S-Fold SCFTs and Supersymmetry Enhancement

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This talk is based on:

- [1810.12323] with Gabriele Lo Monaco and Noppadol Mekareeya,
- [1901.10493] with Gabriele Lo Monaco and Noppadol Mekareeya,
- [1905.07183] with Gabriele Lo Monaco and Noppadol Mekareeya and Matteo Sacchi.

Plan of the talk

1. **3d $\mathcal{N} = 4$ gauge theory and mirror symmetry**

2. **S-fold SCFTs**

3. **Supersymmetry enhancement**

3d $\mathcal{N} = 4$ gauge theory

- A large class of 3d $\mathcal{N} = 4$ gauge theories can be engineered via brane systems involving D3, D5, NS5 [Hanany, Witten '96]

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x				x			
NS5	x	x	x	x	x	x				
D5	x	x	x					x	x	x

- The theories living on the worldvolume of D3 branes have R-symmetry $SU(2)_V \times SU(R)_H$.
- Two branches of the moduli space of vacua:
  - **Higgs Branch**: parametrized by gauge invariant operators built from scalars in the h-plet. It is classically exact.
  - **Coulomb Branch**: Parametrized by dressed monopole operators. It is **not** classically exact, receives **quantum corrections**.

Mirror Symmetry

- In [Intriligator, Seiberg '96] a duality has been discovered involving 3d $\mathcal{N} = 4$ gauge theories. Such a duality acts in the following way:
  - Exchanges $SU(2)_V$ and $SU(2)_H$;
  - exchanges the Higgs and Coulomb branch;
  - exchanges mass terms and Fayet-Iliopulos terms.

  Quantum effects (on the Coulomb branch) in one duality frame appears as **classical** effects on the other frame.
- In HW brane set-up mirror symmetry amounts to an $\mathrm{SL}(2, \mathbb{Z})$ transformation supplemented by a rotation $x^j \to x^{j+4}$, $x^{j+4} \to -x^j$ for $j = 3, 4, 5$.

$T[U(N)]$ theory

- The theory living on 1/2 BPS boundary conditions in $\mathcal{N}=4$ SYM is $T[SU(N)]$ theory, whose quiver description is [Gaiotto, Witten '08]:

$$\underset{1}{\circ} - \underset{2}{\circ} - \cdots - \underset{N-1}{\circ} - \underset{N}{\Box}\ .$$

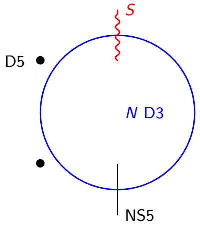
- Variant of this theory: $T[U(N)] = T[SU(N)] \times T[U(1)]$, where $T[U(1)]$ is an *almost empty theory*, given by a $U(1) \times U(1)$ background v-plet plus an $\mathcal{N}=4$ background mixed CS term at level 1 between these two $U(1)$'s.
- $T[U(N)]$ is a *self-mirror* theory whose Higgs and Coulomb branch are given by the nilpotent cone of $SU(N)$:

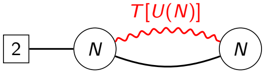
$$\mathcal{N}_{SU(N)} = \{M_{N\times N} : M^N = 0\}, \qquad \dim_{\mathbb{H}} \mathcal{N}_{SU(N)} = \frac{1}{2}N(N-1)$$

A simple example

[IG, G. Lo Monaco, N. Mekareeya '18]

- Let us consider **compact models**, namely we take the direction x^6 in the brane system to be compact. An example is the following





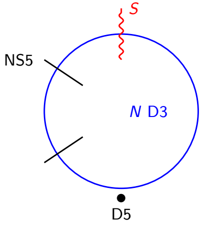
- The dimension of the Higgs branch is computed via the Higgs mechanism:

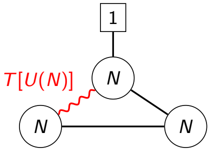
$$\dim_{\mathbb{H}} \mathcal{H} = 2N + 2\left[\frac{1}{2}(N-1)N\right] + N^2 - N^2 - N^2 = N$$

- What about the Coulomb branch of this theory?

