Bruno Touschek and the Bloch Nordsieck problem:

resummation in QED/QCD

Mario Greco - December 9th, 2009

The Infra-Red Radiative Corrections for Colliding Beam (Electrons and Positrons) Experiments.

E. ETIM, G. PANCHERI and B. TOUSCHEK

Nuovo Cimento 51B, 276 (1967)

Energy distribution of radiation from Bloch Nordsieck

$$N\,\mathrm{d}P(\omega) = \beta\,\frac{\mathrm{d}\omega}{\omega}\left(\frac{\omega}{E}\right)^{\beta} \qquad \text{for } \omega < E\,.$$

$$\beta = \frac{4\alpha}{\pi}\left(\lg 2\gamma - \frac{1}{2}\right)$$

$$\mathrm{d}^2\sigma_{\text{exp}} = N^{-1}\left(\frac{\Delta\omega}{E}\right)^{\beta}(1+\lambda)\,\mathrm{d}^2\sigma_0$$

- Infrared divergences in the S-matrix elements are related to the definition of states which are diagonal in the number of photons.
- In the reaction $A+B\to C+D+\ldots+\Gamma,$ the number of soft photons cannot be measured.

Coherent states in QED

A Note on the Infrared Divergence.

M.Greco and G.Rossi, Nuovo Cimento 50B, 168 (1967)

- Realistic definition of final states in the S-matrix elements

$$|f'\rangle = e^{i\Lambda_c}|f\rangle \qquad \Lambda_c = \int d^4x\, j_\mu(x)\, A_\mu(x)$$

$$j_\mu(x) = (2\pi)^{-4} \int d^4k\, j_\mu(k) e^{ikx}$$

$$j_\mu(k) = ie(2\pi)^{-\frac{3}{2}} \sum_i \varepsilon_i P_{i\,\mu} (P_i k)^{-1}$$

Is the classical current associated to the initial and final charged particles.

One can define a new S-matrix element:

$$\overline{M} = (f'|S|i) = (f|e^{-i\Lambda_c} T(e^{iL})|i)$$

which is finite (no infrared divergences) to all orders of perturbation theory, separable, and directly comparable to the observed cross section.

$$M = \frac{1}{\gamma^{\beta/2}} \frac{1}{\sqrt{\Gamma(1+\beta)}} \left(\frac{\Delta\omega}{E}\right)^{\beta/2} M_E$$

$$d\sigma = \frac{1}{\gamma^\beta} \frac{1}{\Gamma(1+\beta)} \left(\frac{\Delta\omega}{E}\right)^\beta d\sigma_E$$

which exactly agrees with Etim, Pancheri, Touschek.

- Very powerful method for resummation.
- Production of Narrow Resonances in e+e- Annihilation (J/Psi, Z, …)

  M. Greco, G. Pancheri, Y. Srivastava, Nucl.Phys.B101:234,1975,

  M. Greco, G. Pancheri, Y. Srivastava, Nucl.Phys.B171:118,1980.

New phenomena: for Γ << $\Delta\omega$ the width provides a natural cutoff and the soft photon emission is governed by an intrinsic physical quantity.

Basic approach for the analysis of J/Psi, … and all LEP/SLC experimental data.

Very precise evaluation of subtle radiative effects, particularly for interference terms with pure QED contribs.

Modification of initial state classical current due to exact energy momentum conservation

$$j_\mu^{R}(k) \doteq \frac{(W-M)+\frac{1}{2}i\Gamma}{(W-M-k)+\frac{1}{2}i\Gamma}\; I_\mu(k)+F_\mu(k)$$

$$\Lambda_R = \int d^4x\, j_\mu^{R}(x)\, A_\mu(x)$$

$$|f''\rangle = \frac{1}{\sqrt{N}}\, e^{i\Lambda_R}\, |f\rangle$$

$$\overline{M_R} = \frac{1}{\sqrt{N}}\, \langle f|\, e^{-i\Lambda_R^{+}} S|i\rangle$$

- More subtleties for interference effects.

