Light nuclei in an extended Skyrme model with rho mesons

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Topological Solitons, Nonperturbative Gauge Dynamics and Confinement 2

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[based on work in collaboration with P. Sutcliffe:

JHEP **1805** (2018) 174 & Phys. Rev. Lett. **121**, 232002 (2018)]

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QCD: Fundamental theory of strong interactions

- **Non-perturbative regime:**
  - Confinement
  - Chiral symmetry breaking
  - Masses of particles (hadrons, nuclei, binding energies, excitations)
  - Nuclear matter properties (EoS, compressibility, chemical potential)
  - Neutron stars
- **Two approaches:**
  - Lattice QCD $\rightarrow$ problematic for high densities
  - EFT $\rightarrow$ mean-field limit usually needed

- **Tony Hilton Royle Skyrme (1922-1987)**

EFT: The Skyrme model (as one of the most popular and succesful)

- Baryons as "vortices" in a mesonic fluid $\Rightarrow$ topological solitons, i.e., emergent, *non-perturbative* objects

$$\underbrace{\mathscr{L} = \mathscr{L}_0 + \underbrace{\lambda_2\mathscr{L}_2 + \lambda_4\mathscr{L}_4}_{\text{massless } \mathscr{L}_{\text{Skyrme}}}}_{\text{massive } \mathscr{L}_{\text{Skyrme}}} + \lambda_6\mathscr{L}_6$$

$$\mathscr{L}_2 = -\mathrm{Tr}(L_\mu L^\mu), \quad \mathscr{L}_4 = \mathrm{Tr}([L_\mu, L_\nu]^2), \quad \mathscr{L}_6 = -\mathcal{B}_\mu \mathcal{B}^\mu$$

$$\mathcal{B}^\mu = \frac{1}{24^2}\mathrm{Tr}(\epsilon^{\mu\nu\rho\sigma} L_\nu L_\rho L_\sigma), \quad L_\mu = U^\dagger \partial_\mu U, \quad U \in \mathrm{SU}(2) \cong \mathbb{S}^3$$

- Baryon number $\equiv$ topological charge: $B = \int d^3x \mathcal{B}^0$
- Success in the 80's with the study of the nucleon:
  - **Massless:** G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. B **228**, 552 (1983).
  - **Massive:** G. Adkins and C. Nappi, Nucl. Phys. B **233**, 109 (1984).

But where are the mesons?

- **Pions:** small perturbations of the Skyrme field

$$U = \exp\left(\frac{2i}{F_\pi}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right) = 1 + \frac{2i}{F_\pi}\boldsymbol{\pi}\cdot\boldsymbol{\tau} + \frac{1}{2!}\left(\frac{2i}{F_\pi}\right)^2\boldsymbol{\pi}^2 + \dots$$

$$\partial_\mu U = \frac{2i}{F_\pi}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau} + \dots$$

Translated into the contributions to the Lagrangian

$$\mathcal{L}_2 = \frac{1}{16}F_\pi^2 \mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + O(\pi^4)$$

$$\mathcal{L}_0 = \frac{1}{8}m_\pi^2 F_\pi^2(\mathrm{Tr}U - 2) = -\frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + O(\pi^4)$$

$$\mathcal{L}_4 = \frac{1}{32e^2}\mathrm{Tr}\left([(\partial_\mu U U^\dagger, \partial_\nu U U^\dagger)]^2\right) = O(\pi^4)$$

$$\boxed{\Rightarrow \mathcal{L} = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + O(\pi^4)}$$

Is the Skyrme model so great? Almost! Unphysical high binding energies and lack of clustering structures for light nuclei.

Looking for improvement, different approaches.

- The lightly bound model: Inclusion of a term proportional to $\mathrm{Tr}(1-U)^4$

  M. Gillard, D. Harland, M. Speight, Nucl. Phys. B **895** (2015) 272-287
- BPS models (lower topological energy bound).

Several realizations:

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A Skyrme-type proposal for baryonic matter

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ABSTRACT

The Skyrme model is a low-energy effective field theory for QCD, where the baryons emerge as soliton solutions. It is, however, not so easy within the standard Skyrme model to reproduce the almost exact linear growth of the nuclear masses with the baryon number (topological charge), due to the lack of Bogomolny solutions in this model, which has also hindered analytical progress. Here we identify a submodel within the Skyrme-type low-energy effective action which does have a Bogomolny bound and exact Bogomolny solutions, and therefore, at least at the classical level, reproduces the nuclear masses by construction. Due to its high symmetry, this model qualitatively reproduces the main features of

Is the Skyrme model so great? Almost! Unphysical high binding energies and lack of clustering structures for light nuclei.

