Holographic Correlators and the Information Paradox

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References

Based on 1606.01119, 1705.09250, 1710.06820, 1812.06479

with A. Bombini, A. Galliani, E. Moscato, R. Russo, C. Wen

Overview and Motivations

Black holes and holography

Main goal:

study black hole microstates using string theory and holography

- In some cases, a black hole is dual to an ensemble in a 2D CFT (with large c)

$$\text{Black hole} \xleftrightarrow{\text{holography}} \text{CFT}$$

- A b.h. microstate is dual to a "heavy" operator ($\Delta_H \sim c$) and (in some limit) is described by a 10D classical geometry

$$ds^2_H \longleftrightarrow O_H$$

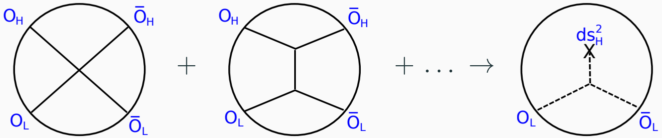
- Microstates can be probed by "light" operators ($\Delta_L \sim O(c^0)$)

$$\langle O_L(z)\bar{O}_L(1)\rangle_{ds^2_H} \longleftrightarrow \langle \bar{O}_H(\infty)O_H(0)O_L(z)\bar{O}_L(1)\rangle$$

HHLL correlators can diagnose information loss vs. unitarity

Holographic correlators

- Holographic correlators of single-trace operators (like O_L) are usually computed by summing Witten diagrams
- This technique has not been extend to correlators with multi-trace operators (like O_H)
- Even for single-trace correlators, Witten diagrams in AdS_3 are subtle: no holographic correlator in a 2D CFT has ever been computed before
- Our approach bypasses Witten diagrams:



- In a certain limit: $\langle \bar{O}_H O_H O_L \bar{O}_L \rangle \to \langle \bar{O}_L O_L O_L \bar{O}_L \rangle$

Correlators and information loss

A version of the information paradox (Maldacena '01)

- A black hole with temperature T_H is dual to a thermal ensemble

$$ds^2_{BH} \quad \longleftrightarrow \quad \rho_\beta = \sum_H e^{-\beta E_H} |H\rangle\langle H| \quad \text{with} \quad \beta = T_H^{-1}$$

- The 2-point function of operators of dimension Δ in a black hole background vanishes at large t

$$\langle O(t)\, \overline{O}(0) \rangle_{\text{b.h.}} \sim e^{-4\pi \Delta T_H\, t}$$

- This follows from the fact that solutions of the wave equation in the black hole background have complex frequencies (quasi-normal modes)
- This is not what one expects for the correlator in the thermal state in a unitary theory with finite entropy

Correlators in unitary theories (Dyson, Lindesay, Susskind; Barbon, Rabinovici)

- In a unitary theory with finite entropy and hence a discrete spectrum

$$\mathcal{C}_\beta(t) \equiv \langle O(t)\,\overline{O}(0)\rangle_\beta = Z_\beta^{-1}\mathrm{Tr}\,\left[e^{-\beta H} O(t)\,\overline{O}(0)\right]$$
$$= Z_\beta^{-1}\sum_{ij} e^{-\beta E_i}|\langle i|O(0)|j\rangle|^2 e^{i(E_i-E_j)t}$$

- The long-time average of the correlator is

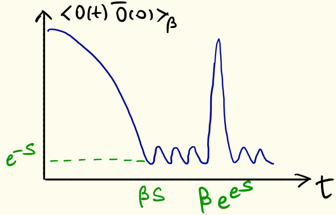
$$\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\,|\mathcal{C}_\beta(t)|^2 \sim \frac{Z_{2\beta}}{Z_\beta^2} \sim e^{-S}$$

- Hence $\mathcal{C}_\beta(t)$ cannot be exponentially vanishing at late times

Qualitative behaviour of $\mathcal{C}_\beta(t)$

$$\mathcal{C}_\beta(t) = Z_\beta^{-1} \sum_{ij} e^{-\beta E_i} |\langle i|O(0)|j\rangle|^2 e^{i(E_i - E_j)t}$$

- For $t \ll \langle E_i - E_j \rangle^{-1}$ the spectrum can be approximated as continuous and $\mathcal{C}_\beta \sim e^{-t/\beta}$
- For $t \sim \beta S$ the correlator is of the order of its long-time average e^{-S}: it oscillates irregularly and no longer decreases
- For $t \sim \beta e^{e^S}$ most of the phases are again of order 1 and hence $\mathcal{C}_\beta \sim O(1)$



The D1-D5 system

A microscopic BH model: D1-D5-P (Strominger, Vafa)

