(Yes, we can we learn about) Cold Dense Phases of Matter from Holography

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mostly arXiv:1807.09712 and work in progress
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Motivation

- Holography as a means to understand strongly coupled field theories
- Study the characteristics of a particular type of phase: those with spontaneous breaking of the gauge group (CSC) at strong coupling
- Of interest in astrophysical setups: neutron stars

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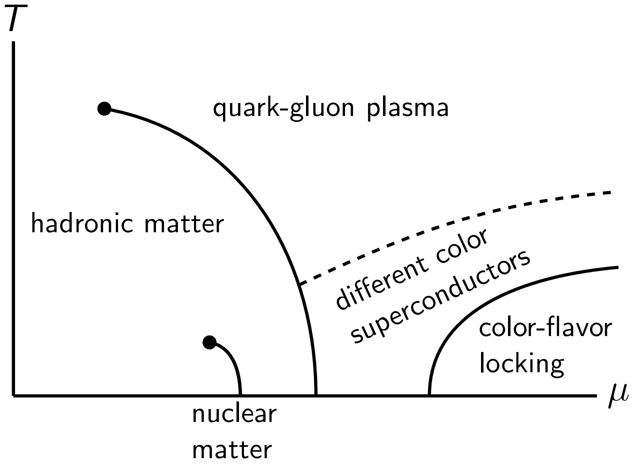
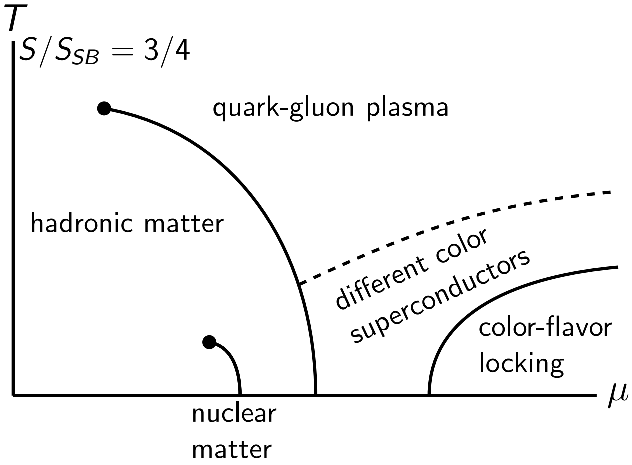
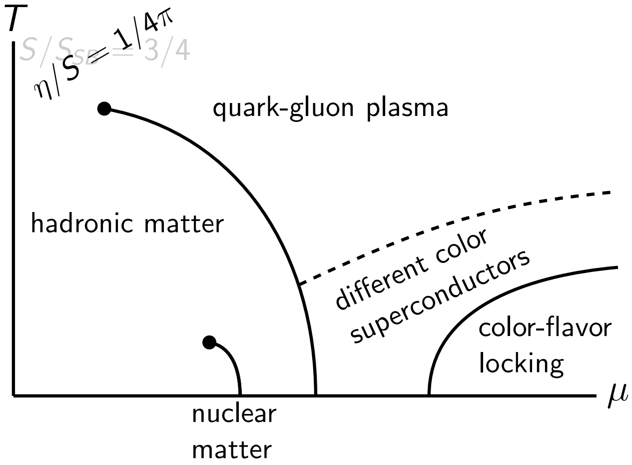
Applications of holography to QCD?



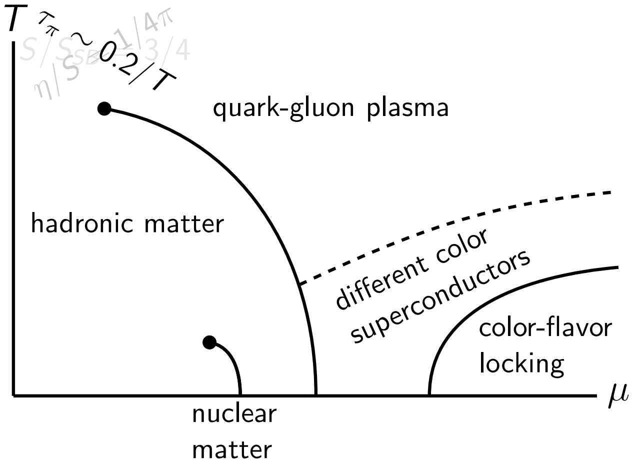
Applications of holography to QCD?



