

Dark energy after GW170817

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with P. Creminelli 1710.05877, + M. Lewandowski and G. Tambalo, 1809.03484
+ V. Yingcharoenrat, in progress

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The Universe accelerates

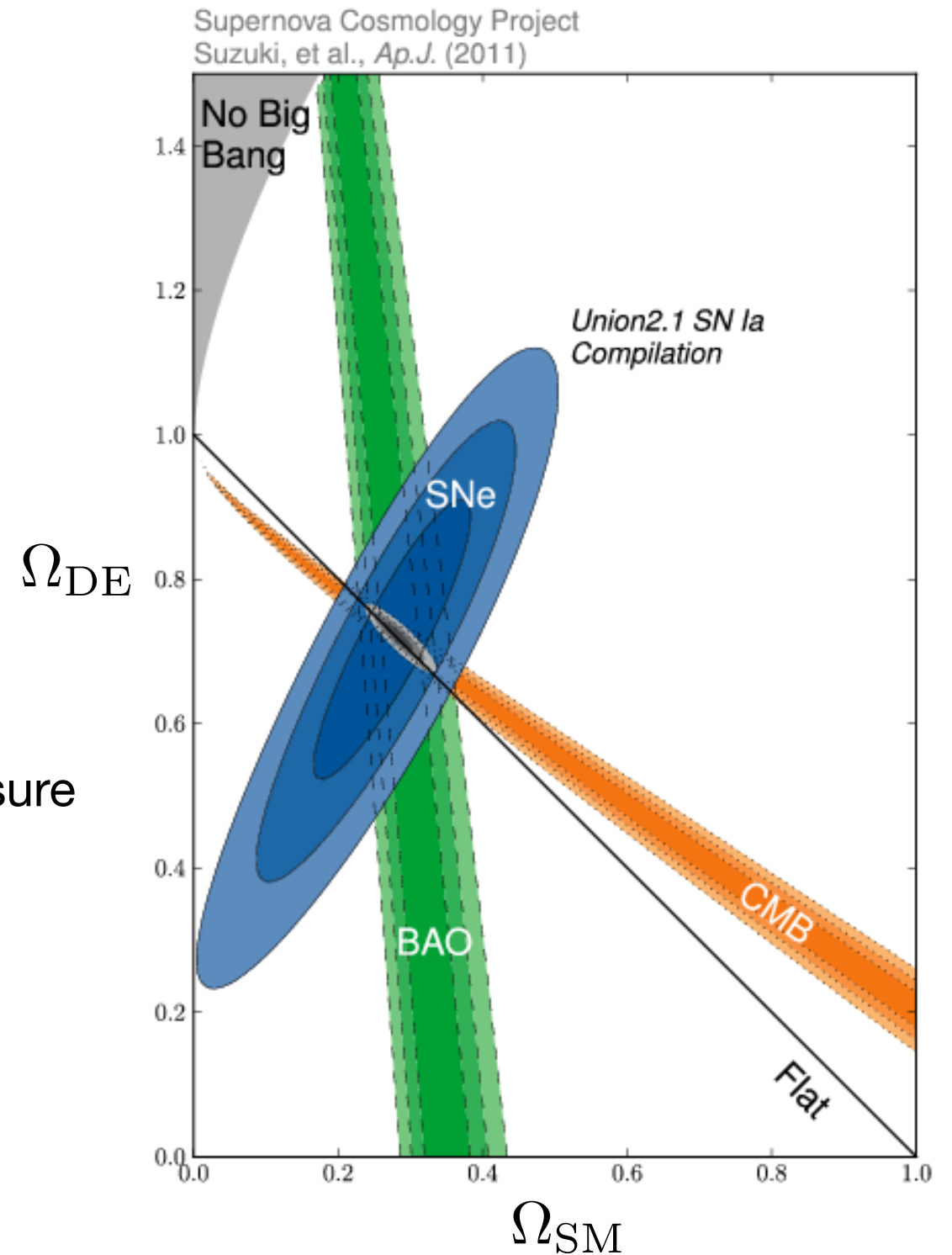
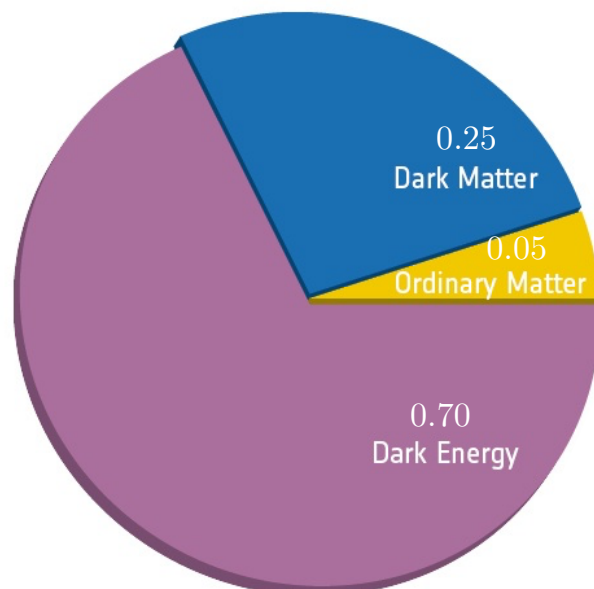
FRW metric: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$

Einstein's equation: $G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\text{matter})}$

Accelerated expansion:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Dark energy: new form of matter with negative pressure



Cosmological Constant: Λ CDM

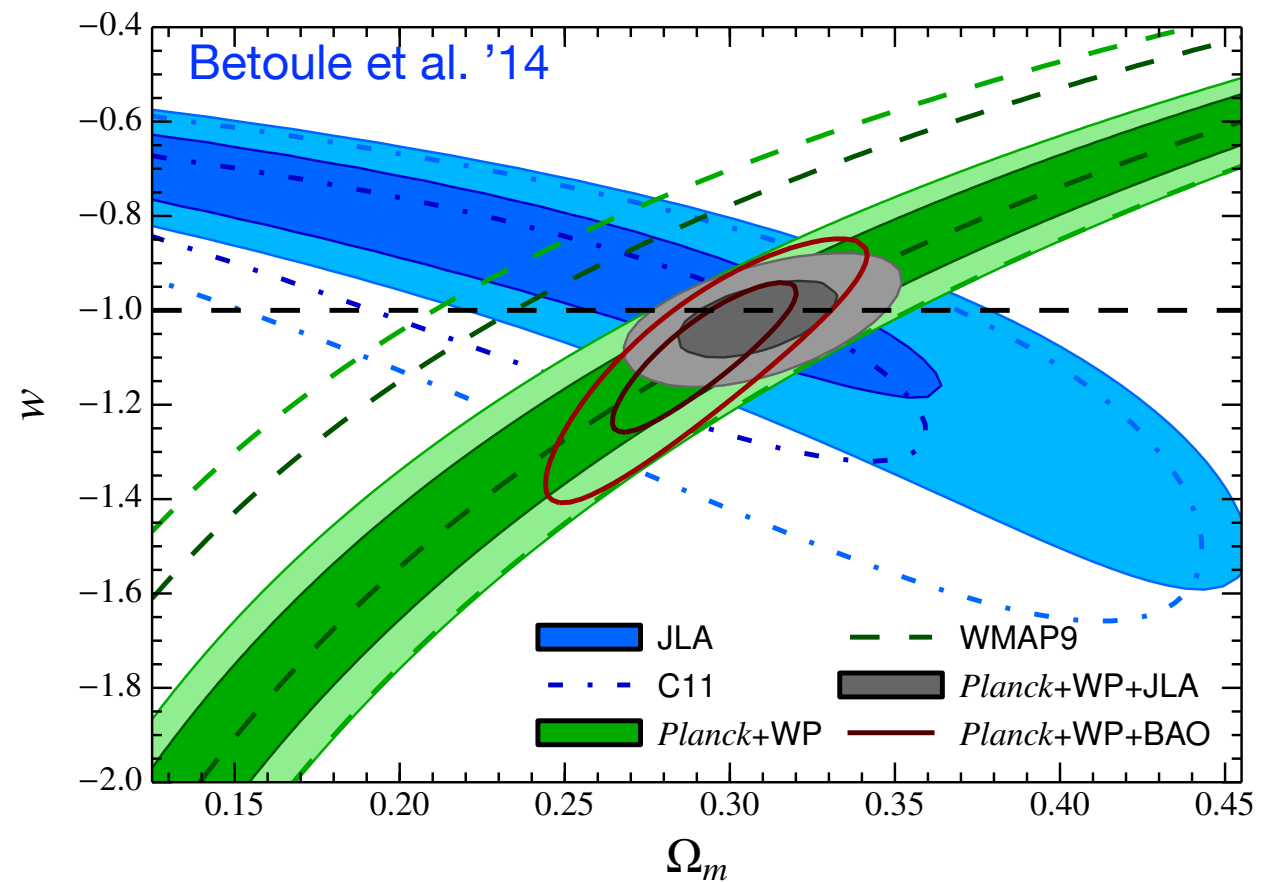
Equation of state very close to -1

$$w \equiv p/\rho$$

$$\rho \propto a^{-3(1+w)}$$

Cosmological constant: vacuum energy,
simplest explanation, consistent with all data

$$T_{\mu\nu}^{(\text{C.C.})} = -\Lambda g_{\mu\nu}$$



Cosmological Constant: Λ CDM

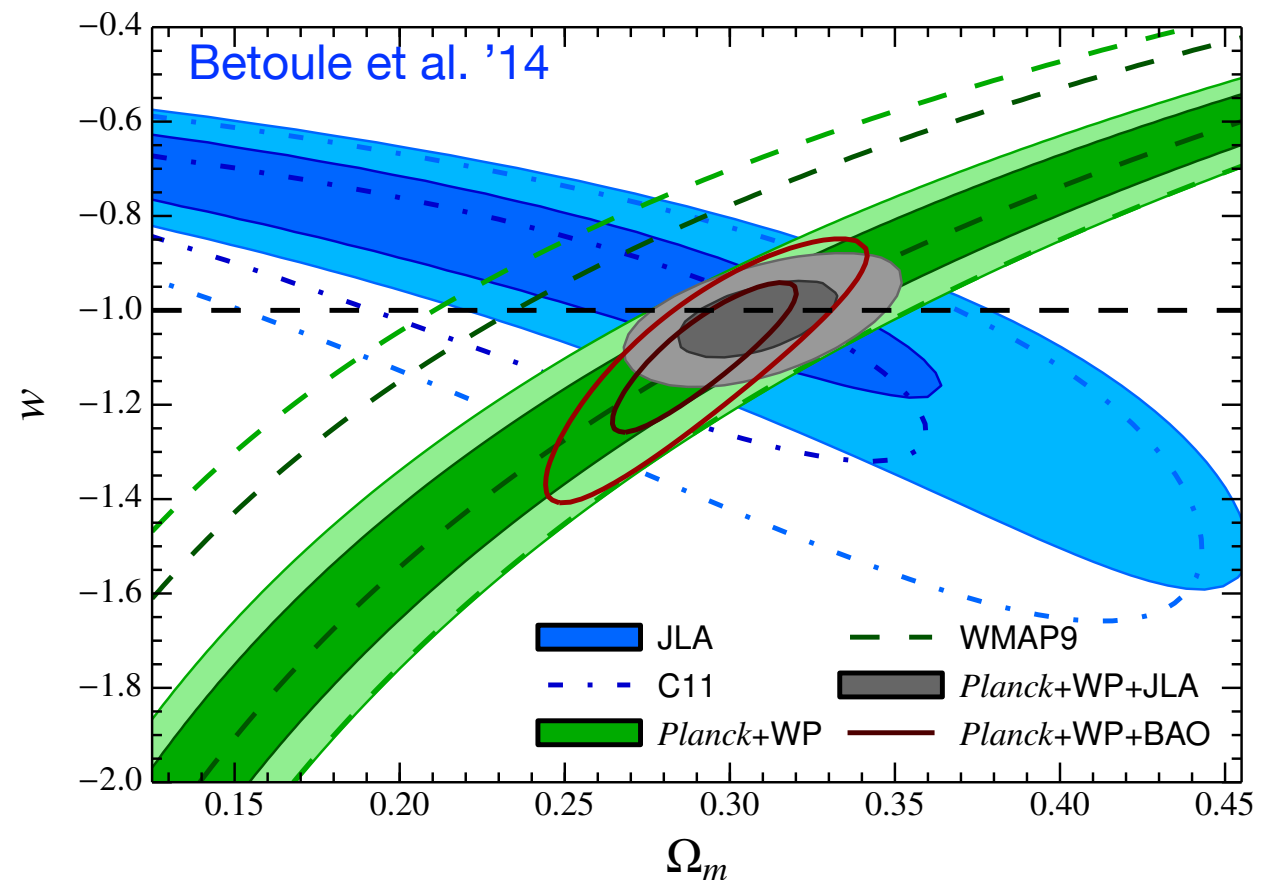
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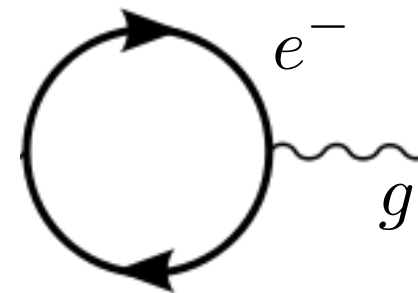
$$T_{\mu\nu}^{(\text{C.C.})} = -\Lambda g_{\mu\nu}$$



But $\Lambda = \rho_{\text{de}} \simeq (10^{-3} \text{eV})^4$ unnaturally small

Very UV sensitive. Cancellation with vacuum energy of each particle at any loop-order in perturbation theory

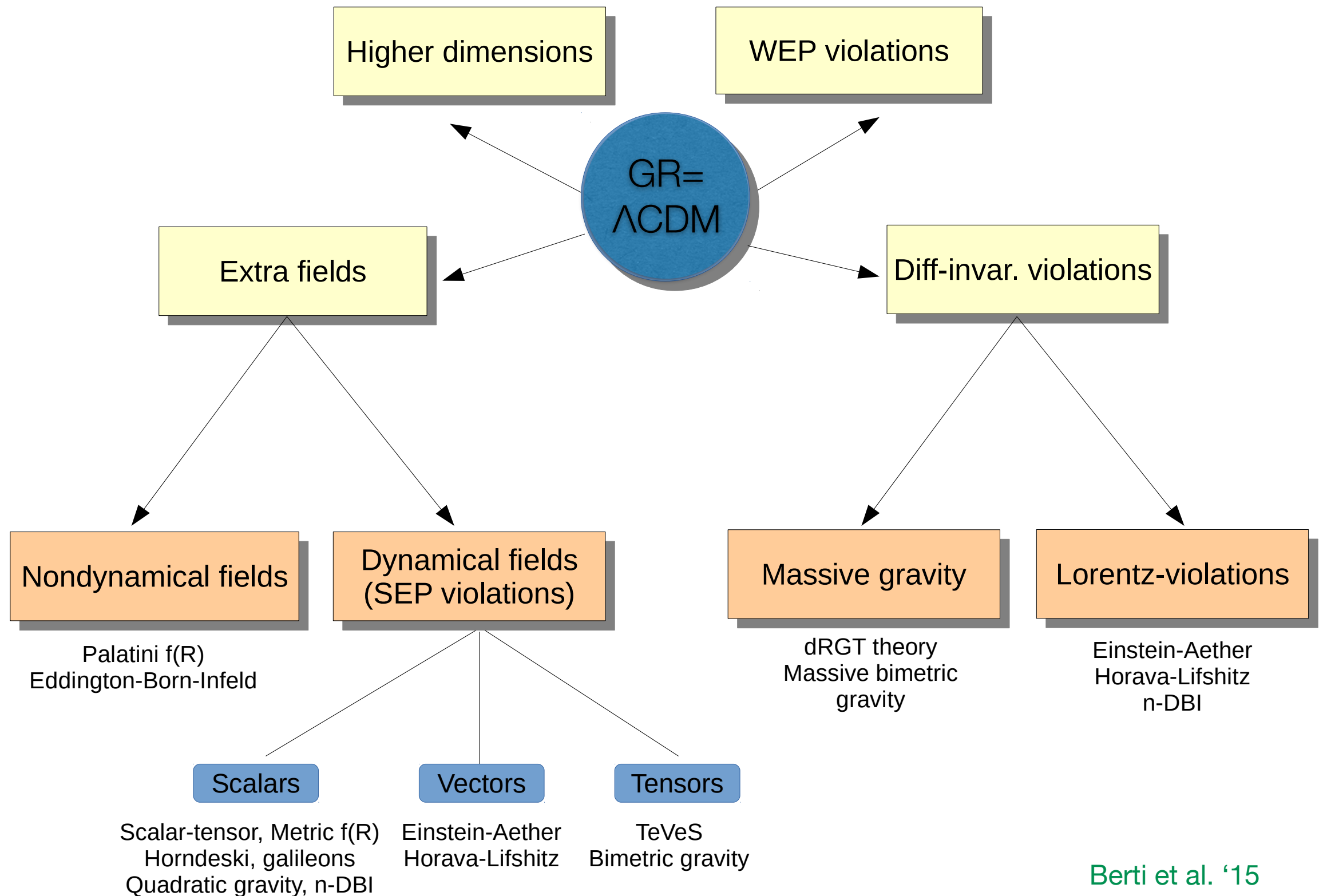
$$\Lambda_{\text{obs}} = \Lambda_{\text{bare}} + \sum_i c_i m_i^4$$



e.g. Burgess 13; Padilla 15

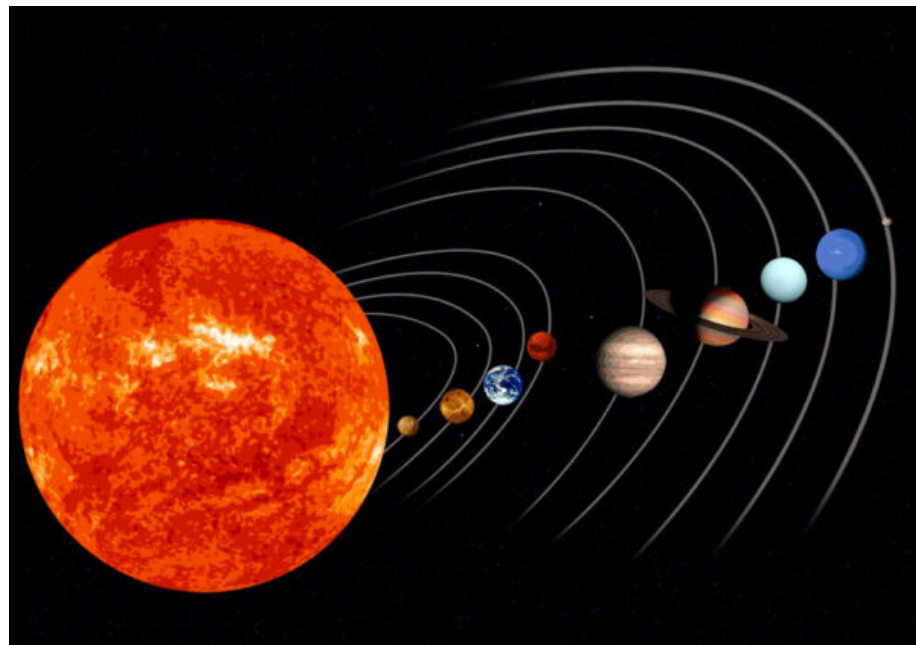
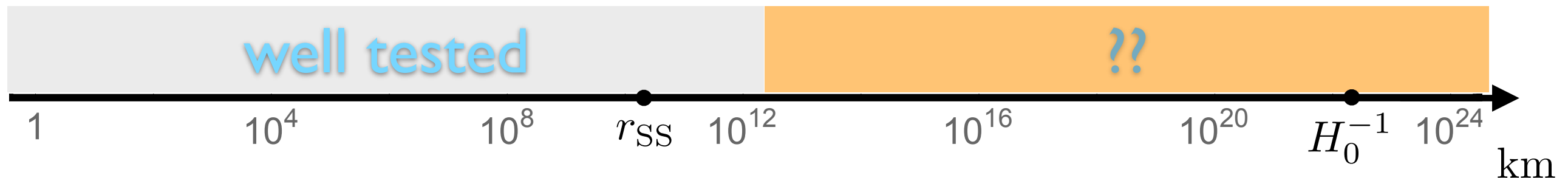
Cosmological constant problem

