EPOS results and future developments

Sophys Gabriel ATER in Theory Group at Subatech Second LHCb Heavy Ion Workshop

with K.Werner, M.Jafarpour, J.Jahan, M.Stefaniak and G.Pokropska





EPOS story: EPOS 3 ≠ EPOS-LHC

B. Guiot, I. Karpenko, S. Ostapchenko, T. Pierog, S. Porteboeuf, K. Werner

J. Jahan, G. Prokropska, G. Sophys, M. Stefaniak

EPOS 1.99 (public 2009)

- □ Effective flow, parametrized
 - Tuned to fit data from SPS, RHIC, Tevatron

EPOS LHC (public 2012)

- □ Effective flow
- □ Tuned to fit pp and pA data up to early LHC data (→ cosmic rays)

EPOS 2.xx (semi public)

□ True 3D+1 ideal hydro+ hadronic cascade (→collective effects)

EPOS 3.xx (to be public in 202x ?)

- □ True 3D+1 viscous hydro (slow) **OR** (fast) effective flow treatment, new implementation of saturation (HM pp, pA and AA)
- All data from LHC run 1 and others...

EPOS philosophy

We try to understand all aspects of hadronic interactions (hadron-hadron, hadron-nucleus, nucleus-nucleus).

The starting point is the Regge Theory:

- Study of the S matrix
- □ Calculation of elastic cross section

The theory inside EPOS is the Parton-Based-Gribov-Regge-Theory:

- Combination of Parton Model and Gribov Regge Theory with energy sharing
- □ Hard/Soft processes, elastic/inelastic cross section
- Energy conservation by multiple Pomeron exchange
- Particle production

How do we construct an event ?

Same procedure applies, based on several stages:

Initial Conditions

- Core-Corona Approach
- Viscous hydrodynamic expansion
- Statistical hadronization
- Final state hadronic cascade

EPOS initial state

Conceptually very different compared to other models (Pythia, Herwig...)

Heavily based on S-matrix theory (S related to T via $S_{if} = \delta_{fi} + i(2\pi)^4 \delta(p_f - p_i)T_{fi}$)

- Lorentz invariance
- Unitarity
- $\Box Analyticity (\Rightarrow crossing symmetry)$

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Asymptotic form of T:

 $T(s,t) = \beta(t)s^{\alpha(t)} \approx \beta(t)s^{\alpha(0) + \alpha'(0)t}$

After Fourier transform:

- □ soft contribution: $T_{soft} = \beta(b)(x^+x^-s_{pp})^{\alpha(b)}$
- **QCD** contribution: $T_{ladder} = \sum \beta(b) (x^+ x^- s_{pp})^{\alpha(b)}$

Main theory in EPOS

Parton-Based-Gribov-Regge-Theory (PBGRT)



Initial Conditions
 Core-Corona Approach

- Interaction between partons = **Pomeron**: treated by Quantum Field Theory
- Energy conserved by partonic participants and remnants



H. J. Drescher, M. Hladik, S. Ostapchenko, T. Pierog and K. Werner, Phys. Rept. **350**, 93 (2001)

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Core-Corona Evolution



One string for one scattering

Few scatterings \rightarrow we *can* treat **independently** each string



GDRE2012, Nantes, Jul 2012, Klaus WERNER, Subatech, Nantes

More scatterings \Rightarrow

Core-Corona Evolution



One Lund string for one scattering

A lot of scatterings \rightarrow we **cannot** treat *independently* each string

We can observe different string densities?



B. Guiot and K. Werner, J. Phys. Conf. Ser. 589 (2015) no.1

Sophys Gabriel

Core-Corona Evolution

Initial Conditions
 Core-Corona Approach

High density: we use hydrodynamics → the Core is treated as a fluid. Low density: we do nothing → Corona becomes hadrons !



B. Guiot and K. Werner, J. Phys. Conf. Ser. 589 (2015) no.1

In high multiplicity pp, we can create some **fluid** with this procedure.

