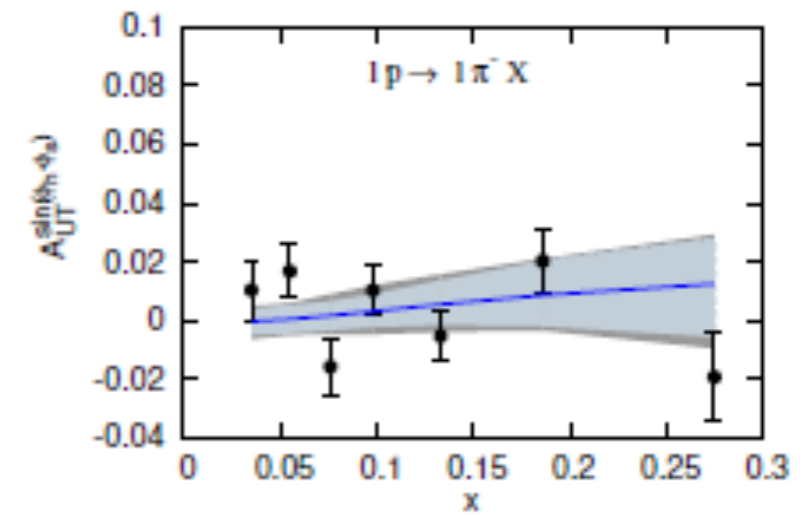
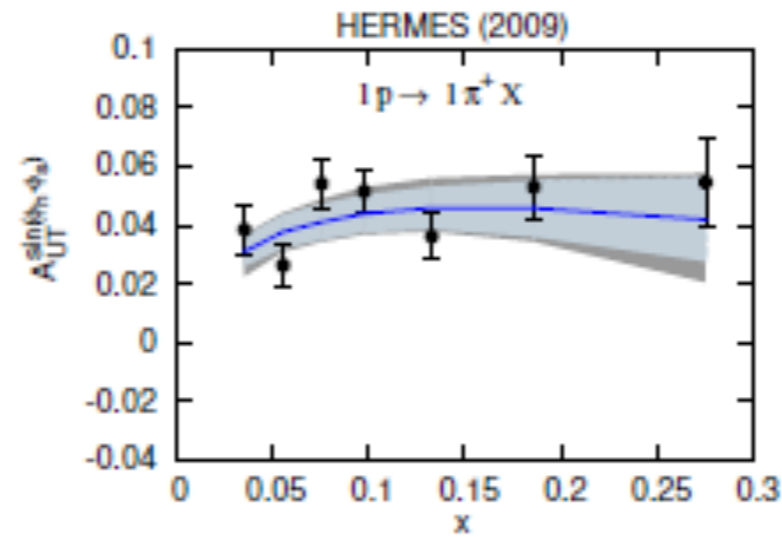
$$F_{UU}$$

**Unpolarized TMDFF & TMDPDF**

**TMD Transversity & Collins function**

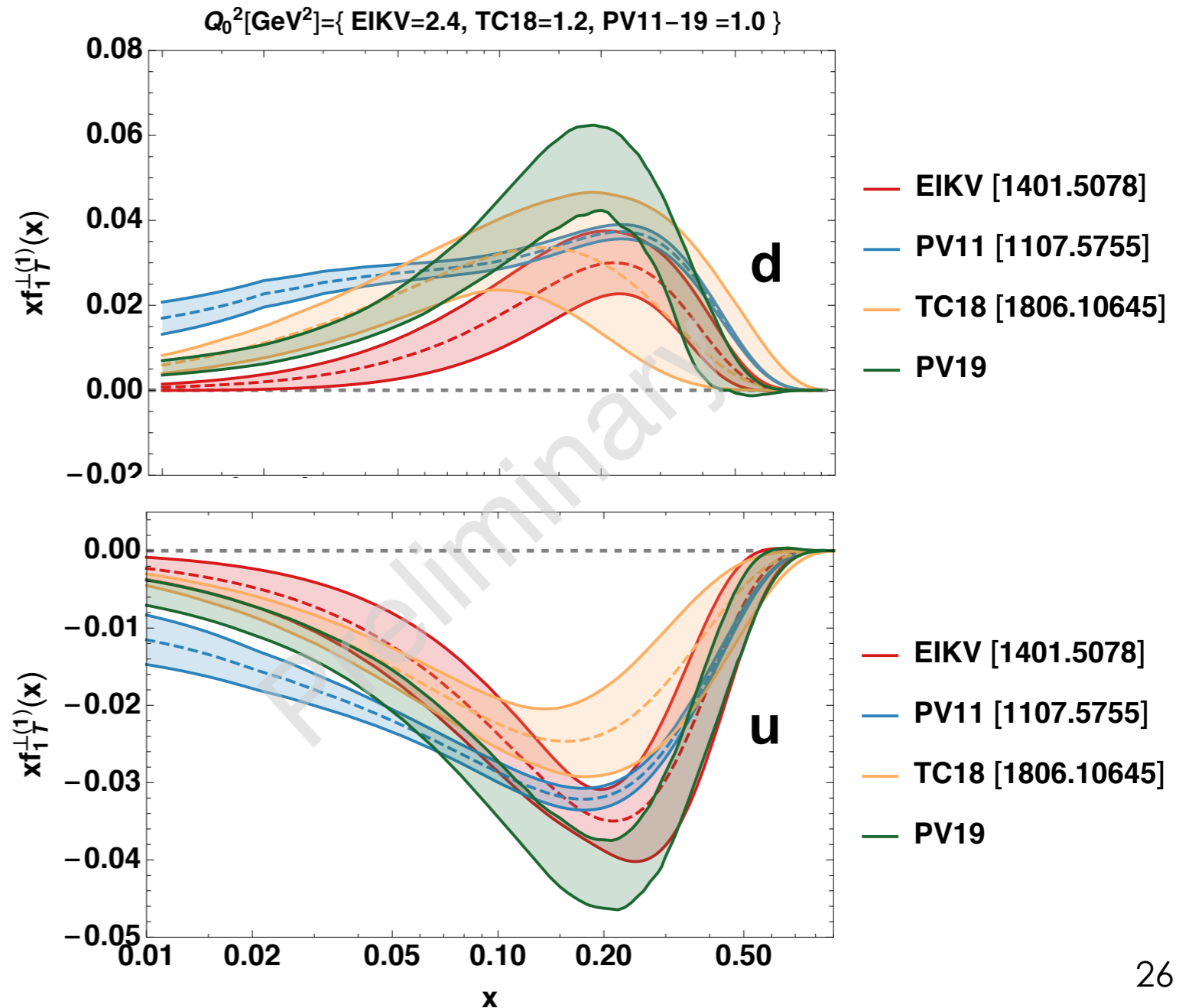
# SIDIS Sivers Asymmetry

Unpolarized  
Functions  
needed as input



# Results comparison

Differences partly due to different assumptions on unpolarized functions, not necessarily from type of picture, i.e.  
Gaussian Ansatz  
CSS  
SCET...



**How well can we extract unpolarized  
TMDs from SIDIS data?  
(challenges)**



# Theoretical Framework: Factorization theorems

$W$  (TMD region)

$$\sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\}$$

Fourier Transform of:

$$\begin{aligned} \tilde{F}_j(x, b_T, Q, \zeta_F) = & \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \underbrace{\tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2)}_{\text{pQCD}} \underbrace{f_i(\hat{x}, \mu_b)}_{\text{Input (extraction from collinear cross section)}} \\ & \times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \underbrace{\gamma_F(\mu; 1)}_{\text{pQCD}} - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \underbrace{\gamma_K(\mu)}_{\text{pQCD}} \right) \right\} \\ & \times \exp \left\{ \underbrace{-g_P(x, b_T)}_{\text{Non-perturbative functions to extract from data.}} - \underbrace{g_K(b_T)}_{\text{Non-perturbative functions to extract from data.}} \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\}, \end{aligned}$$



pQCD

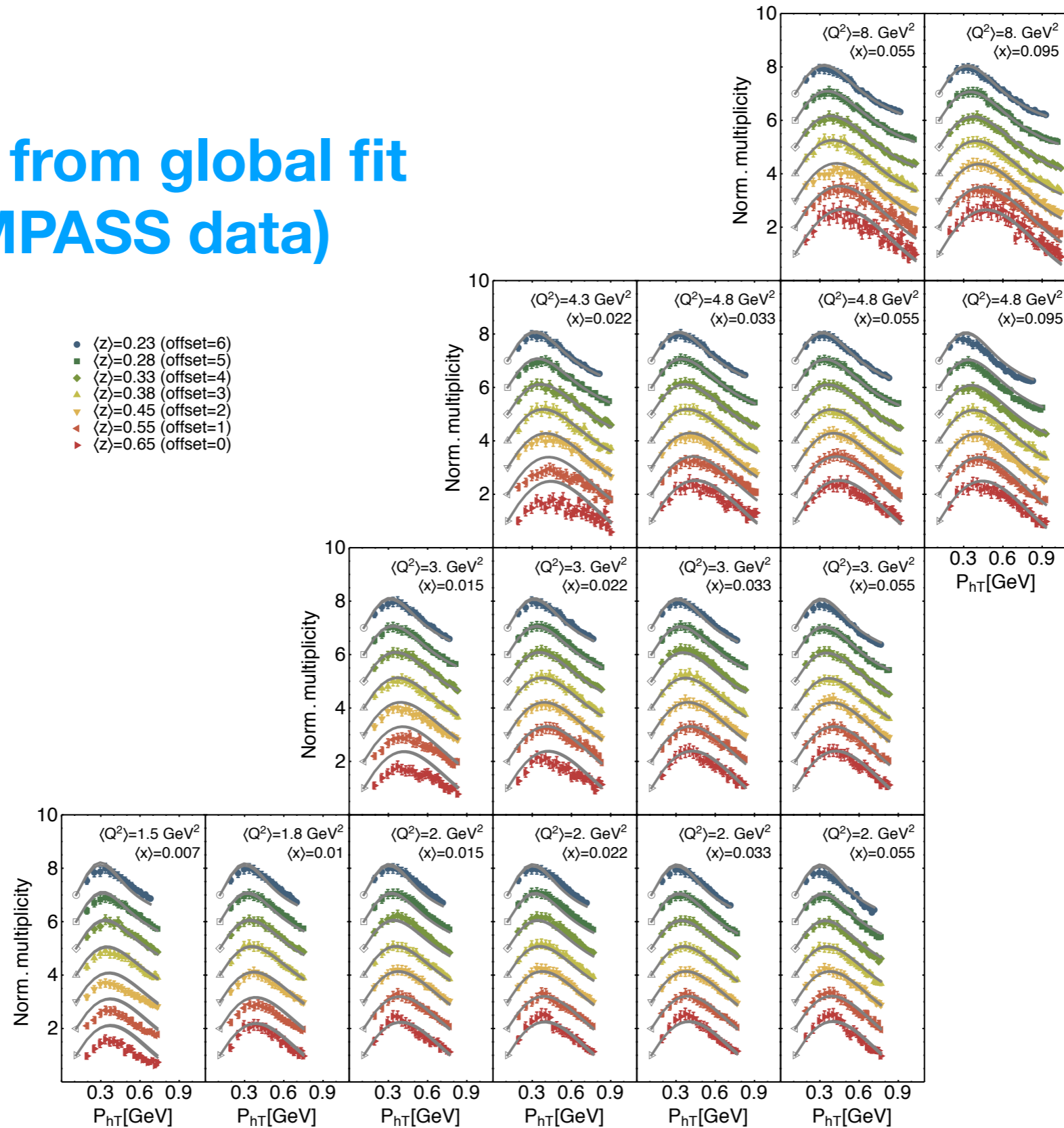


Input (extraction from collinear cross section)



