Stability Issues with Broken SUSY and a Cosmological Clue

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ASI/COSMOS Meeting on ΛCDM
U. Roma "Tor Vergata", May 28-29, 2019

The Problem

Vacuum Fluctuations in Field Theory

- BOSE (FERMI) HARMONIC OSCILLATOR: $$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2 \longrightarrow \mathcal{E}_0 = (-)\frac{\hbar \omega}{2}$$

- QUANTUM FIELD THEORY: $$\frac{\mathcal{E}_0}{V} = \sum_i (-1)^{F_i} \int \frac{d^d \mathbf{k}}{(2\pi)^d} \frac{\hbar c}{2} \sqrt{\mathbf{k}^2 + m_i^2}$$

- THE COSMOLOGICAL CONSTANT ISSUE: $$\frac{\mathcal{E}_0}{V} \sim \frac{\mathcal{E}_{Pl}}{V_{Pl}} \sum_i (-1)^{F_i} + \ldots$$

(Zeldovich, 1968)

- SUPERSYMMETRY (SUSY) as a regulator: (Exact, GLOBAL) SUSY removes problem

(Zumino, 1975)

Vacuum Configurations & String Theory

- NO SUSY → TACHYONS !

- NO – TACHYONS ? MAJOR RESULT → G. S. O. & SUSY-SUGRA

(Gliozzi, Scherk, Olive, 1977)

- (SLIGHLY) MORE GENERALLY NO TACHYONS ? YES !

❖ 3 D=10 SUSY STRINGS: – SO(16)xSO(16) (HETEROTIC)

(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)

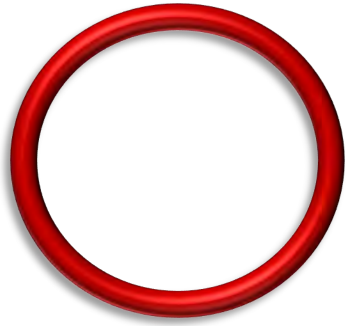
– U(32) O'B (ORIENTIFOLD)

(AS, 1995)

– Usp(32) (ORIENTIFOLD) → "Brane SUSY Breaking" (HIDDEN SUSY)

(Sugimoto, 1999)

SUSY Breaking in String Theory

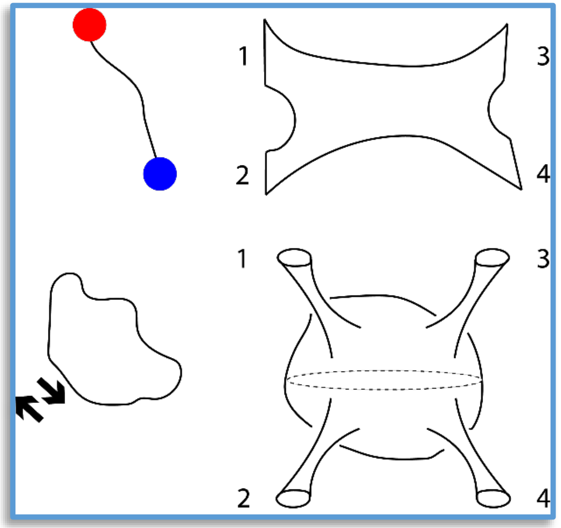


- (CIRCLE) SCHERK-SCHWARZ: MODIFY KALUZA-KLEIN FOR FERMIONS

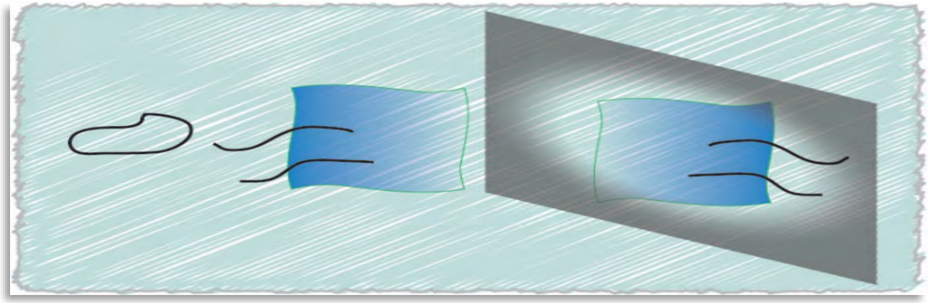
$$p_B = \frac{m}{R} \quad [\psi_B(x + 2\pi R) = \psi_B(x)] \qquad p_F = \frac{m+\frac{1}{2}}{R} \quad [\psi_F(x + 2\pi R) = -\psi_F(x)]$$