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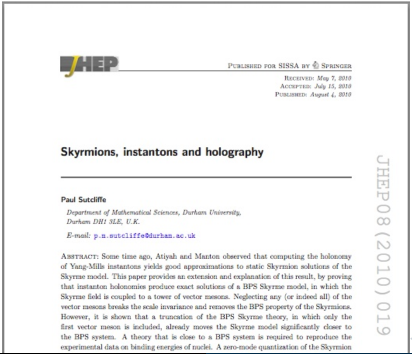

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Skyrmions, instantons and holography

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Abstract: Some time ago, Atiyah and Manton observed that computing the holonomy of Yang-Mills instantons yields good approximations to static Skyrmion solutions of the Skyrme model. This paper provides an extension and explanation of this result, by proving that instanton holonomies produce exact solutions of a BPS Skyrme model, in which the Skyrme field is coupled to a tower of vector mesons. Neglecting any (or indeed all) of the vector mesons breaks the scale invariance and removes the BPS property of the Skyrmions. However, it is shown that a truncation of the BPS Skyrme theory, in which only the first vector meson is included, already moves the Skyrme model significantly closer to the BPS system. A theory that is close to a BPS system is required to reproduce the experimental data on binding energies of nuclei. A zero-mode quantization of the Skyrmion

JHEP08(2010)019

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The origins:

P. Sutcliffe, JHEP **08** (2010) 019

- Based on Atiyah and Manton's idea: Skyrme field generated from the holonomy of a Yang-Mills instanton in $\mathbb{R}^4$.

$$U(\mathbf{x}) = \pm \mathcal{P} \exp \int_{-\infty}^{\infty} A_4(\mathbf{x}, x_4) dx_4$$

- Yang-Mills energy given by ($I = 1, 2, 3, 4$, and $z = x_4$):

$$E = -\frac{1}{8} \int \mathrm{Tr}(F_{IJ} F_{IJ}) d^3x\, dz$$

- Fixing the gauge $A_z = 0 \Rightarrow$ gauge field expanded in terms of Hermite functions, ψ_n

$$A_i = -\partial_i U U^{-1} \psi_+(z) + \sum_{n=0}^{\infty} V_i^n(\mathbf{x}) \psi_n(z)$$

Resulting theory: Skyrme field coupled to an infinite tower of vector mesons with a BPS bound.

Skyrme, are you there?

Neglecting vector fields

$$A_i = -\partial_i U U^{-1} \psi_+(z)$$

with

$$\psi_+(z) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}(z/\sqrt{2}), \qquad \mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\xi^2} d\xi$$

- Energy

$$E_{\mathrm{S}} = \int \left(-\frac{c_1}{2}\mathrm{Tr}(R_i R_i) - \frac{c_2}{16}\mathrm{Tr}([R_i, R_j]^2) \right) d^3x$$

where

$$c_1 = \frac{1}{4\sqrt{\pi}}, \qquad c_2 = \int_{-\infty}^{\infty} 2\psi_+^2(\psi_+ - 1)^2 dz = 0.198$$

- Energy bounds

$$E_{\mathrm{S}} \geq 12\pi^2 \sqrt{c_1 c_2}|B| = 2.005\pi^2|B|, \qquad E_{YM} \geq 2\pi^2|B|$$

The first in the queue: the ρ meson

Gauge field with one vector meson ($V_i^0 = V_i$)

$$A_i = -\partial_i U\, U^{-1} \psi_+(z) + V_i \psi_0(z) \qquad \left(\psi_0(z) = \frac{1}{\pi^{1/4}} e^{-z^2/2} \right)$$

and field strength

$$F_{zi} = -R_i \frac{\psi_0}{\sqrt{2}\pi^{1/4}} - V_i \frac{\psi_1}{\sqrt{2}},$$

$$F_{ij} = [R_i, R_j]\psi_+(\psi_+ - 1) + (\partial_i V_j - \partial_j V_i)\psi_0 + [V_i, V_j]\psi_0^2 - ([R_i, V_j] - [R_j, V_i])\psi_+\psi_0$$