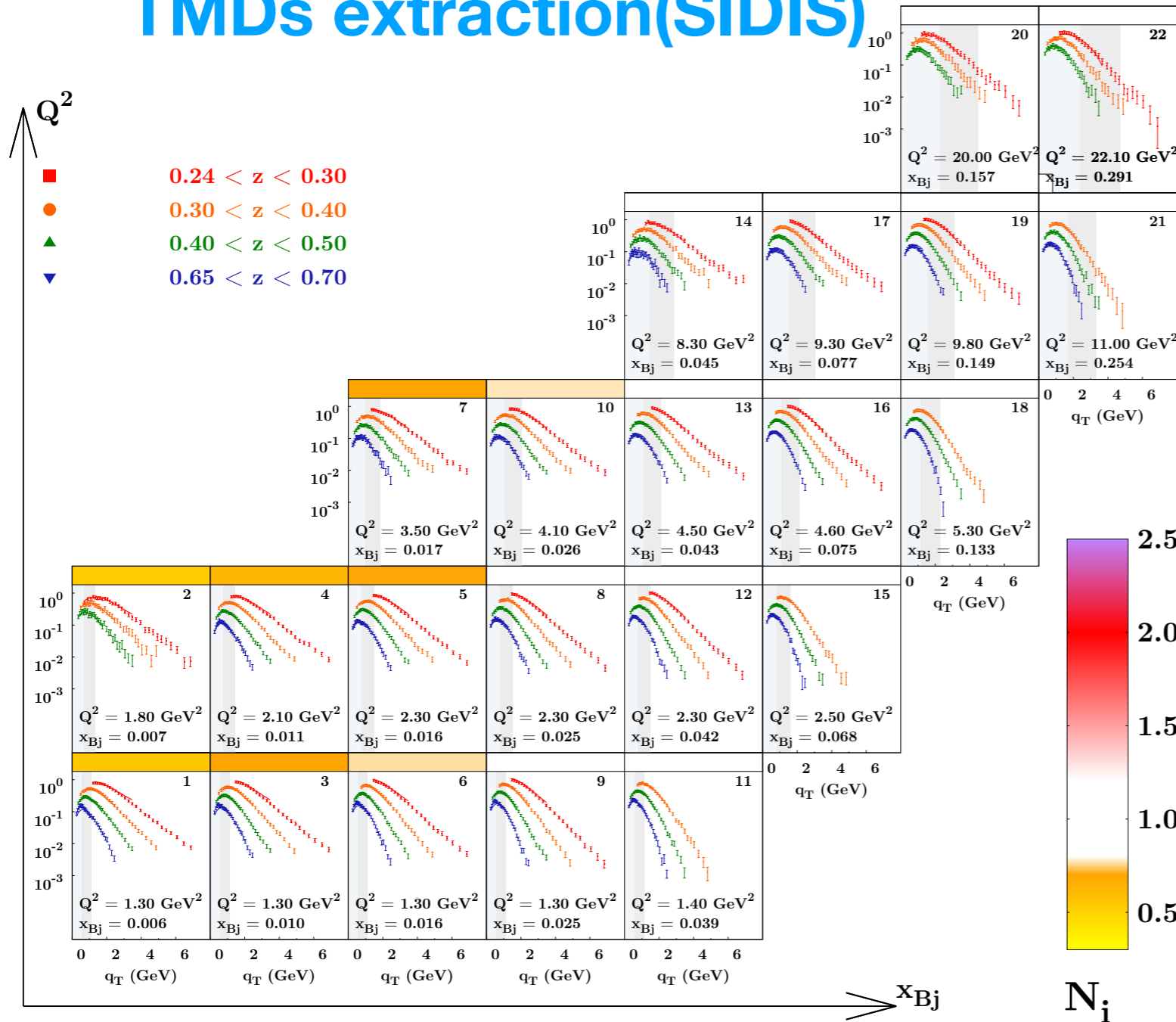
Non-perturbative functions to extract from data.

# Example: extraction from global fit (2013 COMPASS data)

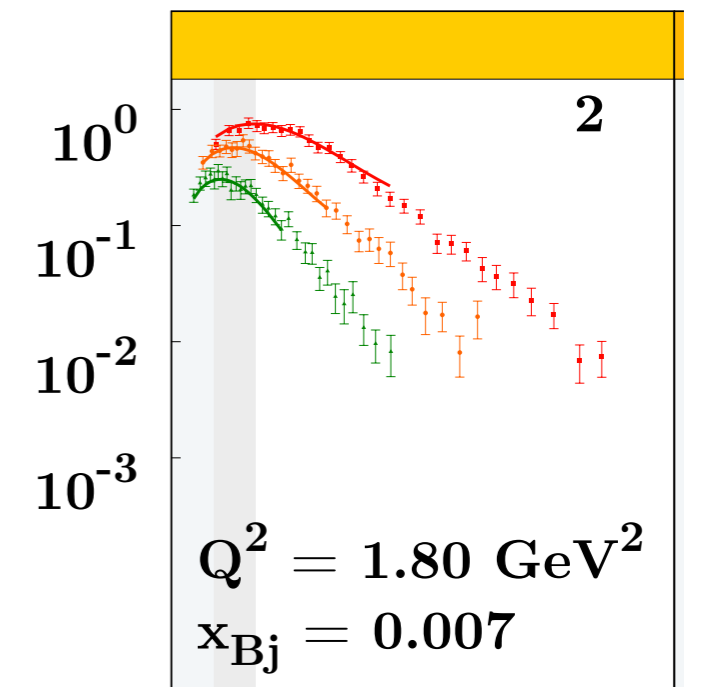
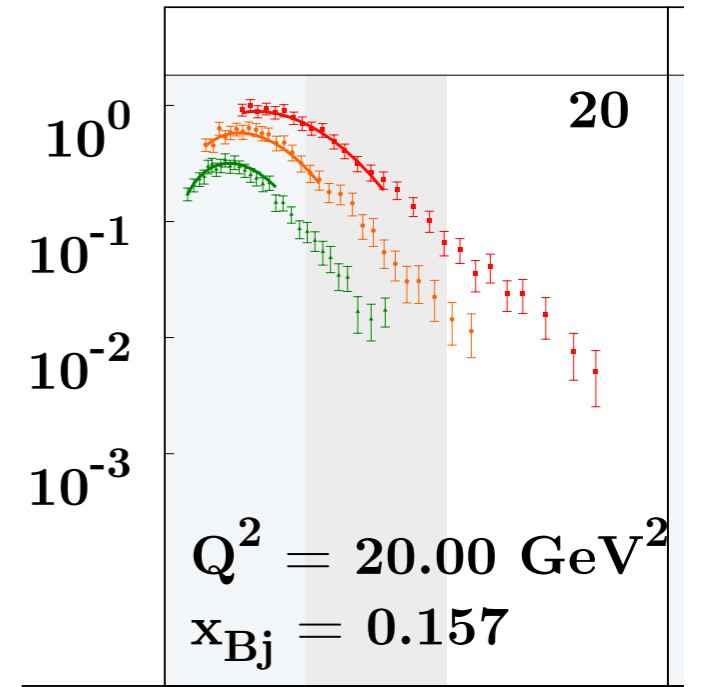


**Bacchetta, Delcarro, Pisano, Radici, Signori**  
**JHEP 1706 (2017) 081**

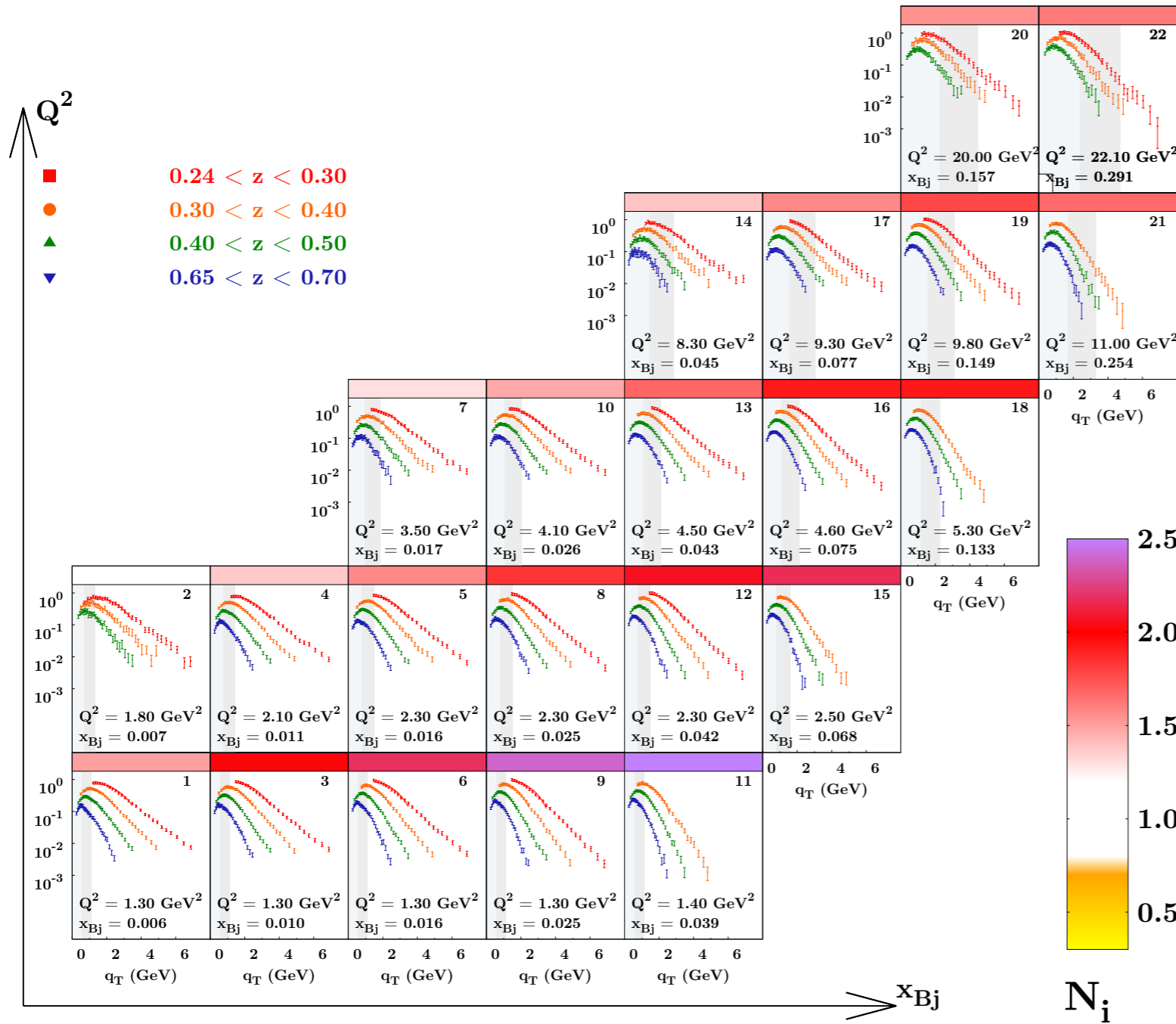
# Some issues with unpolarized TMDs extraction (SIDIS)



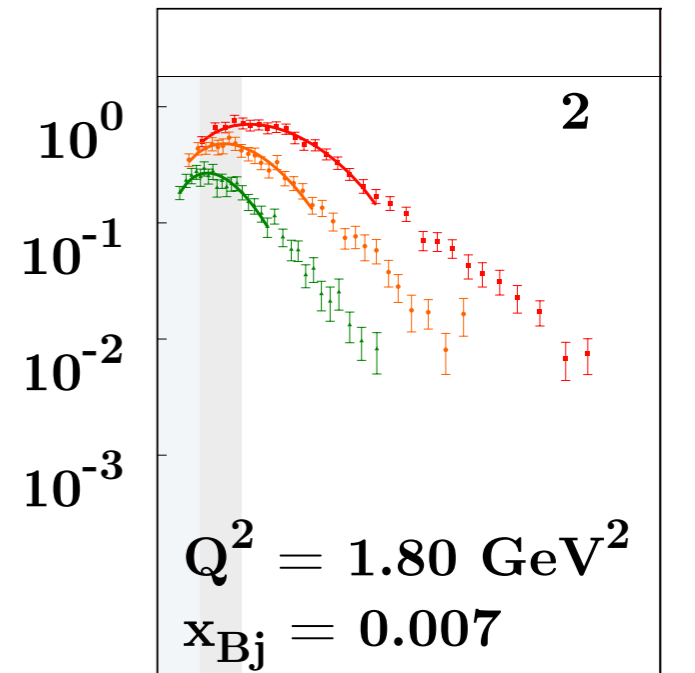
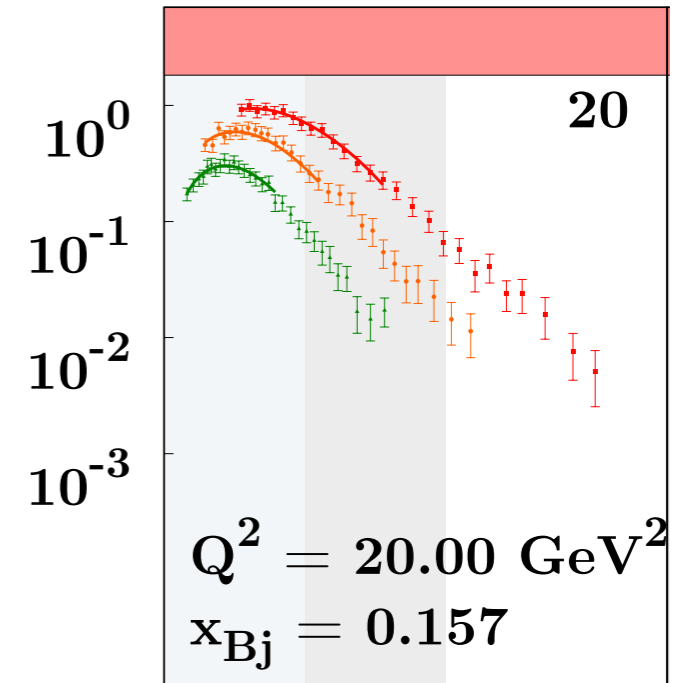
$$\mathcal{O}(\alpha_s^0)$$



$$\tilde{F}_j = f(x, \mu_b) \exp \left\{ g_P(x, b_T) - g_K(b_T) \ln \left( \frac{Q}{Q_0} \right) \right\}$$



