

Gauging one-form \mathbb{Z}_K center symmetries in some simple $SU(N)$ gauge theories

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18/07/2019

work in progress

Strongly interacting gauge theories with chiral coupling with fermions can play a role in physics!



We need to know what happens in IR :

- ▶ Does the theory confine, or does it become conformal?
- ▶ How are the global symmetries realized? How many vacua are there?
- ▶ Do the theory abelianize?

't Hooft Anomaly matching

1-form $(\mathbb{Z}_K^C)^{(1)}$ symmetry



mixed $[(\mathbb{Z}_K^C)^{(1)} - (\mathbb{Z}_M^\psi)^{(0)}]$ 't Hooft anomaly

Chiral symmetry : (ψ in R , $T(R) = \text{Dynkin index}$)

$$\frac{1}{8\pi^2} \int \text{tr}_R \{ F(a) \wedge F(a) \} = 2T(R)k \quad , \quad k \in \mathbb{Z}$$

↓

$$U^X(1) \rightarrow \mathbb{Z}_{2T(R)}^X$$

Center symmetry

$$\psi \rightarrow z\psi = \psi \quad \text{for } z \in \mathbb{Z}_K^C \subset SU(N)$$

↓

Exact \mathbb{Z}_K 1-form symmetry!

Gauging a 1-form symmetry

Let's gauge the \mathbb{Z}_K^C 1-form symmetry

$$\underbrace{A}_{\in su(N)} \rightarrow a = \underbrace{A + \frac{B^{(1)}}{K} \mathbb{1}}_{\in u(N)}$$

Introducing ($\underbrace{B^{(1)}}_{U(1) \text{ connection}}, B^{(2)}$)

$$B^{(2)} = \frac{dB^{(1)}}{K} \rightarrow \int_{C, \partial C = \emptyset} \overbrace{B^{(2)}}^{\text{like Dirac monopole}} = \frac{2\pi m}{K} ; m \in \mathbb{Z}$$

with :

$$\begin{cases} B^{(1)} \rightarrow B^{(1)} + K\lambda \\ B^{(2)} \rightarrow B^{(2)} + d\lambda \end{cases}$$

$$SU(N)\text{-bundle} \rightarrow \frac{SU(N)}{\mathbb{Z}_K}\text{-bundle}$$

Mixed anomaly!

$$\frac{1}{8\pi^2} \int \text{tr}_R(F(A) \wedge F(A)) \rightarrow \frac{1}{8\pi^2} \int \overbrace{\text{tr}_R((F(a) - B^{(2)}) \wedge (F(a) - B^{(2)}))}^{U(N) \text{ and 1-form invariant}}$$
$$\underbrace{\frac{2T(R)}{8\pi^2} \int \text{tr}(F(a) \wedge F(a))}_{\mathcal{A}_0 = 2T(R)\mathbb{Z}} - \underbrace{\frac{2T(R)N}{8\pi^2} \int B \wedge B}_{\mathcal{A}_1 = \frac{2T(R)\mathbb{Z}N}{K^2}}$$

Then if

$$\begin{cases} \psi \rightarrow e^{2\pi i \frac{k}{2T(R)}} \psi \\ \mathcal{Z}[B^{(1)}, B^{(2)}] \rightarrow \mathcal{Z}[B^{(1)}, B^{(2)}] \underbrace{e^{2\pi i \frac{k}{2T(R)} * \mathcal{A}_0}}_{=1} \underbrace{e^{2\pi i \frac{k}{2T(R)} * \mathcal{A}_1}}_{\neq 1} \end{cases}$$

SU(6) self-adjoint model, symmetries

$SU(6)$ with single chiral $\psi \in \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array}$
Discrete \mathbb{Z}_6^ψ chiral symmetry ($2T(R) = 6$)
Discrete \mathbb{Z}_3^C center symmetry.

$$\int_C B^{(2)} = \frac{2\pi k}{3}$$

then :

$$\frac{2T(R)}{8\pi^2} \int B^{(2)} \wedge B^{(2)} = \frac{2T(R)N}{K^2} m = \frac{6 * 6}{9} m = 4m$$

$$Z[B^{(1)}, B^{(2)}] \rightarrow Z[B^{(1)}, B^{(2)}] e^{4i\alpha} ; \alpha = \frac{2\pi k}{6} ; k = 1, 2, \underline{3}, 4, 5, \underline{6}$$

$$\mathbb{Z}_6 \rightarrow \mathbb{Z}_2 ; 3 \text{ vacua}$$

Yamaguchi, "t Hooft anomaly matching condition and chiral symmetry breaking without bilinear condensate"