Different contributions to the energy

$$E = E_{\mathrm{S}} + E_{\mathrm{V}} + E_{\mathrm{I}} = 1.060 \times 2\pi^2$$

Vector meson energy:

$$E_{\mathrm{V}} = \int -\mathrm{Tr}\left\{\frac{1}{8}(\partial_i V_j - \partial_j V_i)^2 + \frac{1}{4}m^2 V_i^2 + c_3(\partial_i V_j - \partial_j V_i)[V_i, V_j] + c_4[V_i, V_j]^2\right\} d^3x,$$

Interaction energy:

$$\begin{aligned} E_{\mathrm{I}} = & \int -\mathrm{Tr}\Bigg\{ c_5([R_i, V_j] - [R_j, V_i])^2 - c_6[R_i, R_j](\partial_i V_j - \partial_j V_i) - c_7[R_i, R_j][V_i, V_j] \\ & + \frac{1}{2}c_6[R_i, R_j]([R_i, V_j] - [R_j, V_i]) - \frac{1}{8}([R_i, V_j] - [R_j, V_i])(\partial_i V_j - \partial_j V_i) \\ & - \frac{1}{2}c_3([R_i, V_j] - [R_j, V_i])[V_i, V_j] \Bigg\} d^3x, \end{aligned}$$

where the constants are

$$m = \frac{1}{\sqrt{2}}, \qquad c_3 = \frac{1}{2\sqrt{6}\pi^{\frac{1}{4}}}, \qquad c_4 = \frac{1}{8}\sqrt{\frac{1}{2\pi}},$$

$$c_5 = 0.038, \qquad c_6 = \frac{\pi^{\frac{1}{4}}}{12\sqrt{2}}, \qquad c_7 = 0.049$$

Instanton approximation

From the single instanton solution we get:

- Associated holonomy

$$U = \exp(i f(r) \hat{x}_a \tau_a), \qquad f(r) = -\frac{\pi r}{\sqrt{\lambda^2 + r^2}}$$

- Vector meson

$$V_{ia} = \rho(r)\, Z_{ia}, \qquad Z_{ia} = \epsilon_{ija}\, \hat{x}_j,$$

with profile function

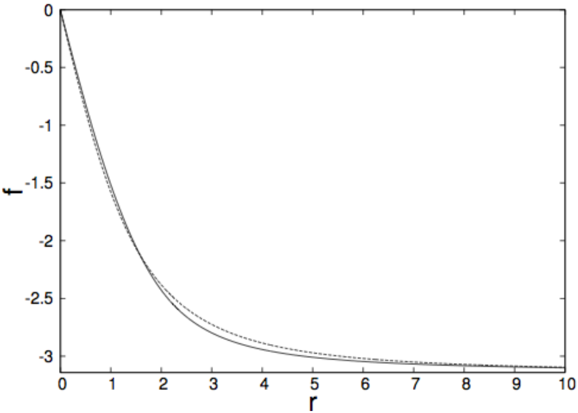
$$\rho(r) = \int_{-\infty}^{\infty} \left\{ \eta \sin H + \left(\zeta + \frac{1}{2r} \right) \cos H - \frac{1}{2r} \cos f \right\} \psi_0 dz,$$

and

$$\eta(r, z) = \frac{z}{\lambda^2 + r^2 + z^2}, \qquad \zeta = -\frac{r}{\lambda^2 + r^2 + z^2},$$

$$H = -\frac{2r}{\sqrt{\lambda^2 + r^2}} \tan^{-1} \left(\frac{z}{\sqrt{\lambda^2 + r^2}} \right)$$

- **Instanton vs. Skyrmion:** No vector mesons



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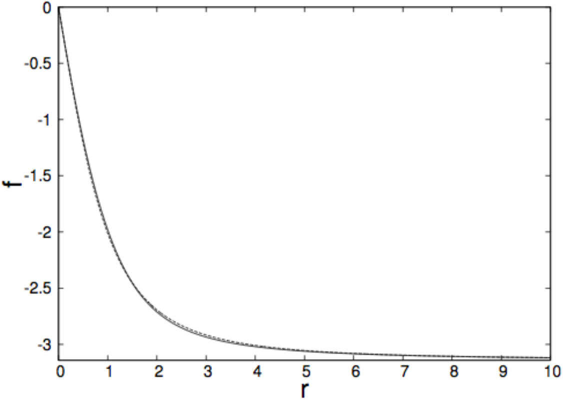
- **Instanton vs. Skyrmion:** One vector meson added



