Infrared enhancement of supersymmetry in four dimensions

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Lagrangians for nonlagrangian theories

RG flows exhibiting supersymmetry enhancement are interesting because they can e.g. provide lagrangian descriptions of strongly coupled theories with extended supersymmetry.

This happens for $\mathcal{N} = 2$ Argyres-Douglas (AD) theories:

- 4d $\mathcal{N} = 2$ SCFT's describing vector multiplets interacting with electrons and monopoles.
- Their Coulomb Branch coordinates have fractional dimension, so they do not have a lagrangian description.
- Originally found at singular points in the CB of $\mathcal{N} = 2$ gauge theories.

The lagrangian for (A_1, A_3) Argyres-Douglas

Consider $SU(2)$ adjoint SQCD with one flavor and the following superpotential: Maruyoshi,Song '16.

$$\mathcal{W} = \alpha_0 \tilde{q} q + \alpha_1 \tilde{q} \Phi q + \alpha_2 \tilde{q} \Phi^2 q$$

$\mathrm{Tr}\,\Phi^2$, α_1 and α_2 fall below the unitarity bound and decouple.

It turns out that the IR fixed point is the A_3 AD theory and the lowest component of α_0 is the corresponding CB operator of dimension $4/3$!

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Description of the RG flow

- Start from a $\mathcal{N}=2$ SCFT with a non abelian global symmetry G_F.
- Add a chiral multiplet M in the adjoint of G_F and turn on the superpotential $\mathcal{W}=\mu M$.
- Turn on a nilpotent vev for M.

The UV SCFT has $U(1)\times SU(2)$ R-symmetry and the vev always leaves two $U(1)$ symmetries unbroken ($R-2\rho$ and I_3)

$$R^{IR}_{\mathcal{N}=1}=\frac{1+\epsilon}{2}(R-2\rho)+(1-\epsilon)I_3$$

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Pattern of supersymmetry enhancement

- For $SU(N)$ or $USp(2N)$ SQCD the maximal and next-to-maximal nilpotent vevs induce susy enhancement.
- For $SO(N)$ SQCD enhancement never occurs.
- For the unitary quivers

$$\boxed{k} - SU(n+k) \ldots SU(mn+k) - \boxed{mn+n+k}$$

susy enhances only for $k = 0, 1$ and maximal nilpotent vev.

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Anomaly matching condition

If supersymmetry enhances, we have $U(2)$ R-symmetry in the infrared and the Cartan generators can be written in terms of $R - 2\rho$ and I_3 (assuming no accidental symmetries in IR).

Extended supersymmetry then implies: Kuzenko, Theisen '99.

1. $\mathrm{Tr}(I_3)^3_{IR} = 0$
2. $\mathrm{Tr}\, R_{IR} = 48(a_{IR} - c_{IR}); \quad \mathrm{Tr}\, R_{IR}(I_3)^2_{IR} = 8a_{IR} - 4c_{IR}$
3. $\mathrm{Tr}\, R^2_{IR}(I_3)_{IR} = 0; \quad \mathrm{Tr}(R_{IR})^3 - \mathrm{Tr}\, R_{IR} = 0$

In all known cases of susy enhancement the CB operators of the IR theory are either CB operators of the UV SCFT or singlets (i.e. components of M). All other singlets and UV CB operators decouple from the theory. **We assume this holds true in general.**

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Mapping R-symmetry generators

In $\mathcal{N}=2$ SCFT's the $R_{\mathcal{N}=1}$ symmetry s.t. $\Delta(\mathcal{O})=\frac{3}{2}R_{\mathcal{N}=1}(\mathcal{O})$ for all chiral operators $\mathcal{O}$ is the combination $R/3+4I_3/3$.

$$R^{IR}_{\mathcal{N}=1}=\frac{1+\epsilon}{2}(R-2\rho)+(1-\epsilon)I_3=\frac{1}{3}R_{IR}+\frac{4}{3}(I_3)_{IR}.$$

The independent combination $R_{IR}-2(I_3)_{IR}$ is a global symmetry w.r.t. the manifest $\mathcal{N}=1$ subalgebra (in particular all the components of a multiplet have the same charge).
For all CB operators in the IR (i.e. singlets or UV CB operators) u_i

$$(R_{IR}-2(I_3)_{IR})(u_i)=3R^{IR}_{\mathcal{N}=1}(u_i).$$

Combining the constraints above we find

$$R_{IR}=\frac{3+3\epsilon}{2}(R-2\rho)+(1+3\epsilon)I_3;\quad (I_3)_{IR}=I_3.$$

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The rank condition

For every 't Hooft anomaly the following have to match:

- Contribution from UV SCFT + singlets
- Contribution from IR SCFT + decoupled operators

The contribution to Tr I_3^3 from UV and IR SCFT's is trivial. The singlets and decoupled operators are free fields and their fermions have charge $-1/2$ under I_3. The anomaly matching condition reads

$$-\frac{1}{8}\text{number of singlets} = -\frac{1}{8}\text{number of decoupled operators}$$

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SQCD with SO(N) gauge group

The charge of UV CB operators and singlets under the trial R-symmetry is $(1+\epsilon)k$.
A-maximization fixes ϵ and operators with $k < k_c \equiv \frac{2}{3+3\epsilon}$ decouple.

Consider $SO(7)$ SQCD with 5 hypers in the $\mathbf{7}$ ($G_F = USp(10)$).
In the case of principal nilpotent vev:
for UV CB operators $k = 2, 4, 6$
and for the singlets $k = 2, 4, 6, 8, 10$.

Regardless of the value of ϵ the rank condition cannot be satisfied!

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Enhancement mechanism

Consider the $\mathcal{N}=1$ subalgebra of the UV SCFT preserved by the flow. Any CB multiplet decomposes into 4 chirals Dolan, Osborn '02.

$$U_{r,0},\ (U_\alpha)_{r-1,\frac{1}{2}},\ U_{r-2,1}$$

If $U_{r,0}$ decouples in the IR, the other multiplets should combine with another chiral to rebuild the $\mathcal{N}=2$ multiplet. Its charge under R_{IR} should be that of $(U_\alpha)_{r-1,\frac{1}{2}}$ plus one, leading to the map

$$u_k^{UV} \to u_{k'}^{IR}; \quad k'-k=-\frac{1+3\epsilon}{3+3\epsilon}.$$

From this we can predict the value of ϵ:

$$\frac{3}{2}(1+\epsilon)=\frac{1}{k_{s_{max}}+1-D_{max}}.$$

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Central charges of the IR SCFT

From anomaly matching equations for Tr R_{IR} and Tr $R_{IR} I_3^2$ we find:

$$a^{IR} = \frac{24a^{UV} + 5r(k_{s_{max}} - D_{max})}{24(k_{s_{max}} + 1 - D_{max})},$$

$$c^{IR} = \frac{6c^{UV} + r(k_{s_{max}} - D_{max})}{6(k_{s_{max}} + 1 - D_{max})}.$$

In particular we find the relation

$$(6c^{IR} - r)(4a^{UV} - 5c^{UV}) = (6c^{UV} - r)(4a^{IR} - 5c^{IR}).$$

The other two 't Hooft anomalies

Given a $\mathcal{N}=2$ SCFT consider $\Delta \equiv 8a-4c-\sum_i(2D_i-1)$.
The matching condition for $\mathrm{Tr}\, R_{IR}^2 I_3$ can be written as follows:

$$(1+3\epsilon)\Delta^{UV}=0; \quad (1+3\epsilon)\Delta^{IR}=0.$$

The matching condition for $\mathrm{Tr}\, R_{IR}^3 - \mathrm{Tr}\, R_{IR}$ reads (in the case of maximal nilpotent vev)

$$6c^{UV}-r=\frac{3k_{G_F}I_\rho-12(h_{G_F}-D_{max})\sum_i(D_i-1)^2}{2(h_{G_F}-D_{max})(h_{G_F}+2-D_{max})}.$$

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A criterion for enhancement

Consider the susy enhancing flow for a $\mathcal{N} = 2$ SCFT satisfying $8a - 4c - \sum_i (2D_i - 1) = 0$. Check that:

- By setting $\frac{3}{2}(1+\epsilon) = \frac{1}{k_{s_{max}} + 1 - D_{max}}$ the number of operators hitting the unitarity bound is equal to the number of singlets.
- The matching condition for $\mathrm{Tr}\, R_{IR}^3 - \mathrm{Tr}\, R_{IR}$ is satisfied.

Conjecture: If these conditions are satisfied supersymmetry enhances in the infrared.

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Using 't Hooft anomaly matching it is possible to understand RG flows in 4d with infrared supersymmetry enhancement.

It would be important to clarify the meaning of the matching condition $\mathrm{Tr}\, R_{IR}^3 = \mathrm{Tr}\, R_{IR}$ and find a derivation of the following observations:

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