Probing gluon TMDs through quarkonia production

Miguel G. Echevarría

INFN Pavia Division

SarWorS 2019 - Sardinian Workshop on Spin studies
Cagliari, Italy, 8-10 July 2019

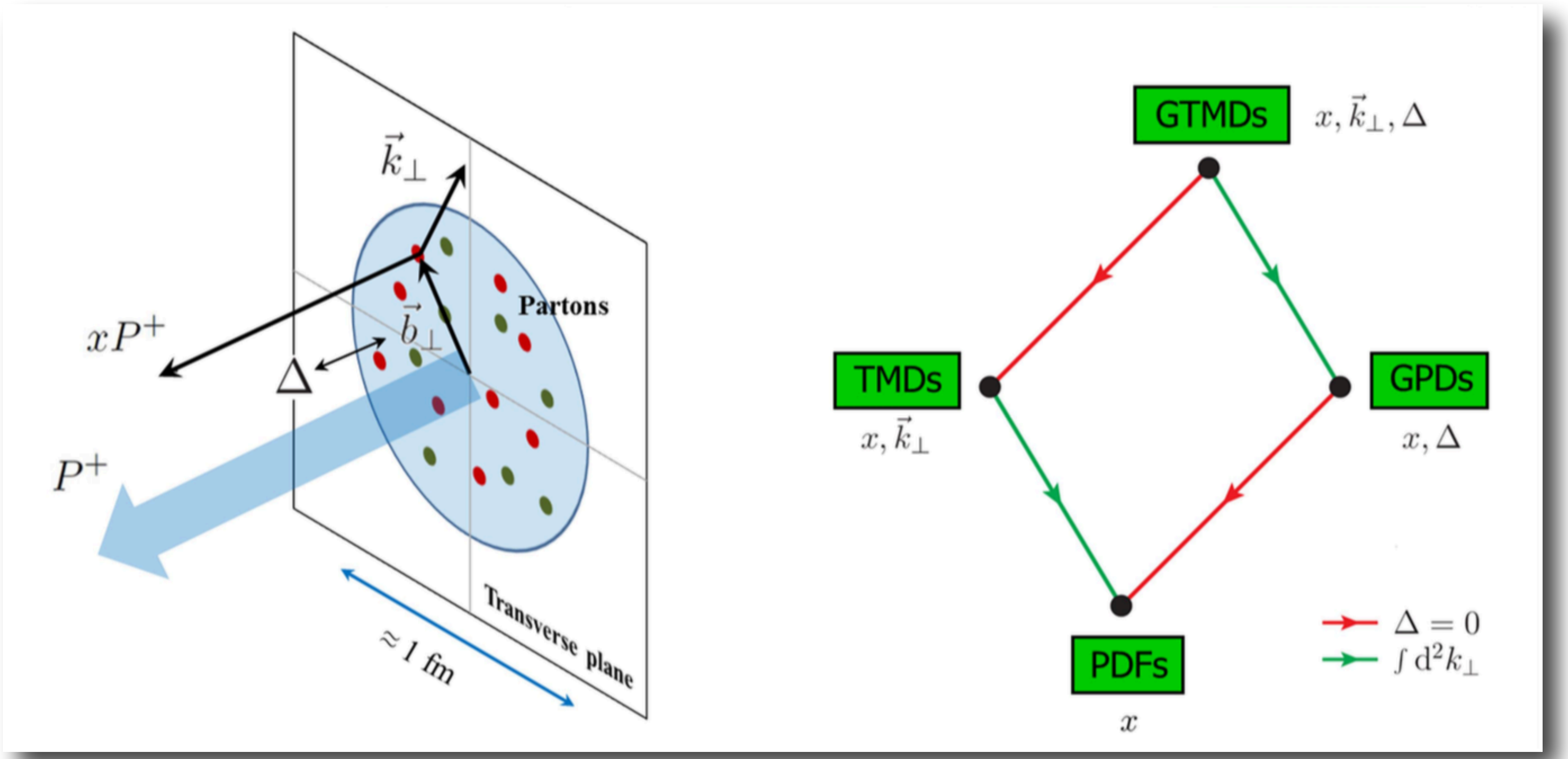
1. **Motivation**
2. **Etac hadro-production**
3. **(Jpsi lepto-production)**
4. **Conclusions & Outlook**

Based on ongoing work…

[MGE arXiv:19XX.XXXXX]

Quarkonia as tools to probe gluon TMDs

- **Goal: understand 3D/spin structure of hadrons**

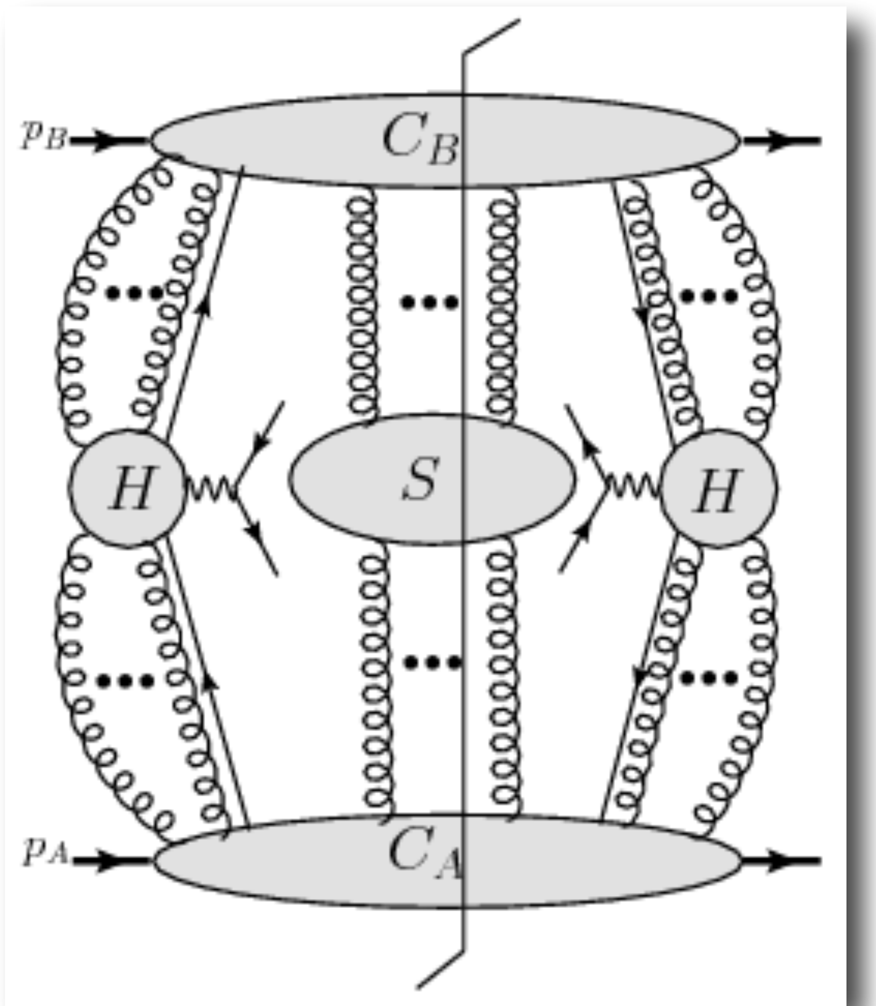


- In theory, all information contained in: $\langle PS|\,\mathcal{O}(\bar{\psi}, \psi, A^\mu)\,|PS\rangle$
- In practice, color confinement prevents us from calculating them. So?
- Lattice or **FACTORIZATION THEOREMS**

$$\sigma = \sigma_{partonic} \otimes [\text{PDFs / FFs / Jets / etc}] + \text{power suppressed}$$

Factorization theorems

$$h_A(P, S_A) + h_B(\bar{P}, S_B) \to H(q_T) + X$$



$m_t \gg m_H \gg q_T \geq \Lambda_{QCD}$

QCD

$C_t(m_t^2/\mu^2)$ **QCD** **n_f=5**

$C_t(m_t^2/\mu^2) H(m_H^2/\mu^2)$ **SCET$_{qT}$**

$C_t(m_t^2/\mu^2) H(m_H^2/\mu^2) \tilde{C}^{[pol]}_{g/j}(x_A, b_T^2\mu^2, m_H^2/\mu^2)$ **SCET$_{II}$**

- - - IR is the same! - - -

Proof main steps:

1. Pinch singularities
2. Power counting: leading contributions
3. Unitarity, Ward identities* (decouple regions)

   * Careful with Glauber gluons!!

