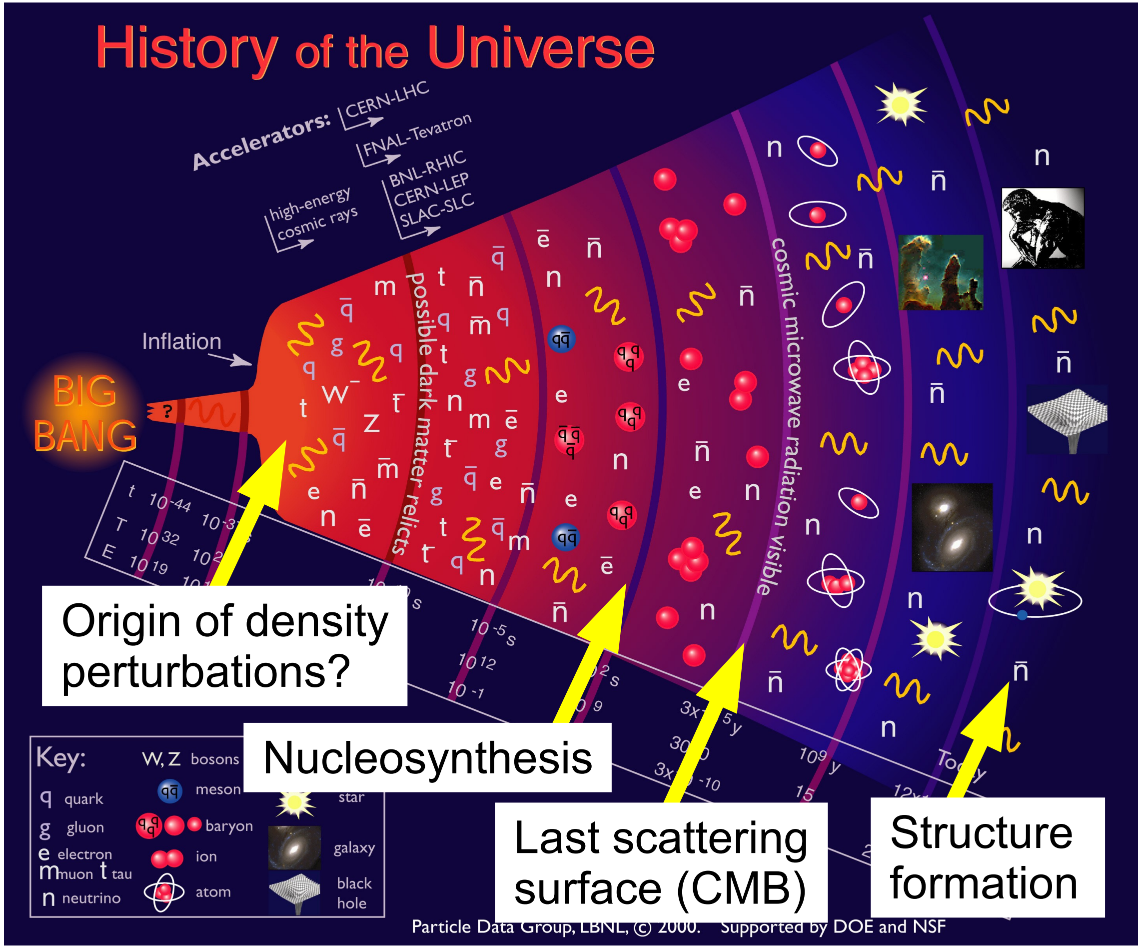
Massive neutrinos and cosmology

Yvonne Y. Y. Wong
RWTH Aachen

Theory colloquium, Padova, November 18, 2009



Relic neutrino background:

- **Temperature**:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB},0} = 1.95\ \mathrm{K}$$

- **Number density** per flavour:

$$n_{\nu,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T_{\nu,0}^3 = 112\ \mathrm{cm}^{-3}$$

- **Energy density** per flavour:

If $m_\nu > 1$ meV

$$\Omega_\nu h^2 = \frac{m_\nu}{93\ \mathrm{eV}}$$

Neutrino dark matter

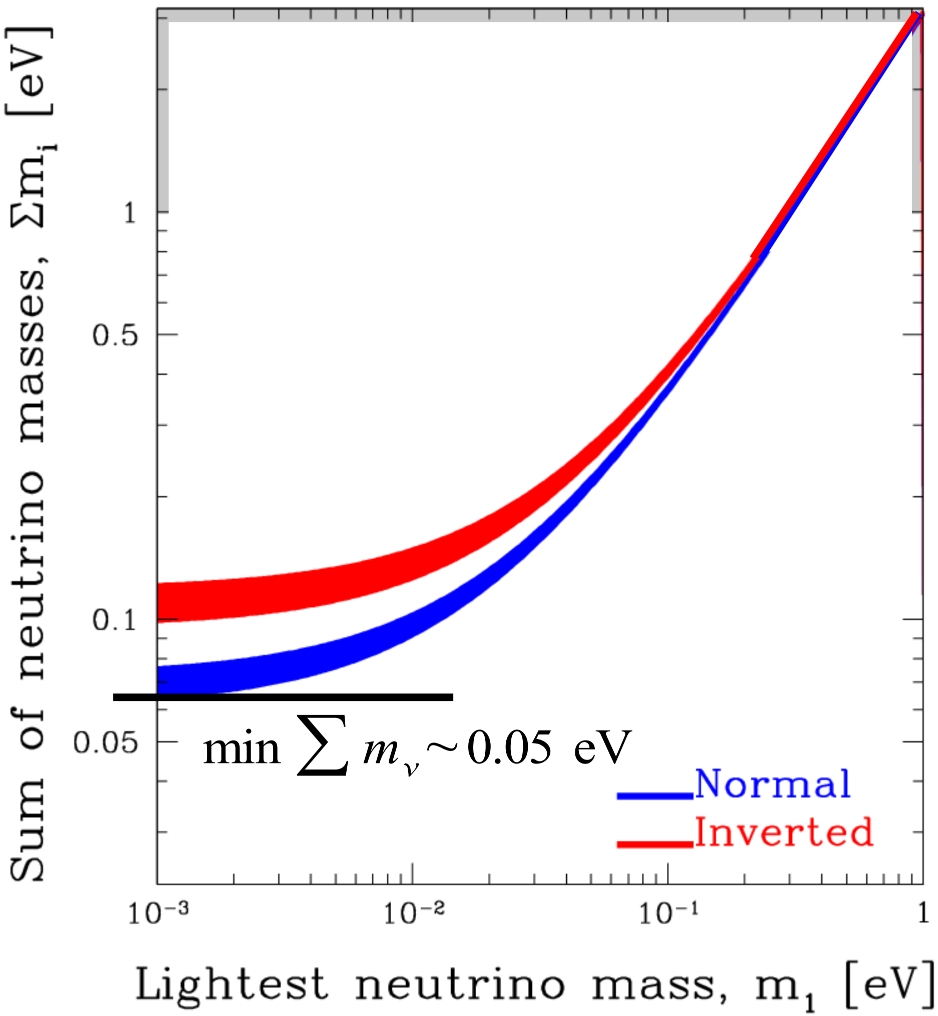
Neutrino dark matter...

- Evidence for neutrino masses from neutrino oscillations experiments :

$$\left|\Delta m^2_{\mathrm{atm}}\right| \sim 2\times 10^{-3}\ \mathrm{eV}^2 \qquad \Delta m^2_{\mathrm{sun}} \sim 8\times 10^{-5}\ \mathrm{eV}^2$$

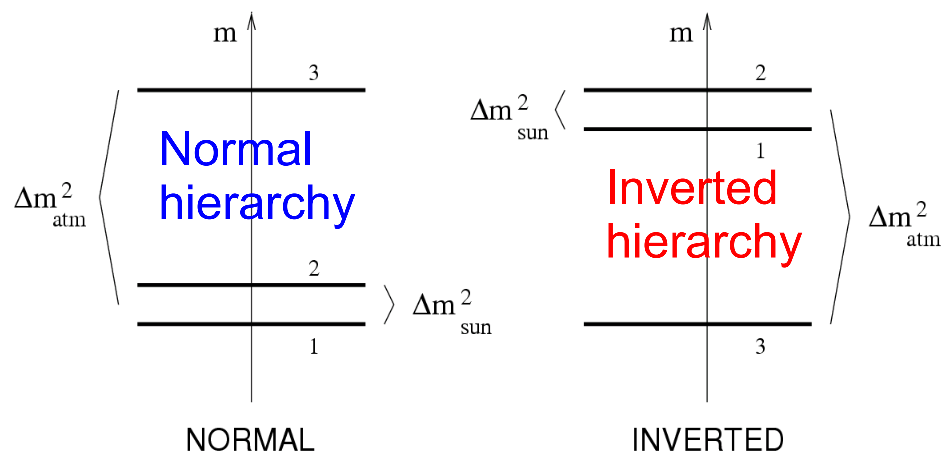
- Large mixing between flavours:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} 0.82 & 0.54 & <0.21 \\ -0.31 & 0.47 & 0.69 \\ 0.47 & -0.71 & 0.69 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$



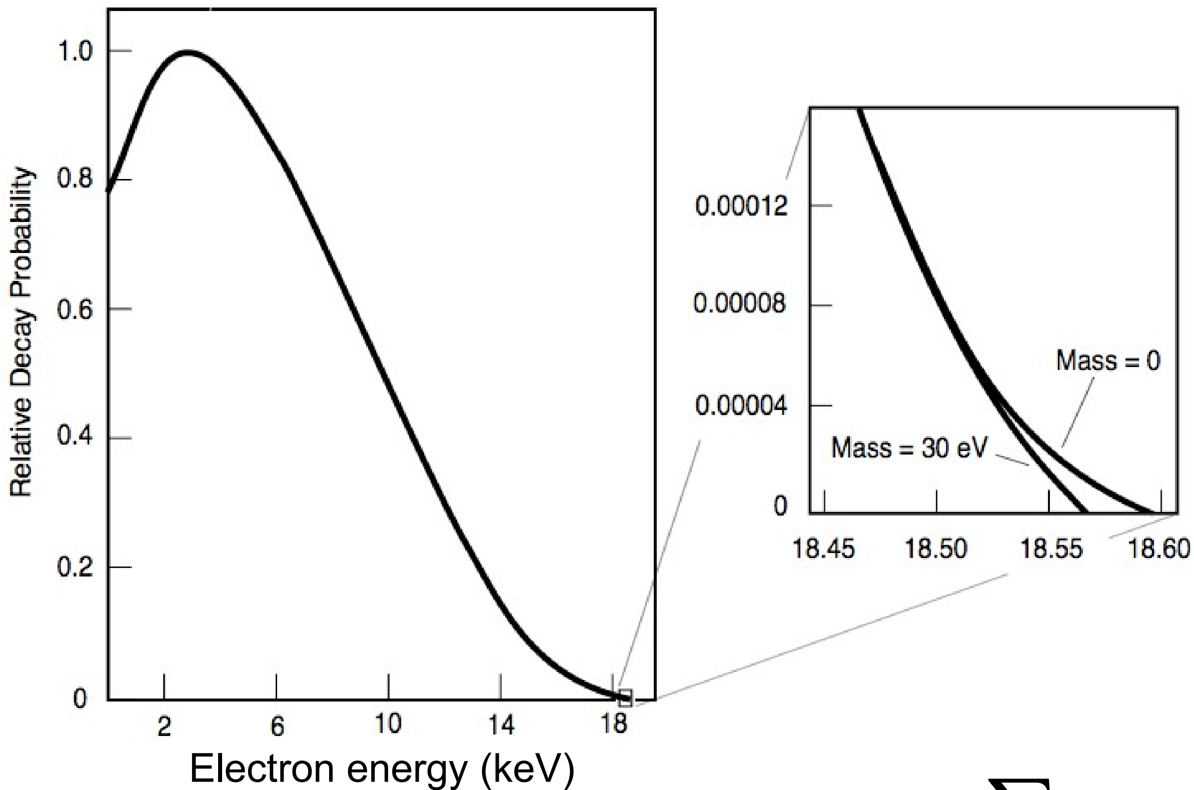
- Oscillation data allow for two mass hierarchies:

$$|\Delta m^2_{\rm atm}| \sim 2\times 10^{-3}\ {\rm eV}^2$$
$$\Delta m^2_{\rm sun} \sim 8\times 10^{-5}\ {\rm eV}^2$$



Mininum amount of neutrino dark matter → $\min \sum m_\nu \sim 0.05\ {\rm eV} \rightarrow \min \Omega_\nu \sim 0.1\,\%$

- Upper limit on neutrino masses from tritium β-decay:



Large mixing means

$|U_{ei}|^2 \sim O(0.1 \to 1)$

$$m_e \equiv \left(\sum_i |U_{ei}|^2 m_i^2 \right)^{1/2} < 2.2 \text{ eV}$$

$$\max \sum m_\nu \sim 7 \text{ eV} \to \max \Omega_\nu \sim 12\%$$

Light neutrinos cannot be the only dark matter component

Neutrino dark matter is hot...

- **Large velocity dispersion**:

$$\langle v_{\text{thermal}} \rangle \simeq 81(1+z)\left(\frac{\text{eV}}{m_\nu}\right) \text{ km s}^{-1}$$

  - A dwarf galaxy has a velocity dispersion of 10 km s^{-1} or less, a galaxy about 100 km s^{-1}.

  - Sub-eV neutrinos have too much thermal energy to be packed into galaxy-size self-gravitating systems.

    - Neutrinos **cannot** be the *dominant* Galactic dark matter.

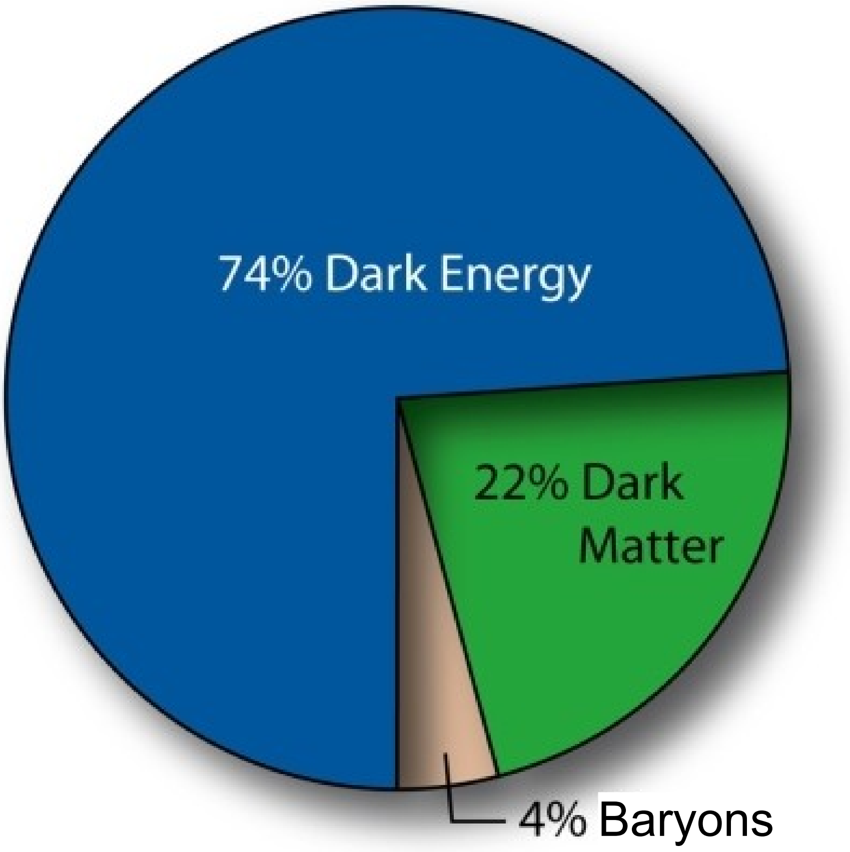
Why study neutrinos in cosmology...

- Hot dark matter leaves a distinctive imprint on the large-scale structure distribution.
  - We can learn about neutrino properties from cosmology.

- Cosmological probes are getting ever more precise:
  - Even a small neutrino mass can **bias** the inference of other cosmological parameters.

The concordance framework...

- We work within the ΛCDM framework extended with a subdominant component of massive neutrino dark matter.
  - Flat geometry.
  - Main dark matter is cold.
  - Initial conditions from single-field slow-roll inflation.



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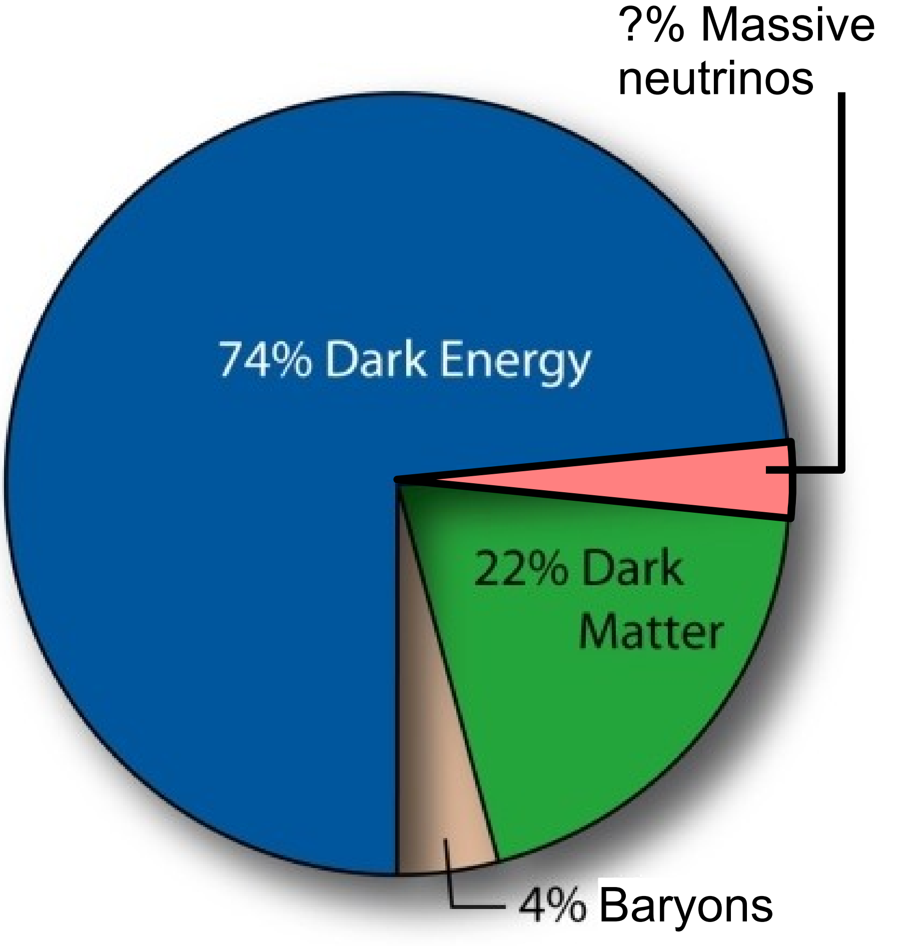


Plan...

- What we can do **now**
- What we can do **in the future**
- The **nonlinear matter power spectrum**

1. What we can do now...

Two effects of massive neutrinos...

- On the **background**:
  - Shift in time of matter radiation equality.

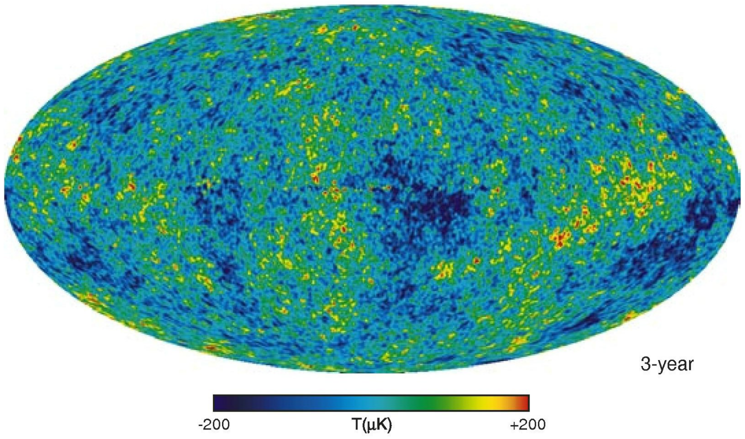
- On the **perturbations**:
  - Suppression of growth.

