Principal Component Analysis of the Primordial Tensor Power Spectrum

(arXiv:1905.08200)

Paolo Campeti
In collaboration with **Davide Poletti** and **Carlo Baccigalupi**

SISSA - Scuola Internazionale Superiore di Studi Avanzati

The Quest for Primordial Gravitational Waves

- **Primordial B-modes in the CMB polarization produced by tensor perturbations are predicted by Inflation**
- **IF Detected Primordial Gravitational waves** will give:
  - "Smoking gun" for inflation
  - Identify energy scale of inflation **for the simplest models!** (single scalar field slow-roll)
- **What about more complex models? Beyond the standard model of Early Universe? Lots of Physics to be understood in this primordial signal!**

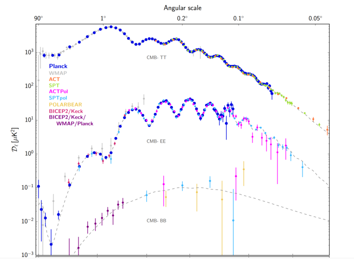


Figure 1: *From Planck results 2018*

Goals and Motivations

Goals and Motivations

Our Goals and Motivations:

1. Examples of **non-standard B-mode emission in the literature**:
   - ***massive gravity inflation*** (Domenech et al. 2017)
   - ***open inflation***(Yamauchi et al. 2011)
   - ***topological defects/cosmic strings*** (Lizarraga et al. 2014)
   - ***multifield inflation*** (Price et al. 2015)
   - ***modified speed of cosmological gravitational waves*** (Raveri et al. 2014)
   - ***rolling axion*** (Namba et al. 2016)
   - ***SU(2)- axion model*** (Dimastrogiovanni et al. 2016)
   - ***high-scale inflation*** (Baumann et al. 2016)
   - ...

Goals and Motivations

Our Goals and Motivations:

1. Examples of **non-standard B-mode emission in the literature**:
    - ***massive gravity inflation*** (Domenech et al. 2017)
    - ***open inflation***(Yamauchi et al. 2011)
    - ***topological defects/cosmic strings*** (Lizarraga et al. 2014)
    - ***multifield inflation*** (Price et al. 2015)
    - ***modified speed of cosmological gravitational waves*** (Raveri et al. 2014)
    - ***rolling axion*** (Namba et al. 2016)
    - ***SU(2)- axion model*** (Dimastrogiovanni et al. 2016)
    - ***high-scale inflation*** (Baumann et al. 2016)
    - ...
2. Establish the constraining power of **future B-mode probes** on the **shape** of primordial tensor power spectrum

Goals and Motivations

Our Goals and Motivations:

1. Examples of **non-standard B-mode emission in the literature**:
   - ***massive gravity inflation*** (Domenech et al. 2017)
   - ***open inflation***(Yamauchi et al. 2011)
   - ***topological defects/cosmic strings*** (Lizarraga et al. 2014)
   - ***multifield inflation*** (Price et al. 2015)
   - ***modified speed of cosmological gravitational waves*** (Raveri et al. 2014)
   - ***rolling axion*** (Namba et al. 2016)
   - ***SU(2)- axion model*** (Dimastrogiovanni et al. 2016)
   - ***high-scale inflation*** (Baumann et al. 2016)
   - ...
2. Establish the constraining power of **future B-mode probes** on the **shape** of primordial tensor power spectrum
3. Sensitivity to **features, deviations from power-law behaviour**

Goals and Motivations

Our Goals and Motivations:

1. Examples of **non-standard B-mode emission in the literature**:
    - ***massive gravity inflation*** (Domenech et al. 2017)
    - ***open inflation***(Yamauchi et al. 2011)
    - ***topological defects/cosmic strings*** (Lizarraga et al. 2014)
    - ***multifield inflation*** (Price et al. 2015)
    - ***modified speed of cosmological gravitational waves*** (Raveri et al. 2014)
    - ***rolling axion*** (Namba et al. 2016)
    - ***SU(2)- axion model*** (Dimastrogiovanni et al. 2016)
    - ***high-scale inflation*** (Baumann et al. 2016)
    - ...
2. Establish the constraining power of **future B-mode probes** on the **shape** of primordial tensor power spectrum
3. Sensitivity to **features, deviations from power-law behaviour**
4. We use **Principal Component Analysis** on Tensor Power Spectrum for a **model independent** approach

Power spectra, Parameters and Observations

Standard Parametrization

Primordial Tensor Power Spectrum (Standard Power-Law)

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_0}\right)^{n_T}$$

Standard Parametrization

Primordial Tensor Power Spectrum (Standard Power-Law)

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_0}\right)^{n_T}$$

- **Tensor-to-scalar ratio**

$$\boxed{r = \frac{A_T}{A_s}}$$

Standard Parametrization

Primordial Tensor Power Spectrum (Standard Power-Law)

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_0}\right)^{n_T}$$

- **Tensor-to-scalar ratio**

$$\boxed{r = \frac{A_T}{A_s}}$$

- Actual observables are **Temperature and Polarization angular power spectra** C_ℓ

Standard Parametrization

Primordial Tensor Power Spectrum (Standard Power-Law)

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_0}\right)^{n_T}$$

- **Tensor-to-scalar ratio**

$$\boxed{r = \frac{A_T}{A_s}}$$

- Actual observables are **Temperature and Polarization angular power spectra** C_ℓ
- **Tensor contribution**

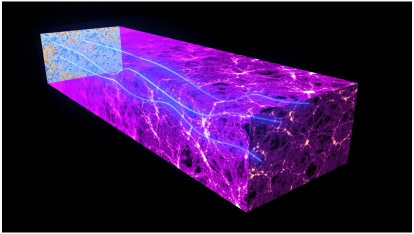
$$\boxed{C_{\ell,t}^{XX'} \propto \mathcal{P}_T(k)}$$

$X, X' \in \{T, E, B\}$

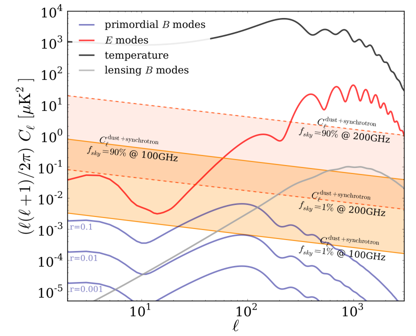
Noise, Lensing and Foregrounds Contribution

$$C_\ell^{XX'} = C_\ell^{XX',\,prim} + C_\ell^{XX',\,noise} + \lambda C_\ell^{XX',\,lens} + C_\ell^{XX,\,fgs}$$

- Total $C_\ell^{XX'}$ contains:
  1. **Primordial spectrum** $C_\ell^{XX',\,prim}$
  2. **Instrumental noise after Component Separation** $C_\ell^{XX',\,noise}$
  3. **CMB lensing contribution** $\lambda C_\ell^{XX',\,lens}$ (λ **delensing factor**)
  4. **Foregrounds contribution** $C_\ell^{XX,\,fgs}$



Foregrounds and Lensing



- **Lensing** dominant at **intermediate – small scales**
- **Foregrounds** → Dominant at **large scales**
- **Dust** + **Synchrotron**
- **Parametric maximum-likelihood component separation** → subtract foregrounds from data using multi-freq observations
- Leaves **residual foregrounds** in the maps → **residuals power spectrum** C_ℓ^{fgs}
- **FGBuster** Code (Poletti & Errard)

Introduction to Principal Component Analysis

Principal Component Analysis (PCA)

Principal Component Analysis (PCA)

