

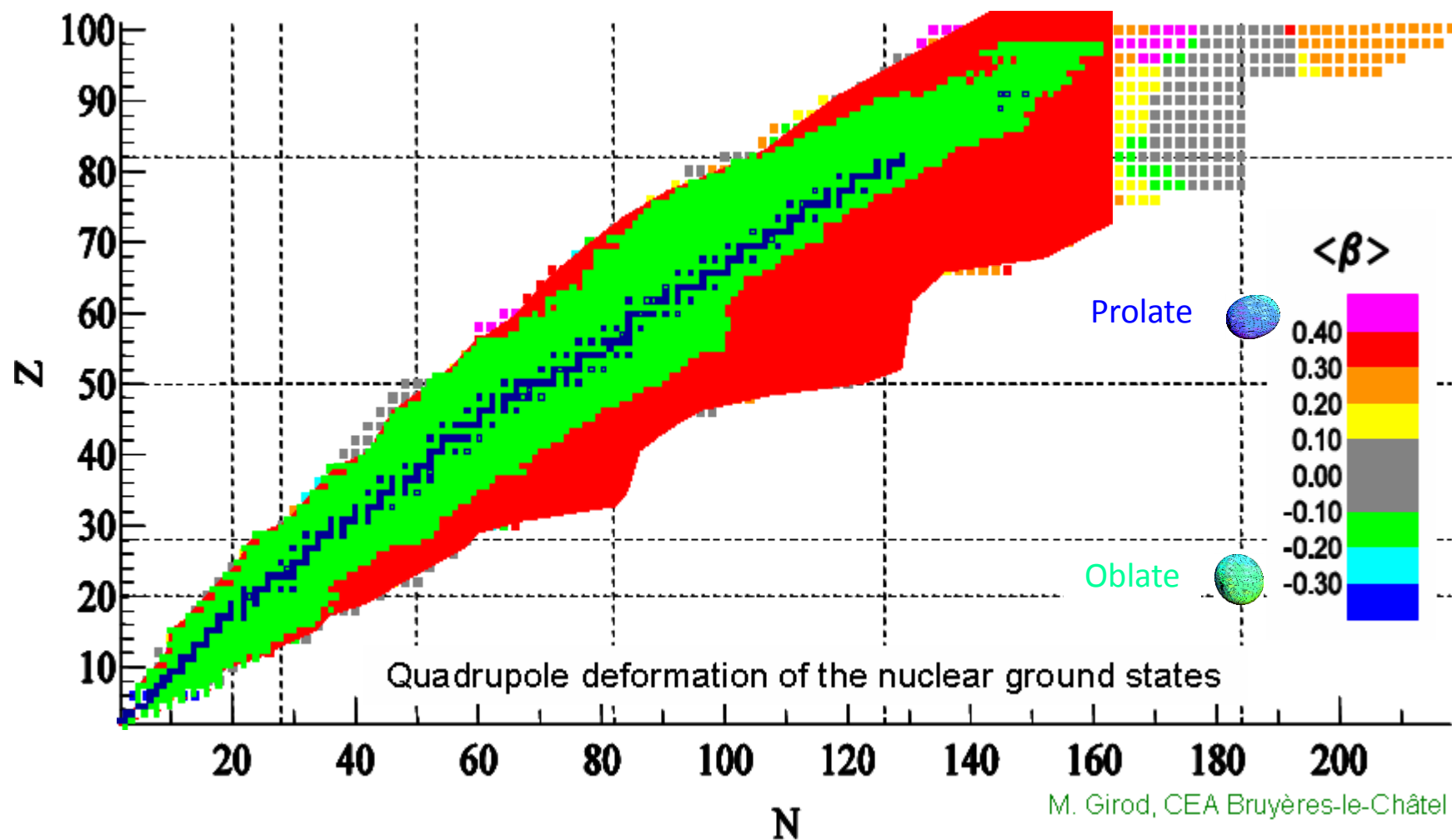
Nuclear Shapes

Lectures I and II

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Selected Topics in Nuclear and Atomic Physics
Fiera di Primiero
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Why study nuclear shapes ?