Effective theories don't give a proof
But can be checked order by order in pQCD
Factorization = multi-step matching

Factorization theorems are the key element!

Definition of TMDs

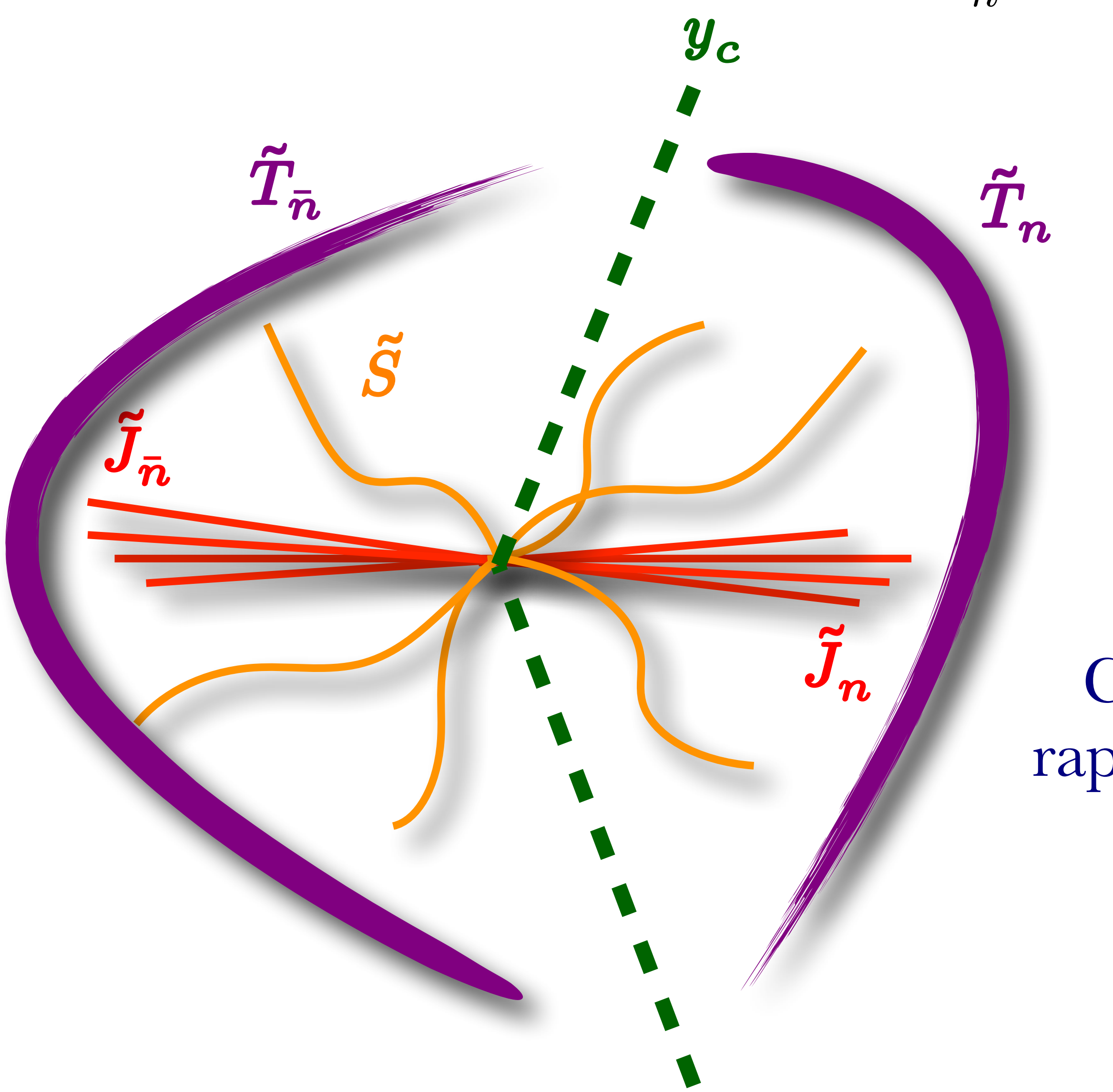
$$k_n \sim Q(1,\lambda^2,\lambda)$$
$$k_{\bar n} \sim Q(\lambda^2,1,\lambda)$$
$$k_s \sim Q(\lambda,\lambda,\lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

Different rapidities (mixed under boosts)

$$k_n^2 \sim k_{\bar n}^2 \sim k_s^2 \;\sim\; Q^2\lambda^2$$

Same invariant mass!



Cancel spurious rapidity divergences

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A,\vec{b}_\perp,S_A;\zeta_A,\mu) = \tilde{J}_n\sqrt{\tilde{S}}$$
$$\tilde{T}_{\bar n}(x_B,\vec{b}_\perp,S_B;\zeta_B,\mu) = \tilde{J}_{\bar n}\sqrt{\tilde{S}}$$

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[Collins' book '11]
[MGE, Idilbi, Scimemi 1111.4996, 1211.1947, 1402.0869]
[MGE, Kasemets, Mulders, Pisano 1502.05354]

We need this quantities to appear in a cross-section in order to talk about gluon TMDs!

TMDs: evolution

TMDs depend on two scales: renormalization and rapidity scales

- The dependence on the **renormalization** scale is:

$$\frac{d}{d\ln\mu}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops.
Numerical at 4-loops

$$\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)=-\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2}-\gamma^j_{nc}(\alpha_s(\mu))$$

[Moch, Vermaseren, Vogt JHEP08(2005)049, NPB688(2004)]
[Moch, Ruijl, Ueda, Vermaseren, Vogt JHEP10(2017)041]

- The dependence on the **rapidity** scale is:

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu)=-D_j(b_T;\mu)$$

Known at 3-loops (almost 4-loops)

$$\frac{dD_j}{d\ln\mu}=\Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp does not completely determine Dj

Indirect NLO: *[Becher, Neubert EPJC71(2011)]*
Direct NLO: *[MGE, Scimemi, Vladimirov PRD93(2016)]*

Direct NNLO:
[Li, Zhu PRL118(2017)]
[Vladimirov PRL118(2017)]

TMDs: re-factorization

- TMDs contain perturbative information when transverse momentum is large:

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,b_T;\zeta,\mu) \otimes t_{j\leftrightarrow A}(x;\mu) + O(b_T\Lambda_{QCD})$$

- For each TMD we have a different OPE. For example:

$$\tilde{f}_1^{q/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^f_{q/j}(\bar{x},b_T;\zeta,\mu)\, f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^h_{g/j}(\bar{x},b_T;\zeta,\mu)\, f_{j/A}(x/\bar{x};\mu)$$

$$\tilde{g}_{1L}^{g/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^g_{g/j}(\bar{x},b_T;\zeta,\mu)\, g_{j/A}(x/\bar{x};\mu)$$

$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1,\bar{x}_2,b_T;\zeta,\mu)\, T_{F\,j/A}(x_1/\bar{x}_1,x_2/\bar{x}_2;\mu)$$

