Novel finite density phase of Chern-Simons scalar theory

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Topological Solitons, Nonperturbative Gauge Dynamics and Confinement 2

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Introduction and Motivation

- Classification of possible states of matter remains one of the outstanding, fascinating challenges.
- In lower dimensional QFTs, particularly 2+1 D, novel effects arise from inclusion of Chern-Simons interactions
- Appearance of Bose-Fermi and particle-vortex dualities in 3D suggests potentially rich vacuum and thermodynamic phase structure. [Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin (2012); Aharony-Gur-Ari-Yacoby (2012); Aharony (2015); Seiberg-Senthil-Wang-Witten (2016); Karch-Tong (2016); Murugan-Nastase (2016) ...]

Introduction and Motivation

- Non-Abelian C-S and Bose-Fermi dualities, e.g:

  $SU(N)_k + N_f\text{-scalars} \leftrightarrow U(k-N)_{-k+\frac{N_f}{2},-N+\frac{N_f}{2}} + N_f\text{-fermions}$

- Theory with massless bosons(fermions) $\leftrightarrow$ Gauged critical fermions (bosons)

- Large-N 't Hooft limit: Fixed line of theories with $\lambda \equiv N/k$, and $0 \leq \lambda \leq 1$.

- Duality maps theory A with λ to theory B with $1-\lambda$

Thermodynamics

- Large-N free energy of $U(N)$ plus single scalar/fermion matched at high T $(\mathbb{R}^2 \times S^1_\beta)$. [Aharony-Giombi-Gur-Ari-Maldacena-Yacoby (2013)]

- Large-N thermal partition functions on $S^2 \times S^1_\beta$. [Jain-Minwalla-et al (2013)]

$$\left\langle e^{T^2 V_{\rm eff}(U)}\right\rangle_{N,\lambda} = \left\langle e^{T^2 \tilde{V}_{\rm eff}(U)}\right\rangle_{k-N,1-\lambda}$$

  $U =$ Polyakov loop/holonomy.

- What happens to the theories at zero temperature and finite density ?

$SU(2)$ Chern-Simons with a scalar

- $\mathcal{L} \,=\, -D_\mu\Phi^\dagger D^\mu\Phi \,-\, U(|\Phi|) \,+\, S_{\rm CS}$

- Signature: $(-++)$ and $D_\mu \equiv \partial_\mu \,+\, A_\mu$;

- $$S_{\rm CS} \,=\, \frac{k}{4\pi}\int d^3x\,\epsilon^{\mu\nu\rho}\,{\rm Tr}\left(A_\mu\partial_\nu A_\rho \,+\, \frac{2}{3}A_\mu A_\nu A_\rho\right)\,.$$

- Global $U(1)_B$ symmetry ("baryon" number):

$$U(1)_B: \qquad \Phi \,\to\, e^{i\vartheta}\Phi\,.$$

- Conserved current:

$$j^\mu_B \,=\, i\left[(D^\mu\Phi)^\dagger\Phi \,-\, \Phi^\dagger D^\mu\Phi\right]$$

Chemical potential for $U(1)_B$

- Chemical potential for $U(1)_B \leftrightarrow$ Background temporal gauge field:

$$\boxed{D_0 \to D_0 + i\mu_B}$$

- Gauge interactions $\sim \frac{1}{\sqrt{k}}$

- Perturbative regime $k \gg 1$; decoupling limit: $k \to \infty$

- Pure scalar theory has the classical effective potential:

$$V_{\rm scalar}(\mu_B, k \to \infty) = U(|\Phi|) - \mu_B^2 \, \Phi^\dagger \Phi$$

- Generically, $U(|\Phi|) = m^2|\Phi|^2 + g_4|\Phi|^4 + g_6|\Phi|^6 \implies$ Bose condensation for $\mu_B > m$.
(No stable ground state if $g_4 = g_6 = 0$)

The (semiclassical) ground state for $k \gg 1$

- What happens to Bose condensed state for finite (large) k ?

