Higher-form global symmetries,

and applications in hydrodynamic limit of strongly interacting QFTs

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Based on series of works: with Sašo Grozdanov 1707.04182, 1801.03199
with Sašo Grozdanov and Andy Lucas 1810.10016;
work in progress with Nabil Iqbal; with Bartek Benenowski and Sašo Grozdanov

Plan

- What is hydrodynamics? and why should QFT people care?
    - Gradient expansions of Noether currents ⇔ Global symmetries
    - Brief mentioned of application in QGP and cond-mat systems
    - Their relations with holographic principle and AdS/CFT

- Incorporating higher-form global symmetry
    - What is a higher-form global symmetries?
    - Hydrodynamics with anti-symmetric 2-form Noether currents
    - Holographic implementation
    - Application in plasma and a certain bad metal phenomenology

and if I have time: a new hydrodynamics of 2-group: arises from fusing ordinary and higher-form symmetry.

What is hydrodynamics ?

What is hydrodynamics ?

say, from engineer's and some mathematician's textbook

Most people will think about it as Euler or Navier-Stokes equation

$$\partial_t n + \vec{\nabla} \cdot (n\vec{v}) = 0,$$

$$n \left[\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} \right] = -\vec{\nabla} p - \zeta \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) + \eta \left(\nabla^2 \vec{v} + \tfrac{1}{3} \vec{\nabla}(\vec{\nabla} \cdot v) \right)$$

Bulk viscosity

Shear viscosity

This is an ultimate cheating tool!

- Only need $p(n)$ and η, ζ
- Qualitatively applicable to most systems across various length scale at large scale + late time.

What is hydrodynamics ?

If a theory has global continuous symmetries, coupled to background fields $\{\mathfrak{a}\}$ s.t. the generating function $Z[\{\mathfrak{a}\}]$ satisfy

$$Z[\{\mathfrak{a} + d\lambda_{\mathfrak{a}}\}] = Z[\{\mathfrak{a}\}]$$

- The conserved currents is

$$\langle \mathfrak{J}^{\mu\nu\dots} \rangle = \frac{1}{Z[0]} \frac{\delta Z[\{\mathfrak{a}\}]}{\delta \mathfrak{a}_{\mu\nu\dots}} \bigg|_{\{a\}=0} \quad \text{where} \quad \partial_\mu \langle \mathfrak{J}^{\mu\nu\dots} \rangle = 0$$

- Write the constitutive relation
  - ⇛ Macroscopic ‘proxy’ i.e.hydrodynamics variables:

$$u^\mu(t,x)\,,\quad T(t,x), \mu(t,x)\,,\quad g_{\mu\nu}(t,x), a_\mu(t,x), \dots$$

fluid velocity

local thermodynamics

background fields

  - ⇛ Then gradient expand

$$\langle \mathfrak{J}^{\mu\nu\dots} \rangle = \mathfrak{J}_0^{\mu\nu\dots}(u^\mu, T, g, a) + \mathfrak{J}_1^{\mu\nu\dots}(\partial u, \partial T, \partial a, \dots) + \mathcal{O}(\partial^2)$$

Ideal hydrodynamics

first order hydrodynamics

What is hydrodynamics ?

- Conformal relativistic hydrodynamics

Eckart frame

Landau frame

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \; 2q^{(\mu}u^{\nu)} \; + \; \pi^{\mu\nu} \; + \mathcal{O}(\partial^2)$$

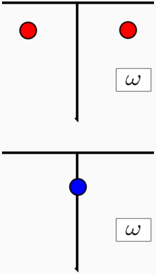
where $q^\mu(\partial u, \partial T)$ and $\pi^{\mu\nu}(\partial u, \partial T)$ satisfy $u^\mu q_\mu = 0$ and $\pi^{\mu\nu} u_\mu = 0$

  - With low energy excitations
    sound : $\omega = \pm c_s k - i\Gamma_s k^2$, diffusion : $\omega = -iDk^2$

- Classifications of low energy data e.g.

$$\pi^{\mu\nu} = \sum_{\text{all tensors}} \alpha_n \; \text{Tensor}^\mu \Big[\partial u, \partial T, \ldots \Big]$$

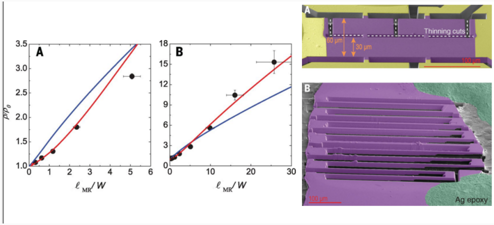
Transport coefficients



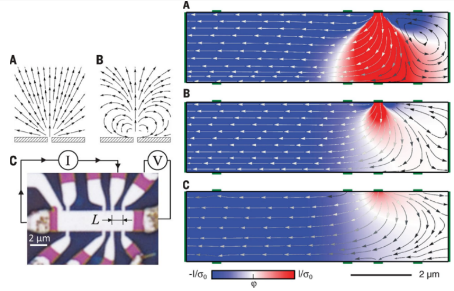
- Hydro works well when these poles are isolated.