Dark energy and modified gravity



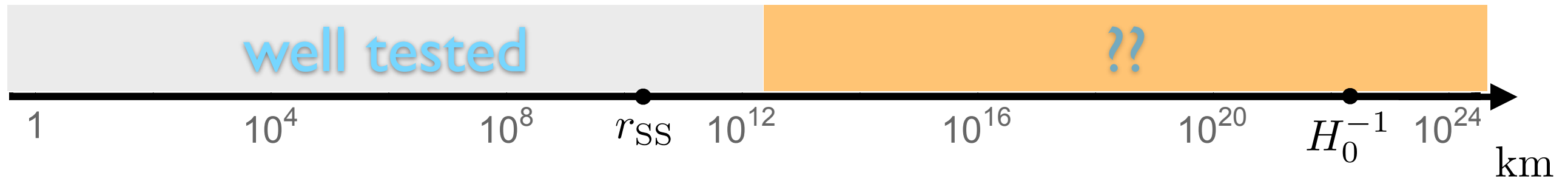
Motivations

General relativity extrapolated over large ranges of scales and masses. Cosmology is a window for testing it on very large distances. Distinguish among models and discover new physics. Cosmological precision tests of Λ CDM (precision tests of the Standard Model at the LHC)



Motivations

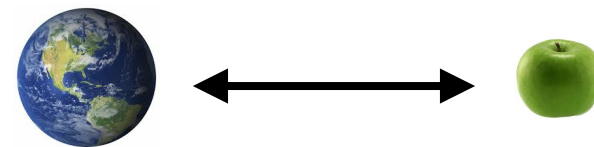
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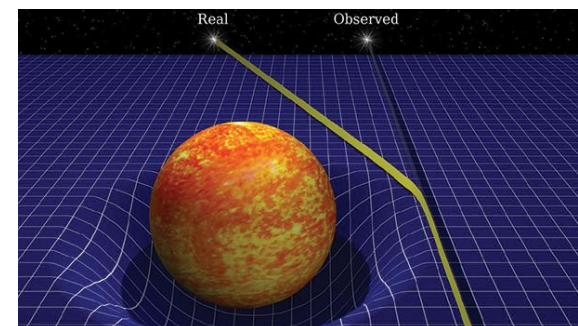
$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

$$\nabla^2\Phi = 4\pi G \mu \delta\rho_m$$

$$\nabla^2(\Phi + \Psi) = 8\pi G \Sigma \delta\rho_m$$



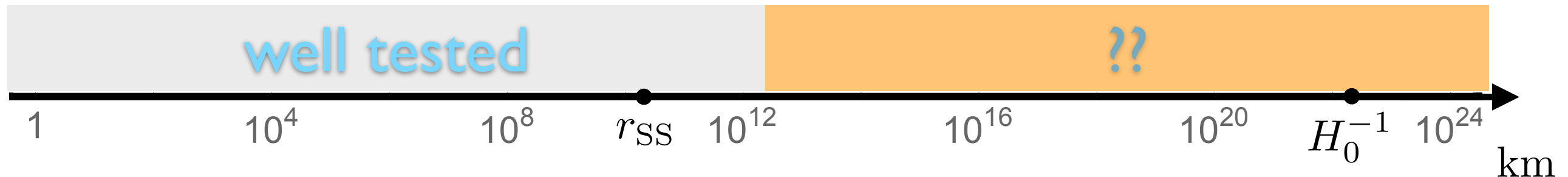
fifth force



anomalous light bending

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Will '14 $|\mu - 1| < 10^{-3} \div 10^{-6}$
 $|\Sigma - 1| < 10^{-5}$

Solar System scales

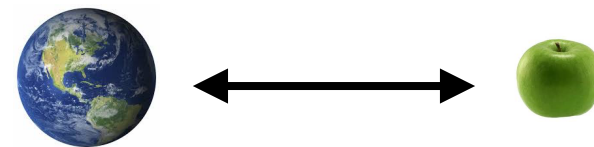
DES '18 $|\mu - 1| < 8 \times 10^{-2}$
 $|\Sigma - 1| < 4 \times 10^{-1}$

Cosmological scales

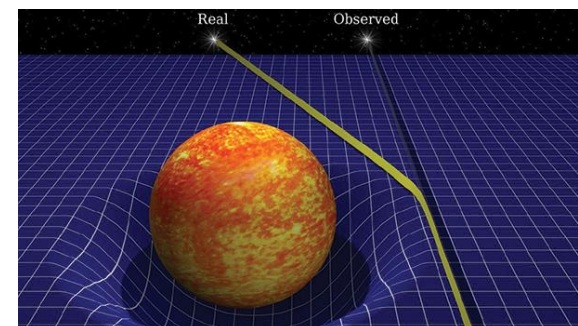
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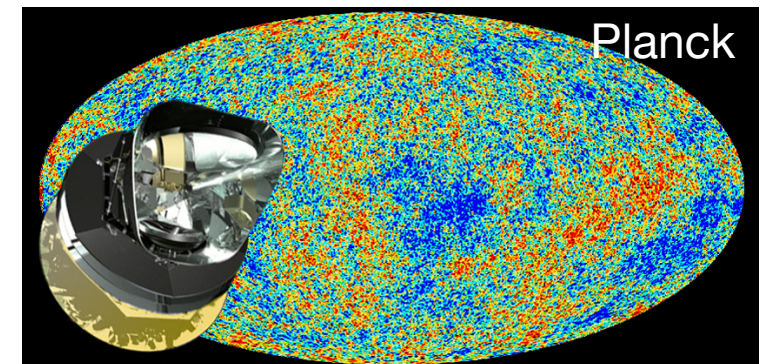
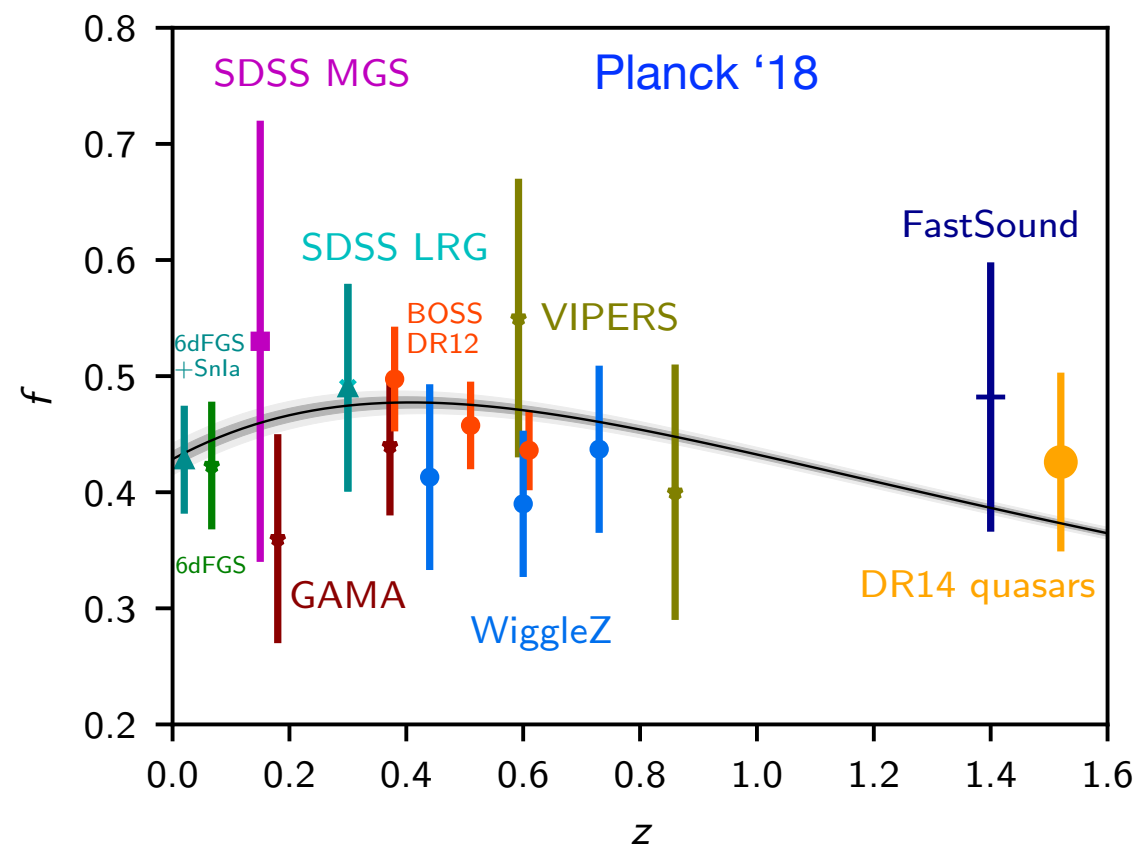
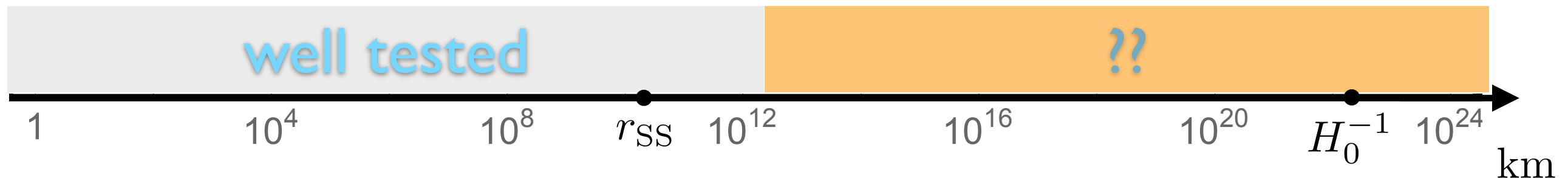
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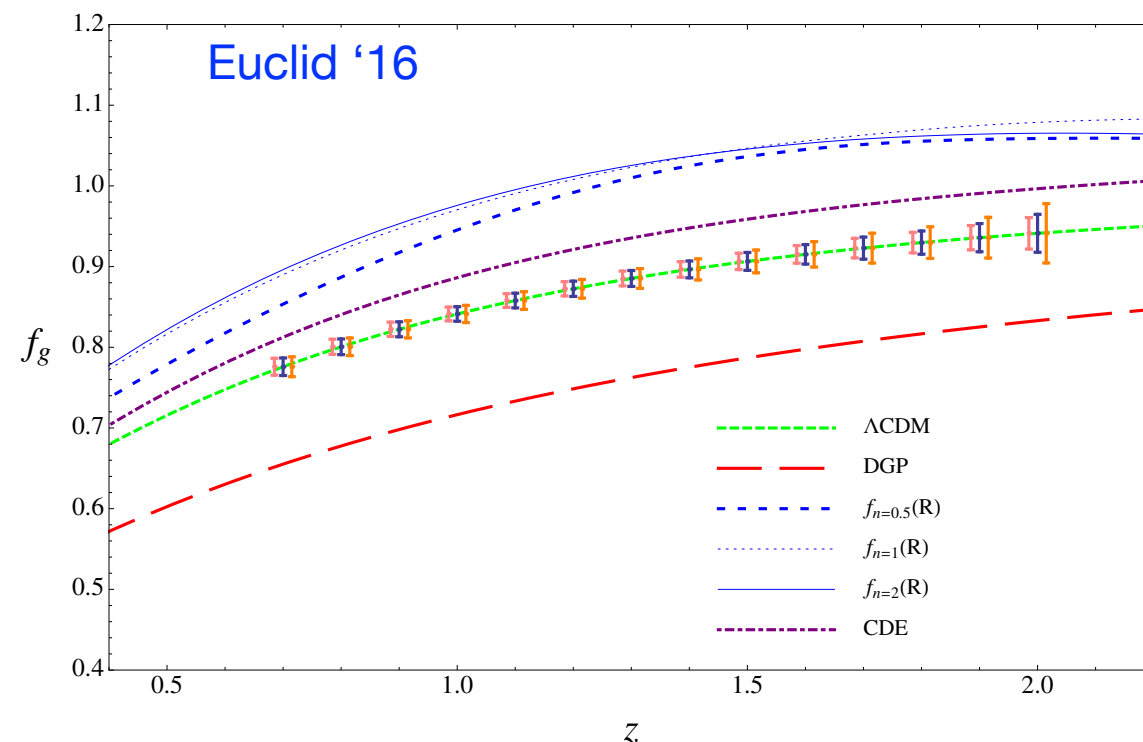
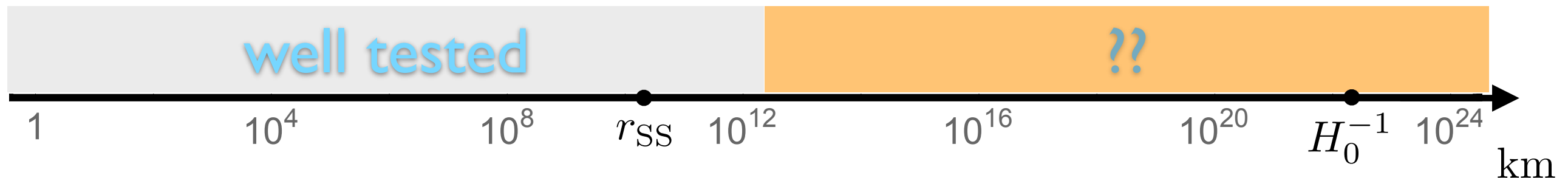
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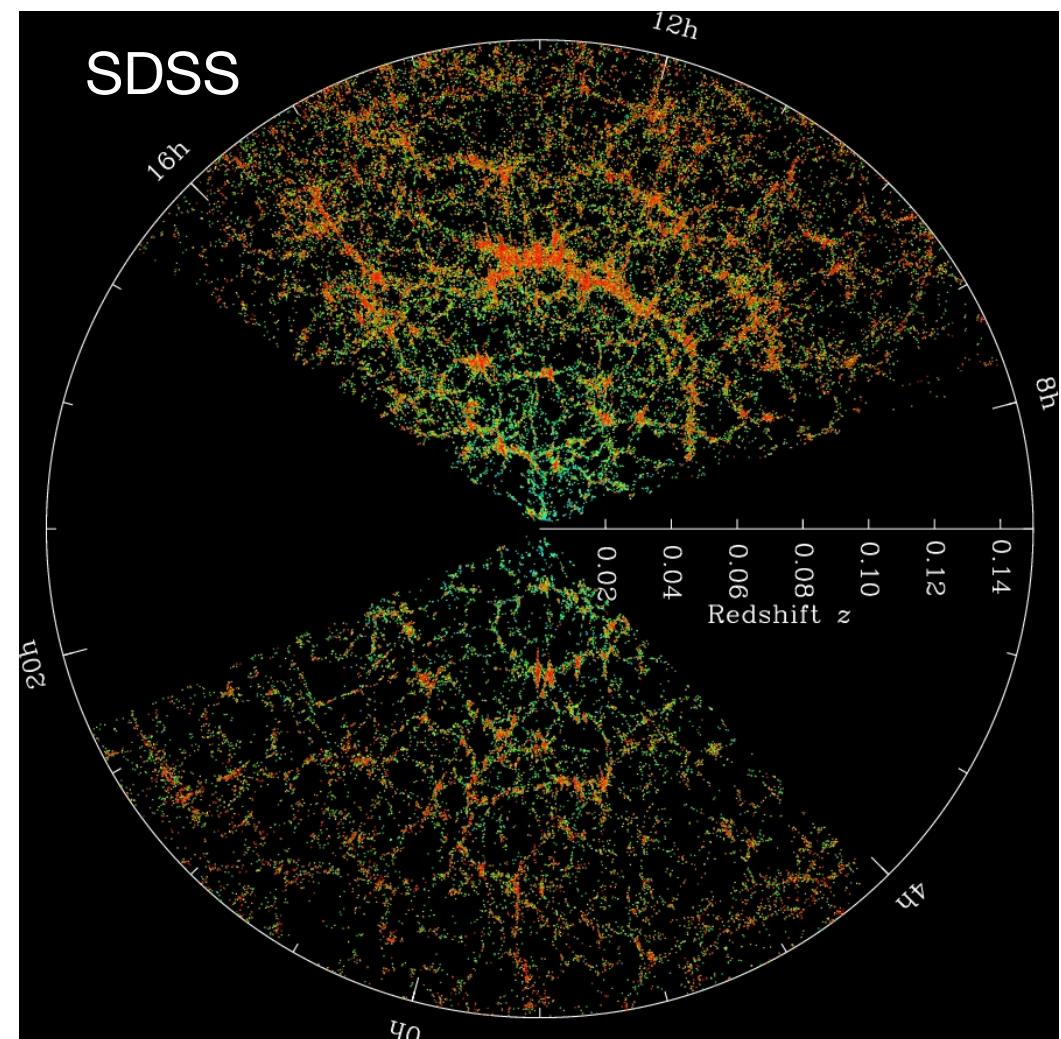
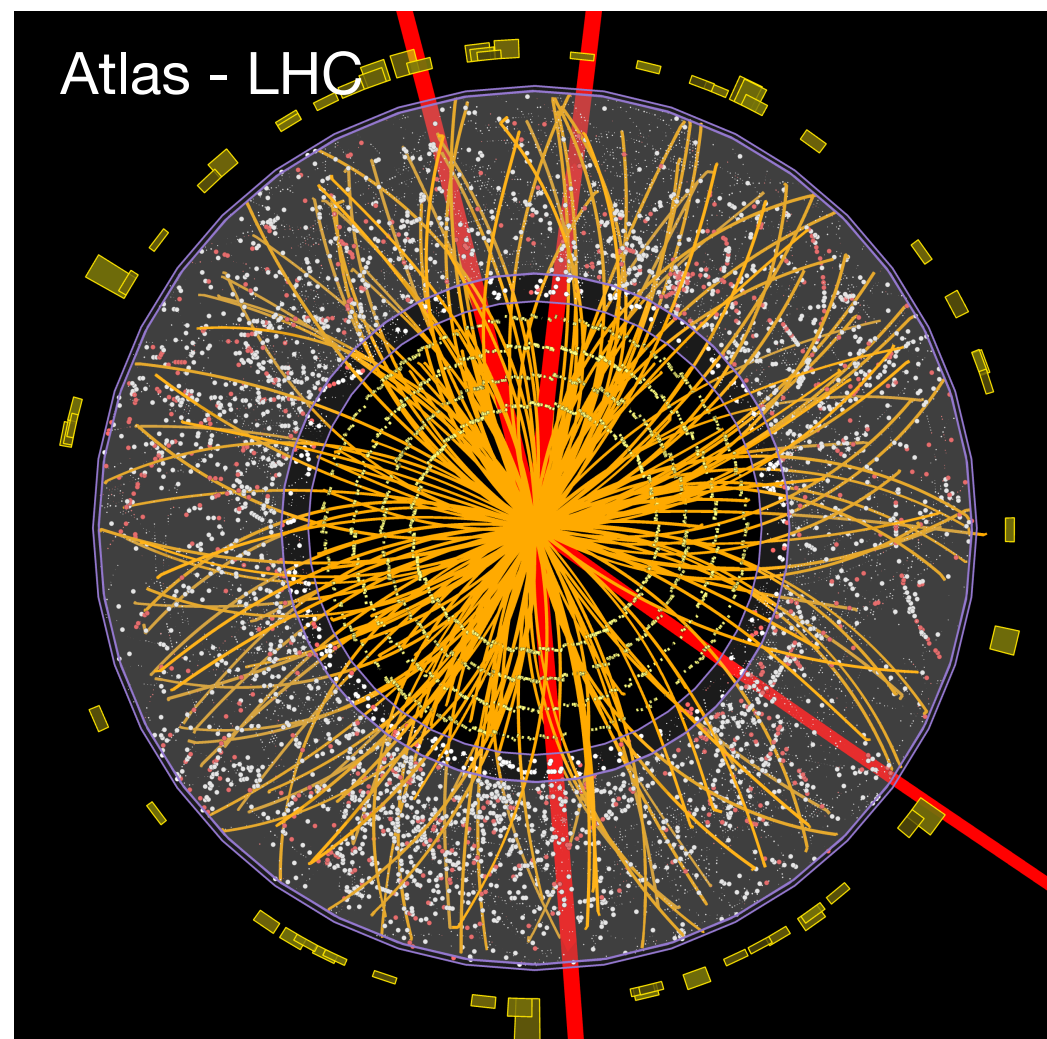
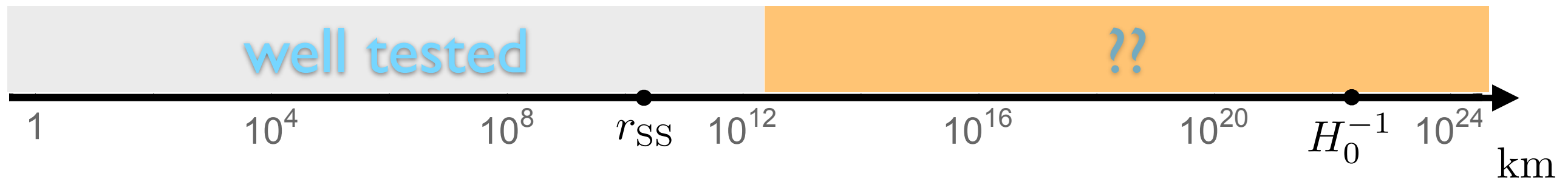
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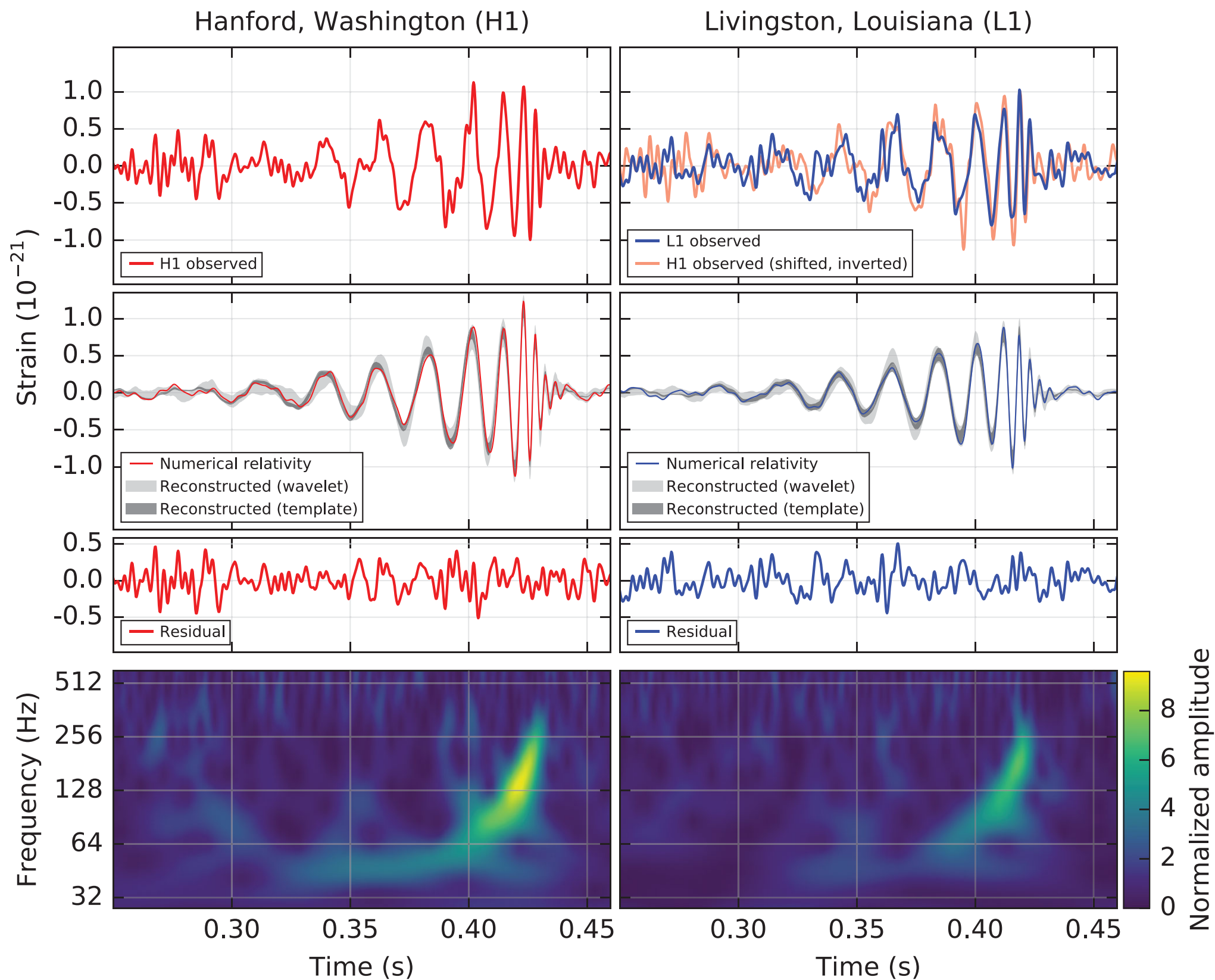
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GW140915: Gravitational Waves