fundamental property of the nucleus
➤ sensitive to the nuclear structure

benchmarks for nuclear theory
➤ shape evolution with N , Z , energy, spin
➤ shape coexistence
➤ evolution of shell structure far from stability

Program

Lecture 1:

- quadrupole moments
- low-energy Coulomb excitation
- lifetime measurements
- neutron-deficient Kr isotopes

Lecture 2:

- E0 transitions
- shape mixing and coexistence
- neutron-deficient Se isotopes
- intermediate-energy Coulomb excitation

Lecture 3:

- neutron-rich Sr and Zr isotopes
- lifetime measurements in fission fragments
- odd-mass nuclei
- triaxiality around ^{110}Ru and ^{140}Sm

Lecture 4:

- triaxial superdeformation
- wobbling motion
- Jacobi shape transition
- extreme deformation

How can we describe the shape of a nucleus?

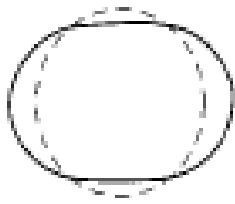
expansion in spherical harmonics:

$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t) Y_{\lambda\mu}(\vartheta, \varphi) \right]$$

a_{00} describes the nuclear volume

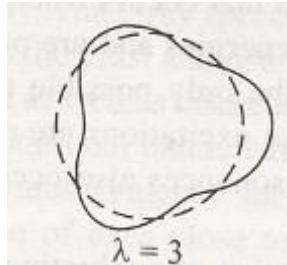
$\lambda = 1$: translation of the whole system

quadrupole



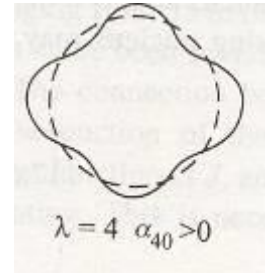
$\lambda = 2$

octupole



$\lambda = 3$

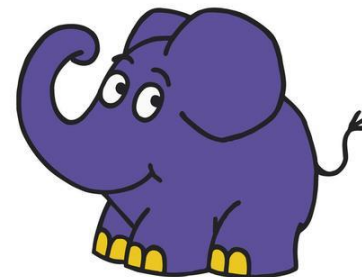
hexadecapole



$\lambda = 4 \quad \alpha_{40} > 0$

most important shape

sufficiently large λ :



Quadrupole shapes

$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t) Y_{\lambda\mu}(\vartheta, \varphi) \right]$$

for $\lambda = 2$: five parameters $a_{2\mu}$

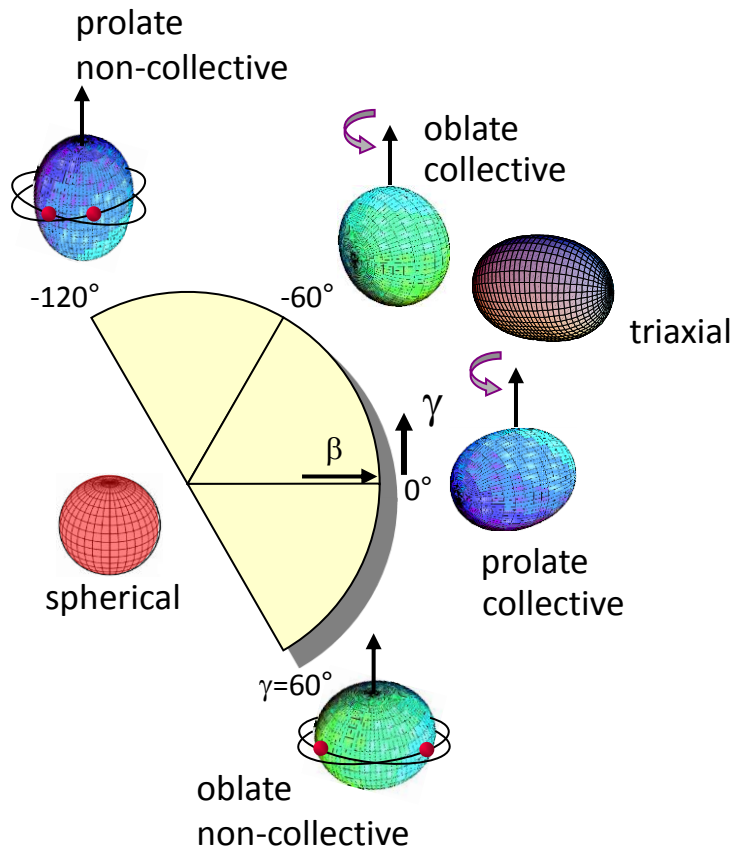
- only two parameters describe the shape
- three describe the orientation in space (Euler angles)

