Merging of fixed points in Chern-Simons-QCD_3

Sergio Benvenuti, SISSA

Pisa, 18-7-2019

based on:
arXiv:1812.01544 & arXiv:1902.05767, w/ Hrachya Khachatryan
arXiv:19??.??, w/ Guillermo Arias-Tamargo and Diego Rodriguez-Gomez

Relativistic QFT's in $2+1$ dimensions

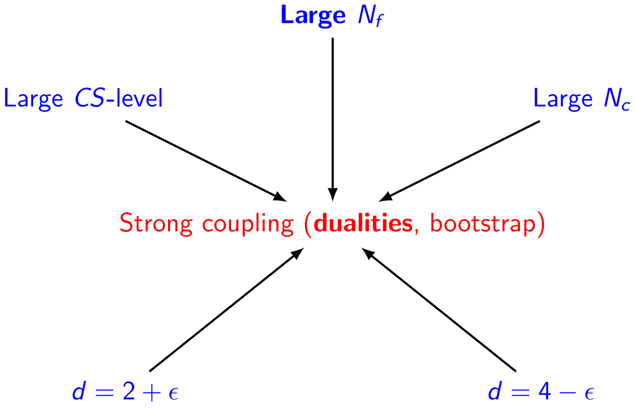
Interesting, for instance, for:

- theoretical laboratory for QFT's in general
- domain walls / boundary conditions of $4d$ QFT's (QCD)
- quantum phase transitions in condensed-matter
- Quantum Hall Effects
- graphene: multilayers, magic angles, ...
- emergent minimal supersymmetry

Lots of recent progress (last 3 years) in understanding dynamics, dualities.

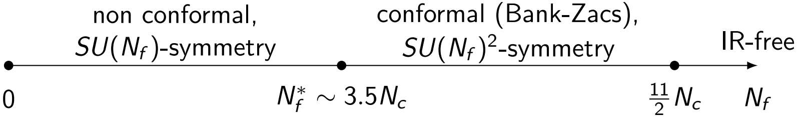
Dynamics of gauge theories in $2+1$ dimensions

Five "simple" or weak-coupling limits:



Question: is there a CFT in the IR? Let's look at QCD_4

In $4d$ QCD, $SU(N_c)$ with N_f Dirac flavors, standard scenario:



Chiral symmetry breaking, triggered by a quartic fermion operator **crossing marginality**

$\Delta[\Psi^4] = 4$ in $4d$: large anomalous dimension, strong coupling

$$\mathcal{L}_{int} = -\Psi^4 \quad \Rightarrow \quad \langle \Psi^2 \rangle \neq 0$$

Ψ^4 is $SU(N_f)^2$-invariant but $\langle \Psi^2 \rangle$ breaks $SU(N_f)^2 \to SU(N_f)$

TWO REAL CFT'S "ANNIHILATE" INTO TWO COMPLEX CFT'S

A controlled example: bosonic QED in $4-\epsilon$ dimensions

$$\mathcal{L} = \sum_{i=1}^{N_f} |D_\mu \phi_i|^2 + \frac{1}{e^2} F_{\mu\nu}^2 - \lambda (\sum_{i=1}^{N_f} |\phi_i|^2)^2$$

One-loop beta functions:

$$\beta_{e^2} = -\epsilon e^2 + \frac{N_f}{6} e^4$$

$$\beta_\lambda = -\epsilon \lambda + \frac{N_f + 8}{6} \lambda^2 - 6e^2 \lambda + 18 e^4$$

Two charged fixed points ($\mathbb{CP}^{N_f-1}$-model and tricritical QED):

$$(e^2)^* = \frac{6\epsilon}{N_f} \qquad \lambda^* = \frac{3N_f + 54 \pm 3\sqrt{N_f^2 - 180N_f - 540}}{2N_f(N_f+4)} \epsilon$$

If $N_f < 6\left(15 + 4\sqrt{15}\right) \sim 182.95$ solutions are complex

Two loop completely changes qualitative behavior: fate in $d=3$ not clear

Let us try to tackle this question using **large** N_f **in** $d=3$ instead!