Abbott et al. '16

first detection: 09/14, 2015



Wave equation

Gravitational wave equation:

$$ds^2 = -dt^2 + a^2(t) [\delta_{ij} + \gamma_{ij}] d\vec{x}^i d\vec{x}^j , \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij} , \quad H = \dot{a}/a$$

$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + k^2\gamma_{ij}^{\lambda} = 16\pi G S_{ij}^{\lambda} , \quad \lambda = +, \times$$

Wave equation

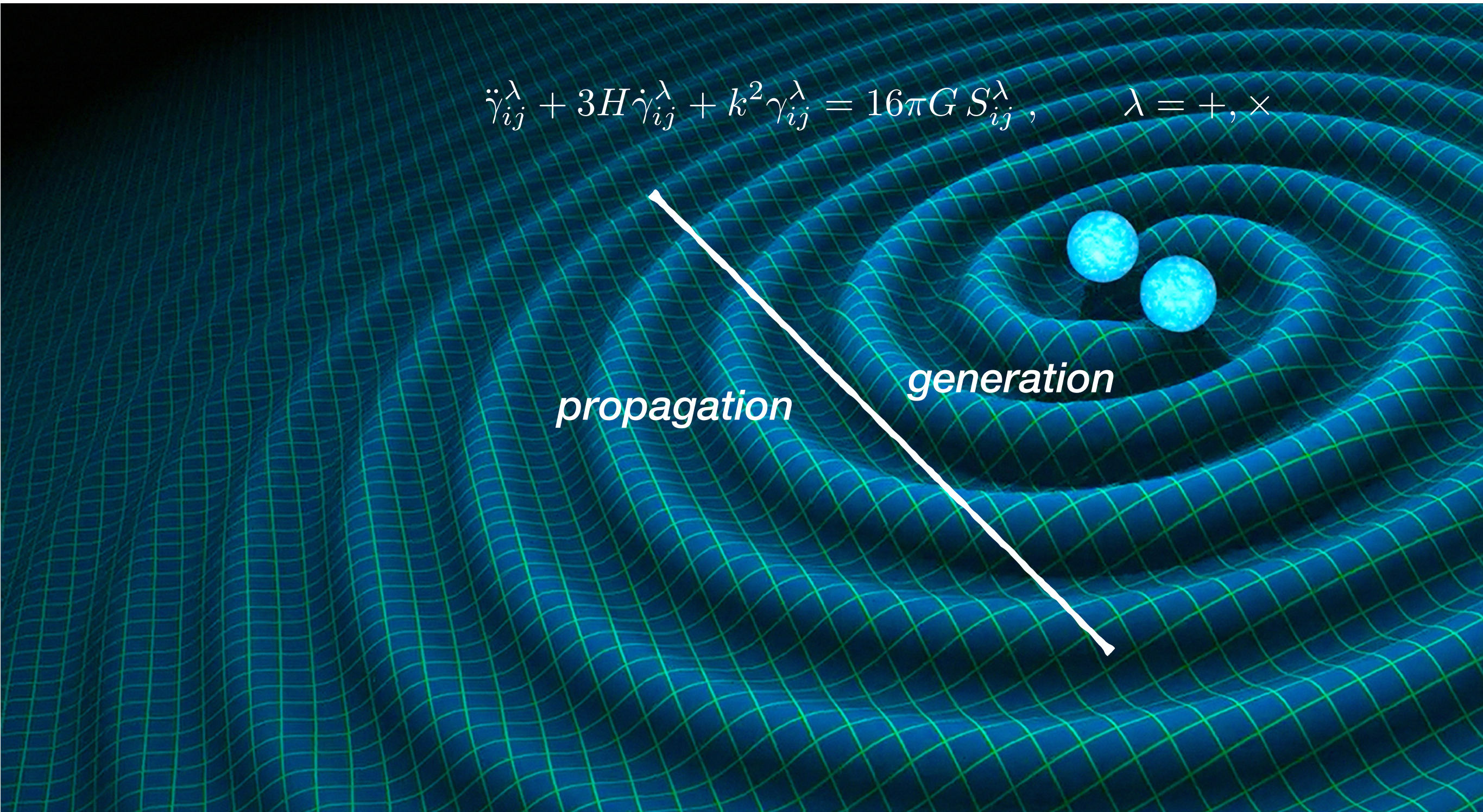
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propagation

generation



Modified wave equation

Gravitational wave equation:

$$ds^2 = -dt^2 + a^2(t) [\delta_{ij} + \gamma_{ij}] d\vec{x}^i d\vec{x}^j, \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij}, \quad H = \dot{a}/a$$

$$\ddot{\gamma}_{ij}^{\lambda} + H \left[3 + \dots \right] \dot{\gamma}_{ij}^{\lambda} + \left[c_T^2 k^2 + \dots \right] \gamma_{ij}^{\lambda} = 16\pi G S_{ij}^{\lambda}, \quad \lambda = +, \times$$

damping dispersion source polarizations

Modified wave equation

Gravitational wave equation:

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$$\ddot{\gamma}_{ij}^{\lambda} + H [\text{damping}] \dot{\gamma}_{ij}^{\lambda} + [\text{dispersion}] \gamma_{ij}^{\lambda} = 16\pi G \text{source } S_{ij}^{\lambda}, \quad \text{polarizations } \lambda = +, \times$$



Modifications in the wave equation are related to modifications of gravity in the LSS:

$$\mu = \mu(\dots), \quad \Sigma = \Sigma(\dots)$$

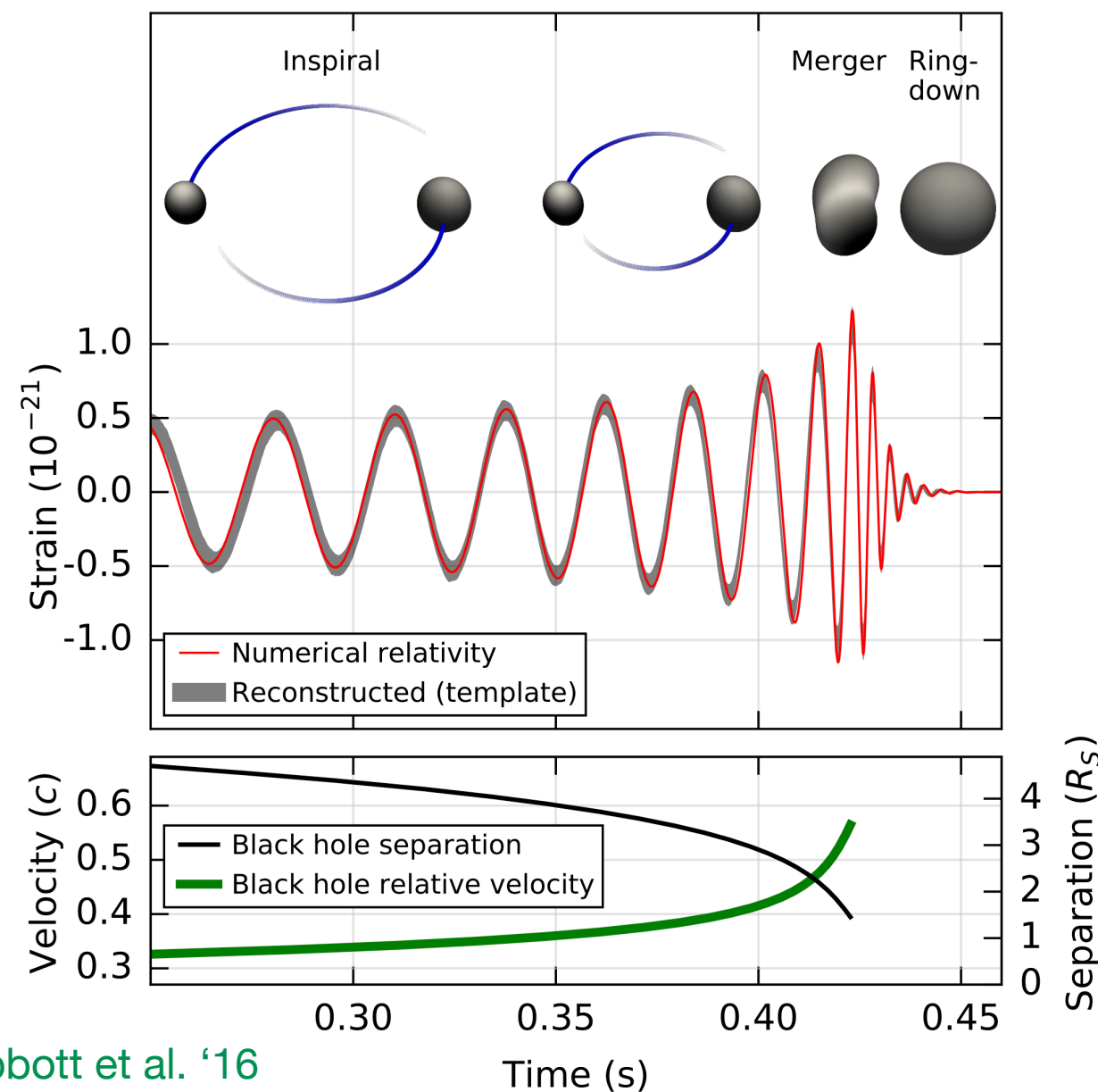
$$\nabla^2 \Phi = 4\pi G \mu \delta\rho_m$$

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Source

Modifications of the motion of inspiral objects and thus of the production mechanism. Affects GW phase (and amplitude)

$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + k^2\gamma_{ij}^{\lambda} = 16\pi G S_{ij}^{\lambda}, \quad \lambda = +, \times$$