with a rotation we can achieve

$$a_{21} = a_{2,-1} = 0$$

$$a_{22} = a_{2,-2}$$

leaving two independent parameters a_{20} and a_{22}



$$a_{20} = \beta \cos \gamma$$

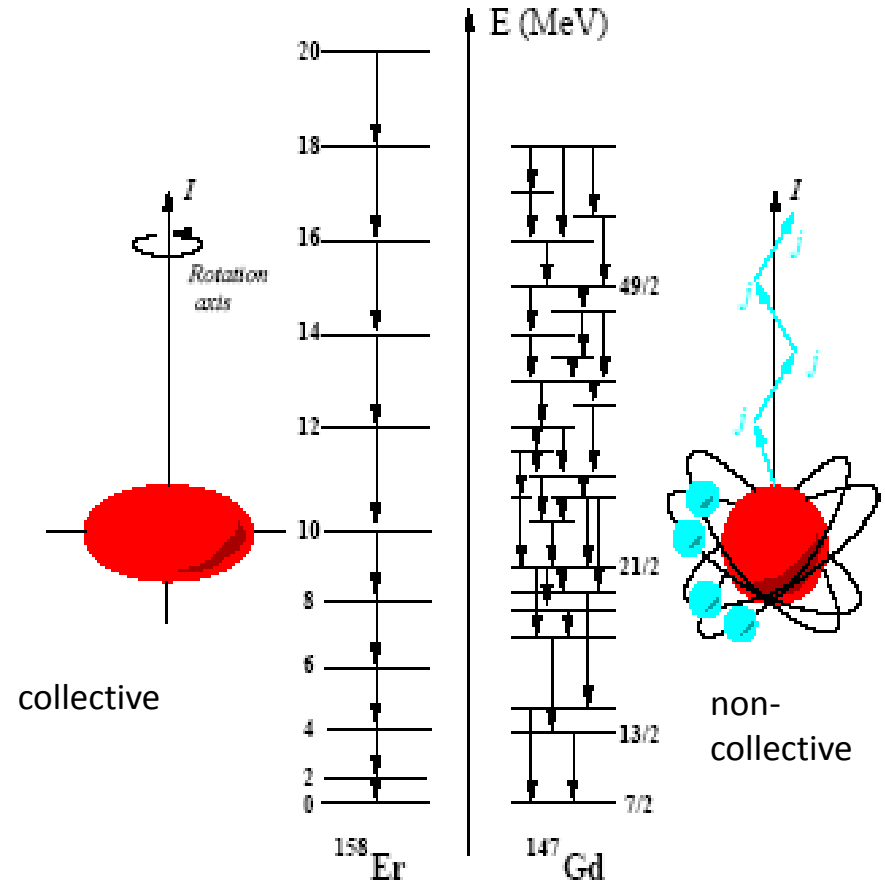
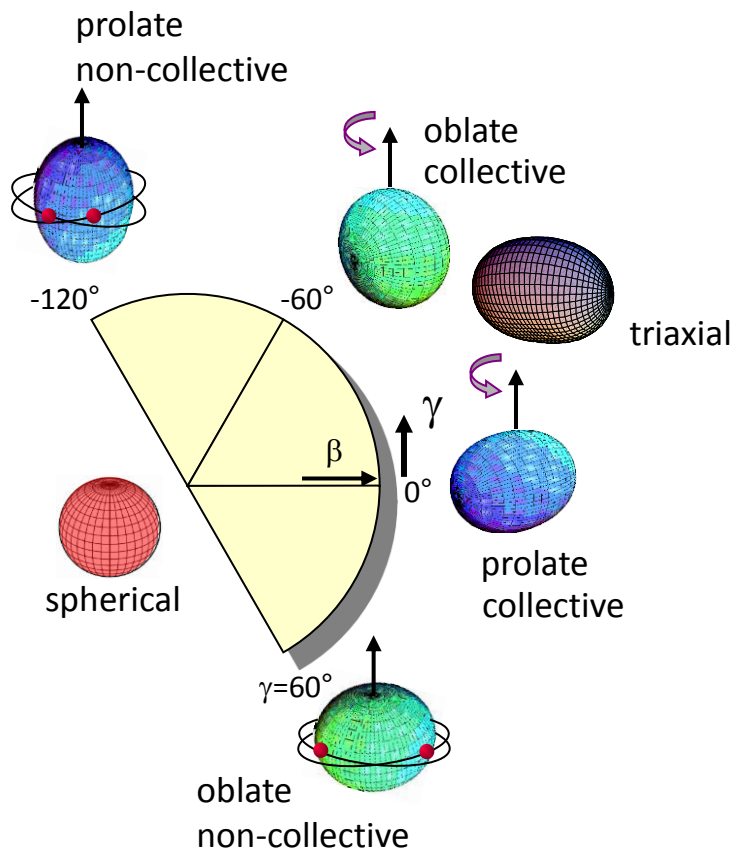
$$a_{22} = \frac{1}{\sqrt{2}} \beta \sin \gamma$$