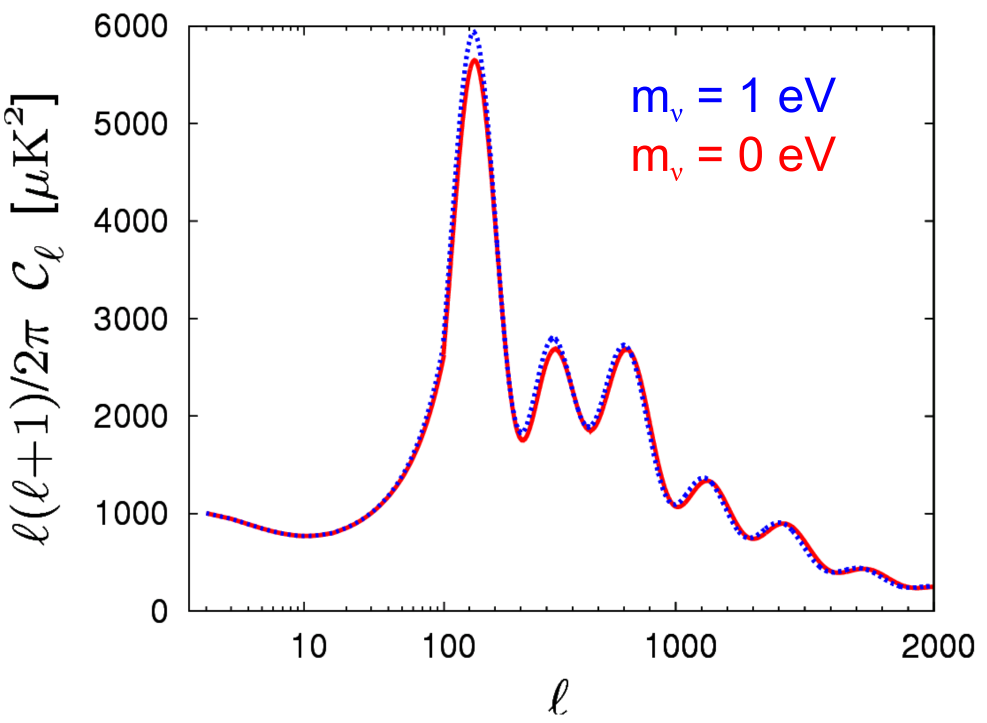
Background...

- Sub-eV neutrinos become nonrelativistic at **z<1000**:
  - Radiation at early times.
  - Matter at late times.

Comoving matter density today ≠ Comoving matter density before recombination

  - Shift in matter-radiation equality relative to model with zero neutrino mass.





Two effects of massive neutrinos...

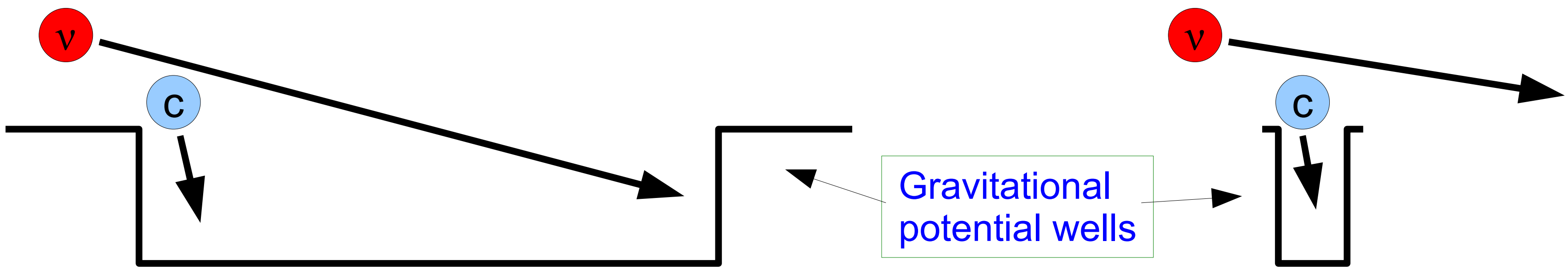
- On the **background**:
  - Shift in time of matter radiation equality.

- On the **perturbations**:
  - Suppression of growth.

Perturbations...

- At low redshifts, neutrinos become nonrelativistic:.
  - But still have large thermal speed: $c_\nu \simeq 81(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right)\ \mathrm{km\ s^{-1}}$

    → hinder ν clustering on small scales.



- Free-streaming length scale & wavenumber:

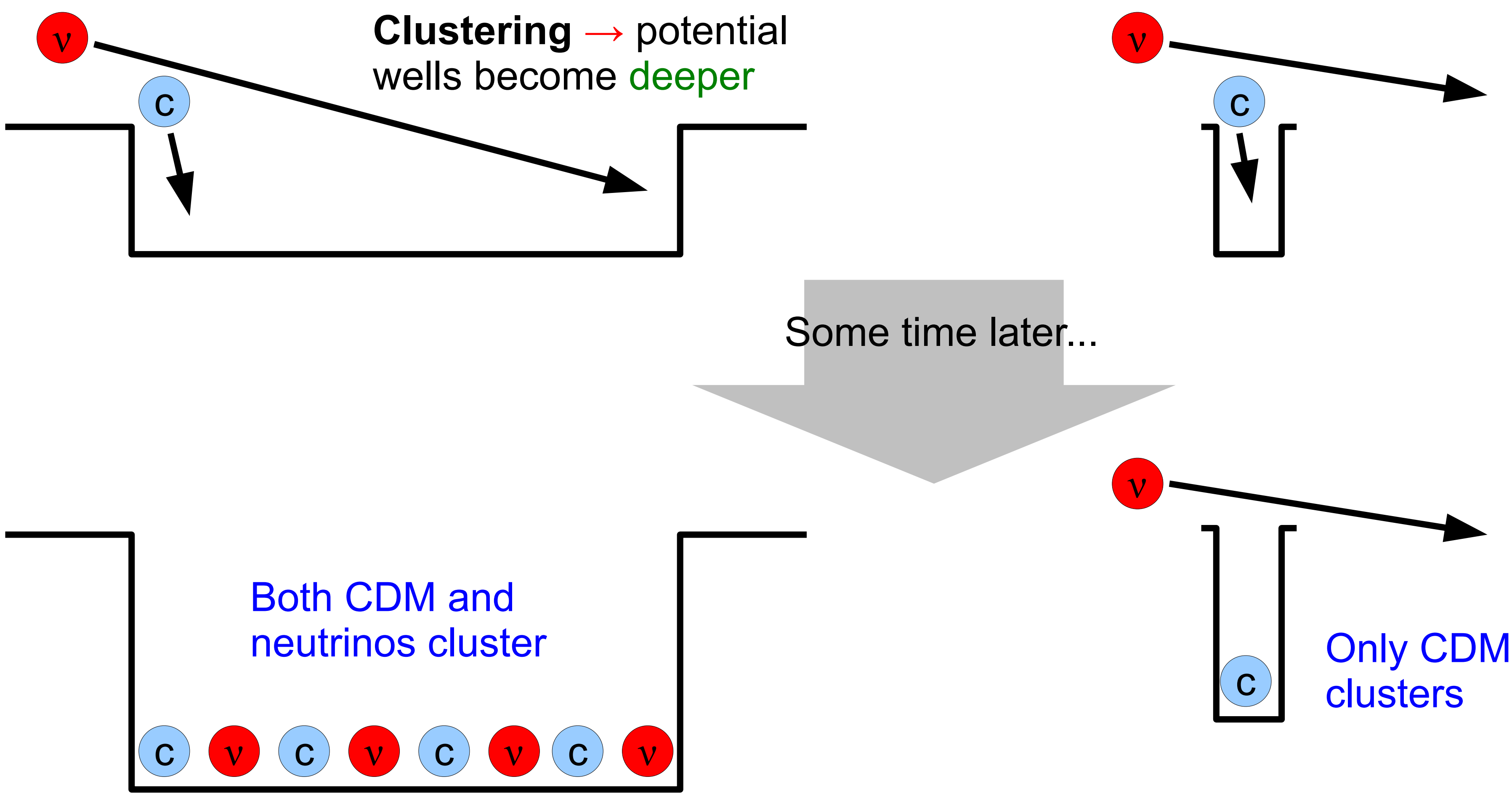
$$\lambda_{\mathrm{FS}} \equiv \sqrt{\frac{8\pi^2 c_\nu^2}{3\Omega_m H^2}} \simeq 4.2\sqrt{\frac{1+z}{\Omega_{m,0}}}\left(\frac{\mathrm{eV}}{m_\nu}\right)\ h^{-1}\ \mathrm{Mpc}$$

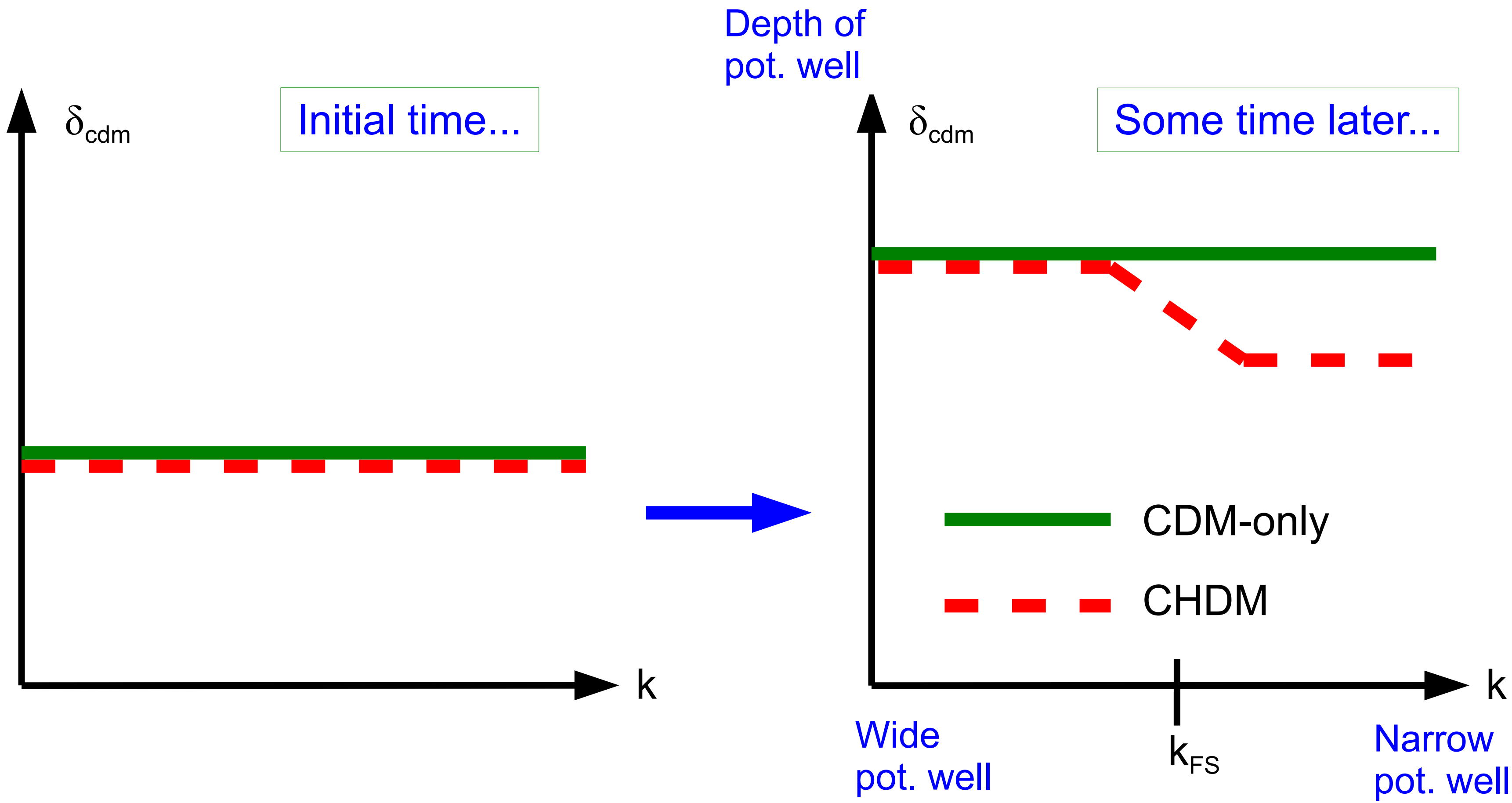
$$k_{\mathrm{FS}} \equiv \frac{2\pi}{\lambda_{\mathrm{FS}}}$$

$\lambda \gg \lambda_{\mathrm{FS}}$, $k \ll k_{\mathrm{FS}}$: Clustering

$\lambda \ll \lambda_{\mathrm{FS}}$, $k \gg k_{\mathrm{FS}}$: Non-clustering

- In turn, free-streaming (non-clustering) neutrinos slow down the growth of gravitational potential wells on **scales** $\lambda << \lambda_{FS}$ or **wavenumbers** $k >> k_{FS}$.





- The presence of HDM slows down the growth of CDM perturbations at large wavenumbers k.
  - The density perturbation spectrum acquires a step-like feature.

Describing perturbations: CDM...

- Cold dark matter = collisionless, pressureless fluid:

Density perturbations

Continuity eqn $$\dot{\delta}_c + \theta_c = 0$$

Gravitational source

Euler eqn $$\dot{\theta}_c + H\theta_c + \nabla^2 \Phi = 0$$

Velocity divergence

Expansion

$f_\nu \equiv \frac{\Omega_\nu}{\Omega_m}$ Neutrino fraction

Poisson eqn $$\nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m [f_c \delta_c + f_\nu \delta_\nu]$$

Describing perturbations: Neutrinos...

- Free-streaming neutrinos **cannot** be described by a perfect fluid.
  - Must solve (linearised) collisionless Boltzmann equation:

Nonrelativistic neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0 + \delta f$$

Phase space density

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$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0 + \delta f$

Phase space density

  - Momentum moments:

$$\delta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, (\delta f)$$ Density perturbation

$$\theta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, \frac{p_i}{a\, m_\nu} \partial_i (\delta f)$$ Velocity divergence

$$\sigma_{ij} \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, \frac{p_i p_j}{a^2 m_\nu^2} (\delta f)$$ Pressure and anisotropic stress

$$\vdots$$

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$f(\boldsymbol{x},\boldsymbol{p},\tau)=f_0+\delta f$

Phase space density

  - Momentum moments:

$$\delta_\nu \equiv \frac{1}{\bar{\rho}_\nu}\int d^3\boldsymbol{p}\,(\delta f)$$

Density perturbation

$$\theta_\nu \equiv \frac{1}{\bar{\rho}_\nu}\int d^3\boldsymbol{p}\,\frac{p_i}{a\, m_\nu}\partial_i(\delta f)$$