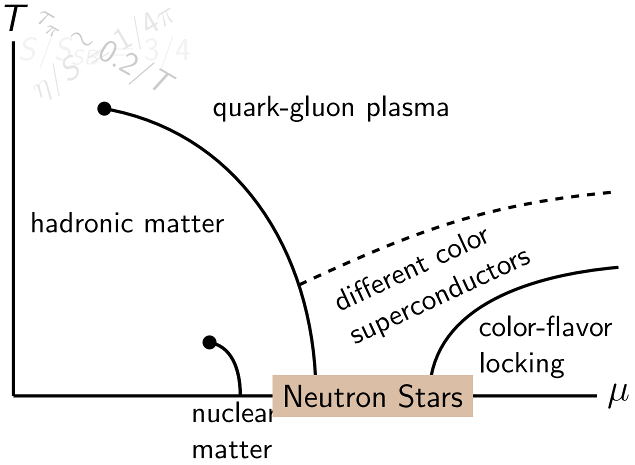
Applications of holography to QCD?



Applications of holography to QCD?



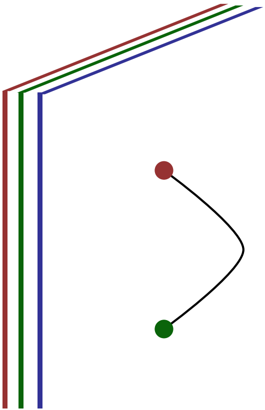
Applications of holography to QCD?



Strings ho!

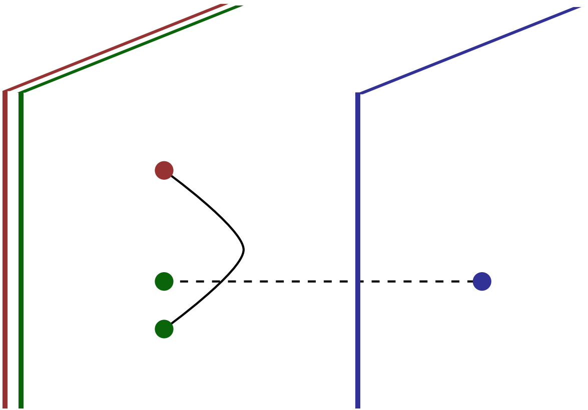
- I describe results from top-down models, where we extremize type IIB SUGRA, DBI and WZ actions
- There will be (horrendous) actions, but will try to keep technical details to a reasonable(?) amount
- Why top-down? To have a clear interpretation of the system under scrutiny

Geometric mechanism of color superconductivity



Coincident D3-branes, SU(N)
Length of strings vanishes (massless gluons)

Geometric mechanism of color superconductivity



Separate one D3-brane, SU(N-1)
Length of string is finite (massive gluons)

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A supersymmetric
CSC
from holography
with isospin

- Zero temperature
- Higgsed phase, ~~$SU(N_c)$~~
- Strong coupling
- Not baryon density!

Setup

- $D3/D7$ system is the dual of $N = 4$ SYM with charged matter in the fundamental (not QCD)

- Work in the probe approximation [hep-th/0205236]

$$N_f \ll N_c$$

- We take $N_f = 2$ D7-branes: non-abelian DBI action and Wess–Zumino [hep-th/9910053]

- The background of the D3-branes is, famously, $AdS_5 \times S^5$

$$\mathrm{d}s^2 = h^{-\frac{1}{2}} \mathrm{d}x^\mu \mathrm{d}x_\mu + h^{\frac{1}{2}} \left[\mathrm{d}r^2 + r^2 w^a w^a + \mathrm{d}(x^8)^2 + \mathrm{d}(x^9)^2 \right]$$

with an RR flux $F \sim N_c \Omega_5 +$ self-dual: rank of gauge group!