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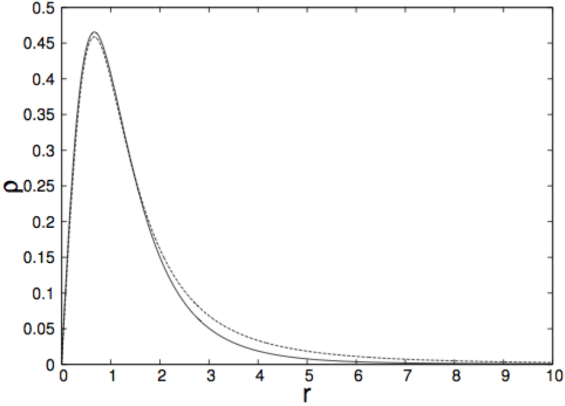

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Numerical setup

- **How?**
- **Where?**
- **When?**

Numerical setup

- **How?**

  Annealing with higher order derivatives

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- **When?**

Numerical setup

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- **Where?**

  Regular lattice:

$$n = 130, \qquad\qquad h = 0.08$$

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Numerical setup

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- **Where?**

  Regular lattice:

$$n = 130, \qquad\qquad h = 0.08$$

- **When?**

  After a reasonable running time.

Numerical setup

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  Regular lattice:

$$n = 130, \qquad h = 0.08$$

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- **Really?**

Numerical setup

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  Annealing with higher order derivatives

- **Where?**

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$$n = 130, \qquad h = 0.08$$

- **When?**

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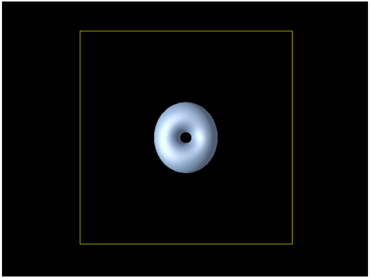
- **Really?**

  Parallel programming: MPI

Product Ansatz for B = 2

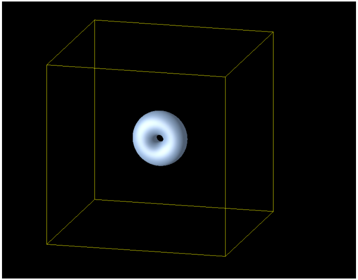
- Product ansatz:

$$n = 130, \qquad h = 0.08$$



- Rational map:

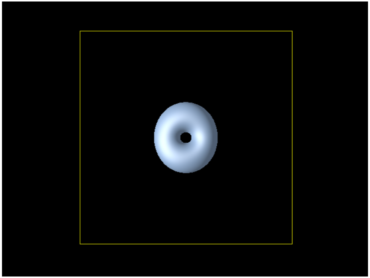
$$n = 130, \qquad h = 0.08$$



Product Ansatz for B = 2

- Product ansatz:

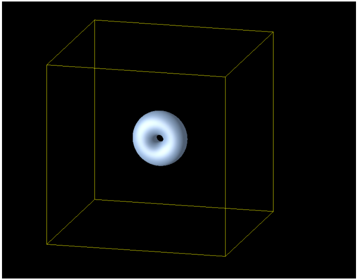
$$n = 130, \qquad h = 0.08$$



E=1.0476

- Rational map:

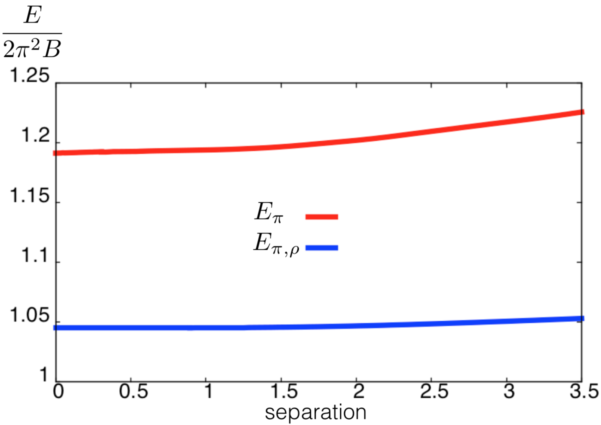
$$n = 130, \qquad h = 0.08$$



E=1.0475

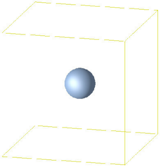
Product Ansatz for B = 2

- Energy per baryon (in units of $2\pi^2$) as a function of the distance

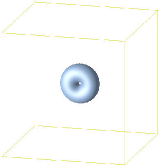


Solutions B = 1 - 4: n = 130, h=0.08

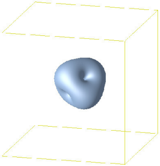
C. Naya and P. Sutcliffe, JHEP **1805** (2018) 174