- Aim $\rightarrow$ **Identify uncorrelated variables and rank them according to uncertainty**
- Diagonalize Fisher matrix
$$\boldsymbol{F} = \boldsymbol{S}^T \boldsymbol{E} \boldsymbol{S}$$
- Rows of **S** $\rightarrow$ **eigenvectors** of **F** (**PCA modes**)
- $\boldsymbol{E} = diag(e_i) \rightarrow e_i$ **eigenvalues** of **F** ordered from largest to smallest
- New set of parameters m_a (**PCA amplitudes**) linear combination of original parameters p_i
$$\boldsymbol{m} = \boldsymbol{S}\boldsymbol{p}$$
uncorrelated (diagonal covariance matrix $\boldsymbol{E}^{-1}$)
- Uncertainties on $m_i \rightarrow \sigma_i = e_i^{-1/2}$
- Natural basis for free parameters for given experimental configuration $\rightarrow$ best measured linear combinations of original parameters
- **Compression of information** in the first best measured modes

Principal Components of Tensor Power Spectrum

- PCA of **scalar power spectrum** already performed by Hu & Okamoto 2004 and Leach 2006 before **Planck** mission

Principal Components of Tensor Power Spectrum

- PCA of **scalar power spectrum** already performed by Hu & Okamoto 2004 and Leach 2006 before **Planck** mission
- First application to tensor power spectrum Farhang & Sadr 2018

Principal Components of Tensor Power Spectrum

- PCA of **scalar power spectrum** already performed by Hu & Okamoto 2004 and Leach 2006 before **Planck** mission
- First application to tensor power spectrum Farhang & Sadr 2018

Our goal (Campeti, Poletti and Baccigalupi arXiv:1905.08200)

apply **PCA** to **Primordial Tensor Power Spectrum** and make realistic forecasts for future CMB **B-mode** probes (**LiteBIRD, SO, CMB-S4**)

Principal Components of Tensor Power Spectrum

- PCA of **scalar power spectrum** already performed by Hu & Okamoto 2004 and Leach 2006 before **Planck** mission
- First application to tensor power spectrum Farhang & Sadr 2018

Our goal (Campeti, Poletti and Baccigalupi arXiv:1905.08200)

apply **PCA** to **Primordial Tensor Power Spectrum** and make realistic forecasts for future CMB **B-mode** probes (**LiteBIRD, SO, CMB-S4**)

- Discretize tensor power spectrum

$$\boxed{\mathcal{P}_T(k) = A_s \sum_i p_i W_i(\ln k)}$$

Principal Components of Tensor Power Spectrum

- PCA of **scalar power spectrum** already performed by Hu & Okamoto 2004 and Leach 2006 before **Planck** mission
- First application to tensor power spectrum Farhang & Sadr 2018

Our goal (Campeti, Poletti and Baccigalupi arXiv:1905.08200)

apply **PCA** to **Primordial Tensor Power Spectrum** and make realistic forecasts for future CMB **B-mode** probes (**LiteBIRD, SO, CMB-S4**)

- Discretize tensor power spectrum

$$\mathcal{P}_T(k) = A_s \sum_i p_i W_i(\ln k)$$

- **Fisher Information Matrix for CMB**

$$F_{ij} = f_{sky} \sum_{\ell=2}^{\ell_{\max}} \frac{2\ell+1}{2} \mathrm{Tr}\left[\mathbf{D}_{\ell i}\mathbf{C}_\ell^{-1}\mathbf{D}_{j\ell}\mathbf{C}_\ell^{-1}\right]$$

PCA modes for Model Testing: Fisher Approach

- Obtain $\mathcal{S}_a(k) \rightarrow$ **basis** for tensor spectrum

PCA modes for Model Testing: Fisher Approach

- Obtain $\mathcal{S}_a(k) \rightarrow$ **basis** for tensor spectrum
- **Detectability of specific theoretical model** $\rightarrow$ 2 approaches:

PCA modes for Model Testing: Fisher Approach

- Obtain $\mathcal{S}_a(k) \rightarrow$ **basis** for tensor spectrum
- **Detectability of specific theoretical model** $\rightarrow$ 2 approaches:

1. **Fisher approach:** **Projecting** model $\mathcal{P}_{model}$ over PCA modes:

$$\boxed{m_a = \int d\ln k\, \mathcal{S}_a(k)\mathcal{P}_{model}(k)\,.}$$

Uncertainties $\boldsymbol{\sigma_{Fisher}}$ on m_a from Fisher matrix

PCA modes for Model Testing: Fisher Approach

- Obtain $\mathcal{S}_a(k) \rightarrow$ **basis** for tensor spectrum
- **Detectability of specific theoretical model** $\rightarrow$ 2 approaches:

1. **Fisher approach:** **Projecting** model $\mathcal{P}_{model}$ over PCA modes:

$$\boxed{m_a = \int d\ln k\, \mathcal{S}_a(k)\mathcal{P}_{model}(k)\,.}$$

Uncertainties $\boldsymbol{\sigma_{Fisher}}$ on m_a from Fisher matrix

- **Advantages** $\rightarrow$ fast & easy

PCA modes for Model Testing: Fisher Approach

- Obtain $\mathcal{S}_a(k) \rightarrow$ **basis** for tensor spectrum
- **Detectability of specific theoretical model** $\rightarrow$ 2 approaches:

1. **Fisher approach:** **Projecting** model $\mathcal{P}_{model}$ over PCA modes:

$$\boxed{m_a = \int d\ln k\, \mathcal{S}_a(k)\mathcal{P}_{model}(k)\,.}$$

Uncertainties $\boldsymbol{\sigma_{Fisher}}$ on m_a from Fisher matrix

- **Advantages** $\rightarrow$ fast & easy
- **Caveats** $\rightarrow$ lower bound, insensitive to non-Gaussianity, **insensitive to physicality prior** $P_T > 0$ (linear combinations of power spectrum parameters giving unphysical behaviour, such as $P_T < 0$)

PCA modes for Model Testing: MCMC Approach

2. **MCMC approach** $\rightarrow$ constrain m_a using simulated C_ℓ spectra (or data)

$$\boxed{\mathcal{P}_T(k) = A_s \sum_{a=1}^{N} m_a \mathcal{S}_a(k)}$$

where N is the number of PCA modes we keep

PCA modes for Model Testing: MCMC Approach

2. **MCMC approach** $\rightarrow$ constrain m_a using simulated C_ℓ spectra (or data)

$$\boxed{\mathcal{P}_T(k) = A_s \sum_{a=1}^{N} m_a \mathcal{S}_a(k)}$$

where N is the number of PCA modes we keep

- Parameter space: **Cosmological + Power spectrum parameters**

$$\{m_1, ..., m_n, A_s, n_s, \tau, \Omega_b h^2, \Omega_D h^2, \theta\}$$

PCA modes for Model Testing: MCMC Approach

2. **MCMC approach** $\rightarrow$ constrain m_a using simulated C_ℓ spectra (or data)

$$\boxed{\mathcal{P}_T(k) = A_s \sum_{a=1}^{N} m_a \mathcal{S}_a(k)}$$

where N is the number of PCA modes we keep

- Parameter space: **Cosmological + Power spectrum parameters**

$$\{m_1, ..., m_n, A_s, n_s, \tau, \Omega_b h^2, \Omega_D h^2, \theta\}$$

- **Impose $P_T > 0$ in MCMC**

PCA modes for Model Testing: MCMC Approach

2. **MCMC approach** $\rightarrow$ constrain m_a using simulated C_ℓ spectra (or data)

$$\boxed{\mathcal{P}_T(k) = A_s \sum_{a=1}^{N} m_a \mathcal{S}_a(k)}$$

where N is the number of PCA modes we keep

- Parameter space: **Cosmological + Power spectrum parameters**

$$\{m_1, ..., m_n, A_s, n_s, \tau, \Omega_b h^2, \Omega_D h^2, \theta\}$$

- **Impose $P_T > 0$ in MCMC**
- **Advantages** $\rightarrow$ evaluate **impact of physicality priors** comparing $\boldsymbol{\sigma}_{Fisher}$ and $\boldsymbol{\sigma}_{MCMC}$, **correlations**, **Gaussianity**