- Assume a scalar VEV: $$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Useful to adopt an internal SO(3) vector notation for gauge fields:

$$A_\mu = \frac{i}{2}\sigma^a A_\mu^{(a)}, \qquad \mathbf{A}_\mu = (A_\mu^{(1)}, A_\mu^{(2)}, A_\mu^{(3)})$$

- Equations of motion:

$$v^2 \mathbf{A}_y = \frac{k}{2\pi}\mathbf{A}_0 \times \mathbf{A}_x, \qquad v^2 \mathbf{A}_x = \frac{k}{2\pi}\mathbf{A}_y \times \mathbf{A}_0,$$

$$-v^2\left(\mathbf{A}_0 - 2\mu_B \mathbf{e}^3\right) = \frac{k}{2\pi}\mathbf{A}_x \times \mathbf{A}_y,$$

$$\frac{v}{2}\left[\left(\mathbf{A}_0 - 2\mu_B \mathbf{e}^3\right)^2 - (\mathbf{A}_x)^2 - (\mathbf{A}_y)^2\right] = \frac{\partial V}{\partial v}.$$

Bose condensed ground state

- $\mu_B < m$: Only trivial solution $v = 0$ allowed.
- $\mu_B > m$: $v \neq 0$ and $\langle \mathbf{A}_\mu \rangle \neq 0$
- $\langle \mathbf{A}_x \rangle$, $\langle \mathbf{A}_y \rangle$ and $\langle \mathbf{A}_0 \rangle$ are mutually orthogonal in internal space:

$$\boxed{\langle \mathbf{A}_0 \rangle = \frac{2\pi v^2}{k} \mathbf{e}^3 \quad \langle \mathbf{A}_x \rangle = a_1 \, \mathbf{e}^1 + a_2 \, \mathbf{e}^2 \quad \langle \mathbf{A}_y \rangle = \left(-a_2 \, \mathbf{e}^1 + a_1 \, \mathbf{e}^2\right)}$$

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$$(a_1)^2 + (a_2)^2 = \frac{4\pi v^2}{k} \left(\mu_B - \frac{v^2 \pi}{k} \right) , \qquad \mu_B > \frac{\pi v^2}{k} .$$

Bose-condensed ground state

- Unique solution for VEV v (set $g_6 = 0$ for simplicity):

$$v^2 = \frac{k}{3\pi}\left(\frac{g_4 k}{\pi} + 2\mu_B - \sqrt{\left(\frac{g_4 k}{\pi} + 2\mu_B\right)^2 - 3(\mu_B^2 - m^2)}\right)$$

- Set $m = 0$ for simplicity. Weak coupling limit:

$$k \gg 1 \qquad g_4 \ll 1 \qquad \frac{\mu_B}{g_4 k}\ \text{fixed.}$$

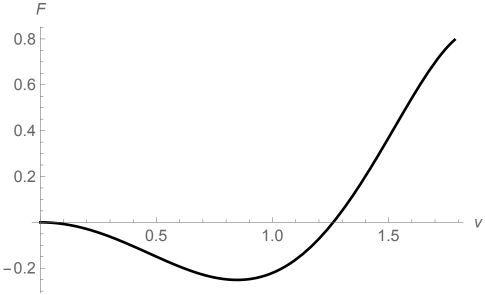
- Theory with no scalar potential i.e. $m = g_4 = g_6 = 0$ also exhibits a Bose condensed state with

$$v^2 = \frac{\mu_B k}{3\pi}\,.$$

C-S interaction stabilizes the ground state $\Longrightarrow$ Bose condensate in scale-invariant theory.