Unpolarized quark/gluon TMD distribution and fragmentation functions at NNLO in

[MGE, Scimemi, Vladimirov JHEP09(2016)004]

Transversely polarized TMDs at NNLO in

[Gutiérrez-Reyes, Scimemi, Vladimirov JHEP07(2018)172]
[Gutiérrez-Reyes, Leal-Gómez, Scimemi, Vladimirov 1907.03780]

TMDs in full glory

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,\hat{b}_T;\mu_b^2,\mu_b) \otimes t_{j\leftrightarrow A}(x;\mu_b)$$
$$\times \exp\left[\int_{\mu_b}^{\mu} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta}{\hat{\mu}^2}\right)\right] \left(\frac{\zeta}{\mu_b^2}\right)^{-D_j(\hat{b}_T;\mu_b)}$$
$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x,b_T;\zeta)$$

- General philosophy: **only parametrize what cannot be calculated**
- **Nonperturbative** part of **Dj is universal** (for all (un)polarized TMDs)
- **Higher-order** calculations allow better determination of nonpert. ingredients
- At large and low bT we need **cutoffs** (qT<Lambda and qT>Q regions)
- There are subtleties with the **evolution path** ***[Scimemi, Vladimirov JHEP08(2018)003]***
- The **determination of nonperturbative pieces is not easy** (Fourier transform mixes regions, overlap of perturbative and non-perturbative)

The higher the theoretical precision, the better!

Processes sensitive to gluon TMDs (probably not complete…)

$p+p \to \eta_{c,b}+X$ factorization **ansatz** @ NLO

$p+p \to \chi_{c,b}+X$ factorization **ansatz** @ NLO, KO beyond NLO!

$p+p \to H^0+X$ N[3]LL+NNLO, factorization **proof**

$p+p \to \gamma+\gamma+X$ factorization @ LO

$p+p \to J/\psi+\gamma^*+X$

$p+p \to J/\psi+Z+X$

$p+p \to J/\psi+J/\psi+X$

$p+p \to \eta_c+\eta_c+X$

$e+p \to e+c+\bar{c}+X$

$e+p \to e+J/\psi+jet+X$

$e+p \to e+J/\psi+\pi+X$

$e+p \to e+J/\psi+X$

$e^+ + e^- \to J/\psi+\pi+X$

(all of the above:) factorization **ansatz** @ LO (if any)

- These processes are very promising to access gluon TMDs (but with proper formalism!)
- With polarized protons one could access different TMDs (Sivers, etc)
  The most basic, the unpolarized gluon TMDPDF, still not fixed…
- No pheno studies (predictions) beyond LO

[Lansberg, Pisano, Scarpa, Schlegel PLB784(2018)]

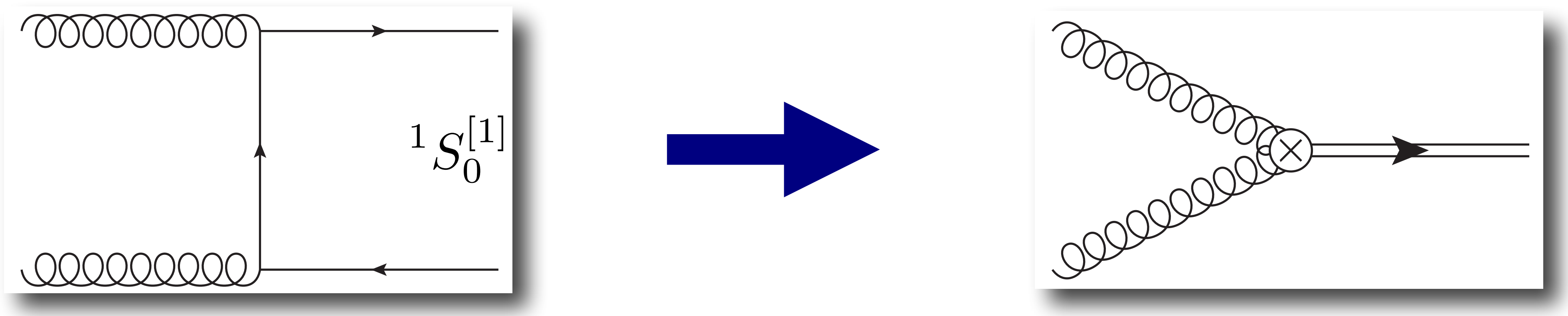
Ansatz: factorization of 2 soft mechanisms in the processes: soft gluon resummation and formation of bound state

$$m_Q v \sim q_T$$

TMD factorization for

$$pp \to \eta_c X$$

Effective operator (in SCET+NRQCD)



+ crossed diagram

$$\mathcal{O}(\xi) = -2q^2 C_H(-q^2;\mu^2) \left[\psi^\dagger(\xi)\,\Gamma_{\mu\nu}\,\chi(\xi)\right] \left[B^{\mu,b}_{\bar{n}\perp}(\xi)\,\mathcal{Y}^{\dagger ba}_{\bar{n}}(\xi)\,\mathcal{Y}^{ac}_{n}(\xi)\,B^{\nu,c}_{n\perp}(\xi)\right]$$

$$B^\mu_{n\perp} = B^{\mu,a}_{n\perp} t^a = \frac{1}{g}[W_n^\dagger i D_n^{\perp\mu} W_n] = \frac{1}{\bar{n}\cdot\mathcal{P}} i\bar{n}_\alpha g^\mu_{\perp\beta} W_n^\dagger F_n^{\alpha\beta} W_n = \frac{1}{\bar{n}\cdot\mathcal{P}} i\bar{n}_\alpha g^\mu_{\perp\beta} t^a (\mathcal{W}_n^\dagger)^{ab} F_n^{\alpha\beta,b}$$

$$W_n(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\,\bar{n}\cdot A_n^a(\xi+\bar{n}s)t^a\right],$$

$$Y_n(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\,n\cdot A_s^a(\xi+ns)t^a\right]$$

Tree-level amplitude matching gives:

$$\Gamma_{\mu\nu} = \frac{i\pi\alpha_s 2\sqrt{2}}{N_c\sqrt{M^5}}\epsilon_{\perp\mu\nu}$$

Caligraphic means: $(t^a)^{bc} = -if^{abc}$

Hard-Soft-Collinear factorization (in SCET+NRQCD)

Cross-section given by:

$$d\sigma = \frac{1}{2s}\frac{d^3q}{(2\pi)^3 2E_q}\int d^4\xi\, e^{-iq\cdot\xi}\sum_X \left\langle PS_A,\bar{P}S_B\right|\mathcal{O}^\dagger(\xi)\left|X\eta_Q\right\rangle\left\langle \eta_Q X\right|\mathcal{O}(0)\left|PS_A,\bar{P}S_B\right\rangle$$

$m_Q v \sim q_T$

Factorization in EFT
=
decoupling of modes!