PCA modes for Model Testing: MCMC Approach

2. **MCMC approach** $\rightarrow$ constrain m_a using simulated C_ℓ spectra (or data)

$$\boxed{\mathcal{P}_T(k) = A_s \sum_{a=1}^{N} m_a \mathcal{S}_a(k)}$$

where N is the number of PCA modes we keep

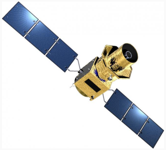
- Parameter space: **Cosmological + Power spectrum parameters**

$$\{m_1, ..., m_n, A_s, n_s, \tau, \Omega_b h^2, \Omega_D h^2, \theta\}$$

- **Impose $P_T > 0$ in MCMC**
- **Advantages** $\rightarrow$ evaluate **impact of physicality priors** comparing $\boldsymbol{\sigma_{Fisher}}$ and $\boldsymbol{\sigma_{MCMC}}$, **correlations**, **Gaussianity**
- **Disadvantages** $\rightarrow$ slow convergence

Future Probes Specifications

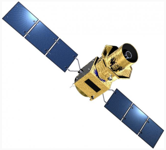
Future Probes Specifications



LiteBIRD

- Satellite
- Timescale 2027
- 15 frequency bands [40-402 GHz]
- Noise [36.1 - 4.7 μK-arcmin]
- Beams FWHM [69.2-9.7 arcmin]
- Sky fraction $f_{sky} = 60\%$
- 20% Delensing
- Multipole range $\ell \sim 2 - 1350$

Future Probes Specifications



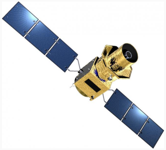
LiteBIRD

- Satellite
- Timescale 2027
- 15 frequency bands [40-402 GHz]
- Noise [36.1 - 4.7 μK-arcmin]
- Beams FWHM [69.2-9.7 arcmin]
- Sky fraction $f_{sky} = 60\%$
- 20% Delensing
- Multipole range $\ell \sim 2 - 1350$

Simons Observatory (SO)

- Ground-based
- Timescale 2022
- 6 frequency bands [27-280 GHz]
- Noise [35.3 - 2.7 μK-arcmin]
- Beams FWHM [91-9 arcmin]
- Sky fraction $f_{sky} = 10\%$
- 50% Delensing
- Multipole range $\ell \sim 30 - 4000$

Future Probes Specifications

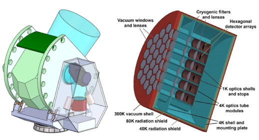


LiteBIRD

- Satellite
- Timescale 2027
- 15 frequency bands [40-402 GHz]
- Noise [36.1 - 4.7 μK-arcmin]
- Beams FWHM [69.2-9.7 arcmin]
- Sky fraction $f_{sky} = 60\%$
- 20% Delensing
- Multipole range $\ell \sim 2 - 1350$

Simons Observatory (SO)

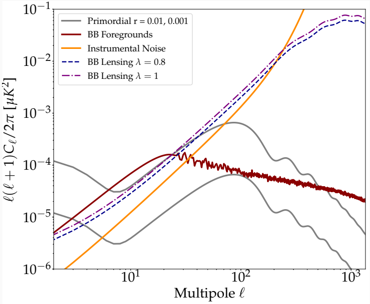
- Ground-based
- Timescale 2022
- 6 frequency bands [27-280 GHz]
- Noise [35.3 - 2.7 μK-arcmin]
- Beams FWHM [91-9 arcmin]
- Sky fraction $f_{sky} = 10\%$
- 50% Delensing
- Multipole range $\ell \sim 30 - 4000$



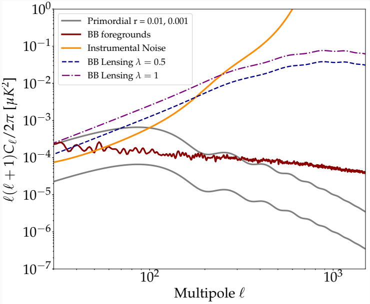
CMB-S4

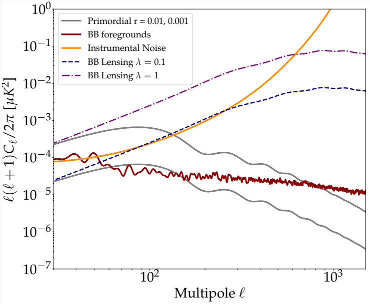
- Ground-based
- Timescale 2027
- 9 frequency bands [20-270 GHz]
- Noise [14 - 1.3 μK-arcmin]
- Beams FWHM [76.6-8.5 arcmin]
- Sky fraction $f_{sky} = 3\%$
- 90% Delensing
- Multipole range $\ell \sim 30 - 4000$

Experiments: LiteBIRD, SO, CMB-S4



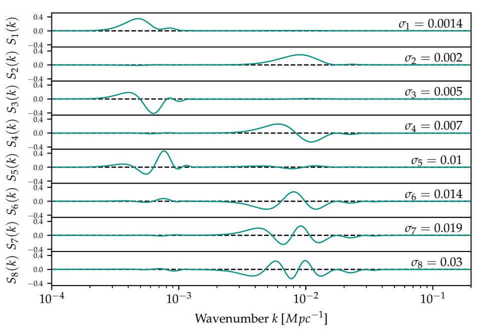
- **LiteBIRD**(top), **SO**(bottom left) and **CMB-S4** (bottom right)
- **For the first time in a PCA of tensor power spectrum include foregrounds residuals and $1/f$ noise!**
- **Complementarity** of experiments!





PCA Basis

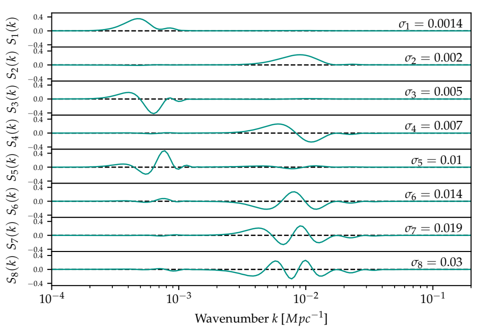
PCA Basis: Problems



Tensor PS need special care with respect to Scalar PS!

- For **scalar spectrum** $\mathcal{P}_\mathcal{R} \to$ PCA describes small deviations around large, well constrained amplitude A_s

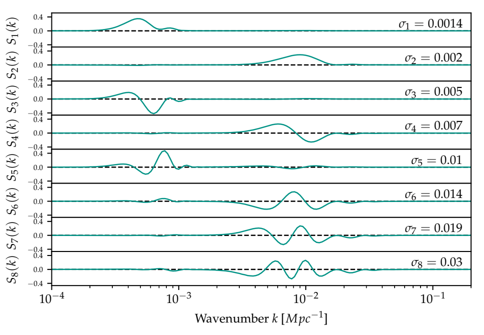
PCA Basis: Problems



Tensor PS need special care with respect to Scalar PS!

- For **scalar spectrum** $\mathcal{P}_\mathcal{R}$ $\rightarrow$ PCA describes small deviations around large, well constrained amplitude A_s
- For tensor spectrum ***r* not yet measured**

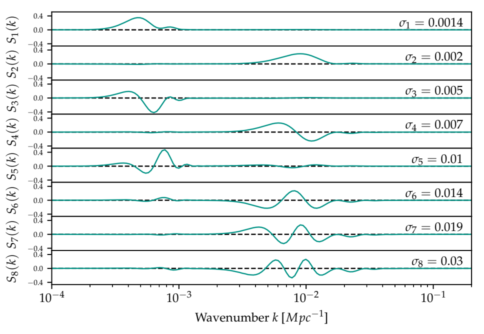
PCA Basis: Problems



Tensor PS need special care with respect to Scalar PS!