- ORIENTIFOLDS: OPEN AND CLOSED STRINGS

(AS, 1987)
[+Bianchi, Pradisi, 1988-96]
[+Stanev, 1994-96]
[+ Angelantonj, Fioravanti]



- (EXTENDED) SOLITONS IN VACUUM

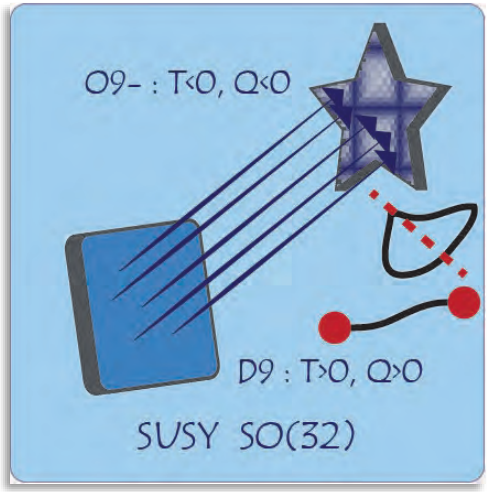


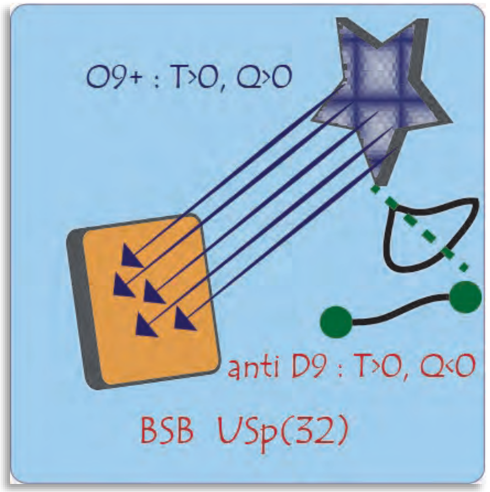
(Polchinski, 1995)

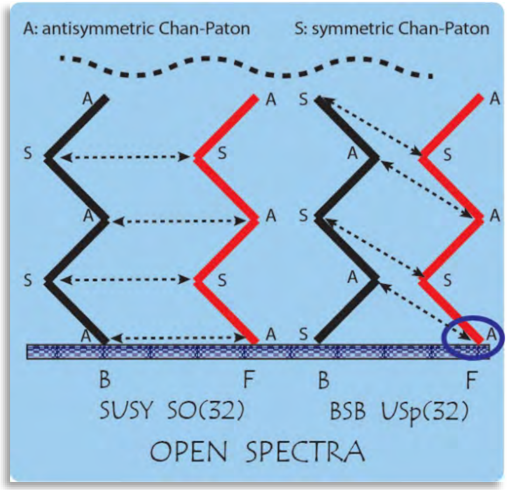
Brane SUSY Breaking in String Theory

- (Non-linear) SUSY
- NO TACHYONS

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)







$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12} \mathcal{H}_3^2 - \frac{1}{4} e^{-\phi} \operatorname{tr} \mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

VACUUM ENERGY → POTENTIAL

NOTE:

- Expansion in powers of $\alpha' R$
- Expansion in powers of $g_s = e^{\phi}$

"Vacuum" Solutions

Cosmological Potentials

- What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \quad \Longrightarrow \quad \ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- **If V does not vanish :** convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\,dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$V\,e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{(d-1)}{2(d-2)}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ Quadratic potential? Far away from origin *(Linde, 1983)*

