Istituto Nazionale di Fisica Nucleare
Sezione di Padova

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di Padova

Constraining Dark Energy parameters with needlets

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Outline

- Integrated Sachs Wolfe effect and cross-correlation;
- Needlets-based estimator;
- ISW detection and Dark energy parameter estimation with needlets;
- Conclusion and future prospects.

Integrated Sachs-Wolfe effect

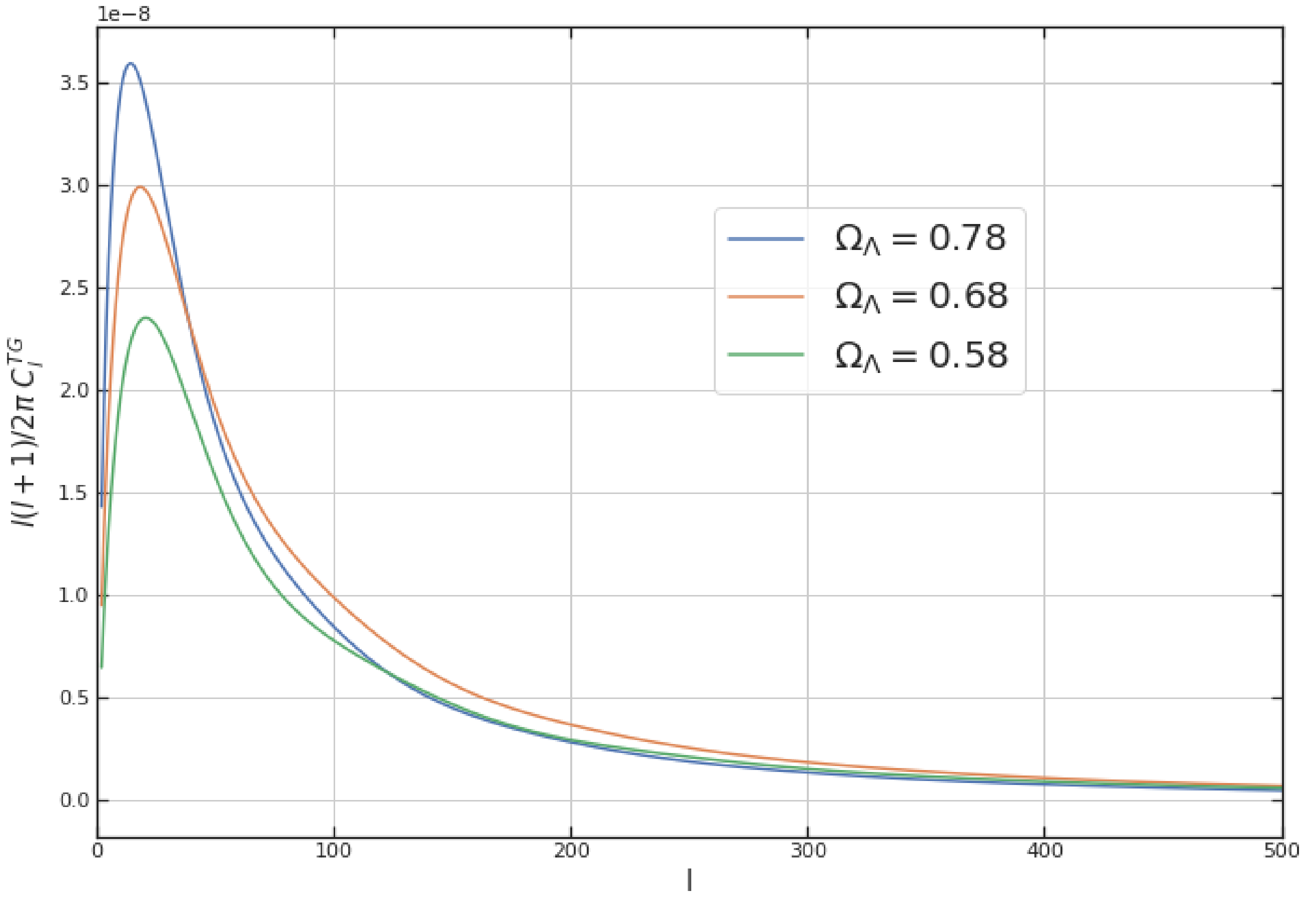
- The ISW is due to the **evolution** of the gravitational potential $\boldsymbol{\Psi}$ after recombination;
- Gravitational potential **decays** between the era of transition (from matter to dark energy dominated universe), giving rise to temperature **anisotropy** in the CMB;
- ISW happens at $z \lesssim 1$ and at extremely large scale $\ell \lesssim 30$;
- It is related to the **Dark Energy** parameters.

Cross correlation with LSS

- At low ℓ, CMB signal is dominated by **cosmic variance**;
- ISW anisotropy is **suppressed** with respect to primary anisotropy (before recombination);
- The most direct way to detect this effect it to **cross-correlate** CMB photons with the Large Scale Structure (LSS) at low ℓ;
- The angular power spectrum in the harmonics space C_ℓ is the most common **observable**.

Cross correlation with LSS

- Cross-correlation angular power spectrum at low ℓ is dependent on **cosmological parameter** $\Omega_\Lambda = 1 - \Omega_\mathrm{m}$ and $w = \frac{p}{\rho}$ of DE;
- Ω_Λ changes the **amplitude** of the power spectrum;