A simple example

- Let us consider the mirror theory (by definition the S-duality wall is invariant)



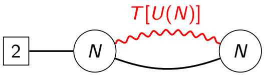


- The dimension of the Higgs branch is:

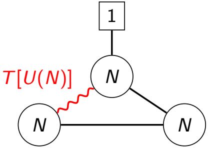
$$\dim_{\mathbb{H}} \mathcal{H} = N^2 + N^2 + N + 2\left[\frac{1}{2}(N-1)N\right] - 3N^2 = 0$$

A simple example

- Using mirror symmetry we learn that:



$\dim_{\mathbb{H}} \mathcal{C} = 0$

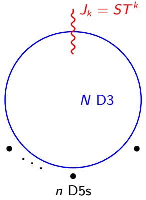


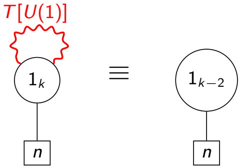
$\dim_{\mathbb{H}} \mathcal{C} = N$

- With respect to the case of the same theories with only usual hypermultiplets and no $T[U(N)]$-links we find that the Coulomb branch is much smaller (by $2N$ for both cases).
- **Freezing rule:** Nodes connected by $T[U(N)]$-links have "frozen" scalars in the corresponding vector multilplets. [IG, G. Lo Monaco, N. Mekareeya '18]
- Segments of D3 branes that are *cut* by an S-duality wall are "stuck" and do not posses Coulomb moduli.

The case of Chern-Simons

- Consider an Abelian theory in the presence of CS level k and a $T[U(1)]$ link





- $T[U(1)]$ contributes with a shift of -2 of the CS level; the superpotential and vacuum equations are

$$W = \tilde{Q}_i \varphi Q^i + \frac{1}{2}(k-2)\varphi^2, \quad i = 1, \ldots, n$$

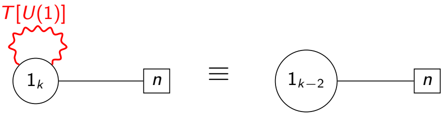
F-terms: $\tilde{Q}_i Q^i + (k-2)\varphi = 0,$

$$\tilde{Q}_i \varphi = \varphi Q^i = 0, \;\; Q^i \sigma = \sigma \tilde{Q}_i = 0$$

D-terms: $(Q^\dagger)_i Q^i - \tilde{Q}_i (\tilde{Q}^\dagger)^i = (k-2)\sigma$

The case of Chern-Simons

- The moduli space of



depends on n and k:

- $k = 2$: the superpotential is the one of the $U(1)$ with n flavours. The Higgs branch is

$$\mathcal{H} = \{M^i_j = Q^i \tilde{Q}_j \mid \text{rank}(M) \leq 1 \text{ and } M^2 = 0\} = \bar{\mathcal{O}}^{SU(n)}_{\text{min}}$$

The Coulomb branch is

$$\mathcal{C} = \{(V_\pm, \varphi) \mid V_+ V_- = \varphi^n\} = \mathbb{C}^2/\mathbb{Z}_n$$

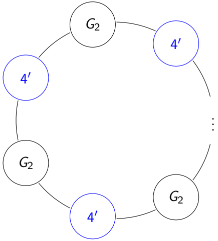
- $k \neq 2$: If $(\varphi, \sigma) \neq 0$ F-terms implies $Q = \tilde{Q} = 0$, but this is in contraddiction with D-terms, hence $\mathcal{C} = \{0\}$. The Higgs branch is the same as before.

- $k \neq 2 \;\; n = 1$: trivial moduli space.