❖ Exponential potential? YES or NO

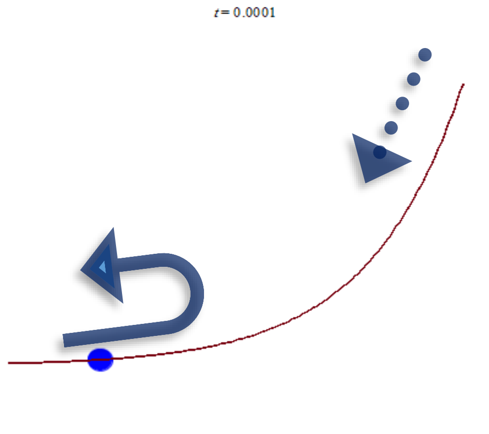
$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

1. Climbing Scalars

- γ < 1 ? Both signs of speed

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \gamma\,(1 + \dot{\varphi}^2) = 0$$



a. "Climbing" solution (φ climbs, then descends → e^{ϕ} BOUNDED)
b. "Descending" solution (φ only descends)

Limiting τ- speed (LM attractor): $v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$

(Lucchin and Matarrese, 1985)

γ = 1 is "critical": LM attractor & descending solution disappear there and beyond !

CLIMBING : in ALL asymptotically exponential potentials with γ≥1 !

BSB in STRING THEORY HAS PRECISELY γ = 1 → WEAK coupling ($g_s = e^{\varphi}$) !!

- γ = 1 :

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$
$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Pre-Inflationary Relics in the CMB?

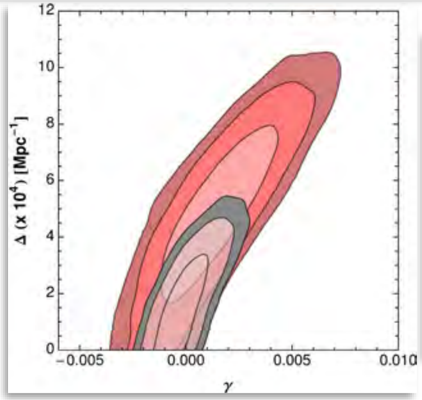
(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)

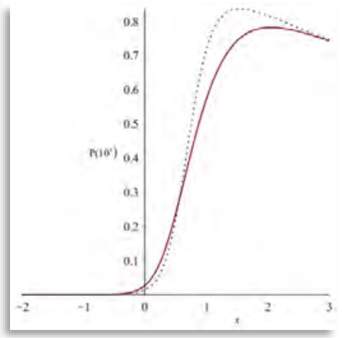
- Extend ΛCDM to allow for low-ℓ suppression:

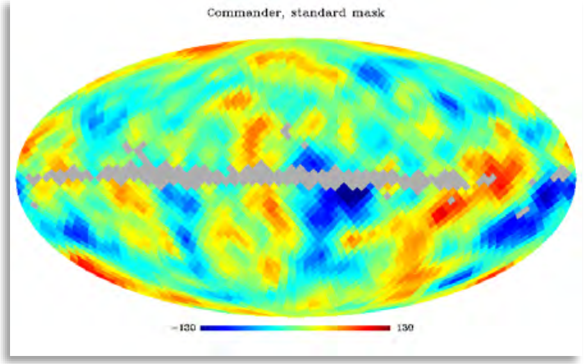
$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\rightarrow\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$

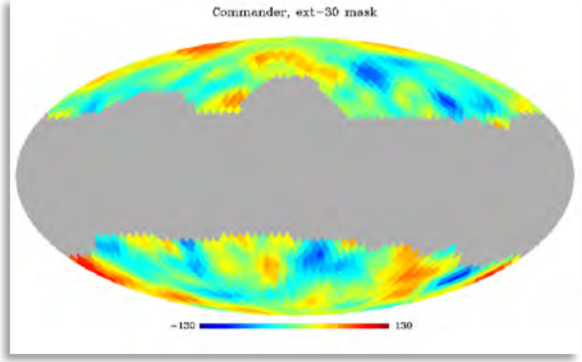
❖ NO effects on standard ΛCDM parameters (6+16 nuisance)