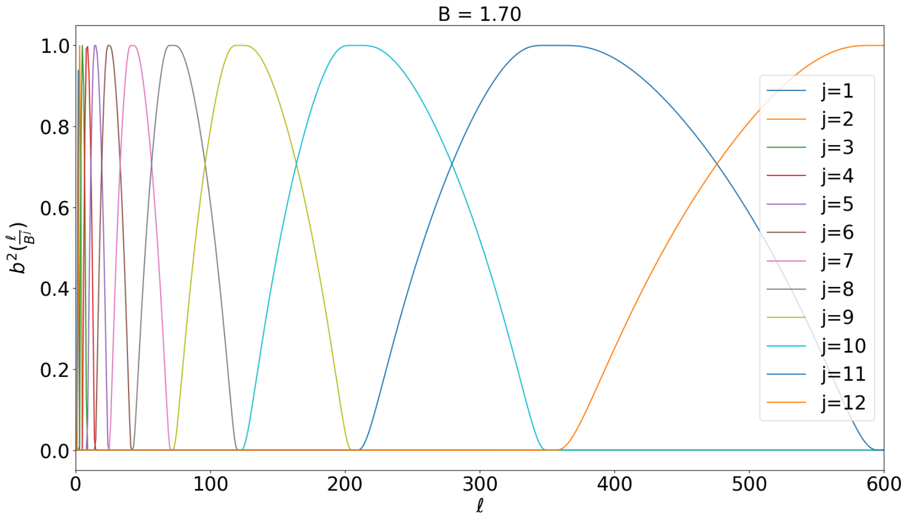


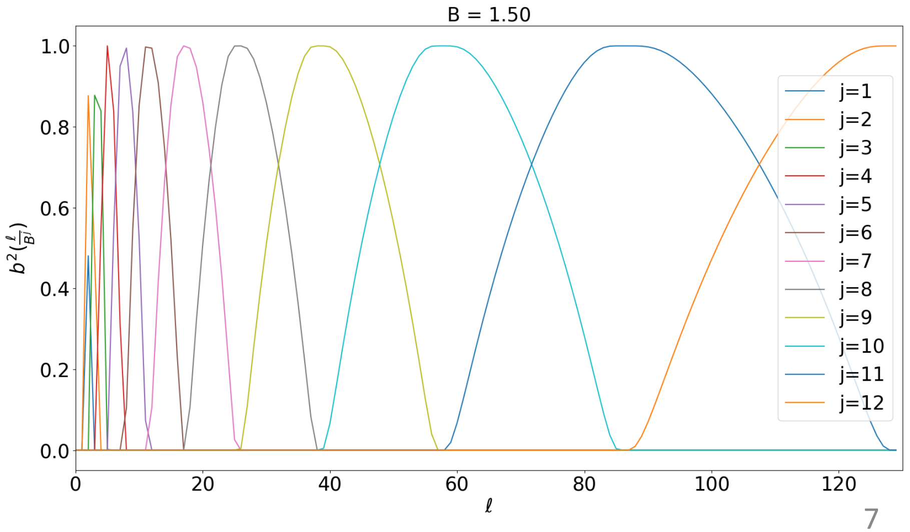
Other estimators?

- Harmonic estimator exploits **orthogonal** properties in harmonic space;
- It possess the **minimal** variance in case of no mask, spatially uniform noise;
- Building new **estimators** to deal with a more **realistic** scenario.

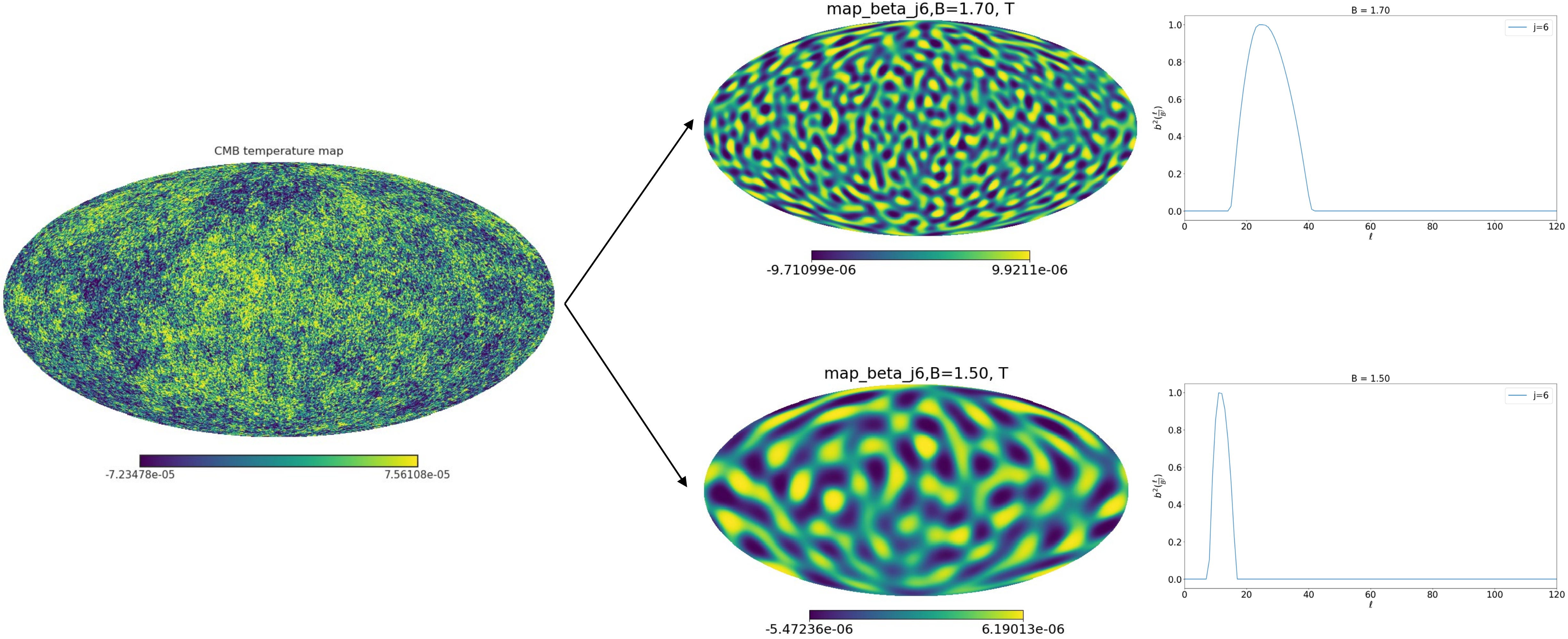
Needlets

- **Needlet**-based **estimator** of the cross-correlation power spectrum between CMB photons and LSS;
- Needlets are a form of **spherical wavelet**;
- Needlets are a **convolution** of spherical **harmonics** and a suitably chosen **window function**;
- Needlets are **localized** both on **real** space and on **harmonic** space;
- Needlets are also used for **testing** the presence of **systematics**.