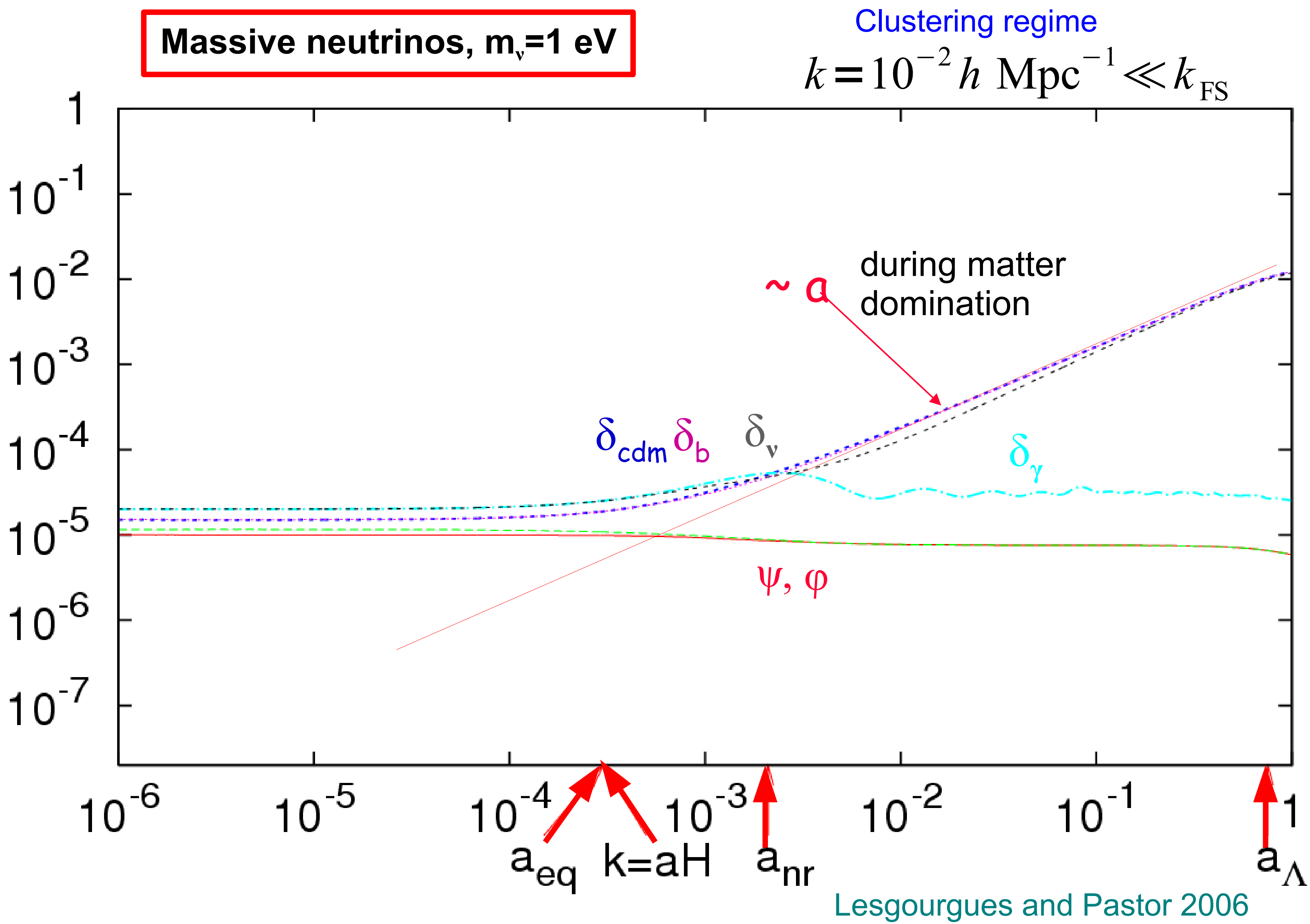
Velocity divergence

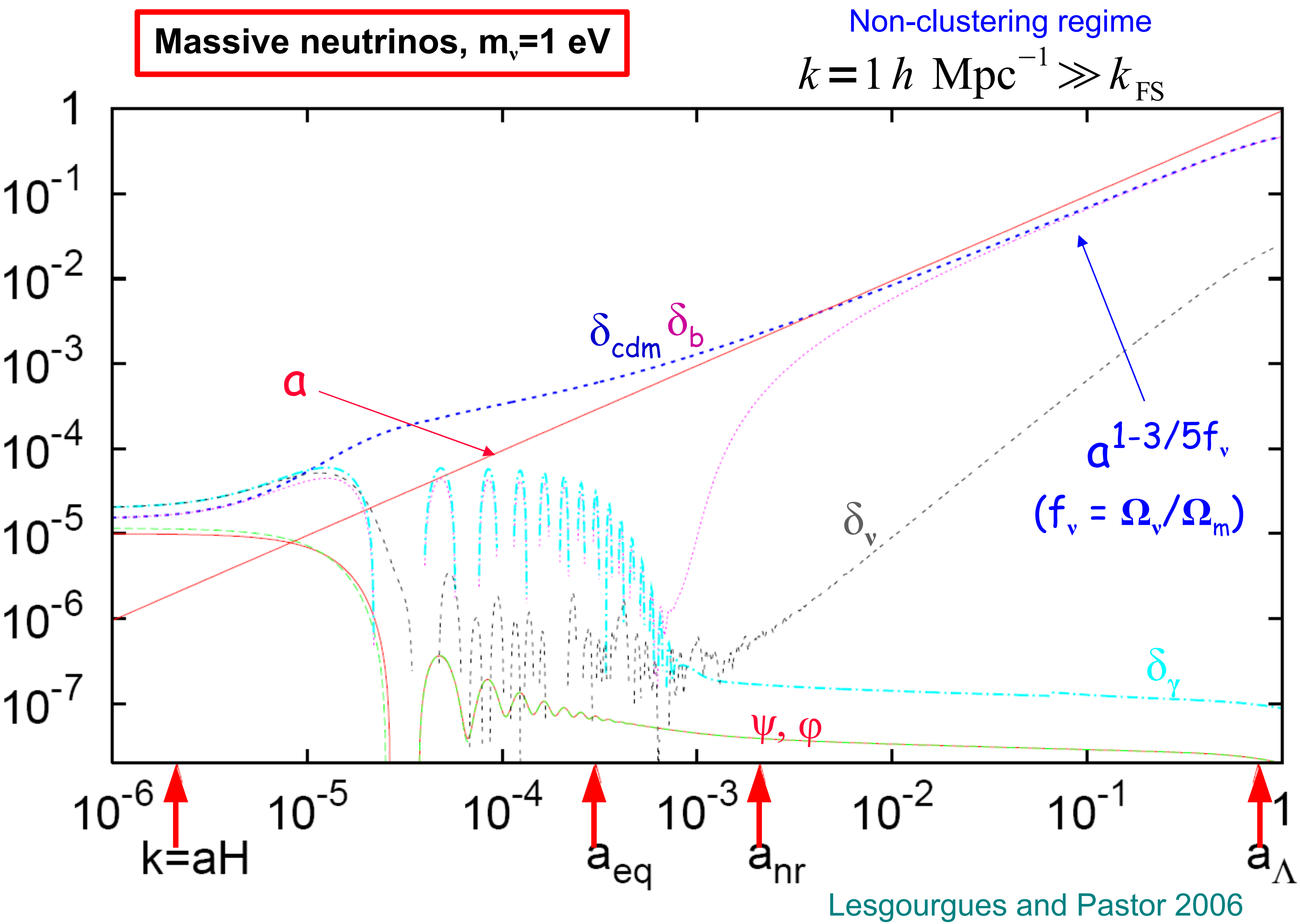
Give rise to free-streaming behaviour

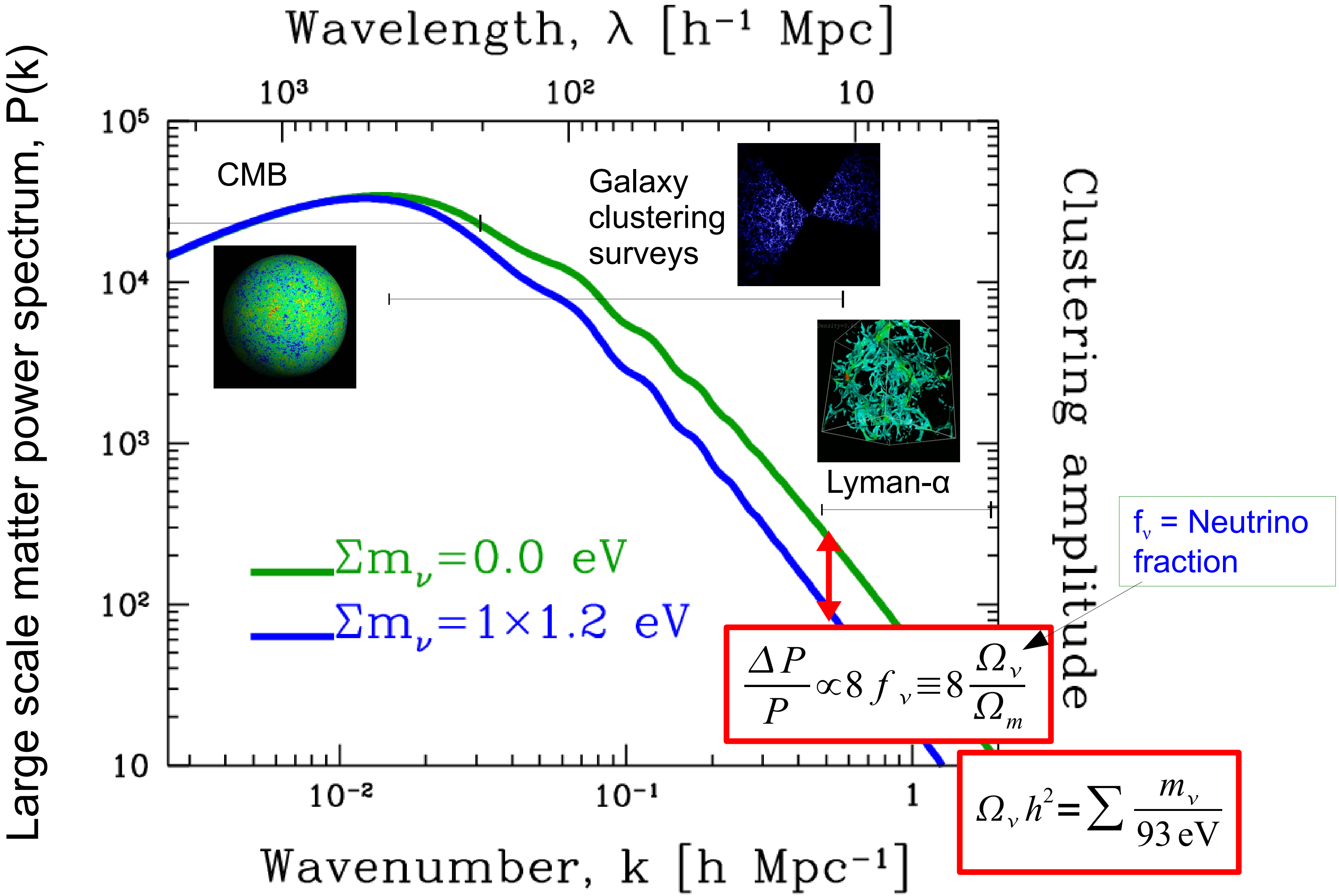
$$\sigma_{ij} \equiv \frac{1}{\bar{\rho}_\nu}\int d^3\boldsymbol{p}\,\frac{p_i p_j}{a^2 m_\nu^2}(\delta f)$$

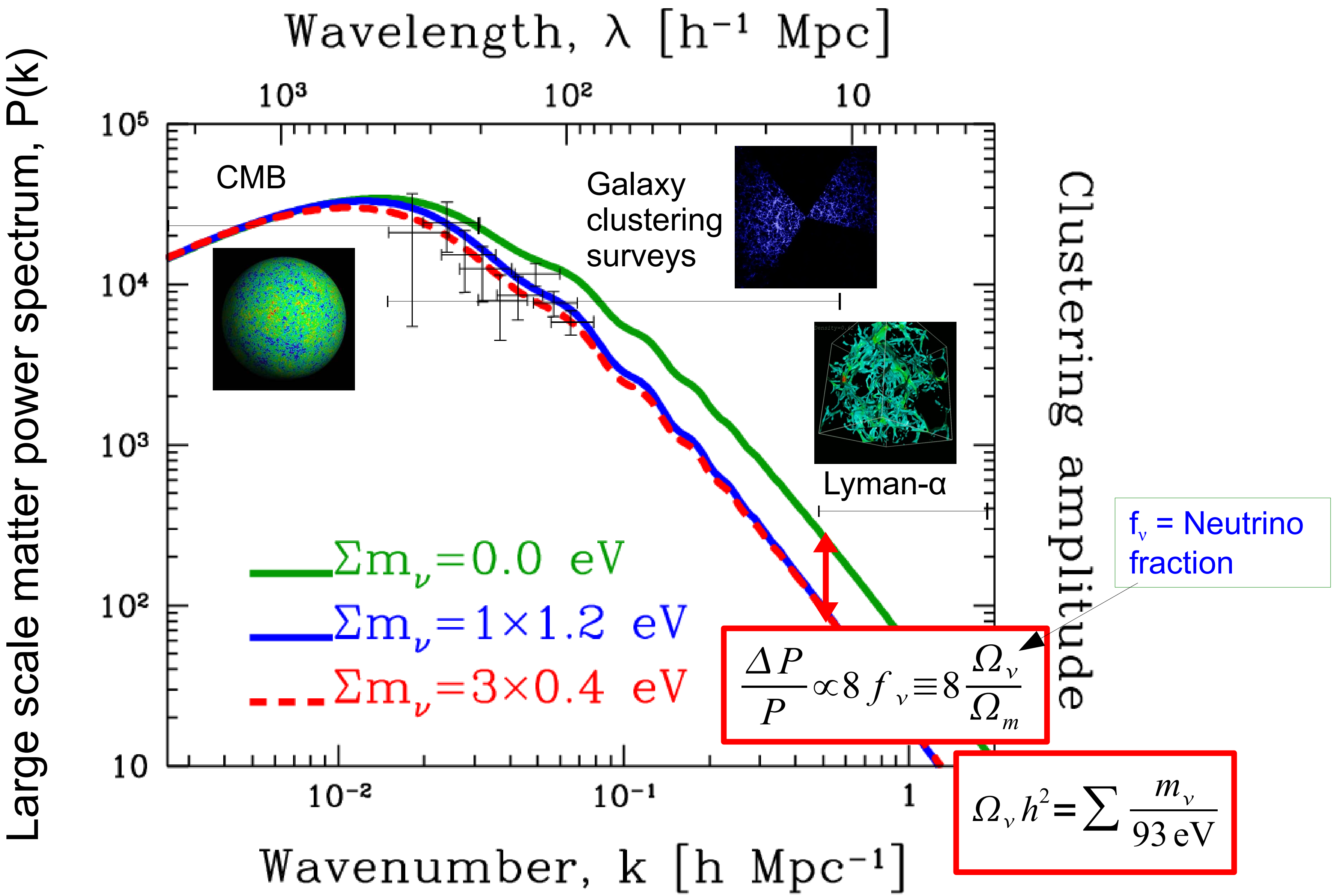
Pressure and anisotropic stress

$$\vdots$$

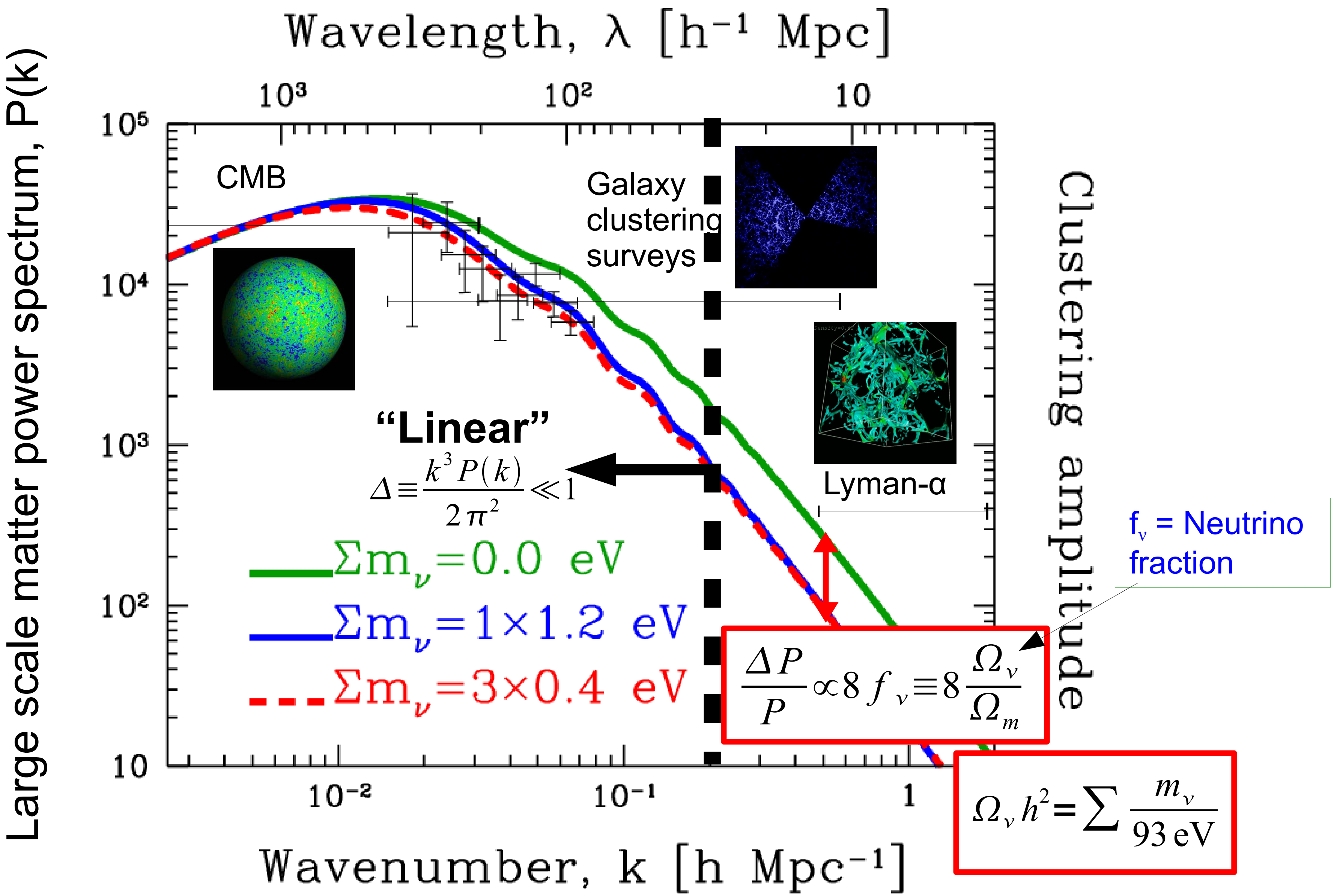

Massive neutrinos, m_ν=1 eV
Clustering regime
$k = 10^{-2}\, h\ \mathrm{Mpc}^{-1} \ll k_{FS}$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
~ a
during matter
domination
δ_{cdm} δ_b
δ_ν
δ_γ
ψ, φ
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
a_{eq}
k=aH
a_{nr}
a_Λ
Lesgourgues and Pastor 2006


Massive neutrinos, m_ν=1 eV
Non-clustering regime
k = 1 h Mpc^-1 ≫ k_FS
δ_cdm δ_b
a
a^(1-3/5f_ν)
(f_ν = Ω_ν/Ω_m)
δ_ν
δ_γ
ψ, φ
1
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
k=aH
a_eq
a_nr
a_Λ
Lesgourgues and Pastor 2006


Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
Large scale matter power spectrum, P(k)
10⁵
10⁴
10³
10²
10
CMB
Galaxy clustering surveys
Lyman-α
Clustering amplitude
Σm_ν=0.0 eV
Σm_ν=1×1.2 eV
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]

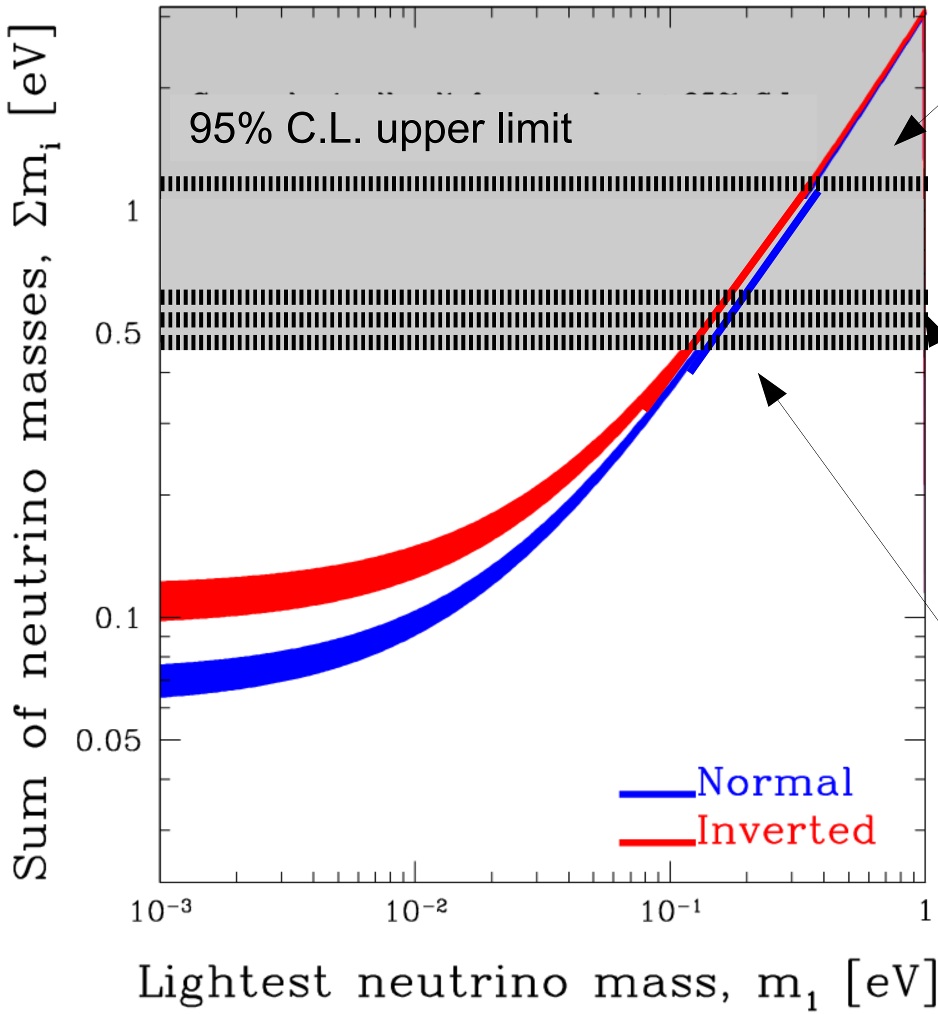


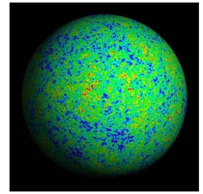
Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
Large scale matter power spectrum, P(k)
10⁵
10⁴
10³
10²
10
CMB
Galaxy clustering surveys
Lyman-α
Clustering amplitude
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1\times1.2$ eV
$\Sigma m_\nu = 3\times0.4$ eV
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]



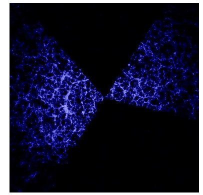
Wavelength, λ [h⁻¹ Mpc]
10^3
10^2
10
10^5
10^4
10^3
10^2
10
CMB
Galaxy clustering surveys
Lyman-α
"Linear"
$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1\times1.2$ eV
$\Sigma m_\nu = 3\times0.4$ eV
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
Clustering amplitude
Large scale matter power spectrum, P(k)
10^{-2}
10^{-1}
1
Wavenumber, k [h Mpc⁻¹]

Present status...