- Our brane configuration

	$\mathbb{R}^{1,3}$	$\mathbb{R}^4$	x^8	x^9
D3	$\bullet$			
D7	$\bullet$	$\bullet$	$+$	
D7	$\bullet$	$\bullet$	$-$	
		$SU(2)_L \times SU(2)_R$		

Intersection breaks explicitly $SU(2)_F \times U(1)_B \to U(1)_{iso}$

Action for two probe D7-branes

- The action for probe D7 branes is

$$S \sim -\frac{1}{4\pi^2}\int_{D7} e^{-\phi}\, tr\sqrt{-|G + H^{\frac{1}{2}}(D\,X)^2 + F|} + \frac{1}{8\pi^2}\int_{D7} C_4 \wedge (tr\, F \wedge F)$$

- Where

$$X = X^a \sigma^a \ , \qquad F^a = \mathrm{d}A^a + i\epsilon^{abc} A^b A^c$$

Action for two probe D7-branes

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- We take

$$X = \phi(r)\sigma^3 \;, \qquad A = \phi(r)\mathrm{d}t\,\sigma^3 + a(r) w_a \sigma^a \;,$$

and

$$F_{ij}^{(mag)} = \frac{1}{2}\epsilon_{ijkl}F_{kl}^{(mag)} \qquad D_i D_i \phi = 0 \qquad (\text{indices in } \mathbb{R}^4)$$

[hep-th/9907014]

Action for two probe D7-branes

- The action for probe D7 branes is

$$S \sim -\frac{1}{4\pi^2}\int_{D7} e^{-\phi}\, tr\, \sqrt{-|G + H^{\frac{1}{2}}(D\,X)^2 + F|} + \frac{1}{8\pi^2}\int_{D7} C_4 \wedge (tr\, F \wedge F)$$

- We take

$$X = \phi(r)\sigma^3\ , \qquad A = \phi(r)\mathrm{d}t\,\sigma^3 + a(r)w_a\sigma^a\ ,$$

and non trivial $a(r)$ implies global symmetry breaking

$$U(1)_{iso} \times SU(2)_R \to U(1)_{\mathrm{diagonal}}$$

Instanton configuration and scale of spontaneous symmetry breaking

- Self-duality determines the solution for the isospin function

$$a = -\frac{\Lambda^2}{r^2 + \Lambda^2} \quad \Rightarrow \quad k = \frac{1}{8\pi^2}\, tr\, F \wedge F = 1$$

for Higgsing with $M_q = 0$ [hep-th/0504151, hep-th/0703094]

- Asymptotic behavior of a field relates to the $\Delta = 2$ dual op.

$$a = \text{source}\frac{\log[r]}{r^2} + \frac{\text{vev}}{r^2}$$

Spontaneous condensation of $Q\sigma^a Q^\dagger$!

Quark mass and isospin chemical potential

- The solution for the scalar equation of motion is simply... [hep-th/9907014]

$$\phi = M_q \frac{r^2}{r^2 + \Lambda^2}$$

- Where M_q is the source of the dual operator: mass of scalars AND isospin density; we can read the vev from

$$\phi = \text{source} - \frac{\text{vev}}{r^2} + \cdots$$

to be

$$\text{vev} = \langle u u^\dagger - d d^\dagger \rangle = \langle n_u - n_d \rangle = M_q \Lambda^2$$

Holographic CSC mechanism

- A D3-brane, tautologically, carries D3-brane charge

$$S_{D3} = -N_c \int \sqrt{-g}\, \mathrm{d}t\mathrm{d}^3x + N_c \int_{t,\vec{x}} C_4$$

- D7-branes also carry D3-brane charge!

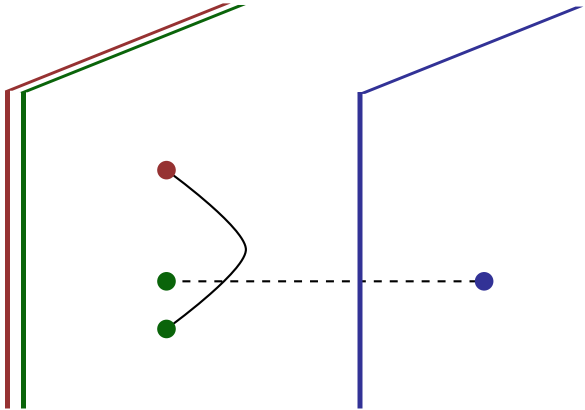
$$S_{D7} \supset \int_{t,\vec{x}} C_4 \wedge \left(\frac{1}{8\pi^2} \int_{\mathbb{R}^4} tr\, F \wedge F \right)$$

- This modifies the flux of F_5 with a radial dependence. One obtains

$$IR: \; N_c \;, \qquad UV: \; N_c + k$$

so branes are pulled out dynamically if $k \neq 0$!

