

# Mini-workshop on supersymmetric Wilson loops and related topics



Wednesday, 15 May 2019 - Thursday, 16 May 2019

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## Scientific Programme

## Wilson lines and superconformal defects

In this talk we discuss some universal results for superconformal defects in three and four spacetime dimensions. In particular, we identify the defect conformal data that are relevant for the study of shape and geometric deformations. We then apply these results to the case of supersymmetric Wilson loops, where we are able to prove two existing conjectures for the energy emitted by a heavy probe in  $N=2$  theories.

## A matrix model for a three-point function in ABJM

I consider a limit in which the three-point function of twist-one chiral primary operators in ABJM can be perturbatively related to the expectation value of a supersymmetric Wilson loop. Using the matrix model description of the latter I derive an all-order expression for the structure constant and test it against a perturbative computation of the complete three-point function.

## The fate of the circular Wilson Loops in $N=4$ dCFT

We consider circular Wilson loops in a defect version of  $N=4$  super-Yang-Mills theory which is dual to the D5-D3 brane system. When the loops are parallel to the defect, we can construct both BPS and non-BPS operators. At strong 't Hooft coupling we observe, in the non BPS case, a Gross-Ooguri-like phase transition in the dual gravitational theory: the familiar disk solution dominates when the operator is far from the defect while a cylindrical string worldsheet, connecting the boundary loop with the probe D5-brane, is favoured below a certain distance. In the BPS case, the cylindrical solution does not exist for any choice of the physical parameters, suggesting that light-modes supergravity exchanges with the disk solution always saturate the expectation value at strong coupling. We study the double-scaling limit for large  $k$  and large 't Hooft coupling that allow us to compare perturbative and non-perturbative results. We discuss, in the BPS case at weak coupling, the OPE expansion of the Wilson loop finding consistency with the known results for the one-point function of scalar composite operators.

## Entropy of supersymmetric AdS5 black holes from SCFT partition functions

The talk will present progress towards a microscopic derivation of the entropy of  $1/16$  BPS black holes in AdS<sub>5</sub>. I will describe a new BPS limit of black hole thermodynamics which first focuses on a supersymmetric family of complexified solutions and then reaches extremality. In this limit the chemical potentials obey a constraint that is interpreted as a regularity condition in the Euclidean bulk geometry, and the on-shell gravitational action precisely reproduces the entropy function that controls the Bekenstein-Hawking entropy of the black holes. The gravitational analysis instructs us that the holographic dual  $N=1$  superconformal field theory is defined on a twisted  $S^1 \times S^3$  with complexified chemical potentials obeying the constraint, and localization allows to compute the corresponding partition function exactly. This computation leads to a slightly modified superconformal index as well as a generalization of the supersymmetric Casimir energy. I will discuss how the black hole entropy is encoded in these quantities.

## Surface operators in the 6d $N=(2,0)$ theory

The mysterious 6d  $N=(2,0)$  theory has surface operators which can be thought as its analog of Wilson loop operators. We have many tools to study Wilson loops in  $N=4$  SYM and in ABJM theory, and I am examining which of them would work also in six dimensions. In the talk I will recall the anomalies associated to a surface observable and calculate it in two simple settings of free fields on flat space and via holography. I will also discuss some aspects of BPS observables, both as a local property and globally BPS observables, for which we may hope to find exact results.

## **TBA (and beyond for Wilson loops/gluon scattering amplitudes)**

We derive the 2D S-matrix for the quantum excitations over the GKP string vacuum. These particles live in a flux tube defining the (null) polygonal Wilson loops of planar  $N=4$  SYM theory via a (non-local) Operator Product Expansion (OPE) in the collinear limit. In specific the OPE series is made up of Pentagonal transitions, which are nothing but the Form Factors of some twist fields for the integrable 2D S-matrix above. Thanks to this observations we are able to write down the pentagon transition (up to complicated integrations), and we are left with the problem of summing the OPE series for the 4D MHV gluon scattering amplitudes (or null polygonal Wilson loops). In the strong coupling limit we can reproduce the classical string Thermodynamic Bethe Ansatz, and speculate about the one-loop corrections. Actually, this approach manifest clearly a computable non-perturbative contribution in the scalar sector of the same order. Besides the importance in itself, this contribution opens the way to explicit computations of the pentagon transitions at all couplings, a clear improvement also in the milieu of integrable Form Factor theory. Eventually, but very importantly is sheds new light on the ODE/IM correspondence from a very new perspective and towards  $N=2$  theories.