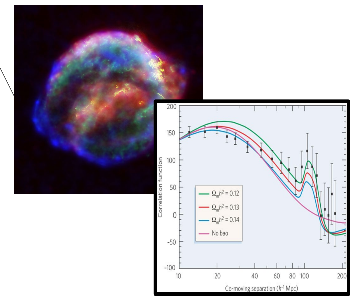




WMAP5 only
Dunkley et al. 2008

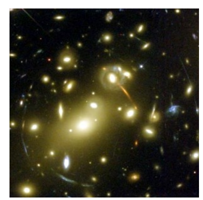


+ Galaxy clustering
Reid et al. 2009



+ Galaxy + SN + HST
Reid et al. 2009

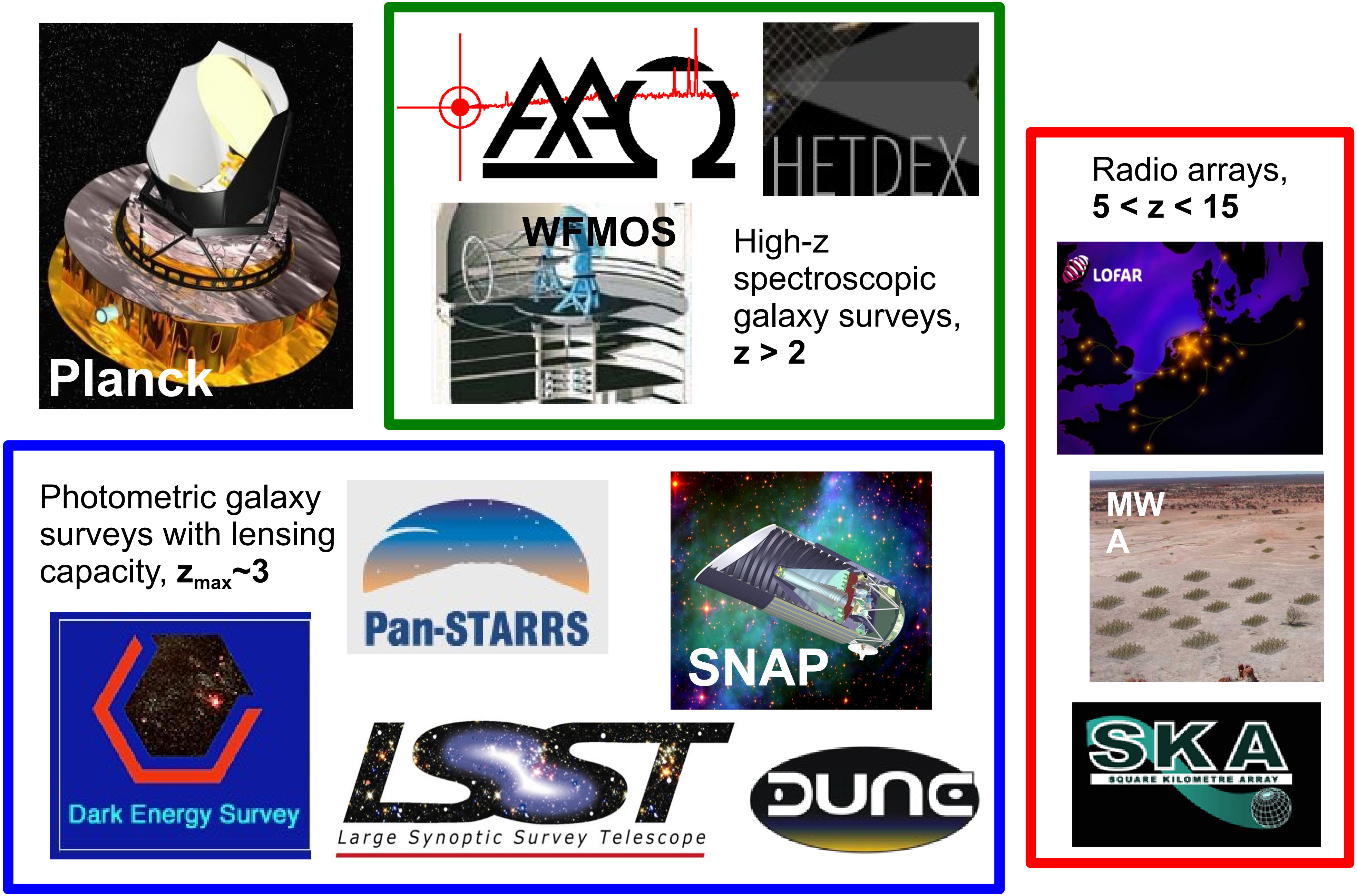
Break degenercies



+ Weak lensing
Tereno et al. 2008
Ichiki et al. 2008

... and many more.

2. What we can do in the future...


Planck
WFMOS
HETDEX
High-z
spectroscopic
galaxy surveys,
z > 2
Radio arrays,
5 < z < 15
LOFAR
MW
A
SKA
SQUARE KILOMETRE ARRAY
Photometric galaxy
surveys with lensing
capacity, z_{max}~3
Pan-STARRS
SNAP
Dark Energy Survey
LSST
Large Synoptic Survey Telescope
DUNE

Possible new techniques...

- **<u>Weak lensing</u>**
  - <u>of galaxies</u>

    Song & Knox 2004
    Hannestad, Tu & Y^3W 2006
    Kitching et al. 2008

  - of the CMB

    Lesgourgues et al. 2006
    Perotto, Lesgourgues, Hannestad, Tu & Y^3W, 2006

- **21 cm emission**

  Mao et al. 2008
  Pritchard & Pierpaoli 2008
  Metcalf 2009

- **ISW effect**

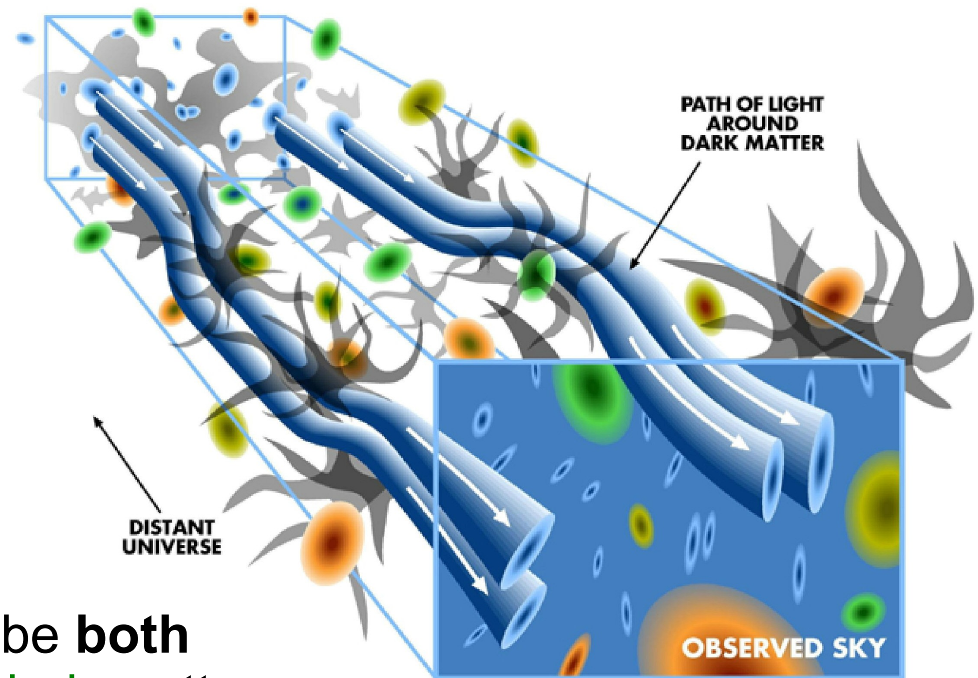
  Ichikawa & Takahashi 2005
  Lesgourgues, Valkenburg & Gaztañaga 2007

- **Cluster abundance**

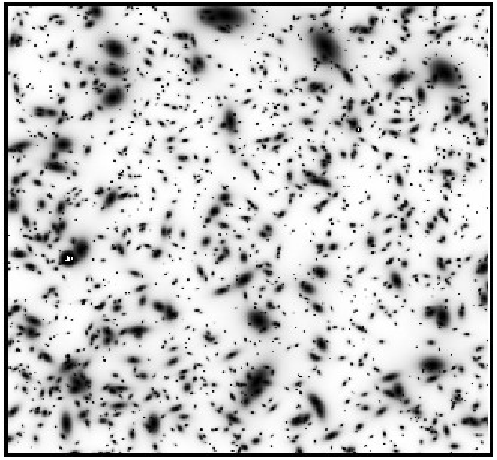
  Wang et al. 2005

Weak lensing of galaxies/Cosmic shear...

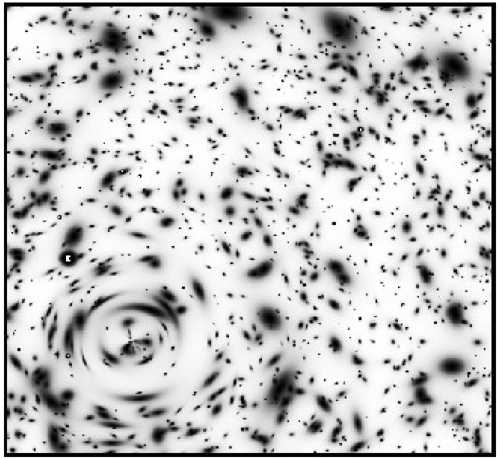
- Distortion (magnification or stretching) of distant galaxy images by **foreground matter**.



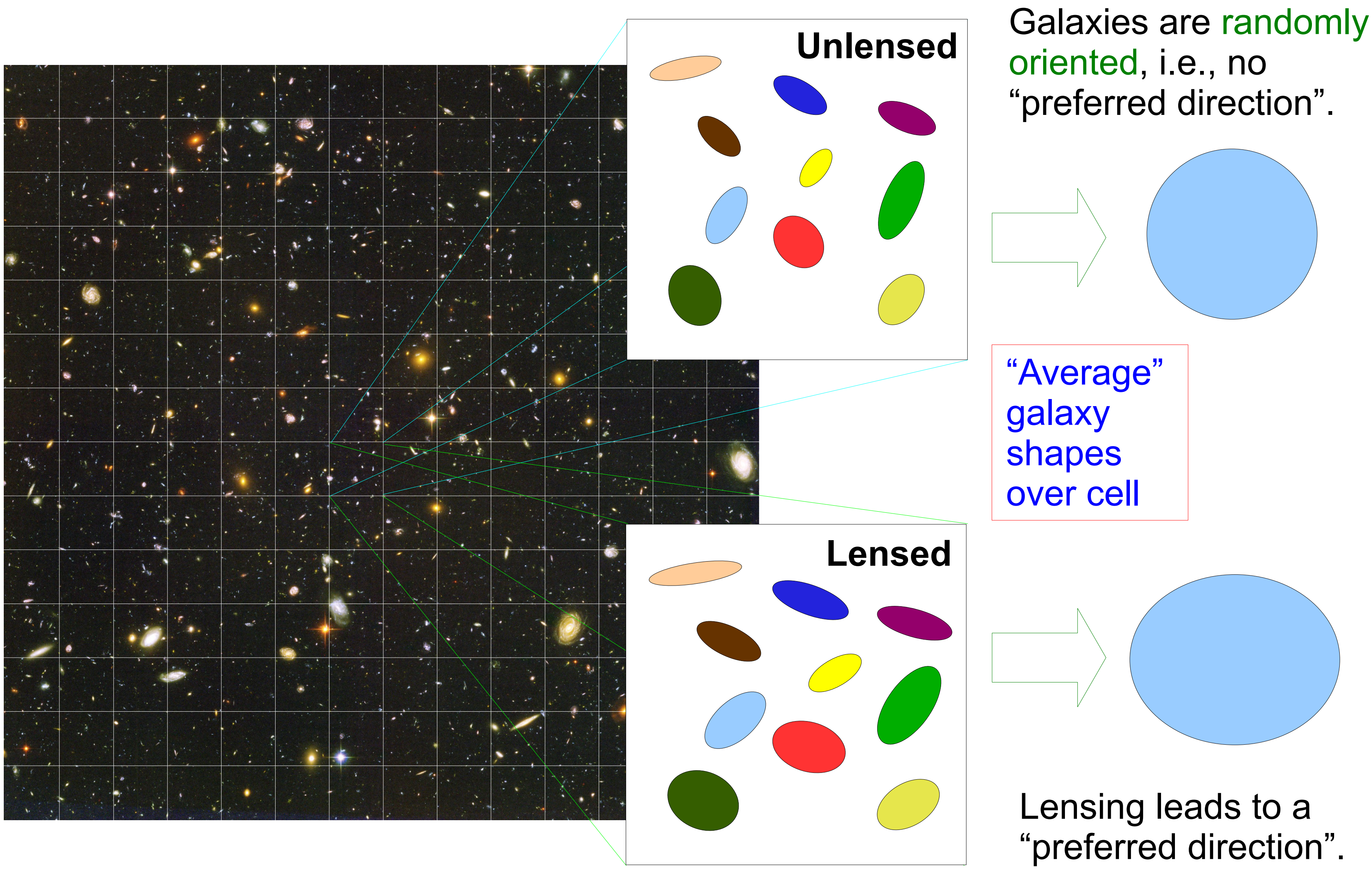
Distortions probe **both** luminous and dark matter (no galaxy bias problem!)



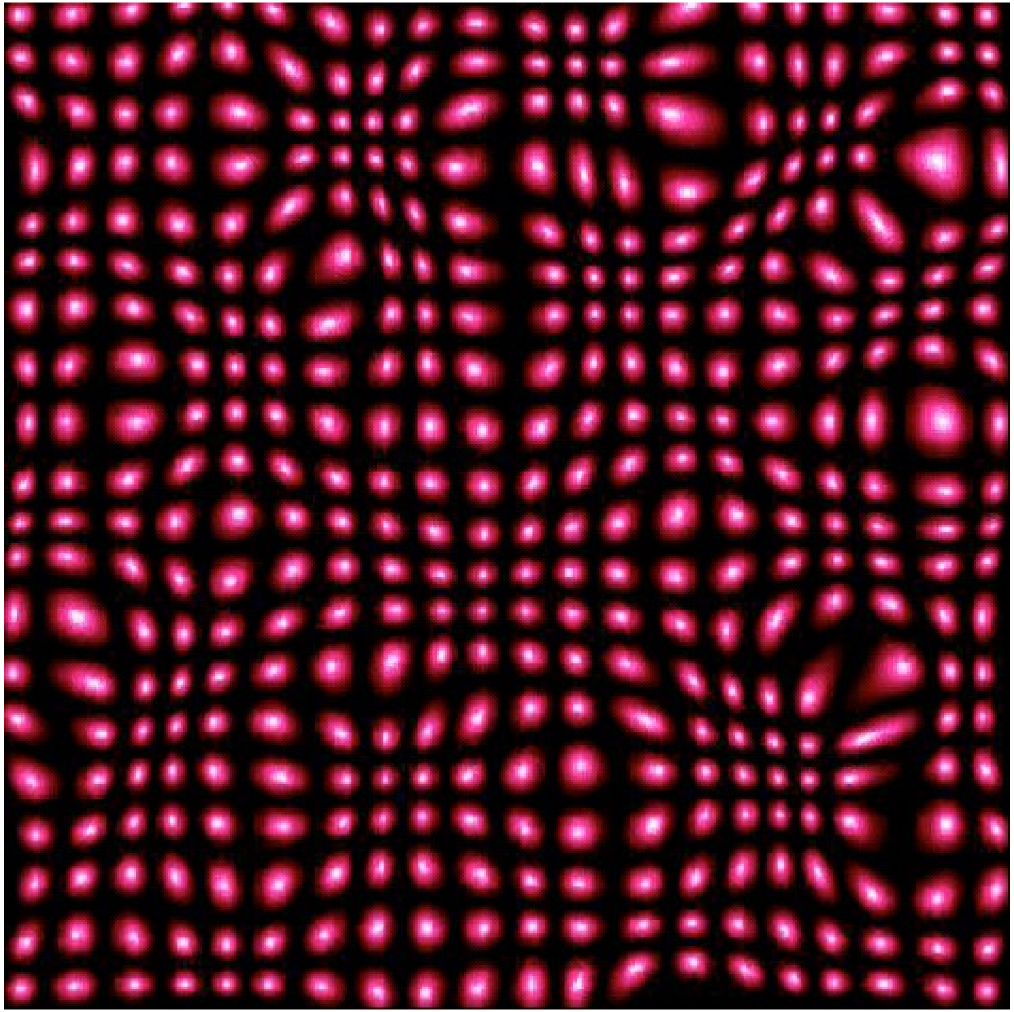
Unlensed



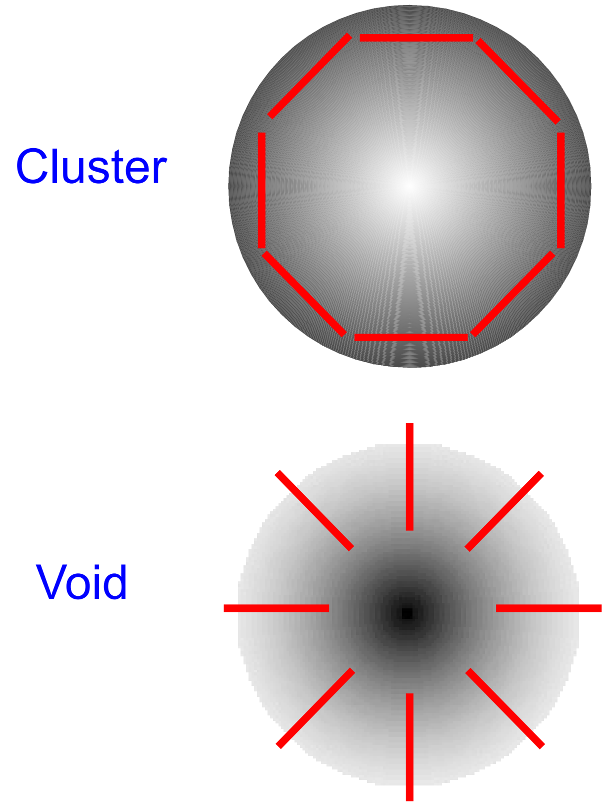
Lensed


Galaxies are randomly oriented, i.e., no "preferred direction".
Unlensed
"Average" galaxy shapes over cell
Lensed
Lensing leads to a "preferred direction".