Result

Unified Approach



Result

Current activities

Current activities

- Statistical hadronization, results pp, PbPb K. Werner werner@subatech.in2p3.fr
- Consistent implementation of hard processes (crucial for Heavy Flavor)

K. Werner werner@subatech.in2p3.fr

- Equation of state for BES energies M. stefaniak stefaniak@subatech.in2p3.fr
- Low Energies (BES, later NICA) G. Sophys: sophys@subatech.in2p3.fr
- EPOS and Rivet (analyzing STAR data)
 J. Jahan: jahan@subatech.in2p3.fr & G. Pokropska: g.pokropska@gmail.com
- EPOS + PHSD

E. Bratkovskaya: brat@fias.uni-frankfurt.de & M. Jafarpour: jafarpou@subatech.in2p3.fr

Toward the release of EPOS

Proper language: EPOS running

How to use EPOS?

- **()** just event simulation, output as "events" into root files: for users
- **2** online analysis, during simulation, with event storing: for developers
- South

For developers: a proper language

EPOS input works like a shell, with command lines. Thus we can "predefine" what kind of histogram we want in the end, as EPOS shell script, with very simple commands.

Result

Low Energies My Ph.D ©

Low Energies

My Ph.D ©

Proper language

beginhist	0
histogram	
pt	!variable name (x-axis)
numpt	l !what is counted (yaxis)
12	!normalisation
0 20	!range for x variable
100	!number of bins
idcode	120 idcode -120 !define particle species
endhisto	

Generalization for much complex observables

A large number of analysis are done to create a new database with EPOS analysis especially concerning the collectivity observables.

Framework independent of further EPOS's developments \rightarrow can test a newest version directly, nothing needs to be done, we just submit the jobs and get the new plots.

Result

EPOS + Rivet

What is Rivet?

EPOS + Rivet

What is Rivet?

Robust Independant Validation of Experiment and Theory: https://rivet.hepforge.org/

Purpose: offer a simple and standardised independent tool to compare simulations from Monte-Carlo generators to experimental data



EPOS + Rivet

What is Rivet?

Software based on C++ libraries, installed with different packages:

- YODA: Python libraries and classes used to manage plotting
- HepMC: simulations recording and reading for analyses
- Fastjet: recombination algorithms, mainly used for jet analyses

Contains many analyses based on publications from many different experiments, with experimental results included.

Develops thanks to users community contributions

Advantage: provides huge and constantly growing library of data and analyses



"Rivet users & developpers workshop", Andy Buckley (27/05/2019)

EPOS + Rivet

Why Rivet?

- Easy and standardised way to analyse simulated heavy-ion collisions.
- Automatisation of the comparison process between results from EPOS (or event generators more generally) and experimental data.
- Access to huge library of experimental data to compare with.
- □ It is planned to make new public release of EPOS usable with Rivet, to facilitate user's work.



J. Jahan & G. Prokropska

Result

EPOS + PHSD

What is PHSD?

EPOS + PHSD?

Questions? Ask P. Moreau or M. Jafarpour

- Microscopic covariant transport model that combines effective partonic and hadronic degrees of freedom and dynamical description of the hadronization process.
- □ The theoretical foundation of PHSD is consisting of the DQPM and equation of motion based on Kadanoff-Baym equations.
- DQPM: dynamics in partonic phase
- □ HSD approach: hadronic part
- □ Kadanoff-Baym equations: describing the dynamics of the strongly interacting systems far out-of equilibrium
- □ The particle production is based on Lund string model EPOS strings can be a new initial condition for PHSD!

Result

EPOS + PHSD

Combining of EPOS and PHSD Models



Preliminary Results





We did nothing outside put EPOS at initial state of PHSD and these first results seems extraordinary good!

EPOS Low Energies

Preliminaries

40-80 %

10-40 %

0-10%

0-80 9

EPOS Low Energies

Preliminaries



Calculated using Eta-sub method





- We have a separation of Baryons and Mesons for all energies
- No energy dependence observed.

L. Adamczyk et al. Phys. Rev., C93(1):014907,2016.

$$m_T - m_0 = \sqrt{p_T^2 + m_0^2} - m_0$$

\$\approx 100K events

Au-Au collisions

~ 1001 CV

Separation Baryon-Meson

Calculated using Eta-sub method





• Energy dependence observed!