Abbott et al. '16

$$v^2 = \frac{G_N m}{r}$$

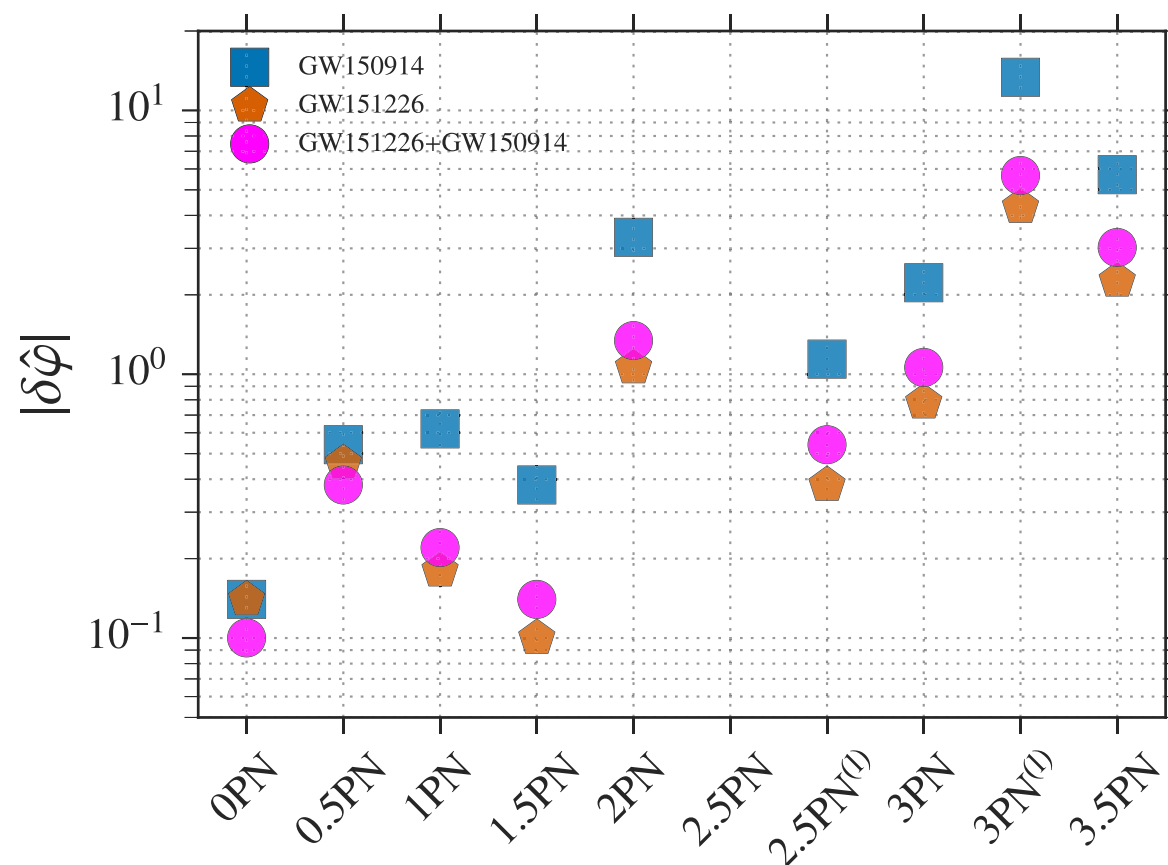
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Consistency check of general relativity:

$$\varphi = \varphi_0 - \frac{1}{32} \left(\frac{GM_c \omega}{c^3} \right)^{-5/3} \left[1 + 1\text{PN} + 1.5\text{PN} + \dots + 3.5\text{PN} + \mathcal{O} \left(\frac{1}{c^8} \right) \right]$$



90% CL upper bound
on relative deviations on PN
parameters

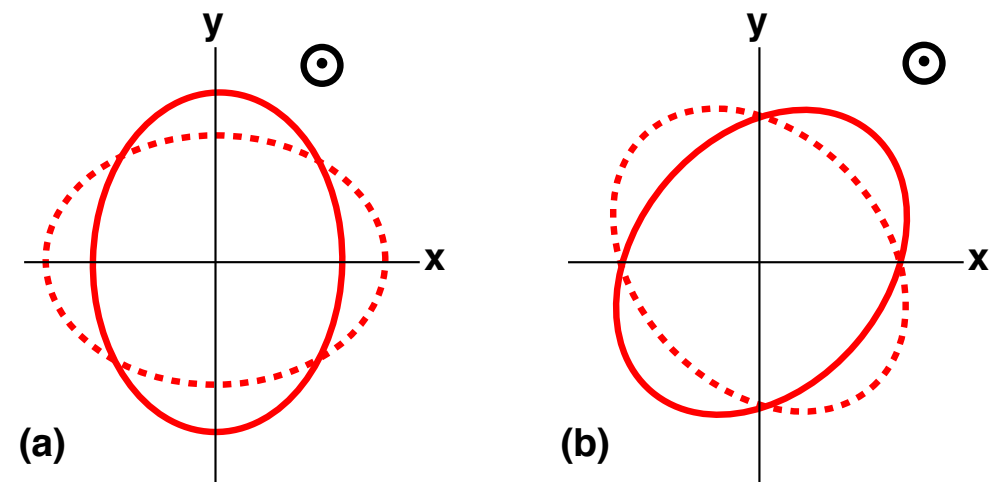
Abbott et al. '16

Polarizations

Different polarizations can be generated or propagate differently: **parity violations** (ex. Cherns-Simons gravity)

Jackiw & Pi '03

$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + [k^2 + f^{\lambda}(k)]\gamma_{ij}^{\lambda} = 16\pi G S_{ij}^{\lambda}, \quad \lambda = +, \times$$



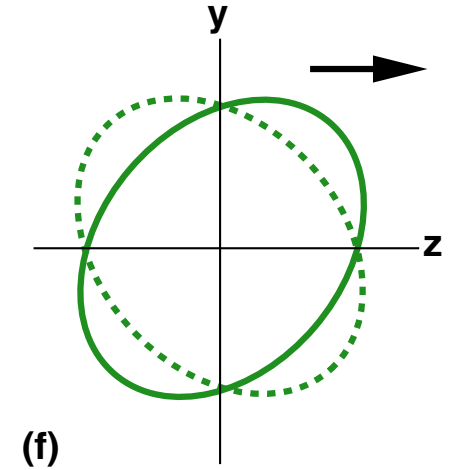
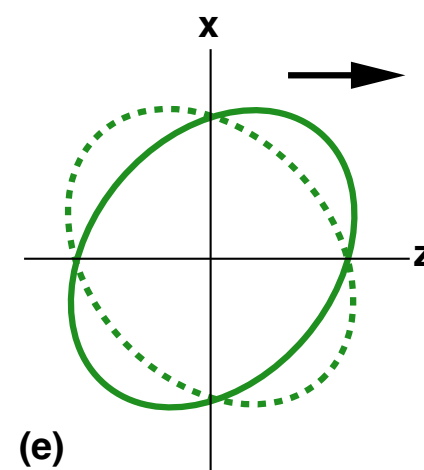
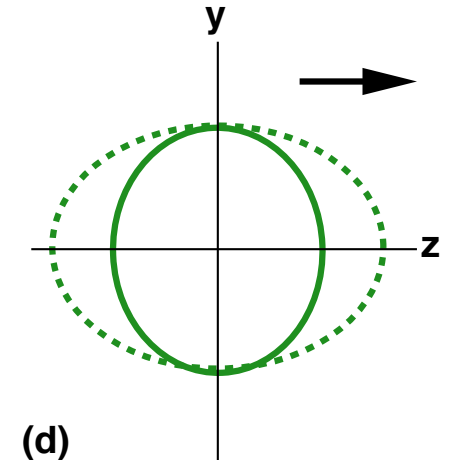
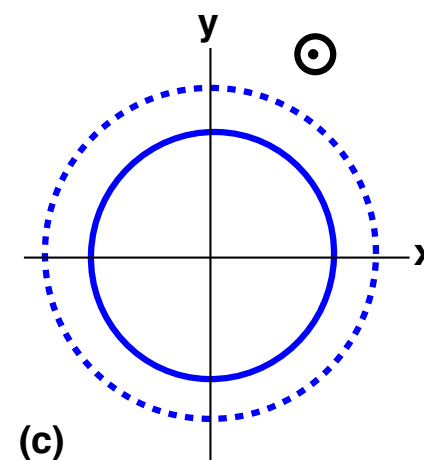
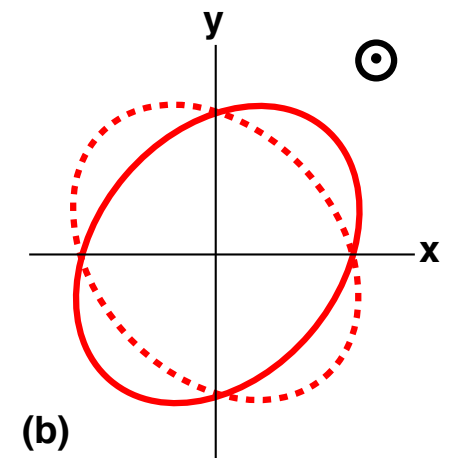
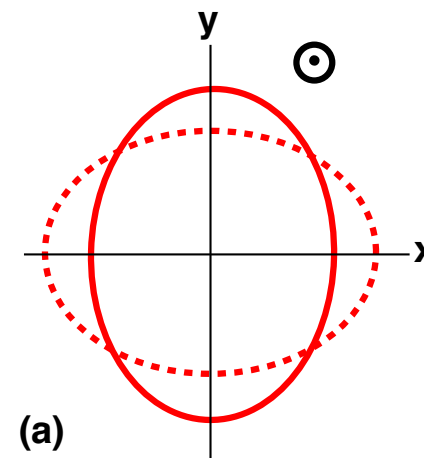
$$\gamma_{ij} = \begin{pmatrix} A_{+} & A_{\times} & 0 \\ A_{\times} & -A_{+} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Polarizations

Modified gravity can involve different degrees of freedom: **scalar, vectors, extra tensors**

Will '14

$$\gamma_{ij} = \begin{pmatrix} A_S + A_+ & A_\times & A_{V1} \\ A_\times & A_S - A_+ & A_{V2} \\ A_{V1} & A_{V2} & A_L \end{pmatrix}$$



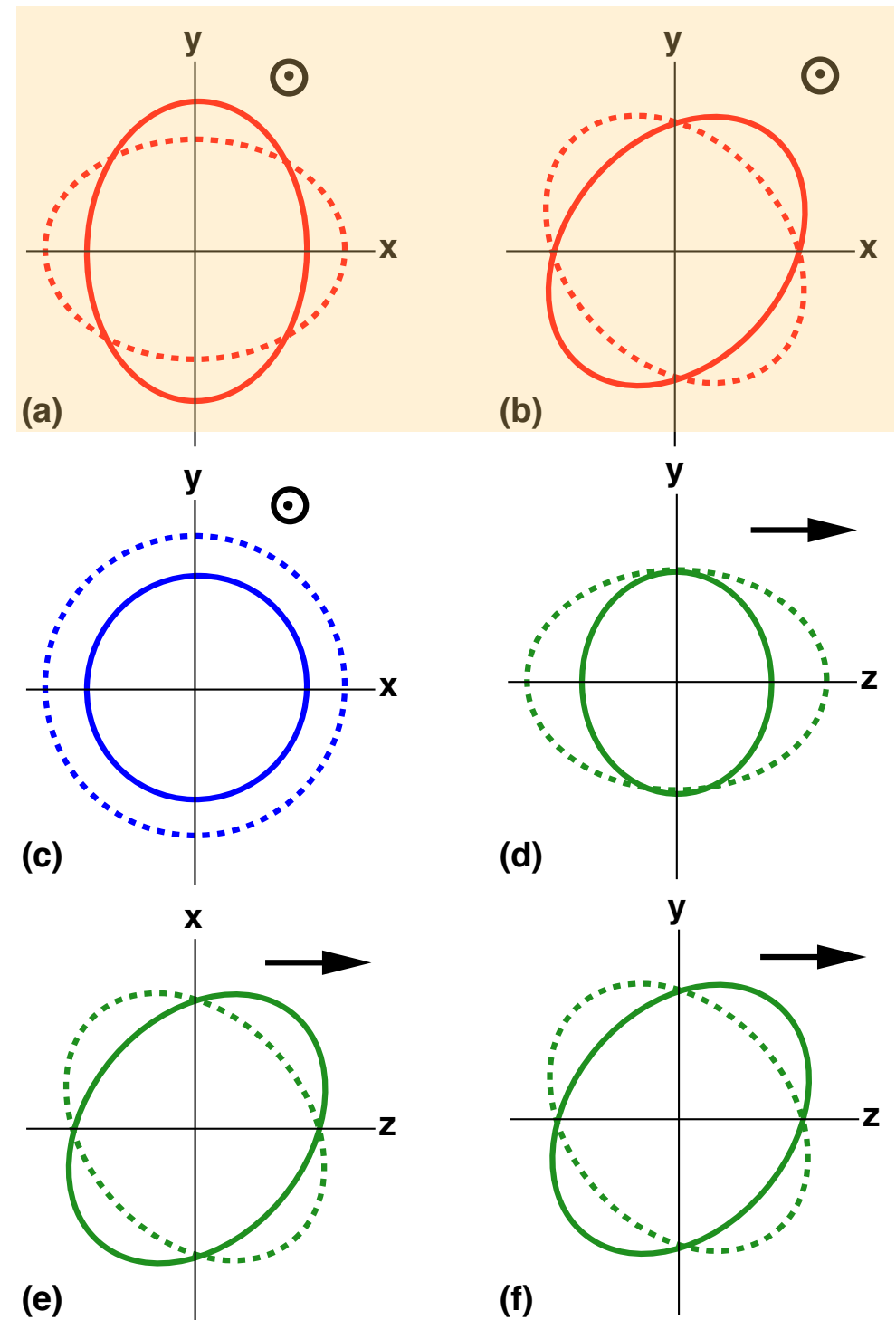
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Will '14

Standard GR: 2 transverse modes (spin-2)

$$\gamma_{ij} = \begin{pmatrix} A_+ & A_\times & 0 \\ A_\times & -A_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Polarizations

Modified gravity can involve different degrees of freedom: **scalar, vectors, extra tensors**

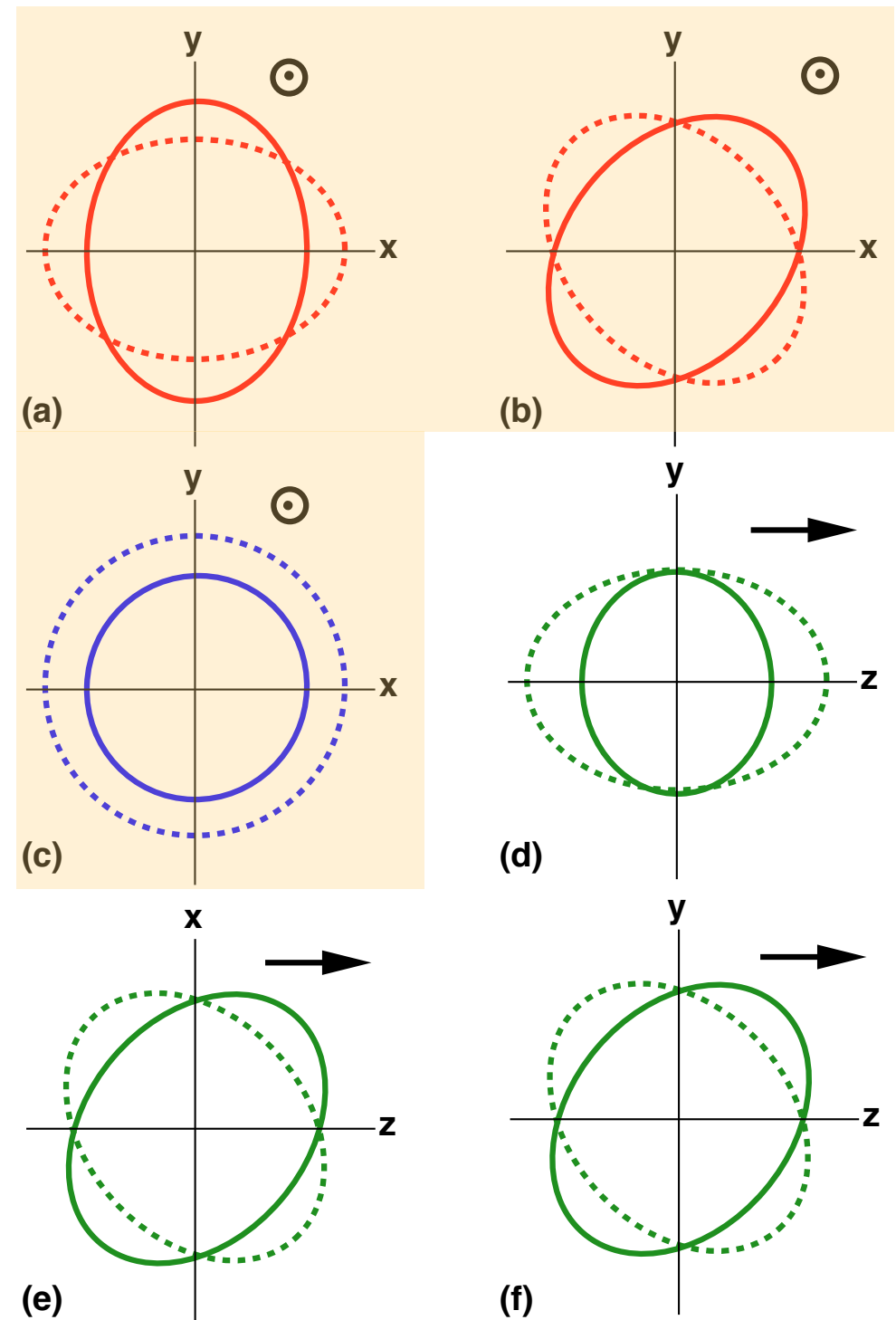
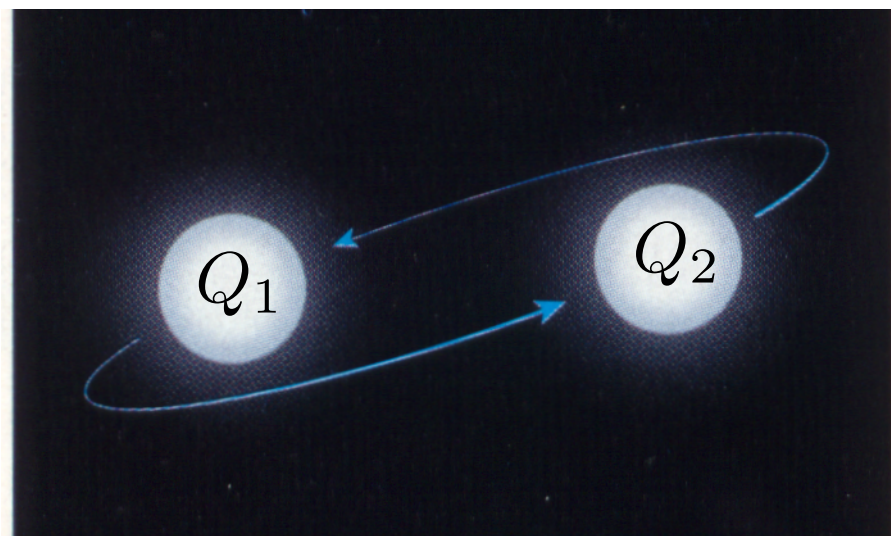
Will '14

