A Galilean Wess-Zumino model in three dimensions

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R. Auzzi, S. Baiguera, G. Nardelli, SP, arXiv:1904.08404

Motivations

We have studied at quantum level a non-relativistic version of the Wess-Zumino model in (2+1)D with $\mathcal{N} = 2$ SUSY

- In models describing CM systems SUSY has been observed to be an emergent symmetry, that is it appears in the effective theory describing the low-energy modes. On the other hand, in this regime the effective theory may be in a non-relativistic setting.

Therefore, it is physically relevant to construct NR SUSY models

- Non-relativistic holography: Non-relativistic generalisation of the AdS/CFT is of interest for the holographic description of CM systems. Which is the role of supersymmetry in NR gauge/gravity correspondence? D. Son, PRD78 (2008) 046003;

K. Balasubramanian, J. McGreevy, PRL101 (2008) 061601

State of the art

NR models with NO SUSY have been extensively studied, which have

- Galilean symmetry $(H, \vec{P}, \vec{J}, \vec{G})$ or Bargmann symmetry ($U(1)$ central extension) in (3+1)D J-M. Lévy-Leblond, CMP6 286 (1967)
- Schroedinger symmetry = NR version of conformal symmetry C. R. Hagen, PRD5 (1972) 377
- Lifshitz symmetry $(H, \vec{P}, \vec{J}, D)$ E. Lifshitz, ZETF11 (1941) 255, 269

NR SUSY models have been extensively studied, in

- (3+1)D $\Rightarrow$ WZ model, QED, Lifshitz models

  R. Puzalowski, Acta Phys. Austriaca 50 (1978) 45; T. E. Clark and S. T. Love, NPB 231 (1984); J. A. de Azcarraga and D. Ginestar, JMP 32 (1991); R. Dijkgraaf, D. Orlando, and S. Reffert, 0903.0732; ....

- (2+1)D $\Rightarrow$ $\mathcal{N} = 2$ Chern-Simons-matter theory (enhanced Schroedinger symmetry)

  M. Leblanc, G. Lozano, H. Min, AP 219 (1992) 328; O. Bergman, C. B. Thorn, PRD 52 (1995) 5997

From a theoretical point of view there are interesting questions:

- Which are the renormalization properties of NR SUSY theories?
- Does SUSY conspire with the NR space-time symmetry to mild UV divergences?
- Do non-renormalization theorems still work ?

We focus on (2+1)D field theories

Plan of the talk

1) Construction of the NR $\mathcal{N} = 2$ supersymmetric Galilean algebra in (2+1)D via DLCQ procedure

2) NR $\mathcal{N} = 2$ Superspace

3) NR Wess-Zumino Model. Perturbative analysis, renormalization properties, one-loop exactness

4) Conclusions and future directions

SUSY Extended Galilean algebra

There are different ways to obtain the extended Galilean algebra in (d+1)D

- Taking the Inönü-Wigner contraction of the (d+1)D Poincaré $\otimes$ $U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing a ((d+1)+1)D relativistic theory along a null direction (DLCQ procedure)

Similarly, we can construct the Super-Galilean algebra in (d+1)D

- Completing the Galilean algebra with a set of fermionic generators and impose constraints on the algebra
- Taking the Inönü-Wigner contraction of the (d+1)D super-Poincaré $\otimes$ $U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing a ((d+1)+1)D relativistic SUSY theory along a null direction (**DLCQ procedure**)

$\Rightarrow$ To construct a NR Superspace the most convenient approach is DLCQ

$\mathcal{N}=2$ SUSY Galilean algebra in (2+1)D

$$[P_j, G_k] = i\delta_{jk} M\,, \qquad [H, G_j] = iP_j\,,$$
$$[P_j, J] = -i\epsilon_{jk} P_k\,, \qquad [G_j, J] = -i\epsilon_{jk} G_k\,, \qquad j,k = 1,2$$

$$[Q_1, J] = \tfrac{1}{2} Q_1\,, \qquad \{Q_1, Q_1^\dagger\} = \sqrt{2} M\,,$$
$$[Q_2, J] = -\tfrac{1}{2} Q_2\,, \quad [Q_2, G_1 - iG_2] = -iQ_1\,, \quad \{Q_2, Q_2^\dagger\} = \sqrt{2} H\,,$$
$$\{Q_1, Q_2^\dagger\} = -(P_1 - iP_2)\,, \quad \{Q_2, Q_1^\dagger\} = -(P_1 + iP_2)$$

Null reduction: We start from the (3+1)D super-Poincarè algebra realized on the spacetime

$$(x^+, x^-, x^{i=1,2}) \qquad x^\pm = \frac{x^3 \pm x^0}{\sqrt{2}} \qquad \text{light} - \text{cone coords.}$$

⋆ compactify x^- on a tiny circle of radius R and rescale $x^+ \to x^+/R$, $x^- \to Rx^-$.