GeV



Differential flow versus p_T

Calculated using Q-Cumulant method



L. Adamczyk et al. Phys. Rev. C. 86, 054908 (2012).

$$p_T = \sqrt{p_x^2 + p_y^2}$$

 ≈ 100 K events

Au-Au collisions

- Jet-matter interaction not include
 Good reproduction for all energies since p_T < 2 GeV
- No energy dependence observed.

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Before 2019 for EPOS



Early 2020

Inverse Wishlist



Outlook about the work in 2018-2019

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- \Box Large database in EPOS at low energies to check all new updates easily \checkmark
- \Box First merging of EPOS with RIVET to have a fast comparison data/theory \checkmark
- □ First merging of EPOS with PHSD to test another approach of model using EPOS at initial condition ✓
- □ Work with D. Fuseau to have a kind of EPOS@PNJL with EPOS at initial conditions for PNJL model
- U Work about microcanonical equation to have consistent pp collisions
- Consistent implementation of hard processes for Heavy Flavor
- □ From me, I working about the nuclear modification factor and propose other factors to distinguish nuclear and QGP effects:

 $r_{AA} \propto \frac{\text{Yield in A} + A_{\text{central}}}{\text{Yield in A} + A_{\text{periph}}}, R'_{AA} \propto \frac{\text{Yield in A} + A}{\text{Yield in pp}_{\text{high mult}}} \text{ and } R_{\text{pp}} = \frac{\text{Yield in pp}_{\text{high mult}}}{\text{Yield in pp}_{\text{min bias}}}$

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Thank for your attention!

How can we use Rivet?

Each analysis composed of 4 files :

#include "Rivet/Tools/Percentile.hh"
#include <complex>
#include <iostream>
#include <string>

namespace Rivet {

"ANALYSIS_NAME.cc" (analysis code)

BEGIN PLOT /STAR_2017_PRC68_044094/d2a+x04-y01 Title=p/Q1*2 (Au+a) 7.5 @vi/s Title=p/Q1*2 (Au+a) 7.5 @vi/s Title=p/Q1*2 (Au+a) 7.5 @vi/s Wika=0.6 Wika=0.6 Wika=0.6 Wika=0.6 Wika=0.6 Wika=0.6 Defines 2 Les Buy defineal plot settings you might like, see make-plots documentation Euro PLOT

"ANALYSIS_NAME.plot" (plotting options)

Name: STAR_2012_PRC50_044094 Year: 2017 Summary: Bulk Properties of the medium produced in Relativistic Heavy-Ion Collisic Experiment: STAR Inspire[101: 31:053 Status: UMALIDATED Authors: - Johannes Jahan cjohannes.jahan@etu.univ-nantes.fr> - Gabricala @AkTopska.go.pokrogska@gmall.com - Marias Bifamida karaia.stafamida@kubakteb.logJp.fr>

"ANALYSIS_NAME.info" (information about paper, beam, author...)

```
BEGIN YODA SCATTER2D V2 /REF/STAR 2017 PRC96 044904/d20-x04-y01
Variations: [**]
IsRef: 1
Path: /REF/STAR 2017 PRC96 044904/d20-x04-v01
Title: -
Type: Scatter2D
# xval xerr- xerr+ yval
                               yerr- yerr+
               4 400000++00
                               4 400000++00
                                               4.410000e-01
                                                              6.0000004.02
                                                                              6.00000e-03
1 390000e+01
2.560008e+01
               7.100000c+00
                                               4.560000e-01
                                                               6.0000000c-02
                                                                               6,000000c-02
                               8.788008e+AA
                                              4.910000e-01
7.180000e+01
                1.020000e+01
                               1.020000e+01
                                               5.300000e-01
                                                               6.900008e-02
                                                                               6.900000e-02
1.8998886+82
               1.100000+01
                               1.1888886+81
                                              5.858888e-81
                                                               7.4888886-82
                                                                               7.400000-02
1.682008e+82
               1.020880e+81
                               1.020008c+01
                                              5.730880e-01
                                                               7.388888e+82
                                                                              7.388888e-82
2.262000e+02
               7.900000e+00
                               7.900008e+00
                                               5.680000e-01
                                                               7.100000e-02
                                                                               7.100000e-02
3.374008c+02
                               2.1000000c+00
                                               5.880000e-01
                                                               7.500000c 02
END YODA SCATTER2D V2
```

"ANALYSIS_NAME.yoda" (experimental data)