A model with G_2 gauge group

[IG, G. Lo Monaco, N. Mekareeya '19]

- Consider a circular quiver with alternating $G_2/USp(4)'$ gauge nodes:



$2n$ alternating $G_2/USp'(4)$ circular nodes

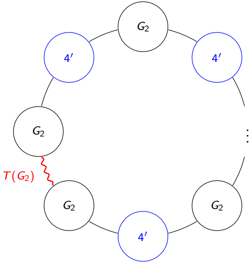
- The dimensions of Higgs and Coulomb branch are equal:

$$\dim_{\mathbb{H}}\mathcal{H} = \dim_{\mathbb{H}}\mathcal{C} = 4n$$

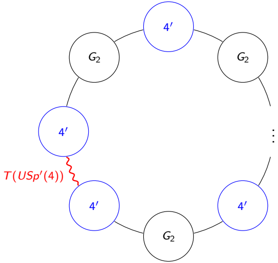
- No brane construction.
- $USp(4)'$ is consistent with the S-folding procedure in that nodes yielding to another self-mirror quiver.

S-folding the G_2 model

- It is possibile to perform the S-folding procedure on the previous quiver both on the G_2 and on the $USp(4)'$ nodes to get other two self-mirror models



n blue nodes + $(n-1)$ G_2 usual circular nodes + 2 G_2 nodes connected by $T(G_2)$



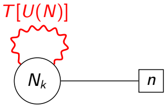
n G_2 circular nodes + $(n-1)$ blue usual circular nodes + 2 blue nodes connected by $T(USp'(4))$

$$\dim_{\mathbb{H}}\mathcal{H} = \dim_{\mathbb{H}}\mathcal{C} = 2(2n-1)$$

$$\dim_{\mathbb{H}}\mathcal{H} = \dim_{\mathbb{H}}\mathcal{C} = 2(2n-1)$$

Supersymmetry enhancement

- The effective description of S-fold SCFTs amounts to gauge both Higgs and Coulomb branch symmetries of $T[U(N)]$, leading to a naive breaking of the $SU(2) \times SU(2)$ R-symmetry to the diagonal $SU(2)$, namely the theory should actually have $\mathcal{N} = 3$ supersymmetry.
- In [Assel, Tomasiello '18] the following statements are made on the supersymmetry of this model **at large** N



  - $k = 0$: the theory has $\mathcal{N} = 3$ supersymmetry
  - $k \geq 3$ and $n = 0$: the theory has $\mathcal{N} = 4$ supersymmetry.
- In this talk we will discuss can study the actual amount of supersymmetry of this theory in the IR at **finite** N (low rank) via the **supersymmetric index**.

Supersymmetric Index

- The 3d supersymmetric index is defined as the partition function on $S^2 \times \mathbb{R}$

  [Bhattacharya, Bhattacharyya, Minwalla, Raju, '08],...

  $$\mathcal{I}(x, \boldsymbol{\mu}) \, = \, \text{Tr}\left[(-1)^{2J_3} x^{\Delta + J_3} \prod_i \mu_i^{T_i}\right]$$

  where

  - Δ: energy in units of the S^2 radius (for SCFTs is related to the R-charge)
  - J_3: Cartan of the $SO(3)$ isometry of S^2
  - T_i: charges for the global non-R symmetries.

- Expanding in series of x one can recast the index in the form

  $$\mathcal{I}(x, \boldsymbol{\mu}, \boldsymbol{n} = 0\}) \, = \sum_{p=0}^{\infty} \chi_p(\boldsymbol{\mu}) \, x^p \, = 1 + a_1 x + a_2 x^2 + \ldots$$

  where $\chi_p(\boldsymbol{\mu})$ is the character of a certain representation of the global symmetry.