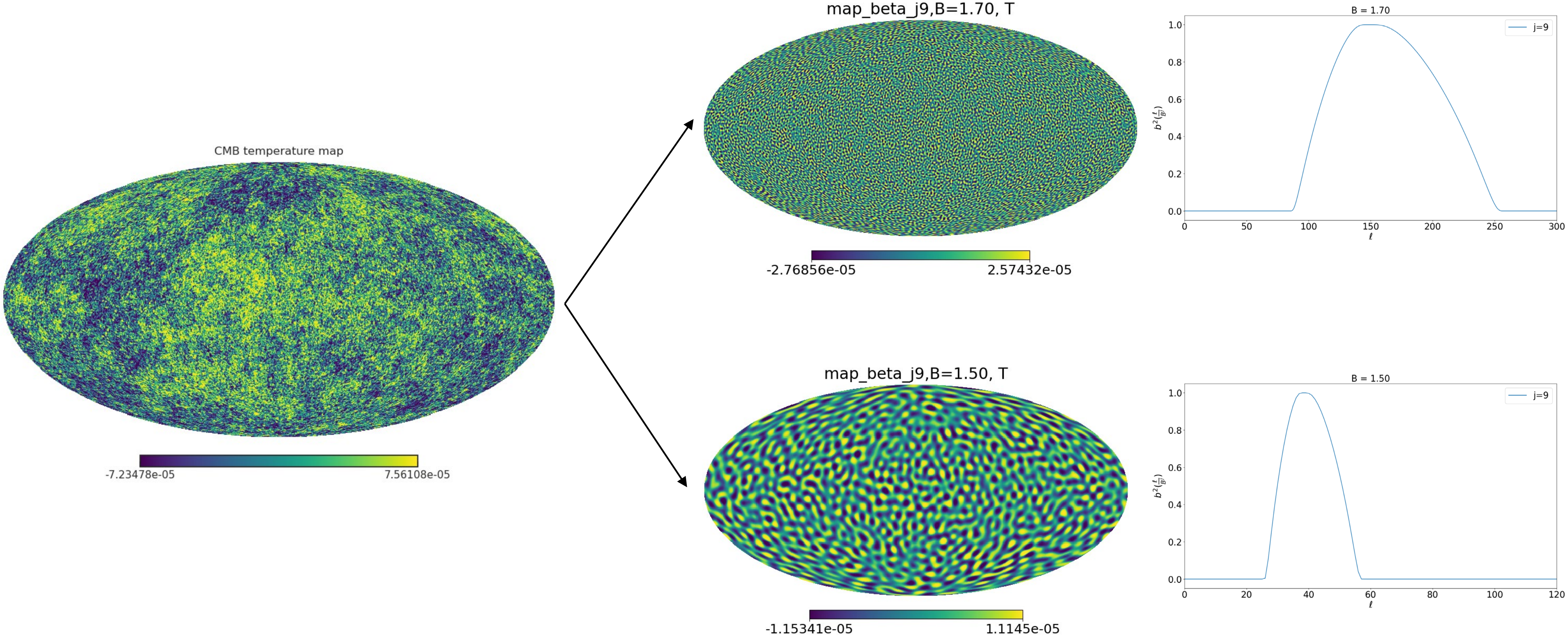




Needlets


CMB temperature map
-7.23478e-05
7.56108e-05
map_beta_j6,B=1.70, T
-9.71099e-06
9.9211e-06
B = 1.70
j=6
map_beta_j6,B=1.50, T
-5.47236e-06
6.19013e-06
B = 1.50
j=6
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
120
ℓ
$b^2(\frac{\ell}{B^j})$

Needlets



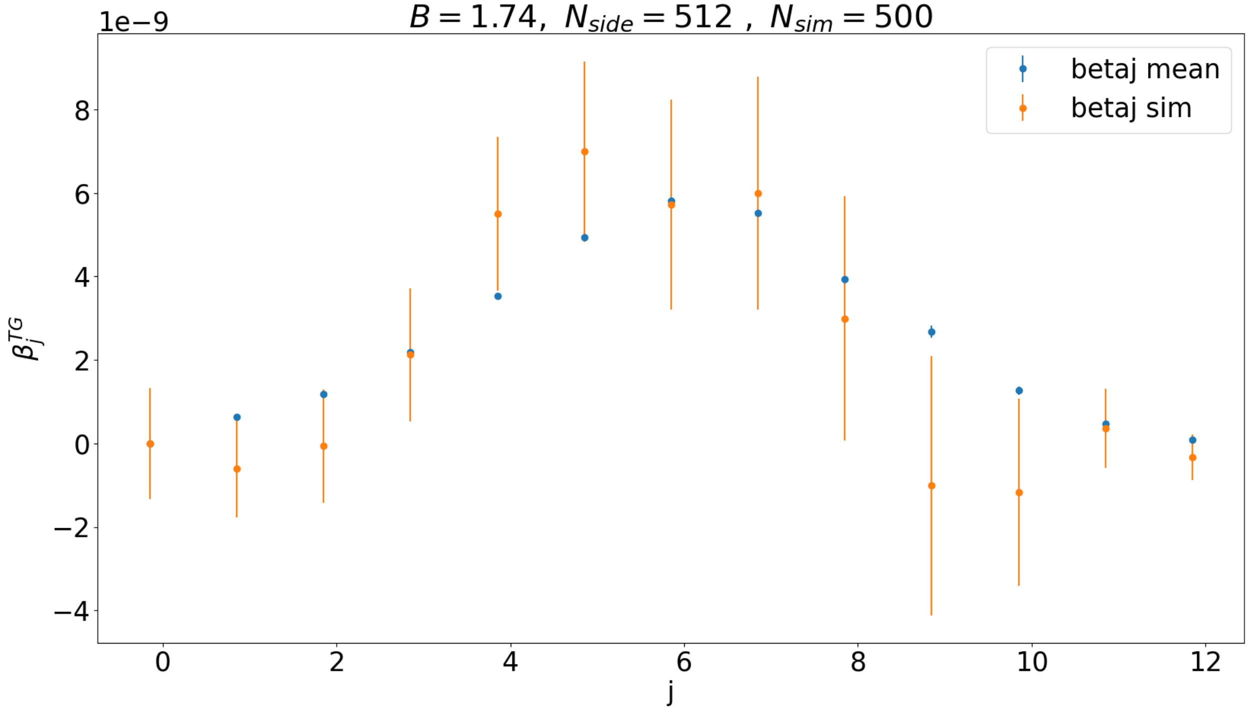
CMB temperature map
-7.23478e-05
7.56108e-05
map_beta_j9,B=1.70, T
-2.76856e-05
2.57432e-05
B = 1.70
j=9
map_beta_j9,B=1.50, T
-1.15341e-05
1.1145e-05
B = 1.50
j=9