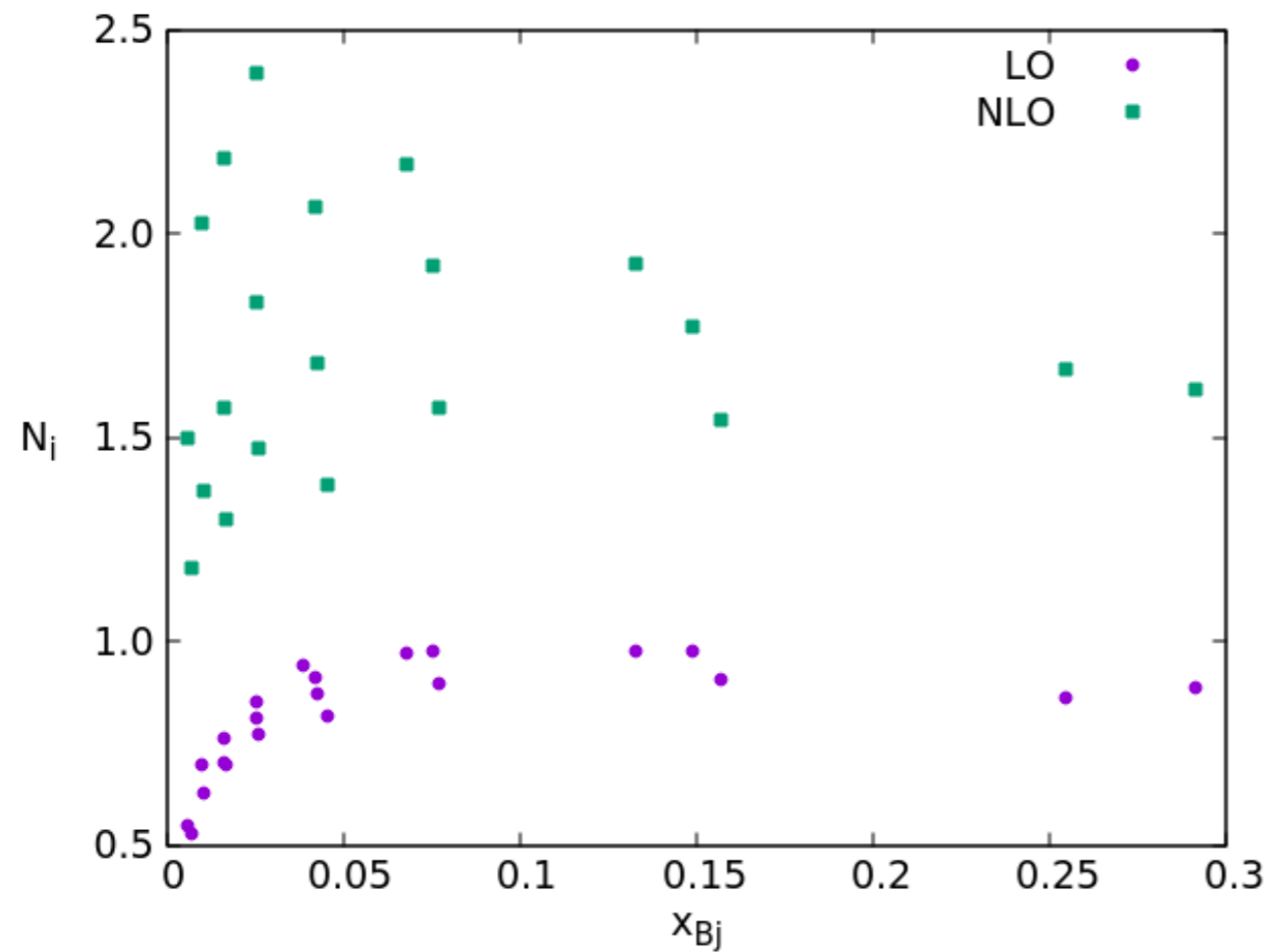
$\mathcal{O}(\alpha_s)$



$$\begin{aligned}
 \tilde{F}_j(x, b_T, Q, \zeta_F) = & \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right) \frac{\tilde{K}(b_*, \mu_b)}{\sum_j} \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b) \\
 & \times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \\
 & \times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\},
 \end{aligned}$$

## Some challenges:

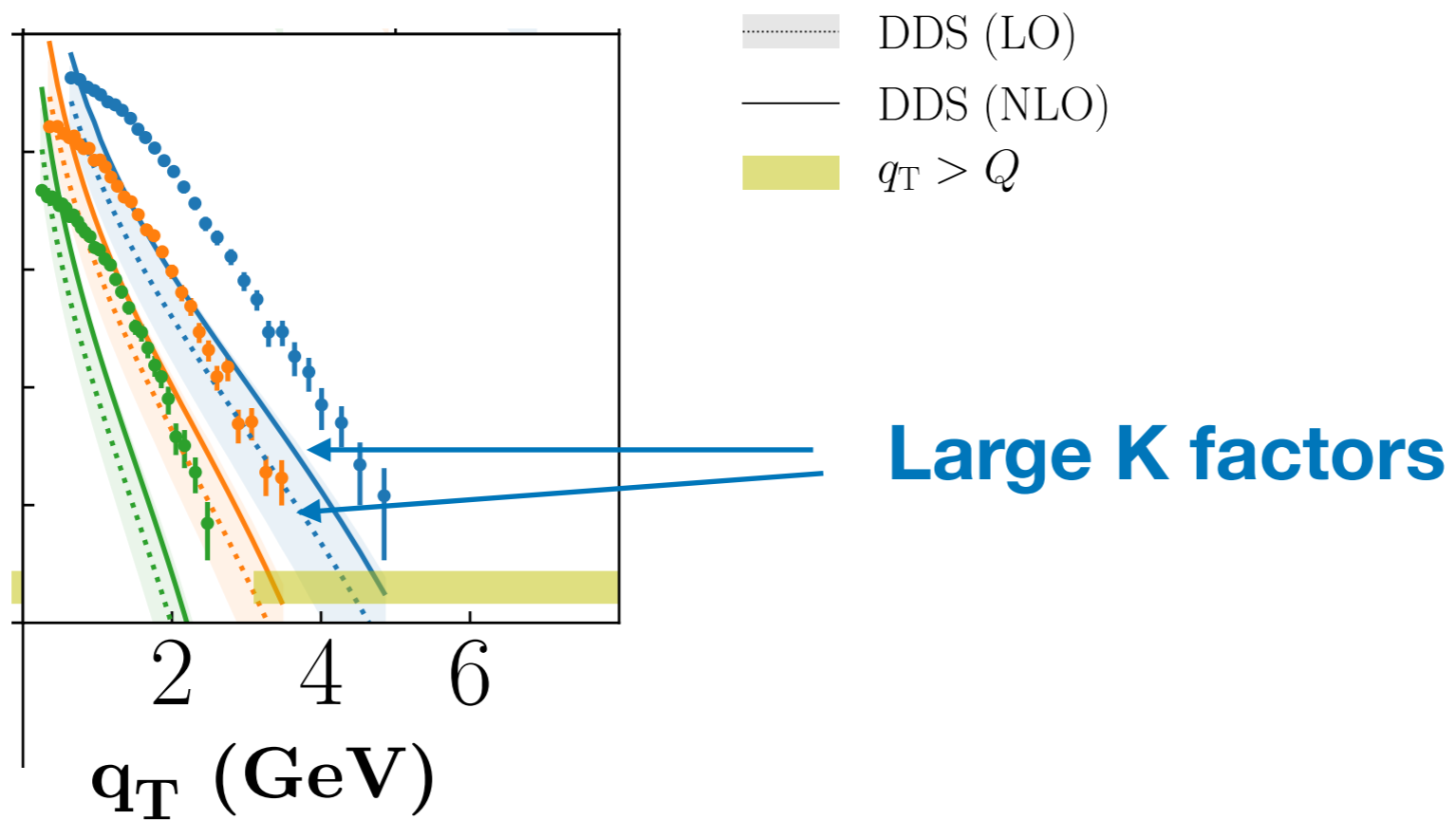
Large (spurious) normalizations have to be introduced to described data



**A bit counter-intuitive**

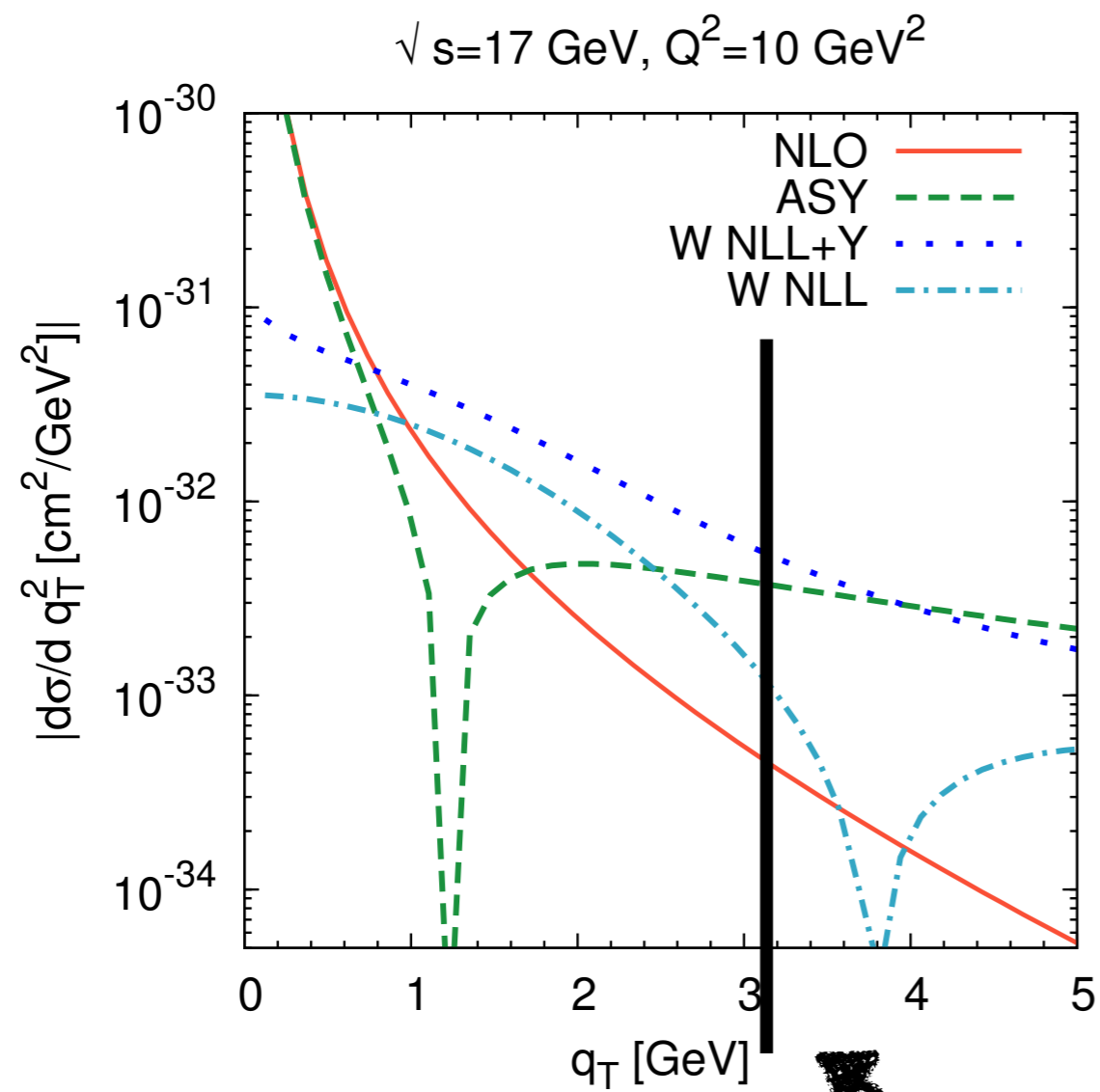
## Some challenges:

Large  $q_T$  region cannot be described with modern  
Collinear PDF sets



# Some challenges:

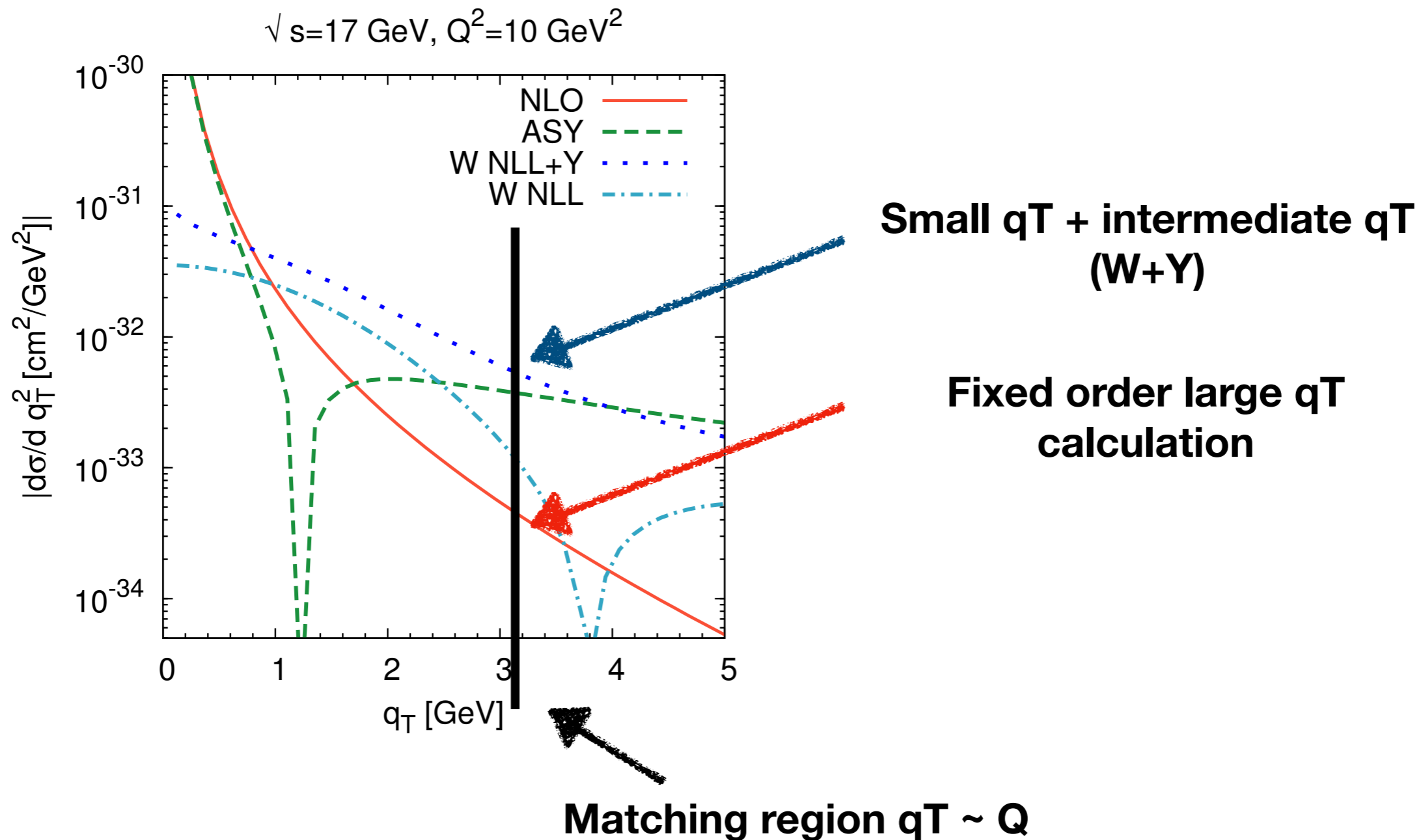
## Matching between small and large $q_T$ regions



Matching region  $q_T \sim Q$

# Some challenges:

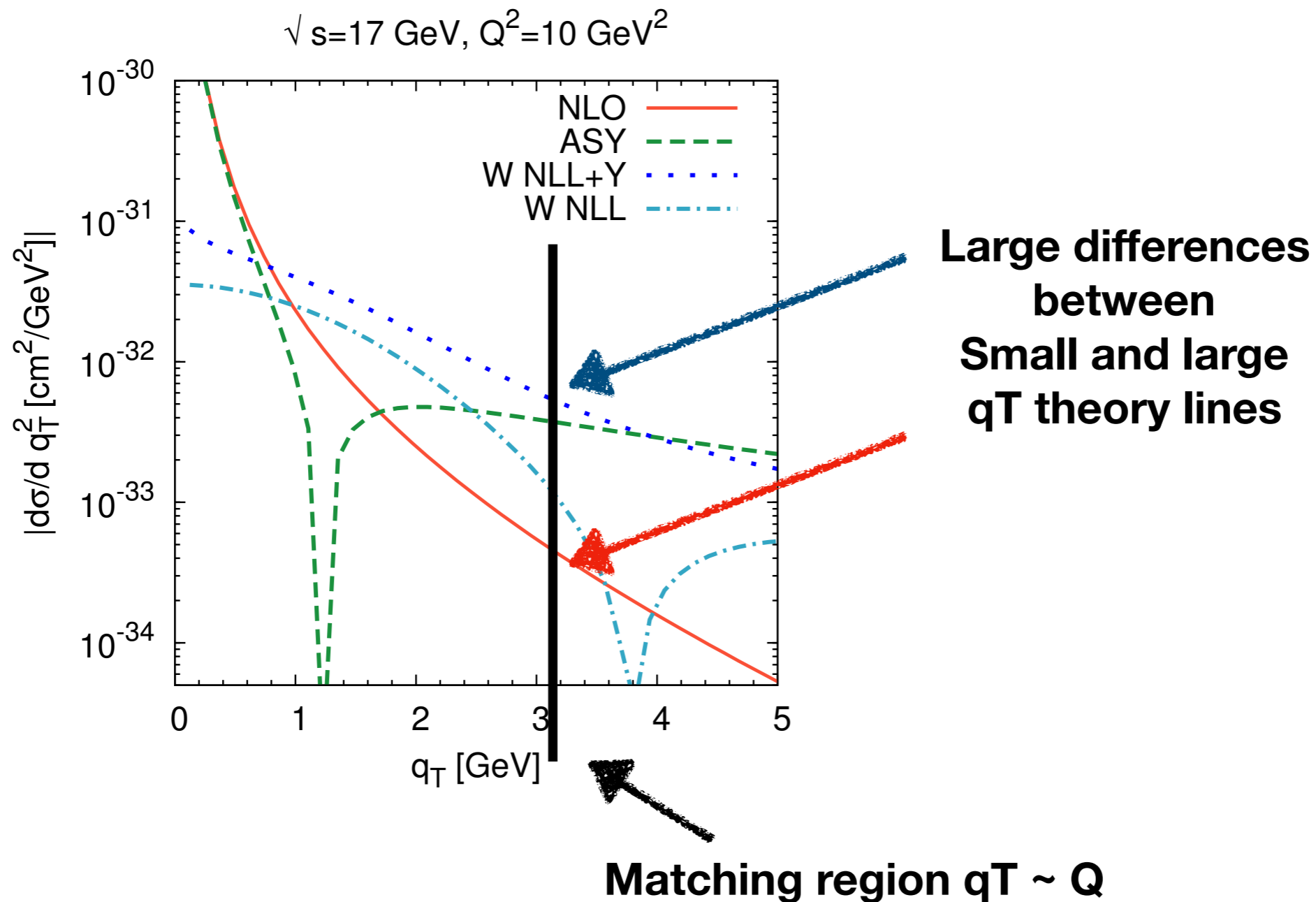
## Matching between small and large $q_T$ regions





# Some challenges:

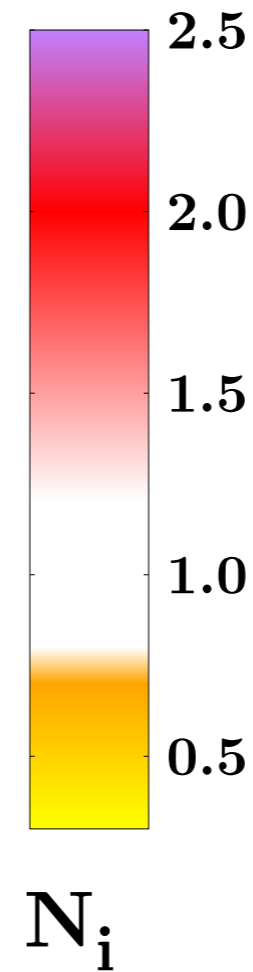
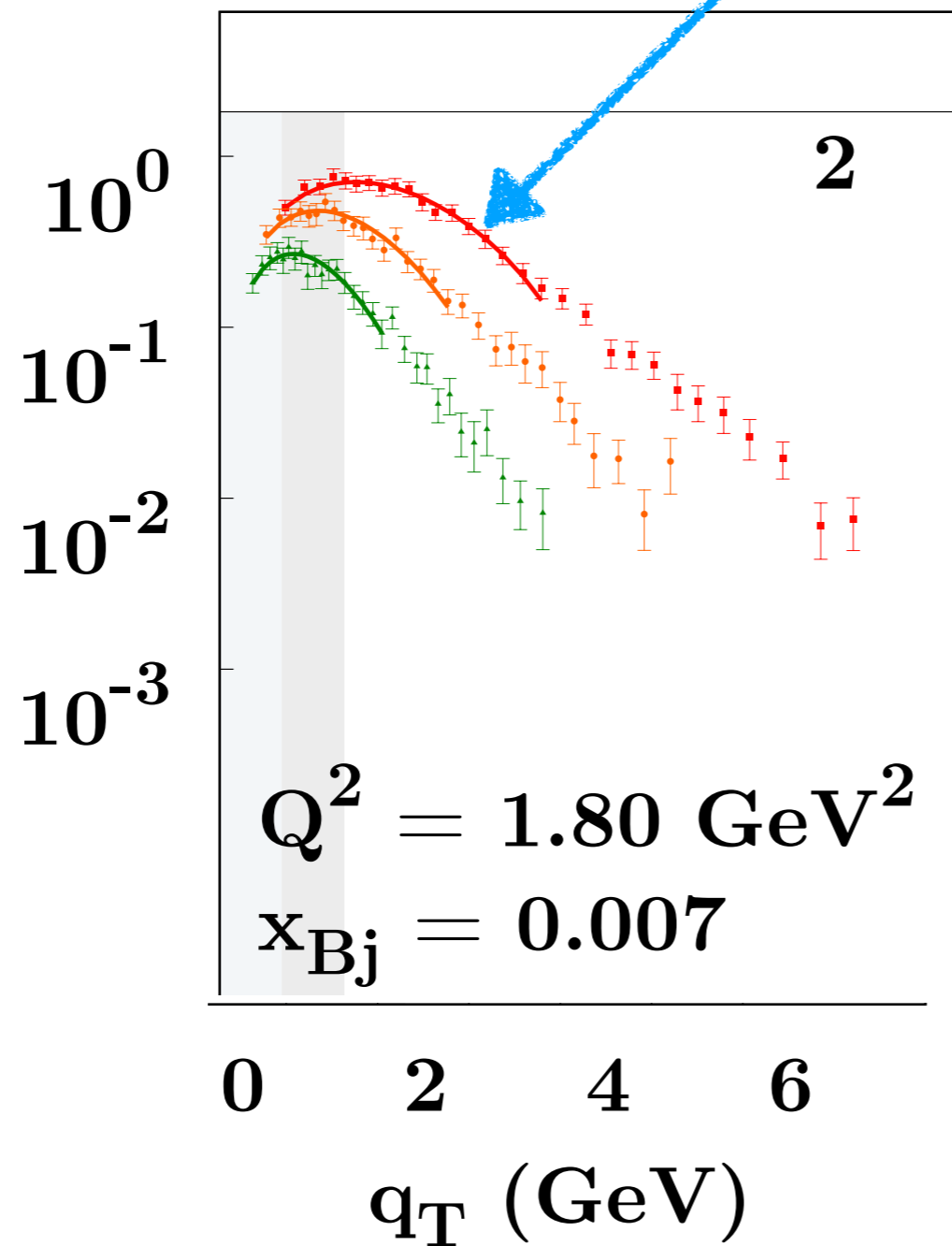
## Matching between small and large $q_T$ regions