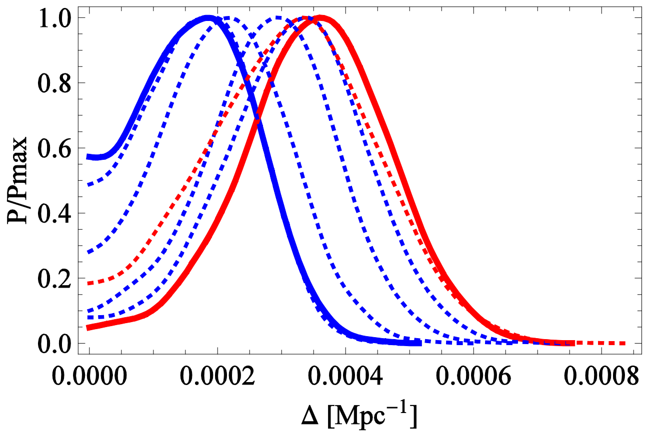
❖ A new scale Δ. Preferred value? Depends on GALACTIC MASK











$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

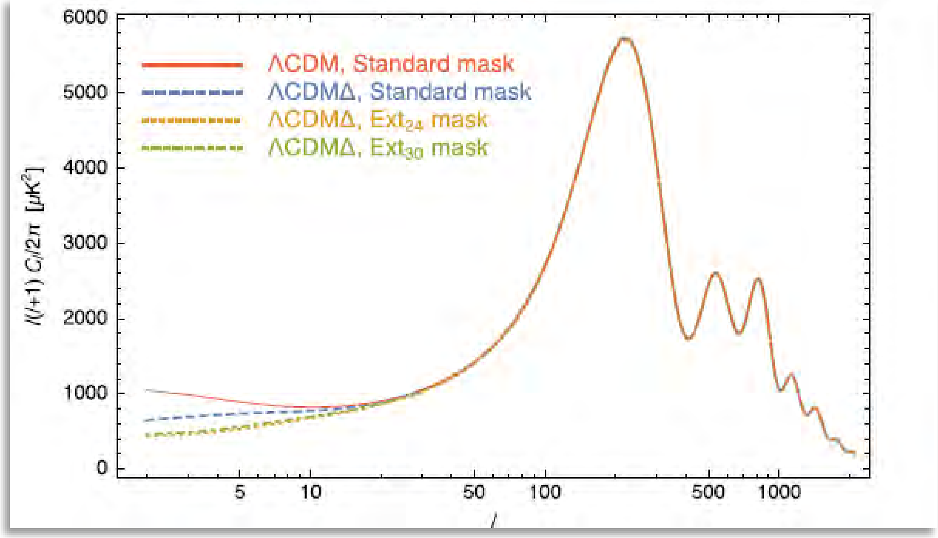
RED : + 30−degree extended mask

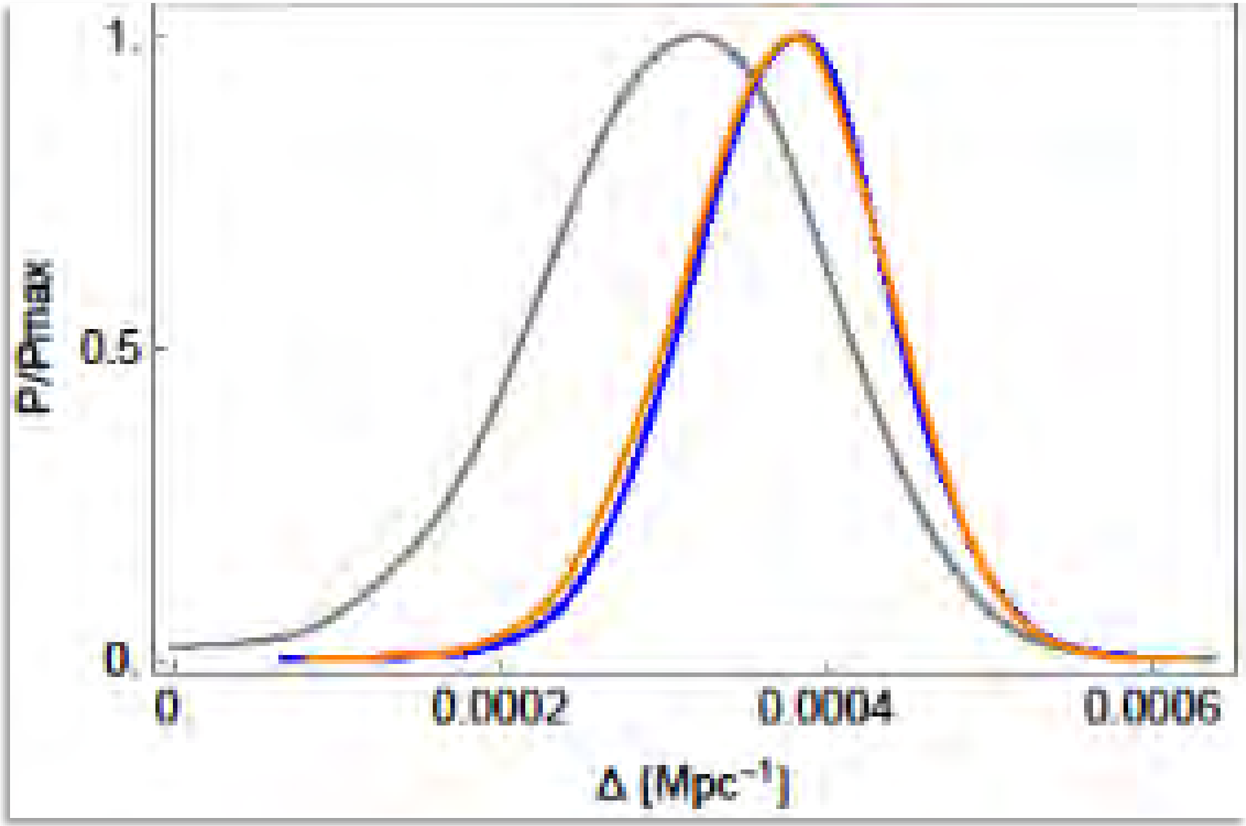
$> 99\%$ confidence level

$$\Delta^{Infl} \;\sim\; 2.4 \times 10^{12}\; e^{N-60}\;\mathrm{GeV} \;\sim\; 10^{12} - 10^{14}\mathrm{GeV} \quad \text{for} \quad N \;\sim\; 60-65$$

Future Prospects

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)





- Δ does not affect standard ΛCMB parameters

WHAT NEXT?

POLARIZATION

- cosmic-variance limited E-mode could lead to a 5-6 σ detection of Δ (or rule it out)

2: 9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\,\Big\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\Big\}$$

1. 9D solution that (in string frame) describes an S_1/Z_2 (interval) compactification.

   Finite 9D Planck mass and gauge coupling, but singularities at the ends

$$ds^2 = \left|\alpha\,y^2\right|^{\frac{2}{9}}\,e^{\frac{1}{2}\phi_0}\,e^{\frac{1}{4}\alpha\,y^2}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + \left|\alpha\,y^2\right|^{-\frac{1}{3}}\,e^{-\phi_0}\,e^{-\frac{3}{4}\alpha\,y^2}\,dy^2$$
$$e^{\phi} = e^{\phi_0}\left|\alpha\,y^2\right|^{\frac{1}{3}}\,e^{\frac{3}{4}\alpha\,y^2}$$

[Applies to both Usp(32) and U(32), similar results also for heterotic SO(16) x SO(16)]

HOWEVER :

a) string loop corrections: determined by second equation, grow out of control for y→ ∞;

b) curvature corrections: same problem near y=0.