Scalar-Tensor: 2 transverse modes (spin 2) + 1 transverse mode (spin 0)

$$\gamma_{ij} = \begin{pmatrix} A_S + A_+ & A_\times & 0 \\ A_\times & A_S - A_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P_\phi \propto (Q_1 - Q_2)^2 \omega^{8/3}$$

dipole



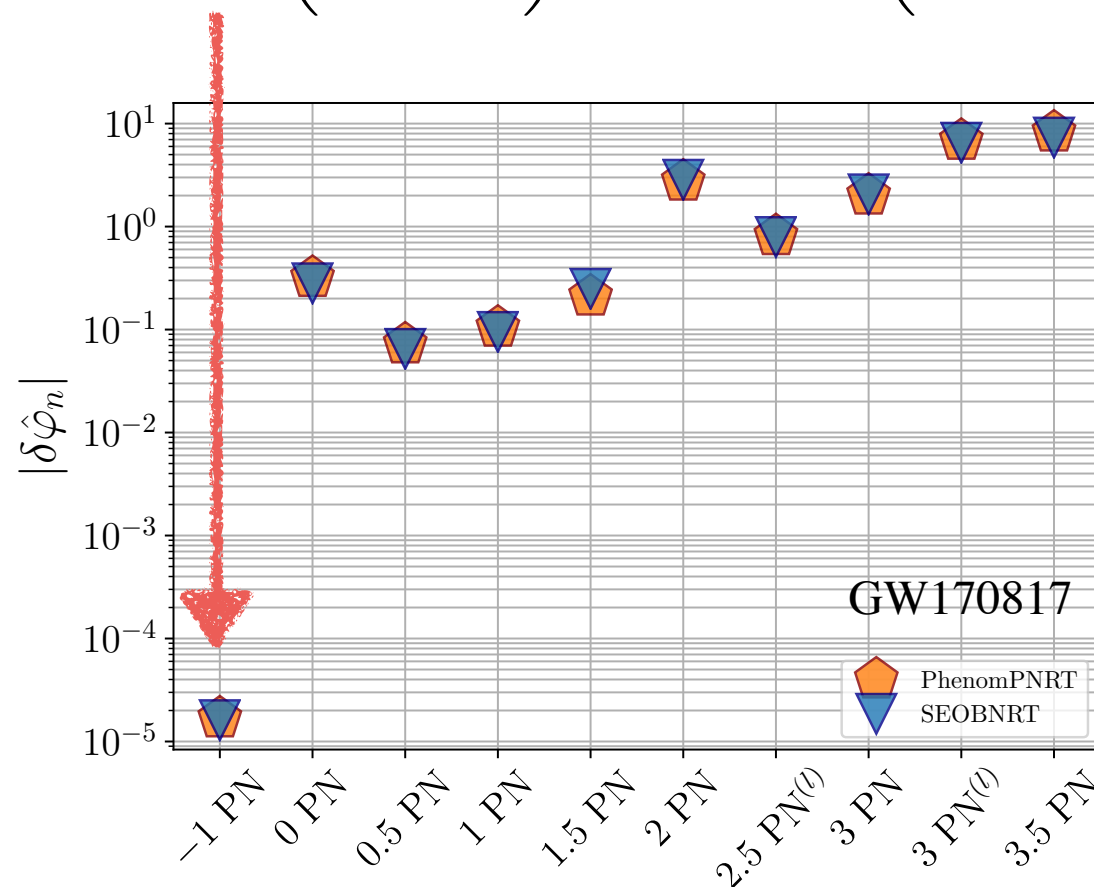
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$$\ddot{\gamma}_{ij}^{\lambda} + 3H\dot{\gamma}_{ij}^{\lambda} + k^2\gamma_{ij}^{\lambda} = 16\pi G S_{ij}^{\lambda}, \quad \lambda = +, \times$$

Consistency check of general relativity:

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90% CL upper bound
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Abbott et al. '18

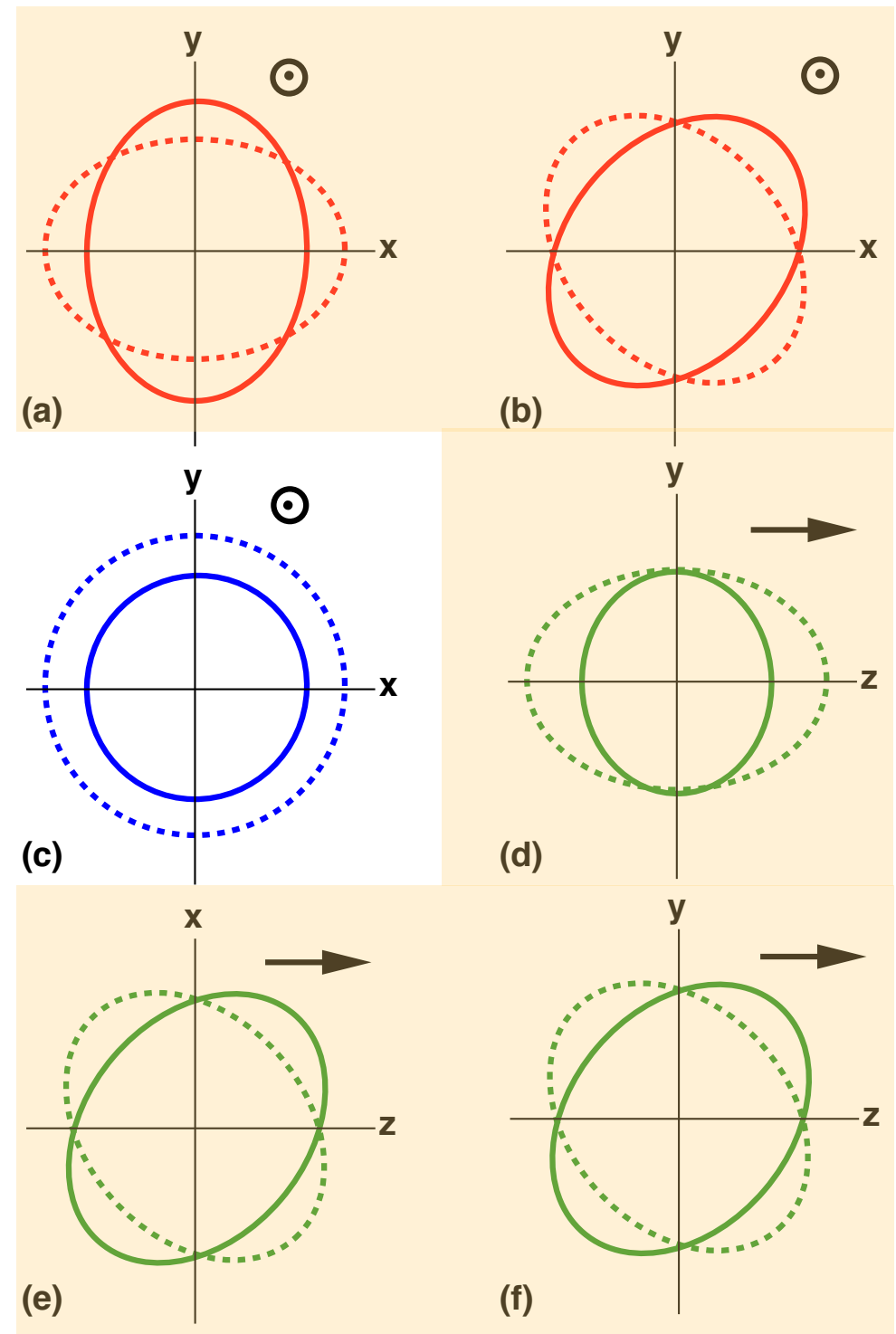
Polarizations

Modified gravity can involve different degrees of freedom: **scalar, vectors, extra tensors**

Will '14

Massive graviton: 2 transverse modes (spin-2) + 2 longitudinal modes (spin 1) + 1 longitudinal mode (spin 0)

$$\gamma_{ij} = \begin{pmatrix} A_+ & A_\times & A_{V1} \\ A_\times & -A_+ & A_{V2} \\ A_{V1} & A_{V2} & A_L \end{pmatrix}$$



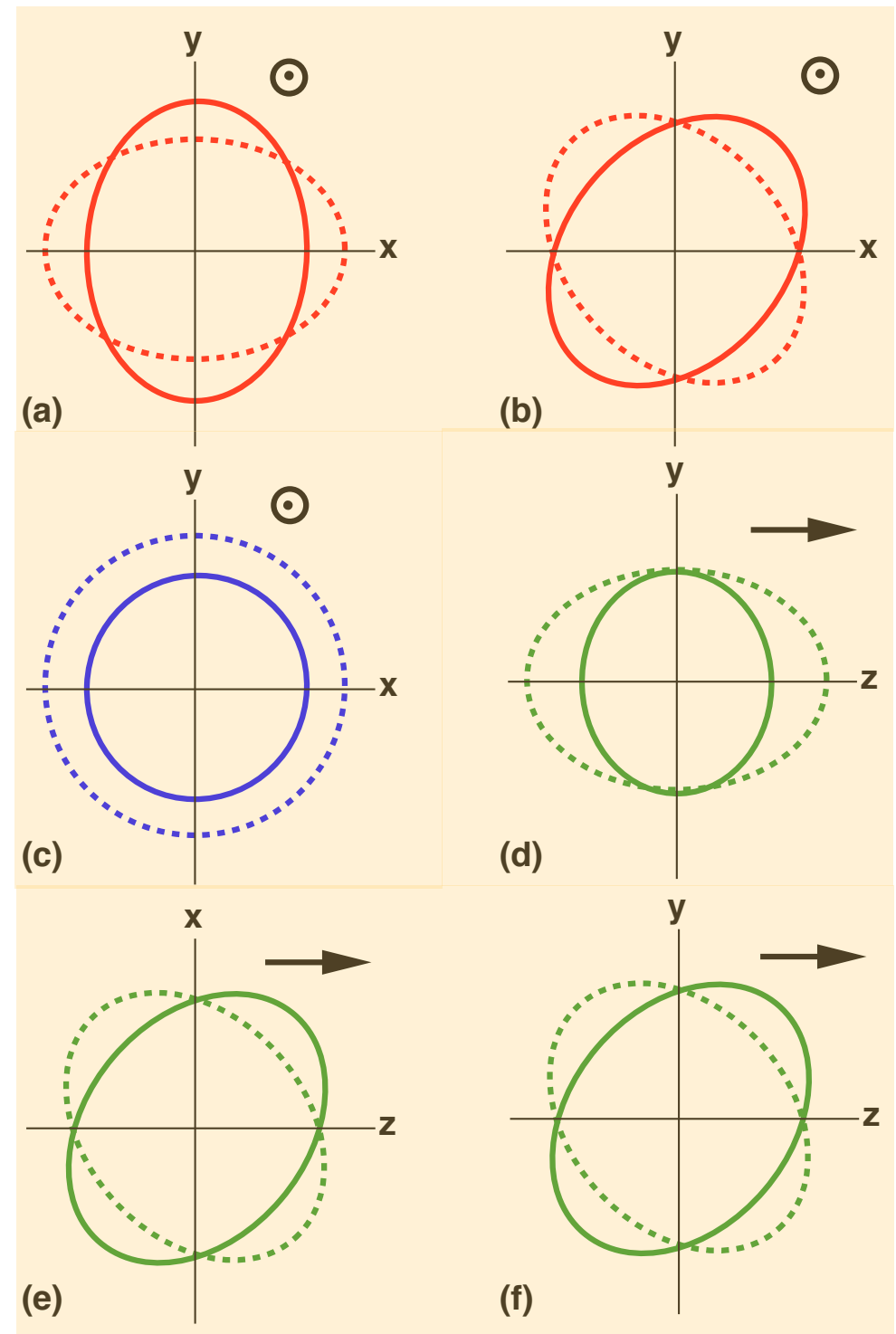
Polarizations

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Will '14

Scalar-Vector-Tensor: 2 transverse modes (spin-2)
+ 1 transverse mode (spin 0) + 2 longitudinal
modes (spin 1) + 1 longitudinal mode (spin 0)

$$\gamma_{ij} = \begin{pmatrix} A_S + A_+ & A_\times & A_{V1} \\ A_\times & A_S - A_+ & A_{V2} \\ A_{V1} & A_{V2} & A_L \end{pmatrix}$$



Polarizations

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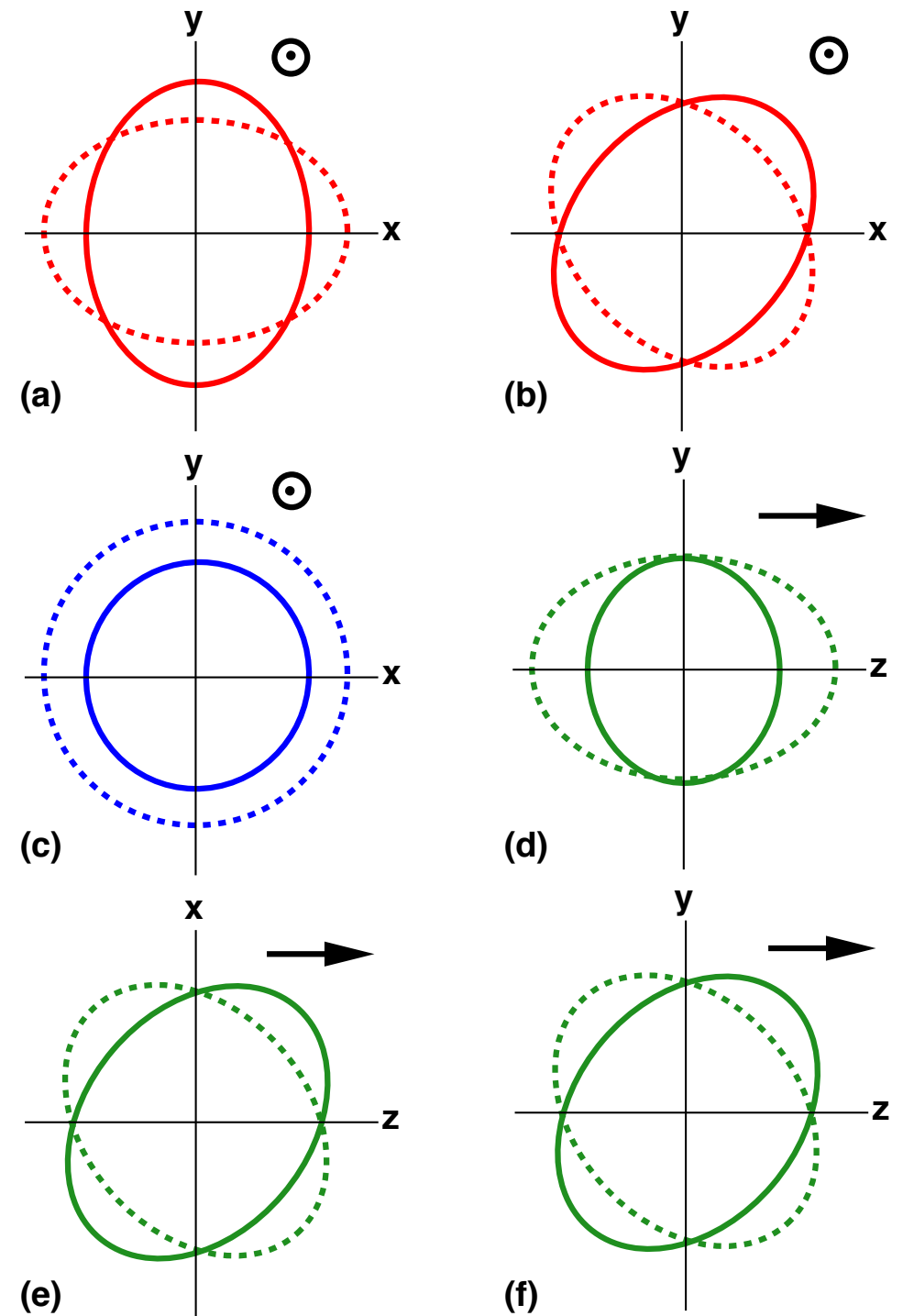
Will '14

6 polarizations + 2 directions = 8 unknowns

Assuming only transverse polarizations and known positions = 3 detectors enough

- Signal consistent with the two standard longitudinal tensor modes
- No evidence for extra polarizations (in the unmixed case)

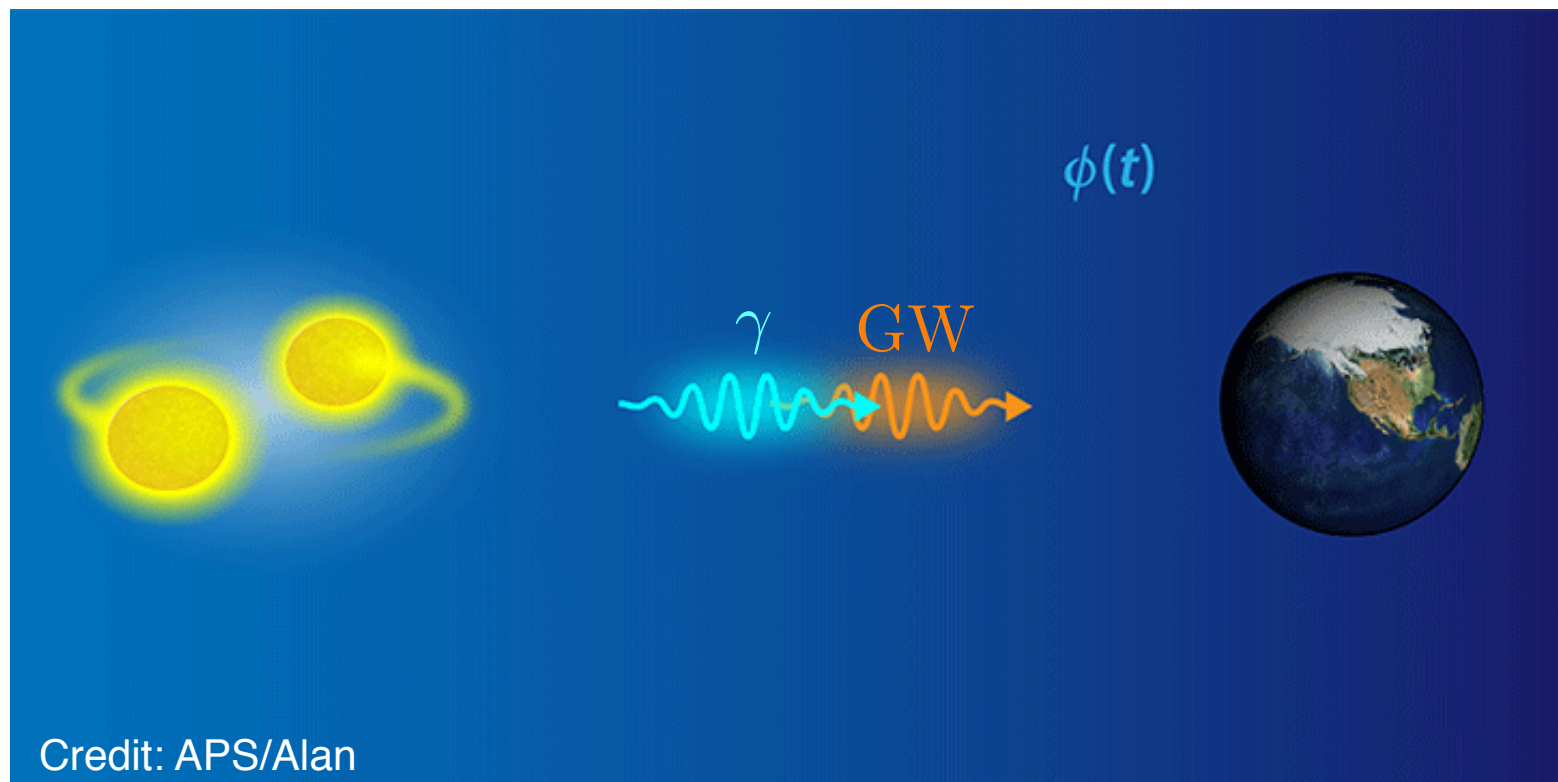
Abbott et al. '16, '18



Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed. Effects accumulate on long time-scale.