Applications to strongly interacting QFTs

- Isotropisation of QGP (cit???)
- Non-Fermi liquid systems e.g. $PdCO_2$ and graphene at Dirac point



[Moll et. al. '16]



[Bandurin et. al. '16]

Relation to holographic duality

Holography is an effective theory contructed from global symmetry

$$\boxed{\begin{aligned} Z_{QFT}[g_{\mu\nu}, a_\mu] &= \left\langle e^{i\int d^{d+1}\sqrt{-g}\left(T^{\mu\nu}g_{\mu\nu}+j^\mu A_\mu\right)}\right\rangle , \\ &= \exp\left(iS_{\text{gravity}}[G_{ab}, A_a]\right) \end{aligned}}$$

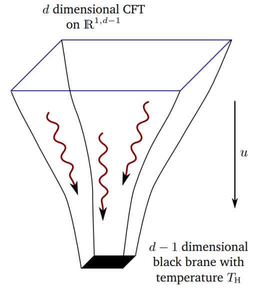
where

$$\boxed{G_{\mu\nu}(u,x^\mu)\Big|_{u\to 0} \sim g_{\mu\nu}(x^\mu), \qquad A_\mu(u,x^\mu)\Big|_{u\to 0} \sim A_\mu}$$

Implying that

$$\partial_\mu\langle T^{\mu\nu}\rangle = 0, \qquad \partial_\mu\langle j^\mu\rangle = 0$$

- ∗ Toy models for strongly coupled QFT
- ∗ Very good at compute 2pt functions
- ∗ Contain info of infinite series of hydrodynamic expansion



Hydrodynamics and holography
with higher-form global symmetries

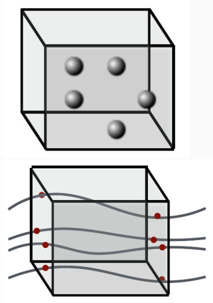
Ordinary (0-form) charge vs 1-form charge

- Global symmetries transformation of point vs line/surface (gauge invariant) operators

$$\mathcal{O}(x) \to \mathcal{O}(x)e^{i\theta} \qquad \Rightarrow \qquad W(\gamma) \to W(\gamma)\mathcal{P}\exp\left[i\int_\gamma \theta_\mu dx^\mu\right]$$

- Ordinary charge is a function spatial volume and count number of point particle charged under it

$$Q_0(\mathcal{M}) = \int d(\text{volume})_\mu j^\mu$$



- Higher-form charge defined on a surface and count number of lines

$$Q_1(\mathcal{S}) = \int d(\text{surface})_{\mu\nu} J^{\mu\nu}$$

- This means that $\partial_\mu j^\mu$ vs $\partial_\mu J^{\mu\nu} = 0$. Both are abundance in physics

[Gaiotto, Kapustin, Seiberg & Willet '14][Lake '18]

What hydrodynamics with 2-form currents good for?

- ❄ Maxwell theory coupled to a (gauged) charge matter

$$\partial_\mu F^{\mu\nu} = j^\mu, \qquad \partial_\mu(\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}) := \partial_\mu J^{\mu\nu} = 0$$

⇛ When $j^\mu = 0$, solution can be solve with $F = dA$ and A is massless (photon). Both $E^i = F^{ti}$ and $B^i = J^{ti}$ are good quantum number.

⇛ When $j^\mu \neq 0$, $F^{\mu\nu}$ no longer conserved: A is massive and E^i decays with time (screening).
In a really deep IR, only B^i is a good quantum number.

- ❄ Can we formulate EFT for this in the following way?
  - Forget that $F = dA$: A_μ is not a good IR EFT variables anyway
  - Treat $dF = d \star J = 0$ as genuinely conserved currents (same footing as energy momentum)
  - See how far we can push it?

[Olesen '96],...
[Caldarelli, Emparan & van Pol'11]
[Schbring '14],[Grozdanov, Hofman & Iqbal '16']

Why is this even a good idea?