3. Orientifold Flux Vacua with BSB

(Mourad, AS, 2016)

- In this setting the field equations from

$$S = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

reduce to

$$(\star):\ T\,e^{\gamma_E\,\phi} = -\,\frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-2\,C} = \frac{h^2\left(p + 1 - \frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}}{(7-p)}$$

$$(A')^2 = k\,e^{-2\,A} + \frac{h^2}{16(p+1)}\left(7 - p + \frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

- (*): Dilaton Eq: constraint from positivity of l.h.s. (β_E < 0 for orientifolds & T>0, NEED H_3 fluxes)
- Third Eq: determines for k=0 A ~r, and thus AdS in Poincaré coordinates (or in other slicings for k ≠ 0)

❖ WIDE REGIONS where the two couplings $\alpha'\,R$ and $g_s = e^{\phi}$ are SMALL

Stability ?

Stability & Broken SUSY?

(Basile, Mourad, AS, 2018)

- Basic Equation:

$$g_{\mu\nu}\,A \;+\; \nabla_\mu\,\nabla_\nu\,B \;=\; 0 \;\longrightarrow\; A=0\,, \quad B=0$$

- Scalar Perturbations:

$$\begin{aligned} b_{\mu\nu} &= \sqrt{-\lambda}\;\epsilon_{\mu\nu\rho}\,\nabla^\rho\,B \\ h_{\mu\nu} &= \lambda_{\mu\nu}\,A\,, \\ h_{\mu i} &= R^2\,\nabla_\mu\,\nabla_i\,D\,, \\ h_{ij} &= \gamma_{ij}\,C\,, \end{aligned}$$

- Breitenlohner-Freedman Bound:

$$M^2 \;\geq\; -\,\frac{(d-1)^2}{4\,R_{AdS}^2}$$

(Breitenlohner, Freedman, 1982)

- OK for ℓ=0:

$$\begin{aligned} \Box\,B \;&+\; 4\left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right)(\beta\,\varphi \;+\; 14\,C) \;=\; 0\,, \\ \Box\,\varphi \;&-\; V_0''\,\varphi \;-\; \left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right)(2\,\beta^2\,\varphi \;+\; 14\,\beta\,C) \;=\; 0\,, \\ \Box\,C \;&-\; C\left(\frac{7}{R_{AdS}^2} + \frac{9}{R^2}\right) - \frac{1}{2}\,\beta\,\varphi\left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right) \;=\; 0\,, \end{aligned}$$

Stability & Broken SUSY?

(Basile, Mourad, AS, 2018)

- $\ell \neq 0$:

$$R^2_{AdS}\,\mathcal{M}^2 \;=\; \frac{L_3}{3}\,(\sigma_3-1)\,\mathbf{1}_3 + \begin{pmatrix} 4+3\sigma_3 & -\frac{7}{2}\,\beta\,\sigma_3 & \frac{L_3}{2}\,(\sigma_3-1) \\ -2\,\beta\,\sigma_3 & 2\,\beta^2\,\sigma_3+\tau_3 & -\frac{\beta\,L_3}{3}\,(\sigma_3-1) \\ 8\,\sigma_3 & -4\,\beta\,\sigma_3 & 0 \end{pmatrix},$$

$$\sigma_3 \to \frac{3}{2}\,, \qquad \tau_3 \to \frac{9}{2}\,, \qquad L_3 \to \ell(\ell+6)$$

$$\left[\mathcal{M}^2\,R^2_{AdS} \;\geq\; -1\,?\right]$$

- Breitenlohner-Freedman Bound Violated for ℓ= 2,3,4 :

$$\begin{bmatrix} \frac{\ell(\ell+6)}{6}+4 \\ \frac{(\ell+6)(\ell+12)}{6} \\ \frac{\ell(\ell-6)}{6} \end{bmatrix}$$