- The simplest BPS black hole with a finite-area horizon is

$$\text{D1-D5-P on } \mathbb{R}^{4,1} \times S^1 \times T^4$$

- We take $\mathrm{vol}(T^4) \sim \ell_s^4$ and $R(S^1) \gg \ell_s \Rightarrow$ 2D CFT
- The b.h. has a "near-horizon" limit $\Rightarrow \mathrm{AdS}_3 \times S^3 \times T^4$
- We take $G_N \to 0$ with R_{AdS} fixed $\Rightarrow c = 6 n_1 n_5 \equiv 6N \to \infty$
- The CFT has a 20-dim moduli space:
  - $g_s N \to 0$: free orbifold point $\longleftrightarrow$ $R_{AdS} \ll \ell_s$
  - $g_s N \gg 1$: strong coupling point $\longleftrightarrow$ $R_{AdS} \gg \ell_s$

Goal: understand b.h. microstates at the strong coupling point

The D1-D5 CFT

- Symmetries:
  (4,4) SUSY with $SU(2)_L \times SU(2)_R$ R-symmetry $\longleftrightarrow$ S^3 rotations
- The orbifold point: sigma-model on $(T^4)^N/S_N$
  The elementary fields are 4 bosons, 4 fermions and twist fields
- Chiral primary operators: $O_{(j,\bar{j})}$ with $h = j$, $\bar{h} = \bar{j}$ (and their descendants with respect to L_{-n}, J^-_{-n}, $G^-_{-n-1/2}$) are protected
- Spectral flow:
  - NS $\longrightarrow$ R
  - $j \longrightarrow j + \frac{N}{2}$, $h \longrightarrow h + j + \frac{N}{4}$
  - (anti)CPO $\longrightarrow$ RR ground states with $h = \bar{h} = \frac{N}{4}$

2-charge microstates

- States carrying D1-D5 charges are RR ground states ($h = \bar{h} = \frac{N}{4}$)
  Note: $h, \bar{h} \sim c \;\Rightarrow$ "heavy" states $\Rightarrow$ classical geometry
- A simple example:

$$
\begin{array}{ccc}
|0\rangle_{\mathrm{NS}} & \xrightarrow{\text{spectral flow}} & |N/2, N/2\rangle_R \\
\downarrow & \downarrow & \downarrow \\
\mathrm{AdS}_3 \times S^3 & \phi \to \phi - \tau \,,\; \psi \to \psi - \sigma & \mathrm{AdS}_3 \times' S^3
\end{array}
$$

  with (ϕ, ψ) S^3 coordinates and (τ, σ) AdS_3 coordinates

- The geometry dual to the maximally rotating RR ground state $|N/2, N/2\rangle_R$ is $\mathrm{AdS}_3 \times' S^3$ with S^3 non-trivially fibered over AdS_3

More generic 2-charge states

(Lunin, Mathur)

- Given a CPO O_k of dimension k one can form the "coherent state"

$$|O_k\rangle_{\rm NS} \equiv \sum_p B^p O_k^p\, |0\rangle_{\rm NS}$$

- In the large N limit the p-sum is peaked for $p \approx B^2/k$

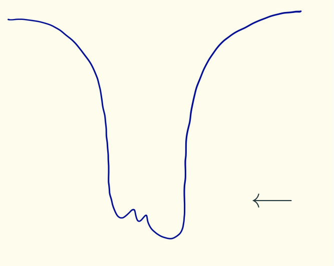
What is the gravitational description of $|O_k\rangle_{\rm NS}$?

- Holography associates to O_k a sugra field ϕ_k : $O_k \longleftrightarrow \phi_k$
- At linear order in B $|O_k\rangle_{\rm NS}$ is a small perturbation of the vacuum

$$|0\rangle_{\rm NS} + B\, O_k\, |0\rangle_{\rm NS} \longleftrightarrow {\rm AdS}_3 \times S^3 + B\, \phi_k$$

where $\phi_k = \left(\frac{1}{\sqrt{\rho^2+1}}\right)^k \sin^k\theta\, e^{ik(\phi+\tau)}$ solves the linearised sugra eqs. around $\mathrm{AdS}_3 \times S^3$

- One can extend the linearised solution to an exact solution of the sugra eqs. valid for $B^2 \sim N$
- The solution is smooth and horizonless
- The solution is asymptotically $\mathrm{AdS}_3 \times S^3$ but in the interior AdS_3 and S^3 are non-trivially mixed
- The R-sector solution can be extended to an asymptotically flat geometry



$\mathbb{R}^{4,1} \times S^1$

$\mathrm{AdS}_3 \times S^3$

$r \sim R_{\mathrm{Hor}}$ no horizon!