## **Circular Wilson loop in general $N=2$ SCFTs**

Starting from Pestun's  $N=2$  matrix model on a four sphere, we generalize it for a superconformal theory with generic matter content. Playing with different numbers of hypermultiplets in fundamental, symmetric or antisymmetric representation of the  $SU(N)$  gauge group, one obtains various conformal theories with interesting properties. We compute the vacuum expectation value a  $1/2$  BPS circular Wilson loop in the matrix model and then we check our result against a perturbative computation up to a four-loops order. We present new techniques to deal with superspace variables, which allow to capture the exact  $\zeta(5)$  coefficient. Finally we mention possible holographic consequences of this generalization

## **The Bremsstrahlung function of $N=2$ SCQCD**

In this talk I will describe how to perform a perturbative computation of the bremsstrahlung function of  $N=2$  superconformal QCD (SCQCD) at three loops. I will show that the corresponding result matches the matrix model prediction given in terms of derivatives of the circular Wilson loop on the ellipsoid. I will introduce the computational technique and discuss possible behaviours at higher loops.

## **Supersymmetric Wilson loops in $N=6$ Super Chern-Simons matter theory: a long and winding road**

Supersymmetric Wilson loops in  $N=6$  Super-Chern-Simons theory with matter differ in many respects from their four-dimensional cousins and much less is known about their relations with integrability, defect CFT's, localizable subsectors and correlation functions. I will review a series of results concerning the generalization of "standard"  $1/2$  BPS (fermionic) and  $1/6$  BPS (bosonic) circular Wilson loops to different shapes, addressing specific calculations for "latitude" loops. The actual proposal of a matrix model computing latitudes, from which an exact expression for the Bremsstrahlung function has been derived, will be also discussed. I will finish presenting a personal point of view on some interesting open problems and directions worth to be explored.

## Exact Bremsstrahlung functions in ABJM theory

We study the Bremsstrahlung functions for the  $1/6$  BPS and the  $1/2$  BPS Wilson lines in ABJM theory. First we use a superconformal defect approach to prove a conjectured relation between the Bremsstrahlung functions associated to the geometric and R-symmetry deformations of the  $1/6$  BPS Wilson line. This result follows from a defect supersymmetric Ward identity. Subsequently, we explore the consequences of this relation for the  $1/2$  BPS Wilson line and, using the localization result for the multiply wound Wilson loop, we provide an exact closed form for the corresponding Bremsstrahlung function. Interestingly, for the comparison with integrability, this expression appears particularly natural in terms of the conjectured interpolating function  $h(\lambda)$ . During the derivation of these results we analyze the protected defect supermultiplets associated to the broken symmetries, including their two- and three-point correlators.

## Deformations of the circular Wilson loop and spectral (in)dependence

We study the expectation value of deformations of the circular Wilson loop in  $N=4$  super Yang-Mills theory. The leading order deformation, known as the Bremsstrahlung function, can be obtained exactly from supersymmetric localization, so our focus is on deformations at higher orders. We find simple expressions for the expectation values for generic deformations at the quartic order at one-loop at weak coupling and at leading order at strong coupling. We also present a very simple algorithm (not requiring integration) to evaluate the two-loop result. We find that an exact symmetry of the strong coupling sigma-model, known as the spectral-parameter independence, is an approximate symmetry at weak coupling, modifying the expectation value starting only at the sextic order in the deformation. Furthermore, we find very simple patterns for how the spectral parameter can appear in the weak coupling calculation, suggesting all-order structures.

## Structure constants of determinant operators

We study three-point functions of a non-BPS single-trace operator and two determinant operators in planar  $N=4$  SYM. The latter operators are dual to maximal giant graviton D-branes in  $AdS_5 \times S^5$ . In the talk we introduce two new methods to efficiently compute such correlators at weak coupling; one based on large- $N$  collective fields and the other based on combinatorics. The results so obtained exhibit a simple determinant structure and indicate that the correlator can be interpreted as an overlap between an integrable boundary state and a state corresponding to the single-trace operator, much like one-point functions in the presence of a domain-wall defect. Time permitting, we will also present the integrability framework to describe the three-point functions at finite 't Hooft coupling.