- For **scalar spectrum** $\mathcal{P}_{\mathcal{R}}$ $\rightarrow$ PCA describes small deviations around large, well constrained amplitude A_s
- For tensor spectrum ***r* not yet measured**
- Generate our **PCA basis** with $r = 0$ $\rightarrow$ first PCA modes are **effective** r

PCA Basis: Problems



Tensor PS need special care with respect to Scalar PS!

- For **scalar spectrum** $\mathcal{P}_\mathcal{R}$ $\rightarrow$ PCA describes small deviations around large, well constrained amplitude A_s
- For tensor spectrum ***r* not yet measured**
- Generate our **PCA basis** with $r = 0$ $\rightarrow$ first PCA modes are **effective** r
- **BUT Information in C_ℓs with Tensors ($r > 0$) can be very different from Information matrix that defined PCA basis!**

PCA Basis: Our Solution

- Tensors not known a priori but number of modes retained is fixed to N from onset!

PCA Basis: Our Solution

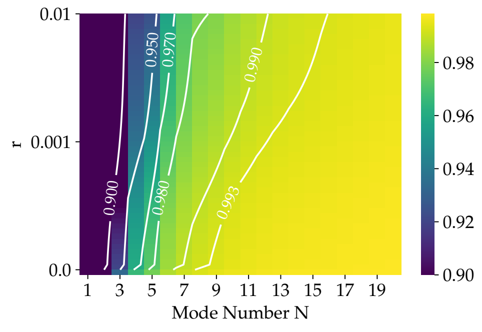
- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r

PCA Basis: Our Solution

- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r
- What fraction of F_r can our basis capture?

PCA Basis: Our Solution

- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r
- What fraction of F_r can our basis capture?

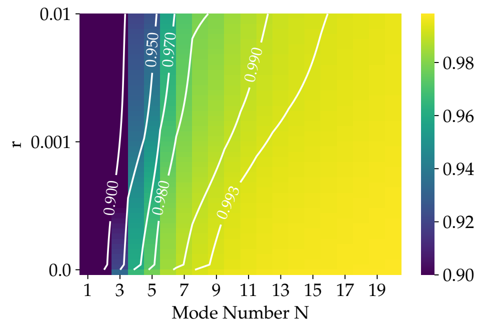


- Study **Information Fraction** for range of r captured by first N modes of our basis:

$$\boxed{I(r,N) = \frac{tr\left(\mathcal{S}_N^T\, F_r\, \mathcal{S}_N\right)}{tr\left(F_r\right)}}$$

PCA Basis: Our Solution

- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r
- What fraction of F_r can our basis capture?



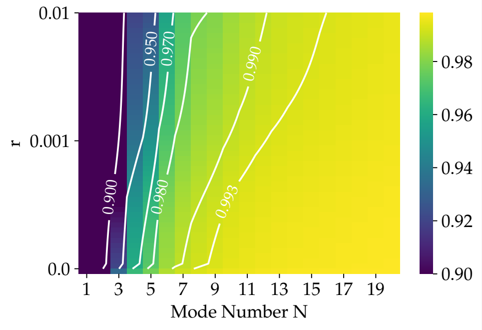
- Study **Information Fraction** for range of r captured by first N modes of our basis:

$$I(r, N) = \frac{tr\left(\mathcal{S}_N^T F_r \mathcal{S}_N\right)}{tr\left(F_r\right)}$$

- **Choose N such that I high enough** $\sim 98\%$

PCA Basis: Our Solution

- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r
- What fraction of F_r can our basis capture?



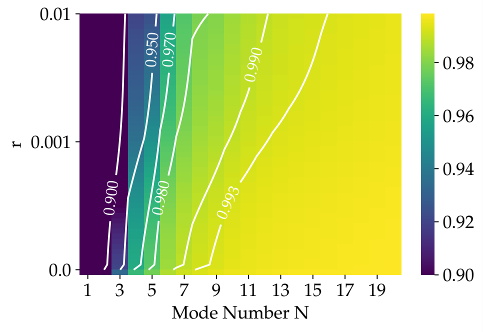
- Study **Information Fraction** for range of r captured by first N modes of our basis:

$$I(r, N) = \frac{tr\left(\mathcal{S}_N^T\, F_r\, \mathcal{S}_N\right)}{tr\left(F_r\right)}$$

- **Choose N such that I high enough** $\sim 98\%$
- **Uncertainties for i-th PCA mode** $\rightarrow \sigma_i^2 = \left(\mathcal{S}_N^T\, F_r\, \mathcal{S}_N\right)^{-1}_{ii}$

PCA Basis: Our Solution

- **Tensors not known a priori but number of modes retained is fixed to N from onset!**
- **Test Stability!** $\rightarrow$ Compute Fisher information F_r for range of possible r
- What fraction of F_r can our basis capture?



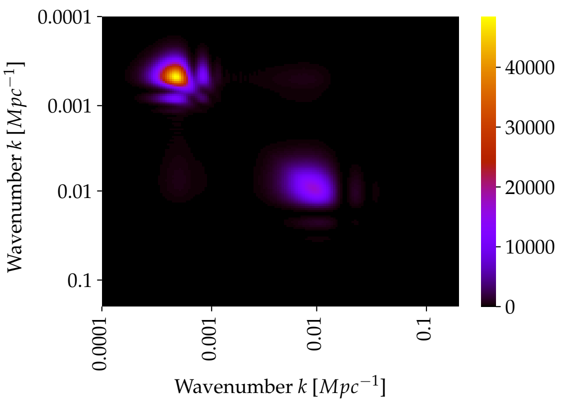
- Study **Information Fraction** for range of r captured by first N modes of our basis:

$$I(r, N) = \frac{tr\left(\mathcal{S}_N^T\, F_r\, \mathcal{S}_N\right)}{tr\left(F_r\right)}$$

- **Choose N such that I high enough** $\sim 98\%$
- **Uncertainties for i-th PCA mode** $\rightarrow \sigma_i^2 = \left(\mathcal{S}_N^T\, F_r\, \mathcal{S}_N\right)^{-1}_{ii}$
- e.g. LiteBIRD $\rightarrow$ set $N = 8$

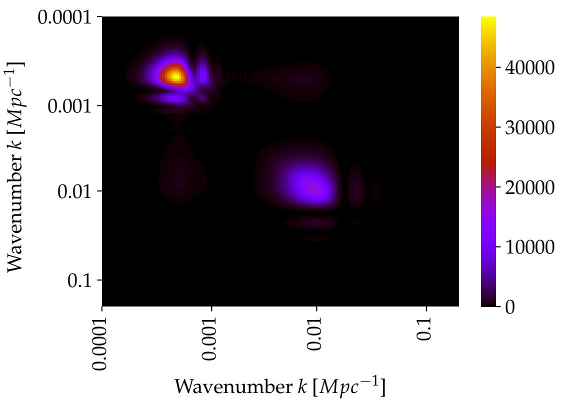
Application to LiteBIRD

Application to LiteBIRD: Fisher Matrix



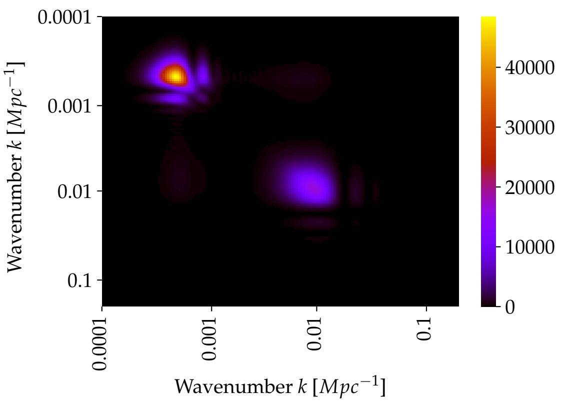
- **Fisher Matrix for LiteBIRD for** $r = 0$