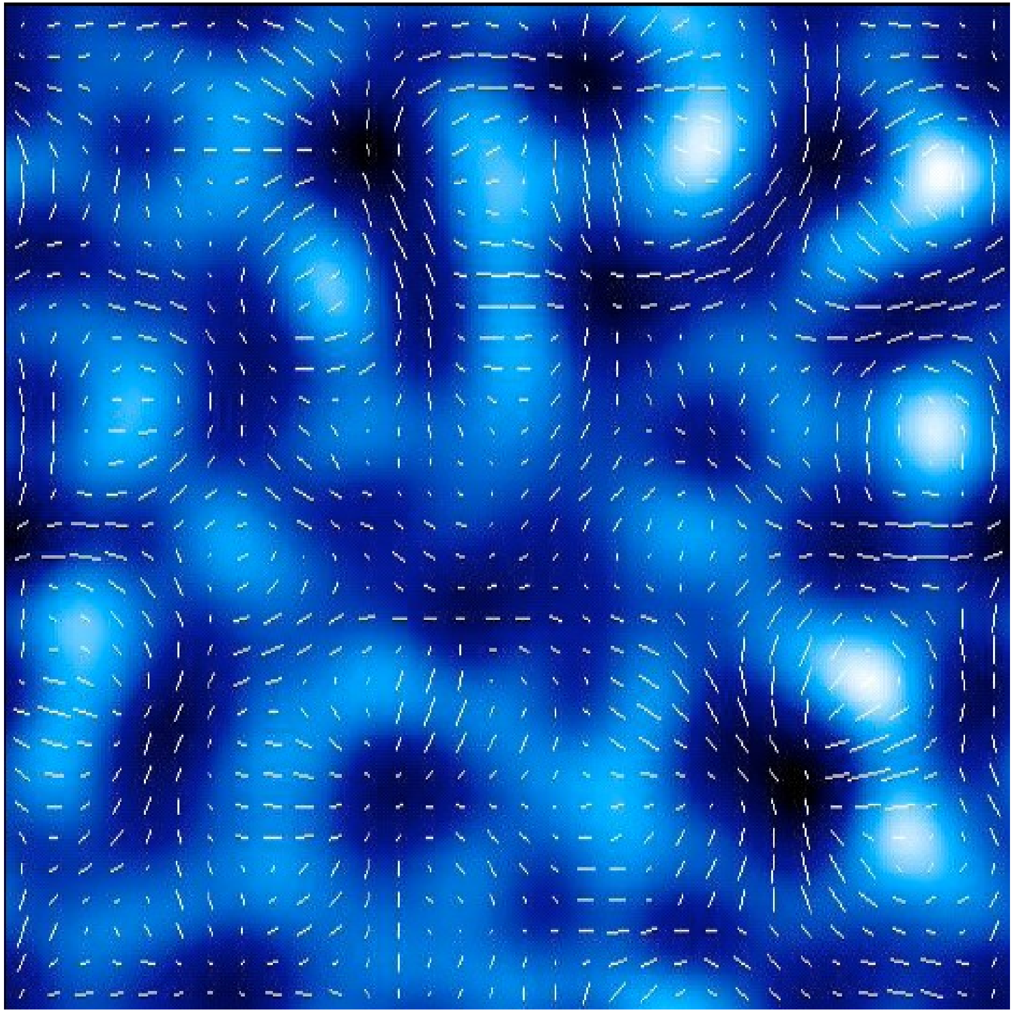
Shear map



Weak lensing theory predicts:



Shear map → Convergence map (projected mass)



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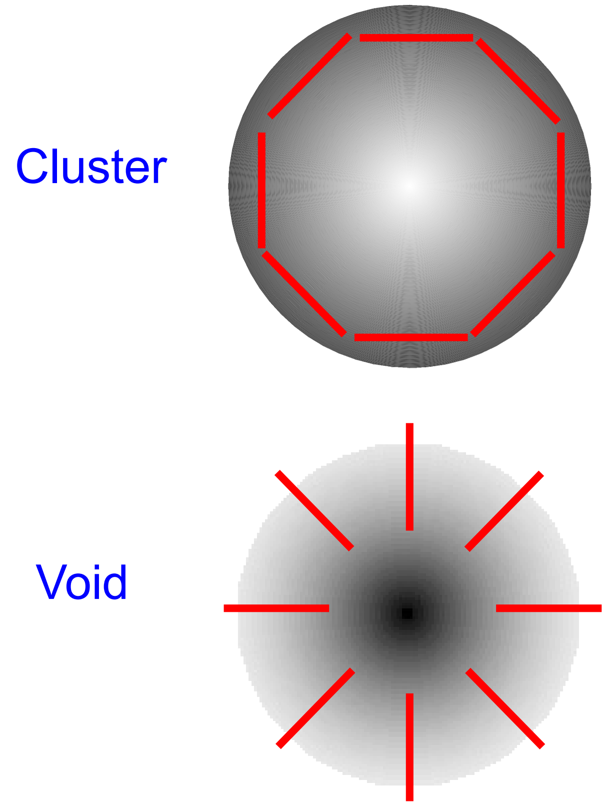
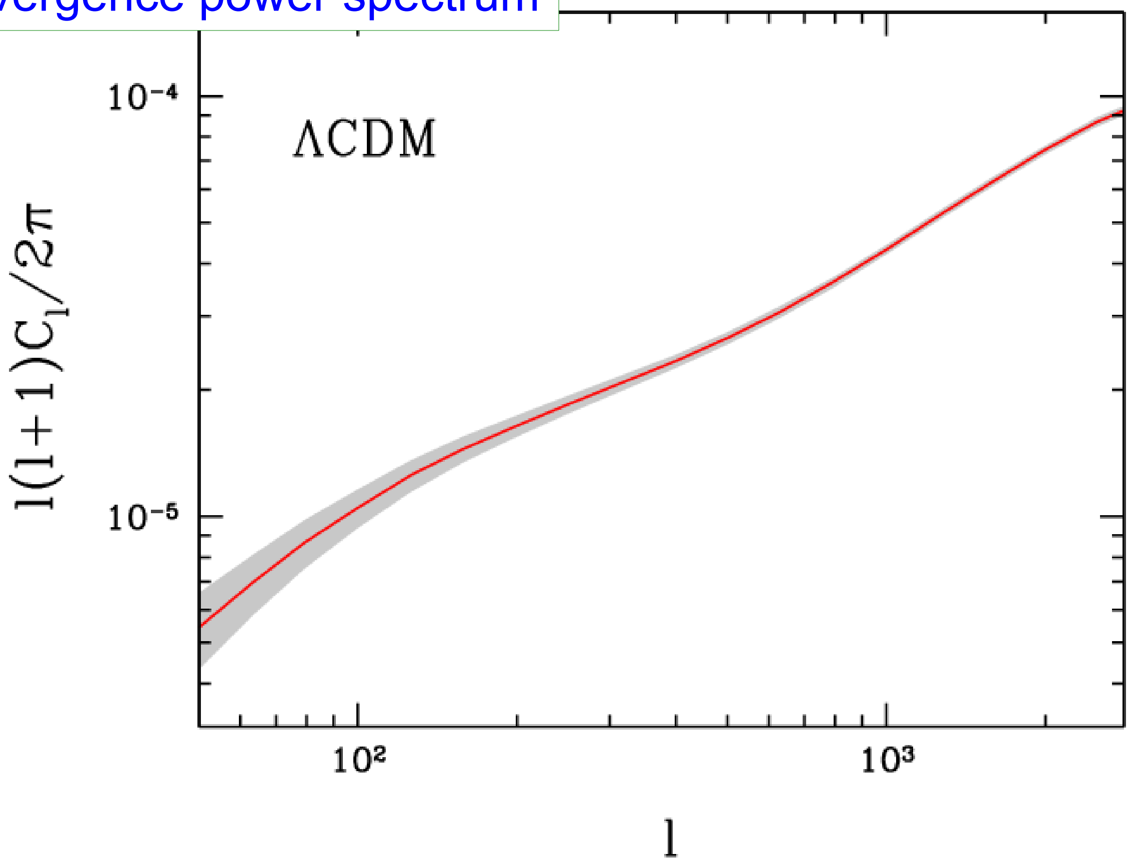
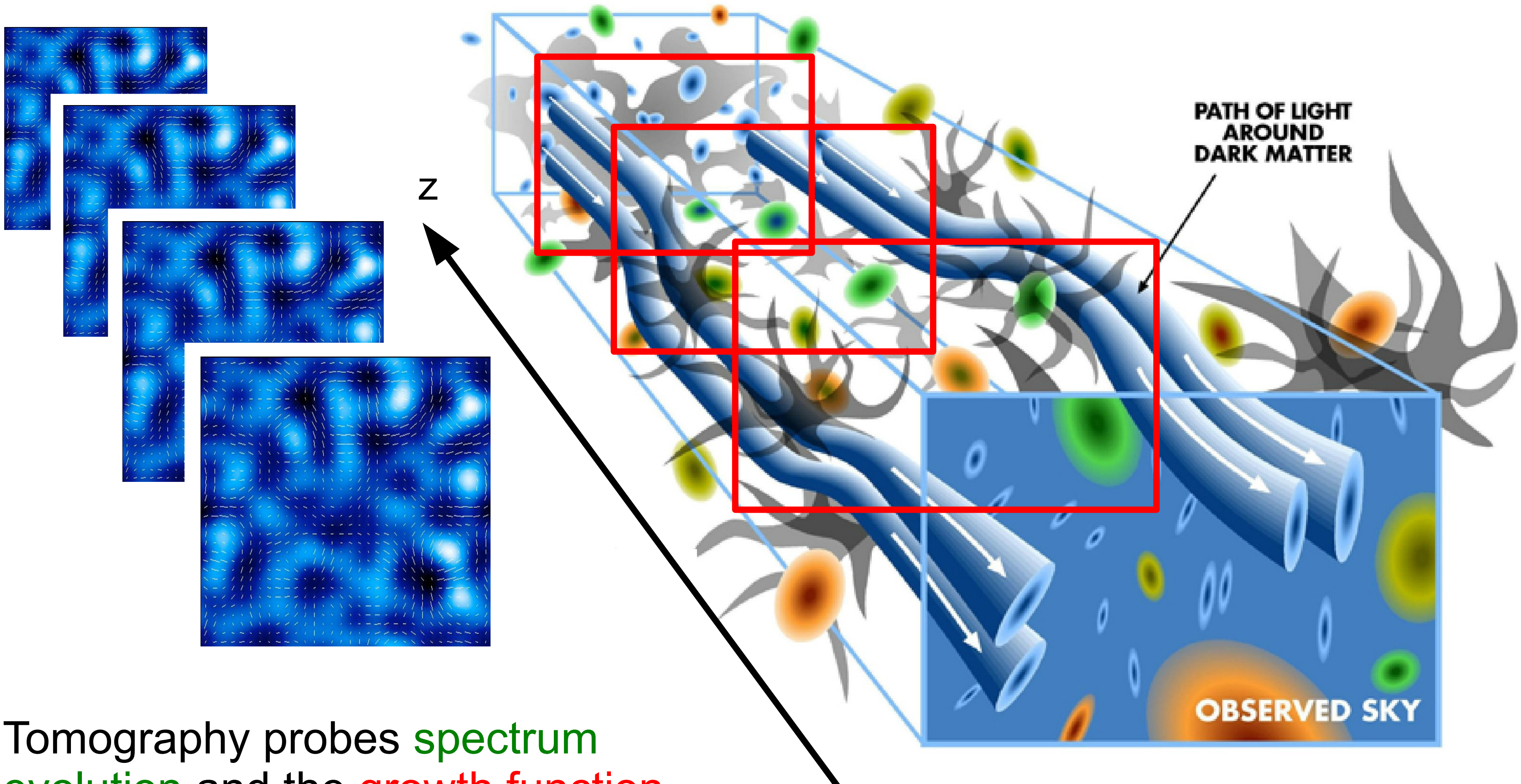


Convergence power spectrum

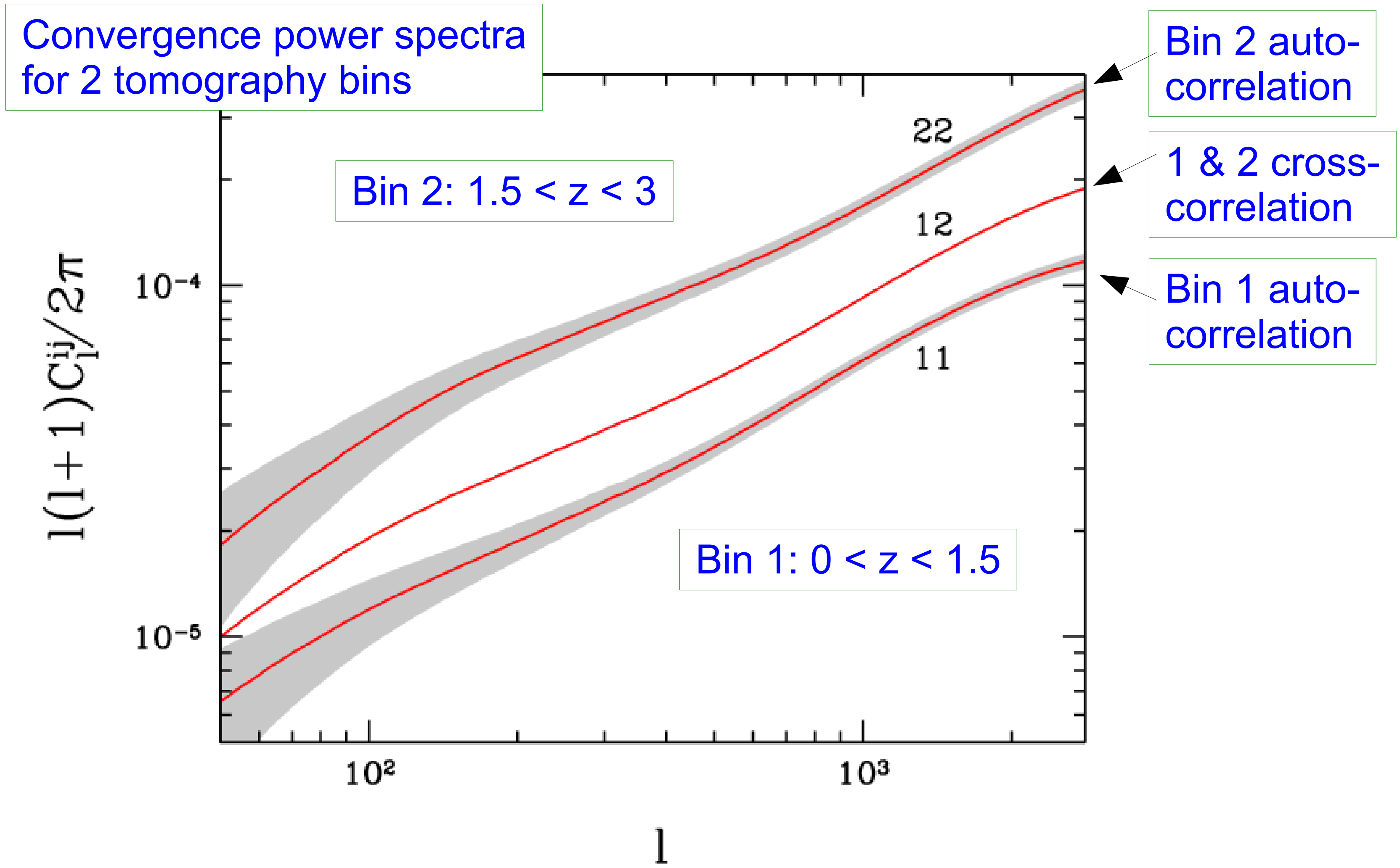


$$C_l \propto \int_0^\infty d\chi\, a^{-2} \left[\int_\chi^\infty d\chi'\, n_{\rm gal}(\chi') \frac{D(\chi'-\chi)}{D(\chi')} \right]^2 P(k = l/D(\chi))$$

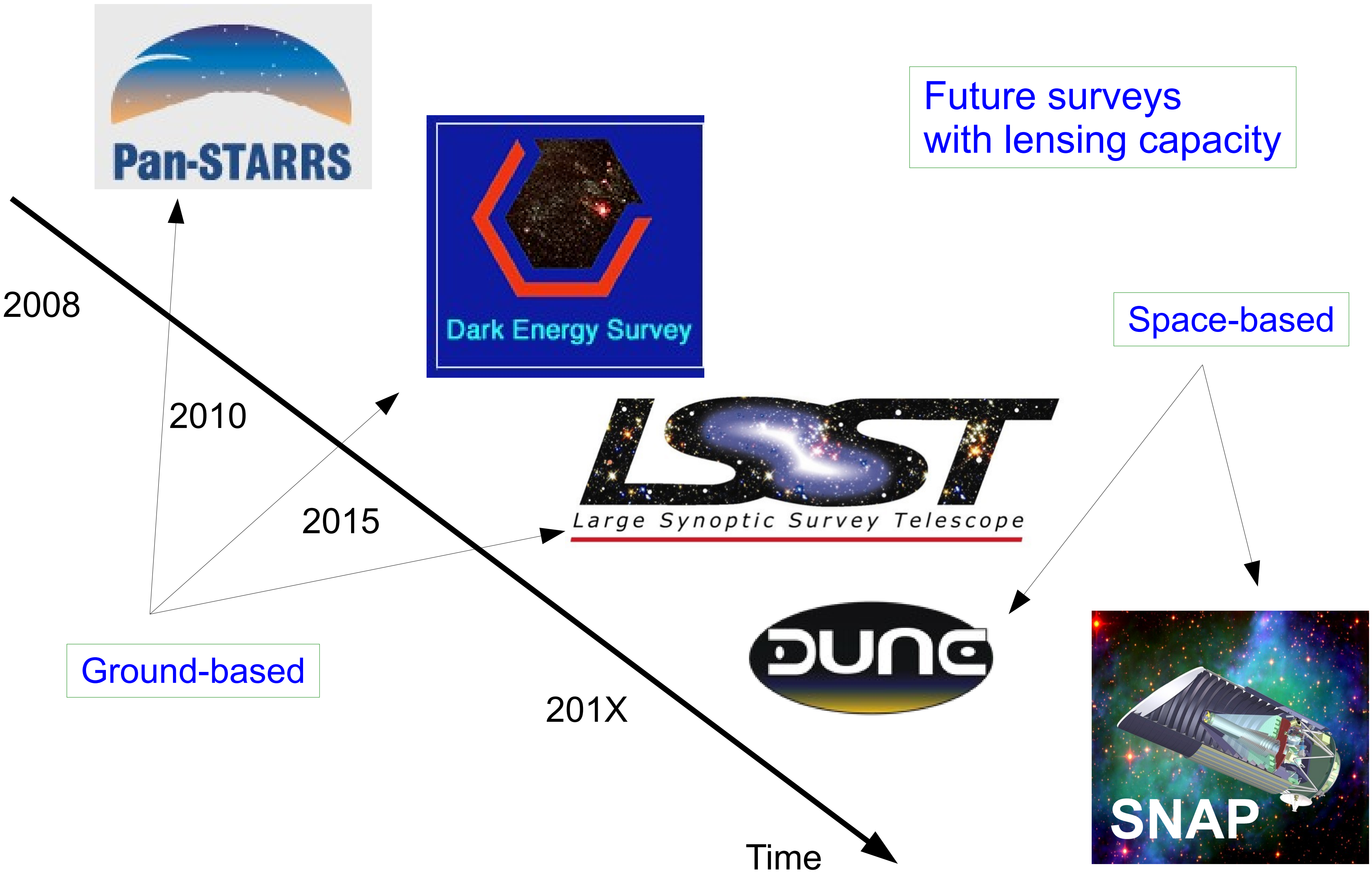
- **Tomography** = bin galaxy images by redshift



Tomography probes spectrum evolution and the growth function.

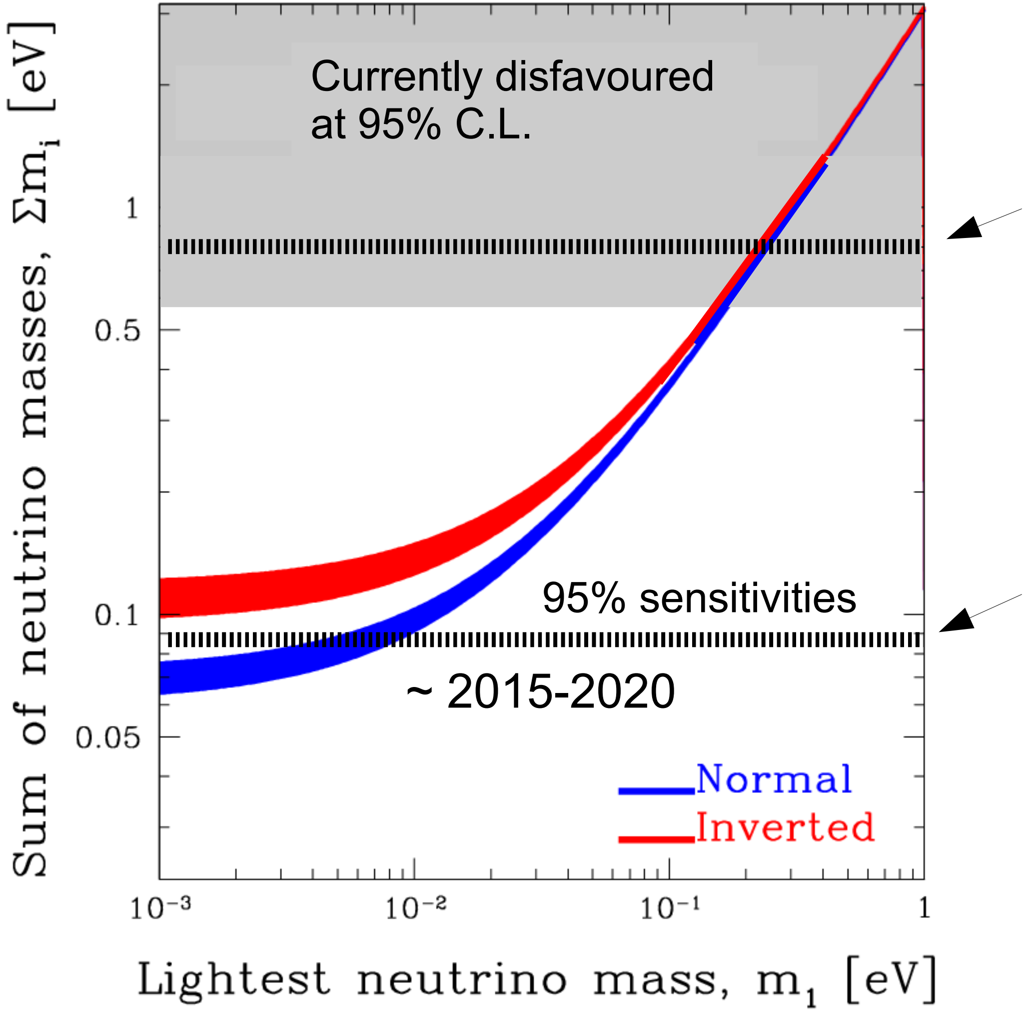


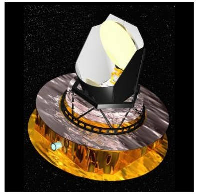
Hannestad, Tu & Y[3]W 2006



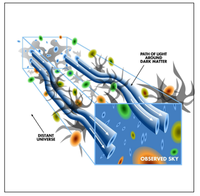
Future surveys
with lensing capacity
Pan-STARRS
Dark Energy Survey
LSST
Large Synoptic Survey Telescope
Space-based
Ground-based
DUNE
SNAP
2008
2010
2015
201X
Time

Projected sensitivities...