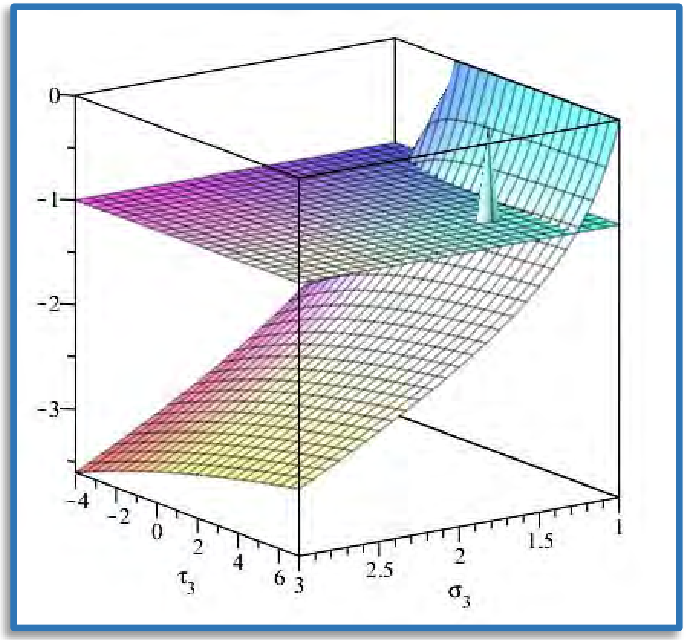
- [Similar result for heterotic SO(16)xSO(16) for ℓ= 1]

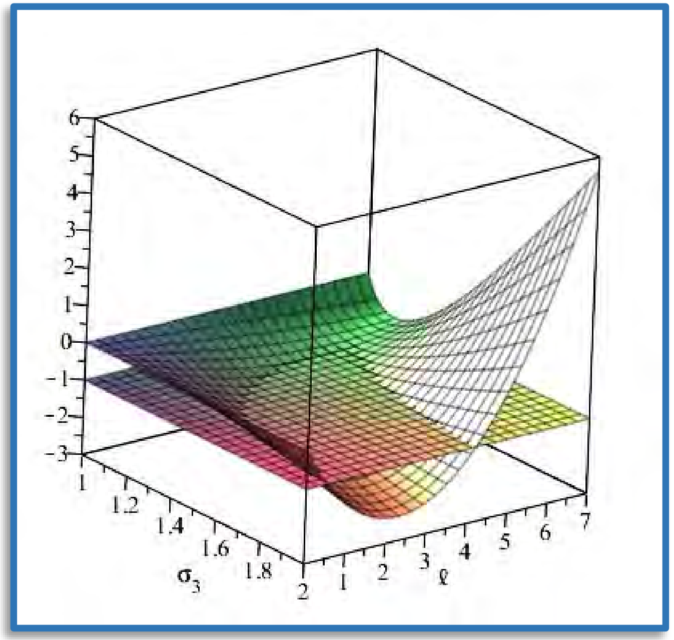
Stability & Broken SUSY?

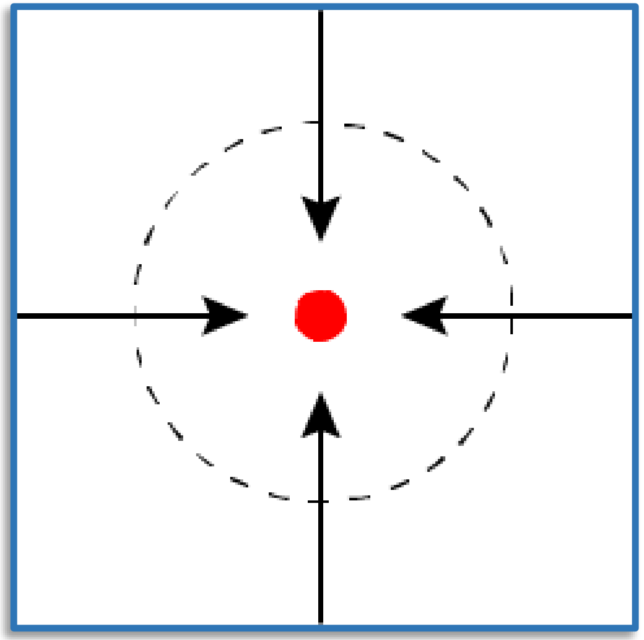
(Basile, Mourad, AS, 2018)

1. $AdS_3 \times S^7$ Orientifold Flux vacua: WEAK coupling but UNSTABLE

- Violation of Breitenlohner-Freedman bound for scalar modes
- HOWEVER: wide nearby regions os stability. From Quantum corrections?