Two hypothesis ($\langle \psi\psi \rangle |_{singlet} = 0$) :

- ▶ $\langle \psi\psi\psi\psi \rangle$, color singlet

$$\mathbb{Z}_6 \rightarrow \mathbb{Z}_2$$

3 vacua

Domain walls

- ▶ $\langle \psi\psi \rangle$, color adjoint

$$\mathbb{Z}_6 \rightarrow \mathbb{Z}_2$$

$$SU(6) \rightarrow U(1)^5$$

3 vacua

5 massless photon and domain walls

Generalizations

$SU(2l)$ with N_f , l -arity, half column representations.

Chiral symmetry :

$$2T(R) = \binom{2l-2}{l-1} \implies \mathbb{Z}_{2T(R)N_f}^\psi$$

Center symmetry :

$$\mathbb{Z}_l^c$$

then :

- ▶ For $N = 4$ nothing new, $\mathbb{Z}_2^\psi \rightarrow \mathbb{Z}_2^\psi$!
- ▶ For $N = 2l = 4k$ (l even) :

$$\mathbb{Z}_{2T(R)N_f}^\psi \rightarrow \mathbb{Z}_{2T(R)N_f/k}^\psi$$

- ▶ For $N = 4k + 2$ (l odd) :

$$\mathbb{Z}_{2T(R)N_f}^\psi \rightarrow \mathbb{Z}_{2T(R)N_f/(2k+1)}^\psi$$

Adjoint QCD

$SU(N)$ theory with N_f adjoint fermions.

Chiral symmetry :

$$\mathbb{Z}_{2N_f N}^\psi$$

Center symmetry :

$$\mathbb{Z}_N^c$$

$$\frac{\overbrace{2N}^{2T(R)} N_f N}{8\pi^2} \int \underbrace{B^{(2)}}_{\frac{1}{N}} \wedge B^{(2)} = 2N_f$$

then :

$$\mathbb{Z}_{2N_f N}^\psi \rightarrow \mathbb{Z}_{2N_f}^\psi$$

$SU(2)$ with 2 fermions

Flavor symmetry is $SU(2) \times \mathbb{Z}_8$. Center symmetry gives $\mathbb{Z}_8 \rightarrow \mathbb{Z}_4$. Two possibilities :

$$\text{Simple scenario } \begin{cases} \langle \psi^{\{I} \psi^{J\}} \rangle \neq 0 \\ \mathbb{Z}_8 \rightarrow \mathbb{Z}_2 \\ SU(2)_f \rightarrow SO(2)_f \end{cases}$$

or

$$\text{Amber and Poppitz } \begin{cases} \langle \psi\psi\psi\psi \rangle \neq 0 \\ \mathbb{Z}_8 \rightarrow \mathbb{Z}_4 \\ SU(2)_f \rightarrow SU(2)_f \end{cases}$$

$[SU(2)_f^3]$ Witten anomaly matched by $\psi\psi\psi$ baryons.
 $[\mathbb{Z}_4^3]$ match!

$$\mathcal{A}_{UV} = N_f * \dim(\text{Adj}) = 2 \pmod{4}$$

$$\mathcal{A}_{IR} = \underbrace{N_f}_{\text{number of baryons}} * \underbrace{3^3}_{\text{charge}} = 2 \pmod{4}$$

SUSY case... $SU(N)$ with $N_f = 1$

1-form gauging

$$\mathbb{Z}_{2N}^\psi \rightarrow \mathbb{Z}_2^\psi$$

then N different vacua.

If $N_f = 1$ then we have $\mathcal{N} = 1$ SUSY multiplet
#(SUSY different vacua), Witten index= N

CONSISTENT!!!