A small black hole

- The statistical ensemble of BPS D1-D5 states (with $P = 0$) is described by the "massless BTZ" geometry

$$\frac{ds^2}{R^2_{\mathrm{AdS}}} = \frac{d\rho^2}{\rho^2} + \rho^2(-d\tau^2 + d\sigma^2) + d\Omega_3^2$$

- It is a singular geometry with $A_{\mathrm{Hor}} = 0$
- Correlators in this geometry still display information loss

$$\langle O(\tau)\bar{O}(0)\rangle_{\mathrm{BTZ}_0} \sim \tau^{-\Delta}$$

Holographic correlators

HHLL correlators

- We want to compute

$$\mathcal{C}_H(z,\bar{z}) \equiv \langle \bar{O}_H(\infty) O_H(0) O_L(z,\bar{z}) \bar{O}_L(1) \rangle$$

with

- $O_H \xrightarrow{\text{spectral flow}} \sum_p B^p O_k^p \xleftrightarrow{\text{holography}} ds_H^2$
- $O_L = O_{k'} \xleftrightarrow{\text{holography}} \phi_{k'}$

We will take $k = k' = 1$ in the following

How to compute correlators in holography

- $O_L(z,\bar{z}) \longleftrightarrow \phi(\rho; z,\bar{z})$
- Solve the linearised e.o.m. for ϕ in the background $ds^2_H \longleftrightarrow O_H$
- Pick the non-normalisable solution such that
  - at the boundary ($\rho \to \infty$)

$$\phi(\rho; z,\bar{z}) \xrightarrow{\rho\to\infty} \underbrace{\delta(z-1)}_{\text{source for } \bar{O}_L(1)}\, \rho^{\Delta-d} + \overbrace{b(z,\bar{z})}^{\text{vev of } O_L(z,\bar{z})}\, \rho^{-\Delta}$$

  - in the interior ($\rho \to 0$) $\phi(\rho; z,\bar{z})$ is regular
- The correlator is given by

$$\mathcal{C}_H(z,\bar{z}) = \langle O_H | O_L(z,\bar{z}) \bar{O}_L(1) | O_H \rangle = b(z,\bar{z})$$

A technical remark

- The e.o.m. for $\phi \longleftrightarrow O_L$ is complicated
- It is simpler to compute

$$\widetilde{\mathcal{C}}_H(z,\bar{z}) \equiv \langle \bar{O}_H(\infty) O_H(0) \widetilde{O}_L(z,\bar{z}) \overline{\widetilde{O}}_L(1) \rangle$$

with

$$\widetilde{O}_L \equiv G\bar{G}O_L \xleftrightarrow{\text{holography}} \tilde{\phi}\ \ \underline{\text{minimally coupled scalar in 6D}}$$

- Since $GO_H = 0$, $\mathcal{C}_H$ and $\widetilde{\mathcal{C}}_H$ are related by the Ward identity

$$\widetilde{\mathcal{C}}_H(z,\bar{z}) = \partial\bar{\partial}\left[|z|\,\mathcal{C}_H(z,\bar{z})\right]$$

- The WI is a non-trivial check on the gravity computation when both $\mathcal{C}_H$ and $\widetilde{\mathcal{C}}_H$ can be computed

Results

The exact HHLL correlator

Gravity

$$\mathcal{C}_H = \alpha\, e^{-i\tau} \sum_{l\in\mathbb{Z}} e^{il\sigma} \sum_{n=1}^{\infty} \frac{\exp\left[-i\alpha\sqrt{(|l|+2n)^2 + \frac{(1-\alpha^2)l^2}{\alpha^2}}\tau\right]}{\sqrt{1+\frac{1-\alpha^2}{\alpha^2}\frac{l^2}{(|l|+2n)^2}}} + N(1-\alpha^2)e^{-i\tau}$$

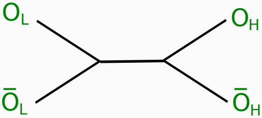
with $z = e^{i(\tau+\sigma)}, \bar{z} = e^{i(\tau-\sigma)}, \ \alpha = \left(1 - \frac{B^2}{N}\right)^{1/2}$

Free CFT

$$\mathcal{C}_H = \frac{1}{|z||1-z|^2} + \frac{B^2}{2N}\frac{|z|^2 + |1-z|^2 - 1}{|z||1-z|^2} + \frac{(N-B^2)B^2}{N}\left(1 - \frac{1}{N}\right)\frac{1}{|z|}$$

The OPE interpretation

- The 4-point function can be reconstructed from the $z \to 1$ OPE



$$O_L(z)\bar{O}_L(1) \sim |1-z|^{-2}(1+(1-z)J+(1-z)^2 T+\ldots)$$
$$+\sum_{n,\ell}(1-z)^{n+\ell}(1-\bar{z})^n :O_L\partial^{n+\ell}\bar{\partial}^n\bar{O}_L:$$
$$+ \text{ non-BPS single-trace operators}$$