# **Signals of non-perturbative dominance**

Here TMD term describes  
data at values

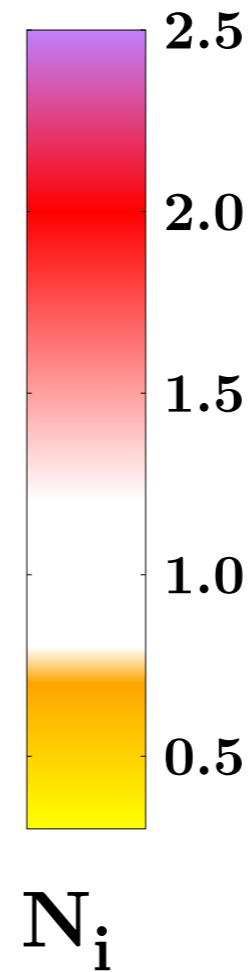
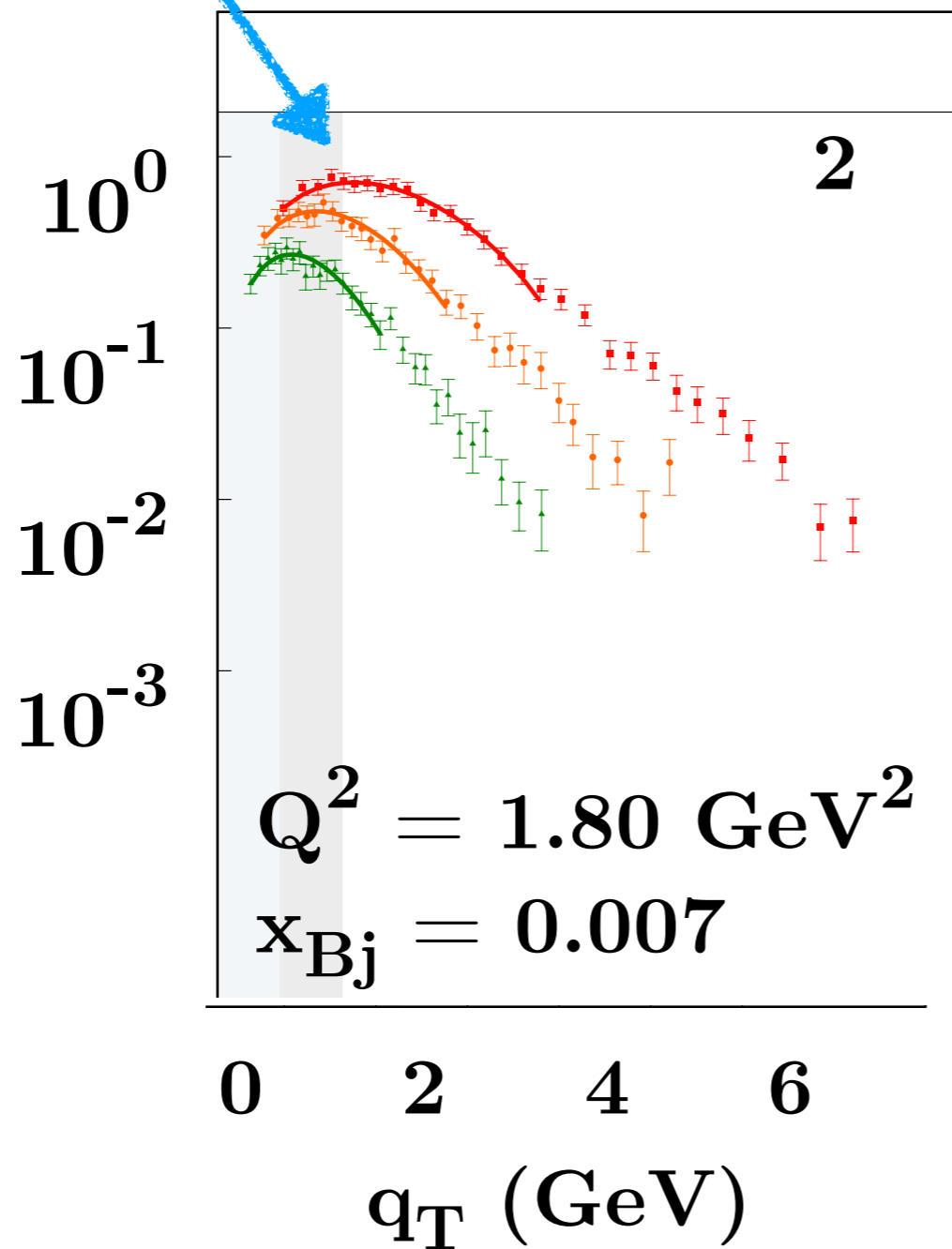
$q_T > Q$



No large  
normalization  
Needed for  
this panel

Formalism only  
justified up to  
 $q_T \sim Q$

Here TMD term describes  
data at values  
 $q_T > Q$

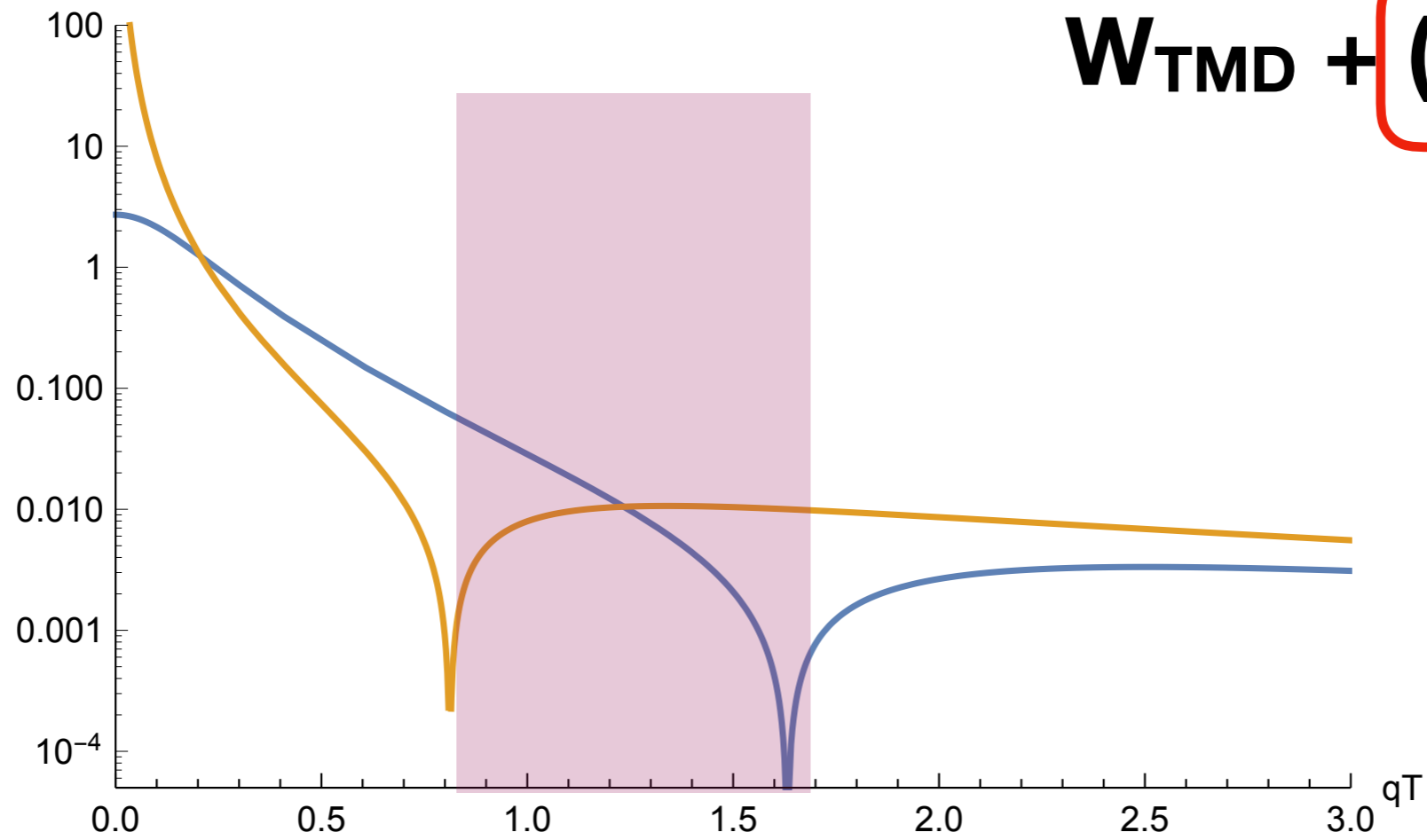


No large  
normalization  
Needed for  
this panel

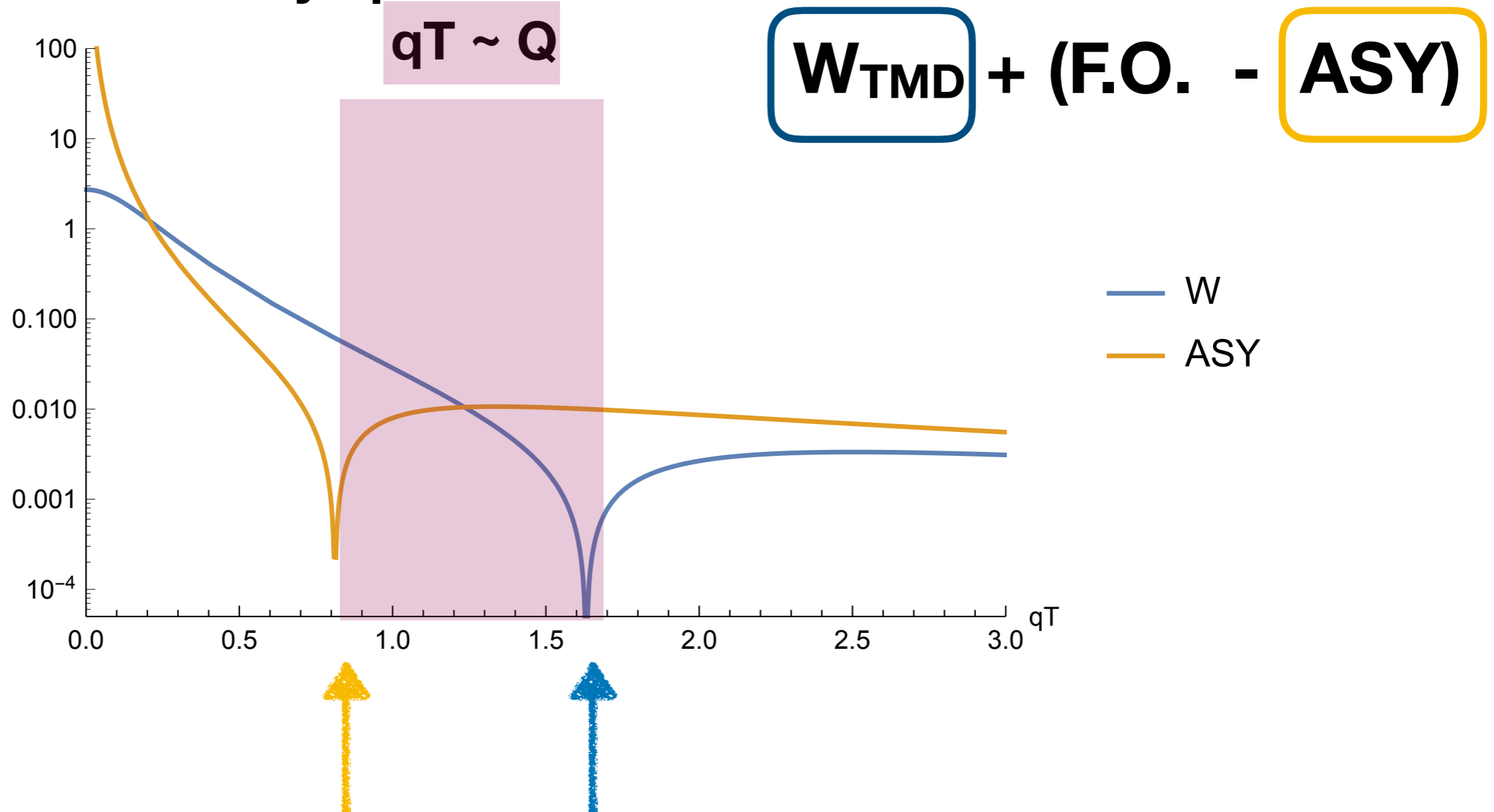
**Corrections extend TMD  
term up to  $q_T \sim Q$**