How can we use Rivet? (2)

To compile the analysis :

rivet-buildplugin RivetANALYSIS_NAME.so ANALYSIS_NAME.cc

Then to run it :

```
rivet /path/to/DATA.hepmc -a --pwd ANALYSIS_NAME:cent=GEN -p
CALIB_NAME.yoda -o ANALYSIS_NAME.yoda
```

--pwd : Rivet will take into account the analysis even if it was not natively provided with the installation Not necessary if it has been added to the rivet --list-analyses by

setenv RIVET_ANALYSIS_PATH /path/to/analysis/files

Command to get plot:

rivet-mkhtml --pwd ANALASIS_NAME.yoda

Focus on Nuclear Modification Factor

The nuclear modification factor used as a probe to the QGP?

To disentangle initial- and final- state effects the nuclear modification factor can be used:

$$R_{AA} = \frac{\text{Yield in A+A}}{\langle N_{\text{coll}} \rangle \times \text{Yield in p+p}}$$

Test whether AA (pA) can be described by incoherent superposition of $\langle N_{coll} \rangle$: binary collisions \rightarrow *Calculated via Glauber Model*

- $R_{AA} \approx 1$ absence of nuclear effects
- $R_{AA} < 1$: suppression in AA collisions \rightarrow QGP? Actually we don't know
- *R_{AA}* > 1: enhancement in AA collisions → some mechanisms: Cronin Effects, Anti-shadowing etc ...

Quantitative analysis

Monte Carlo Glauber Model

• Basic assumptions

- nucleons travel on straight line trajectories
- independant binary nucleon-nucleon collision
- inelastic nucleon-nucleon cross section is independent of number of binary collision of a nucleon underwent before
- Impact parameter is randomly sampled
- Nucleons are randomly distributed inside nuclei, generally by Wood Saxon distribution
- Collision occurred based on the transverse distance between nucleons, and on the measured nucleon-nucleon inelastic cross sections (from PDG)
- Model provides impact parameter (b), number of participants (N_{part}), number of binary collisions (N_{coll}), and their correlations
 - also provides spatial anisotropy, so called "eccentricities"

Limitation: Cannot be used for light nuclei!

How to calculate the number of binary collisions? Introduction to Glauber Model

 N_{coll} cannot be find directly in experiment \Rightarrow Use Glauber Model



M. L. Miller et al, arXiv:nucl-ex/0701025

- The simplest approach to describe the initial condition of nucleus-nucleus collisions
- Widely used to determine centrality, and for initial conditions in hydrodynamical models, event generators

How to calculate the number of binary collisions? (2)



How to calculate the number of binary collisions? (2)



Number of nucleons per unity of surface at \vec{s} position in nucleus A:

$$T_A(\vec{s}) = \int_{-\infty}^{+\infty} dz \, \rho_A \sqrt{s^2 + z^2}$$

with A' number of nucleons in nucleus A

 T_A means the thickness function of the nucleus A with $\int ds^2 T_A(s) = A'$ Between a collision between nuclei A and B, the **Nuclear overlap function** is:

$$T_{AB}(b) = \int ds^2 T_A(s) T_B(|\vec{s} - \vec{b}|) \Rightarrow \boxed{N_{\text{coll}}(b) = \sigma_{NN} T_{AB}(b)}$$



arXiv:nucl-ex/0701025

How to calculate the nuclear modification factor?

- Position of nucleons in each nucleus generated by Monte Carlo following the Wood Saxon distribution as impact parameter
- Determination of N_{coll}
- Histogram of each distribution between the two different collisions

• Use this formulas: $R_{AA} = \frac{\text{Yield in A} + A}{\langle N_{\text{coll}} \rangle \times \text{Yield in p} + p}$

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The value of R_{AA} is not equal to $1 \Rightarrow$ something happens in **AA** more than **pp** and this is the **only thing** that we can say (not necessary a fluid) \rightarrow **Nuclear effects**.