$$|X\eta_Q\rangle = |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\eta_Q\rangle$$

$$\left|PS_A,\bar{P}S_B\right\rangle = |PS_A\rangle \otimes \left|\bar{P}S_B\right\rangle$$

etac couples to soft and ultrasoft

$$\begin{aligned} d\sigma =& \frac{1}{2s}\frac{d^3q}{(2\pi)^3 2E_q}\, 4M^4\, H(M^2,\mu^2)\,\Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}\int d^4\xi\; e^{-iq\xi} \\ &\times \sum_{X_n}\langle PS_A|\, B^{\sigma,c'}_{n\perp}(\xi)\,|X_n\rangle\langle X_n|\, B^{\nu,c}_{n\perp}(0)\,|PS_A\rangle \sum_{X_{\bar{n}}}\left\langle \bar{P}S_B\right|B^{\rho,b'}_{\bar{n}\perp}(\xi)\,|X_{\bar{n}}\rangle\langle X_{\bar{n}}|\,B^{\mu,b}_{\bar{n}\perp}(0)\left|\bar{P}S_B\right\rangle \\ &\times \sum_{X_s}\langle 0|\left[\mathcal{Y}^{\dagger c'a'}_{n}\mathcal{Y}^{a'b'}_{\bar{n}}\chi^\dagger\psi\right](\xi)\,|X_s\eta_Q\rangle\;\langle\eta_Q X_s|\left[\mathcal{Y}^{\dagger ba}_{\bar{n}}\mathcal{Y}^{ac}_{n}\psi^\dagger\chi\right](0)\,|0\rangle \end{aligned}$$

$$H(M^2,\mu^2) = |C_H(-q^2,\mu^2)|^2$$

Homogenous power counting: Taylor expansion

$$q \sim M(1,1,\lambda)$$

Derivatives of matrix elements scale as their own momenta

$$\begin{aligned} d\sigma =& \frac{1}{2s}\frac{d^3q}{(2\pi)^3 2E_q}\frac{4M^4\, H(M^2,\mu^2)}{(N_c^2-1)^2}\,\Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}\int d^4\xi\; e^{-iq\xi} \\ &\times \langle PS_A|\, B^{\sigma,c}_{n\perp}(\xi^-,\xi_\perp)\, B^{\nu,c}_{n\perp}(0)\,|PS_A\rangle\, \left\langle \bar{P}S_B\right|\, B^{\rho,b}_{\bar{n}\perp}(\xi^+,\xi_\perp)\, B^{\mu,b}_{\bar{n}\perp}(0)\left|\bar{P}S_B\right\rangle \\ &\times \langle 0|\left[\mathcal{Y}^{\dagger ca'}_n \mathcal{Y}^{a'b}_{\bar{n}}\chi^\dagger\psi\right](\xi_\perp)\, a^\dagger_{\eta_Q} a_{\eta_Q}\left[\mathcal{Y}^{\dagger ba}_{\bar{n}}\mathcal{Y}^{ac}_n\psi^\dagger\chi\right](0)\,|0\rangle \\ &+ \mathcal{O}(\lambda) \end{aligned}$$

Played with color (beam functions are diagonal) and used completeness relations:

$$\sum_{X_n} |X_n\rangle\langle X_n| = 1\,, \quad \sum_{X_{\bar{n}}} |X_{\bar{n}}\rangle\langle X_{\bar{n}}| = 1\,,$$

$$\sum_{X_s} |X_s\eta_Q\rangle\langle X_s\eta_Q| = a^\dagger_{\eta_Q}\sum_{X_s}|X_s\rangle\langle X_s|\, a_{\eta_Q} = a^\dagger_{\eta_Q} a_{\eta_Q}$$

Factorization theorem

$$\frac{d\sigma}{dy d^2 q_\perp} = \frac{4M^4\, H(M^2,\mu^2)}{2sM^2(N_c^2-1)}\, \Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}(2\pi) \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times\, J_n^{(0)\sigma\nu}(x_A, \boldsymbol{k}_{n\perp}, S_A; \mu; \eta_n)\, J_{\bar{n}}^{(0)\rho\mu}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \mu; \eta_{\bar{n}})\, S_{\mathcal{Q}}^{(0)}(\boldsymbol{k}_{s\perp}; \mu)$$

Rapidity divergences!

$$x_{A,B} = \sqrt{\tau}\, e^{\pm y}$$
$$\tau = (M^2 + q_T^2)/s$$

Pure collinear and bare TMD shape functions defined as:

$$J_n^{(0)\mu\nu} = \frac{x_A P^+}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{-i\left(\frac{1}{2}x_A \xi^- P^+ - \boldsymbol{\xi}_\perp \boldsymbol{k}_{n\perp}\right)}\, \langle PS_A|\, B_{n\perp}^{\mu,a}(\xi^-, \boldsymbol{\xi}_\perp)\, B_{n\perp}^{\nu,a}(0)\, |PS_A\rangle\ ,$$

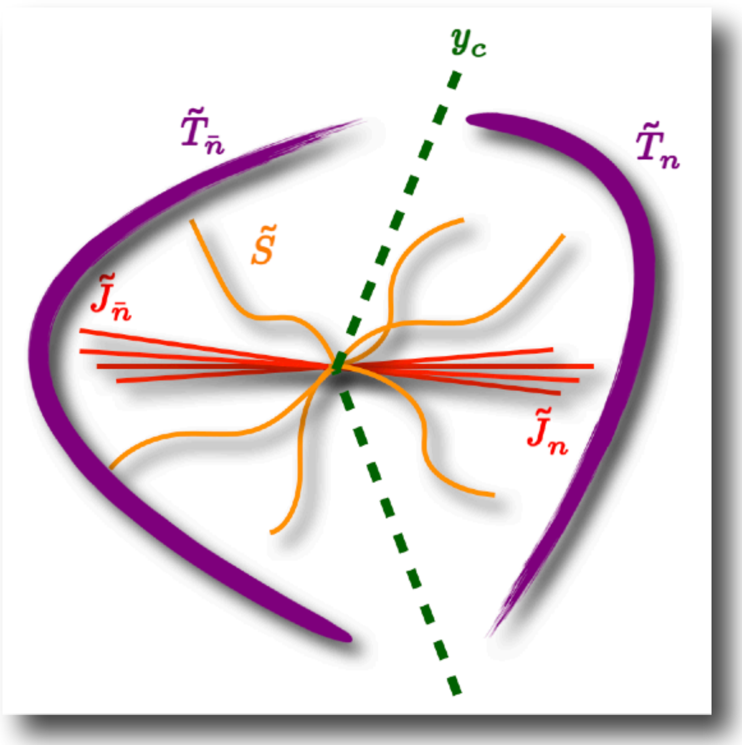
$$J_{\bar{n}}^{(0)\mu\nu} = \frac{x_B \bar{P}^-}{2} \int \frac{d\xi^+ d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, e^{-i\left(\frac{1}{2}x_B \xi^+ \bar{P}^- - \boldsymbol{\xi}_\perp \boldsymbol{k}_{\bar{n}\perp}\right)}\, \left\langle \bar{P}S_B\right|\, B_{\bar{n}\perp}^{\mu,a}(\xi^+, \boldsymbol{\xi}_\perp)\, B_{\bar{n}\perp}^{\nu,a}(0)\, \left|\bar{P}S_B\right\rangle\ ,$$

$$S_{\mathcal{Q}}^{(0)} = \frac{1}{N_c^2-1} \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}}\, \langle 0|\left[\mathcal{Y}_n^{\dagger ab}\mathcal{Y}_{\bar{n}}^{bc}\chi^\dagger\psi\right](\boldsymbol{\xi}_\perp)\, a_{\eta_Q}^\dagger a_{\eta_Q} \left[\mathcal{Y}_{\bar{n}}^{\dagger cd}\mathcal{Y}_n^{da}\psi^\dagger\chi\right](0)\, |0\rangle$$

Definition of gluon TMDs

We need to introduce the relevant soft function (same as for Higgs production):

$$S = \frac{1}{N_c^2 - 1} \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}} \,\langle 0|\, \left[\mathcal{Y}_n^{\dagger ab}\mathcal{Y}_{\bar{n}}^{bc}\right](\boldsymbol{\xi}_\perp)\, \left[\mathcal{Y}_{\bar{n}}^{\dagger cd}\mathcal{Y}_n^{da}\right](0)\, |0\rangle$$



Soft function can be split to all orders in pQCD:

$$\tilde{S}(b_T;\mu;\eta_n,\eta_{\bar{n}}) = \tilde{S}_-\left(b_T;\mu;\eta_n\right)\tilde{S}_+\left(b_T;\mu;\eta_{\bar{n}}\right)$$