Needlets – CMBxG power spectrum

- Validation of the needlet estimator:
  - Calculation of the **auto-** and **cross-** angular power spectra for CMB **temperature** and density of **galaxies** from CAMB;
  - **Simulation** of Healpix **maps** for the correlated T and G from the $C_\ell' s$, in the case of **full-sky** and **zero noise**;
  - Extraction of the needlets **coefficients** $\boldsymbol{\beta_j}$ for the cross-correlation power spectrum from maps;
  - Computation of the **covariance** matrix from simulations;

Needlets - ISW

- From the analysis of these coefficients, it is possible to **predict** the **detectability** of ISW;
- As test of significance, we compute the **chi square** between $\boldsymbol{\beta_j}$ of a single realization and the mean of all the realizations;
- We found $\boldsymbol{\chi^2 = 15.6}$, so for 12 d.o.f. we can exclude the null hypothesis (no ISW) with **79%** confidence.

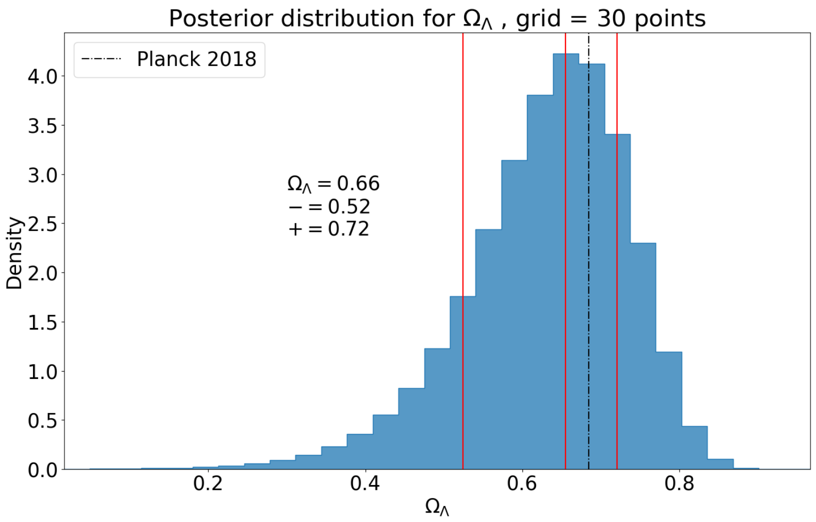


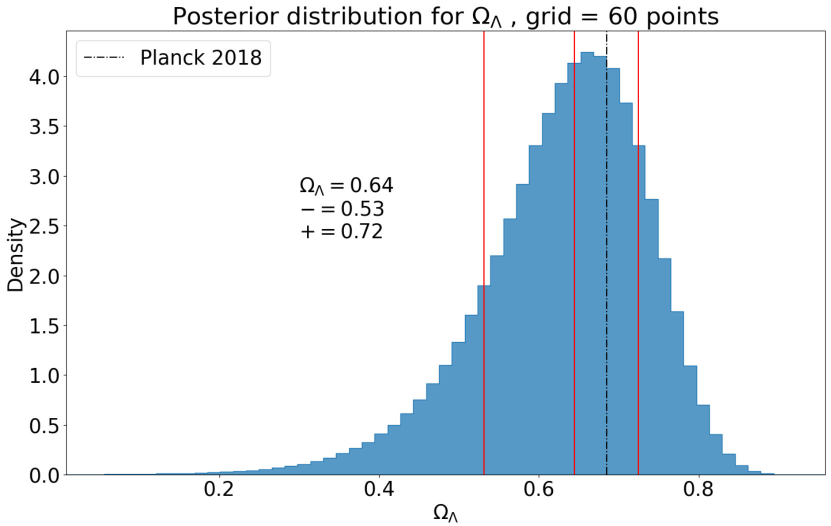
Needlets – Dark Energy parameter estimation

- We produced a **grid** of spectra for different values of $\boldsymbol{\Omega_\Lambda}$;
- In each point we evaluated a Gaussian **likelihood** for a single realization, with mean equal to β_j in that point and covariance from the simulations;
- We **sampled** from the normalized likelihood in order to find the **posterior** distribution for Ω_Λ.
- We calculated the **median** of the distribution and confidence interval at **68%** confidence level.

Needlets – Dark Energy parameter estimation

- We repeated that procedure for $\boldsymbol{\beta_j}$ from different realizations and for 30 and 60 values of Ω_Λ;
- We compared our result with the **Planck** best-fit value taken as fiducial;
- We found that the estimator is **unbiased**;





Conclusion and future prospective

- We found that it is possible to **detect** the ISW from a needlets-based estimator of the cross-correlation power spectrum between CMB temperature and galaxies at low ℓ;
- We also demonstrate that it is possible to **constrain** Dark Energy parameters from the β_j spectrum;
- Verification with **masked** sky and catalogues of galaxies in order to forecast some results for the **Euclid** survey.
- Comparison with other **estimators.**

Thank you for the attention!