Classical effective (grand) potential

- Effective potenatial favours the Bose condensed phase ($g_4 = g_6 = 0$) whenever $\mu_B > m$



Color-flavor locked symmetry

- Scalar VEV breaks $U(1)_B$ and $SU(2)_{\rm Colour}$, but left invariant by $U(1)_{B+C}$:

$$U(1)_B:\ \langle\Phi\rangle\ \to\ e^{i\vartheta/2}\langle\Phi\rangle \qquad U(1)_C:\ \langle\Phi\rangle\ \to\ U(\vartheta)\langle\Phi\rangle$$

$$U(\vartheta)\ \equiv\ e^{i\vartheta\sigma_3/2}$$

- $U(1)_C$ acts on $\langle A_i\rangle$ exactly as rotation (R) in $x-y$ plane.
- Ground state left invaraint by $U(1)_{B+C+R}$
- Gauge invariant operators made from field strengths are rotationally invariant:

$$\langle {\rm Tr}\,(F_{0i})^2\rangle\ =\ -\frac{2\pi^3 v^6}{k^3}\left(\mu_B-\frac{v^2\pi}{k}\right)$$

$$\langle {\rm Tr}\,(F_{ij})^2\rangle\ =\ -\frac{8\pi^2 v^4}{k^3}\left(\mu_B-\frac{v^2\pi}{k}\right)^2.$$

Fluctuation Spectrum

- **Limit** $k \to \infty$: Gauge fields decouple; $O(4)$ global symmetry with a $U(1)_B$ chemical potential $\implies$ 2 gapped and 2 gapless modes:

$$\omega_{\mathrm{I}(-)} = \frac{|\mathbf{p}|}{\sqrt{3}} + \ldots\,, \qquad \omega_{\mathrm{II}(-)} = \frac{\mathbf{p}^2}{2\mu_B} + \ldots$$

Linear dispersion $\leftrightarrow$ phonon $\to$ Goldstone mode for broken $U(1)_B$. Ground state *not* a superfluid due to second gapless mode with quadratic dispersion relation. (Landau criterion)

- **Finite, large** k: $U(1)_B$ broken spontaneously $\implies$ One massless phonon

Fluctuation Spectrum

- Expand action to quadratic order in fluctuations

$$A_\mu = \langle A_\mu \rangle + \mathcal{A}_\mu\,, \quad \Phi = \langle \Phi \rangle + \delta\Phi\,, \quad \delta\Phi \equiv \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

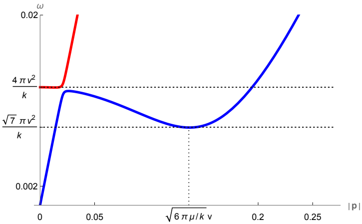
- Include R_ξ gauge-fixing term

$$\mathcal{L}^{(2)} \to \mathcal{L}^{(2)} + \mathcal{L}_{\rm gf}\,,$$
$$\mathcal{L}_{\rm gf} = \frac{1}{2\xi}\mathrm{Tr}\left(\mathcal{D}_\mu\mathcal{A}^\mu - \xi\langle\Phi\rangle\delta\Phi^\dagger + \xi\delta\Phi\langle\Phi^\dagger\rangle\right)^2 .$$

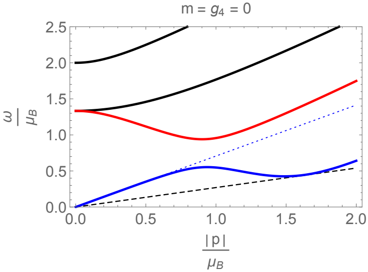
- Compute determinant of fluctuation matrix: ξ-independent zeroes yield the physical state dispersion relations

Fluctuation Spectrum

- Spectrum exhibits phonon branch with a “roton” minimum



Large-k case



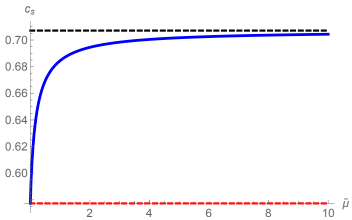
Scale-invariant case

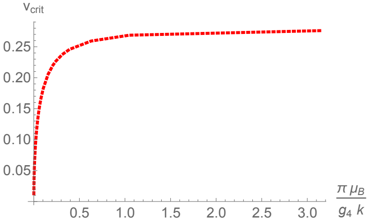
- Roton minimum governs Landau critical velocity, above which superfluidity ceases (non-relativistic expression):

$$v_{\rm crit} = \min_{|\mathbf{p}|}\left(\frac{\omega(\mathbf{p})}{|\mathbf{p}|}\right)$$

Rotons and critical velocity

- Above Landau critical velocity, rotons are expected to "condense" giving rise to some periodic structure. [Pitaevskii (1984)]





- Sound speed c_s and $v_{\rm crit}$ as a function of $\mu_B/(g_4 k)$.