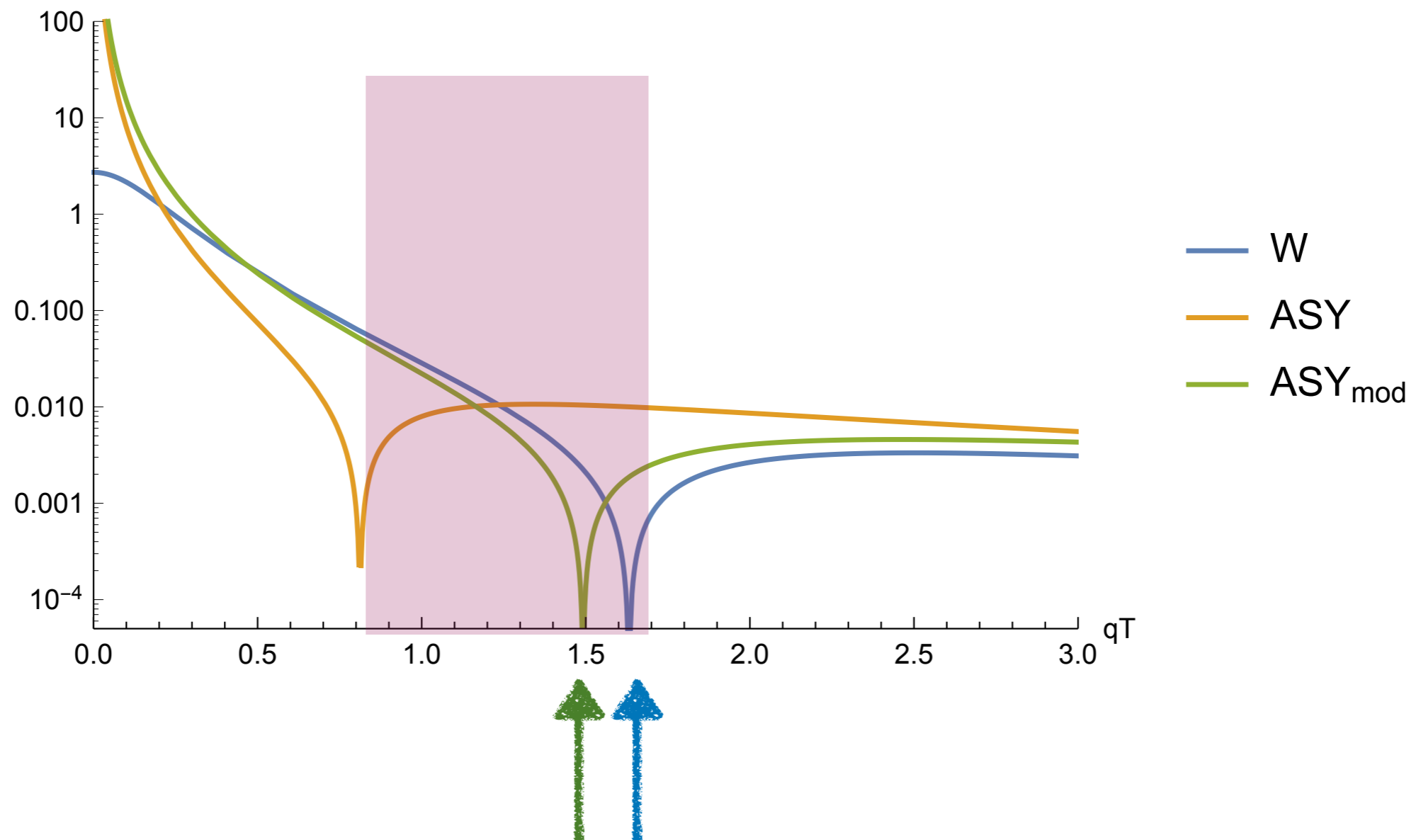
$$W_{\text{TMD}} + (\text{F.O.} - \text{ASY})$$



One of the issues  
with the matching can be  
traced to large differences  
Between TMD ( $W$ ) and  
asymptotic term at



**Large differences due  
to non-perturbative effects.  
Here, model with  
mass parameter  $M/Q \sim 0.2$**

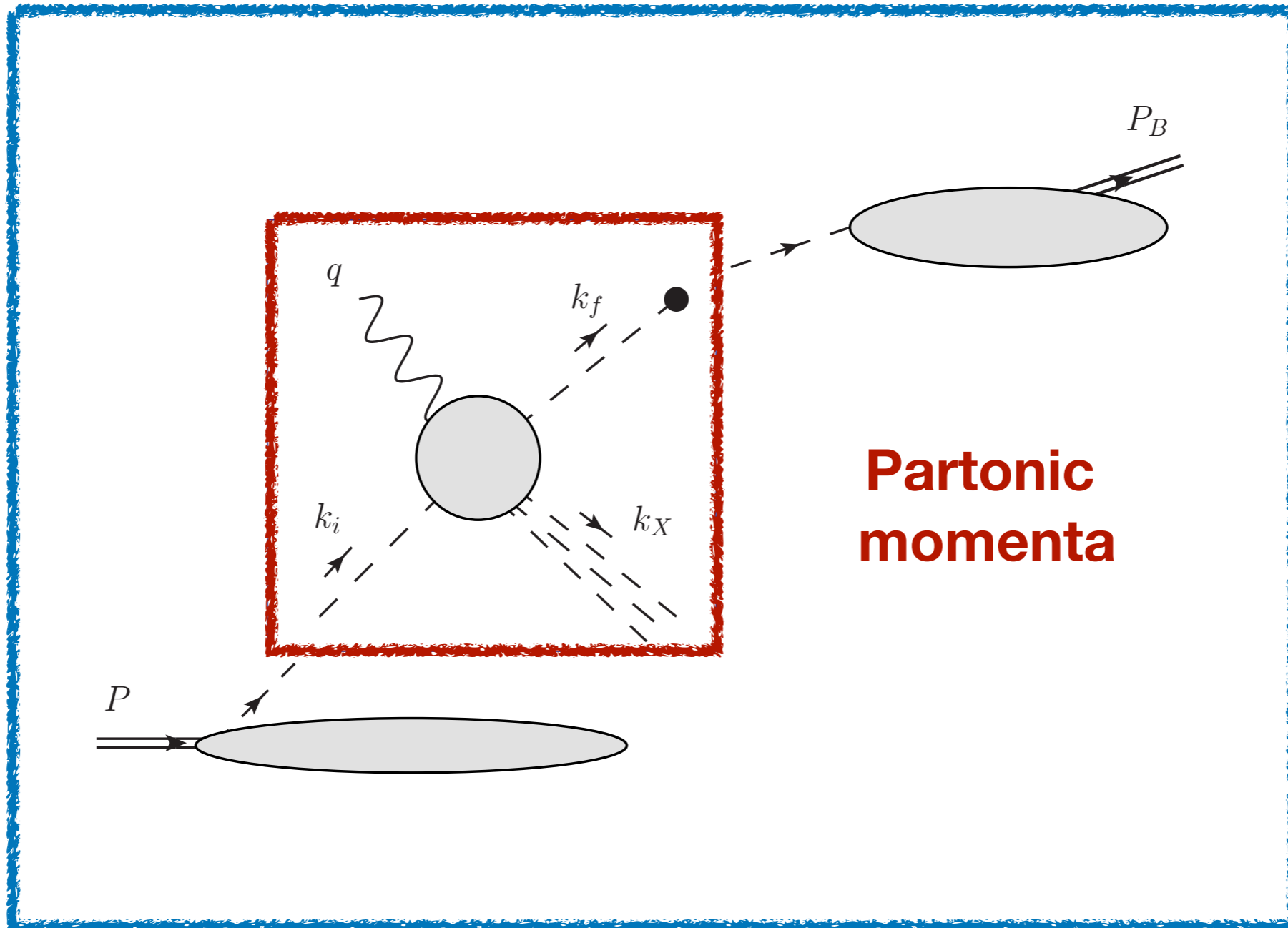


**Testing the kinematics where factorization  
theorems hold**

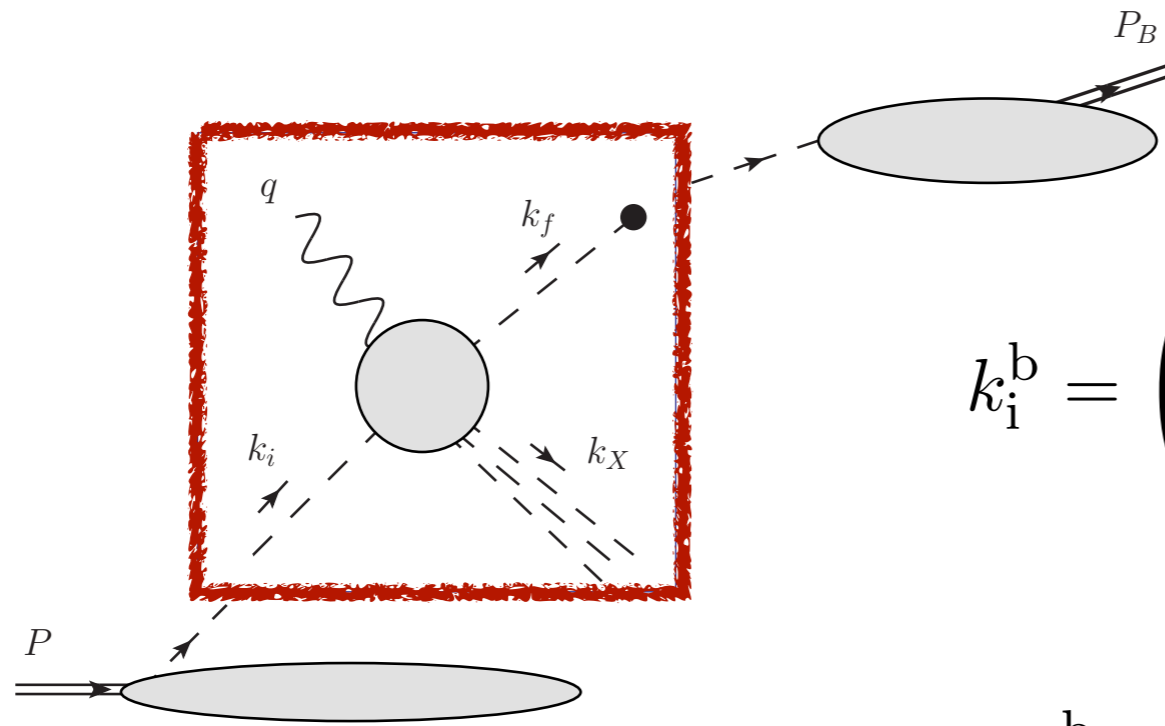


# Different types of approximations

## External momenta kinematics



# Partonic kinematics



$$k_i^b = \left( \frac{Q}{\hat{x}_N \sqrt{2}}, \frac{\hat{x}_N (k_i^2 + \mathbf{k}_{i,b,T}^2)}{\sqrt{2} Q}, \mathbf{k}_{i,b,T} \right)$$