In plasma physics (MHD) textbooks, this is what you will find

[Friedberg CUP '14; Goedbloed & Poedts CUP '10]

$$\begin{aligned}
0 &= \partial_t n + \nabla \cdot (n\vec{v}) \longleftarrow \text{Conservation of density} \\
(\partial_t + \vec{v}\cdot\nabla)\vec{v} &= -\nabla p + \vec{J} \times \vec{B} \longleftarrow \text{Euler equation + Lorentz force} \\
J/\sigma &= -\partial_t \vec{B} + \nabla \times (\vec{v} \times \vec{B}) \longleftarrow \text{Ohm's law with } \sigma \to \infty \\
0 &= (\partial_t + \vec{v}\cdot\nabla)(p/n)^\gamma \longleftarrow \text{Adiabatic equation of state} \\
p &= p_0(T, n) \longleftarrow \text{Fluid's pressue independent of } B \\
\partial_t \vec{E} &= 0 \quad \Rightarrow \quad J \approx \nabla \times \vec{B} \longleftarrow \text{Ampere's law}
\end{aligned}$$

- ❄ Not clear how to systematically gradient expand
- ❄ Restricted equation of state (justified only when $T^2/B \gg 1$)

[Fortov Springer '11][Fortov & Mintsev '13]

Why is this even a good idea?

What if one systematically gradient expand $T^{\mu\nu}$ and $J^{\mu\nu}$?

- Generating functions for this system is

$$Z[g_{\mu\nu}, b_{\mu\nu}] = \left\langle \exp\left[i \int d^4x \sqrt{-g} \left(\frac{1}{2} T^{\mu\nu} g_{\mu\nu} + J^{\mu\nu} b_{\mu\nu} \right) \right] \right\rangle$$

- Hydrodynamics “proxies” variables are

u^μ, T μ h^μ $g_{\mu\nu}, b_{\mu\nu}$

conjugate of string density

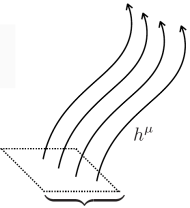
Direction of string

new background fields

- New constitutive relations obtained systematically

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\text{ideal}} - \mu\rho\, h^\mu h^\nu + \mathcal{O}(\partial) \\ J^{\mu\nu} &= \rho\, (u^\mu h^\nu - u^\nu h^\mu) + \mathcal{O}(\partial) \end{aligned}$$

[Schbring ’14],[Grozdanov, Hofman & Iqbal ’16]
See also [Hernandez & Kovtun ’17][Amas & Jain ’18]
[Glorioso & Son ’18]



ρ lines per unit area
each with chem. pot. μ

Why is this even a good idea?

[Goldreich & Julian '69]

[Blanford & Znajek '77]

- Puzzle in how cold magnetic object loss energy
  - ⇛ For a cold neutron star/black hole to emit strong radiation

fast rotating + strong $\vec{B}$ field ⇒ energy loss ⇒ Radiation

- A known EFT for magnetically dominated system:

  Force-free electrodynamics

$$0 = \nabla_{[\mu}F_{\nu\rho]} \longleftarrow \text{Conservation of magnetic flux}$$

$$0 = F_{\mu\nu}\nabla_{\lambda}F^{\lambda\nu} \longleftarrow F_{\mu\nu}j^{\nu} = 0 : \text{ Force free configuration}$$

$$0 = \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \longleftarrow \vec{E}\cdot\vec{B} = 0 : \text{ screening of } \vec{E} \text{ field}$$

- Hinted that one should think of this as dynamics of strings

$$F_{\mu\nu} = \partial_{\mu}X^{1}\partial_{\nu}X^{2} - \partial_{\nu}X^{1}\partial_{\mu}X^{2}$$

[Uchida '97]

- Above formalism interpolate MHD to FFE!

  but without $\vec{E}\cdot\vec{B} = 0$

[Gralla & Iqbal '18]

[Glorioso & Son '18]

[Benenowski, Grozdanov & NP to appear]

Why is this even a good idea?

- Global symmetry can be anomalous
  - ⇒ Anomaly matching
  - ⇒ New transport phenomena due to anomaly
  - ⇒ Protected phase

[Aharony, Seiberg & Tachikawa '13],...,
[Benini, Hsin & Seiberg '17]
[Komargodski, Thorngren, Sharon & Zhou '17]
[Kapustin & Thorngren '13],...

- Controlled framework when global symmetry is weakly broken
  - ⇒ Memory matrix formalism

conjugate of ρ_a

$$\partial_t\langle\rho_a\rangle+\partial_i\langle J^i_a(\rho_a)\rangle = 0 \quad \Rightarrow \quad \partial_t\langle\rho_a\rangle+\partial_i\langle J^i_a(\rho_a)\rangle = -M_{ab}\ \ \mu_b$$

with $M_{ab} = \lim_{\omega\to 0} G^R_{\dot\rho_a\dot\rho_b}(\omega, \vec{k}=0)$

[Grozdanov, Lucas & NP '18]

- Very straightforward to construct holographic dual (toy model for strongly coupled QFT)

$$Z[g_{\mu\nu}, b_{\mu\nu}] = \exp\Big(-iS_{\text{grav}}\Big)$$

with

$$S_{\text{grav}} = \int d^{d+2}X\sqrt{-G}\left(R - 2\Lambda - \frac{1}{3}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right) + S_{\text{bnd}}\,,$$

with $H = dB$ and $B_{\mu\nu}|_{bnd} \sim b_{\mu\nu}$

[Grozdanov & NP '17-'18]
[Hofman & Iqbal '17]

What is about in lower dimension ?