Geometric mechanism of color superconductivity



Massive spectrum

- The solution just presented has a finite isospin density. At zero density the spectrum of fluctuations is

$$\omega^2 - k^2 = 8\pi^2 \frac{M_q{}^2}{\lambda}(n+1)(n+2)$$

- at finite density, charged fields uplift the degeneracy in the spectrum

$$(\omega \pm 2\mu_{iso})^2 - k^2 = 8\pi^2 \frac{M_q{}^2}{\lambda}(n+1)(n+2)$$

- Furthermore, some fluctuations get an offset: strings connecting both D7-branes

$$(\omega \pm 2\mu_{iso})^2 - k^2 = 4M_q{}^2 + 8\pi^2 \frac{M_q{}^2}{\lambda}(n+1)(n+2)$$

[to be published]

Massive spectrum: pseudogoldstone

- When $M_q \ll \Lambda$ there is a pseudogoldstone mode: explicit breaking of scale symmetry with energy scale much lower than the sponatenous breaking scale

$$\omega \simeq 2M_q + \mathcal{O}(k)$$

[to be published]

Goldstone bosons

- Global symmetry broken

$$SU(2)_R \times U(1)_{iso} \to U(1)_{\text{diagonal}}$$

- We find two massless modes with dispersion relation

$$\omega \simeq \pm \frac{1}{2\mu_{iso}} k^2 + \mathcal{O}(k^3)$$

(recall $\mu_{iso} = M_q$)

[to be published]

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A non-susy
normal phase
from holography
w/ baryon density

- Finite temperature
- not a Higgsed phase
- Strong coupling
- No isospin

Setup

- $D3/D7$ system is the dual of $N = 4$ SYM with charged matter in the fundamental (not QCD)

- Work in the Veneziano limit

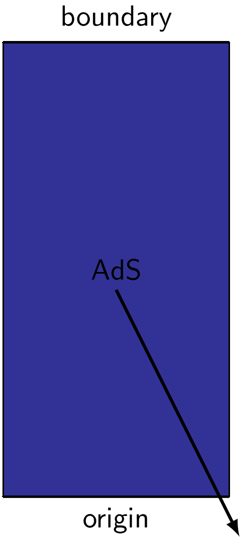
$$N_f/N_c \text{ is finite}$$

- Yet ${N_c}^{1/3} < N_f < N_c$ (and flavors are massless)

Qualitative description of the solution [1101.3560]

$$
\begin{aligned}
L = e^{-2\phi} & \left[R * 1 - \frac{1}{2} H \wedge * H - \frac{1}{2} \mathrm{d}\phi \wedge * \mathrm{d}\phi \right] \\
& - \frac{1}{2} F_1 \wedge * F_1 - \frac{1}{2} F_3 \wedge * F_3 - \frac{1}{4} F_5 \wedge * F_5 \\
& - \frac{1}{2} C_4 \wedge H \wedge F_3 \\
& - \frac{N_f}{N_c} \lambda \, e^{-\phi} \sqrt{-|G + \mathrm{d}A + B|} \wedge \Xi_2 \\
& + \frac{N_f}{N_c} \lambda \, e^{\mathrm{d}A + B} \left[C_8 - C_6 + C_4 - C_2 \right] \wedge \Xi_2
\end{aligned}
$$