I. Affleck, M. Dine and N. Seiberg, '1985).

V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, '1985

D. Amati, K. Konishi, Y. Meurice, G. C. Rossi and G. Veneziano, '1988

N. M. Davies, T. J. Hollowood, V. V. Khoze and M. P. Mattis, '1999

"QCD" with quarks in symmetric representation

Consider now $SU(N)$, with N even and N_f pairs of

$$\psi, \tilde{\psi} = \square \oplus \bar{\square}$$

Center symmetry : \mathbb{Z}_2^c

Chiral symmetry : $\mathbb{Z}_{2N_f(N+2)}^\psi$

1-form gauging :

$$\frac{2 \overbrace{T(R)}^{N+2} N_f N}{8\pi^2} \int B^{(2)} \wedge B^{(2)} = 2$$

$$\mathbb{Z}_{2N_f(N+2)}^\psi \rightarrow \mathbb{Z}_{N_f(N+2)}^\psi$$

$\langle \psi \tilde{\psi} \rangle \neq 0$ possible!

$SU(N)$ gauge theories with :

$$\frac{N-4}{k} \psi^{\{i,j\}} \oplus \frac{N+4}{k} \bar{\chi}^{[i,j]}$$

Two examples :

- ▶ $SU(6)$ with

$$\psi \in \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \oplus 5 \chi \in \begin{array}{|c|} \hline \bar{\square} \\ \hline \square \\ \hline \end{array}$$

- ▶ $SU(8)$ with

$$\psi \in \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \oplus 3 \chi \in \begin{array}{|c|} \hline \bar{\square} \\ \hline \square \\ \hline \end{array}$$

$SU(6)$ chiral model

$SU(6)$ model with :

$$\psi \in \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \oplus 5 \chi \in \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

Flavor symmetries :

$$\mathbb{Z}_8^\psi \times \mathbb{Z}_{20}^\chi \times \frac{\overbrace{U(1)_{\psi\chi}}^{Q(\psi)=5, Q(\chi)=-2}}{\mathbb{Z}_{40} \times \mathbb{Z}_6} \times \frac{SU(5)}{\mathbb{Z}_5}$$

Center symmetry :

$$\mathbb{Z}_2^c$$

Standard 't Hooft anomaly matching

Standard $[SU(5)^3]$, $[SU(5)^2U(1)_{\psi\chi}]$, $[U(1)_{\psi\chi}^3]$ 't Hooft anomalies cannot be matched with baryons.

Two hypothesis :

- ▶ Color singlet $\langle \psi\chi\psi\chi \rangle$ condensate, still difficult to match.
- ▶ Break also color group,

$$MAC \implies \langle \psi\chi \rangle$$

adjoint of color, fundamental of $SU(5)$.

Stuart Raby, Savas Dimopoulos and Leonard Susskind, "Tumbling gauge theories"

Discrete [1-form -χral] anomaly

For simplicity we gauge $\mathbb{Z}_8^\psi \times \mathbb{Z}_{20}^\chi$

$$\mathbb{Z}_2^c \rightarrow (B_c^{(1)}, B_c^{(2)})$$

$$\mathbb{Z}_8^\psi \rightarrow A_\psi \quad \int A_\psi = \frac{2\pi m}{8}$$

$$\mathbb{Z}_{20}^\chi \rightarrow A_\chi \quad \int A_\chi = \frac{2\pi k}{20}$$

6-D anomaly :

$$P = \frac{1}{24\pi^2} \int tr_{sym} \{(\tilde{F} - B_c^{(2)}) - dA_\psi\}^3 + \frac{1}{24\pi^2} \int tr_{anti} \{(\tilde{F} - B_c^{(2)}) - dA_\chi\}$$

then..

$$P = -\frac{8}{8\pi^2} \int tr \{(\tilde{F} - B_c^{(2)})\}^2 dA_\psi - \frac{20}{8\pi^2} \int tr \{(\tilde{F} - B_c^{(2)})\}^2 dA_\chi$$

$$-\frac{8}{8\pi^2} \int tr\{(\tilde{F} - B_c^{(2)})\}^2 dA_\psi$$

$$\downarrow P_{2n+2} = dQ_{2n+1}$$

$$-\frac{8}{8\pi^2} \int tr\{(\tilde{F} - B_c^{(2)})\}^2 A_\psi$$

$$\downarrow \delta Q_{2n+1} = dQ_{2n}^{(1)}$$

$$-\frac{8 * N}{8\pi^2} \left(\int B^{(2)} \wedge B^{(2)} \right) \Delta A_\psi = \frac{2\pi m}{8} \frac{8N}{8\pi^2} \frac{8\pi^2}{4} = \frac{2\pi * 3 * k}{2}$$

therefore

$$\mathbb{Z}_8^\psi \rightarrow \mathbb{Z}_4^\psi$$

In the same way

$$\mathbb{Z}_{20}^\chi \rightarrow \mathbb{Z}_{10}^\chi$$

The $\langle \psi\chi \rangle$ adjoint scenario satisfies all of the constraints.

