Near-BPS baby Skyrmions as nuclear model

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work in progress with
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Topological Solitons, Nonperturbative Gauge Dynamics and Confinement
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Outline

- Skyrme model as nuclear model
- Motivation for near-BPS Skyrme-like models
- 2D near-BPS baby Skyrme model
- Analytic and numerical investigation
- Conclusions

Introduction

QCD

Quark confinement

Bound states: baryons, mesons, etc.

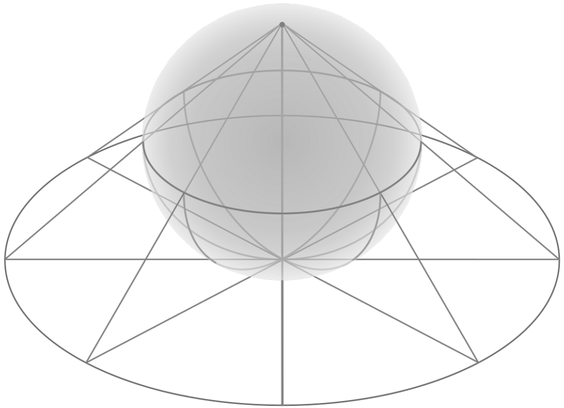
Field Theory for quarks and gluons

Effective Field Theory for baryons and nuclei

Skyrme model in 3+1 dim

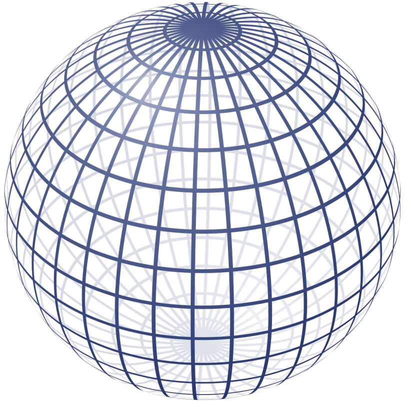
$$\mathcal{L} = \frac{f_\pi^2}{16\pi^2} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2} Tr\left(\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2\right)$$

$$U(t, \mathbf{x}) \in SU(2) \qquad G : SU(2) \times SU(2) \to SU(2)$$



$\mathbb{R}^3 \cup \{\infty\}$

map



$SU(2) \simeq S^3$

Skyrmions

Topological solitons:
Skyrmions

⟷

Baryons and nuclei

Topological degree B

$$\pi_3(S^3) = \mathbb{Z}$$

⟷

Baryon number B

‘60: Baryons as vortices in a “mesonic fluid”

‘80: Large-N QCD as an effective meson theory

Results from Skyrme model

Input: $m_p\,,m_\Delta$ → $f_\pi\,,\,r_N\,,\,g_{\pi NN}\,,\,\ldots$

Accuracy 30%

Problem

Too large binding energy

$$\frac{E_B}{N} \sim 10 \times \frac{E_B}{N}\Big|_{exp}$$

New Skyrme-like models

$$\mathcal{L}(U,\partial U) = a\mathcal{L}_0 + b\mathcal{L}_2 + c\mathcal{L}_4 + \ldots$$

Motivation for Near-BPS models

Small binding energy
for nucleons

$$\frac{|E_B|}{N} << m_N$$

$$E_{nuclei} \approx m_n |N| - \delta$$

BPS model

$$E = c|N|$$

no interactions

Near-BPS model

weak interactions

Near-BPS model in n-dimensions

$$\mathcal{L} = \mathcal{L}_{BPS} + \varepsilon\mathcal{L}'$$

3D Near-BPS Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_0 + \mathcal{L}_6}_{\text{BPS}} + \varepsilon\mathcal{L}'$$

2D Near-BPS baby Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_0 + \mathcal{L}_4}_{\text{BPS}} + \varepsilon\mathcal{L}'$$

2D near-BPS baby Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_{4,0}}_{\text{BPS}} + \varepsilon \mathcal{L}_2$$

$$\mathcal{L}_{4,0} = -\frac{1}{4}(\partial_\mu \vec{\phi} \times \partial_\nu \vec{\phi}) \cdot (\partial^\mu \vec{\phi} \times \partial^\nu \vec{\phi}) - m^2 (1 - \phi_3)$$