Qualitative description of the solution



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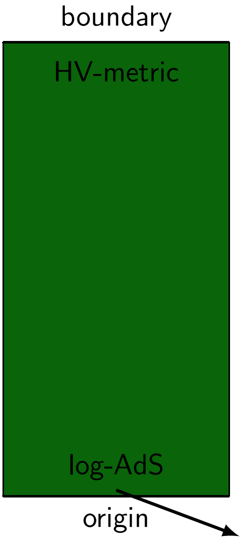
\end{aligned}
$$

$$
\mathrm{d}s^2 = -r^2 \mathrm{d}t^2 + r^2 \mathrm{d}\vec{x}^2 + r^{-2} \mathrm{d}r^2
$$

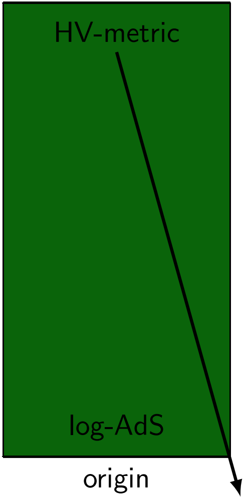
Qualitative description of the solution [hep-th/0612118][1611.05808]



$$L = e^{-2\phi}\left[R * 1 - \frac{1}{2} H \wedge * H - \frac{1}{2}\mathrm{d}\phi \wedge * \mathrm{d}\phi\right]$$
$$- \frac{1}{2} F_1 \wedge * F_1 - \frac{1}{2} F_3 \wedge * F_3 - \frac{1}{4} F_5 \wedge * F_5$$
$$- \frac{1}{2} C_4 \wedge H \wedge F_3$$
$$- \frac{N_f}{N_c}\lambda\, e^{-\phi}\sqrt{-|G + \mathrm{d}A + B|} \wedge \Xi_2$$
$$+ \frac{N_f}{N_c}\lambda\, e^{\mathrm{d}A+B}\left[C_8 - C_6 + C_4 - C_2\right] \wedge \Xi_2$$

$$\mathrm{d}s^2 = \log r^{1/3}\left(-r^2\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2\right) + r^{-2}\mathrm{d}r^2$$

Qualitative description of the solution [1611.05808]



$$L = e^{-2\phi}\left[R * 1 - \frac{1}{2}H \wedge *H - \frac{1}{2}\mathrm{d}\phi \wedge *\mathrm{d}\phi\right]$$
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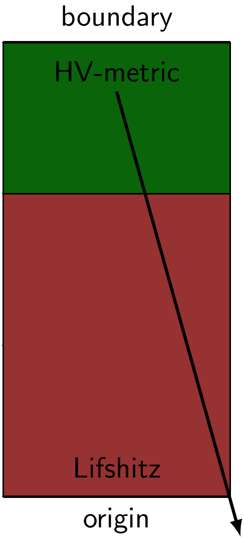
$$\mathrm{d}s^2 = r^{-7/3}\left(-r^2\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + r^{-2}\mathrm{d}r^2\right)$$

Qualitative description of the solution [1707.06989]



$$L = e^{-2\phi}\left[R * 1 - \frac{1}{2} H \wedge * H - \frac{1}{2}\mathrm{d}\phi \wedge * \mathrm{d}\phi\right]$$

$$- \frac{1}{2} F_1 \wedge * F_1 - \frac{1}{2} F_3 \wedge * F_3 - \frac{1}{4} F_5 \wedge * F_5$$

$$- \frac{1}{2} C_4 \wedge H \wedge F_3$$

$$- \frac{N_f}{N_c} \lambda\, e^{-\phi} \sqrt{-|G + \mathrm{d}A + B|} \wedge \Xi_2$$

$$+ \frac{N_f}{N_c} \lambda\, e^{\mathrm{d}A + B} \left[C_8 - C_6 + C_4 - C_2\right] \wedge \Xi_2$$

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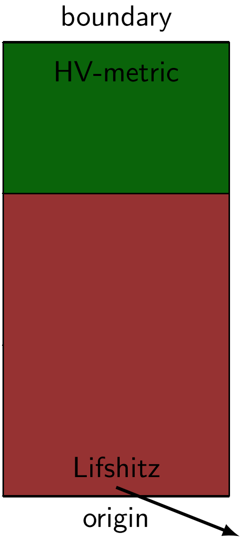
Qualitative description of the solution [1707.06989]