Dependence on the width Γ : result for J/psi hadronic final state.

$$\tilde{\sigma}(M) = \frac{6\pi^2 \Gamma_e \Gamma_h}{\sqrt{2\pi}\,\sigma M^2 \Gamma} \left(\frac{\Gamma}{M}\right)^{\beta_i} \left(1 + \frac{\Gamma^2}{8\sigma^2}\right) \left\{1 - \frac{\Gamma}{\sqrt{2\pi}\,\sigma} + \beta_i \left[\ln \frac{2\sqrt{2}\sigma}{\Gamma} - \frac{1}{2}\gamma\right]\right\} (1 + C_F^{res})$$

Different result obtained by D.R.Yennie, Phys. Rev. Letts.34 (1975) 239.

$$\tilde{\sigma}(M) \simeq \frac{6\pi^2}{\sqrt{2\pi}\,\sigma} \frac{\Gamma_e \Gamma_h}{M^2 \Gamma} \left(\frac{2\sqrt{2}\sigma}{M}\right)^{\beta_i} (1 + 0.79\,\beta_i)$$

Due to the wrong rad. corrections formulae used for the data analysis SLAC has published for many years wrong results for the total and partial widths of J/Psi, Psi',… For SLC the right formulae have been applied.

Theoretical activity in QCD.

- Many ideas and techniques previously developed in QED have been extended to QCD leading to quite fundamental results for the phenomenology of strong interaction processes.

- Exponentiation and resummation of the leading perturbative effects to all orders: K-factors, physics of jets, parton distributions at large x, transverse momentum distributions of Drell-Yan pairs, of W and Z bosons,…which are still at the basis of the phenomenology of Tevatron and LHC processes.

- Frascati and the roman area playing a central role in this field and giving a fundamental contribution to QCD phenomenology:

  Altarelli, Curci, Greco, Martinelli, Pancheri, Parisi, Petronzio, Srivastava,…

Coherent states in QCD

M.Greco, F. Palumbo, G. Pancheri, Y. Srivastava, Phys.Lett.B77:282,1978.
G.Curci, M.Greco, Phys.Lett.B79:406,1978.
G.Curci, M.Greco, Y.Srivastava, P.R.L.43:834,1979; Nucl.Phys. B159:451,1979.

$$|\tilde{\mathrm{i}}\rangle = \mathrm{e}^{\mathrm{i}\Lambda_\mathrm{i}}|\mathrm{i}\rangle, \quad |\tilde{\mathrm{f}}\rangle = \mathrm{e}^{\mathrm{i}\Lambda_\mathrm{f}}|\mathrm{f}\rangle$$

$$\Lambda = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 k\, j_\mu^c(k) A_\mu^c(-k).$$

$A_\mu^c(x)$ is the quantized gluon field of color c

$j_\mu^c(k)$ Is the “classical” current

$$j_\mu^c(k) = \frac{\mathrm{i}}{(2\pi)^{3/2}} \bar{g}(k) \sum_\alpha \frac{\eta_\alpha p_{\alpha\mu}}{(p_\alpha k)} t^c(\alpha)$$

$t^c(\alpha)$ are the appropriate generators of the color group for the αth particle and $\bar{g}(k)$ is the effective coupling constant

One can define new S-matrix elements which have similar infrared properties as in QED, can include collinear hard gluon terms and generalize in QCD the resummation of the leading perturbative contributions for various processes.

A few examples:

- DIS: Quark distributions at x close to 1, Gribov-Lipatov relation. Modification of Altarelli-Parisi eq.

- Drell-Yan processes: exponentiation of leading corrections in the K-factor.

- Jet physics: transverse momentum distributions, confirmations at Petra.

- Transverse momentum distributions of W and Z bosons due to soft gluon emission: major th. result, at the basis of the vector bosons phenomenology at hadron colliders.

Conclusions.

- Many theoretical activities have been developed following Bruno's ideas about the soft radiation problems at Adone.

- In QED they led to the exact and detailed treatment of the problem, for non resonant processes as well as the J/Psi,.. and Z production in e+e- colliding beam experiments.

- Similarly in QCD the simple ideas of exponentiation have generated the basis for many resummation results in pertub. theory, which are fundamental for the phenomenology at hadron colliders and now at LHC.