Classification of multiplets

- $\mathcal{N}=2$ multiplets
  [Razamat, Zafrir' 08], [Cordova, Dumitrescu, Intriligator '16]

Multiplet	Contribution to the modified index	Comment
$A_2\bar{B}_1[0]^{(1/2)}_{1/2}$	$+x^{1/2}$	free fields
$B_1\bar{A}_2[0]^{(-1/2)}_{1/2}$	$-x^{3/2}$	free fields
$L\bar{B}_1[0]^{(1)}_{1}$	$+x$	relevant operators
$L\bar{B}_1[0]^{(2)}_{2}$	$+x^2$	marginal operators
$A_2\bar{A}_2[0]^{(0)}_{1}$	$-x^2$	conserved currents

- Decomposition of $\mathcal{N}=3$ multiplets in $\mathcal{N}=2$ ones

Type	$\mathcal{N}=3$ multiplet	Decomposition into $\mathcal{N}=2$ multiplets
Flavour current	$B_1[0]^{(2)}_{1}$	$L\bar{B}_1[0]^{(1)}_{1}+B_1\bar{L}[0]^{(1)}_{-1}+A_2\bar{A}_2[0]^{(0)}_{1}$
Extra SUSY-current	$A_2[0]^{(0)}_{1}$	$A_2\bar{A}_2[0]^{(0)}_{1}+A_1\bar{A}_1[1]^{(0)}_{3/2}$
Stress tensor	$A_1[1]^{(0)}_{3/2}$	$A_1\bar{A}_1[1]^{(0)}_{3/2}+A_1\bar{A}_1[2]^{(0)}_{2}$

- Observe that

$$(-a_2)-a_1=\#(\text{extra SUSY current multiplets})$$

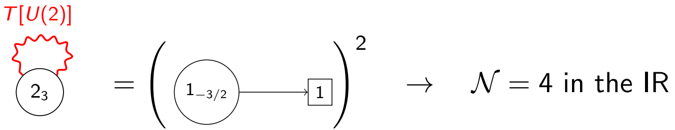
An $\mathcal{N} = 5$ model

- Let us consider the abelian case for $k = 1$ and $n = 1$, corresponding to a $\mathcal{N} = 3$ $U(1)_{-1}$ with 1 flavour. The index is:
  $\mathcal{I}(x;\omega) = 1 + x - x^2\left(\omega + \omega^{-1} + 1\right) + x^3\left(\omega + \omega^{-1} + 2\right) + \dots$
- Interpretation of the various terms:
  - $+x$: there is an $\mathcal{N} = 3$ flavour current $B_1[0]_1^{(2)}$, decomposing as
  $$B_1[0]_1^{(2)} \to L\bar{B}_1[0]_1^{(1)} \oplus A_2\bar{A}_2[0]_1^{(0)}$$
  $$(+x) \to (+x) \oplus (-x^2)$$
  - $+x^2[-(1 + \omega + \omega^{-1})]$: there are two sets of $\mathcal{N} = 3$ extra SUSY current multiplets $A_2[0]_1^{(0)}$, each carrying fugacities ω and ω^{-1}.
- From the analysis of the index it arises that the $U(1)_{-1}$ theory with 1 flavour gets enhanced from $\mathcal{N} = 3$ to $\mathcal{N} = 5$ in the IR.

$U(2)_k$ with no flavour

- Consider the theory

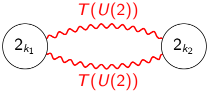
- In [IG, Lo Monaco, Mekareeya, Sacchi '19] it is shown that the gauge group can also be taken to as $SU(2)$.
- In particular, in [Gang, Yamazaki '18] it was shown that



- Supersymmetry is enhanced to $\mathcal{N} = 4$ for all values of k such that $|k| \geq 4$ [IG, Lo Monaco, Mekareeya, Sacchi '19]

A model with two T-links

- Let us take the theory

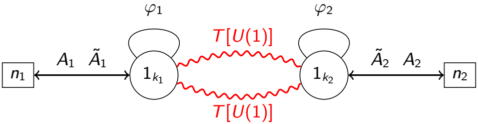


- The Z_{S^3} at large N for the theory $U(N)_{k_1} \times U(N)_{k_2}$ with $\pm(k_1 k_2 - 2) > 2$ shows $\mathcal{N} = 4$ supersymmetry [Assel, Tomasiello '18]

- For $N = 2$ and different values of k_1 and k_2 the index shows that [IG, Lo Monaco, Mekareeya, Sacchi '19]:
  - the model with $(k_1, k_2) = (2, 3)$ is a new SCFT with $\mathcal{N} = 4$ supersymmetry
  - the theory with $(2, -1)$ is the same as the one in the previous slide for $k = \pm 4$.