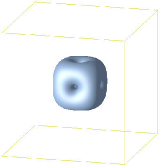
B=1: O(3) $\Rightarrow$ E=1.0624



B=2: $\mathrm{D}_{\infty \mathrm{h}}$ $\Rightarrow$ E=1.0475



B=3: T_{d} $\Rightarrow$ E=1.0372



B=4: O_{h} $\Rightarrow$ E=1.0286

Solutions B = 5 - 8: n = 130, h=0.08

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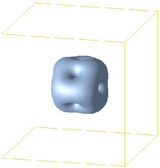
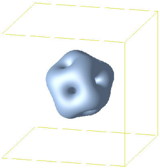
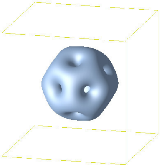


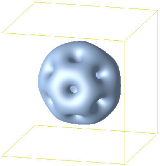
B=5: D_{2d} ⇒ E=1.0288



B=6: D_{4d} ⇒ E=1.0260



B=7: Y_h ⇒ E=1.0216



B=8: D_{6d} ⇒ E=1.0233

Solutions B = 9 - 12: n = 130, h=0.08

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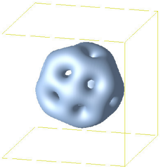
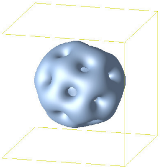
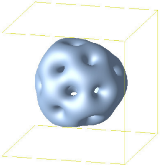


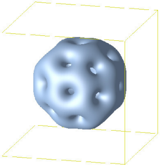
B=9: $D_{4d} \Rightarrow$ E=1.0234



B=10: $D_{4d} \Rightarrow$ E=1.0225



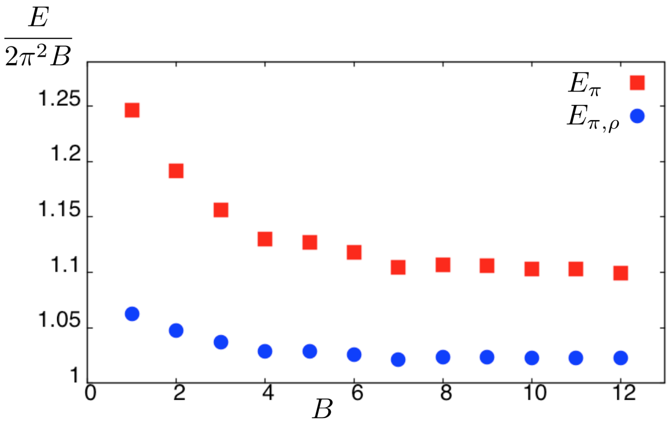
B=11: $D_{3h} \Rightarrow$ E=1.0231



B=12: $T_d \Rightarrow$ E=1.0225

Comparing with the standard Skyrme Model

- Energy per baryon (in units of $2\pi^2$) for Skyrmions with B=1-12.



Feeding the family: the pion mass

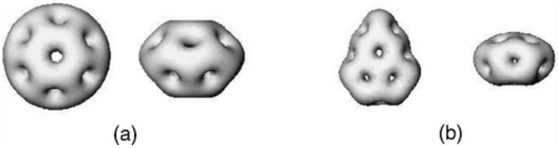
Pion mass term added to the Skyrme contribution to the energy as usual

$$E_{\rm S} = \int \left(-\frac{c_1}{2}\mathrm{Tr}(R_i R_i) - \frac{c_2}{16}\mathrm{Tr}([R_i, R_j]^2) + \hat{m}^2 \mathrm{Tr}(1 - U) \right) d^3x,$$

where $\hat{m}^2 = m^2 \frac{c_1^2}{c_2}$ and $m = \frac{2m_\pi}{F_\pi e} = 0.526$

Usual massive Skyrme model as function of the pion mass

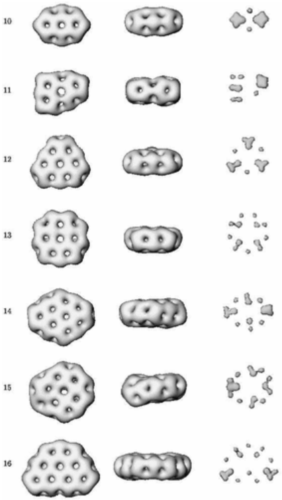
- For high enough pion mass, unstable shell like structures.
- Flatter configurations are preferred.