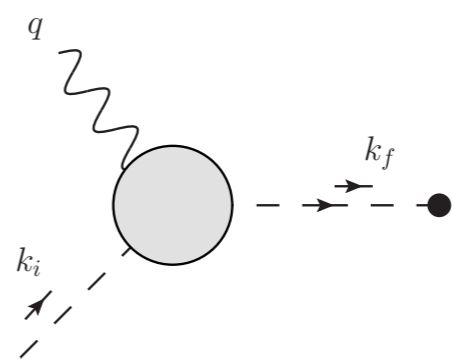
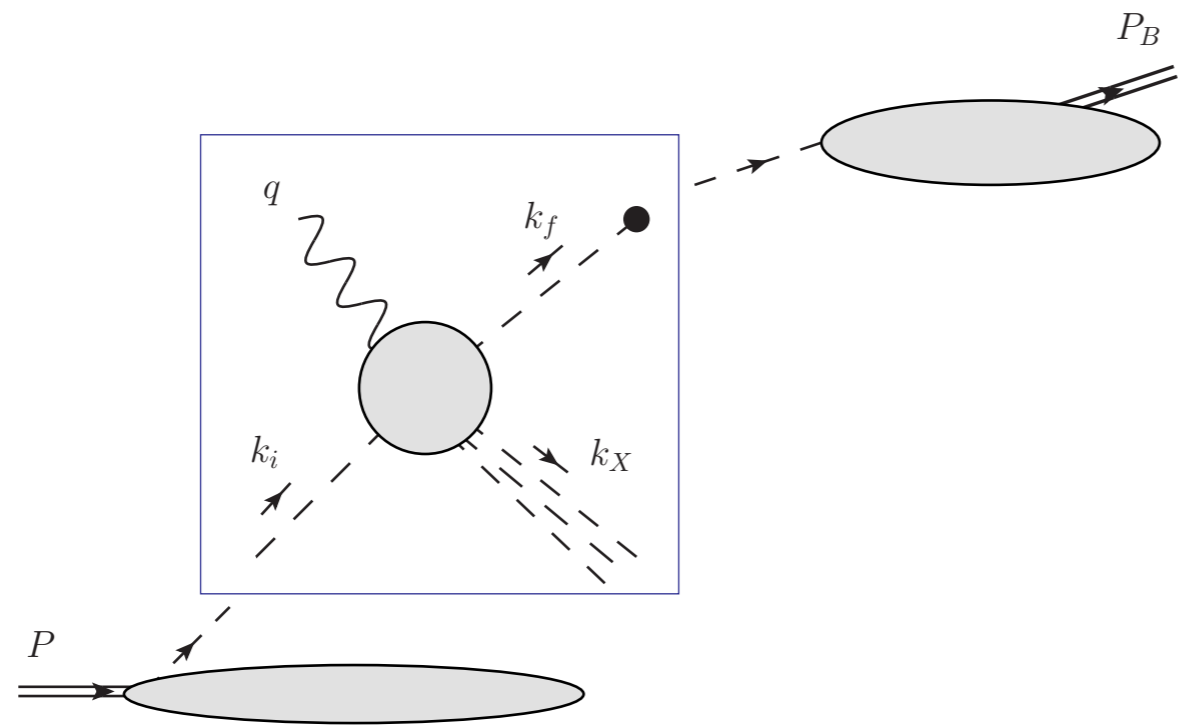
$$k_f^b = \left( \frac{\mathbf{k}_{f,b,T}^2 + k_f^2}{\sqrt{2} \hat{z}_N Q}, \frac{\hat{z}_N Q}{\sqrt{2}}, \mathbf{k}_{f,b,T} \right)$$

$$k_i^+ \equiv \xi P_b^+$$

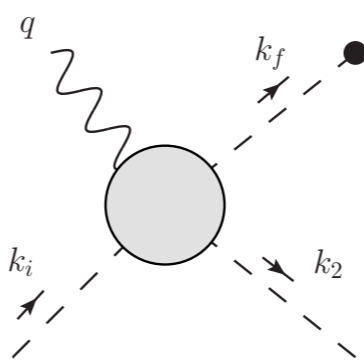
$$\hat{x}_N \equiv -\frac{q_b^+}{k_{i,b}^+} = \frac{x_N}{\xi}$$

$$P_{B,b}^- \equiv \zeta k_f^-$$

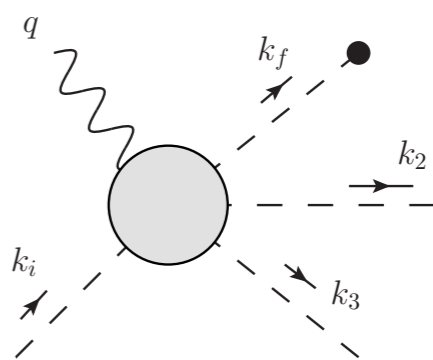
$$\hat{z}_N \equiv \frac{k_{f,b}^-}{q_b^-} = \frac{z_N}{\zeta}$$



(a)

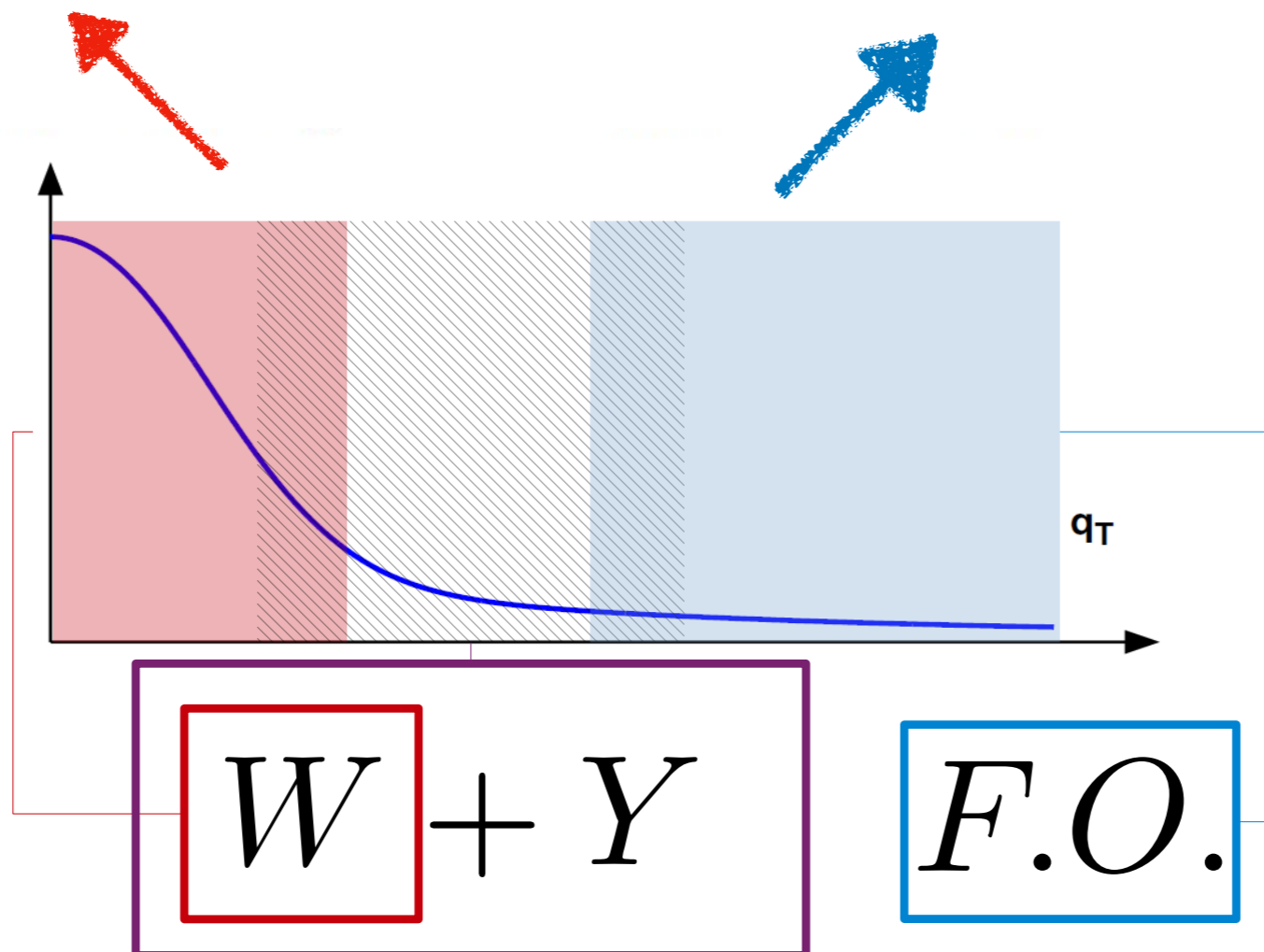
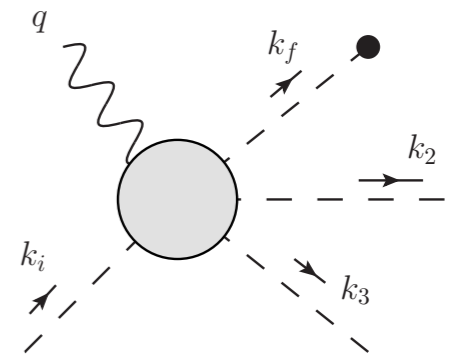
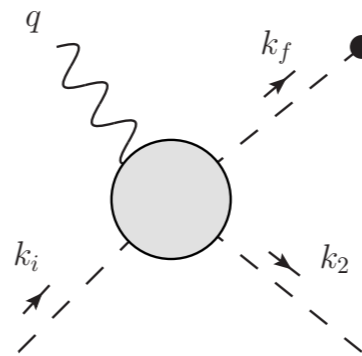
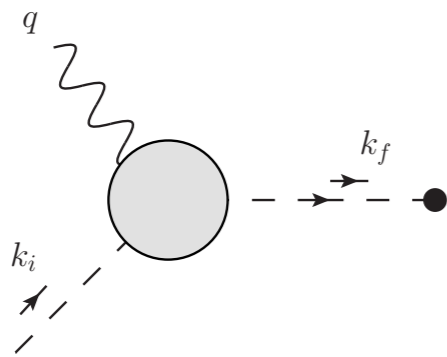


(b)

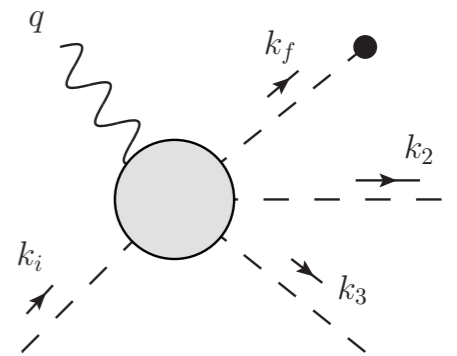
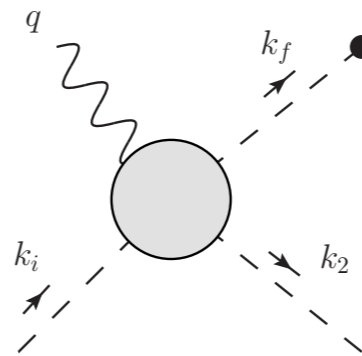
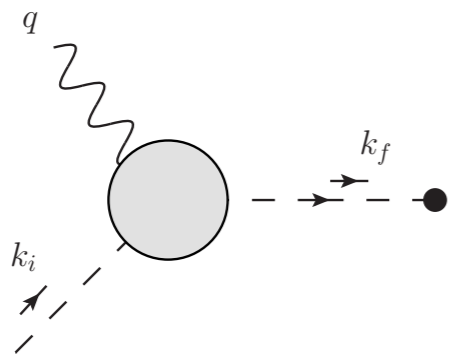