Moduli space of flavoured abelian J-fold theories

- Take the quiver (in $\mathcal{N}=2$ language)



with superpotential

$$W = -\mathrm{tr}(A_1\varphi_1\tilde{A}_1 + A_2\varphi_2\tilde{A}_2) + \frac{1}{2}(k_1\varphi_1^2 + k_2\varphi_2^2) - 2\varphi_1\varphi_2\,.$$

- The vacuum equations are

$$\begin{aligned} A_1\varphi_1 &= \tilde{A}_1\varphi_1 = 0\,, \quad A_2\varphi_2 = \tilde{A}_2\varphi_2 = 0 \\ k_1\varphi_1 - 2\varphi_2 &= (A_1)_a(\tilde{A}_1)^a, \quad k_2\varphi_2 - 2\varphi_1 = (A_2)_i(\tilde{A}_2)^i \end{aligned}$$

where $a, b, c = 1, \ldots, n_1$ and $i, j, k = 1, \ldots, n_2$.

Moduli space of flavoured abelian J-fold theories

- For $\varphi_1 = \varphi_2 = 0$ and for every choice of (k_1, k_2) the vacuum equations admit a Higgs branch

$$\mathcal{H} = \{(M_1)_a^b = (A_1)_a(\tilde{A}_1)^b,\ (M_2)_i^j = (A_2)_i(\tilde{A}_2)^j \mid \\ \text{rank}(M_{1,2}) \leq 1\ ,\ \ M_{1,2}^2 = 0\} = \bar{\mathcal{O}}_{\text{min}}^{SU(n_1)} \times \bar{\mathcal{O}}_{\text{min}}^{SU(n_2)}$$

- The branch fo which $\varphi_1 \neq 0$ and $\varphi_2 \neq 0$ is described by the equations

$$k_1\varphi_1 = 2\varphi_2\,, \qquad k_2\varphi_2 = 2\varphi_1\,, \qquad k_1 k_2 - 4 = 0\,;$$

admitting solutions only for $(k_1, k_2) = (2, 2)$ or $(1, 4)$.

Moduli space of flavoured abelian J-fold theories

- Take $(k_1, k_2) = (2, 2)$. From the vacuum equations

$$\varphi_1 = \varphi_2 \equiv \varphi\,, \quad m_1 = m_2 \equiv m$$

- The R-charge and gauge charge of $V_{(m,m)}$ are:

$$R[V_{(m,m)}] = \frac{1}{2}(n_1 + n_2)|m|\,, \quad q_1[V_{(m,m)}] = 0\,, \;\; q_2[V_{(m,m)}] = 0\,.$$

- $V_{(m,m)}$ are gauge neutral and hence this branch is

$$\mathcal{C} = \{(V_{\pm(1,1)}, \varphi) \mid V_{(1,1)} V_{-(1,1)} = \varphi^{n_1+n_2}\} = \mathbb{C}^2/\mathbb{Z}_{n_1+n_2}$$

- For $(k_1, k_2) = (2, 2)$ the moduli space admit a clear separation into Higgs and Coulomb branch as $\mathcal{N} = 4$ theories, due to the fact that monopole operators are gauge neutral.