Baryon number B = 8 with (a) $m = 1$ and (b) $m = 2$.

Extracted from: R. Battye and P. Sutcliffe, Phys. Rev. C **73** (2006) 055205.

Skyrmions with no vector mesons for B = 10 - 16: Pion mass $m = 1$



Extracted from: R. Battye and P. Sutcliffe, Phys. Rev. C **73** (2006) 055205.

Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

  Regular lattice:

$$n = 128, \qquad h = 0.08$$

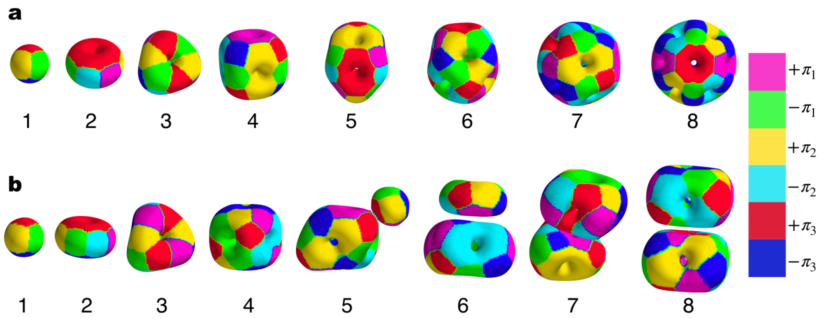
- **When?**

  After a reasonable running time.

- **Really?**

  Parallel programming: MPI

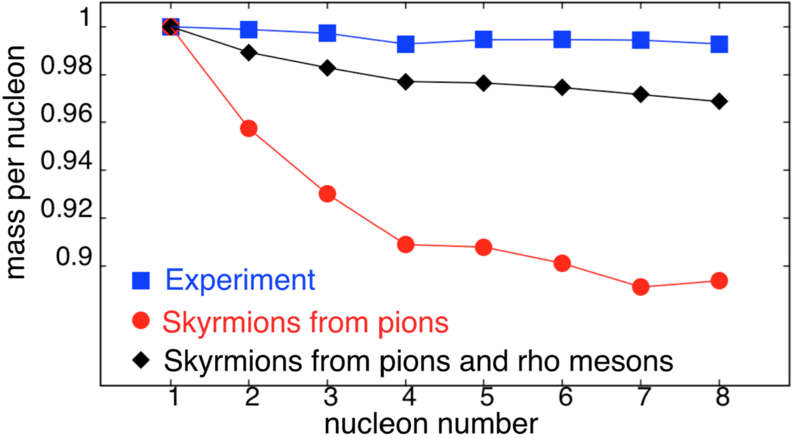
Solutions B = 1 - 8: n = 128, h=0.08, m = 0.526



Baryon density isosurfaces: (a) Standard Skyrme model (b) Extended version with massive pions and rho mesons.

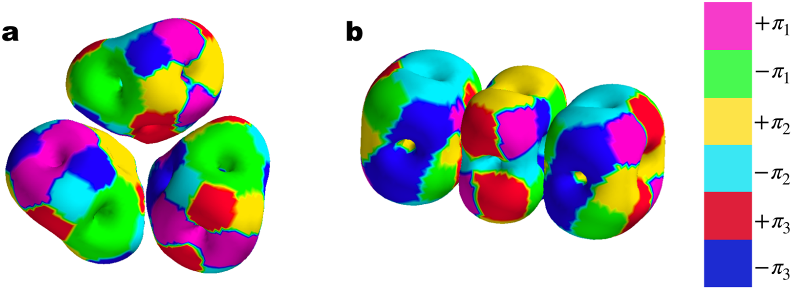
C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

Mass per nucleon normalised by the nucleon (proton) mass:
n = 128, h=0.08, m = 0.526



C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

Solution B = 12: n = 128, h=0.08, m = 0.526



Baryon density isosurfaces: (a) 3α triangular structure (^{12}C ground state) (b) 3α linear chain (Hoyle state).

C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

Conclusions

- The Skyrme model as an effective field theory in the low energy limit of QCD.
- Baryons as excitations of the mesonic degrees of freedom (pions).
- Further mesons can be included.
- BPS model with the Skyrme field coupled to an infinite tower of vector mesons.
- Explanation of Atiyah-Manton's instanton approximation.
- Even the first vector meson moves the energy closer to the bound in a significative way.
- Dramatic effect of pion mass even at the usual value.
- Two main problems solved: lower binding energies and clustering!


obrigado
Dank U
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Děkuju
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