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An example: *Au* and *Pb* nuclei have a majority number of **neutrons**. *np* or *nn* collisions do not produced the same thing than *pp* collisions (isospin effect)

New problem arises: high multiplicity pp collisions with some heavy ions physic behaviors

My Problem with this observable



Collectivity in pp collisions

Evolution of point of view

- Before 2010: pp is only a **reference** for heavy ions collisions as an elementary collision
- 2010: Something happens!



multiplicity

Collectivity in pp collisions

Evolution of point of view

- Before 2010: pp is only a **reference** for heavy ions collisions as an elementary collision
- 2010: Something happens! Ridge also on small system collisions !



multiplicity

- Currently, several workshops/conferences or session at QM have the title: **Collectivity at small system?**
- Can we continue to use pp collisions as reference when we don't know everything of this collision?

My plan to better understand these collisions

- Using EPOS to generate event of *pp*, *pA* and *AA* collisions
- **2** Generate events for pp collisions activating fluid or not
- Plot the yields of each collision
- Calculate the N_{coll} with EPOS
- Plot the R_{AA} using the two pp collisions and check firstly the differences
- **()** Calculate the R_{AA} for different centralities of AA collisions.
- **②** Calculate a new r_{aa} not to study nuclear effect but only to study

the QGP: $r'_{AA} \propto \frac{\text{Yield in A} + A_{\text{central}}}{\text{Yield in A} + A_{\text{periph}}}$

Yields in pp collisions



🛑 full 🔺 core ★ corona

Successful production of Υ for *ma* analysis.

Behavior equivalent about the yields of both analysis.

As expected, in *ma* analysis (without fluid activation), we have same value for corona part and full part.

Less corona part for each particles except for J/Ψ

Yields activating or not the fluid in pp collisions



Generally more particles created without fluid. Heavier anti-particles created with fluid (because of strange quarks?). No suppression observed for J/Ψ particles with creation of fluid in EPOS. Big difference between proton produced by collision with or without fluid.



Approximately same behavior for each collision for anti-particles. More particles produced by fluid in Pb-Pb collisions.

Even with normalization, less particles are produced for smaller system.

Preliminary Results about Nuclear Modification Factor

Previous works thanks to Abdellatif EL JAAFARI (internship student in Master 1)

Preliminary Results about Nuclear Modification Factor

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- Results come from Abdellatif El Jaafari
- Data come from my Ph.D stocked in root file, old version of EPOS (3.239)
- N_{coll} not stocked in root file \Rightarrow need to take this variable in article: arXiv:1710.07098v3 (edited in February 2019).
- Consequently, we firstly start by analysis the difference between the two R_{AA} with or without core-corona approach before to check quantitatively results with the factor 1.
- We have 1.6M events for *pp* without fluid / 100K events for *pp* with fluid and 40K events for *Pb-Pb* collisions
- In the following *ma* means without fluid and *my* means with fluid in *pp* collisions.

Preliminary Results about Nuclear Modification Factor

Previous works thanks to Abdellatif EL JAAFARI (internship student in Master 1)



Quantitative differences between pp collisions with fluid or without fluid. When we don't activate the fluid in pp collision, the **bump** have a **bigger** amplitude while the **plateau** is a bit **smaller** than R_{AA} with fluid in pp.

Trend do not depend on the QGP? (in pp)

Outlook and Perspectives

Outlook and Perspectives

- **1** Using EPOS to generate events of pp, pA and AA collisions \checkmark
- 2 Generate events for pp collisions activating fluid or not \checkmark
- \bigcirc Plot the yields of each collision \checkmark
- 4 Calculate the N_{coll} with EPOS
- S Plot the R_{AA} using the two pp collisions and check firstly the differences
- **6** Calculate the R_{AA} for different centralities of AA collisions.
- ⁽²⁾ Calculate a new r_{aa} and R'_{AA} not to study nuclear effect but only to study the QGP or opposite: $r_{AA} \propto \frac{\text{Yield in A} + A_{\text{central}}}{\text{Yield in A} + A_{\text{periph}}}$ and $R'_{AA} \propto \frac{\text{Yield in A} + A}{\text{Yield in p}_{\text{high mult}}}$

When the plan will be completed, I want to calculate a kind of R_{pp} means:

 $\overline{R_{\rm pp}} = \frac{\rm Yield \ in \ pp_{\rm high \ mult}}{\rm Yield \ in \ pp_{\rm min \ bias} \times (N_{\rm coll} = 1)}$

Raa of charmonia particles



Not enough particles to say something.