⋆ Write the (3+1)D anticommutator $\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\beta}\} = i\sigma^\mu_{\alpha\dot\beta}\partial_\mu$ in terms of the light-cone coordinates

$$\{\mathcal{Q}, \bar{\mathcal{Q}}\} = i \begin{pmatrix} \sqrt{2}\partial_+ & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\sqrt{2}\partial_- \end{pmatrix}$$

⋆ Reduce a generic field as

$$\Phi(x^\mu) = e^{imx^-}\tilde{\Phi}(x^+ \equiv t, x^i) \qquad m \to M - \text{eigenvalue } (\textbf{adimensional})$$

⋆ Identify $\partial_+ \to \partial_t\,,\ \partial_- \to im \qquad \mathcal{Q}_\alpha \to Q_\alpha, \quad \bar{\mathcal{Q}}_{\dot\alpha} \to Q^\dagger_\alpha$

NR Superspace

Since the null reduction does not affect the fermionic coordinates

$$
\begin{array}{ccc}
(3+1)\ \mathcal{N}=1 \text{ relativistic superspace} & \Longrightarrow & (2+1)\ \mathcal{N}=2 \text{ NR superspace} \\
(x^+, x^-, x^i, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) & \Longrightarrow & (x^+, x^i, \theta^1, \theta^2, (\theta^1)^\dagger, (\theta^2)^\dagger)
\end{array}
$$

Reduction of a generic superfield

$$\Phi(x^M, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) = e^{imx^-}\,\tilde{\Phi}(t, x^i, \theta^1, \theta^2, (\theta^1)^\dagger, (\theta^2)^\dagger)$$

⋆ Covariant derivatives

$$
\begin{cases}
\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\bar{\theta}^{\dot{\beta}}\partial_{\alpha\dot{\beta}} \\
\bar{\mathcal{D}}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \frac{i}{2}\theta^\beta\partial_{\beta\dot{\alpha}}
\end{cases}
\Longrightarrow
\begin{cases}
D_1 = \frac{\partial}{\partial\theta^1} - \frac{i}{2}\bar{\theta}^2(\partial_1 - i\partial_2) - \frac{i}{\sqrt{2}}\bar{\theta}^1\partial_t \\
\bar{D}_1 = \frac{\partial}{\partial\bar{\theta}^1} - \frac{i}{2}\theta^2(\partial_1 + i\partial_2) - \frac{i}{\sqrt{2}}\theta^1\partial_t \\
D_2 = \frac{\partial}{\partial\theta^2} - \frac{i}{2}\bar{\theta}^1(\partial_1 + i\partial_2) - \frac{1}{\sqrt{2}}\bar{\theta}^2 M \\
\bar{D}_2 = \frac{\partial}{\partial\bar{\theta}^2} - \frac{i}{2}\theta^1(\partial_1 - i\partial_2) - \frac{1}{\sqrt{2}}\theta^2 M
\end{cases}
$$

⋆ <u>(Anti)chiral superfields</u> $\bar{D}_\alpha \Sigma = 0, \qquad D_\alpha \bar{\Sigma} = 0$

$$\Sigma(x_L^\mu, \theta^\alpha) = \varphi(x_L^\mu) + \theta^\alpha \tilde{\psi}_\alpha(x_L^\mu) - \theta^2 F(x_L^\mu)$$

$$\bar{\Sigma}(x_R^\mu, \bar{\theta}^\beta) = \bar{\varphi}(x_R^\mu) + \bar{\theta}_\gamma \bar{\tilde{\psi}}^\gamma(x_R^\mu) - \bar{\theta}^2 \bar{F}(x_R^\mu) \qquad x^\mu_{L,R} = x^\mu \mp i\theta^\alpha (\bar{\sigma}^\mu)_{\alpha\beta}\bar{\theta}^\beta$$

⋆ <u>Berezin integration</u>

$$\int d^4x d^4\theta\, \Phi = \int d^4x\, \mathcal{D}^2 \bar{\mathcal{D}}^2 \Phi \Big|_{\theta=\bar{\theta}=0} \longrightarrow \int d^3x d^4\theta\, \tilde{\Phi} \; \times \frac{1}{2\pi}\int_0^{2\pi} dx^- \, e^{imx^-}$$

$$\int d^3x d^4\theta\, \tilde{\Phi} \equiv \int d^3x D^2 \bar{D}^2 \tilde{\Phi} \Big|_{\theta=\bar{\theta}=0} \qquad \Downarrow$$