Application to LiteBIRD: Fisher Matrix



- **Fisher Matrix for LiteBIRD for $r = 0$**
- Main features: **recombination bump** ($k \approx 6 \times 10^{-3}\mathrm{Mpc}^{-1}$) and **reionization bump** ($k \approx 6 \times 10^{-4}\mathrm{Mpc}^{-1}$)

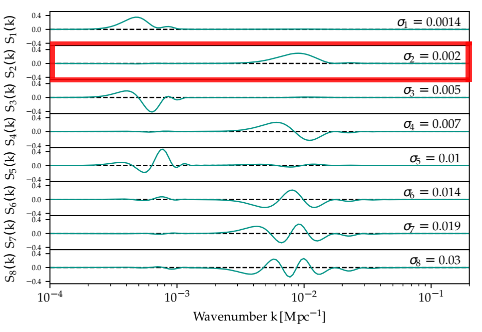
Application to LiteBIRD: Fisher Matrix



- **Fisher Matrix for LiteBIRD for** $r = 0$
- Main features: **recombination bump** ($k \approx 6 \times 10^{-3}\mathrm{Mpc}^{-1}$) and **reionization bump** ($k \approx 6 \times 10^{-4}\mathrm{Mpc}^{-1}$)
- Most information for $r = 0$ comes from reionization peak

Application to LiteBIRD: σs

Experiment	PCA mode	$r=0$	$r=0.001$		$r=0.01$	
			σ_a	S/N	σ_a	S/N
LiteBIRD	1st	0.0014	0.005	1.0	0.03	2.0
	2nd	0.002	0.002	3.0	0.003	16.0
	3rd	0.005	0.01	0.09	0.06	0.14
	4th	0.007	0.007	0.13	0.009	1.02
	5th	0.01	0.019	0.14	0.09	0.3
	6th	0.014	0.014	0.17	0.02	1.2
	7th	0.019	0.019	0.02	0.02	0.2
	8th	0.03	0.03	0.08	0.04	0.6

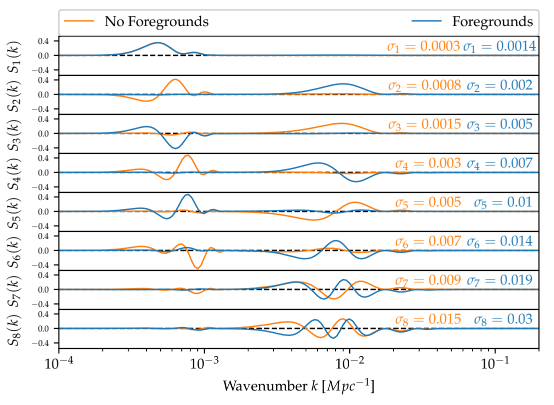


- Modes shift in importance as r changes!
- For $\mathbf{r > 0}$

$$\boxed{\sigma_2 < \sigma_1}$$

$\Rightarrow$ **most information from recombination bump rather than reionization!**

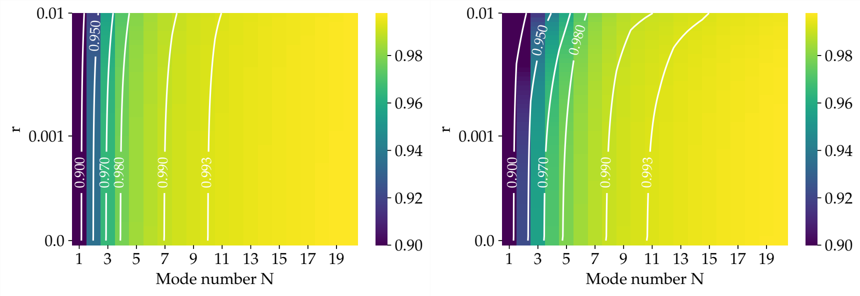
Why Foregrounds are important?



- Factor ~ 4 on σ_r!
- **Factor ~ 5 on σ_1 and ~ 3 on σ_2 due to foregrounds!**
- Adding foregrounds changes relative importance of reionization and recombination peak
- Reionization peak loses importance
- $\Rightarrow$ **Foregrounds cannot be neglected!**

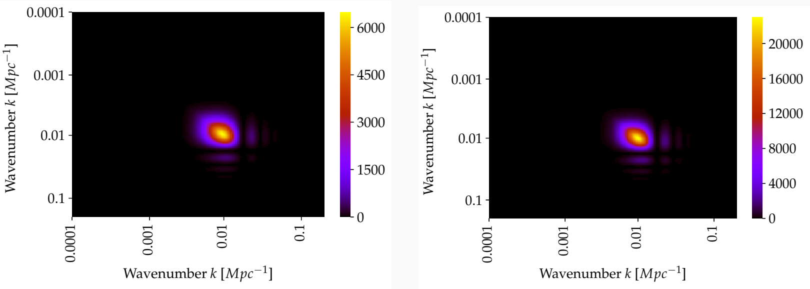
Application to SO and CMB-S4

Application to SO and CMB-S4



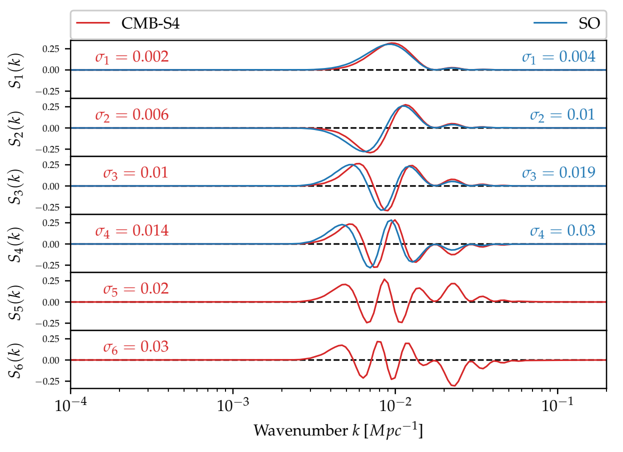
- Study **Information fraction** $I(r, N)$ **also for SO and CMB-S4**
- Weaker dependence on r w.r.t. LiteBIRD $\Rightarrow$ **only need $\mathbf{N = 6}$ for CMB-S4 and $\mathbf{N = 4}$ for SO to capture** 98% **of information**

Application to SO and CMB-S4



- **Fisher Matrices for SO (left) and CMB-S4 (right)** for $r = 0$
- Features in Fisher matrices → **recombination bump**
- **No reionization bump**, but higher resolution and delensing → some information **beyond first acoustic peak!**

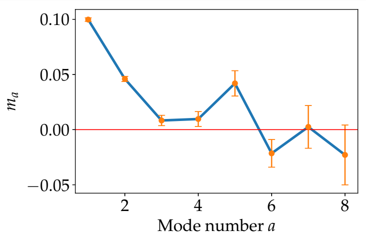
Application to SO and CMB-S4

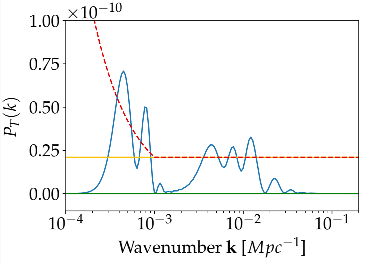


- **PCA modes for SO and CMB-S4**
- Features in PCA modes $\rightarrow$ **recombination bump** and some information beyond
- **No reionization bump!**
- σ_{CMB-S4} are half of σ_{SO}