TMDs are defined as (Rapidity divergences free):

$$\tilde{G}^{\mu\nu}_{g/A}(x_A,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = \tilde{J}^{(0)\mu\nu}_n(x_A,\boldsymbol{b}_\perp,S_A;\mu;\eta_n)\,\tilde{S}_-(b_T;\mu;\eta_n)\,,$$

$$\tilde{G}^{\mu\nu}_{g/B}(x_B,\boldsymbol{b}_\perp,S_B;\zeta_B,\mu) = \tilde{J}^{(0)\mu\nu}_{\bar{n}}(x_B,\boldsymbol{b}_\perp,S_B;\mu;\eta_{\bar{n}})\,\tilde{S}_+(b_T;\mu;\eta_{\bar{n}})$$

Final factorization theorem

$$\frac{d\sigma}{dy\, d^2q_\perp} = \frac{4M^4\, H(M^2,\mu^2)}{2sM^2(N_c^2-1)}\, \Gamma^*_{\rho\sigma}\Gamma_{\mu\nu} \langle \mathcal{O}\big({}^1S_0^{[1]}\big)\rangle\, (2\pi) \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times\, \tilde{G}^{\sigma\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, \tilde{G}^{\rho\mu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_{s\perp}; \mu)$$

TMD ShF!

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\big\langle \mathcal{O}\big({}^1S_0^{[1]}\big)\big\rangle\, \tilde{S}(\boldsymbol{y}_\perp)}$$

Normalized to one when integrated over

$$\big\langle \mathcal{O}\big({}^1S_0^{[1]}\big)\big\rangle = \langle 0|\, \chi^\dagger \psi\, a^\dagger_{\eta_c} a_{\eta_c}\, \psi^\dagger \chi\, |0\rangle$$

- Here we have all (un)polarized TMDs
- TMD shape function is spin-independent
- TMD shape function depends on the quarkonium state
- TMD shape function universal for future/past pointing Wilson lines

Factorization theorem for unpolarized pp collisions

$$\frac{d\sigma}{dy d^2 q_\perp} = \sigma_0(\mu) H(M^2, \mu^2) \left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle \left[\mathcal{C}[f_1^g f_1^g S_{\mathcal{Q}}] - \mathcal{C}[w_{UU} h_1^{\perp g} h_1^{\perp g} S_{\mathcal{Q}}]\right]$$

$$\sigma_0 = \frac{(4\pi\alpha_s)^2\pi}{N_c^2(N_c^2-1)sM^3}$$

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_\perp, S_A; \zeta_A, \mu) \equiv \frac{1}{2}\left[-g_\perp^{\mu\nu}\tilde{f}_1^g(x_A, b_T^2; \zeta_A, \mu) - \frac{M_p^2}{2} b_T^{\mu\nu}\tilde{h}_1^{\perp g(2)}(x_A, b_T^2; \zeta_A, \mu)\right]$$

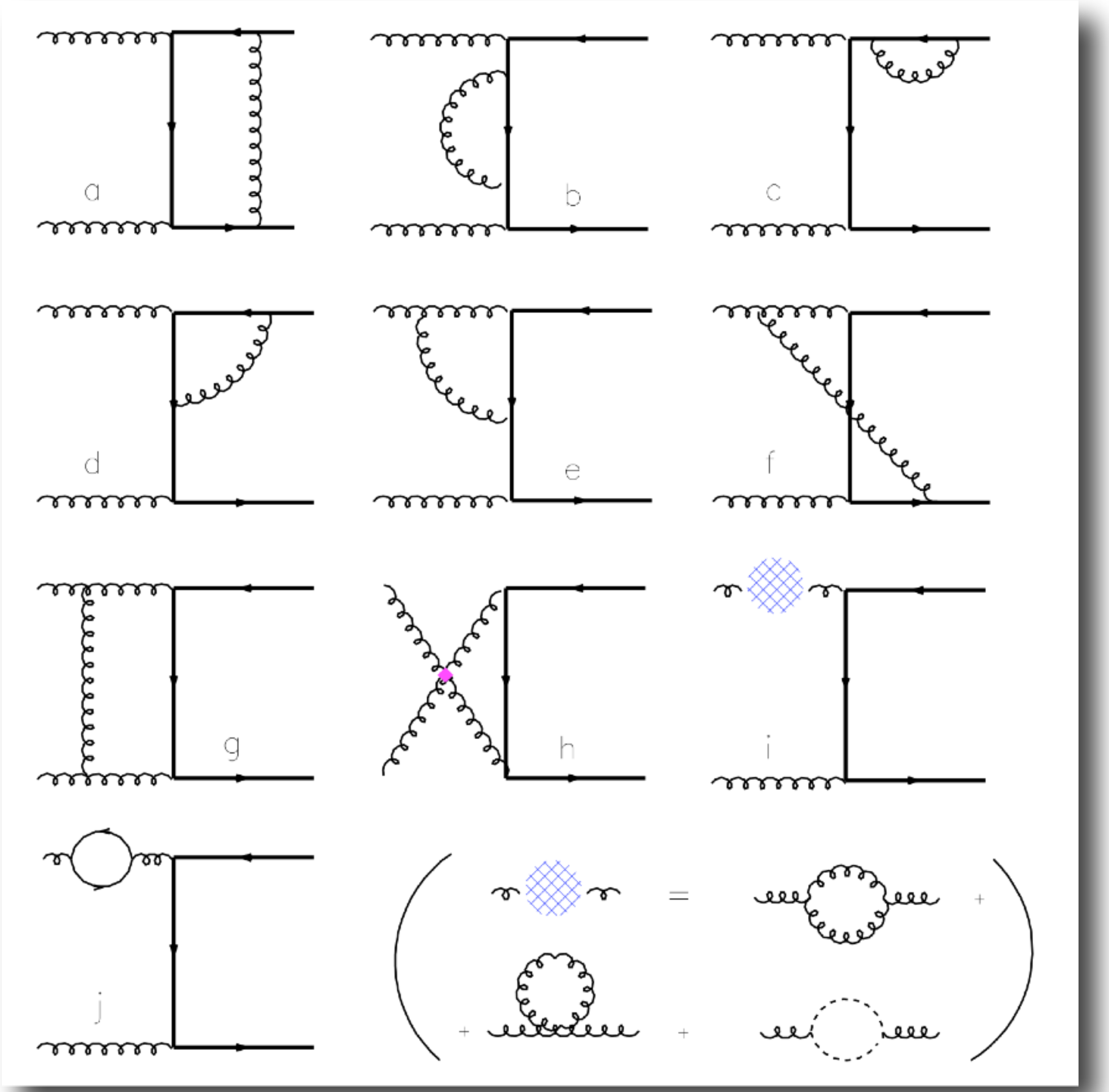
$$\mathcal{C}[w\, f\, f\, S_{\mathcal{Q}}] \equiv \int d^2\boldsymbol{p}_{\perp a} \int d^2\boldsymbol{p}_{\perp b} \int d^2\boldsymbol{k}_\perp\, \delta^2(\boldsymbol{p}_{\perp a} + \boldsymbol{p}_{\perp b} + \boldsymbol{k}_\perp - \boldsymbol{q}_\perp)$$
$$\times\, w(p_{\perp a}, p_{\perp b})\, f(x_A, \boldsymbol{p}_{Ta}^2; \zeta_A, \mu)\, f(x_B, \boldsymbol{p}_{Tb}^2; \zeta_B, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_T^2)$$

$$w_{UU} = \frac{p_{\perp a}^{\mu\nu}\, p_{\perp b\,\mu\nu}}{2M_p^4}$$

Convolution of 2 gluon TMDs and the TMD ShF!