Like Electro- (or Gravito-) static Instability ?

Stability & Broken SUSY?

(Basile, Mourad, AS, 2018)

2. Dudas-Mourad vacua: STABLE but STRONG COUPLING(s)

$$ds^2 = e^{2\Omega(z)} \left[(1+A)\, dx^\mu\, dx_\mu \; + \; 2\, dx^\mu\, dz\, \partial_\mu D \; + \; (1+C)\, dz^2 \right] ,$$

- BUT: D can be gauged away, and then C = - 7 A (looks familiar…)

$$A'' + A' \left(24\, \Omega' \; - \; \frac{2}{\phi'}\; e^{2\Omega}\, V_\phi \right) \; + \; A \left(m^2 \; - \; \frac{7}{4}\; e^{2\Omega}\, V \; - \; 14\, e^{2\Omega}\, \Omega' \, \frac{V_\phi}{\phi'} \right) \; = \; 0$$

- Can turn into a Schödinger-like form

$$\begin{aligned} m^2\, \Psi \; &= \; \left(b \; + \; \mathcal{A}^\dagger\, \mathcal{A} \right) \Psi \\ b \; &= \; \frac{7}{2}\, e^{2\Omega}\, V\, \frac{1}{1 \; + \; \frac{9}{4}\, \alpha_O\, y^2} \; > \; 0 \end{aligned}$$

- NO tachyons in 9D → STABILITY

Stability & Broken SUSY?

(Basile, Mourad, AS, 2018)

3. Cosmological Models: the issue is the time evolution of perturbations

❖ For large η V is negligible and tensor perturbations evolve as

$$
\begin{aligned}
h''_{ij} &+ 8\,\frac{1}{\eta}\,h'_{ij} + \mathbf{k}^2\,h_{ij} = 0 \\
h_{ij} &\sim A_{ij}\,J_0(k\eta) + B_{ij}\,Y_0(k\eta) \qquad (\mathbf{k} \neq 0) \\
h_{ij} &\sim A_{ij} + B_{ij}\,\log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)
\end{aligned}
$$

❖ NOTICE: logarithmic growth for k=0 (instability of isotropy) !!

Stability & Broken SUSY?

(Basile, Mourad, AS, 2018)

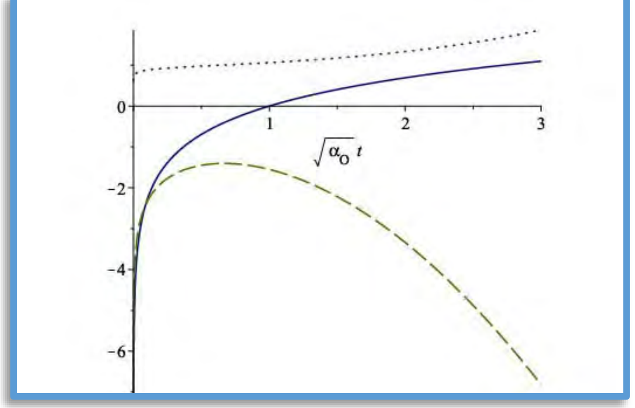
3. HENCE → Cosmological Models Behave BETTER
   a) CLIMBING SCALAR: INSTABILITY of ISOTROPY (k=0 only) in climbing phase
   b) STABLE in descent driven by a mild (brane) potential [Lucchin-Matarrese attractor]

Perfect Fluid Picture:

$$\frac{p}{\rho} = \alpha = \frac{T - V}{T + V}$$

$$ds^2 = \left(\frac{\eta}{\eta_0}\right)^{\frac{4}{(d-1)(1+\alpha)} - 2} \left[- d\eta^2 + d\mathbf{x} \cdot d\mathbf{x}\right]$$

$$h_{ij} = A_{ij} + B_{ij}\, \eta^{\frac{(d-1)(\alpha-1)}{(d-1)(\alpha+1)-2}}$$



❖ RATIONALE for COMPACTIFICATION D=10 → D=4 ?

Thank You