Study of Early Universe Models

Study of Early Universe Models: Tilted spectrum from inflation





- **Ideal application of PCA! $\rightarrow$ detect deviation w.r.t. fiducial case**
- **Red-Tilted** model similar to open inflation with bubble nucleation
- m_a from MCMC show characteristic trend $\Rightarrow$ **Success for PCA!**
- **Reconstructed tensor power spectrum** (blue)

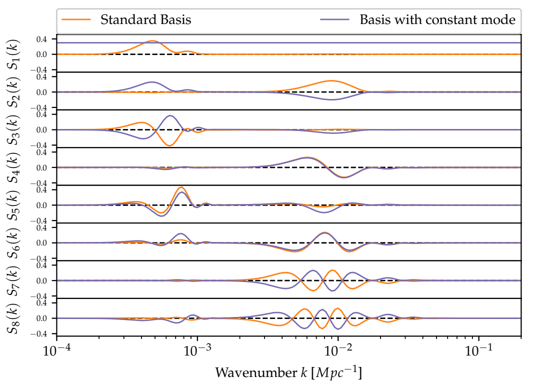
Limitations of the PCA Method and MCMC Exploration

Limitations of PCA

Limitations of PCA

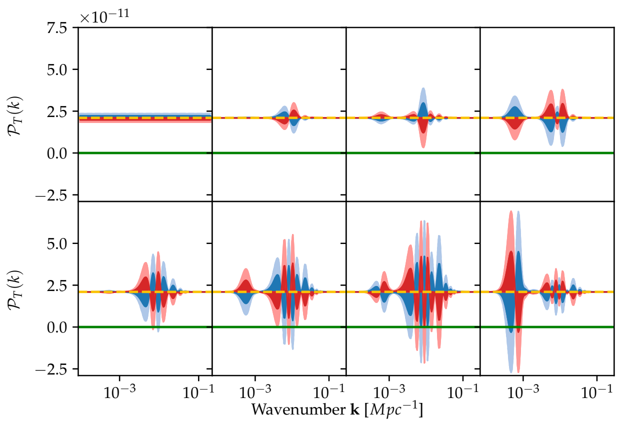
1. **Fisher estimates** for PCA insensitive to **physicality prior** $\mathcal{P}_T > 0$ can be **inconsistent**!
2. Why parametrize $\mathcal{P}_T$ as linear combination of PCA modes $\rightarrow$ **no inclusion of the popular** r?
3. Our basis makes Fisher analysis **more robust against physicality prior**!

Standard PCA Basis vs PCA Basis with Constant Mode



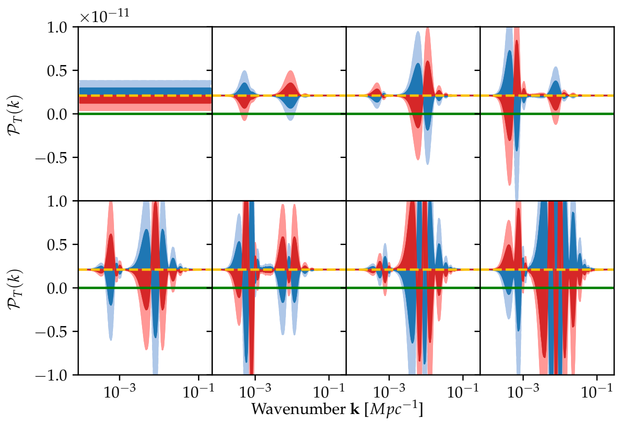
- **Include r in parametrization** $\rightarrow$ **PCA Basis with Constant Mode**
- First constant mode $\Rightarrow m_1 = r$
- Only $\mathcal{S}_1$ can be **positive definite**
- All other modes are **oscillatory**

Physicality Prior: A Visual Argument



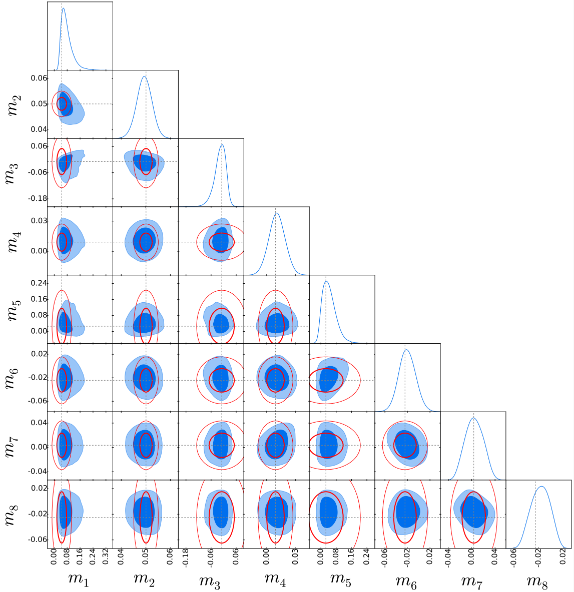
- Assume that σ_{Fisher} **is true**
- **Plot** 2σ **range** in which modes oscillate 95% of the cases
- Some modes **intesect physicality prior! (unphysical)**
- For $r = 0.01$ **only first 3 modes unaffected!**

Physicality Prior: A Visual Argument

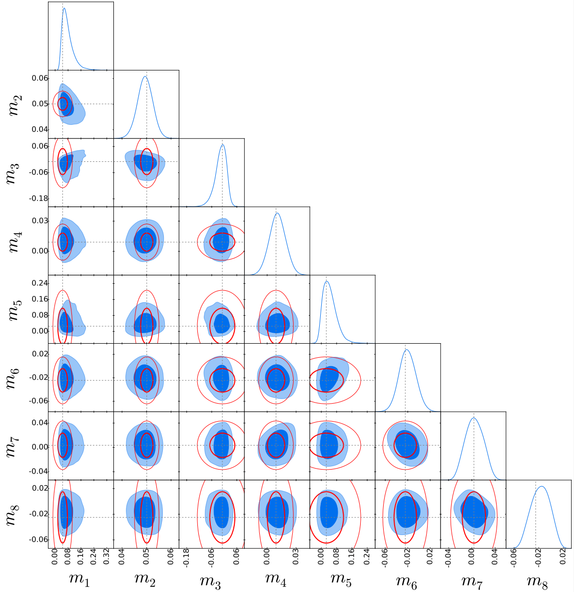


- For $r = 0.001$ **only first mode not affected!**

MCMC Exploration: $r = 0.01$

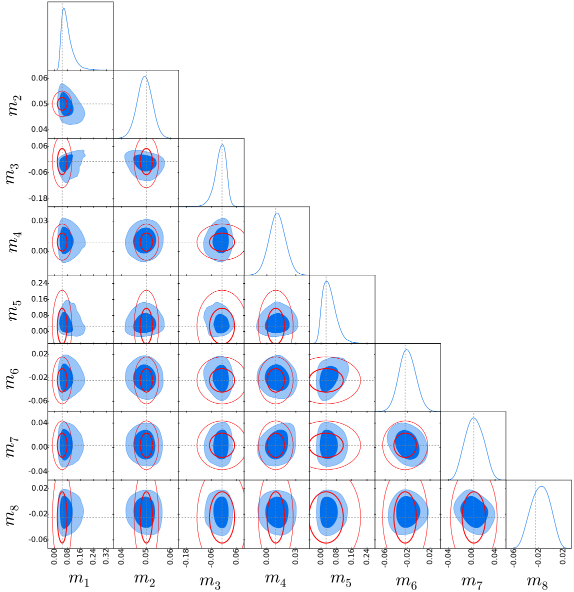


MCMC Exploration: $r = 0.01$



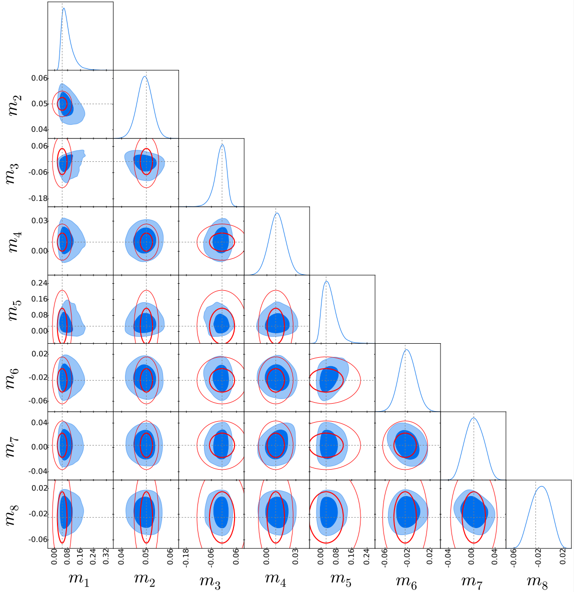
- Must have $\sigma_{\text{MCMC}} \geq \sigma_{\text{Fisher}} \rightarrow$ **true without physicality prior**