Calculation of H: virtual part of cross-section in full QCD

Hard part
=
finite part of virtual diagrams in full QCD (pure dim. reg.)???



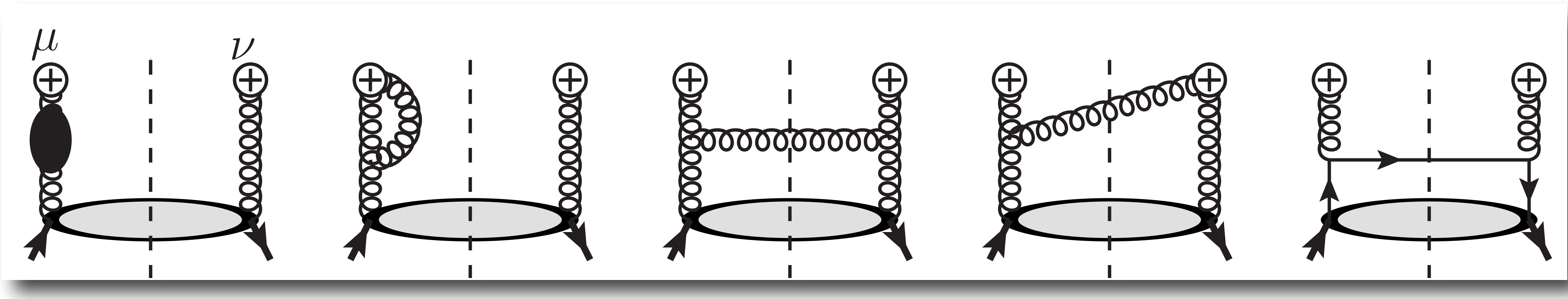
[Hawigara, Kim, Yoshino NPB'81]
[Kuhn, Mirkes Z.Phys.'92]
[Ma, Wang, Zhao PRD'14]
[Petrelli, Cacciari, Greco, Maltoni, Mangano hep-ph/9707223]

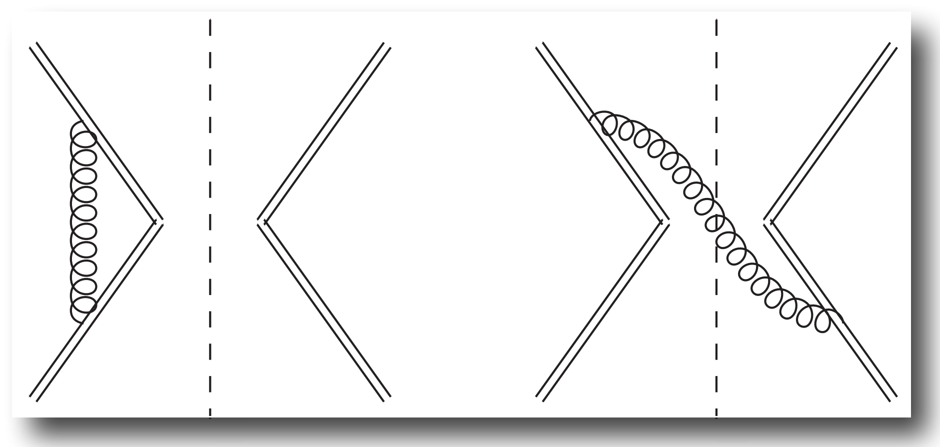
$$\frac{d\sigma}{\sigma_0}\bigg|_v = \delta(1-x_A)\delta(1-x_B)$$

$$+\frac{\alpha_s}{2\pi}\left[C_F\frac{\pi^2}{v} - 2\left(\frac{C_A}{\varepsilon_{\rm IR}^2}+\frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2}+C_A\ln\frac{\mu^2}{M^2}\right) - C_A\ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{6} + 2B^{[1]}_{^1S_0}\right)\right]\delta(1-x_A)\delta(1-x_B)$$

$$B^{[1]}_{^1S_0} = C_F\left(-5+\frac{\pi^2}{4}\right)+C_A\left(1+\frac{5\pi^2}{12}\right)$$

Calculation of H: virtual part of gluon TMDPDF





Virtual part is:

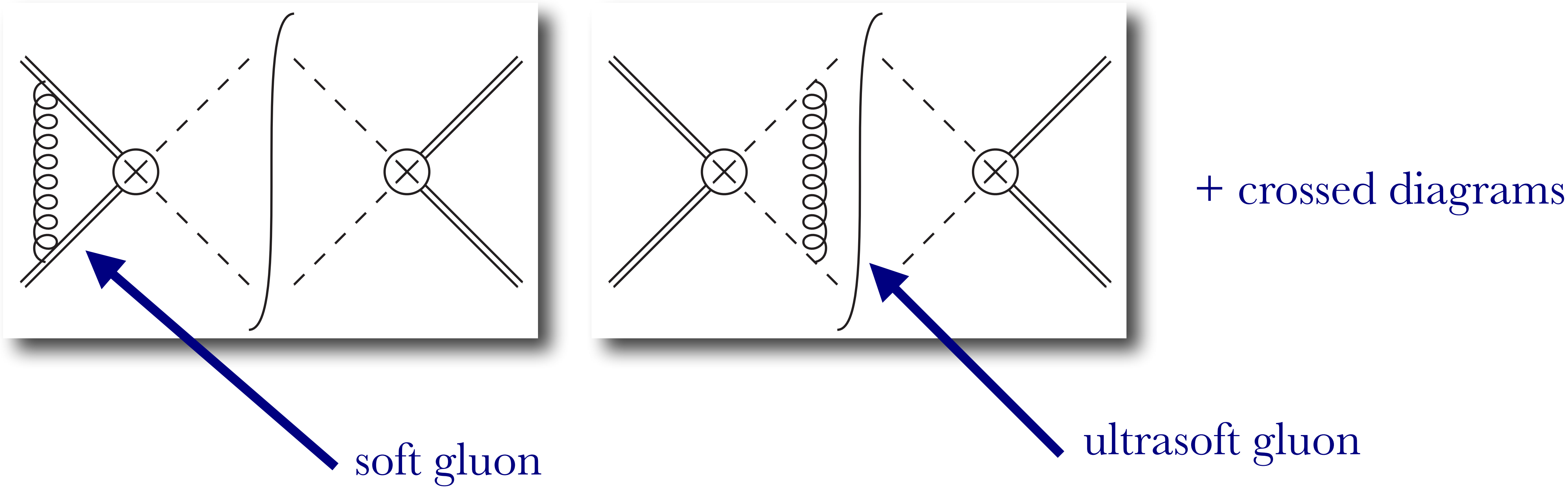
$$\tilde{f}_1^g\Big|_v = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[\frac{C_A}{\varepsilon_{\rm UV}^2} + \frac{1}{\varepsilon_{\rm UV}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{\zeta_A}\right) + \ldots\right]\delta(1-x)$$

[MGE, Kasemets, Mulders, Pisano 1502.05354]

Renormalizing, in dim. reg.:

$$\tilde{f}_1^g\Big|_v = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right)\right]\delta(1-x)$$

Calculation of H: virtual part of the TMD shape function



There are no diagrams which connect at this order the heavy fields with the Wilson lines!