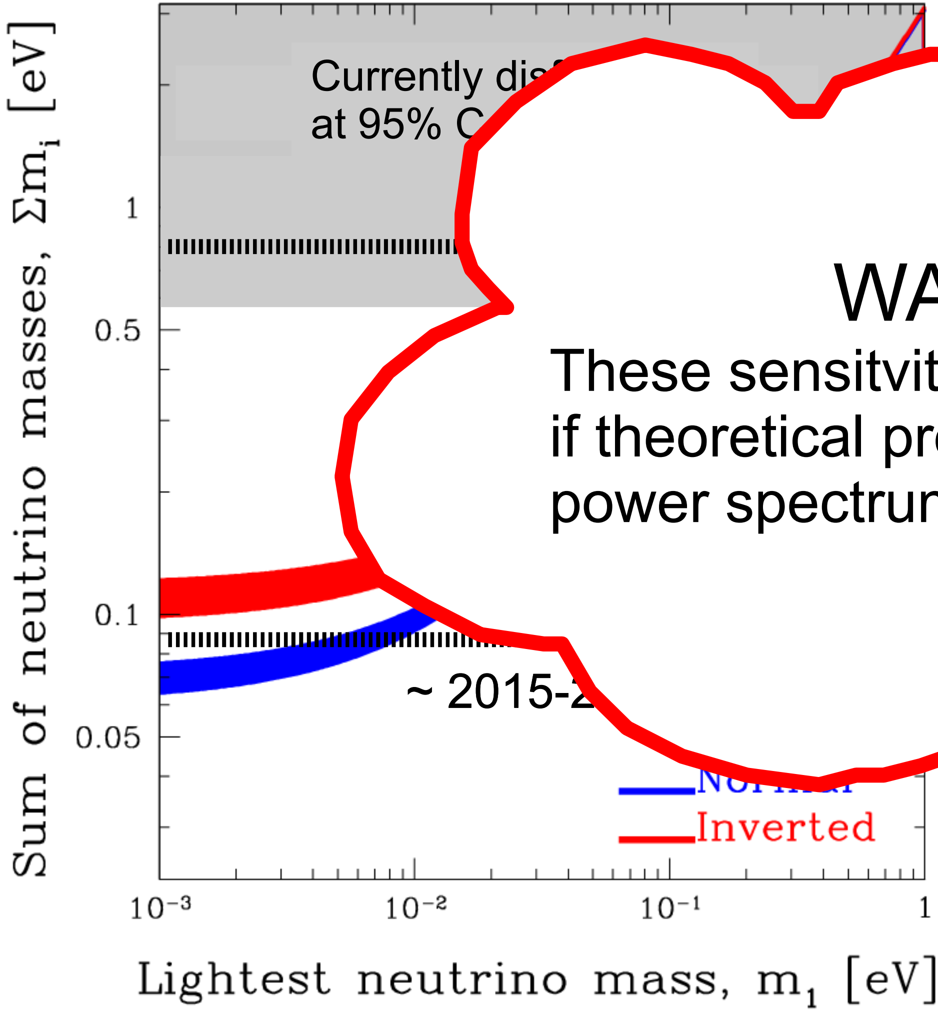


Planck (1 year)
Lesgourgues et al. 2006
Perotto, Lesgouruges,
Hannestad, Tu & Y^{3}W 2006



+ Weak lensing with LSST
(tomography)
Hannestad, Tu & Y^{3}W 2006
Kitching et al. 2008

Projected sensitivities...



1 year)
al. 2006
s,
/ 2006

g with LSST

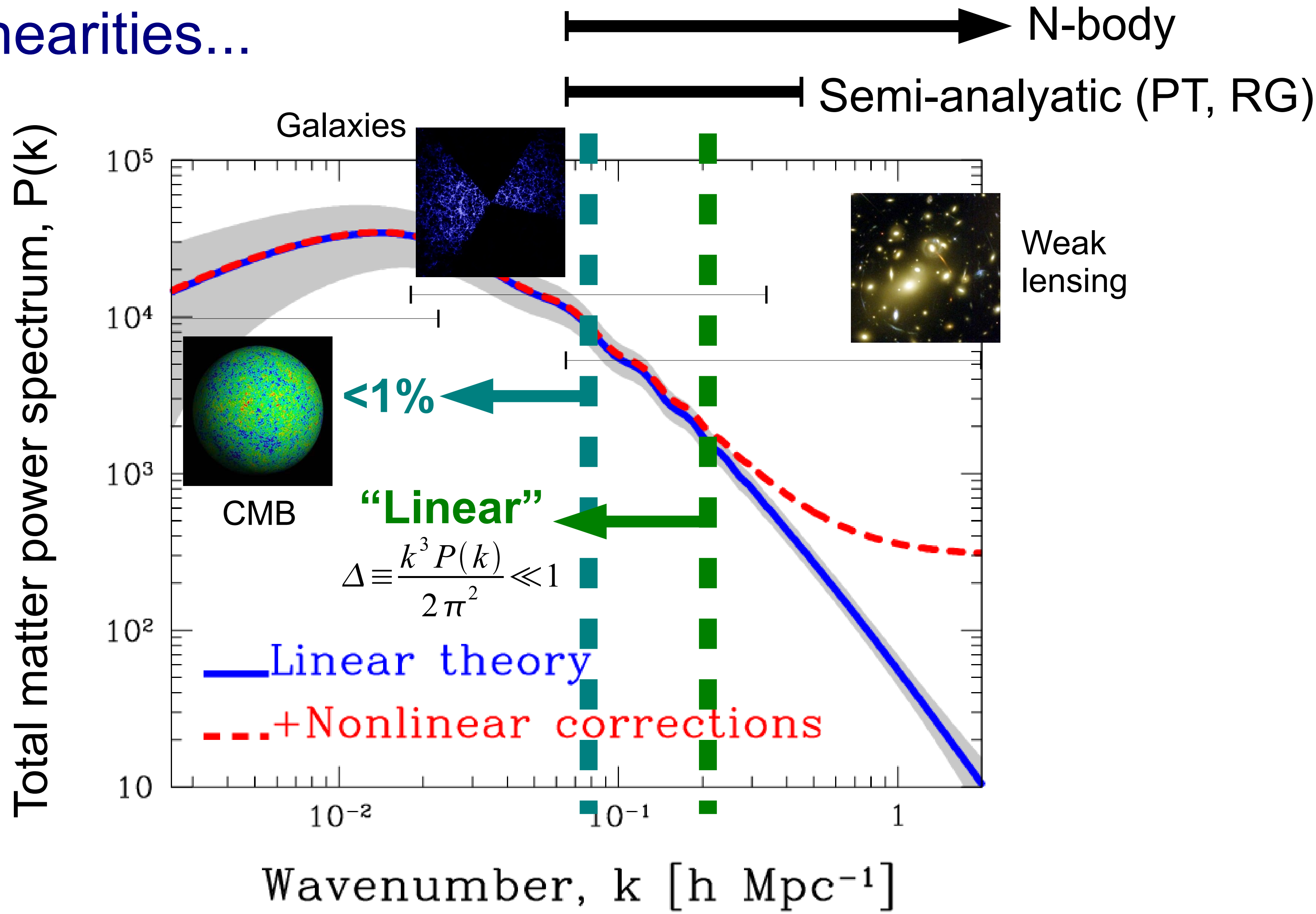
& Y[3]W 2006
al. 2008

WARNING!

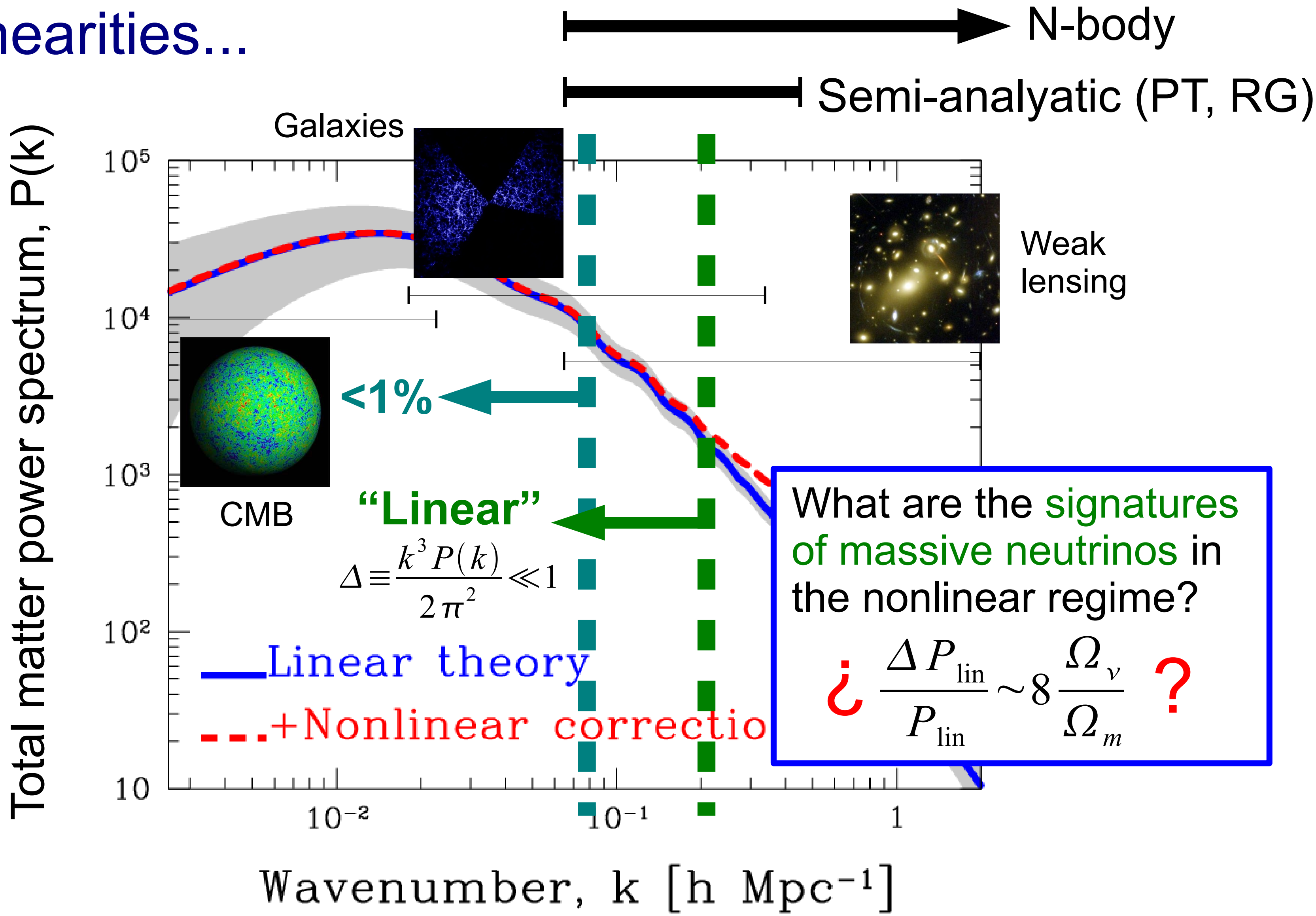
These sensitvities can be achieved only if theoretical predictions of the matter power spectrum are accurate to ~ 1%.

3. The nonlinear matter power spectrum...

Nonlinearities...



Nonlinearities...

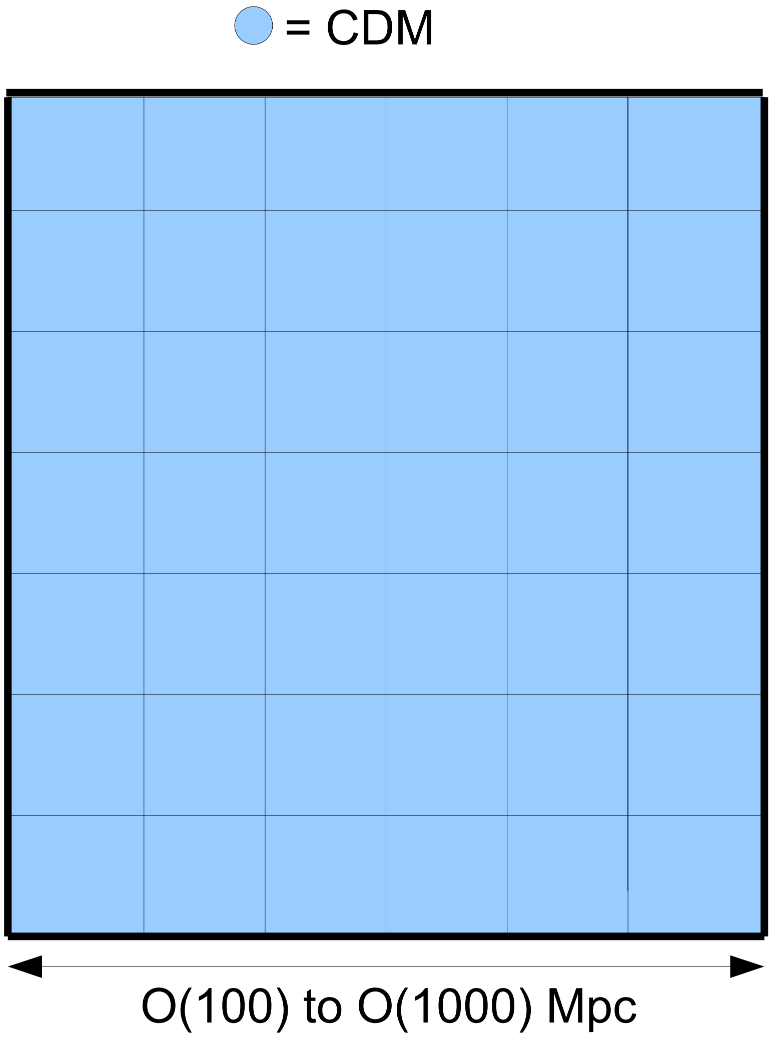


What are the signatures of massive neutrinos in the nonlinear regime?

$$¿\ \frac{\Delta P_{\rm lin}}{P_{\rm lin}} \sim 8 \frac{\Omega_\nu}{\Omega_m}\ ?$$

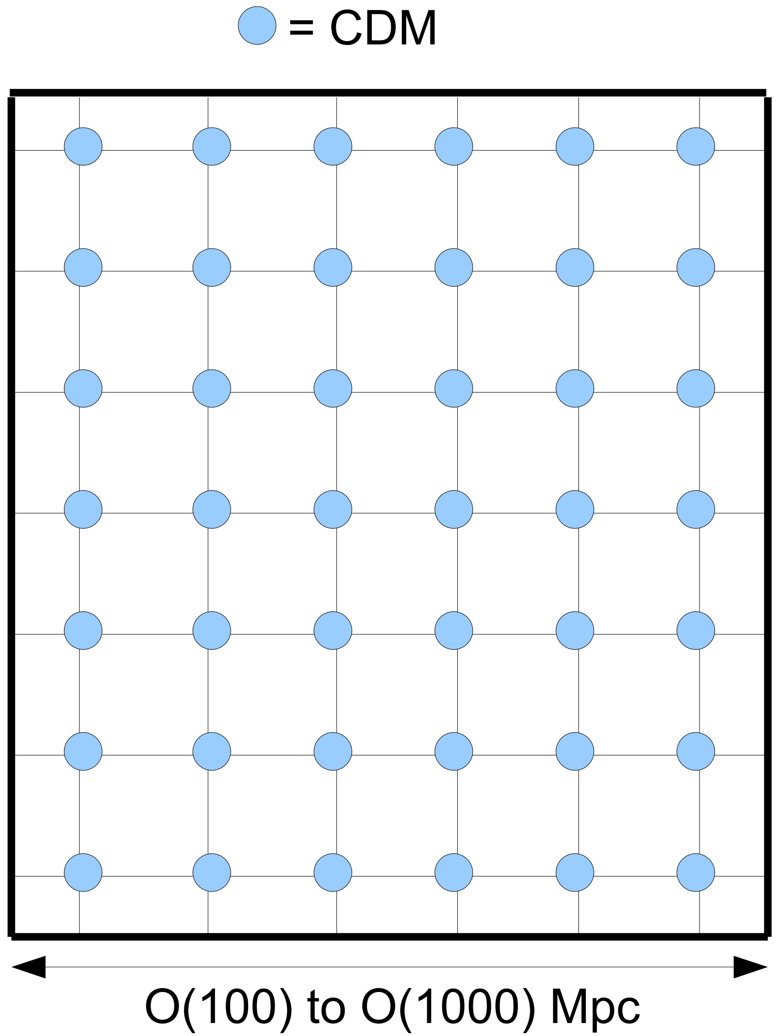
Cosmological N-body simulations...

- Represent fluid



Cosmological N-body simulations...

- Represent fluid by point particles.



Cosmological N-body simulations...

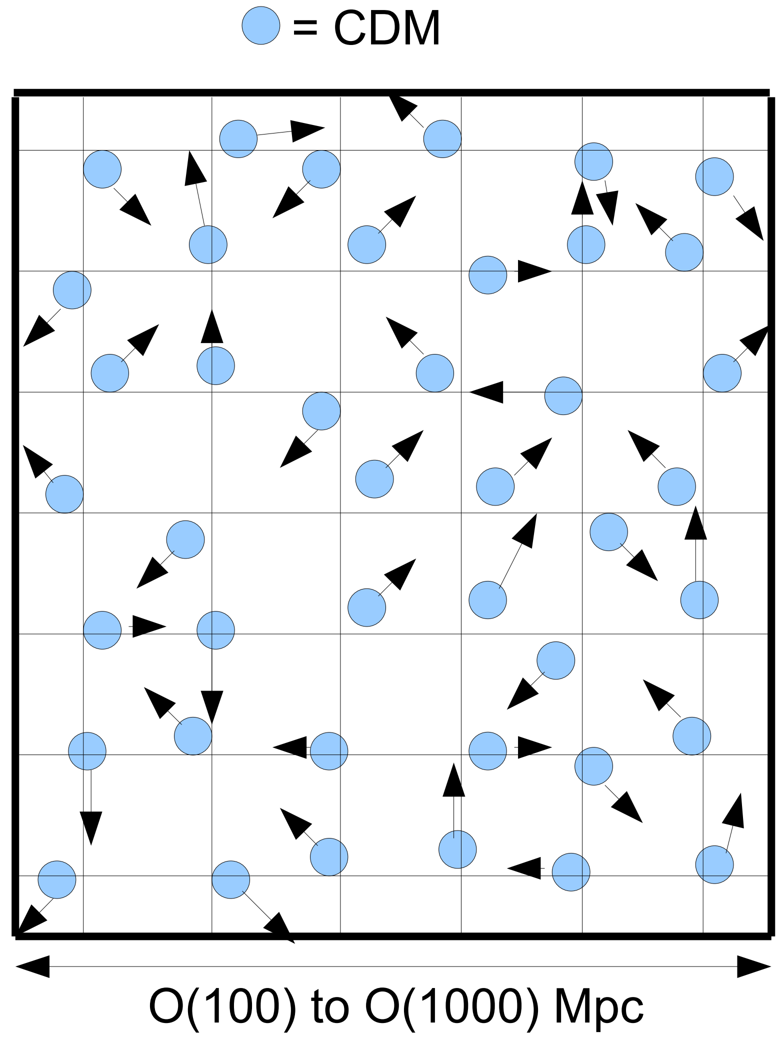
- Represent fluid by point particles.
- Initial conditions from the Zel'dovich approximation:

Initial positions & velocities

$$\boldsymbol{x} = \boldsymbol{x}_i + D(\tau_i)\,\boldsymbol{\psi}(\boldsymbol{x}_i)$$

$$\boldsymbol{v} = \frac{dD}{d\tau}\,\boldsymbol{\psi}(\boldsymbol{x}_i)$$

ψ = random displacement vector (from linear power spectrum)
D = linear growth function



Cosmological N-body simulations...