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi}$$

Target space S^2 $\quad \vec{\phi} \cdot \vec{\phi} = 1$

$G : O(3)$

Topological solitons

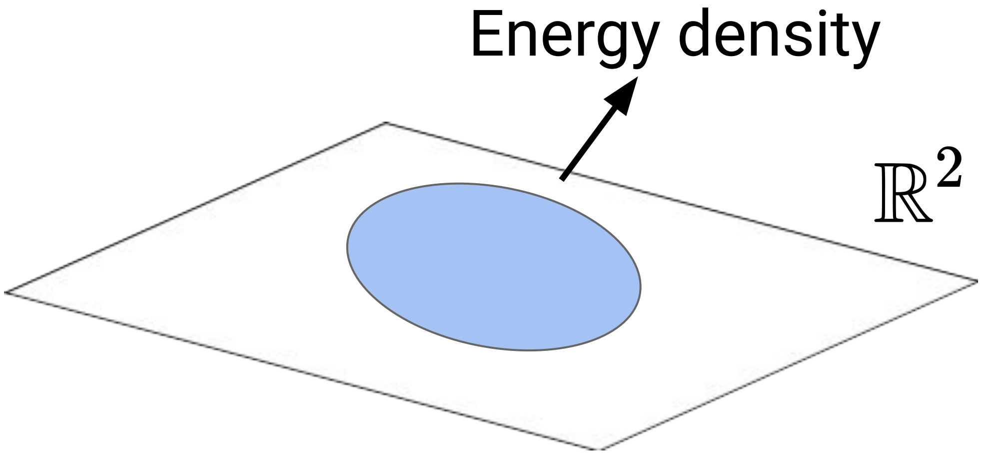
$$\boxed{\pi_2(S^2) = \mathbb{Z}}$$

BPS sector

$$\mathcal{L}_{4,0} = -\frac{1}{4}(\partial_\mu \vec{\phi} \times \partial_\nu \vec{\phi}) \cdot (\partial^\mu \vec{\phi} \times \partial^\nu \vec{\phi}) - m^2(1 - \phi_3)$$

$$E_{BPS} = \frac{16\pi m}{3}|N|$$ N Topological charge

Compacton solution



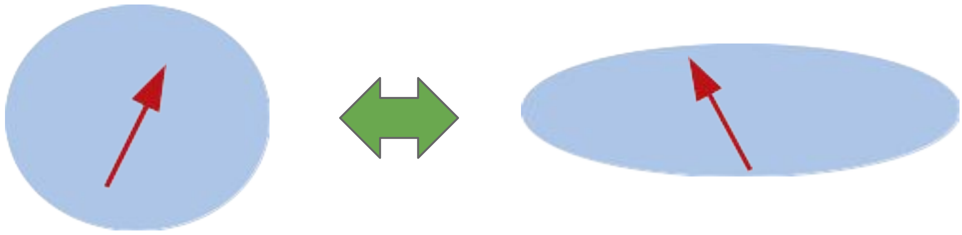
“Solitons in a baby Skyrme model with invariance under volume/area preserving diffeomorphisms ” T.Gisinger et al, Phys. Rev. D55, 7731 (1997)

Moduli space of BPS sector

$$\mathcal{L}_{4,0} = -\frac{1}{4}(\partial_\mu \vec{\phi} \times \partial_\nu \vec{\phi}) \cdot (\partial^\mu \vec{\phi} \times \partial^\nu \vec{\phi}) - m^2(1 - \phi_3)$$

Moduli space $\Rightarrow$ (x_0, y_0) | Phase α | Diff. area preserving

Translations | $SO(2)$ | $x \to x'$, $J = 1$



"*Compact baby Skyrmions*" C. Adam, A. Wereszczynski et al. Phys. Rev. D80, 105013 (2009)

2D near-BPS baby Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_{4,0}}_{\text{BPS}} + \varepsilon \mathcal{L}_2$$