MCMC Exploration: $r = 0.01$



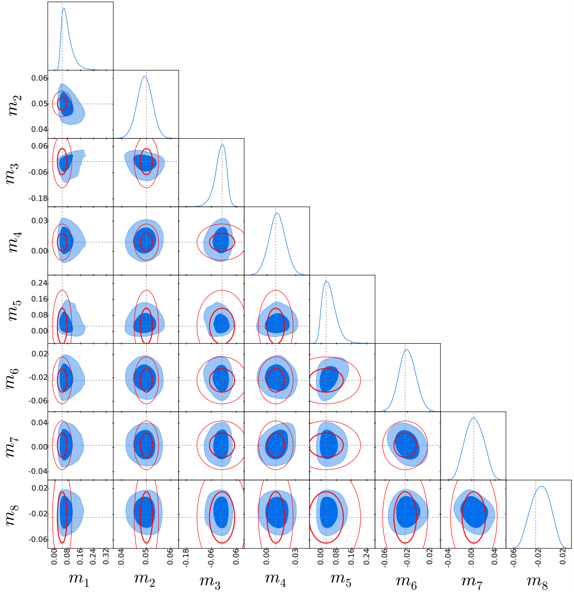
- Must have $\sigma_{\text{MCMC}} \geq \sigma_{\text{Fisher}}$ → **true without physicality prior**
- **With physicality prior** ⇒ $\sigma_{\text{MCMC}} < \sigma_{\text{Fisher}}$ for most modes!

MCMC Exploration: $r = 0.01$



- Must have $\sigma_{\textbf{MCMC}} \geq \sigma_{\textbf{Fisher}}$ $\rightarrow$ **true without physicality prior**
- **With physicality prior** $\Rightarrow \sigma_{\textbf{MCMC}} < \sigma_{\textbf{Fisher}}$ for most modes!
- LiteBIRD $r = 0.001$ our standard basis $\rightarrow$ 2 modes instead of 1 in Constant Mode basis!

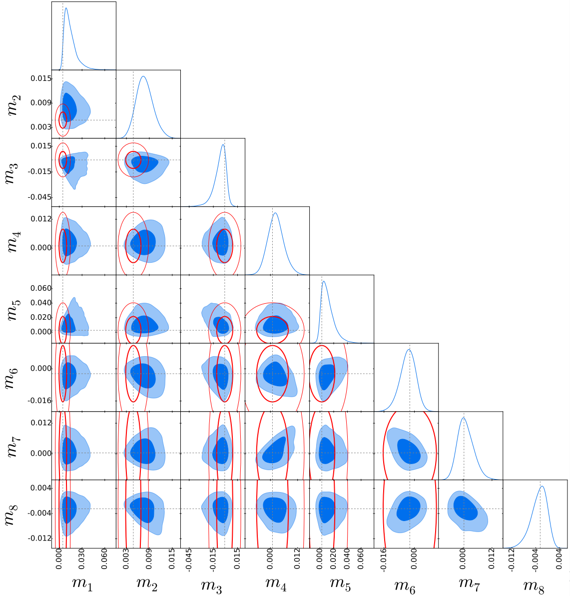
MCMC Exploration: $r = 0.01$



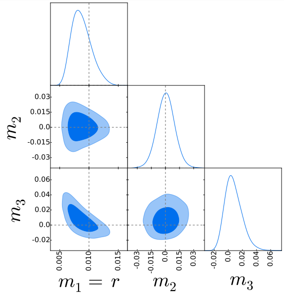
- Must have $\sigma_{\mathbf{MCMC}} \geq \sigma_{\mathbf{Fisher}}$ → **true** **without physicality prior**
- **With physicality prior** $\Rightarrow \sigma_{\mathbf{MCMC}} < \sigma_{\mathbf{Fisher}}$ for most modes!
- LiteBIRD $r = 0.001$ our standard basis → 2 modes instead of 1 in Constant Mode basis!
- **Physicality prior** effect in marginal distributions → **asymmetric, polygonal shapes, very different from Fisher**(red ellipses)

MCMC Exploration: $r = 0.001$

- Physicality prior effect **even more evident** for $r = 0.001$!

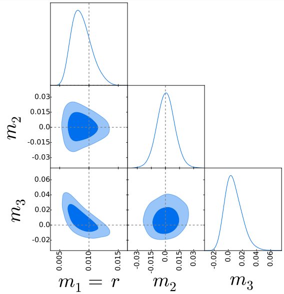


Correlations in the Constant Mode Basis



- **Constant Mode basis** shows $m_1 - m_3$ **correlation**
- **Our Standard basis** $\rightarrow$ **uncorrelated** parameters $\Rightarrow$ preferable!

Correlations in the Constant Mode Basis



- **Constant Mode basis** shows $m_1 - m_3$ **correlation**
- **Our Standard basis** $\rightarrow$ **uncorrelated** parameters $\Rightarrow$ preferable!

CONCLUSION $\rightarrow$ can always use PCA basis to model primordial tensor power spectrum **BUT** Fisher uncertainties rarely accurate! Should be used only for relative comparison!

Conclusions

- Applied **PCA** to **Tensor primordial power spectrum**
- Detect in **B-modes deviations from scale-invariance** in **model-independent** way
- Constraints for **LiteBIRD**, **SO** and **CMB-S4**
- **Foregrounds cannot be neglected!**
- **Our Basis** (**no tensors**) $\rightarrow$ preferable to the **Constant Mode Basis**
- **Fisher uncertainties** can be affected by **Physicality prior**!
- Can be applied to any **Early Universe scenario**

Thanks For Your Attention

Backup Slides

Orthogonalization

- **Degeneracies** between the effect on the C_ℓ due to **cosmological parameters** and the **tensor power spectrum parameters**
- Solution $\Rightarrow$ **orthogonalize** with respect to cosmological parameters the **Fisher matrix**

$$\mathbf{F}_{\mu\nu} = \begin{pmatrix} \mathbf{F}_{ij} & \mathbf{B} \\ \mathbf{B}^{\mathbf{T}} & \mathbf{F}_{ab} \end{pmatrix},$$

$\mathbf{F}_{ab} \rightarrow$ cosmological parameters Fisher matrix
$\mathbf{F}_{ij} \rightarrow$ PCA power spectrum parameters Fisher matrix

Foregrounds: Component Separation

- Foregrounds $\rightarrow$ dominant source of uncertainty on large scales for B-modes
- Two components: **dust** and **synchrotron**
- **Parametric maximum likelihood component separation**: subtract fgs component from data using multi-freq observations **BUT** propagation in uncertainties $\rightarrow$ fgs residuals in the cleaned map (**FGBuster** code reference?)
- On a given sky pixel p

$$\boxed{\mathbf{d}_p = \mathbf{A}\mathbf{s}_p + \mathbf{n}_p}$$

$\mathbf{d}_p \rightarrow$ multi-frequency maps
$\mathbf{s}_p \rightarrow$ multi-component sky signal
$\mathbf{n}_p \rightarrow$ instrumental noise at each frequency