Large N_f expansion: let's try $\mathcal{N}=1$ sQED

Test a "mirror-like" duality for $\mathcal{N}=1$ sQED with 2 flavors

$U(1)$ with 2 flavors $Q_{i=1,2}$
$\mathcal{W}=0$

$\Longleftrightarrow$

$\mathcal{N}=1$ Wilson-Fisher model
with 7 real superfields
$\mathcal{W}=\mu_I M_\alpha(\sigma_I)_{\alpha\beta}M^\dagger_\beta$

$\Delta[\mathfrak{M}^{\pm 1}] \sim 0.362 N_f = 0.724$	$\Delta[M_\alpha] \sim 0.76$
$\Delta[\lvert Q\rvert^2_{spin-1}] \sim 1 - \frac{24}{3\pi^2 N_f} = 0.595$	$\Delta[\mu_I] \sim 0.66$
$\Delta[\lvert Q\rvert^2_{singlet}] \sim 1 - \frac{0}{3\pi^2 N_f} = 1$	$\Delta[-2\sum \mu_I^2 + \sum \lvert M_\alpha\rvert^2] \sim 1$
$\Delta[\lvert Q\rvert^4_{spin-2}] \sim 2 - \frac{48}{3\pi^2 N_f} = 1.19$	$\Delta[\mu_I\mu_J - \frac{\delta_{IJ}}{3}\sum \mu_K^2] \sim 1.33$
$\Delta[\lvert Q\rvert^4_{singlet}] \sim 2 - \frac{0}{3\pi^2 N_f} = 2$	$\Delta[2\sum \mu_I^2 + 3\sum \lvert M_\alpha\rvert^2] \sim 2.33$

large N_f SB, Khachatryan 2018 $4-\epsilon$ expansion SB, Benini 2018

Large N_f works with $\sim 10\%$ accuracy for $2_f + 2_b$ complex flavors!
(similar accuracy in $O(N=8)$-model or $O(N=8)$-Gross-Neveu model)

Bosonic QED's at large N_f in $d = 3$

$$\mathcal{L}_{bQED} = \sum_{i=1}^{N_f} |(\partial_\mu + ia_\mu)\phi_i|^2$$

$$\mathcal{L}_{\mathbb{CP}^{N_f-1}\text{model}} = \sum_{i=1}^{N_f} |(\partial_\mu + ia_\mu)\phi_i|^2 + \sigma \sum_{i=1}^{N_f} |\phi_i|^2$$

Geometric series of bubble diagrams $\rightarrow$ propagators for a_μ and σ

Scaling dimension of simple operators (probably $\sim$ accurate for $N_f \geq 4$):

bQED (tricritical)	$\Delta[\phi^*\phi_{SU(N_f)-adjoint}] = 1 - \frac{64}{3\pi^2 N_f}$ $\Delta[\lvert\phi\rvert^2_{SU(N_f)-singlet}] = 1 + \frac{128}{3\pi^2 N_f}$ $\Delta[\phi_i^*\phi_j^*\phi^k\phi^l - \text{traces}] = 2 - \frac{128}{3\pi^2 N_f}$ $\Delta[\lvert\phi\rvert^4_{SU(N_f)-singlet}] = 2 + \frac{256}{3\pi^2 N_f}$
$\mathbb{CP}^{N_f-1}$ model	$\Delta[\phi^*\phi_{SU(N_f)-adjoint}] = 1 - \frac{48}{3\pi^2 N_f}$ $\Delta[\phi_i^*\phi_j^*\phi^k\phi^l - \text{traces}] = 2 - \frac{48}{3\pi^2 N_f}$ $\Delta[\sigma] = 2 - \frac{144}{3\pi^2 N_f}$ $\Delta[\frac{-5\mp\sqrt{37}}{12}\sigma^2 + F^{\mu\nu}F_{\mu\nu}] = 4 - \frac{32(4\pm\sqrt{37})}{3\pi^2 N_f}$

SB, Khachatryan 2018

Estimates of merging of $\mathbb{CP}^{F-1}$ and tricritical bQED: $N_f^* \sim 10$

SB, Khachatryan 2018

Three somewhat different ways give similar results:

$|\phi|^4_{SU(N_f)-singlet}$ in tricritical QED crosses marginality:

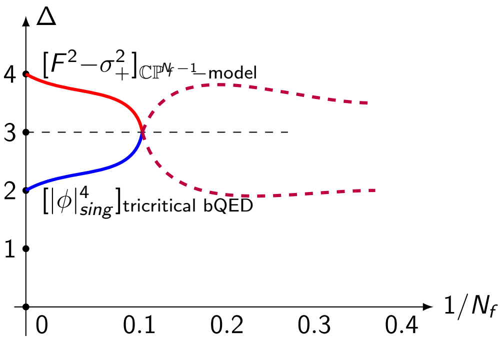
$$\Delta[|\phi|^4_{SU(N_f)-singlet}]_{bQED} = 3 \quad \rightarrow \quad N_f^* \sim \frac{256}{3\pi^2} \simeq 8.6$$