Perturbative expansion in $\varepsilon \ll 1$ $\quad m = 1$

$$\vec{\phi} = \vec{\phi}^{(0)} + \vec{\phi}^{(1)} + \ldots$$

Zero order

$$\begin{cases} \vec{\phi}^{(0)} = \vec{\phi}_{BPS}(\vec{x}_0, \alpha, \lambda_{diff}) \\ E^{(0)} = \frac{16\pi}{3}|N| \end{cases}$$

First order in ε

$$E^{(1)} = \varepsilon E_2\left[\vec{\phi}_{BPS}(\vec{x}_0, \alpha, \lambda_{diff})\right]$$

with $E_2 = \int d^2x\, \partial_i \vec{\phi} \cdot \partial_i \vec{\phi}$

$\delta E^{(1)} = 0$ $\Leftrightarrow$ Minimize E_2 within the moduli space of $\vec{\phi}_{BPS}$

$\vec{\phi}^{\,*}_{BPS}(\vec{x}_0, \alpha, \lambda^*_{diff})$

Restricted harmonic map

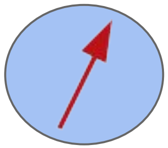
Restricted harmonic maps

λ_{diff} "Infinite parameters"

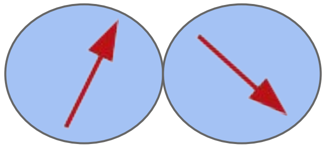
Mathematical criterium

J.M.Speight (2015)

Restricted harmonic



Q= N Axially symmetric + random orientation



Q= N +N 2-Axially symmetric + random orientations

"Near BPS skyrmions and restricted harmonic maps" J.M.Speight, J. Geom.Phys. 92,30 (2015)

Leading-order in ε

$$E = E^{(0)} + E^{(1)} = \frac{16\pi}{3} N + \varepsilon\left(8\pi \log 2 + \frac{7\pi}{6} N^2\right)$$

Stable configuration

$$\frac{d}{dN}\left(\frac{E}{N}\right) = 0 \Rightarrow \boxed{N_* = 4\sqrt{\frac{3\log 2}{7}} \simeq 2.18}$$

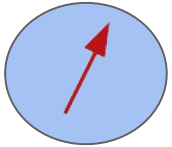
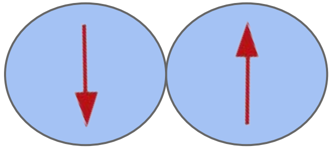
Next order

- Improving approximation $\vec{\phi} = \vec{\phi}^{(0)} + \vec{\phi}^{(1)} + \ldots$
- Interaction $O(\varepsilon^2)$ $E_{N+N}^{(0)+(1)} = E_N^{(0)+(1)} + E_N^{(0)+(1)}$

Second order in ε

$$\mathcal{L}^{(2)} = -\frac{1}{4}(\partial_i \vec{\phi}^{(0)} \times \partial_j \vec{\phi}^{(1)}) \cdot (\partial_i \vec{\phi}^{(0)} \times \partial_j \vec{\phi}^{(1)}) + \ldots$$

Calculating $\vec{\phi}^{(1)}$ for:

- Axially symmetric background Q=N $\quad \vec{\phi}^{(0)} =$ 
- 2-Axially symmetric background Q=N+N $\quad \vec{\phi}^{(0)} =$ 