- The first line gives the affine identity block, which dominates in the light-cone limit ($\bar{z} \to 1$)
- In the second line are non-BPS double-trace operators with

$$h = 1+n+\ell+\frac{\gamma_{n\ell}}{N} \quad , \quad \bar{h} = 1+n+\frac{\gamma_{n\ell}}{N}$$

- The operators in the third line are dual to string modes and have $h, \bar{h} \to \infty$ in the sugra limit

The late-time behaviour

The late time behaviour of the HHLL correlator

- The geometry of O_H depends on the parameter B
  - $B^2 \to 0$: the geometry is a small perturbation of $AdS_3 \times S^3$
  - $B^2 \to N$ (or $\alpha \to 0$): the geometry approximates the "small b.h."
- We focus on the $\alpha \to 0$ limit
- In this limit the series giving $\mathcal{C}_H$ is dominated by terms with $n \gg \frac{|l|}{2\alpha}$:

$$\mathcal{C}_H \sim e^{-i\tau} \left[\frac{1}{1 - e^{i(\sigma-\tau)}} + \frac{1}{1 - e^{-i(\sigma+\tau)}} - 1 \right] \frac{\alpha}{1 - e^{-2i\alpha\,\tau}}$$

- The time-dependence of the correlator is controlled by α:
  - for $\tau \ll \alpha^{-1}$ one recovers the BTZ behaviour $\mathcal{C}_H \sim \tau^{-1}$
  - for $\tau \gtrsim \alpha^{-1}$ $\mathcal{C}_H$ stops decreasing with τ and oscillates

The late-time behaviour of the correlator is consistent with unitarity

LLLL limit

The small B limit

- When $B^2 \ll N$ O_H, spectrally flowed to the NS sector, is light

$$O_H \xrightarrow{\text{spectral flow}} \sum_p B^p O^p \longrightarrow O \equiv O_L \quad \text{for} \quad B^2 = 1$$

- Naively one expects $\langle \bar{O}_H O_H O_L \bar{O}_L \rangle \to \langle \bar{O}_L O_L O_L \bar{O}_L \rangle$ for $B^2 = 1$
- This is not correct! There is an order of limit problem:
  - HHLL: take $N \to \infty$ with B^2/N fixed and then $B^2/N \to 0$
  - LLLL: take $B^2 = 1$ first and then $N \to \infty$
- But it works for $z \to 1$, more precisely

the $B^2 \to 0$ limit of the HHLL correlator correctly captures all the single-trace operators exchanged between O_L and $\bar{O}_L$

Reconstructing the LLLL correlator

One can uniquely reconstruct $\mathcal{C}_L \equiv \langle \bar{O}_L(z_1) O_L(z_2) O_L(z_3) \bar{O}_L(z_4) \rangle$ from

- for $z_1 \to z_2$, $\mathcal{C}_L = \lim_{B^2 \to 1} \mathcal{C}_H$
- $\mathcal{C}_L$ is symmetric under $z_2 \leftrightarrow z_3$ exchange
- $\mathcal{C}_L$ is consistent with the flat space limit ($R_{\mathrm{AdS}} \to \infty$)
- the operator with the lowest dimension exchanged for $z_2 \to z_3$ is protected

One finds

$$\mathcal{C}_L = \left(1 - \frac{1}{N}\right)(1 + |1 - z|^{-2}) + \frac{2}{\pi N}|z|^2(\hat{D}_{1122} + \hat{D}_{1212} + \hat{D}_{2112})$$

where $\hat{D}_{i_1 i_2 i_3 i_4}$ is the Witten contact digram with operators of dimension $i_1, \ldots, i_4$

(The generalisation to generic CPOs is under construction)

Summary and outlook

Summary

- Black hole mircostates can be identified with “heavy” states of a dual CFT
- At strong coupling heavy states are described by smooth horizonless geometries
- HHLL correlators can be extracted from these geometries and can be used to probe unitarity
- If probed for a short time microstates are indistinguishable from the black hole
- For sufficiently long times microstates deviate from the black hole and produce correlators that are consistent with unitarity already at large c

Outlook

- Classical supergravity works well for atypical states in the black hole ensemble
- Deviations from a typical state and the classical black hole should be exponentially suppressed in the entropy
- How much of our analysis can be extended to typical states?
- And what about microstates of black holes with non-degenerate horizons (3-charge, non-BPS)?
- It is possible that classical supergravity probes cannot resolve the structure of typical states
- Does one need to resort to full string theory?