$$SU(5) \times U(1)_{\psi\chi} \rightarrow SU(4) \times U'(1)$$

- ▶ The discrete symmetry is broken strongly :

$$\mathbb{Z}_8^\psi \times \mathbb{Z}_{20}^\chi \rightarrow \mathbb{Z}_4$$

- ▶ The theory abelianizes, $SU(6) \rightarrow U(1)^5$ and $SU(4) \times U'(1)$ continuous anomalies are matched by weakly interacting (massless) fermions.

A $SU(8)$ chiral model

An $SU(8)$ model with :

$$\psi \in \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \oplus 3 \chi \in \begin{array}{|c|} \hline \bar{\square} \\ \hline \end{array}$$

Flavor symmetries :

$$\underbrace{\mathbb{Z}_2^\psi}_{\left\{ \begin{array}{l} \psi \rightarrow -\psi \\ \chi \rightarrow \chi \end{array} \right. \text{ or } \left\{ \begin{array}{l} \psi \rightarrow \psi \\ \chi \rightarrow -\chi \end{array} \right.} \times U(1)_{\psi\chi} \times \frac{SU(5)}{\mathbb{Z}_5}$$

Center symmetry : \mathbb{Z}_2

$$\text{Pick } \mathbb{Z}_2^\psi : \begin{cases} \psi \rightarrow -\psi \\ \chi \rightarrow \chi \end{cases}$$

$$\frac{2T(R)N}{8\pi^2} \int B \wedge B = \frac{10 * 8 * 8\pi^2 k}{8\pi^2 * 4} \in \mathbb{Z}$$

then no new anomaly matching condition!

Same result if we embed in the other way :

$$\mathbb{Z}_2 : \begin{cases} \psi \rightarrow \psi \\ \chi \rightarrow -\chi \end{cases}$$

$SU(3)$ and $U_{\psi\chi}(1)$ anomalies can't be matched in IR
 $MAC \rightarrow adjoint \langle \psi\chi \rangle$ again

$$SU(3) \times U(1) \rightarrow SU(2) \times U(1)$$

Probably dynamical abelianization and perturbative anomalies trivially matched.

Chiral symmetry :

$$\mathbb{Z}_2^\psi \rightarrow 1$$

which is consistent.

- ▶ 1-form- χ ral mixed 't Hooft anomaly matching condition can give interesting insights on the discrete χ ral symmetry realization.
- ▶ If we add the clues given by standard anomaly matching the IR behavior is really constrained.
- ▶ Application on realistic models are very welcome!

BACKUP : Generalities on 1-form symmetries

$D - 1$ sub-manifold M , 1-form current $dJ = 0 \rightarrow Q = \int_M *J$

$$\langle U_g^{(D-1)}(M)V^{(0)}(x) \rangle = R(g) \langle U_g^{(D-1)}(M)V^{(0)}(x') \rangle$$

$D - k$ sub-manifold M , $k+1$ -form current $dJ = 0 \rightarrow Q = \int_M *J$

$$\langle U_g^{(D-1)}(M)V^{(k)}(S) \rangle = \underbrace{R(g)}_{\text{Needs to be a phase!}} \langle U_g^{(D-1)}(M)V^{(k)}(S') \rangle$$

1-forms symmetries acts on Wilson-'t Hooft loops!

Discrete 0-form symmetry bundle \rightarrow insertion of topological
DW g defects

Discrete k -form symmetry bundle \rightarrow insertion of topological
(possibly open) sheets