[Luke, Manohar, Rothstein hep-ph/9910209]

$$\tilde{S}^{(0)}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v} + S\Big|_v^1$$

contains rapidity divergences

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}^{(0)}_{\mathcal{Q}}(\boldsymbol{y}_\perp)}{\langle \mathcal{O}(^1S_0^{[1]}) \rangle \, \tilde{S}(\boldsymbol{y}_\perp)}$$

$$\langle \mathcal{O}(^1S_0^{[1]}) \rangle \, \tilde{S}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v}$$

Calculation of H: putting everything together...

$$\frac{d\sigma}{dy d^2 q_\perp} = \sigma_0(\mu) H(M^2, \mu^2) \left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle \left[\mathcal{C}[f_1^g f_1^g S_{\mathcal{Q}}] - \mathcal{C}[w_{UU} h_1^{\perp g} h_1^{\perp g} S_{\mathcal{Q}}]\right]$$

- Putting everything together:

$$\left.\frac{d\sigma}{\sigma_0}\right|_v = \delta(1-x_A)\delta(1-x_B)$$
$$+ \frac{\alpha_s}{2\pi}\left[C_F \frac{\pi^2}{v} - 2\left(\frac{C_A}{\varepsilon_{\rm IR}^2} + \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right) - C_A \ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{6} + 2B_{^1S_0}^{[1]}\right)\right]\delta(1-x_A)\delta(1-x_B)$$

$$\left.\tilde{f}_1^g\right|_v = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A\ln\frac{\mu^2}{M^2}\right)\right]\delta(1-x)$$

$$\left.\left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle \tilde{S}_{\mathcal{Q}}\right|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v}$$

$$H = 1 + \frac{\alpha_s}{2\pi}\left[-C_A \ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{6} + 2B_{^1S_0}^{[1]}\right]$$

New!

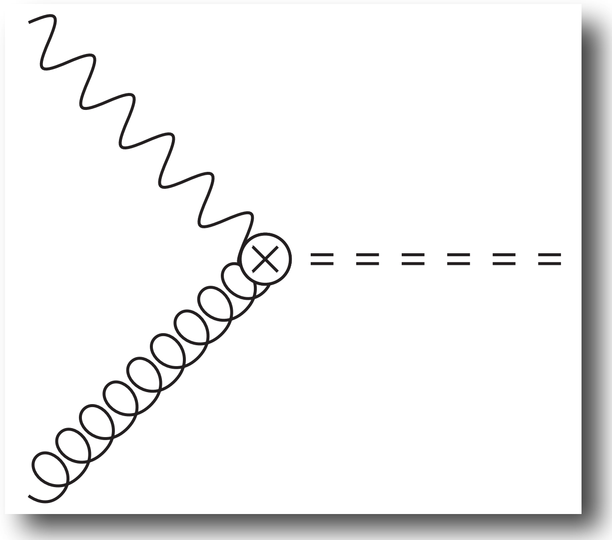
(same log as for Higgs in pp)

TMD factorization for

$$e^- \, p \to e^- \, J/\psi \, X$$

very preliminary...

Effective operator (sketchy…)



$$\mathcal{O}^\mu(\xi) \sim \left[\psi^\dagger(\xi)\,\Gamma^\mu_\nu\, t_a\, \chi(\xi)\right]\,\left[\mathcal{Y}^{ac}_n(\xi)\, B^{\nu,c}_{n\perp}(\xi)\right]$$

Several color-octet LDMEs contribute

[Bacchetta, Boer, Pisano, Taels 1809.02056]

Time-like Wilson lines needed for gauge invariance!

$$\psi^\dagger Y_v^\dagger T^a Y_v \chi = \psi^\dagger \mathcal{Y}_v^{ab} T^b \chi$$

[Nayak, Qiu, Sterman hep-ph/0501235]
[Nayak, Qiu, Sterman hep-ph/0509021]
[Rothstein, Shrivastava, Stewart 1806.07398]

$$Y_v(\xi) = P\exp\left[ig\int_{-\infty}^0 ds\, v\cdot A_s^a(\xi+vs)t^a\right]$$

$$v = (1,0,0,0)\,, \quad v^2 = 1$$

For color singlet there was no issue:

$$\psi^\dagger Y_v^\dagger Y_v \chi = \psi^\dagger \chi$$

Factorization theorem (sketchy…)

Cross-section given by:

$$d\sigma \sim L_{\mu\nu}\, H(M^2,\mu^2)\,\langle \mathcal{O}(^1S_0^{[8]})\rangle\, \Gamma_\alpha^{\mu *}\Gamma_\beta^{\nu}(2\pi) \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{s\perp}\right) \times \tilde{G}_{g/A}^{\alpha\beta}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, S_{\mathcal{Q}}(\boldsymbol{k}_{s\perp};\mu)$$

There is actually a sum over states, with different TMD ShFs, H and Lorentz structures, but let's simplify…

$$S_{\mathcal{Q}}^{(0)} = \frac{1}{N_c^2-1}\int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}}\, \langle 0|\left[\mathcal{Y}_n^{\dagger ab}\mathcal{Y}_v^{bc}\chi^\dagger t^c\psi\right](\boldsymbol{\xi}_\perp)\, a_\psi^\dagger a_\psi \left[\mathcal{Y}_v^{\dagger cd}\mathcal{Y}_n^{da}\psi^\dagger t^c\chi\right](0)\,|0\rangle$$

$$\tilde{G}_{g/A}^{\mu\nu}(x_A,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = \tilde{J}_n^{(0)\mu\nu}(x_A,\boldsymbol{b}_\perp,S_A;\mu;\eta_n)\,\tilde{S}_-(b_T;\mu;\eta_n)$$

$$\tilde{S}(b_T;\mu;\eta_n,\eta_{\bar{n}}) = \tilde{S}_-\left(b_T;\mu;\eta_n\right)\tilde{S}_+\left(b_T;\mu;\eta_{\bar{n}}\right)$$

$$\tilde{S}_{\mathcal{Q}}(\boldsymbol{y}_\perp) = \frac{\tilde{S}_{\mathcal{Q}}^{(0)}(\boldsymbol{y}_\perp)}{\langle \mathcal{O}(^1S_0^{[8]})\rangle\,\tilde{S}_-(\boldsymbol{y}_\perp)}$$

Different from etac case!