Description of used collectivity observable Anisotropic flow

A way of characterizing the various patterns of anisotropic flow is to use a Fourier expansion of the particle distribution function:

$$E\frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi}\frac{d^2}{p_t dp_t dy} \left(1 + 2\sum_{n=1}^{\infty} \frac{\mathbf{v}_n \cos\left[n(\phi - \psi_{RP})\right]}{\left(1 + 2\sum_{n=1}^{\infty} \frac{\mathbf{v}_n$$

E: energy of the particle ; p: momentum ; pt: transverse momentum ; ϕ : azimuthal angle ; y: rapidity ; ψ_{RP} : reaction plane angle.



Elliptic Flow

Anisotropic Flow

Direct evidence of flow: anisotropy in particle momentum distributions correlated with the reaction plane.





Anisotropic Flow: (n=1 :Directed Flow , n=2: Elliptic Flow)

$$v_n(pt,y) = \left\langle \cos\left[n\left(\phi(pt,y) - \psi_{RP}\right)\right]\right\rangle$$

Event plane method

Eta-Sub: Event Plane Method

Event Flow vector (projection of azimuthal angle):

$$Q_{n,x} = \sum_{i} w_i \cos(n\phi_i) = Q_n \cos(n\Psi_n)$$
$$Q_{n,y} = \sum_{i} w_i \sin(n\phi_i) = Q_n \sin(n\Psi_n)$$

The sum goes over all particles *i* used in *the event plane calculation*. ϕ_i and w_i are the lab azimuthal angle and weight for particle *i*

Where Ψ_n is **the event plane angle**:

$$\Psi_n = \frac{1}{n} \tan^{-1} \left(\frac{\sum_i w_i \sin(n\phi_i)}{\sum_i w_i \cos(n\phi_i)} \right)$$

Cumulants

$$\langle 4 \rangle \equiv \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle$$

Four azimuthal correlations

Study of four-particle correlations \Rightarrow flow in order of v_n^4

For combinatorial reasons, the probability that 4 particles are all correlated together is of order $1/M^3$ (M: Multiplicity).

Can Measure flow if $v_n \gg 1/M^{3/4} \rightarrow$ bias from nonflow effects is smaller.



Q-Cumulant Method A. Bilandzic, R. Snellings, and S. Voloshin Phys. Rev. C 83, 044913 – Published 26 April 2011

. .

Q-Cumulant \rightarrow Recent Method to calculate cumulants \rightarrow one loop over data Fast and unbiased Cumulant Method

Flow vector:
$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$
 $\langle 2 \rangle \equiv \langle e^{in(\phi_1 - \phi_2)} \rangle$ $\langle 4 \rangle \equiv \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle$

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Procedure to create cumulants by direct calculations:

- Decompose azimuthal correlations into expressions like $|Q_n|^2$, $|Q_n|^4$... in terms of $\langle 2 \rangle$, $\langle 4 \rangle$...
- Solve system of coupled equations for multi-particle scattering in same harmonic (2), (4) ...
- S Create $\langle \langle 2 \rangle \rangle, \langle \langle 4 \rangle \rangle$, average on all events, taking in account weights of event
- 4 Create Cumulants with terms of $\langle \langle 2 \rangle \rangle, \langle \langle 4 \rangle \rangle$ etc ...

Ex: $\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$

Reduce the contribution of nonflow effects

Q-Cumulant Method

A. Bilandzic, R. Snellings, and S. Voloshin Phys. Rev. C 83, 044913 - Published 26 April 2011

Cumulant coefficients

Cumulants for reference flow:

Reference flow or integrated flow:

Reference Flow: v_2 vs multiplicity or vs centrality

Cumulants for differential flow:

Differential flow:

 $d_n\{2\} = \langle \langle 2' \rangle \rangle \qquad \qquad v'_n\{2\} = d_n\{2\}/\sqrt{c_n\{2\}} \\ d_n\{4\} = \langle \langle 4' \rangle \rangle - 2 \times \langle \langle 2' \rangle \rangle \langle \langle 2 \rangle \rangle \qquad \qquad v'_n\{4\} = -d_n\{4\}/(-c_n\{4\})^{3/4}$

Differential Flow: v_2 vs p_t or vs η