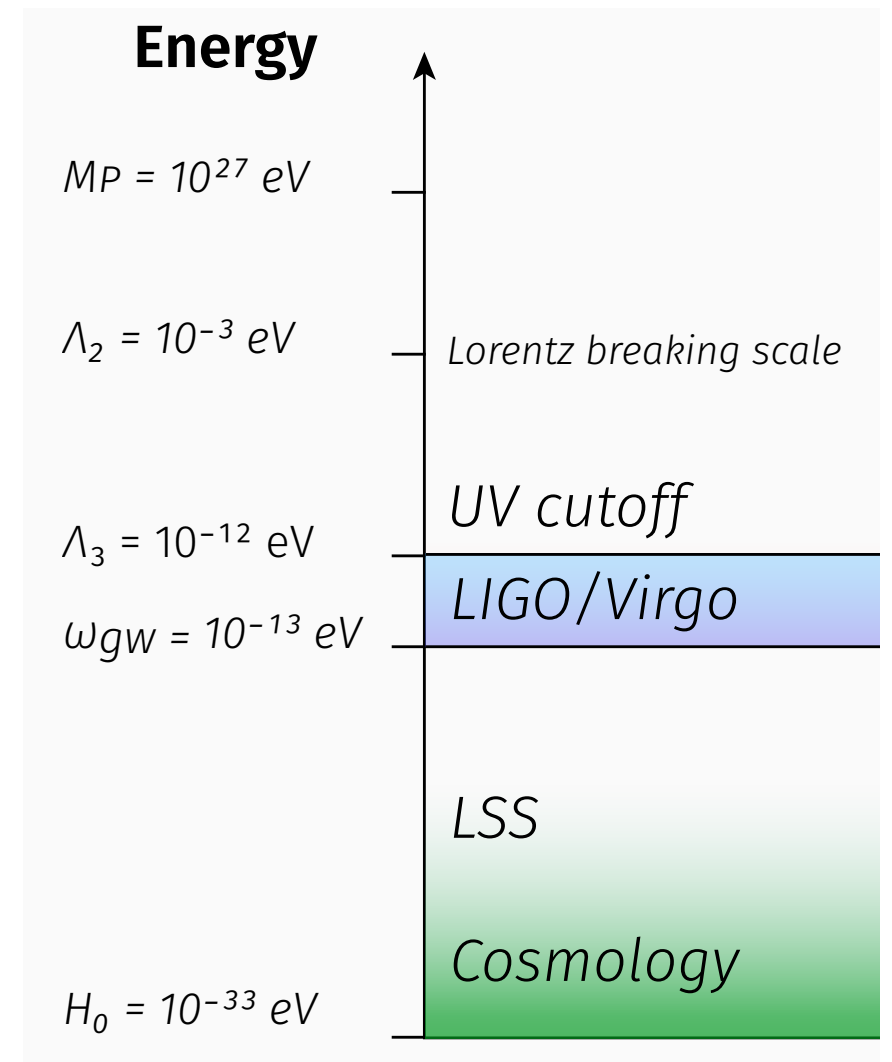
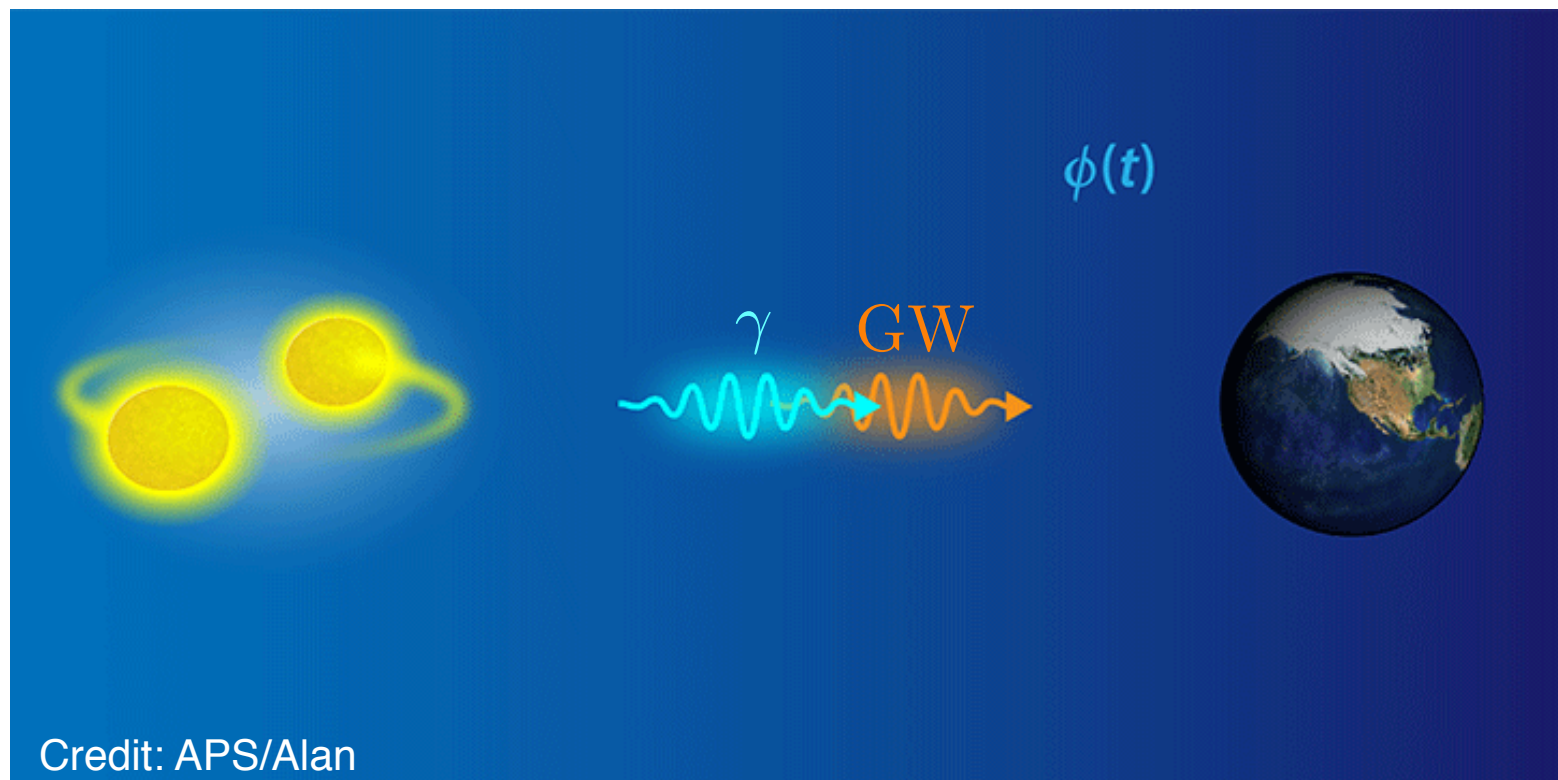
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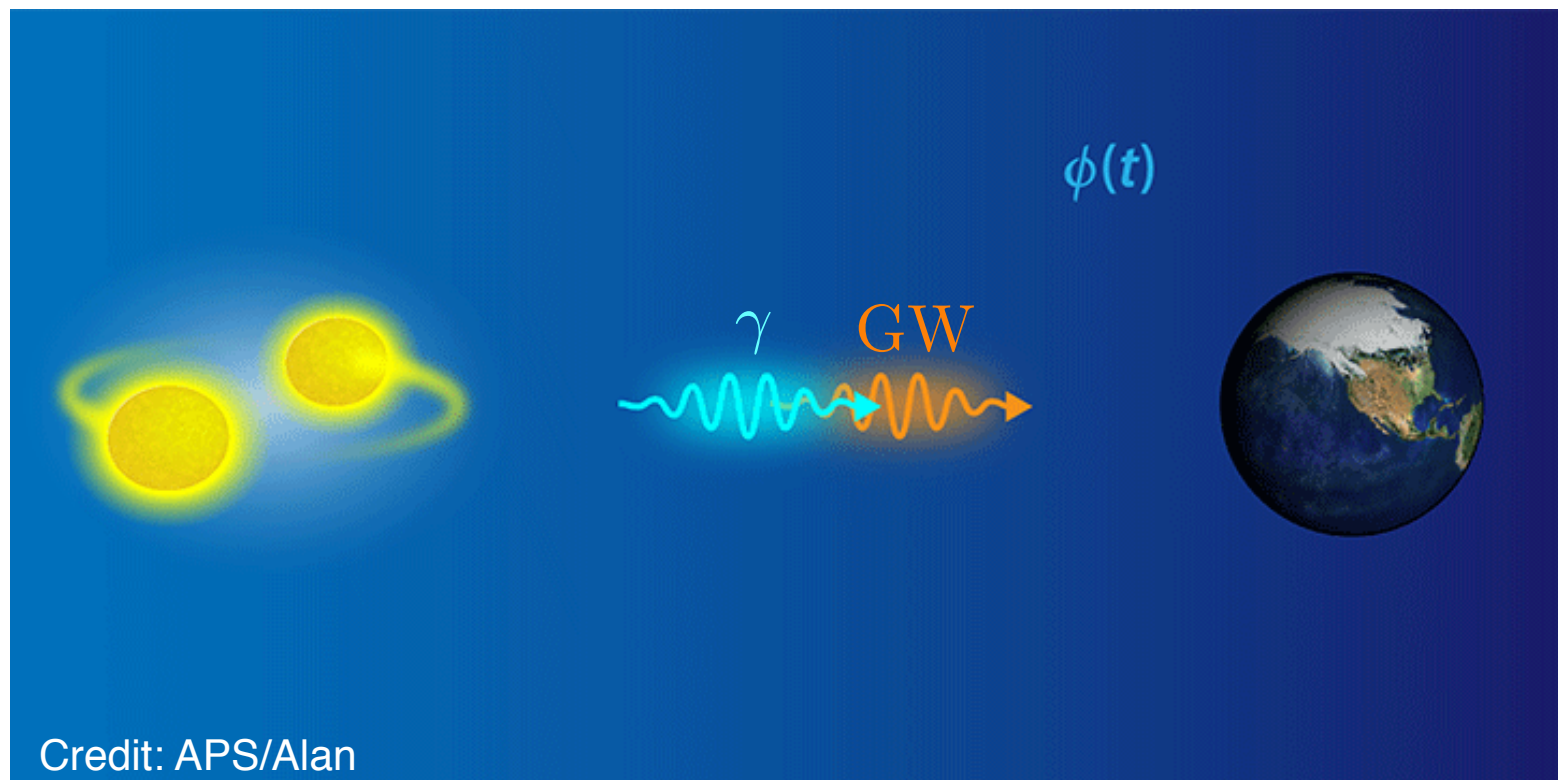
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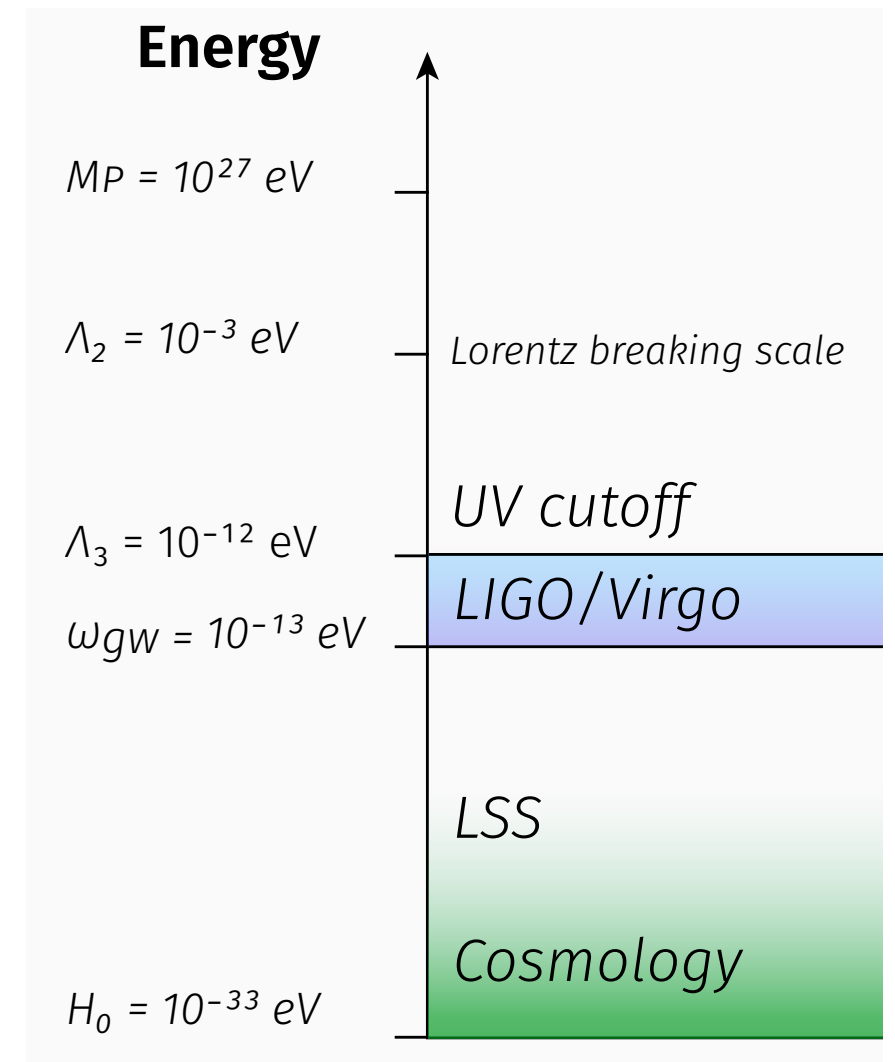
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Similar to GW damping by neutrinos after or damping and modification of the propagation speed by CDM

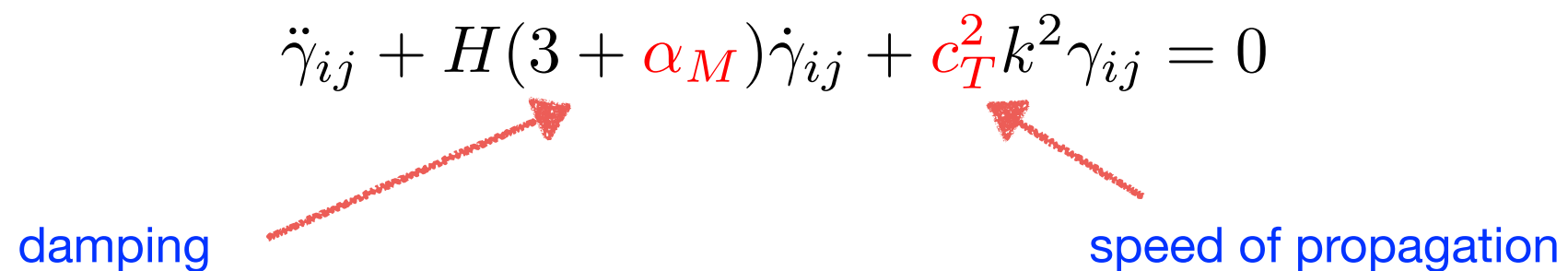
Weinberg '03; Flauger & Weinberg '18



GW propagation

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Frequency independent effects:

$$\ddot{\gamma}_{ij} + H(3 + \alpha_M)\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$


damping

speed of propagation

$$\mu = \mu(\alpha_M, c_T^2, \dots) , \quad \Sigma = \Sigma(\alpha_M, c_T^2, \dots)$$

GW propagation

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damping

speed of propagation

$d_L^{\text{gw}} \neq d_L^{\text{em}}$ different luminosity distances

Deffayet, Menou '07;
Calabrese, Battaglia, Spergel, '16;
Amendola et al. '17, Belgacem et al. '17,
etc...

- Time variation of G
- Extra-dimensional leakage
- Nonlocal models
- ...