- Represent fluid by point particles.
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Initial positions & velocities

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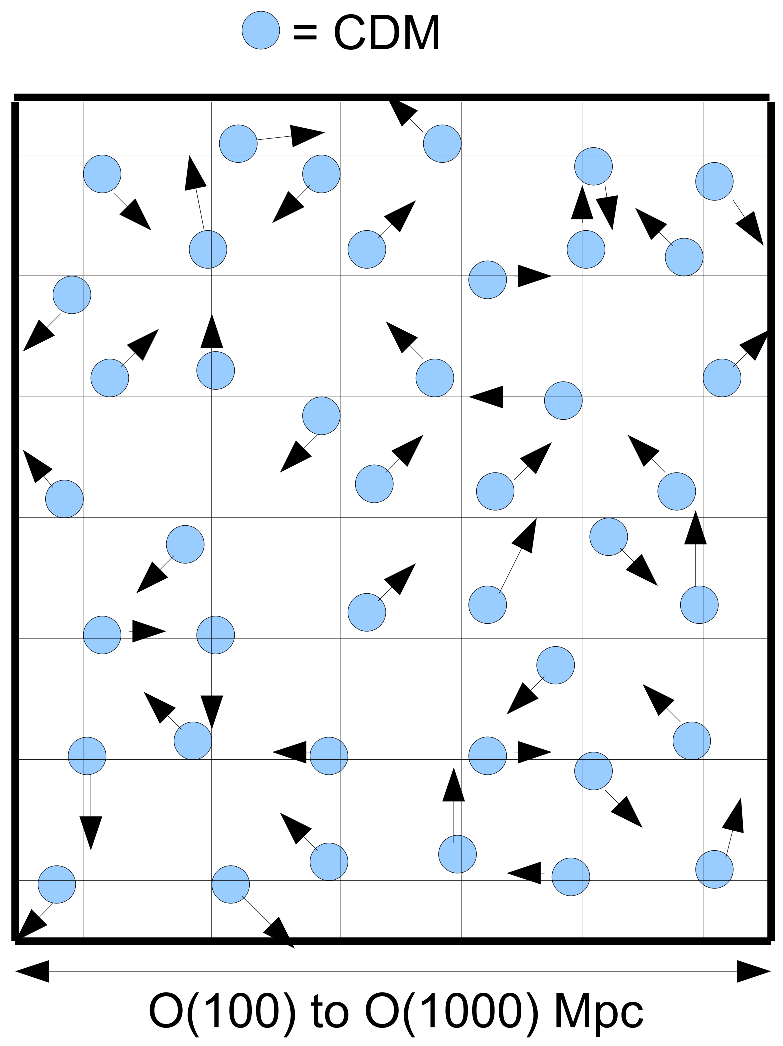
D = linear growth function

- **Evolve** according to:

$$\frac{d\boldsymbol{x}}{d\tau} = \boldsymbol{v}, \quad \frac{d\boldsymbol{v}}{d\tau} + \frac{\dot{a}}{a}\boldsymbol{v} = -\nabla\Phi$$



Simulating massive neutrinos...

- Neutrinos have thermal motion.
  - Unperturbed phase space distribution:

$$f(p) = \frac{1}{\exp(p/T_0)+1}$$

Present day neutrino temperature $T_0 = 1.95\ \mathrm{K}$

  - Average velocity:

$$\langle v \rangle = \left\langle \frac{q}{m_\nu a} \right\rangle \simeq 81(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right)\ \mathrm{km\ s^{-1}}$$

- For m_ν = 1 eV, <v> is 10% the speed of light at z = 100.

- Same equations of motion as for CDM.
- Neutrino thermal motion is factored into the initial conditions.
- **One possible implementation**:
  - Give neutrino particles an initial velocity:

$$\boldsymbol{v} = \frac{dD}{d\tau}\boldsymbol{\psi}(\boldsymbol{x}_i) + \underline{v_{\text{thermal}}\hat{\boldsymbol{r}}}$$

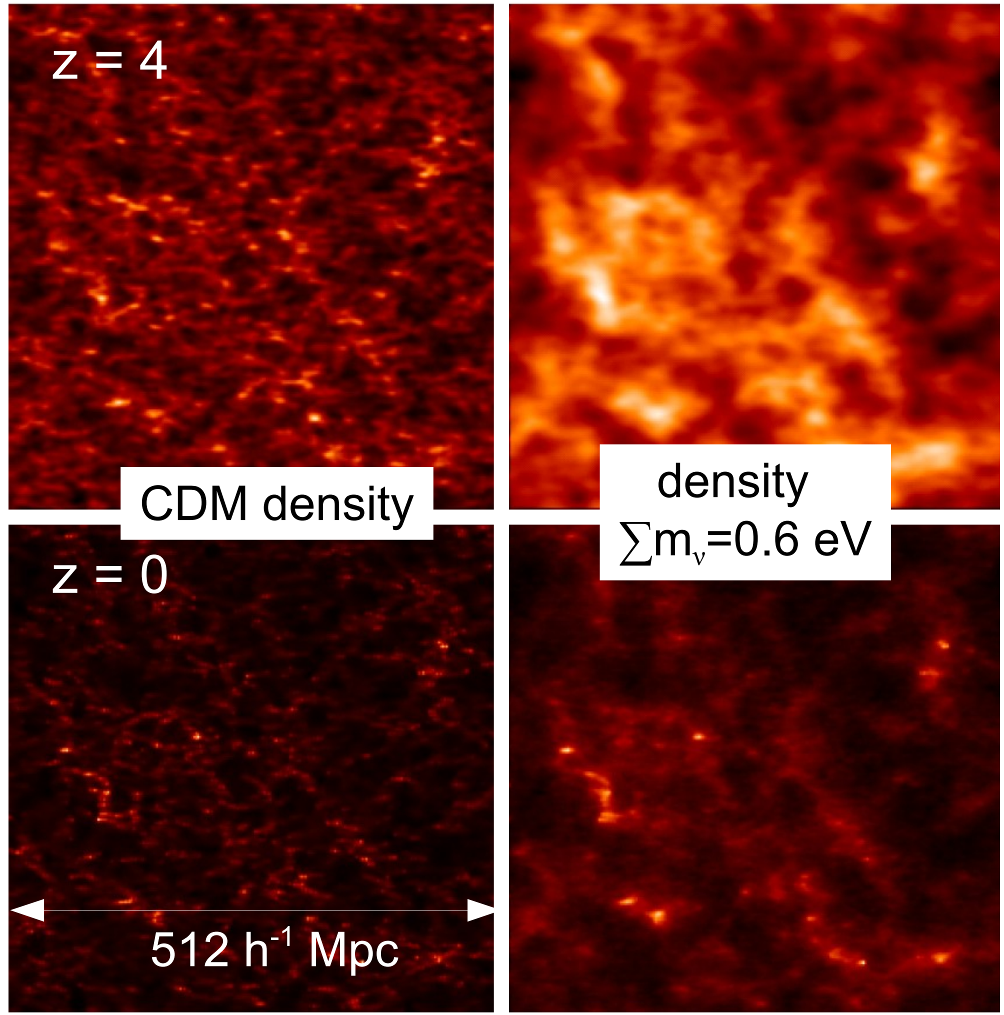
    - $\hat{\boldsymbol{r}}$ = random direction vector
    - v_{thermal} = thermal velocity drawn randomly from a Fermi-Dirac distribution

Brandbyge, Hannestad, Haugbølle & Thomsen 2008

Simulating CDM+massive neutrinos...

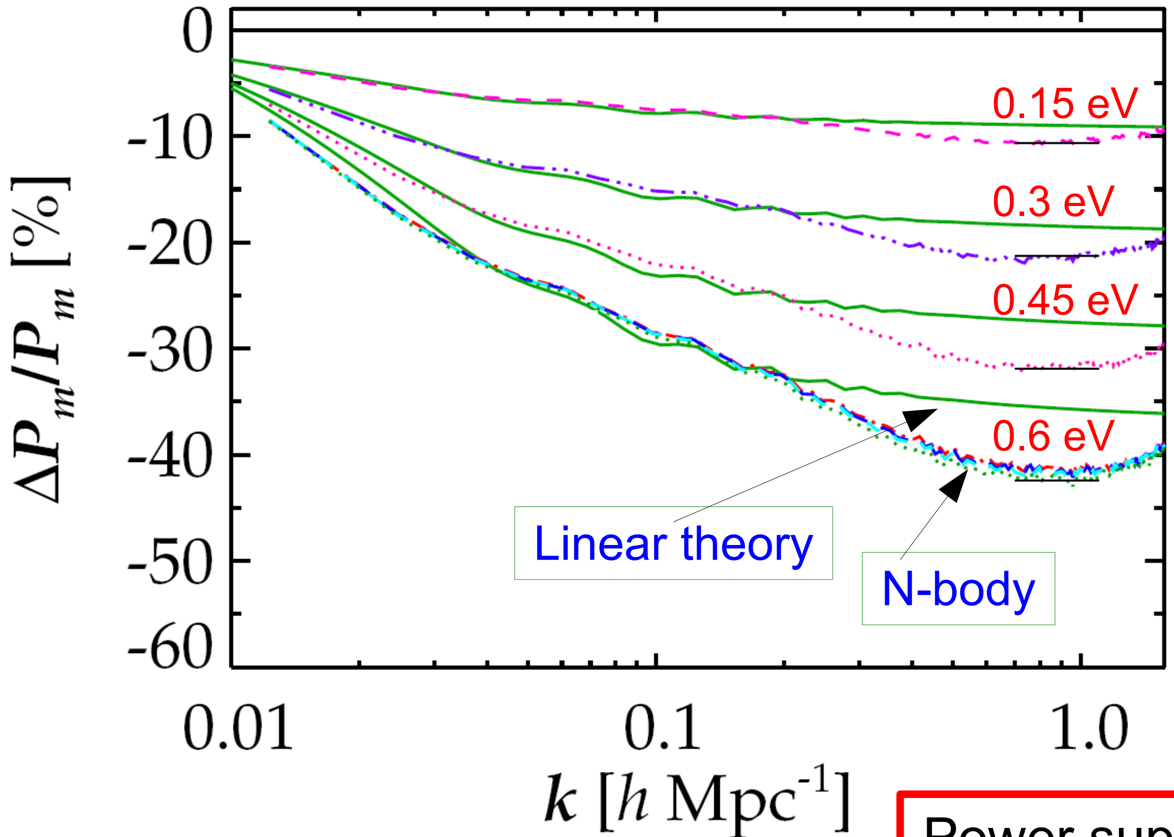
- Particle representation for **both** CDM and neutrinos.
  - Modified GADGET-2.
  - **Neutrino particles** drawn from Fermi-Dirac distribution.

Brandbyge, Hannestad, Haugbølle & Thomsen 2008
Brandbyge and Hannestad 2008, 2009



Change in the total matter power spectrum relative to the $f_\nu = 0$ case:

$$\frac{\Delta P_m}{P_m} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$



Linear perturbation theory:

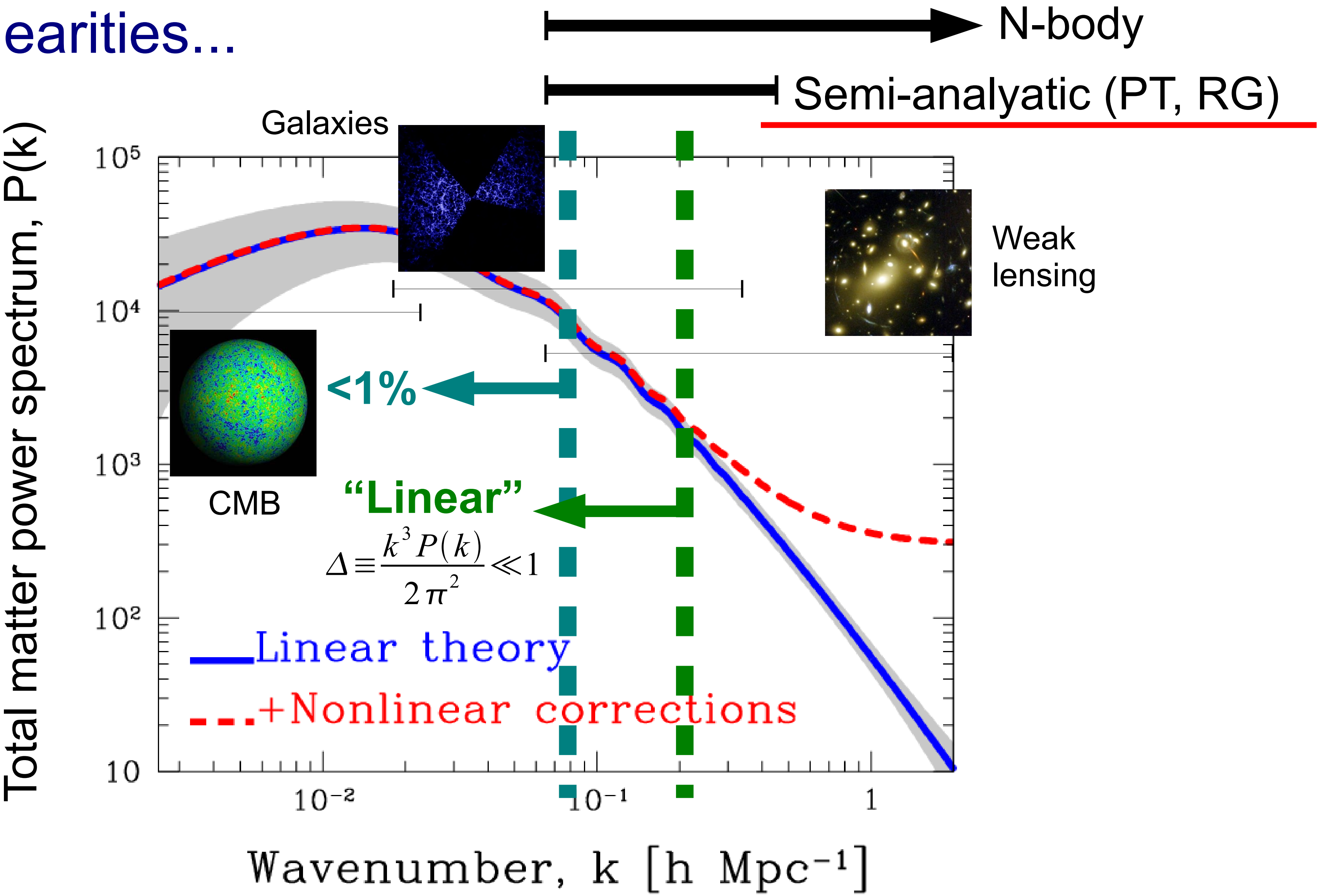
$$\frac{\Delta P_m}{P_m} \sim 8 \frac{\Omega_\nu}{\Omega_m}$$

With nonlinear corrections:

$$\frac{\Delta P_m}{P_m} \sim 9.8 \frac{\Omega_\nu}{\Omega_m}$$

Power suppression due to neutrino free-streaming is larger than predicted by linear perturbation theory.

Nonlinearities...



Semi-analytic techniques...

- Going beyond linear perturbation theory?

✓ Nonlinear correction

CDM

$$\dot{\delta}_c + \theta_c = 0$$

Linearised continuity eqn

$$\dot{\theta}_c + H\theta_c + \nabla^2 \Phi = 0$$

Linearised Euler eqn

× **No** nonlinear correction

Neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

Linearised Vlasov eqn

But see
Shoji & Komatsu 2009
Y^3W in prep

$$\nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m [f_c \delta_c + f_\nu \delta_\nu]$$

Poisson eqn

Corrections to the CDM component...

- Fluid description (linear):

Continuity eqn

$$\dot{\delta}_c(\boldsymbol{k},\tau)+\theta_c(\boldsymbol{k},\tau)= 0$$

Euler eqn

$$\dot{\theta}_c(\boldsymbol{k},\tau)+H\,\theta_c(\boldsymbol{k},\tau)-k^2\Phi(\boldsymbol{k},\tau)= 0$$

Poisson eqn

$$k^2\Phi=-\frac{3}{2}H^2\Omega_m[f_c\delta_c+f_\nu\delta_\nu]$$

δ_c = CDM density perturbations
δ_ν = ν density perturbations
θ_c = Divergence of velocity field

Corrections to the CDM component...

- Fluid description (incl. **some** nonlinear terms):

Vertex

$$\gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\equiv\delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\frac{\boldsymbol{q}_{12}\cdot\boldsymbol{q}_1}{q_1^2}$$

Continuity eqn

$$\dot{\delta}_c(\boldsymbol{k},\tau)+\theta_c(\boldsymbol{k},\tau)=\ -\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2\gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\delta_c(\boldsymbol{q}_2,\tau)$$

Euler eqn

Mode coupling

$$\dot{\theta}_c(\boldsymbol{k},\tau)+H\theta_c(\boldsymbol{k},\tau)-k^2\Phi(\boldsymbol{k},\tau)=\ -\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2\gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\theta_c(\boldsymbol{q}_2,\tau)$$

Poisson eqn

$$k^2\Phi=-\frac{3}{2}H^2\Omega_m[f_c\delta_c+f_\nu\delta_\nu]$$

Vertex

$$\gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\equiv\delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\frac{q_{12}^2(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)}{2q_1^2q_2^2}$$

δ_c = CDM density perturbations
δ_ν = ν density perturbations
θ_c = Divergence of velocity field

Starting point of **most** semi-analytic calculations in the literature.

Standard perturbation theory...