Backup

Late ISW

- The late ISW occurs at late times, $z \lesssim 1$
- It is restricted to extremely large scale $l \lesssim 30$
- It depends on the cosmological parameters related to the Dark Energy
- The most direct way to detect this effect it to cross correlate CMB photons with the Large Scale Structure (LSS) at low redshifts.
- Galaxies and clusters are tracers of the dark matter overdensity δ_m related to the evolution of the gravitational potential Ψ trough the Poisson equation:

$$\nabla^2\Psi = 4\pi G a^2 \rho_m \delta_m(\vec{x}, a)$$

Integrated Sachs-Wolfe effect

$$\Theta_\ell^{ISW} = -2 \int_0^{\eta_0} d\eta \, e^{-\tau} \psi'(k,\eta) j_\ell[k(\eta_0 - \eta)]$$

- The ISW is due to the evolution of the gravitational potential Ψ after recombination;
- The ISW at late time is due to the decay of gravitational potential. This gives rise to temperature anisotropy in the CMB spectrum during the era of transition between a matter dominated universe to a dark energy dominated one;
- ISW happens at $z \lesssim 1$ and it is restricted to extremely large scale $l \lesssim 30$;
- It depends on the cosmological parameters related to the Dark Energy.

Cross correlation with LSS

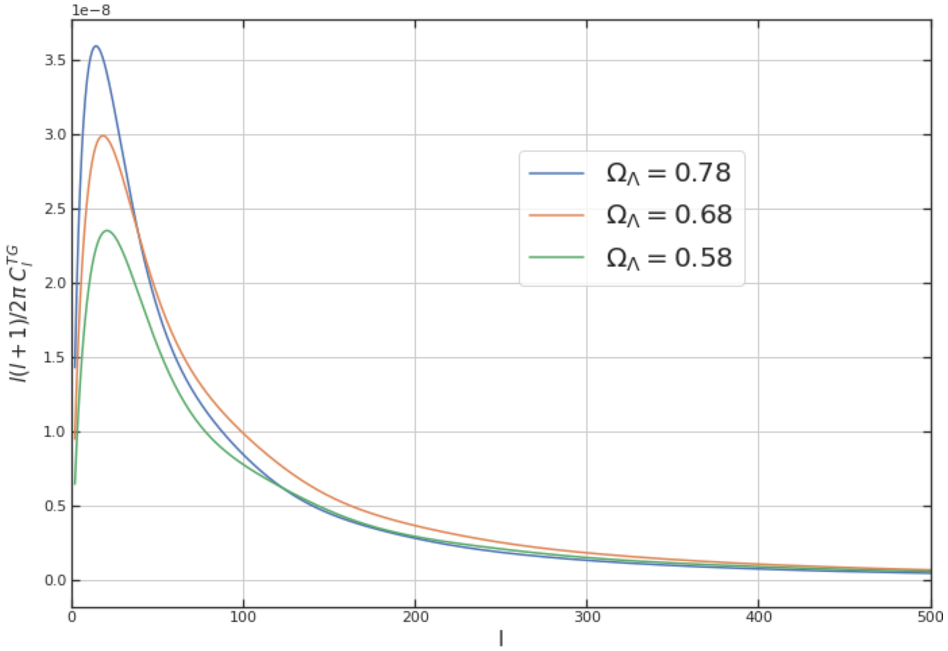
- The most direct way to detect this effect it to cross correlate CMB photons with the Large Scale Structure (LSS) at low. The cross-correlation power spectrum C_ℓ^{TG} is:

$$C_\ell = \frac{2}{\pi}\int_0^\infty dk k^2 P_M(k) \int_0^\infty dz\, W^{ISW}(z) \int_0^\infty dz'\, W^g(z')$$

- $P_M(k)$ is the power spectrum of the matter overdensity today;
- $W^g(z)$ is the distribution of galaxies/clusters in a redshift interval;
- $W^{ISW}(z)$ is the contribution from the CMB temperature anisotropies due to the ISW:

$$W^{ISW}(z) = -3\frac{\Omega_m H_0^2}{k}\frac{1}{D_+(a)}e^{-\tau}\frac{d}{dz}\left[\frac{D_+(a)}{a(z)}\right] j_\ell(k\chi(z))$$

- It depends on the cosmological parameters of the Dark Energy.



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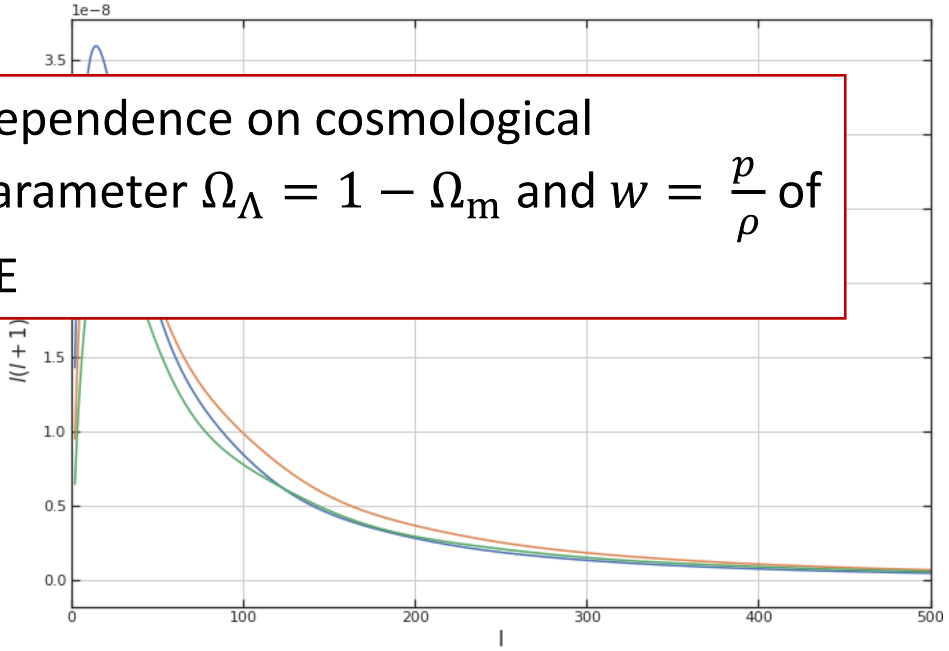
Dependence on cosmological parameter $\Omega_\Lambda = 1 - \Omega_\mathrm{m}$ and $w = \frac{p}{\rho}$ of DE

- It depends on the cosmological parameters of the Dark Energy.