Non-vanishing result only if $M(\Phi) = 0$

Relativistic $\mathcal{N}=1$ WZ model in (3+1)D

$$S=\int d^4x\,d^4\theta\,\bar{\Sigma}\Sigma+\int d^4x\,d^2\theta\left(\frac{\mu}{2}\Sigma^2+\frac{\lambda}{3!}\Sigma^3\right)+\text{h.c.}\qquad \mu=0$$

- **The WZ model is renormalizable**
- Renormalization in Superspace: UV divergent contributions only to the Kähler potential (no chiral divergent terms). Therefore,

$$\mathcal{L}\to\mathcal{L}_{\text{ren}}=\int d^4\theta\,Z_\Sigma(\bar{\Sigma}\Sigma)+\int d^2\theta\,Z_\lambda Z_\Sigma^{3/2}(\frac{\lambda}{3!}\Sigma^3)$$

The absence of chiral divergences implies

$$Z_\lambda Z_\Sigma^{3/2}=1\implies Z_\lambda=Z_\Sigma^{-3/2}$$

- **Non-renormalization theorem**

$\rightarrow$ Perturbative — M.T. Grisaru, W. Siegel, M. Rocek, NPB 159 (1979) 429

$\rightarrow$ Non-perturbative — N. Seiberg, PLB 318 (1993) 469

Holomorphicity, SUSY and R-symmetry

Non-relativistic Wess-Zumino model in (2+1)D

Particle number conservation requires at least two superfields

$$S = \int d^3x d^4\theta \left(\bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2\right) + g \int d^3x d^2\theta \, \Phi_1^2 \Phi_2 + \text{h.c.}$$

$$M(\Phi_1) = m, \; M(\Phi_2) = -2m$$

Manifestly invariant under NR $\mathcal{N} = 2$ SUSY

$$\Phi_1 = \varphi_1 + \theta^1 \xi_1 + \theta^2 \, 2^{\frac{1}{4}} \sqrt{m} \, \chi_1 - \frac{1}{2} \theta^\alpha \theta_\alpha F_1$$

$$\Phi_2 = \varphi_2 + \theta^1 \xi_2 + \theta^2 \, i 2^{\frac{1}{4}} \sqrt{2m} \, \chi_2 - \frac{1}{2} \theta^\alpha \theta_\alpha F_2$$

(ξ_1, F_1) (ξ_2, F_2) $\longrightarrow$ auxiliary (non-dynamical) fields

Technical subtely: $M(\varphi_2) < 0, M(\chi_2) < 0$

$$S = \int d^3x \left[2im\bar{\varphi}_1\partial_t\varphi_1 + \bar{\varphi}_1\partial_i^2\varphi_1 \underbrace{-4im\bar{\varphi}_2\partial_t\varphi_2} + \bar{\varphi}_2\partial_i^2\varphi_2 + \ldots\right]$$

Integrate by parts and exchange $\varphi_2 \leftrightarrow \bar{\varphi}_2$ (the same for fermions)

$$S = \int d^3x \left[\bar{\varphi}_1 \left(2im\partial_t + \partial_i^2\right) \varphi_1 + \bar{\varphi}_2 \left(4im\partial_t + \partial_i^2\right) \varphi_2 \right.$$
$$\left. + \bar{\chi}_1 \left(2im\partial_t + \partial_i^2\right) \chi_1 + \bar{\chi}_2 \left(4im\partial_t + \partial_i^2\right) \chi_2\right] + S_{\text{int}}$$

This action could have been obtained by null reduction of the action in components for the relativistic (3+1)D WZ model

Renormalization in Superspace

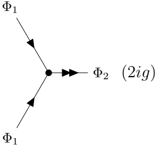
- Superfield propagators

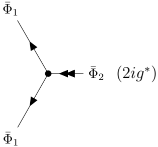
$$\langle \Phi_a(\omega,\vec{p},\theta_1,\bar{\theta}_1)\bar{\Phi}_a(-\omega,-\vec{p},\theta_2,\bar{\theta}_2)\rangle = \frac{i}{2m_a\omega-\vec{p}^{\,2}+i\varepsilon}\,\delta^{(4)}(\theta_1-\theta_2) \qquad a=1,2$$

In configuration space

$$D_a(\vec{x},t) = \int \frac{d^2p\,d\omega}{(2\pi)^3}\, i\,\frac{\delta^{(4)}(\theta_1-\theta_2)}{2m_a\,\omega-\vec{p}^{\,2}+i\varepsilon}\, e^{-i(\omega t-\vec{p}\cdot\vec{x})} = -\frac{i\,\Theta(t)}{4\pi\,t}\, e^{i\frac{m_a\vec{x}^2}{2t}}\,\delta^{(4)}(\theta_1-\theta_2)$$

- Supervertices



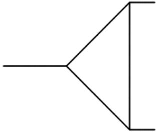
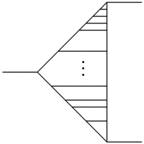
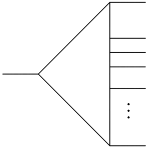


Number of incoming arrows = Number of outgoing arrows

Selection rules

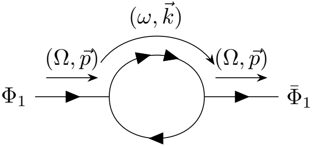
Loop diagrams are formally the same as in the relativistic 2-field WZ model, but....