work in progress

Axially symmetric solution Q=N

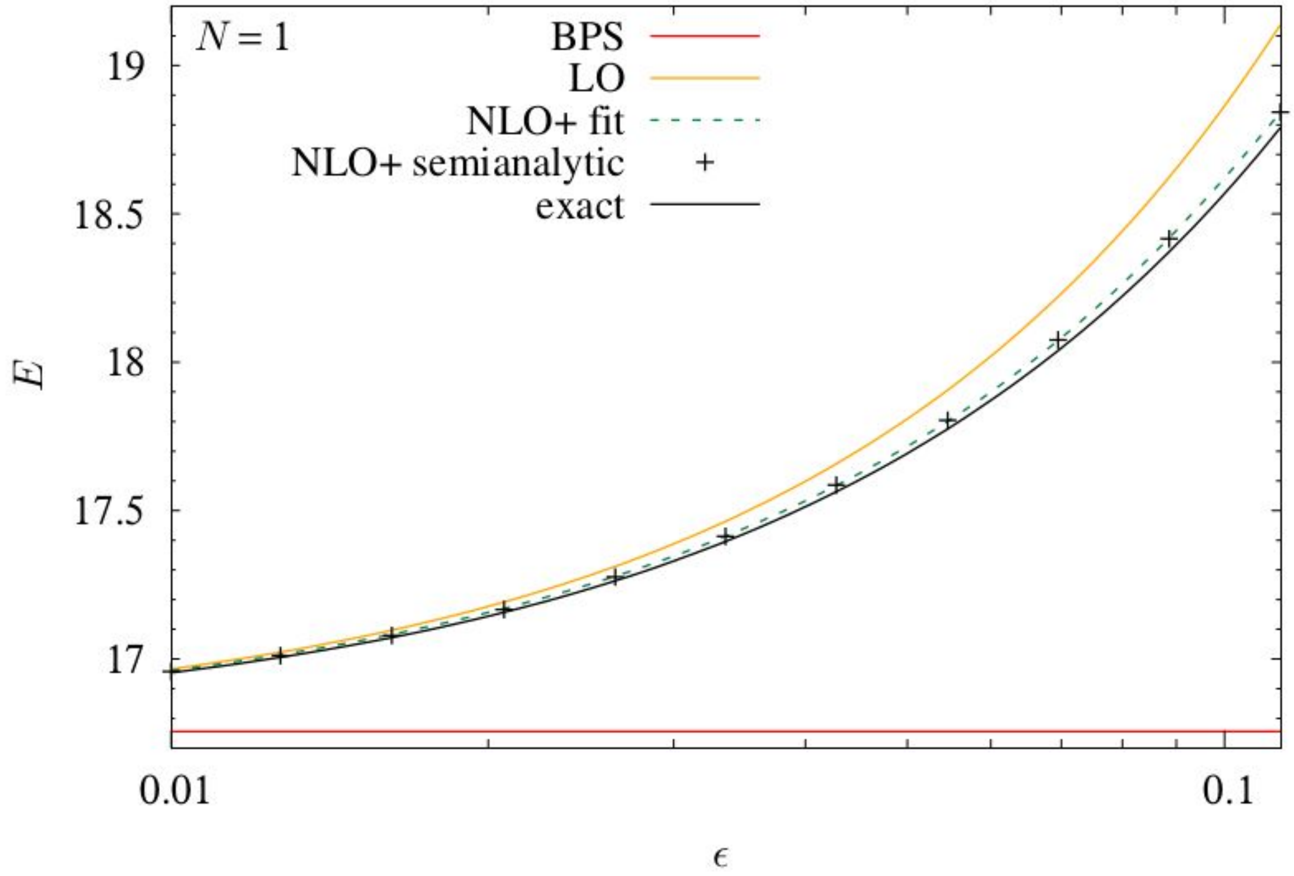
Ansatz $$\vec{\phi}^{(1)}(r,\theta) = c(r)\vec{\varphi}_{\perp}(r,\theta) \qquad \vec{\phi}^{(0)} \cdot \vec{\varphi}_{\perp} = 0$$

Energy $$E = E^{(0)} + E^{(1)} + E^{(2)}$$

$$E^{(2)} = \varepsilon^2(-91.56 + 34.36N - 6.198N^2)$$

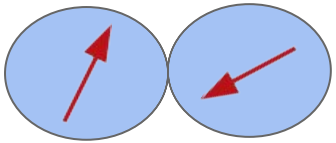
Stable configuration $$\frac{d}{dN}\left(\frac{E}{N}\right) = 0 \quad \Rightarrow \quad N_* \simeq 2 \quad \varepsilon < 0.1$$

Energy axially symmetric solution N=1



Solution Q=1+1

$$\vec{\phi} = \vec{\phi}^{(0)} + \vec{\phi}^{(1)} + \dots$$

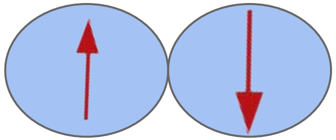


? $\Leftrightarrow$ $\vec{\phi} : \ \varepsilon \to 0 \, , \, \vec{\phi} \to \vec{\phi}_{BPS}$?