σ^2 in $\mathbb{CP}^{N_f-1}$ model crosses marginality:

$$\Delta[\sigma^2 + F^{\mu\nu}F_{\mu\nu}]_{\mathbb{CP}} = 3 \quad \rightarrow \quad N_f^* \sim \frac{32(4+\sqrt{37})}{3\pi^2} \simeq 10.9$$

$|\phi|^4_{SU(N_f)-singlet}$ in tricritical QED meets σ in $bC\mathbb{P}^{N_f-1}$ model:

$$\Delta[|\phi|^2_{singlet}]_{bQED} = 1 + \frac{128}{3\pi^2 N_f} = \Delta[\sigma]_{\mathbb{CP}} = 2 - \frac{144}{3\pi^2 N_f} \quad \rightarrow \quad N_f^* \sim 9.2$$



In particular for $N_f = 2$ no 2^{nd} order phase transition:

- important for celebrated *Deconfined Quantum Critical point*, (VBS $\leftrightarrow$ Neel order transition)
- consistent with numerical simulations Svistunov 2007
- consistent with rigorous numerical bootstrap bounds

Chern-Simons-QCD$_3$: $SU(N_c)_k$ with N_f fermions

Regime $k \geq \frac{N_f}{2}$: Bosonization duality

$$SU(N_c)_k \text{ w/ } N_f\, \psi' s \quad \Longleftrightarrow \quad U(k+\tfrac{N_f}{2})_{-N_c} \text{ w/ } N_f\, \phi' s$$

Aharony 2015

- massive phases match because of level-rank duality for TQFT's
- large-N_c & large-k limit: exact checks $\rightarrow$ CFT   Aharony,Minwalla,...
- $N_f = 1$: holographic duality with Vasiliev higher-spin theory on AdS_4

Regime $k < \frac{N_f}{2}$: possible region with "quantum-phase" and symmetry breaking

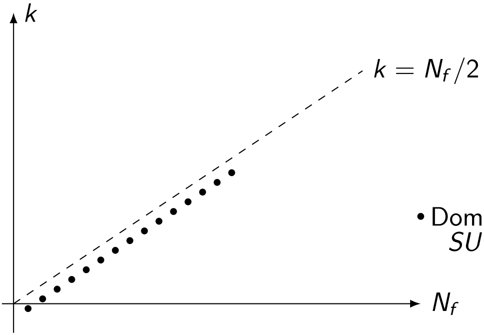
$$U(N_f) \rightarrow U(N_f/2+k) \times U(N_f/2-k)$$

Komargoski,Seiberg 2017

Armoni,Dumitrescu,Festuccia,Komargoski 2018

Argurio,Bertolini,Mignosa,Niro 2018

Chern-Simons-QCD$_3$: phase diagram



• Domain wall theories $SU(N_c)_{-1+N_f/2}$

Domain walls of QCD$_4$ at $\theta_{QCD} = \pi$:
$SU(N_c)_{-1+N_f/2}$ w/ N_f Ψ's

Gaiotto,Komargoski,Seiberg 2017
Argurio,Bertolini,Bigazzi,Cotrone,Niro 2017

What can we say from large N_f expansion?
Compute scaling dimensions of quartic operators and look for $\Delta[\Psi^4] = 3$

CS-QCD$_3$ at large N_f: bilinear and quartic operators

Arias-Tamargo, SB, Rodriguez-Gomez

The bilinears, $\bar{\Psi}_i \Psi^j$, transform in the adjoint plus singlet of $SU(N_f)$:

$$\Delta[\Psi\Psi_{adjoint}] = 2 - \frac{64(N_c^2-1)}{3\pi^2\left(1+(\frac{8k}{\pi N_f})^2\right)N_c N_f} + O(1/N_f^2)$$

$$\Delta[\Psi\Psi_{singlet}] = 2 - \frac{128(N_c^2-1)\left(2(\frac{8k}{\pi N_f})^2-1\right)}{3\pi^2\left(1+(\frac{8k}{\pi N_f})^2\right)^2 N_c N_f} + O(1/N_f^2)$$