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Ω_Λ changes the amplitude of the power spectrum

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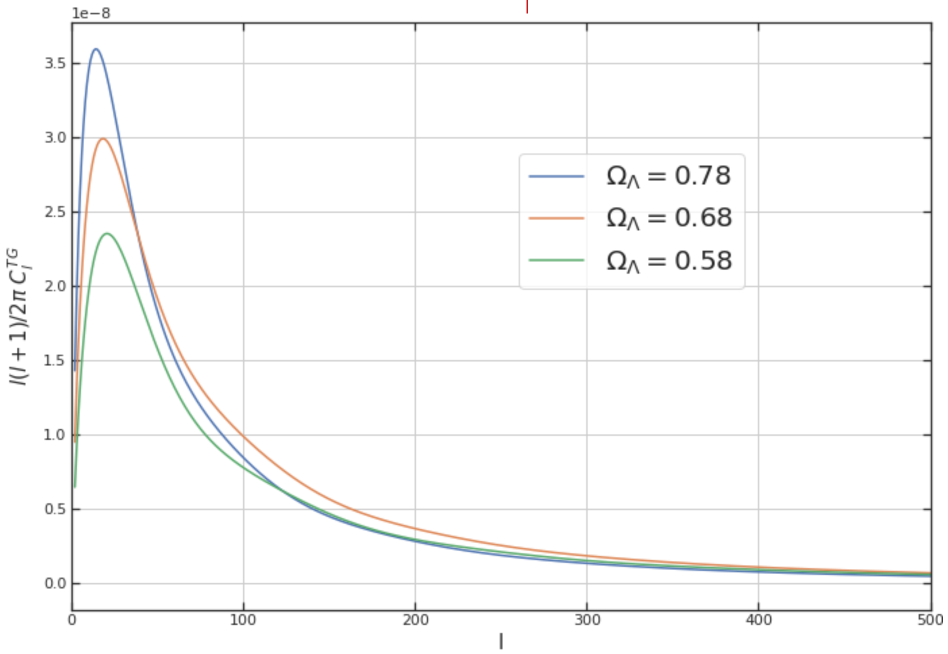


Research activity - Needlets

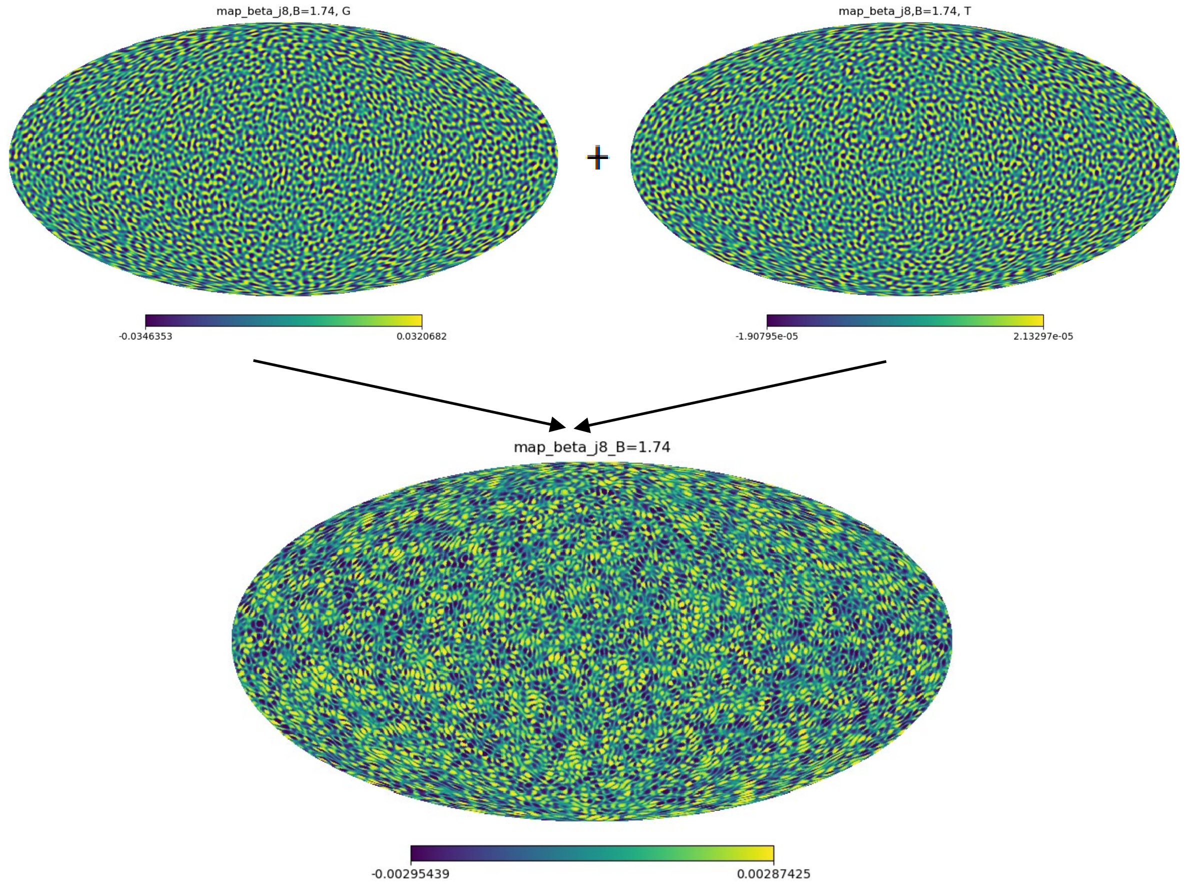
- I am currently working on a needlet-based estimator of the cross-correlation power spectrum between CMB photons and LSS;
- Needlets are a form of spherical wavelet widely study for data analysis of power spectra. Wavelets are useful to handle with rapidly changeable signals; $\psi_{jk}(\hat{\mathrm{n}}) = \sqrt{\lambda_{jk}} \sum_{\ell=[B^{j-1}]}^{[B^{j+1}]} b\left(\frac{\ell}{B^j}\right) \sum_{m=-\ell}^{\ell} Y^*_{\ell m}(\hat{\mathrm{n}}) Y_{\ell m}(\xi_{jk})$
- The parameter B defines the localization in the harmonic space, since $\ell \in [B^{j-1}, B^{j+1}]$, and in the real spaces;
- Needlets-based estimator: $\beta_{jk} = \sqrt{\lambda_{jk}} \sum_{\ell=[B^{j-1}]}^{[B^{j+1}]} b\left(\frac{\ell}{B^j}\right) \sum_{m=-\ell}^{\ell} x_{\ell m} Y_{\ell m}(\xi_{jk})$ $\qquad \hat{\beta}_j^{XY} = \frac{1}{N_{\mathrm{pix}}} \sum_k \beta_{jk}^X \beta_{jk}^Y$
- Mean and variance:

$$\langle \hat{\beta}_j^{XY} \rangle \equiv \beta_j^{XY} = \sum_\ell \frac{2\ell+1}{4\pi} b^2\left(\frac{\ell}{B^j}\right) C_\ell^{XY} \qquad (\Delta\beta_j^{XY})^2 \equiv \mathrm{Var}[\hat{\beta}_j^{XY}] = \sum_\ell \frac{2\ell+1}{16\pi^2} b^4\left(\frac{\ell}{B^j}\right) [(C_\ell^{XY})^2 + C_\ell^{XX} C_\ell^{YY}]$$

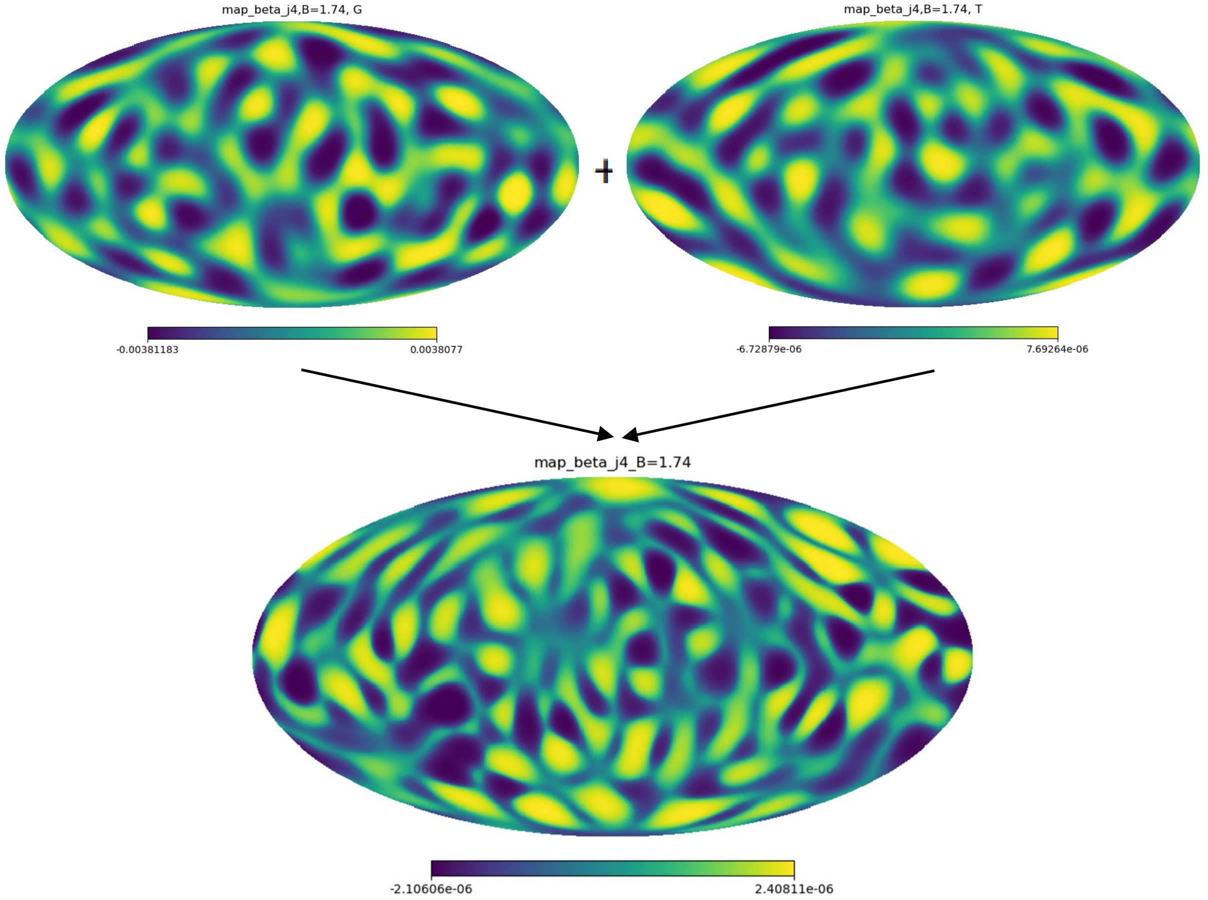
- Covariance: $\mathrm{Cov}_{jj'} \equiv \mathrm{Cov}[\hat{\beta}_j, \hat{\beta}_{j'}] = \langle (\hat{\beta}_j - \langle \hat{\beta}_j \rangle_{\mathrm{MC}})(\hat{\beta}_{j'} - \langle \hat{\beta}_{j'} \rangle_{\mathrm{MC}}) \rangle_{\mathrm{MC}}$

Research activity – Needlets (real space)