- Mixing matrix $\boldsymbol{A}$ parametrized by spectral parameters β

$$\boxed{\mathbf{A} = \mathbf{A}(\beta) = \mathbf{A}(\beta_s, \beta_d, T_d, C_s)}$$

$$\mathbf{A}_{sync}(\nu, \nu_{ref(s)}) = \left(\frac{\nu}{\nu_{ref(s)}}\right)^{\beta_s + C_s \log(\nu/\nu_0)}$$

$$\mathbf{A}_{dust}(\nu, \nu_{ref(d)}) = \left(\frac{\nu}{\nu_{ref(d)}}\right)^{\beta_d + 1} \frac{e^{\frac{h\nu_{ref(d)}}{kT_d}} - 1}{e^{\frac{h\nu}{kT_d}} - 1}$$

Foregrounds: Component Separation

- **Fisher approach**: maximise the **spectral log-likelihood**:

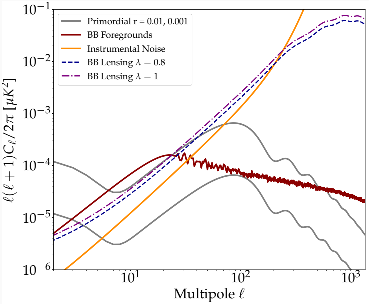
$$\mathbf{\Sigma}^{-1} \simeq -\left\langle \frac{\partial^2 \mathcal{L}}{\partial\beta\partial\beta'} \right\rangle_{noise} \Bigg|_{true\beta}$$

$$= -tr\left\{ \left[\frac{\partial \mathbf{A}^T}{\partial\beta} \mathbf{N}^{-1}\mathbf{A}(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{N}^{-1}\frac{\partial \mathbf{A}}{\partial\beta'} - \frac{\partial \mathbf{A}^T}{\partial\beta}\mathbf{N}^{-1}\frac{\partial \mathbf{A}}{\partial\beta'} \right] \sum_p \mathbf{s}_p\mathbf{s}_p^T \right\}$$

- **Uncertainties** on β spectral parameters:

$$\sigma(\beta) \simeq \sqrt{|\mathbf{\Sigma}|_{\beta\beta}}.$$

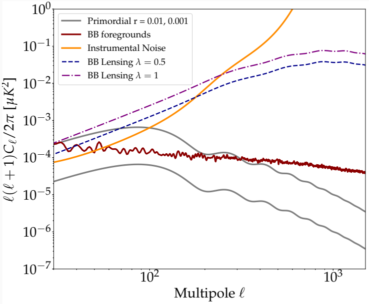
Foregrounds Cleaning: LiteBIRD, SO, CMB-S4

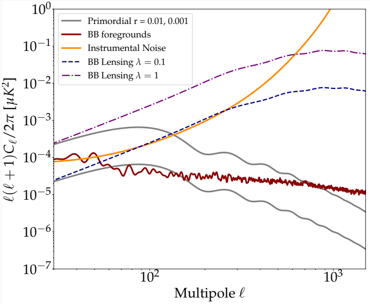


- **Component separation** leaves **residual foregrounds** in the maps (due to propagation of statistical uncertainty in β estimation) $\rightarrow$ **residuals power spectrum**

$$C_\ell^{fgs} \equiv \sum_{k,k'} \sum_{j,j'} \boldsymbol{\Sigma}_{kk'} \alpha_k^{0j} \alpha_{k'}^{0j'} C_\ell^{jj'}$$

- **LiteBIRD**, **SO** and **CMB-S4**





Noise after Component Separation

- **Instrumental Noise** for experiment

$$N_{\ell,\nu}^{XX} = \left[w_{X,\nu}^{-1} \exp\left(\ell(\ell+1) \frac{\theta^2_{FWHM,\nu}}{8 \log 2} \right) \right] \cdot \left[1 + \left(\frac{\ell}{\ell_{knee}} \right)^{\alpha_{knee}} \right]$$

$w_{X,\nu}^{-1/2} \Rightarrow$ **Sensitivity**, white noise level of frequency channel ν
$\theta_{FWHM} \Rightarrow$ **FWHM** of Gaussian beam (radians)

- $\alpha_{knee}, \ell_{knee} \Rightarrow$ **1/f noise**
- **Noise after component separation**:

$$N_\ell^{XX} \equiv \left[\left(\mathbf{A}^T \left(\mathbf{N}_\ell^{XX} \right)^{-1} \mathbf{A} \right)^{-1} \right]_{CMB\ CMB}$$

where $\mathbf{N}_\ell^{XX} \equiv \left(N_\ell^{XX} \right)^{\nu j} \equiv N_{\ell,\nu}^{XX}\, \delta_\nu^j \rightarrow$ matrix containing instrumental noise for each frequency channel

Inflation and CMB Polarization

Inflation and CMB Polarization

- **Inflationary Paradigm**: Vacuum quantum fluctuations of inflaton scalar field produced during inflation⇒ **generation of primordial scalar and tensor perturbations (gravitational waves) of the metric**
- Effect on the CMB:

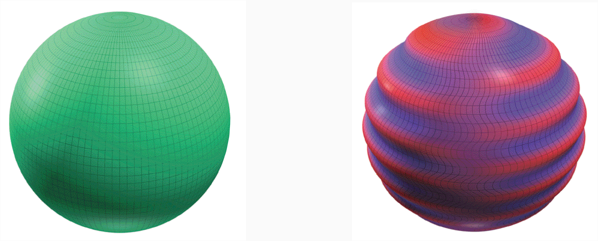
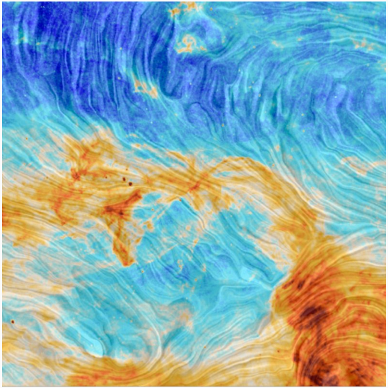


Figure 2: *Kamionkowski & Caldwell 2000*

Inflation and CMB Polarization

- **CMB is polarized!** $\Rightarrow$ CMB polarization is produced by Thomson scattering at recombination
- Polarization state defined by **Stokes parameters** Q **and** U $\Rightarrow$ **Polarization Tensor**
- Can Helmholtz decompose in **Curl** component (**B-modes**) which is divergence-free and **Gradient** component (**E-modes**) which is curl-free



The Quest for Primordial Gravitational Waves

- **E-modes** produced by *both scalar **and** tensor* perturbations
- **Primordial B-modes produced ONLY by tensor perturbations!** **BUT** Big challenge: Tiny signal, Lensing E-modes leaking into B-modes, Foregrounds...
- **IF Detected Primordial Gravitational waves** will give:
  - "Smoking gun" for inflation
  - Identify energy scale of inflation **for the simplest models!** (single scalar field)
- **What about more complex models? Beyond the standard model of Early Universe? Lots of Physics to be understood in this primordial signal!**

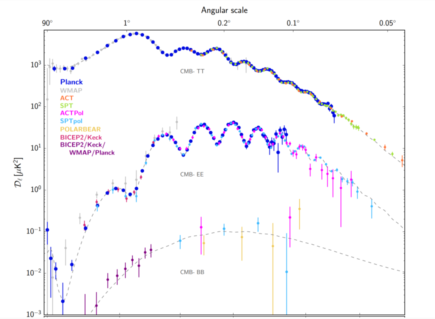


Figure 3: *From Planck results 2018*