(Almost-) Tensionless Confining Strings

and Even More about the Massive Schwinger Model

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Introduction

There is ample evidence (lattice, Seiberg-Witten, gauge/gravity correspondence ...) that 4d (Super-)YM theory confines

$$\langle W \rangle = \exp -\sigma \mathcal{A}$$

with a string tension $\sigma \sim \Lambda^2$.

Interestingly, in 2d QED/QCD with massive matter the string tension is given by

$$\sigma \sim me\,(1 - cos(2\pi Q_{ext}/Q_{dyn}))$$

so it vanishes in the limit of massless electrons $m \to 0$. A cloud of massless electrons can screen an external fractional charge!

A.A., S. Sugimoto, JHEP 1903 (2019) 175.

A.A., Y. Frishman, J. Sonnenschein ,Phys.Rev.Lett. 80 (1998) 420-423

Introduction

This fascinating phenomenon, of confining strings that becomes tensionless when the electron mass becomes zero, is by now fairly understood in terms of field theory dynamics.

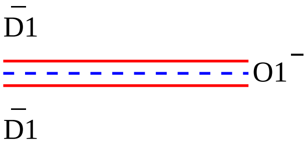
How can we understand this phenomenon from string theory viewpoint? how can a fundamental string become tensionless? it is well known that at weak coupling the F1 tension is given by $\sigma \sim \frac{g_s}{\alpha'}$.

The purpose of this talk is twofold:

- To study the vacuum structure of 2d QED
- To learn about string dynamics at strong coupling

String Theory Realisation

We wish to study 2d QED. A simple realisation of the theory is given by a system of $O1^-$ and an *anti* $D1$ brane



The presence of an *anti* D1 brane near an orientifold plane breaks supersymmetry. The field theory that lives on this system is an $SO(2)$ (or $U(1)$) gauge field, a neutral scalar and a charge 2 electron. The matter content is given by

	$U(1)$ charge	$SO(1,1)$	$SO(8)$
a_μ	0	2	1
ϕ_I	0	1	8_v
ψ^i_R	2	1_+	8_+
ψ^i_L	2	1_-	8_-

String Theory Realisation

The scalar couples to the electron via a Yukawa interaction

$$S_{\mathrm{Yukawa}} = \int d^2x \, \left(y\Gamma^I_{ij}\, \phi_I \psi^{i\dagger}_R \psi^j_L + \mathrm{h.c.} \right)$$

As usual, the vev of the scalar - which gives rise to a mass in field theory, parameterizes the distance between the brane and the orientifold.

In order to study potential between the brane and the orientifold we will study the field theory dynamics as a function of the electron mass.

Let's start with the massless case.

Vacuum structure of 2d QED

The action of 2d QED of charge k massless N_f flavours is given by

$$S_{\mathrm{QED}} = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + i\bar{\psi}_i \gamma^\mu (\partial_\mu + ikA_\mu)\psi^i\right)$$

The theory admits a classical symmetry of the form

$$G_{\mathrm{classical}} = \frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_A/\mathbb{Z}_2}{(\mathbb{Z}_{N_f})_L \times (\mathbb{Z}_{N_f})_R} \ ,$$

The axial symmetry $U(1)_A/\mathbb{Z}_2$ is broken by the anomaly, leading to

$$G = \frac{SU(N_f)_L \times SU(N_f)_R \times \mathbb{Z}_{kN_f}^{axial}}{(\mathbb{Z}_{N_f})_L \times (\mathbb{Z}_{N_f})_R}$$

$\mathbb{Z}_{kN_f}^{axial}$ is further broken spontaneously to $\mathbb{Z}_{N_f}$ resulting in k-vacua.

Bosonization and vacuum structure

The bosonized Hamiltonian of the multi-flavour Schwinger model is given by

$$H = \int dx^1 \left(\frac{2\pi}{N_f} \Pi_\varphi^2 + \frac{N_f}{8\pi} (\partial_1 \varphi)^2 + \frac{e^2}{8\pi^2} (k N_f \varphi - \theta)^2 \right) + H_{\mathrm{WZW}} \ ,$$

where $\exp i\varphi$ is a $U(1)$ and H_{WZW} is the Hamiltonian for the $SU(N_f)$ valued field g.

The field g decouples from the dynamics of the $U(1)$. It means that a condensate $\langle g \rangle$ cannot form. This is consistent with the Coleman-Mermin-Wagner theorem that implies that $SU_L(N_f) \times SU_R(N_f)$ cannot be broken spontaneously and a condensate $\langle \bar{\psi}_L^i \psi_j^R \rangle$, with $i \neq j$ cannot form.

Interestingly, $\langle \exp i N_f \varphi \rangle \neq 0$ which corresponds to $\langle \det \bar{\psi}_L^i \psi_j^R \rangle \neq 0$ is formed.

Quark condensate

The bosonized Hamiltonian implies

$$\langle\theta|e^{iN_f\varphi}|\theta\rangle = e^{i\frac{\theta}{k}}$$

which, together with the 2π periodicity of θ, gives rise to k vacua and a spontaneous breaking of the form $\mathbb{Z}^{axial}_{kN_f} \to \mathbb{Z}_{N_f}$.

Adding a small quark mass

In the presence of a mass term $M_0 \bar{\psi}_j \psi^j$ chiral symmetry is explicitly broken and a chiral condensate is formed

$$\langle \bar{\psi}_j \psi^j \rangle \propto e^{\frac{2}{N_f+1}} M_0^{\frac{N_f-1}{N_f+1}} \ .$$

Using the relation

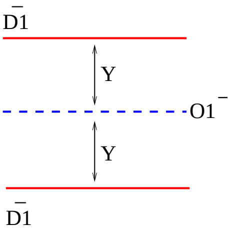
$$\frac{\partial E}{\partial M_0} = \langle \bar{\psi}_j \psi^j \rangle$$

We can calculate the vacuum energy in the presence of an external charge, $\delta S = Q \int F$ and without it.
The string tension is given by

$$\sigma(Q) = E(\theta + 2\pi Q) - E(\theta)$$

Back to the string theory setup

We are interested in the vacuum energy of the system as a function of the distance between the anti D1 and the orientifold, Y



The precise relation between the field theory mass M_0 and the distance Y in the string setup is given by

$$M_0 = y\phi = \frac{Y}{\pi\alpha'} \ . \tag{1}$$

We can also introduce a RR 0-form background and relate it to the θ parameter.

The QCD string tension is related to the difference between the tension of a $(1,-1)$ D-brane (a bound state of an anti D1 brane and F1) and a $(0,-1)$ D-brane.
Using the previous results (with $N_f = 8$) we find at short distances $(Y^2 \ll \alpha')$

$$T_{(1,-1)} - T_{(0,-1)} = C\,\frac{g_s}{\alpha'}\left(\frac{Y^2}{g_s\alpha'}\right)^{\frac{8}{9}} \;,$$

This result is in contrast to the behaviour at long distances $(\alpha' \ll Y^2)$

$$T_{(1,-1)} - T_{(0,-1)} = \frac{g_s}{4\pi\alpha'} \;,$$

Outlook

Several comments are in order:

- At long distance (and weak string coupling) we recover the standard result: the tension is constant. This is similar to $\sigma \sim \Lambda^2$ in 4d.
- At short distance (and strong coupling) the string tension is $\sim Y^{\frac{16}{9}}$. In particular it vanishes at $Y = 0$. The QCD string becomes tensionless and $T_{(1,-1)} = T_{(0,-1)}$.
- At strong string coupling the anti D1 tension $\sim g_s^{\frac{1}{9}}$