BACKUP : "Global" 1-form symmetries in gauge theories, discrete description

$$\text{change of chart } \left\{ \begin{array}{l} \psi_i = R(g_{ij})\psi_j \\ \Omega_j \rightarrow \Omega_i \end{array} \right. \left\{ \begin{array}{l} A_i = g_{ij}A_j g_{ij}^\dagger - g_{ij}dg_{ij}^\dagger \end{array} \right.$$

$$\text{"global" center symmetry } \left\{ \begin{array}{l} g_{ij}g_{jk}g_{ki} = \mathbb{1} \\ R(z)\psi = \psi \text{ for } z \in \mathbb{Z}_K \end{array} \right. \left\{ \begin{array}{l} g_{ij}(x) \rightarrow z_{ij}g_{ij}(x) \\ z_{ij}z_{jk}z_{ki} = \mathbb{1} \end{array} \right.$$

and on a manifold with non-trivial holonomies :

$$W[\gamma] \rightarrow zW[\gamma]$$

D. Gaiotto, A. Kapustin, N. Seiberg and B. Willett, "Generalized Global Symmetries,"
Anton Kapustin and Nathan Seiberg, "Coupling a QFT to a TQFT and Duality"

BACKUP : Gauging, discrete description

$$z_{ij}z_{jk}z_{ki} \neq \mathbb{1} (\in \mathbb{Z}_K)$$

$$g_{ij}g_{jk}g_{ki} = B_{ijk} (\in \mathbb{Z}_K)$$

$$B_{ijk} \xrightarrow{\text{local center symmetry}} z_{ij}z_{jk}z_{ki}B_{ijk}$$

and $\delta B = 0$

$$B \in H^2(M, \mathbb{Z}_K) = \frac{\{\delta B = \mathbb{1}\}}{\{B_{ijk} = z_{ij}z_{jk}z_{ki}\}}$$

Choice of g_{ij} s.t. $g_{ij}g_{jk}g_{ki} = \mathbb{1} \rightarrow$ choice of $SU(N)$ bundle

Choice of g_{ij} s.t. $g_{ij}g_{jk}g_{ki} = B_{ijk} \rightarrow$ choice of $\frac{SU(N)}{\mathbb{Z}_K}$ bundle

Anton Kapustin and Nathan Seiberg, "Coupling a QFT to a TQFT and Duality"

BACKUP : Gauging, continuous description

$$A (\in su(N)) \rightarrow a = A + \frac{B^{(1)}}{K} \mathbb{1} (\in u(N))$$

$$(B^{(1)}, B^{(2)})$$

$$B^{(2)} = \frac{dB^{(1)}}{K} \rightarrow \overbrace{\int_{C, \partial C = \emptyset} B^{(2)}}^{\text{like Dirac monopole}} = \frac{2\pi m}{K} \quad m \in \mathbb{Z}$$

impose :

$$B^{(1)} \rightarrow B^{(1)} + K\lambda$$

$$B^{(2)} \rightarrow B^{(2)} + d\lambda$$

$a \rightarrow a + \lambda \implies$ No new d.o.f. in \mathbb{R}^4 .

Anton Kapustin and Nathan Seiberg, "Coupling a QFT to a TQFT and Duality"

Gauging, line operators

- ▶ $W_o[\gamma] = \text{Tr}[P \exp(i \int_\gamma A)]$ is not $U(N)$ gauge invariant!
- ▶ $W[\gamma] = \text{Tr}[P \exp(i \int_\gamma a)] \xrightarrow{\lambda} W[\gamma] \exp(\int_\gamma \lambda)$ not 1-form invariant!
- ▶ $W[\gamma, S] = W[\gamma] \exp(\int_S B^{(2)})$ is $U(N)$ and 1-form gauge invariant!

$W[\gamma, S]$ depends on the surface S :

$$\underbrace{\int_S B^{(2)} - \int_{S'} B^{(2)}}_{\text{as Dirac monopoles...}} = \frac{2\pi m}{K}$$

But W^K not $\rightarrow SU(N)/\mathbb{Z}_K$ theory defined by

local fields + $(B^{(1)}, B^{(2)})/\{1\text{-form symmetry } \lambda\}$

Gauging, coupling with fermions and anomalies

The coupling with fermions becomes :

$$\partial_\mu - igA_\mu \rightarrow \partial_\mu - ig\left(a_\mu - \frac{B_\mu^{(1)}}{K}\right)$$

then :