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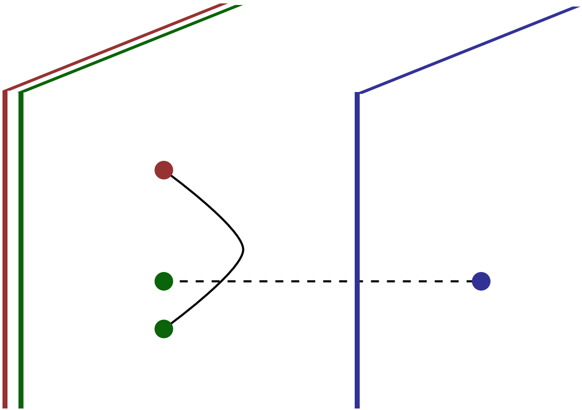
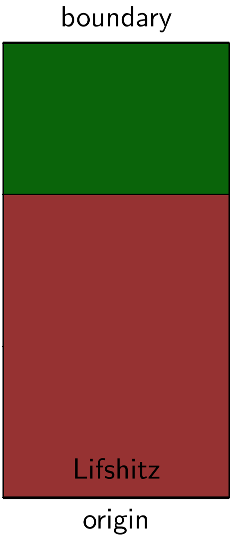
$$\mathrm{d}s^2 = -r^{14}\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + r^{-2}\mathrm{d}r^2$$

Geometric mechanism of color superconductivity



Instability towards color superconduction work in progress



$$L = e^{-2\phi}\left[R * 1 - \frac{1}{2}H \wedge *H - \frac{1}{2}\mathrm{d}\phi \wedge *\mathrm{d}\phi\right]$$

$$- \frac{1}{2}F_1 \wedge *F_1 - \frac{1}{2}F_3 \wedge *F_3 - \frac{1}{4}F_5 \wedge *F_5$$

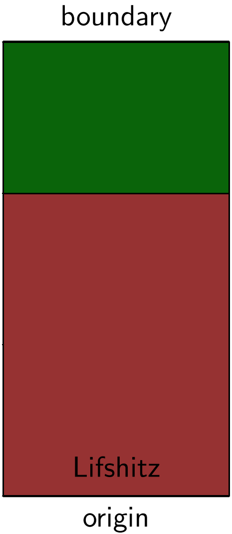
$$-\frac{1}{2}C_4 \wedge H \wedge F_3$$

$$- \frac{N_f}{N_c}\lambda\, e^{-\phi}\sqrt{-|G + \mathrm{d}A + B|}$$

$$+ \frac{N_f}{N_c}\lambda\, e^{\mathrm{d}A + B}\left[C_8 - C_6 + C_4 - C_2\right]$$

$$A = A_t(r)\mathrm{d}t$$

Instability towards color superconduction work in progress



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$$+ \frac{N_f}{N_c} \lambda\, e^{\mathrm{d}A + B} \left[C_8 - C_6 + C_4 - C_2\right]$$

$$A = A_t(r)\mathrm{d}t + \psi(r)\, w^3(\theta s)$$

Instability towards color superconduction work in progress

- The BF bound in Lifshitz is

$$m^2 \geq -\frac{(3+7)^2}{4} = -25$$

- ψ has mass below the BF bound, so it needs to condense to avoid dynamic instability

- Backreaction of the mode in the supergravity fields affects the Gauss law for D3-branes

$$\int_{S^5} F_5 \sim N_c + \frac{1}{2} N_f \int_0^r \psi(r')^2 \mathrm{d}r'$$

the color branes are separated: ~~$SU(N)$~~.

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Summary and conclusions

- Instability in a setup with finite baryon density and low temperatures towards dynamic Higgsing
- Technically difficult to find end-point of instability: work in progress
- One easier way to obtain a Higgsing is to promote to a non-abelian configuration
  - Pros: Only two extra fields, susy solution exists
  - Cons: Non-abelian DBI

Summary and conclusions

- We have constructed such supersymmetric Higgsed phase at finite isospin density
- Spontaneous condensation of $\mathcal{O}^a \sim Q^\dagger \, \sigma^a \, Q$ and geometric realization of CSC
- The low energy spectrum of the theory contains non-relativistic Goldstone modes due to spontaneous breaking of global symmetries

Thank you