Theory of elasticity

See e.g. [Chaikin & Lubenski '95]
[Beekman, Cvetkovic, Liu et. al. '16]

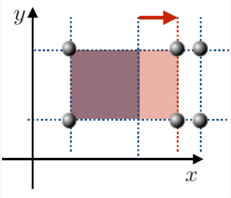
Action of a displacement field $X^I \propto x^i$

$$S = \int d^{2+1} C^{\mu\nu}_{IJ} \partial_\mu X^I \partial_\nu X^J$$

- Conservation of two 1-form current: momentum P^μ_I

$$0 = \partial_\mu P^\mu_I = \partial_\mu C^{\mu\nu}_{IJ} \partial_\nu X^I$$

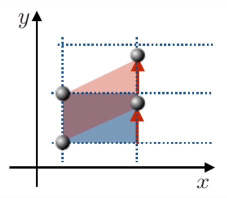
- X^I is massless, some treat it as Goldstone boson for spontaneous TSB

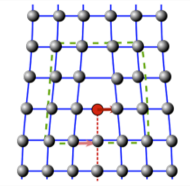


- Another global symmetry

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) X^I = \partial_\mu J^{\mu\nu}_I = 0$$

where $J^{\mu\nu}_I = \epsilon^{\mu\nu\lambda} \partial_\lambda X^I$





Fluid + spontaneous translational order

Mixing of fluid and some solid structure on top
(e.g. stripe or charge density wave)

See e.g. [Martin, Parodi & Pershan '72]
[Zippelius, Halperin & Nelson '80]
[Delacretaz, Hartnoll, Gouteraux & Karlsson '17]

❇ Traditionally, one write

$$\partial_\mu T^{\mu\nu} = 0 \quad ; \qquad T^{\mu\nu} = T^{\mu\nu}_{\text{fluid}}(u, T, \ldots) + T^{\mu\nu}_{\text{solid}}[\partial_j X^I]$$

together derivatives of the "Josephson condition"

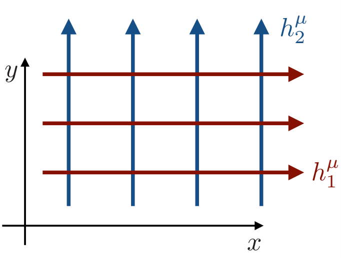
$$\partial_t \delta X^I = \delta u^\mu \delta^I_\mu$$

and add derivative corrections ?

➧ But this is our 2-form current!

$$J^{\mu\nu} = 2\rho u^{[\mu} h_I^{\nu]} \propto \epsilon^{\mu\nu\lambda} \partial_\lambda X^I$$

e.g. $\delta J_2^{xy} = \rho \delta u^x h_2^y \propto \epsilon^{xyt} \partial_t X^I$



[Grozdanov & NP '18]

Holographic dual ?

for holography experts

- Nice and simple holographic dual with analytic background

$$ds^2 = \frac{dr^2}{r^2 f} + r^2 \left(-f dt^2 + dx^2 + dy^2\right)$$

[Grozdanov & NP '18]

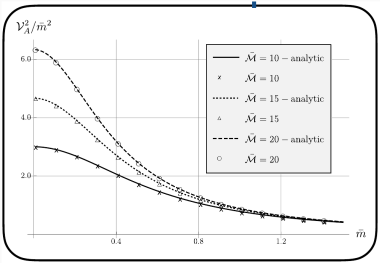
$$f = 1 - \frac{m^2}{2r^2} - \left(1 - \frac{m^2}{2r_h^2}\right) \frac{r_h^3}{r^3}, \qquad H_{1,txr} = H_{2,tyr} = -m$$

with somewhat unusual boundary condition

- Exhibit transverse sound
  and agrees with EFT prediction

$$\langle T^{t\perp} T^{t\perp} \rangle = \frac{\mu \rho k^2}{\omega^2 - \mathcal{V}_A^2 k^2}$$

$$\mathcal{V}_A^2 = \frac{\mu \rho}{\epsilon + p - \mu \rho}$$



- Requires no clever BH engineering

[Esposito et. al. '18]
[Jokela et. al. '18][Andrade et. al. '18]
[Ammon et. al. '18][Amoretti et. al. '18]...

Outlook

- Trying to systematise ways to cheat strongly coupled QFT
- Global symmetry seems to be good idea?

- New hydrodynamics! with systematic way to extend it to more exotic QFTs
- Potential new macroscopic phenomena, both in linearised and non-linear regime
- It is really fun!

THANK YOU VERY MUCH FOR LISTENING!