(c)

$$k \equiv k_f - q$$



$$k^2 = (k_f - q)^2$$



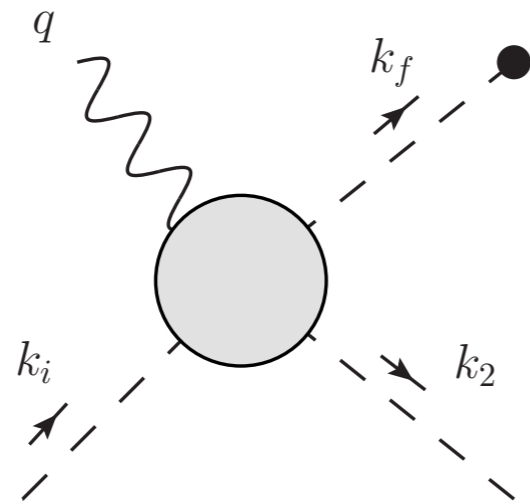
$$k^2 = 0$$



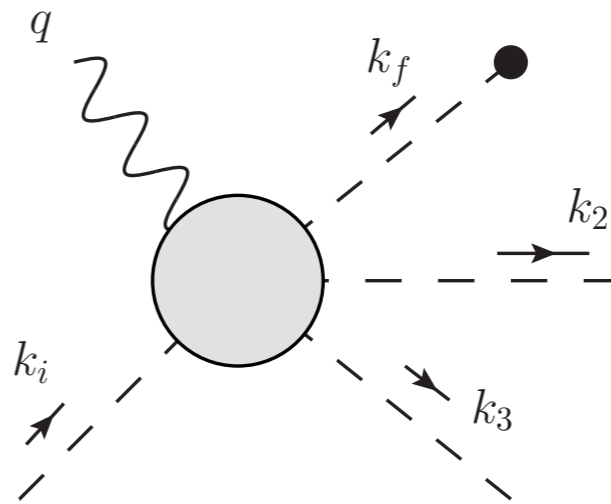
$$k^2 \neq 0$$

**Allows to distinguish  
handbag from real emission  
kinematics**

$$k_X^2 = (k_i + q - k_f)^2$$

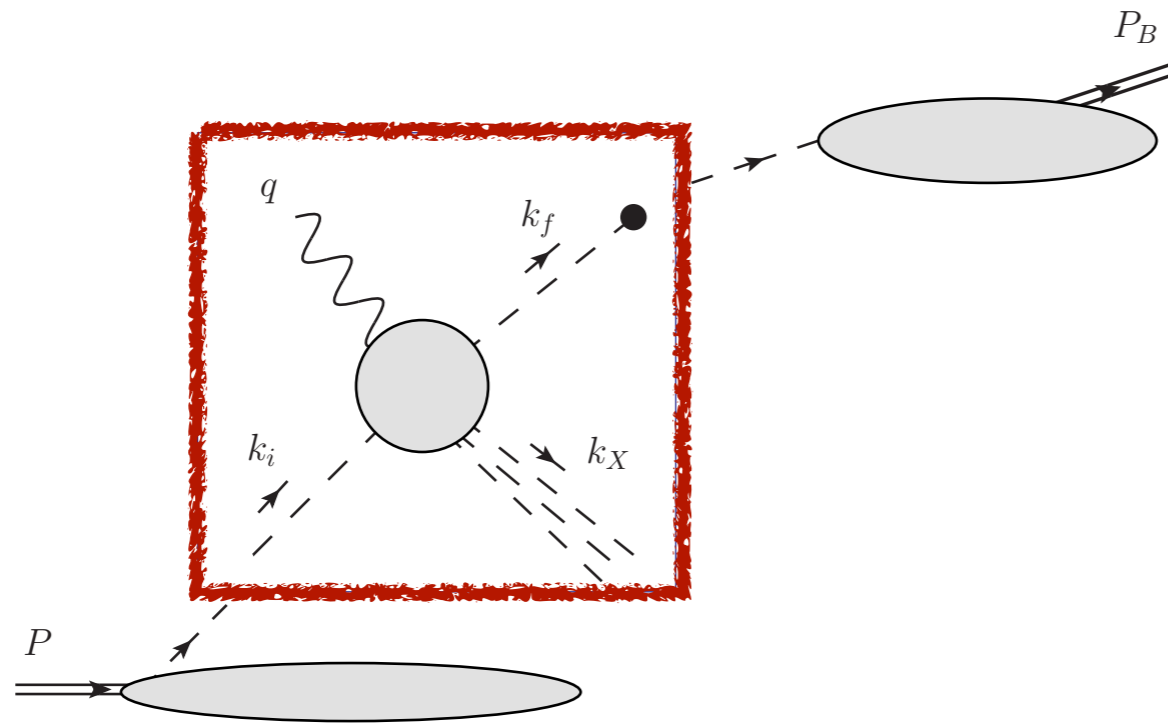


$$k_X^2 = 0$$



$$k_X^2 \neq 0$$

**Higher order pQCD corrections in large  $q_T$  cross section associated to larger values of virtuality spectator**



$$k_X^2 = (k_i + q - k_f)^2$$

$$k \equiv k_f - q$$

$$\text{Collinearity} = R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

$$\text{Transverse Hardness Ratio} = R_2 \equiv \frac{|k^2|}{Q^2}$$

$$\text{Spectator Virtuality Ratio} = R_3 \equiv \frac{|k_X^2|}{Q^2}$$

**The size of these ratios determine partonic configurations (factorization theorem) and map to kinematical regions of the observables**

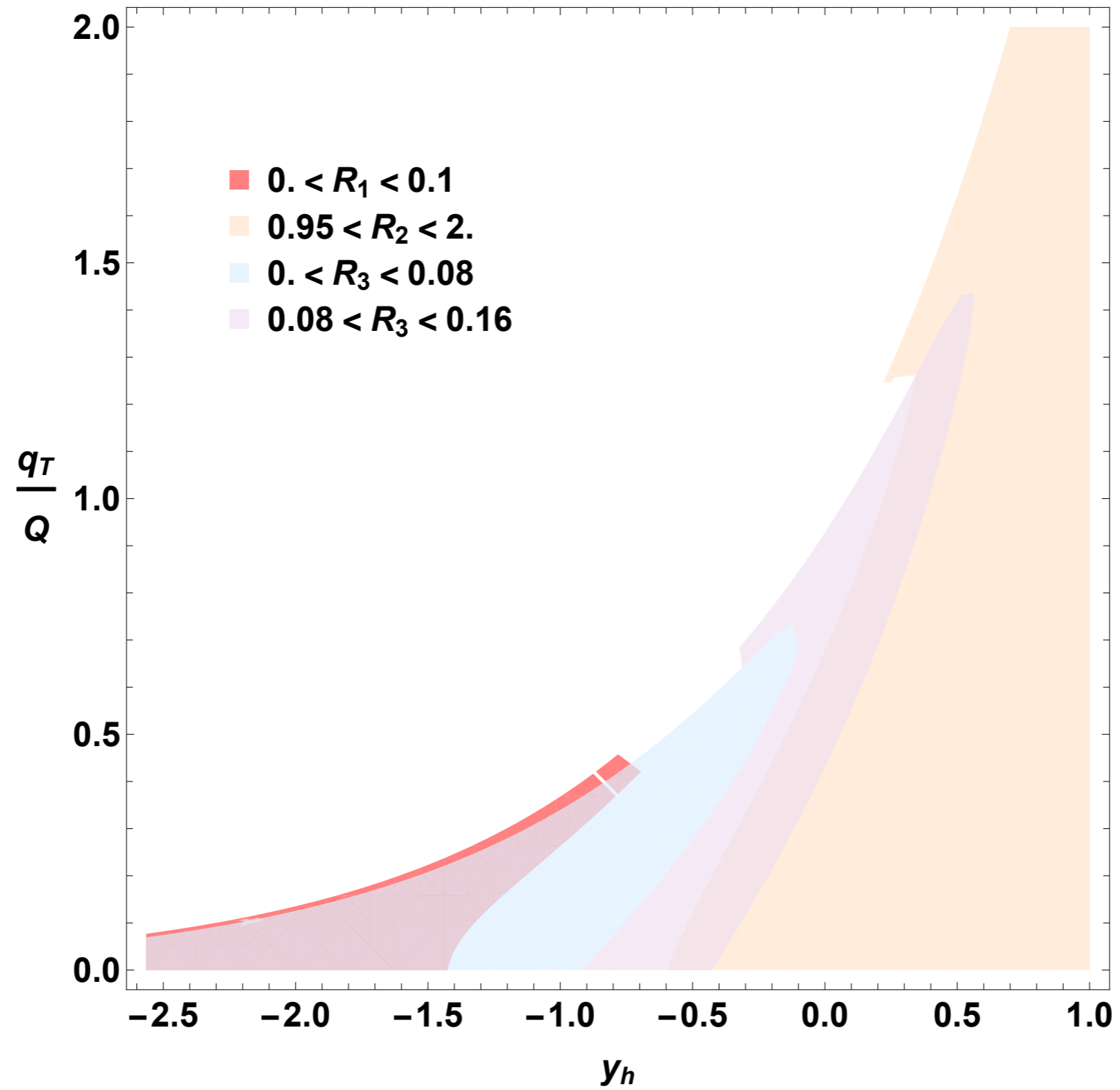
$$\text{Collinearity} = R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i}$$

$$\text{Transverse Hardness Ratio} = R_2 \equiv \frac{|k^2|}{Q^2}$$

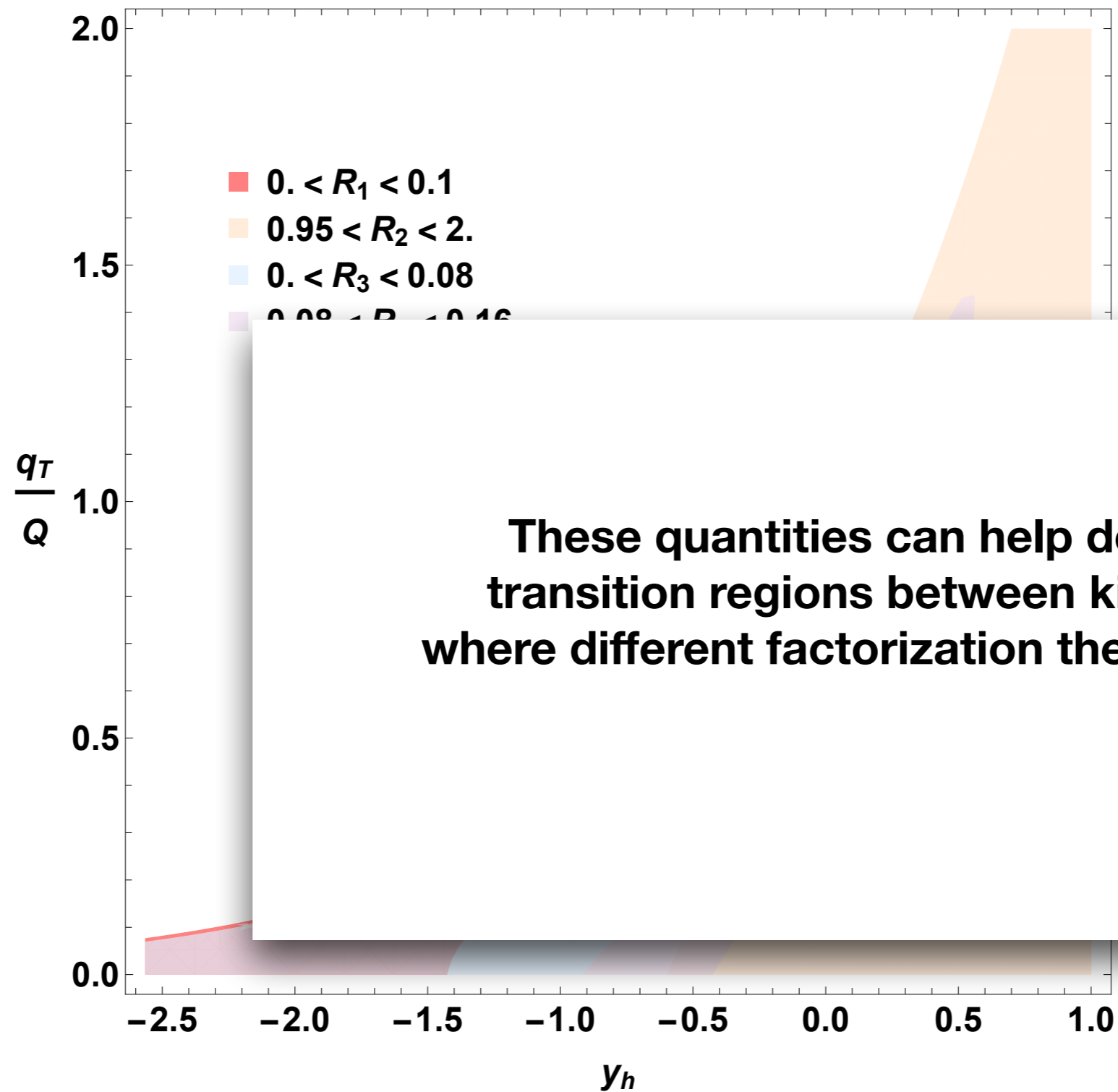
$$\text{Spectator Virtuality Ratio} = R_3 \equiv \frac{|k_X^2|}{Q^2}$$



**(caveat: Parton momenta have to be estimated,  
so this is just an example)**



**(caveat: Parton momenta have to be estimated,  
so this is just an example)**



# Final Remarks

- **Some progress on TMD extraction from SIDIS (evolution)**
- **Currently, some results are counter-intuitive.**
- **Matching is quite important, it is a non-trivial component of the TMD formalism (otherwise must cut data)**
- **Must think of solutions to describe the data: Fixed order large  $q_T$  region a good starting point.**
- **Does non-perturbative dominance compromises the validity of the formalism?**
- **Mapping (optimal) regions of applicability of factorization is crucial (like using  $R_1, R_2, R_3$  shown in this talk.)**

**Thanks**