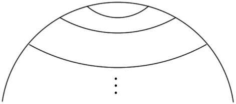
- Selection rule 1 - Particle number conservation at each vertex

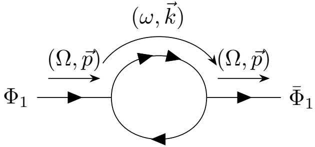




- Selection rule 2 - Arrows inside a Feynman diagram cannot form a closed loop.

O. Bergman, PRD 46 (1992) 5474







⋆ In momentum space

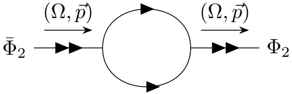
$$i\Gamma_1^{(2)}(\Phi_1, \bar{\Phi}_1) \to 4|g|^2 \int \frac{d\omega\, d^2k}{(2\pi)^3} \frac{1}{\left[4m\omega - \vec{k}^2 + i\varepsilon\right]\left[2m(\omega - \Omega) - (\vec{k} - \vec{p})^2 + i\varepsilon\right]} = 0$$

⋆ In configuration space it would be proportional to $\Theta(t)\Theta(-t) = 0$

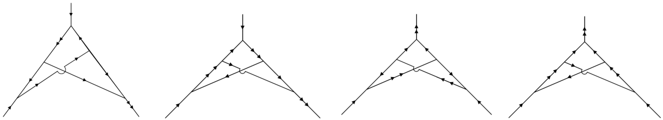
Results

Self-energy corrections - The only non-vanishing diagram at one loop

$$\Gamma_2^{(2)}(\Phi_2, \bar{\Phi}_2) \to \frac{|g|^2}{4\pi m} \frac{1}{\varepsilon} \int d^4\theta \, \Phi_2(\Omega, \vec{p}, \theta) \bar{\Phi}_2(\Omega, \vec{p}, \theta)$$



Vertex corrections - No one-loop. At two loops



They vanish due to circulating loops.

No non-vanishing diagrams arise at higher loops ⇒ One-loop exactness

⋆ Non-relativistic non-renormalization theorem

We have proved the NR perturbative non-renormalization theorem (no vertex corrections allowed)
Seiberg's argument can be easily imported and a non-perturbative non-renormalization theorem holds

⋆ Exact beta-function

$$\mathcal{L} \to \mathcal{L}_{\rm ren} = \int d^4\theta \, \left(Z_1 \bar{\Phi}_1 \Phi_1 + Z_2 \bar{\Phi}_2 \Phi_2 \right) + g \int d^2\theta \, Z_g Z_1 Z_2^{1/2} \Phi_1^2 \Phi_2 + \text{h.c.}$$

with $Z_1 = 1, \quad Z_2 = 1 - \frac{|g|^2}{4\pi m} \frac{1}{\varepsilon}$

There are no UV divergent vertex corrections. Therefore,

$$Z_g Z_1 Z_2^{1/2} = 1 \implies Z_g = Z_2^{-1/2} \sim 1 + \frac{|g|^2}{8\pi m} \frac{1}{\varepsilon} \implies \boxed{\beta_g = \frac{g^3}{4\pi m}}$$

The model is classically scale invariant, but scale invariance is lost due to quantum corrections.

Conclusions

We have studied quantum properties of the simplest NR susy model in (2+1)D, obtained as null reduction of $\mathcal{N} = 1$ WZ model in (3+1)D.

1. The model is One-loop exact. Scale invariance is broken by one-loop effects.
2. At quantum level the model cannot be obtained as the null reduction of the quantum (3+1)D relativistic model. In particular, the (2+1)D NR theory has much nicer properties compared to its (3+1) parent theory.

Future directions

1. Coupling to gauge fields
2. Coupling to gravity. NR supergravity as null reduction of relativistic supergravity? Connection with NR holography
3. Theories with more SUSY (ex: NR ABJM, Y. Nakayama 0902.2267; K.-M. Lee, S. Lee, S. Lee 0902.3857)
4. Opposite limit ($c \to 0$). super-Carroll