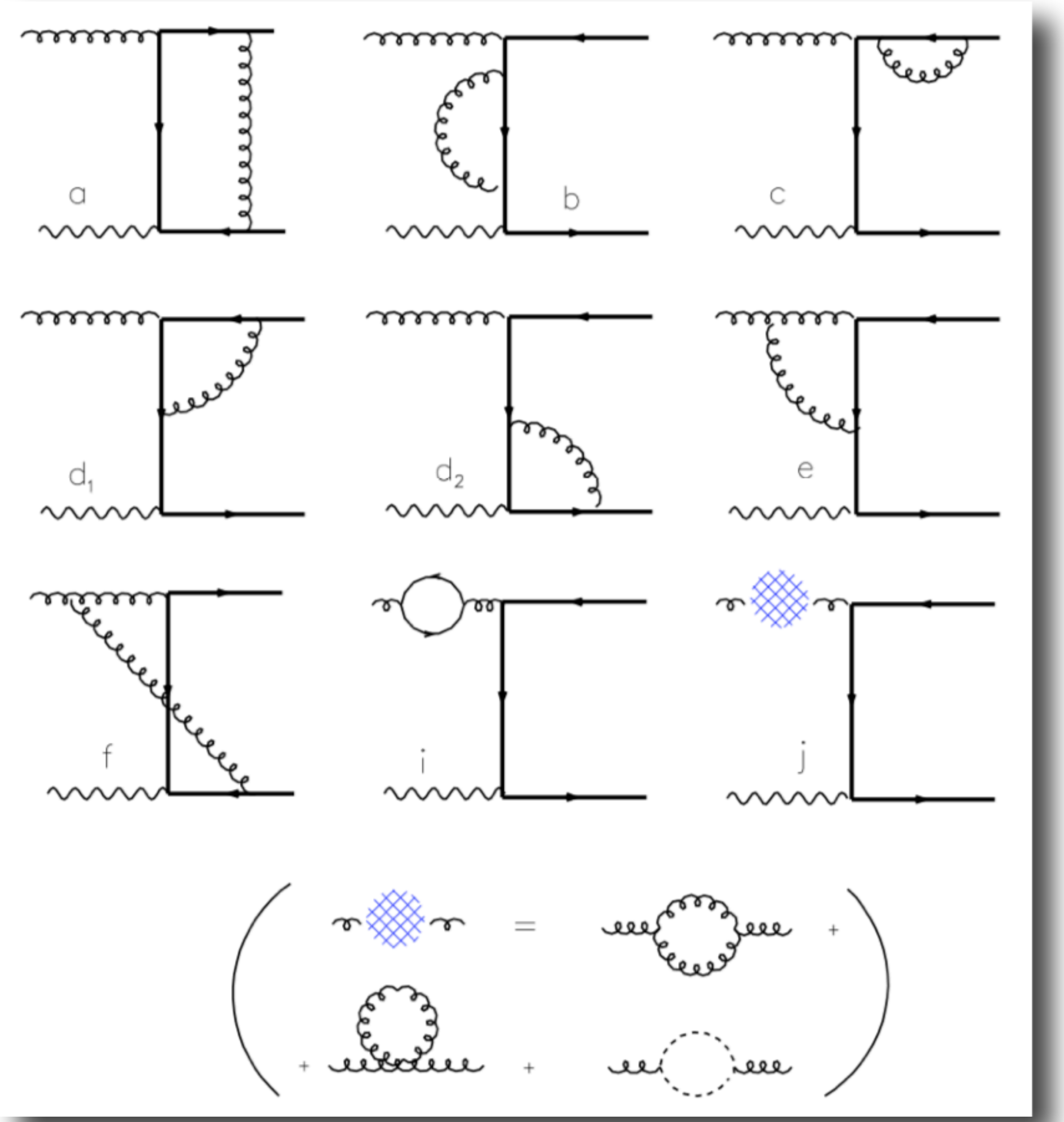
$$\langle \mathcal{O}(^1S_0^{[8]})\rangle = \langle 0|\,\chi^\dagger t^a\psi\, a_{\eta_c}^\dagger a_{\eta_c}\, \psi^\dagger t^a\chi\,|0\rangle$$

This is not full S!

Hard part (sketchy…)

Hard part
=
finite part of virtual diagrams in full QCD (pure dim. reg.)???

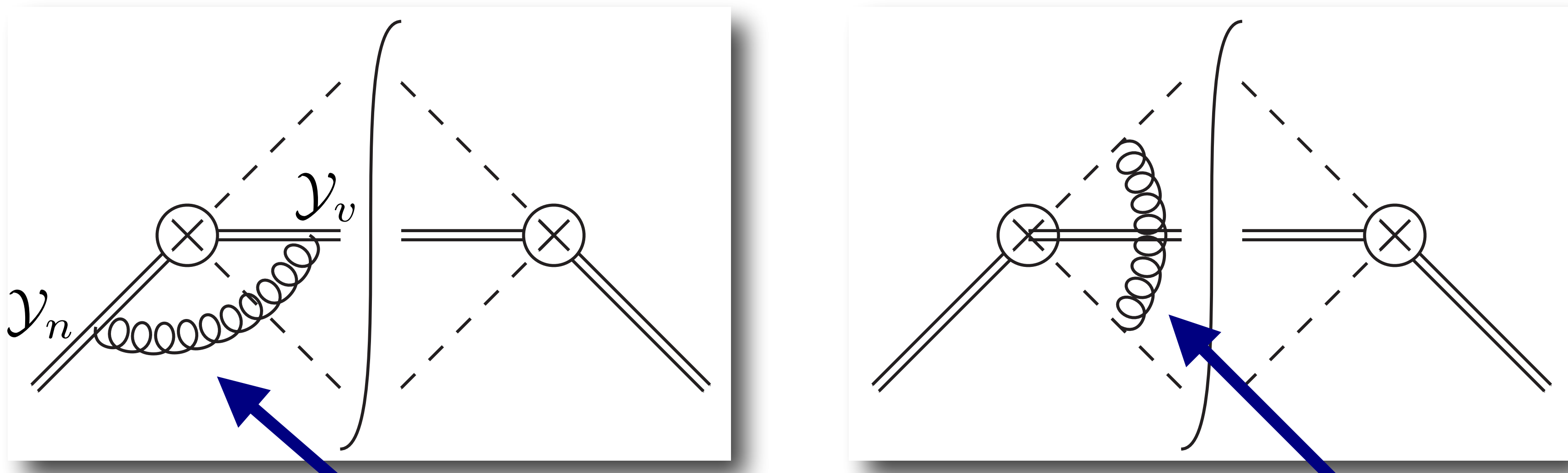
[Maltoni, Mangano, Petrelli hep-ph/9708349]



$$\left.\frac{d\sigma}{\sigma_0}\right|_v = \frac{\alpha_s}{2\pi}\left[\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \left(\frac{C_A}{\varepsilon_{\rm IR}^2} + \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right) - \frac{C_A}{\varepsilon_{\rm IR}} - \frac{C_A}{2}\ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{12} + D^{[8]}_{{}^1S_0}\right)\right]$$

$$D^{[8]}_{{}^1S_0} = C_F\left(-5 + \frac{\pi^2}{4}\right) + C_A\left(\frac{3}{2} + \frac{\pi^2}{12}\right)$$

Virtual part of the TMD shape function (sketchy…)



\+ crossed diagrams

ultrasoft gluon

soft gluon

There are no diagrams which connect at this order the heavy fields with the Wilson lines!

[Luke, Manohar, Rothstein hep-ph/9910209]

$$\tilde{S}_{\mathcal{Q}}^{(0)}\Big|_v = 1 + \frac{\alpha_s}{2\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} + S_-\Big|_v^1 - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}}$$

$$\left\langle \mathcal{O}\left({}^1S_0^{[8]}\right)\right\rangle \tilde{S}_{\mathcal{Q}}\Big|_v = 1 + \frac{\alpha_s}{2\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}}$$

Calculation of H: putting everything together...

$$\left.\frac{d\sigma}{\sigma_0}\right|_v = \frac{\alpha_s}{2\pi}\left[\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \left(\frac{C_A}{\varepsilon_{\rm IR}^2} + \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right)\right) - \frac{C_A}{\varepsilon_{\rm IR}} - \frac{C_A}{2}\ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{12} + D^{[8]}_{{}^1S_0}\Big)\right]$$

$$\left.\tilde{f}_1^g\right|_v = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{\varepsilon_{\rm IR}^2} - \frac{1}{\varepsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right)\right]\delta(1-x)$$

$$\left.\langle \mathcal{O}({}^1S_0^{[8]})\rangle\, \tilde{S}_{\mathcal{Q}}\right|_v = 1 + \frac{\alpha_s}{2\pi}\left(C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{v} - \frac{\alpha_s}{2\pi}\frac{C_A}{\varepsilon_{\rm IR}}$$

$$H = 1 + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{2}\ln^2\frac{\mu^2}{M^2} - C_A\frac{\pi^2}{12} + D^{[8]}_{{}^1S_0}\right]$$

New!

A similar calculation yields the hard part for the other state that contributes

Conclusions & Outlook

●**New factorization theorem derived** for quarkonia production at small q_T (more to come!!)

●**New non-perturbative** matrix elements appear: **TMD Shape Functions**

●**All studies up to now should be revised** (careful with the produced pheno!)

✤Lots of work to do before being able to extract gluon TMDs from quarkonia production: Derive **factorization theorems**, calculate **perturbative ingredients**, etc

✤**Mesure** not only gluon TMDs but also **TMD Shape Functions** (need different processes with the same TMD shape function)

✤**Better fix LDMEs** (currently several incompatible fits exist)

Thank you!