LISA: $\sigma_{\alpha_M} \approx 0.03 - 0.1$

Amendola, Sawicki, Kunz, Saltas '18

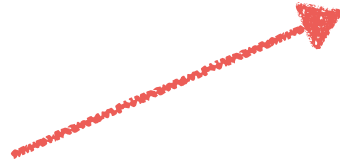
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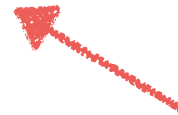
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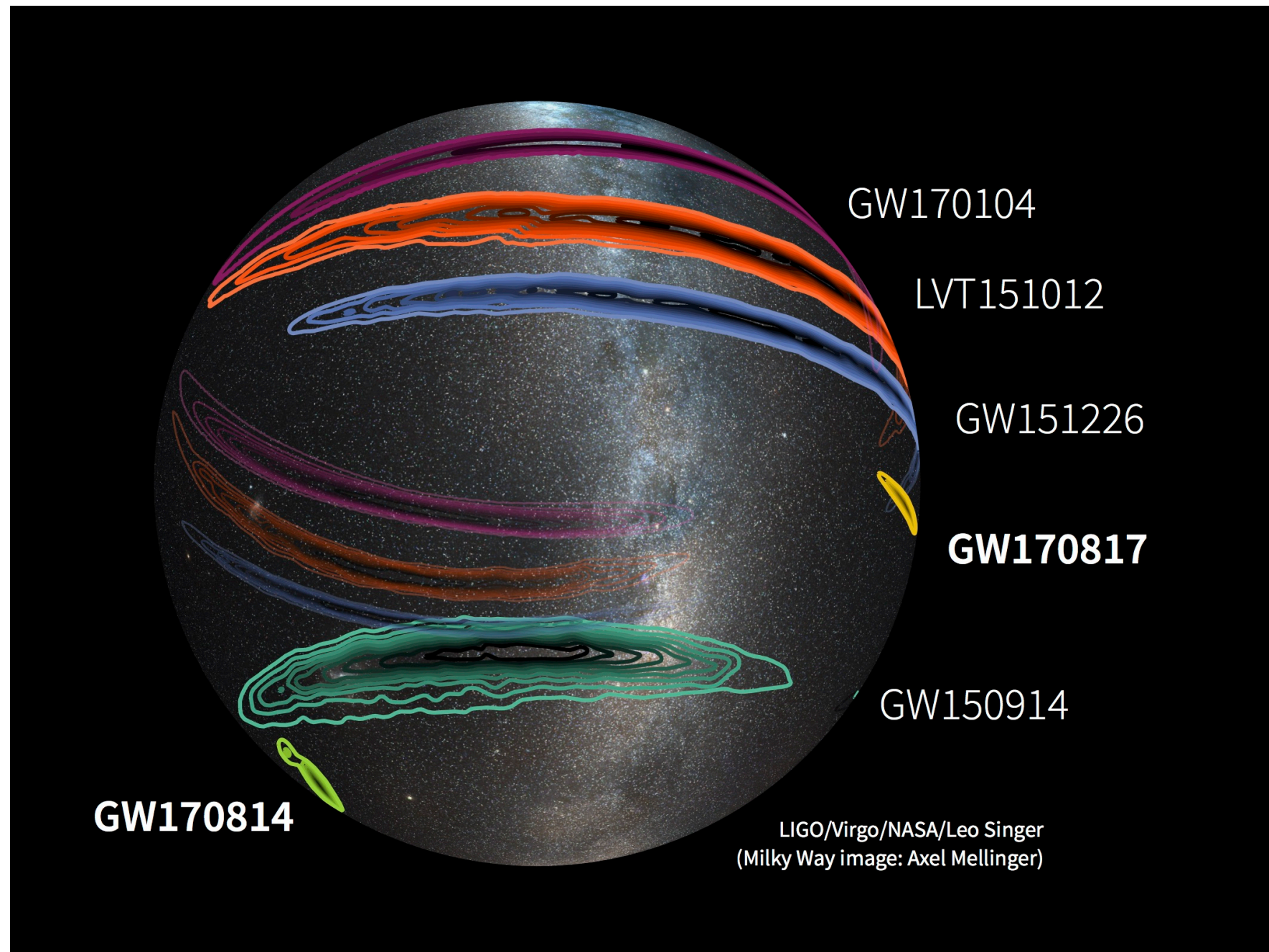
speed of propagation



GW170817: neutron star merger

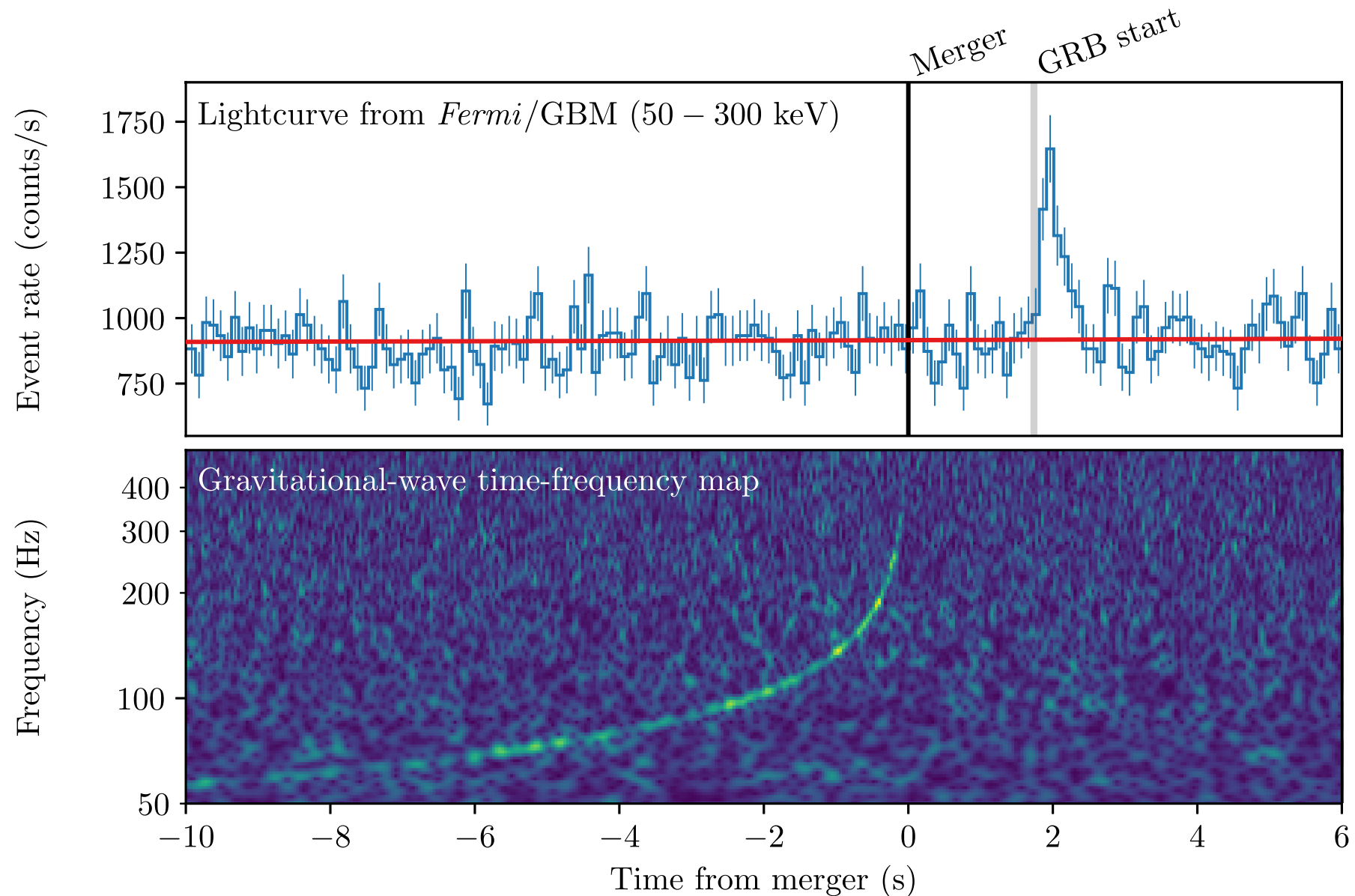


GW170817: neutron star merger



Efficient localization with three detectors

Multi-messenger observation



$$-3 \times 10^{-15} \leq \frac{c_T - c}{c} \leq 7 \times 10^{-16}$$

- Previous (indirect) limit only on GW slower than light.
- Low energy ($\lambda \lesssim 10^4$ km)
- Over cosmological distances

GW propagation

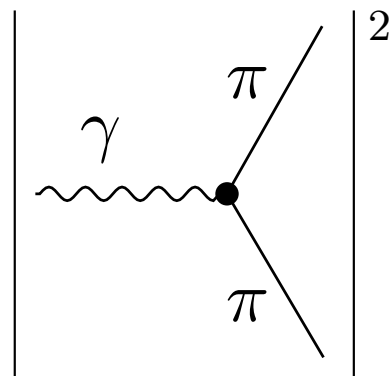
Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed. Effects accumulate on long time-scale.

Frequency **dependent** effects:

Creminelli, Lewandowski, Tambalo, FV '18

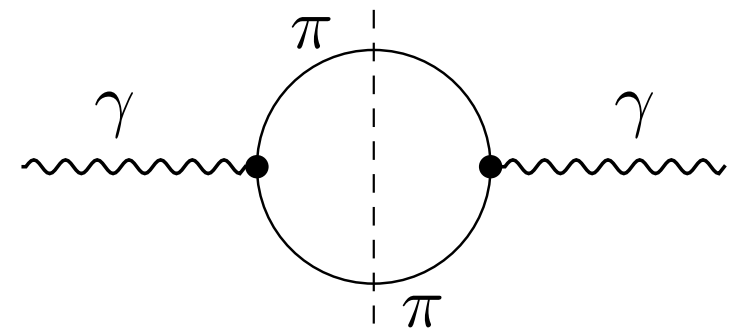
$$\ddot{\gamma}_{ij} + [(3 + \alpha_M)H + \Gamma(k)] \dot{\gamma}_{ij} + [c_T^2 k^2 + f(k)] \gamma_{ij} = 0$$

decay



Similar to Cherenkov radiation

dispersion



Similar to a refractive material

Related by optical theorem:

$$\Gamma(k)\omega(k) = \text{Im} [f(k)]$$

GW propagation

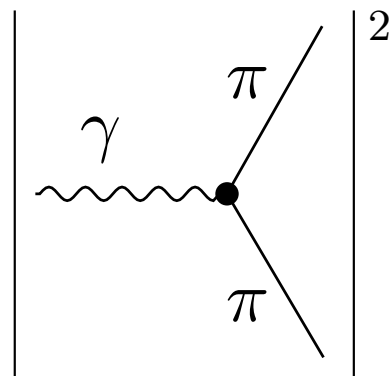
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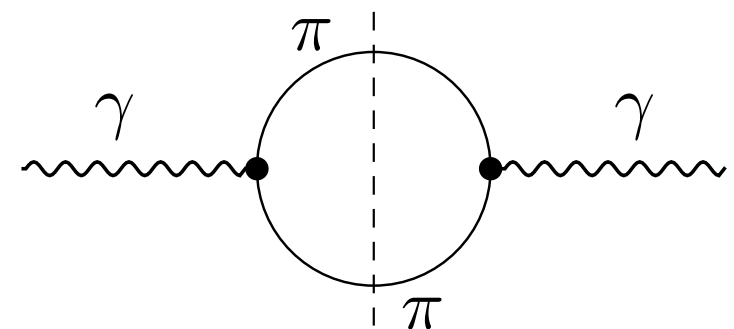
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decay



Similar to Cherenkov radiation

dispersion



Similar to a refractive material

$$\frac{\Gamma(k)}{\omega} \lesssim \frac{1}{d_S \omega}$$

$$\frac{f(k)}{\omega^2} \lesssim \frac{1}{d_S \omega} \sim 10^{-18} \times \frac{2\pi \times 100 \text{ Hz}}{\omega} \frac{40 \text{ Mpc}}{d_S}$$

See e.g. Yunes, Yagi, Pretorius '16; Abbott et al. '17

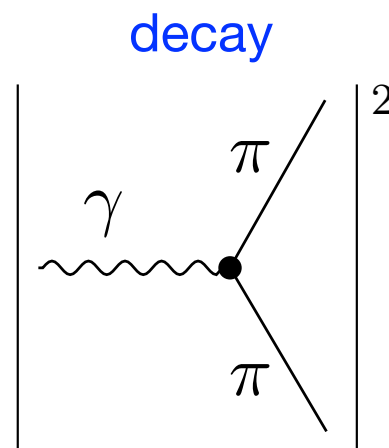
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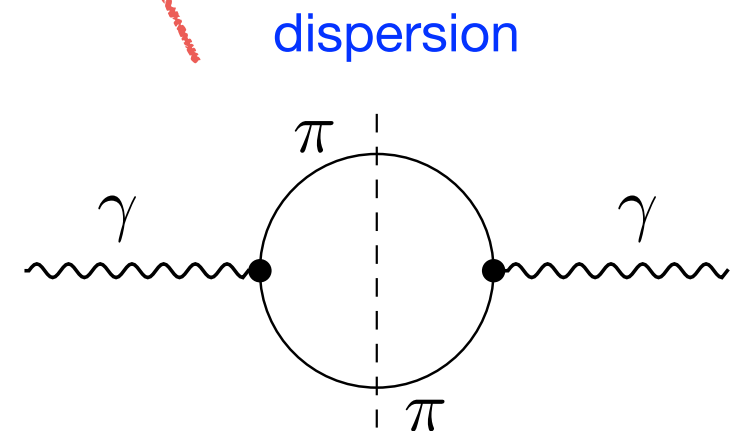
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Similar to Cherenkov radiation

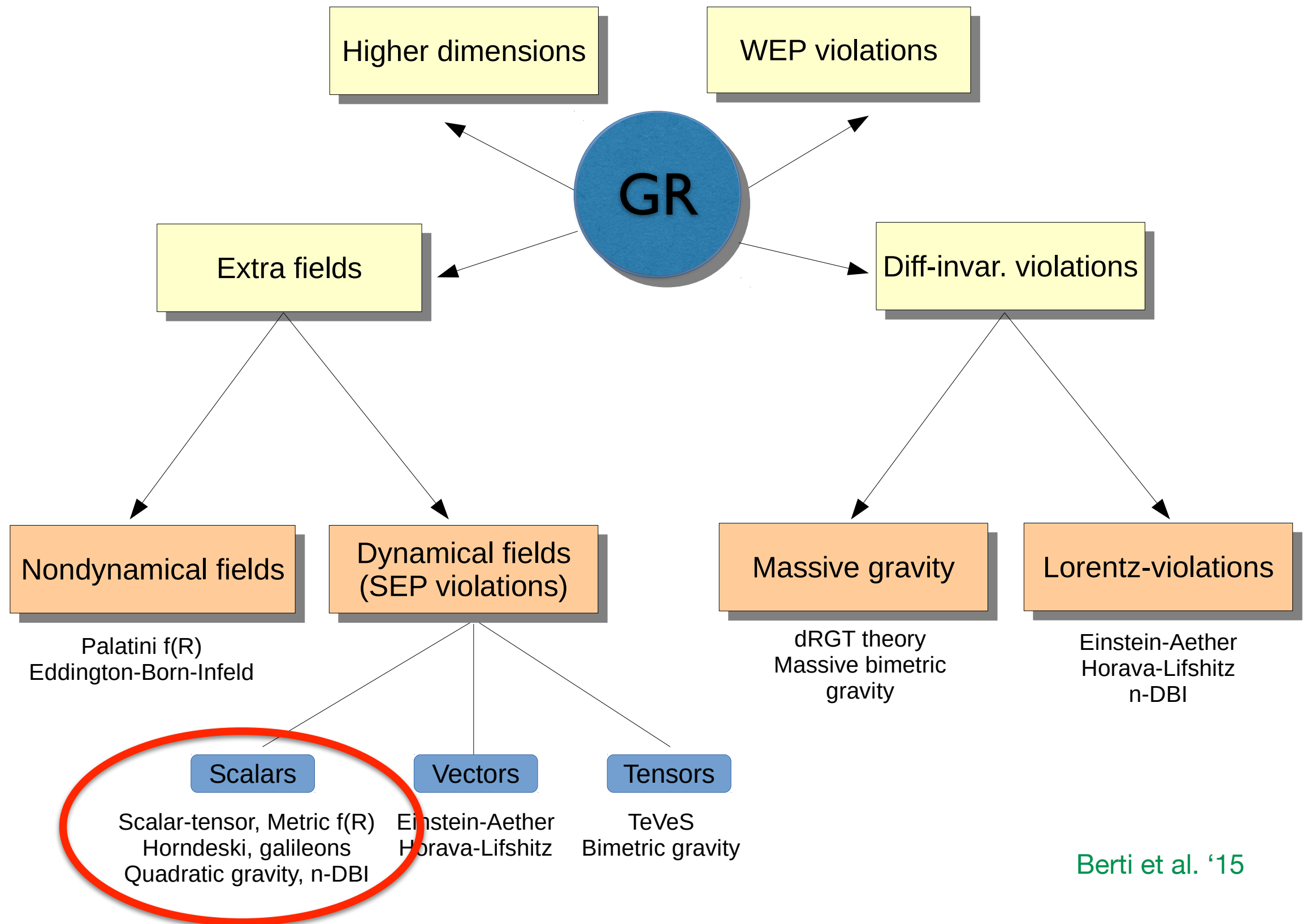


Similar to a refractive material

Simplest case: $f(k) \equiv m_\gamma^2$, $m_\gamma^2/\omega^2 < (d_S \omega)^{-1} \Rightarrow m_\gamma < 10^{-22} \text{ eV}$

See e.g. Yunes, Yagi, Pretorius '16; Abbott et al. '17

Dark energy and modified gravity



Scalar-tensor theories

Simplest models of modified gravity are based on single scalar field (universal coupling)

Ex: $\mathcal{L} = R + V(\phi) - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ quintessence

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

$$w \neq -1$$

Scalar-tensor theories

Simplest models of modified gravity are based on single scalar field (universal coupling)

Ex: $\mathcal{L} = R + G_2(\phi, X)$, $X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ k-essence

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

$$c_s^2 \neq 1 : \text{clustering}$$

Scalar-tensor theories

Simplest models of modified gravity are based on single scalar field (universal coupling)

Ex: $\mathcal{L} = f(\phi)R + G_2(\phi, X)$, $X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ scalar-tensor gravity

$$G_{\mu\nu}^{(\text{modified})} = 8\pi G \left(T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\phi)} \right)$$

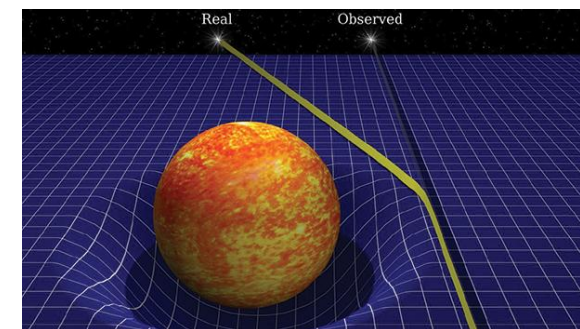
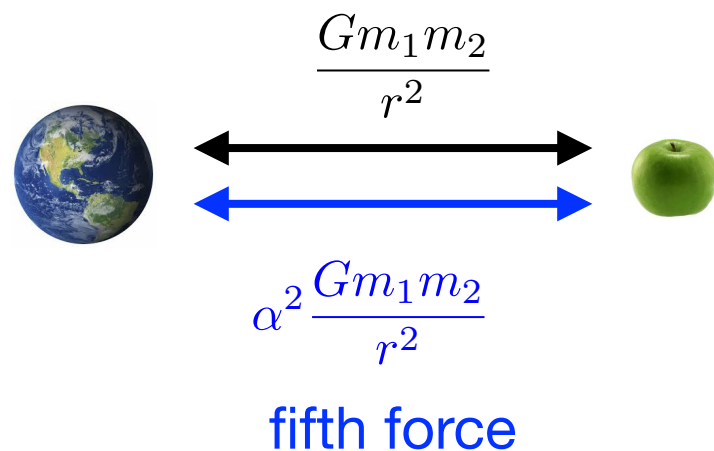
self-acceleration

Modified gravity

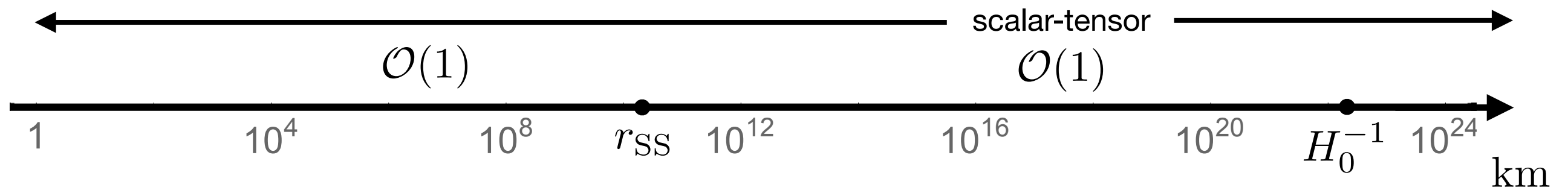
Simplest models of modified gravity are based on single scalar field (universal coupling)