Juszkiewisz 1981, Vishniac 1983, Fry 1984, Goroff et al. 1986

- Solve by perturbative expansion:

$$\varphi(\boldsymbol{k},\tau) \equiv \begin{pmatrix} \delta_c(\boldsymbol{k},\tau) \\ -\theta_c(\boldsymbol{k},\tau)/H \end{pmatrix} \qquad \varphi(\boldsymbol{k},\tau) = \sum_{m=1}^{\infty} \varphi^{(n)}(\boldsymbol{k},\tau)$$

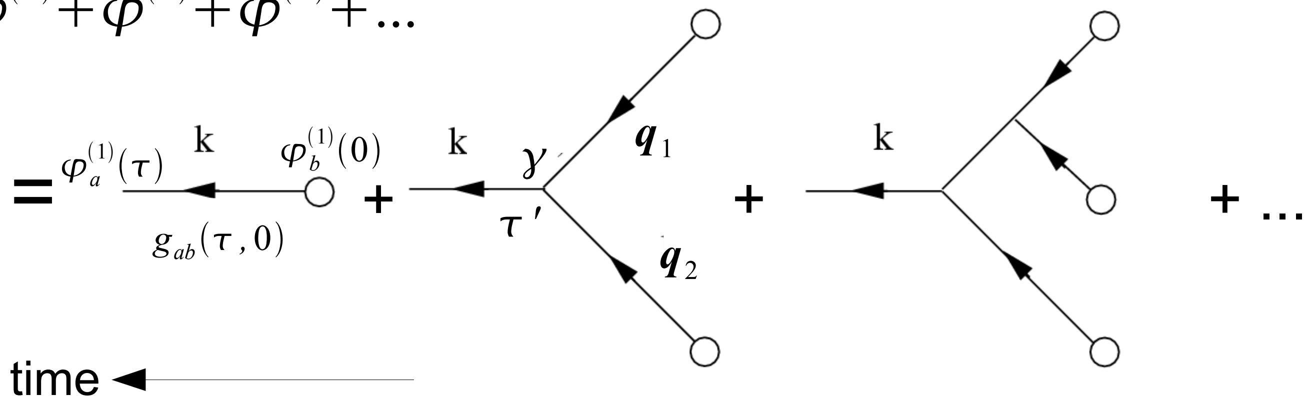
- nth order solution:

$$\varphi_a^{(n)}(\boldsymbol{k},\tau) = g_{ab}(\tau,0)\,\varphi_b^{(n)}(\boldsymbol{k},0) + \int d^3\boldsymbol{q}_1 \int d^3\boldsymbol{q}_2 \int_0^{\tau} d\tau'\, g_{ab}(\tau,\tau')\,\gamma_{bcd}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2) \sum_{m=1}^{n-1} \varphi_c^{(n-m)}(\boldsymbol{q}_1,\tau')\,\varphi_d^{(m)}(\boldsymbol{q}_2,\tau')$$

Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

$$\varphi(\boldsymbol{k}) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \ldots$$

Density/
Velocity =



Linear

$$\varphi_a^{(1)}(\boldsymbol{k}, \tau) = g_{ab}(\tau, 0)\varphi_b^{(1)}(\boldsymbol{k}, 0)$$

Linear propagator
≈ Linear growth function

2nd order

$$\varphi_a^{(2)}(\boldsymbol{k}, \tau) = \int d^3\boldsymbol{q}_1 \int d^3\boldsymbol{q}_2 \int_0^\tau d\tau' g_{ab}(\tau, \tau')\gamma_{bcd}(\boldsymbol{k}, \boldsymbol{q}_1, \boldsymbol{q}_2) \times g_{ce}(\tau', 0)\varphi_e^{(1)}(\boldsymbol{q}_1, 0) g_{df}(\tau', 0)\varphi_f^{(1)}(\boldsymbol{q}_2, 0)$$

Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

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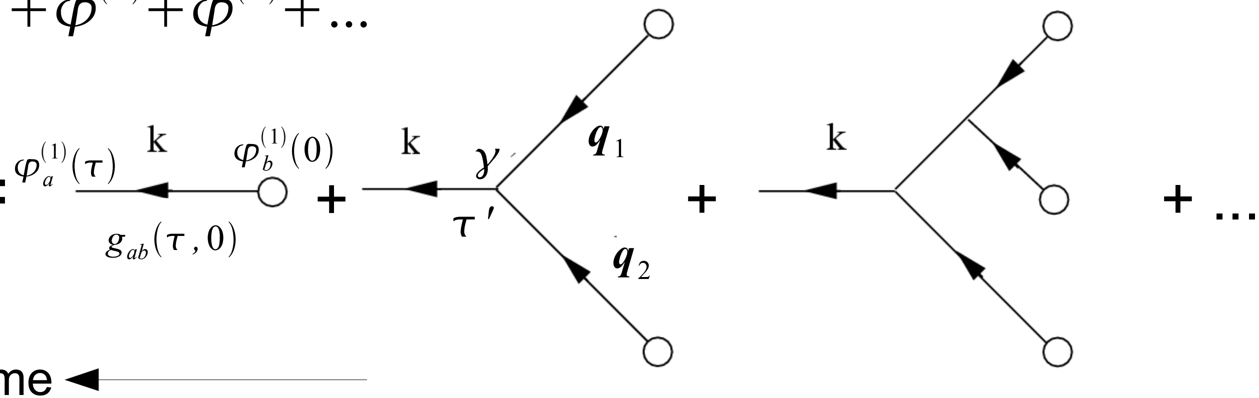
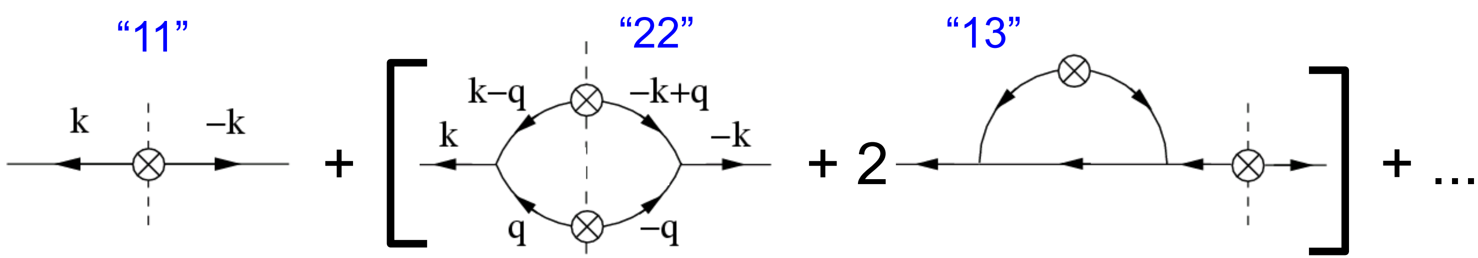


$$P(k)\,\delta_D(\boldsymbol{k}+\boldsymbol{k}') \equiv \langle \varphi(\boldsymbol{k})\varphi(\boldsymbol{k}')\rangle = \langle \varphi^{(1)}\varphi^{(1)}\rangle + [\langle \varphi^{(2)}\varphi^{(2)}\rangle + 2\langle \varphi^{(1)}\varphi^{(3)}\rangle] + \ldots$$

Power spectrum =

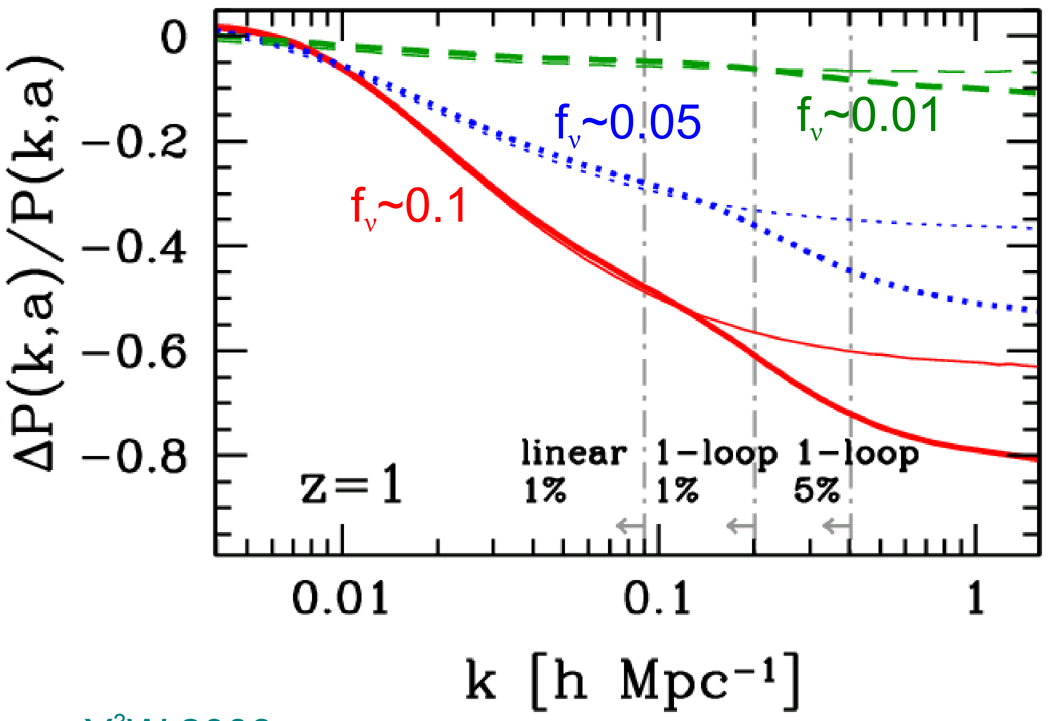


Linear

"One-loop" correction

Free-streaming suppression: One-loop corrected...

Thin lines = linear
Thick lines = 1-loop corrected



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Y[3]W 2008
also Saito et al. 2008, 2009

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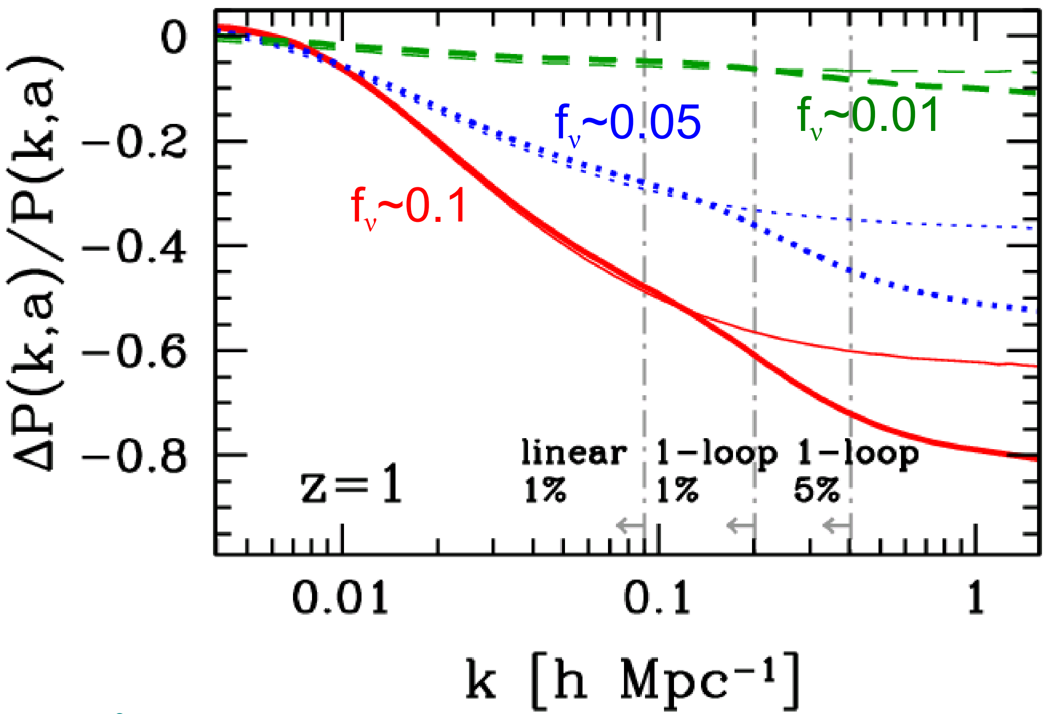
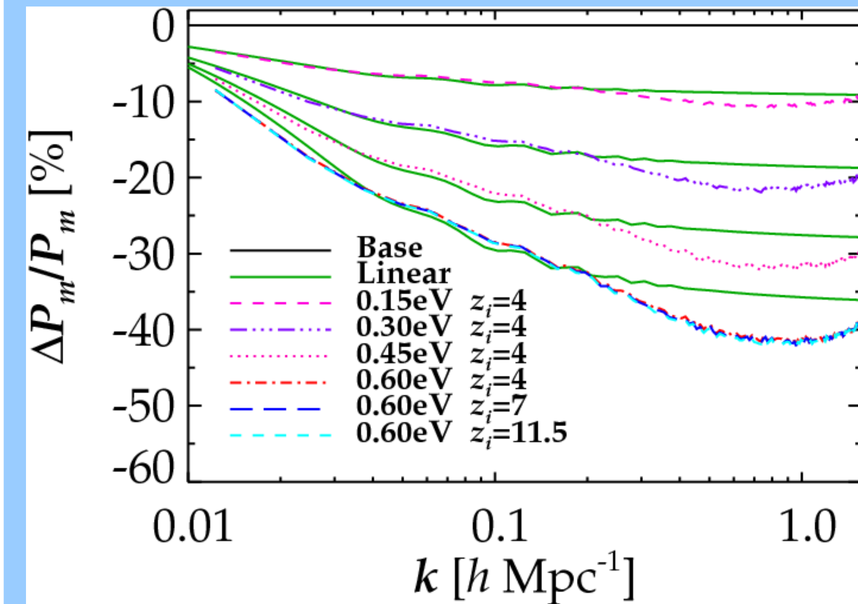




N-body simulations, Brandbyge et al. 2008

Y^3W 2008
also Saito et al. 2008, 2009

Resummation and renormalisation group techniques...

- Many schemes have been proposed that go beyond standard perturbation theory:

  Crocce & Scoccimarro 2006, 2008

  Taruya & Hiramatsu 2007

  McDonald 2007

  Matarresse & Pietroni 2007, 2008

  Matsubara 2008

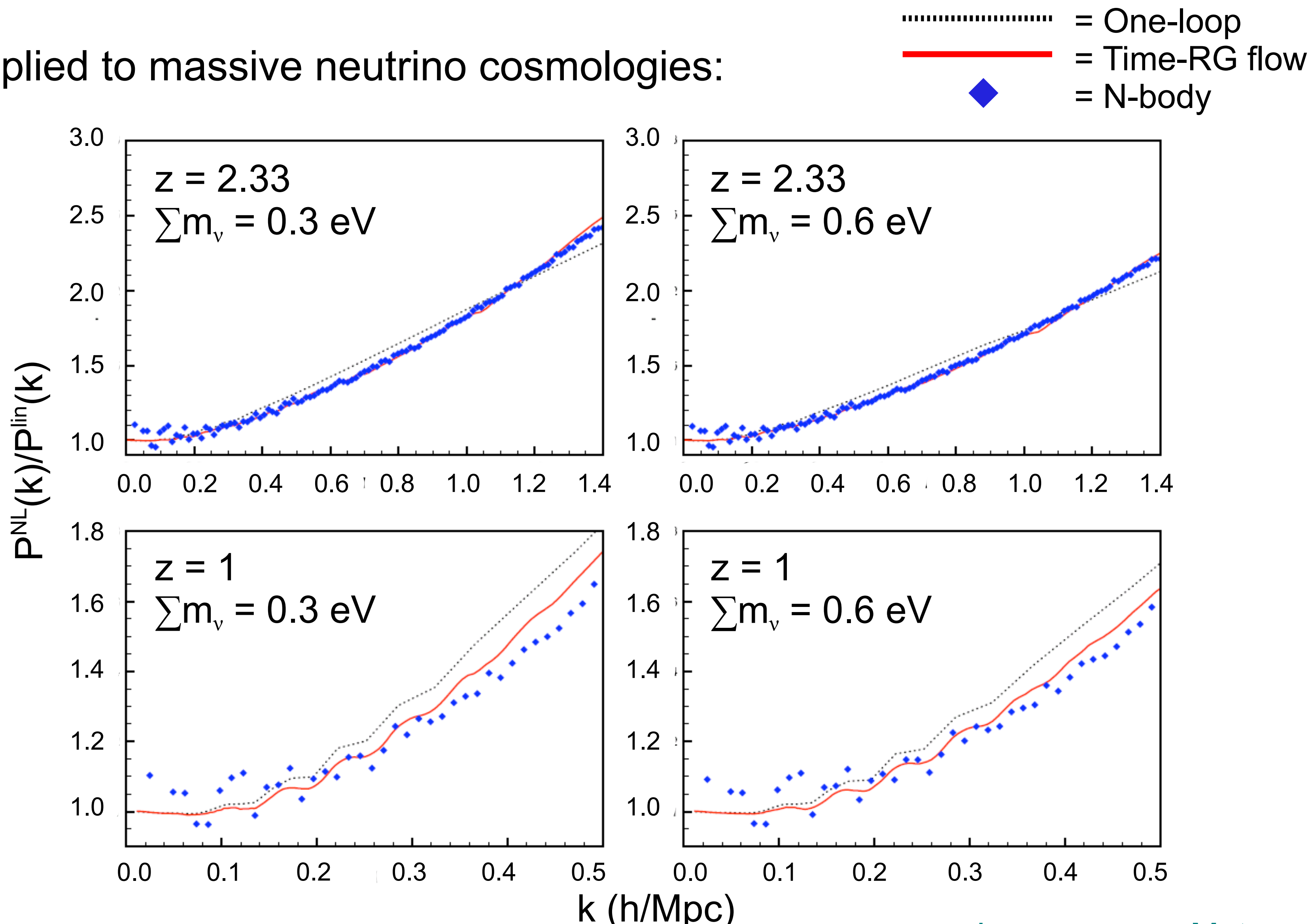
  Valageas 2007

  Pietroni 2008

  Hiramatsu & Taruya 2009

  etc..

- Applied to massive neutrino cosmologies:



Lesgourgues, Matarrese, Pietroni & Riotto 2009

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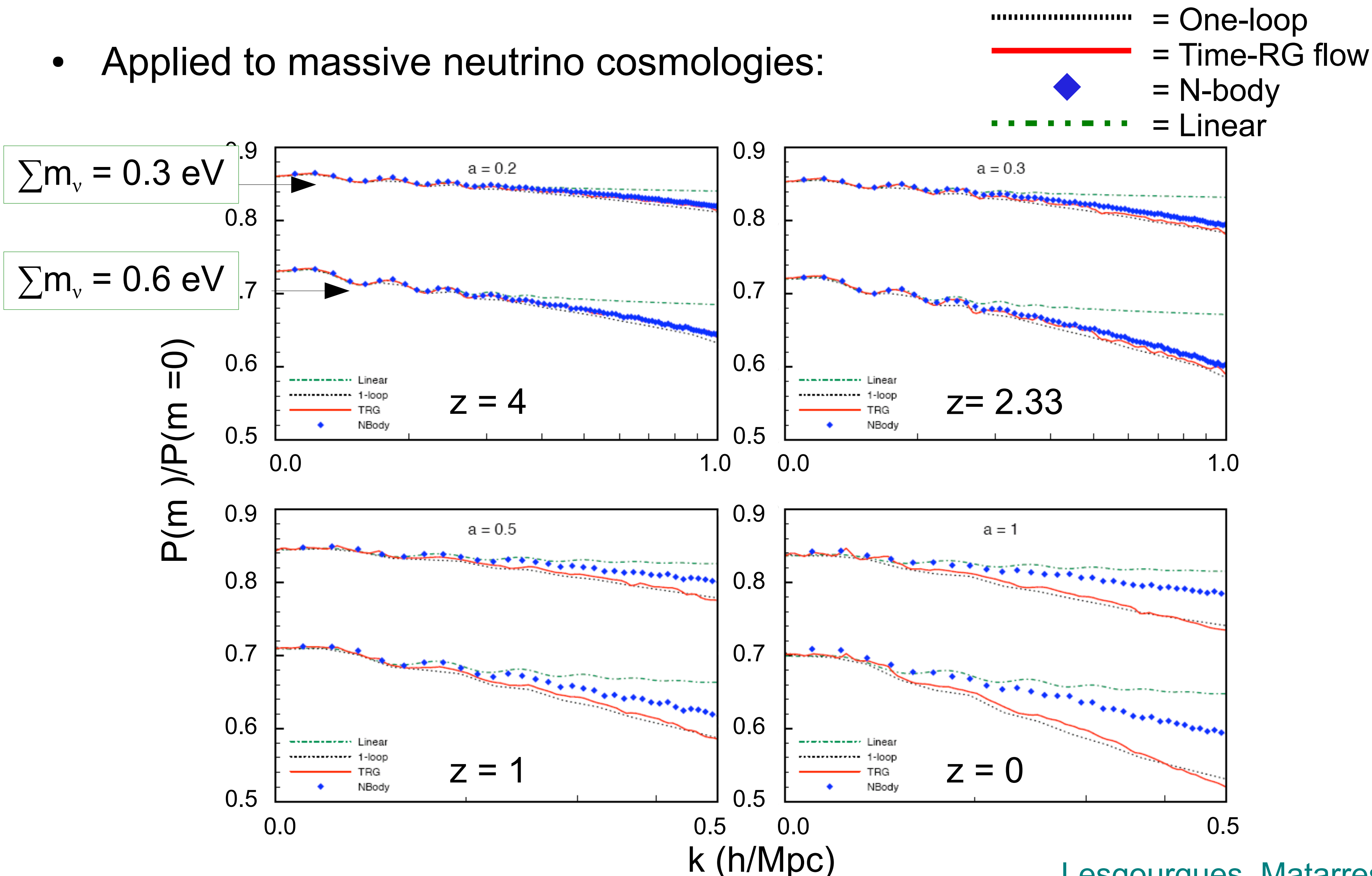

Lesgourgues, Matarrese, Pietroni & Riotto 2009

Summary...

- Using the large-scale structure distribution to probe neutrino physics is still fun.
  - We can do **even better** in the future with forthcoming probes/new techniques.

- But we must make sure our theoretical predictions are reliable (1% accurate) at the (nonlinear) scales of interest.
  - N-body simulations are the definitive way to go.
  - Semi-analytic PT & RG techniques are also of some (limited) use.