Moduli space of flavoured abelian J-fold theories

- Take $\varphi_1 \equiv \varphi \neq 0$ and $\varphi_2 = 0$. The vacuum equations imply that $k_1 = 0$ and that

$$A_1 = \tilde{A}_1 = 0 \qquad (A_2)_i (\tilde{A}_2)^i = -2\varphi.$$

- Monopole fluxes (m_1, m_2) are of the form $(m, 0)$. The R-charge and gauge charge of the monopoles are

$$R[V_{(m,0)}] = \frac{1}{2} n_1 |m|, \qquad q_1[V_{(m,0)}] = 0, \quad q_2[V_{(m,0)}] = 2m$$

- The gauge invariant (dressed) monopole operators are

$$(W^+)^{ij} = V_{(1,0)} (\tilde{A}_2)^i (\tilde{A}_2)^j \ , \qquad (W^-)_{ij} = V_{(-1,0)} (A_2)_i (A_2)_j \ .$$

  and satisfy the quantum relation

$$(W^+)^{ij} (W^-)_{ji} = \varphi^{n_1 + 2} \ .$$

  that is a “mixed” Higgs/Coulomb branch.

Conclusion and Outlook

Summary

- A large class of 3d SCFTs obtained by inserting an SL(2,$\mathbb{Z}$) duality wall into the Type IIB brane system.
- The dynamics of these branes through mirror symmetry
- Supersymmetry in the IR via the supersymmetric index.

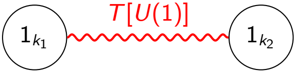
Outlook

- Engineering of the models with G_2 gauge groups either from brane picture or from F-theory.
- Extend these class of theories to lower supersymmetry, *e.g.* $\mathcal{N} = 2$ models.

Thank you!

$T[U(1)]$ theory

- The action for the following quiver [Kapustin, Strassler '99]



in the $\mathcal{N} = 2$ notation, is given by

$$\int d^3x d^4\theta \left(\frac{k_1}{4\pi} \Sigma_1 V_1 + \frac{k_2}{4\pi} \Sigma_2 V_2 - \frac{1}{4\pi} \Sigma_1 V_2 - \frac{1}{4\pi} \Sigma_2 V_1 \right)$$
$$- \int d^3x d^2\theta \left(\frac{k_1}{4\pi} \Phi_1^2 + \frac{k_2}{4\pi} \Phi_2^2 - \frac{1}{2\pi} \Phi_1 \Phi_2 + \text{c.c.} \right) .$$

- The various terms are

$\Sigma_{i=1,2}$	$\mathcal{N} = 2$ linear multiplet
$V_{i=1,2}$	$\mathcal{N} = 2$ vector multiplet
$\Phi_{i=1,2}$	$\mathcal{N} = 2$ χ-plet inside the $\mathcal{N} = 4$ v-plet

$T[SU(2)]$ vs $T[U(2)]$

- Given the index for $T[SU(2)]$ $\mathcal{I}_{T[(SU(2)]}(\{\boldsymbol{\mu},\mathbf{n}\},\{\boldsymbol{\tau},\mathbf{p}\})$, one has to impose the conditions $\mu_1\mu_2=\tau_1\tau_2=1$ and $n_1+n_2=p_1+p_2=0$ on fugacities and fluxes.

- The index of $T[U(2)]$ is

$$\mathcal{I}_{T(U(2))}(\{\boldsymbol{\mu},\mathbf{n}\},\{\boldsymbol{\tau},\mathbf{p}\}) = \left[\prod_{i=1}^{2}\mathcal{I}_{T(U(1))}(\{\mu_i,n_i\},\{\tau_i,p_i\})\right]\times\mathcal{I}_{T(SU(2))}(\{\boldsymbol{\mu},\mathbf{n}\},\{\boldsymbol{\tau},\mathbf{p}\})$$

- No need to impose any constraint
- $T[U(2)]$ self-mirror property translated into invariance of $\mathcal{I}_{T(U(2))}(\{\boldsymbol{\mu},\mathbf{n}\},\{\boldsymbol{\tau},\mathbf{p}\})$ under $\boldsymbol{\mu}\leftrightarrow\boldsymbol{\tau}$, $\mathbf{n}\leftrightarrow\mathbf{p}$.