Guess

Long range potential

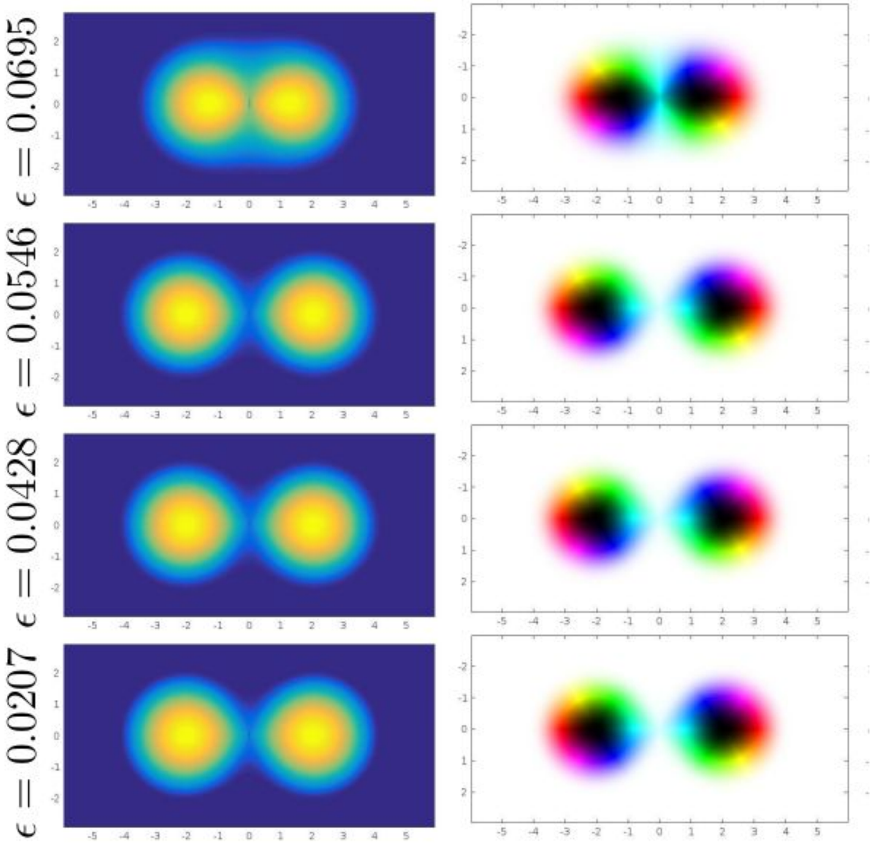
$$V_{1+1} \propto \cos(\alpha - \beta) \frac{e^{-\sqrt{m}r}}{r}$$



“Multisolitons in a Two-dimensional Skyrme Model” B.M.Piette et al. (1994)

Exact Q=1+1 solution for $\varepsilon \to 0$

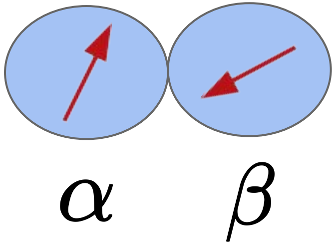
Topological charge density



Orientation color scheme

Solution Q=2+2

$$\vec{\phi} = \vec{\phi}^{(0)} + \vec{\phi}^{(1)} + \dots$$



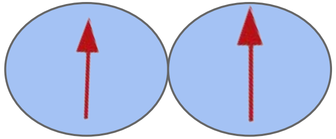
?