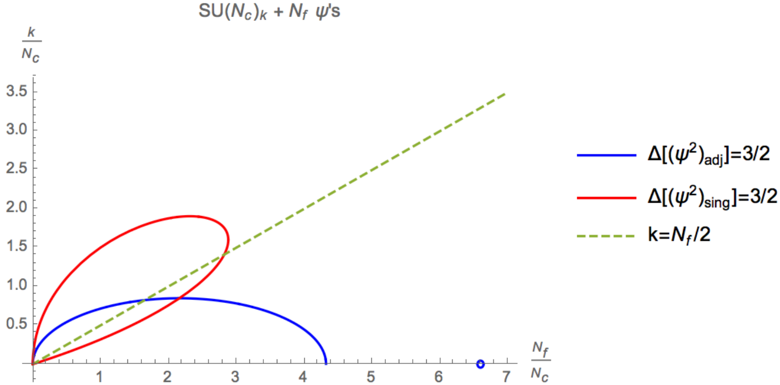
In principle we need to study mixing of four $\Psi^4_{singlet}$ operators (WIP)

In the regime $N_c >> 1$, only two double-trace quartic ops are into play:

$$\Delta[(\Psi\Psi_{adjoint})^2] \sim 2 \times \Delta[\Psi\Psi_{adjoint}] \sim 4 - \frac{c_1(k/N_f)N_c}{N_f} + O(N_c^2/N_f^2)$$

$$\Delta[(\Psi\Psi_{singlet})^2] \sim 2 \times \Delta[\Psi\Psi_{singlet}] \sim 4 - \frac{c_2(k/N_f)N_c}{N_f} + O(N_c^2/N_f^2)$$

Marginality crossing of the two double trace $\Psi^4_{singlet}$

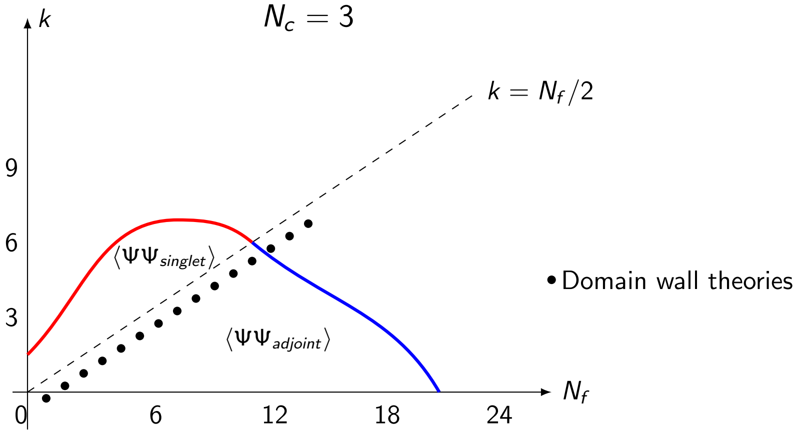


Inside blue curve: $\Delta[(\Psi\Psi_{adjoint})^2] < 3 \quad \Rightarrow \quad \langle \Psi\Psi_{adjoint} \rangle \neq 0$

Inside red curve: $\Delta[(\Psi\Psi_{singlet})^2] < 3 \quad \Rightarrow \quad \langle \Psi\Psi_{singlet} \rangle \neq 0$

Blue dot: comes from $\Delta[\Psi\Psi_{adjoint}]$ at $O(1/N_f^2)$, known for $k = 0$
(important: confirms that merging region is non-empty)

Qualitative picture and relation to domains walls



Conjecture:
symmetry-breaking on the wall $\leftrightarrow$ χ-symmetry-breaking in the $3+1$d bulk

SUMMARY:

non susy $2+1$d gauge theories at strong coupling (small N_f & small k): often **no** 2^{nd} **order phase transition**

Large N_f techniques useful in $d=3$: minimal susy, QED and CS-QCD

Dualities for 1^{st} order phase transitions less powerful (but complex CFT's)

FUTURE DIRECTIONS:

More general $4d$ gauge theories (quiver, rank-2 matter): cf. Poppitz talk
domains walls
$3d$ dualities and IR scenarios (e.g. quantum phases)
$4d$ IR scenarios (many different proposals, domain walls may help)
$4d$ non susy dualities?

Numerical conformal bootstrap:
CFT's with intermediate number of flavors in $3d$ and $4d$?