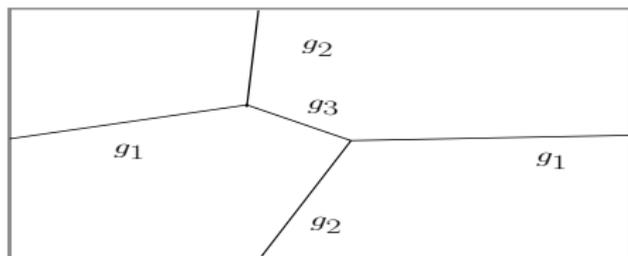
$$\begin{aligned} F(A) &\rightarrow F(a) - B^{(2)} \\ \frac{1}{8\pi^2} \int \text{tr}_R(F(A) \wedge F(A)) &\rightarrow \frac{1}{8\pi^2} \int \text{tr}_R((F(a) - B^{(2)}) \wedge (F(a) - B^{(2)})) \\ &= \underbrace{\frac{2T(R)}{8\pi^2} \int \text{tr}(F(a) \wedge F(a))}_{\mathcal{A}_0 = 2T(R)\mathbb{Z}} - \underbrace{\frac{2T(R)N}{8\pi^2} \int B \wedge B}_{\mathcal{A}_1 = \frac{2T(R)\mathbb{Z}N}{K^2}} \end{aligned}$$

Then :

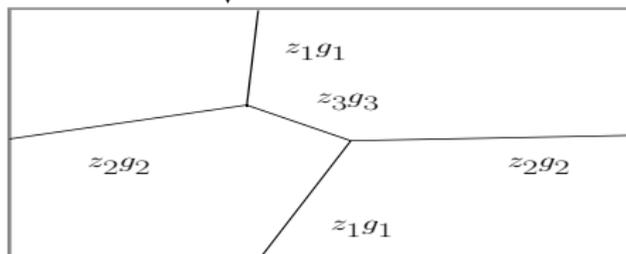
$$\left\{ \begin{array}{l} \psi \rightarrow e^{2\pi i \frac{k}{2T(R)}} \psi \\ \mathcal{Z}[B^{(1)}, B^{(2)}] \rightarrow \mathcal{Z}[B^{(1)}, B^{(2)}] \underbrace{e^{2\pi i \frac{k}{2T(R)} * \mathcal{A}_0}}_{=1} \underbrace{e^{2\pi i \frac{k}{2T(R)} * \mathcal{A}_1}}_{\neq 1} \end{array} \right.$$

BACKUP : 't Hooft twisted boundary conditions

$$SU(N) \rightarrow \frac{SU(N)}{Z_N}, \text{ hand-made.}$$



Who cares about cocycle condition !



BACKUP : Discrete chiral symmetry

Naively the discrete symmetries are :

$$\mathbb{Z}_8 \times \mathbb{Z}_{20} : \begin{cases} \psi \rightarrow e^{2\pi i \frac{l}{8}} \psi & ; l \in \mathbb{Z}_8 \\ \chi \rightarrow e^{2\pi i \frac{m}{20}} \chi & ; m \in \mathbb{Z}_{20} \end{cases}$$

but

$$\mathbb{Z}_8 \times \mathbb{Z}_{20} \cap U(1)_{\psi\chi} = \mathbb{Z}_{40}$$

In particular :

$$(l, m) \in \mathbb{Z}_8 \times \mathbb{Z}_{20} = \underbrace{(l + m + 4N, 0)}_{\in \mathbb{Z}_4} \underbrace{\mathcal{U}}_{\in U(1)_{\psi\chi}}$$

$$\mathbb{Z}_4 : \begin{cases} \psi \rightarrow e^{2\pi i \frac{k}{8}} \psi & ; \text{for } k = 1, 2, 3, 4 \\ \chi \rightarrow \chi \end{cases}$$

BACKUP : Discrete [1-form -χral] anomaly

Doing the smallest \mathbb{Z}_4 transformation :

$$\mathbb{Z}_4 : \begin{cases} \psi \rightarrow e^{2\pi i \frac{k}{8}} \psi & ; \text{ for } k = 1, 2, 3, 4 \\ \chi \rightarrow \chi \end{cases}$$

and $Z[B^{(1)}, B^{(2)}] \rightarrow Z[B^{(1)}, B^{(2)}] e^{i\alpha}$

$$\alpha = \frac{\overbrace{2T(R)}^8 \overbrace{N}^6}{8\pi^2} \int \underbrace{B}_{\frac{1}{2}} \wedge B = \frac{8 * 6}{4} \frac{2\pi k}{8} = \frac{3}{2} 2\pi k$$

Therefore $\mathbb{Z}_4 \rightarrow \mathbb{Z}_2$