B = 1.74 , j = 8

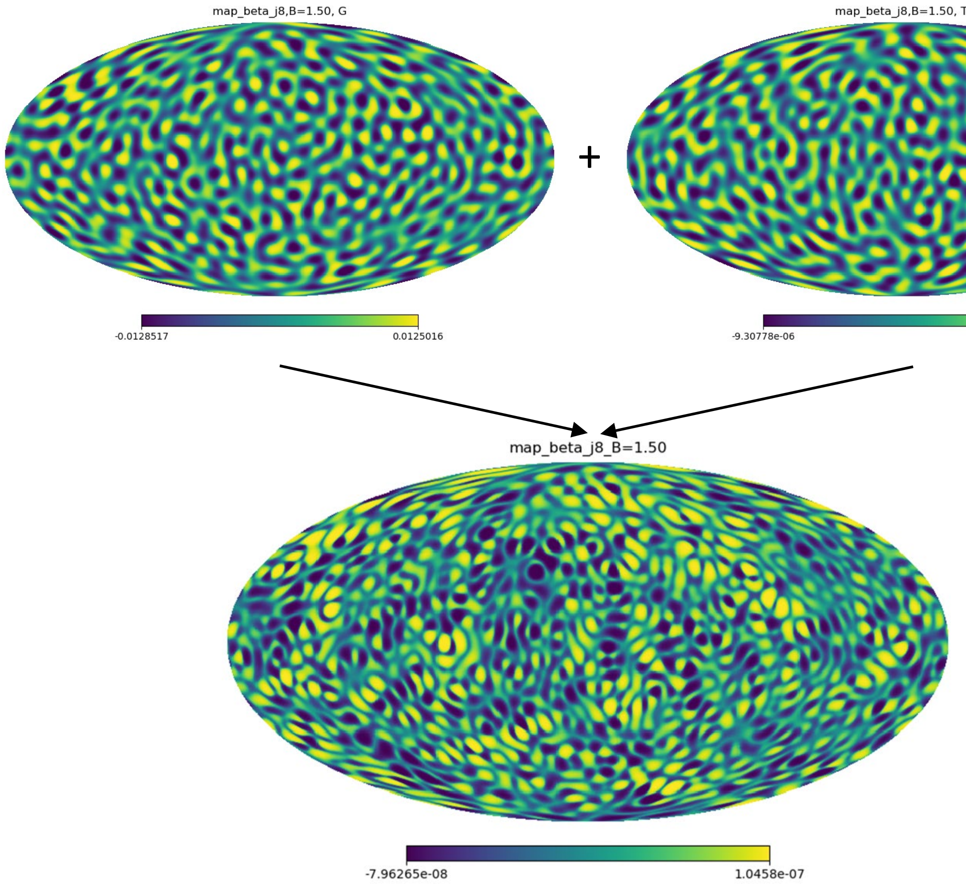


B = 1.74, j = 4

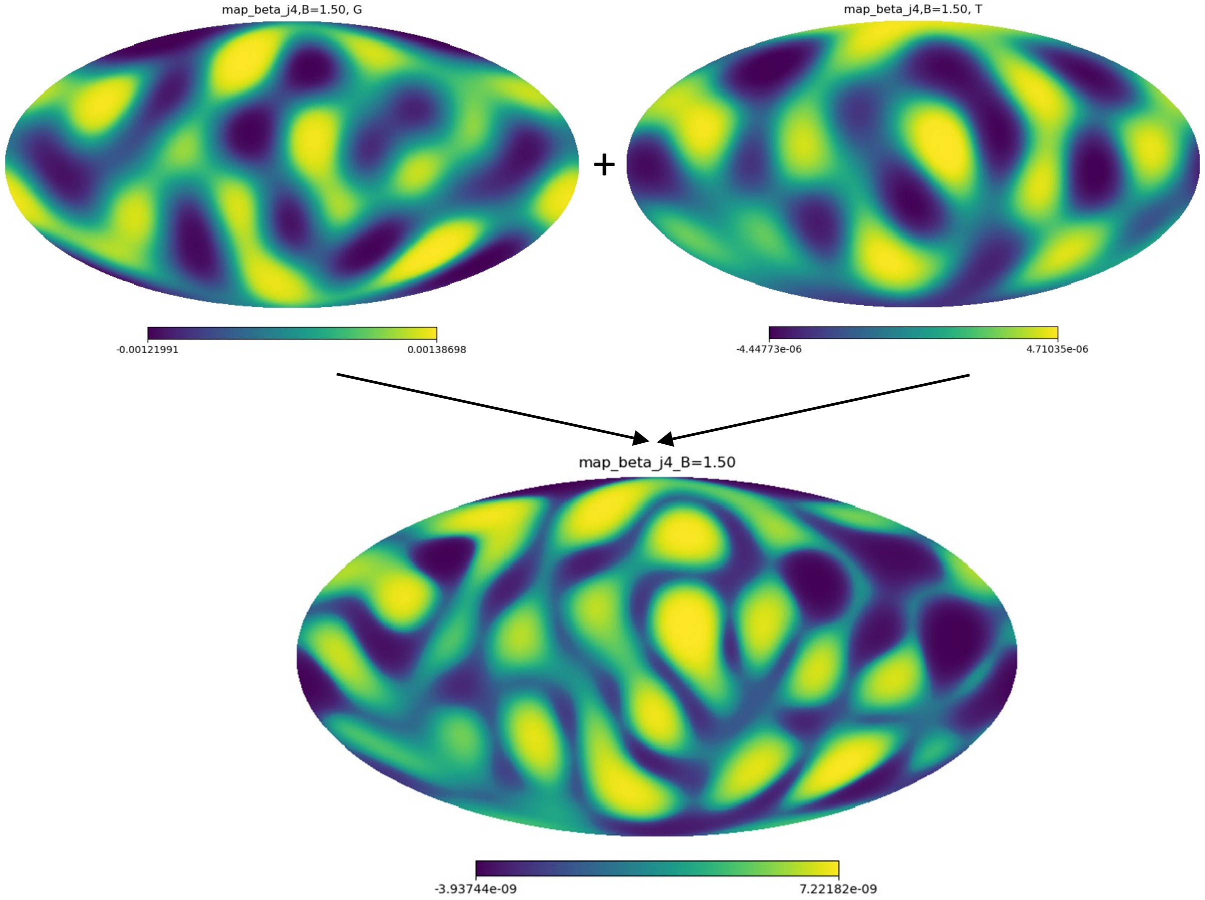


Research activity – Needlets (real space)

B = 1.5 , j = 8



B = 1.5, j = 4



Needlets – Dark Energy parameter estimation

- We repeated that procedure for $\boldsymbol{\beta_j}$ from different realizations and for 30 and 60 values of Ω_Λ;
- We compared our result with the **Planck** best-fit value taken as fiducial;
- We found that the estimator is **unbiased**;
- We found that the **accuracy** on the constrains on the parameter **improves** as the points of the grid are increasing.