Limit $\mu_B/g_4 k \to \infty \to$ Scale invariant limit $c_s = \frac{1}{\sqrt{2}}$.

Generalizing to $SU(N)$

- For $\mu_B > m$, VEV for scalar $\langle\Phi\rangle = (0, 0 \ldots v)^T$
- Gauge field VEVs: $\langle A_\pm\rangle = i\,(\langle A_x\rangle \pm i\langle A_y\rangle)$

$$[\langle A_+\rangle, \langle A_-\rangle] = \#\langle A_0\rangle \sim \left[\begin{array}{c|c} \mathbf{1}_{(N-1)\times(N-1)} & 0 \\ \hline 0 & 1-N \end{array}\right]$$

- $\langle A_+\rangle^N = \langle A_-\rangle^N = 0$. $\langle A_\mu\rangle$ realize finite dimensional representation of oscillator algebra.
- These also describe the noncommutative disc with non-commutativity parameter $\sim 1/N$:

$$(i\langle A_x\rangle)^2 + (i\langle A_y\rangle)^2 = \frac{\#}{N}\mathrm{diag}\,(1\,,\,3\,,5\,,\ldots\,,(2N-3)\,,\,(N-1))\;.$$

Generalizing to $SU(N)$

- The radius of the disc/droplet is bounded at large N:

$$R_{\rm droplet}\big|_{N\to\infty} = 2\beta\sqrt{\frac{\mu_B}{\beta}-1}\,, \qquad \beta = \frac{2\pi N v^2}{k}\,.$$

- Ground state invariant under $U(1)_{B+C+R}$. Spatial rotations, undone by a $U(1)_C \subset SU(N)$:

$$U(1)_C: \quad \langle A_j\rangle \to e^{i\vartheta J_3}\,\langle A_j\rangle\, e^{-i\vartheta J_3}$$

$$J_3 \equiv {\rm diag}\left(-\tfrac{N-1}{2}, -\tfrac{N-3}{2}, -\tfrac{N-5}{2}\ldots, \tfrac{N-3}{2}, \tfrac{N-1}{2}\right)\,.$$

- Gauged matrix quantum mechanics with same background matrices describe the fractional quantum Hall droplet .

[Polychronakos (2001), Susskind (2001), Tong (2003)].

Generalizing to SU(N)

- Physical excitations live on edge of droplet $\leftrightarrow$ N-th row/column of $N \times N$ matrices.
- The ground state appears to "deconstruct" a theory on $\mathbb{R}^3 \times Disc_{\rm noncomm}$
- Gauge-invariant baryon-like order parameters for breaking of $U(1)_B$:

$$\langle B_z \rangle \equiv \langle \Phi^{a_1} \left(D_z\Phi\right)^{a_2} \left(D_z^2\Phi\right)^{a_3} \ldots \left(D_z^{N-1}\Phi\right)^{a_N} \epsilon_{a1a2\ldots a_N} \rangle \neq 0$$

where $z = x + iy$.

Open questions

- Ground state breaks $U(1)_B \implies$ expect superfluid vortex solutions. (Separable ansatz in polar coordinates appear not to work)
- At any non-zero temperature, the effective theory is two dimensional, and vortex-antivortex pairs are excited, and above a critical temperature we expect Kosterlitz-Thouless transition to destroy the condensate.
- Bose-Fermi duality applies in high T phase. Does it also apply in the low T ordered phase ?
- Present goal is to understand the full spectrum of fluctuations at large-N using the connection to the noncommutative quantum Hall droplet picture.