$$\vec{\phi} : \ \varepsilon \to 0\,,\ \vec{\phi} \to \vec{\phi}_{BPS}\ ?$$

Guess

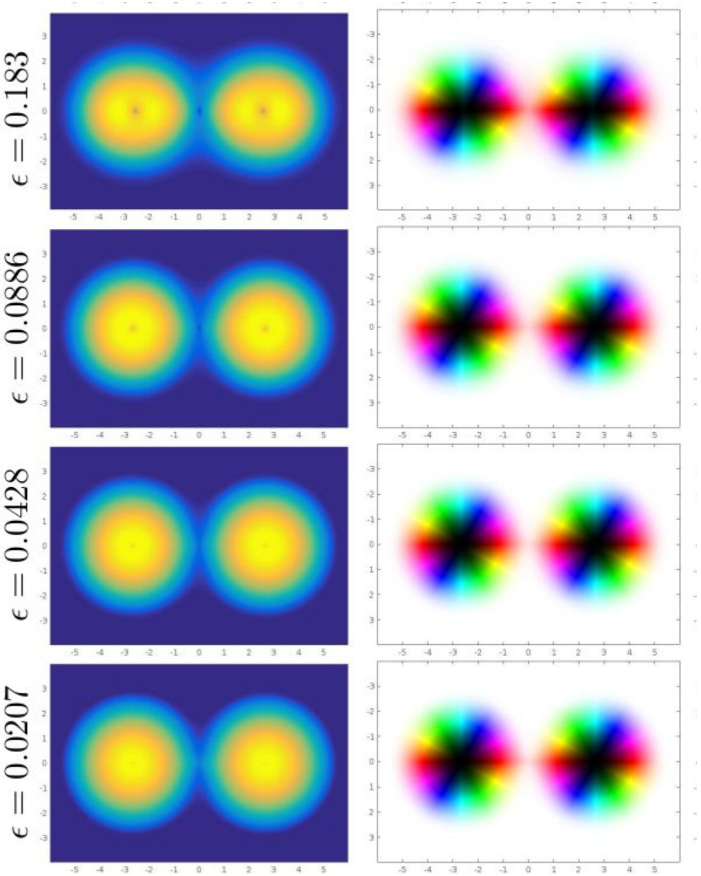
Long range potential

$$V_{2+2} \propto -\cos(\alpha - \beta)\,\frac{e^{-\sqrt{m}r}}{r}$$



Exact Q=2+2 solution for $\varepsilon \to 0$

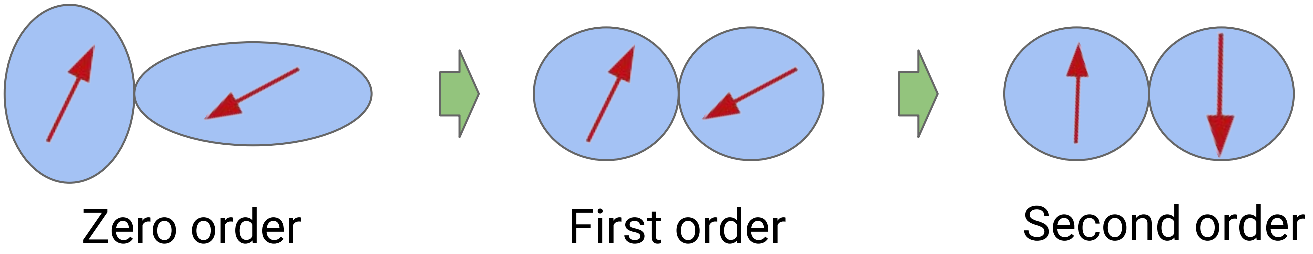
Topological charge density



Orientation color scheme

Solutions Q=N+N on working

$$\vec{\phi}^{(0)} = \vec{\phi}_{BPS}$$



$$\vec{\phi} : \ \varepsilon \to 0 \, , \, \vec{\phi} \to \vec{\phi}_{BPS} \ ?$$

Ansatz

$$\vec{\phi}^{(1)}(r,\theta) = a(r,\theta)\vec{\varphi}_{\perp}^{1} + b(r,\theta)\vec{\varphi}_{\perp}^{2} \qquad \vec{\phi}^{(0)} \cdot \vec{\varphi}_{\perp}^{i} = 0$$

Binding energy on working!

Conclusions

- Near-BPS baby Skyrme model as toy model for nuclear physics
- Investigation of the model using a perturbative expansion in the small parameter $\mathcal{E}$
- Leading order in $\mathcal{E}$ with restricted harmonic condition
- Next-to-leading order for solitons and multi-solitons
- Guess and numerical simulations for two interacting near-BPS baby skyrmions
- Working on the binding energy