Ex: $\mathcal{L} = f(\phi)R + G_2(\phi, X) , \quad X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

$$G_{\mu\nu}^{(\text{modified})} = 8\pi G \left(T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\phi)} \right)$$



$\Psi \neq \Phi$
anomalous light bending



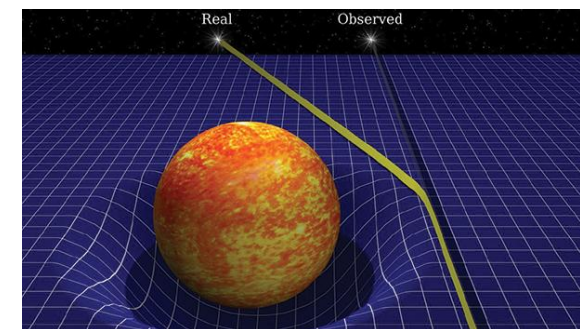
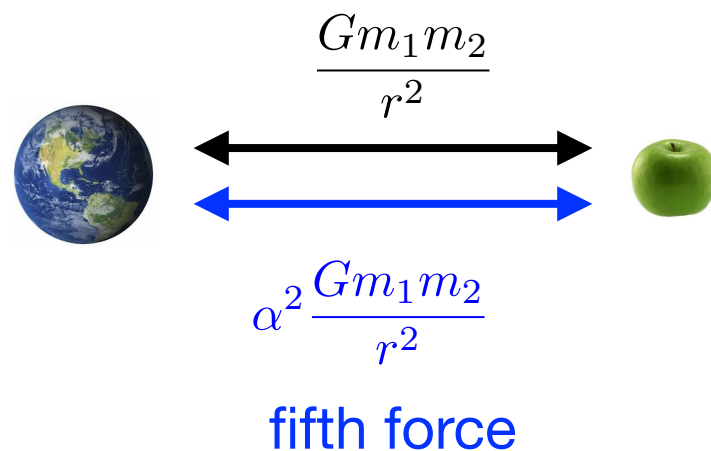
Screening

Simplest models of modified gravity are based on single scalar field (universal coupling)

Ex: $\mathcal{L} = f(\phi)R + G_2(\phi, X) + \frac{\square\phi}{\Lambda_3^3}G_3(\phi, X) \quad \square \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$

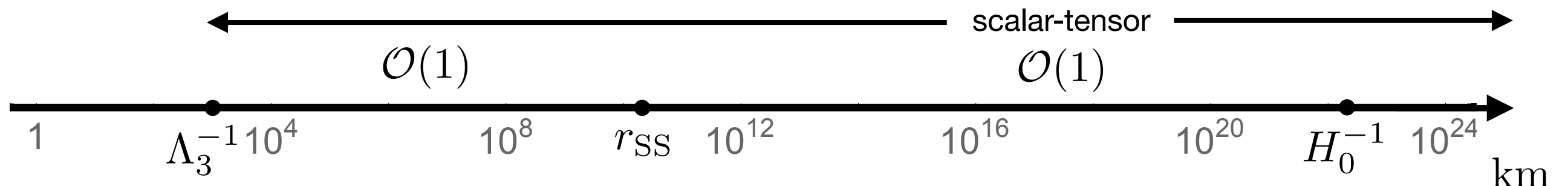
$$\frac{\square\phi}{\Lambda_3^3} \gg 1$$

Vainshtein screening: large classical scalar field nonlinearities



anomalous light bending

$$\Lambda_3 \sim (M_{\text{Pl}} H_0^2)^{1/3} \sim (1000 \text{ km})^{-1}$$



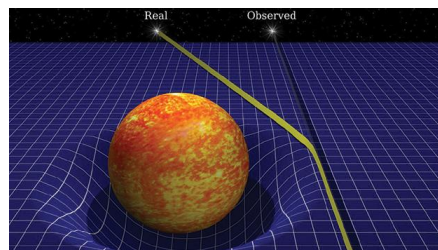
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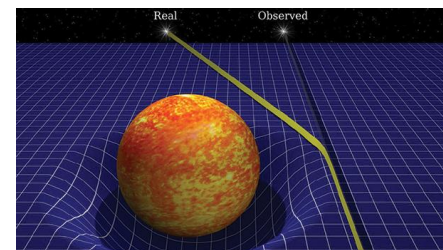
Ex: $\mathcal{L} = f(\phi)R + G_2(\phi, X) + \frac{\square\phi}{\Lambda_3^3}G_3(\phi, X) \quad \square \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$

$$\frac{\square\phi}{\Lambda_3^3} \gg 1$$

Vainshtein screening: large classical scalar field nonlinearities



$$\Psi = \Phi$$

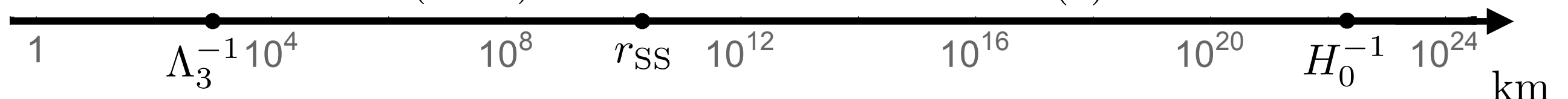


$$\Psi \neq \Phi$$

$$\Lambda_3 \sim (M_{\text{Pl}} H_0^2)^{1/3} \sim (1000 \text{ km})^{-1}$$

almost GR
 $\ll \mathcal{O}(10^{-2})$

scalar-tensor
 $\mathcal{O}(1)$



$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + \frac{\square\phi}{\Lambda_3^3}G_3(\phi, X)$$

Is this the end of the story?

Generalized theories

Most general Lorentz-invariant scalar-tensor theory with 2nd-order EOM.

Horndeski 73
Deffayet et al. 11

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi \quad \Box\phi \equiv \phi_{;\mu}^{\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\ & - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]\end{aligned}$$

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Degenerate theories: most general stable theory.

Langlois, Noui '15; Crisostomi, Koyama, Tasinato '16
Zumalacarregui, Garcia-Bellido '13

Beyond Horndeski theories:

Gleyzes, Langlois, Piazza, FV '14

$$\begin{aligned}- & F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'} \\ - & F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}\end{aligned}$$

$$XG_{5,X}F_4 = 3F_5[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}]$$

Setting $c_T=1$

$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi & \square\phi \equiv \phi_{;\mu}^{;\mu} & X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
 & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\
 & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\
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 \end{aligned}$$

Scalar field play with gravity through higher derivatives:

$$\nabla_{\mu}\nabla_{\nu}\phi \supset \Gamma_{\mu\nu}^{\rho}\partial_{\rho}\phi \quad \Rightarrow \quad \Gamma_{ij}^0\dot{\phi} \supset \dot{\gamma}_{ij}\dot{\phi}$$



$$\mathcal{L}_{\gamma} \sim (\dot{\gamma}_{ij})^2 - c_T^2(\partial_k\gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + XF_4 - 3HX\dot{\phi}F_5$$

Expected from LSS: $|c_T^2 - 1| \lesssim \text{few} \times 0.01$

Setting $c_T=1$

$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \quad \square\phi \equiv \phi^\mu_{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
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 \end{aligned}$$

Scalar field play with gravity through higher derivatives:

$$\nabla_\mu \nabla_\nu \phi \supset \Gamma^\rho_{\mu\nu} \partial_\rho \phi \quad \Rightarrow \quad \Gamma^0_{ij} \dot{\phi} \supset \dot{\gamma}_{ij} \dot{\phi}$$

$$\mathcal{L}_\gamma \sim (\dot{\gamma}_{ij})^2 - c_T^2 (\partial_k \gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + XF_4 - 3HX\dot{\phi}F_5$$



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Scalar field play with gravity through higher derivatives:

$$\nabla_{\mu}\nabla_{\nu}\phi \supset \Gamma_{\mu\nu}^{\rho}\partial_{\rho}\phi \quad \Rightarrow \quad \Gamma_{ij}^0\dot{\phi} \supset \dot{\gamma}_{ij}\dot{\phi}$$



$$\mathcal{L}_{\gamma} \sim (\dot{\gamma}_{ij})^2 - c_T^2(\partial_k\gamma_{ij})^2$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + XF_4 - 3HX\dot{\phi}F_5$$

Most general theory compatible with $c_T=1$: $G_5 = F_5 = 0$, $XF_4 = 2G_{4,X}$

What remains

$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \quad \square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
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 & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'} \\
 & ~~- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}~~
\end{aligned}$$

$$XF_4 = 2G_{4,X}$$

$$\alpha_H \equiv -\frac{X^2 F_4}{G_4}$$

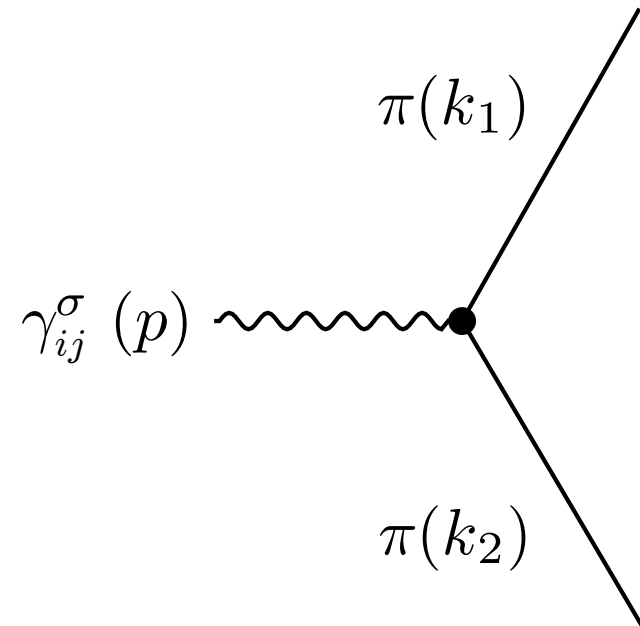
The decay of GW

Creminelli, Lewandowski, Tambalo, FV '18

Beyond Horndeski implies cubic interactions between GW and scalar fluctuations π

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \ddot{\gamma}_{ij}^c \partial_i \pi_c \partial_j \pi_c$$

$$\Lambda_3 \equiv (M_{\text{Pl}} H_0^2)^{1/3} \quad \pi \equiv \delta\phi/\dot{\phi}_0$$



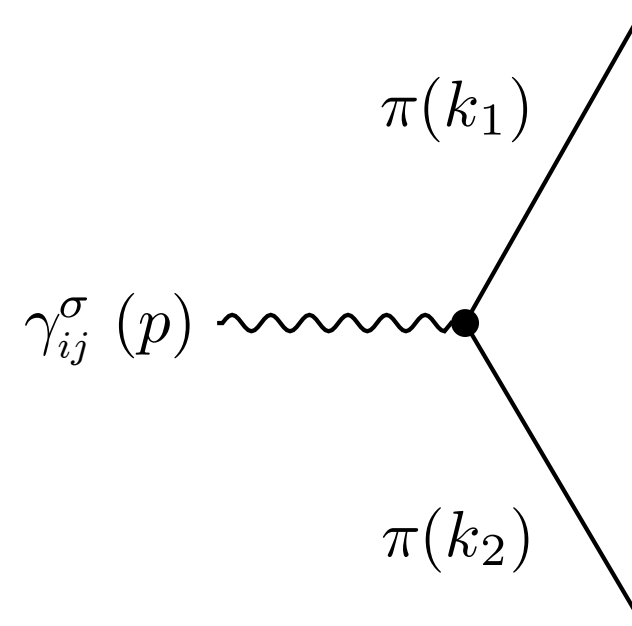
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For $c_s < 1$ (c_s = sound speed of π fluctuations; assume $c_T=1$) GWs can decay into dark energy fluctuations π s. Analogous to Cherenkov radiation

$$\Gamma \simeq \left(\frac{\alpha_H}{\Lambda_3^3} \right)^2 \frac{\omega_{\text{gw}}^7 (1 - c_s^2)^2}{c_s^7} \quad \text{decay rate}$$

$$d_S \Gamma \ll 1 \quad \Rightarrow \quad \alpha_H \ll 10^{-10}$$

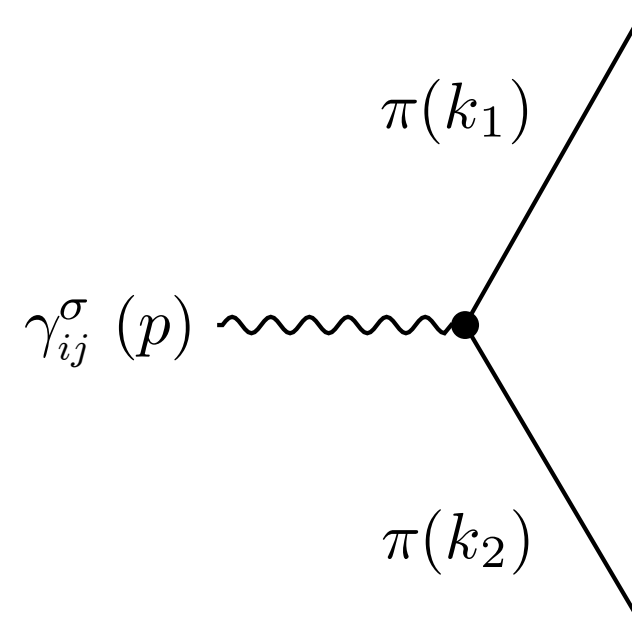
GW dispersion

Creminelli, Lewandowski, Tambalo, FV '18

Spontaneous Lorentz-breaking implies modifications of the dispersion relation

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \ddot{\gamma}_{ij}^c \partial_i \pi_c \partial_j \pi_c$$

$$\Lambda_3 \equiv (M_{\text{Pl}} H_0^2)^{1/3} \quad \pi \equiv \delta\phi/\dot{\phi}_0$$



Graviton self-energy, focussing on the calculable part:

$$\omega^2 = k^2 - \left(\frac{\alpha_H}{\Lambda_3^3} \right)^2 \frac{k^8 (1 - c_s^2)^2}{\pi c_s^7} \log \left(-(1 - c_s^2) \frac{k^2}{\mu^2} \right)$$

The diagram shows a graviton self-energy loop. A wavy line with momentum $q-p$ enters a circular loop from the left. A wavy line with momentum q exits the loop to the right. The loop itself is a solid circle.

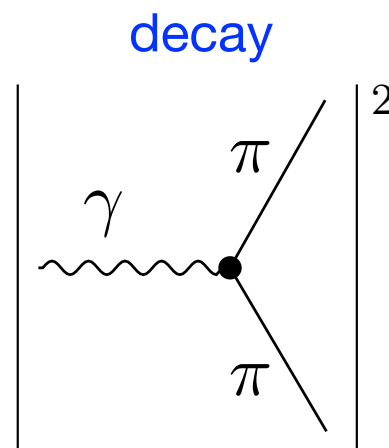
Valid also for $c_s > 1$ (off-shell π s). Gives similar constraints on α_H .

Optical theorem: $\Gamma(k)\omega(k) = \text{Im} [f(k)]$

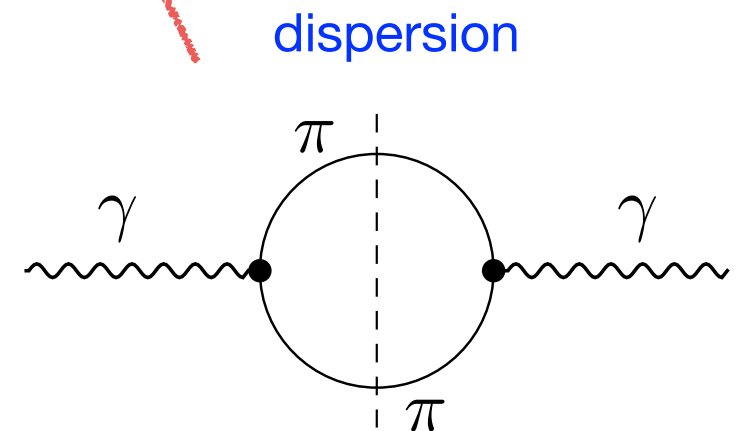
Gravitational Waves propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are absorbed and dispersed.

$$\ddot{\gamma}_{ij} + [(3 + \alpha_M)H + \Gamma(k)] \dot{\gamma}_{ij} + [c_T^2 k^2 + f(k)] \gamma_{ij} = 0$$



Similar to Cherenkov radiation



Similar to a refractive material

$$\mu = \mu(\alpha_M, c_T^2 = 1, \alpha_H = 0, \dots) , \quad \Sigma = \Sigma(\alpha_M, c_T^2 = 1, \alpha_H = 0, \dots)$$

What remains

$$\begin{aligned}
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \quad \square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\
 & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\
 & ~~+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]~~ \\
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 & ~~- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}~~
\end{aligned}$$

$$XF_4 = 2G_{4,X}$$

$$\alpha_H \equiv -\frac{X^2 F_4}{G_4}$$

What remains

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \quad \square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

~~$$2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right]$$~~

~~$$+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]$$~~

~~$$- F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}$$~~

~~$$- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}$$~~

$$XF_4 = 2G_{4,X} = 0$$

$$\alpha_H \equiv -\frac{X^2 F_4}{G_4} = 0$$

What remains

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\square\phi$$

Is this the end of the story?

What remains

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\square\phi$$

Is this the end of the story? **Yes.**

What remains

$$\mathcal{L} = f(\phi)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$

Is this the end of the story? **Yes.**

GW decay rate and modification of the dispersion relation are suppressed

$$\mathcal{L}_{\gamma\pi\pi} \simeq \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij}^c \partial_i \pi_c \partial_j \pi_c$$

$$\Lambda_2 \equiv (M_{\text{Pl}} H_0)^{1/2} \sim 10^{10} \Lambda_3$$

The decay could be enhanced by the large occupation number ($\sim 10^{40}$) of the GW.
Similar to preheating vs reheating.

$$\ddot{\pi} - c_s^2 \nabla^2 \pi + \frac{2\omega M_{\text{Pl}} |h^+|}{\Lambda^2} \cos(\omega(t - z)) (\partial_x^2 - \partial_y^2) \pi = 0$$

Analogous to a Mathieu equation $\Rightarrow$ **parametric resonance**. Can we rule out G_3 as well?

Creminelli, Tambalo, FV, Yingcharoenrat, in progress...

Conclusion

- GWs can test gravity in the (strong) field regime (deviations in the dissipative and conservative dynamics, extra-polarizations, dipole, etc.)
- GWs also dramatically change the prospect for LSS: huge cut in available models
- Because $c_T=1$ and GW do not decay, many theories are ruled out: G_4 , G_5 , F_4 , F_5 are absent