β axial deformation
 γ triaxial deformation

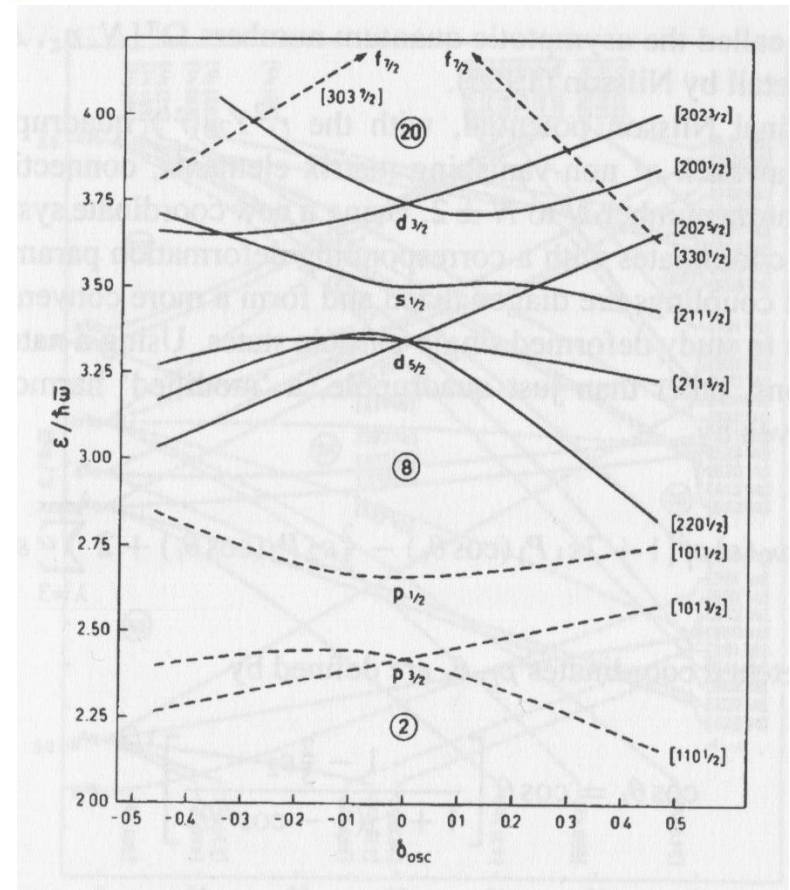
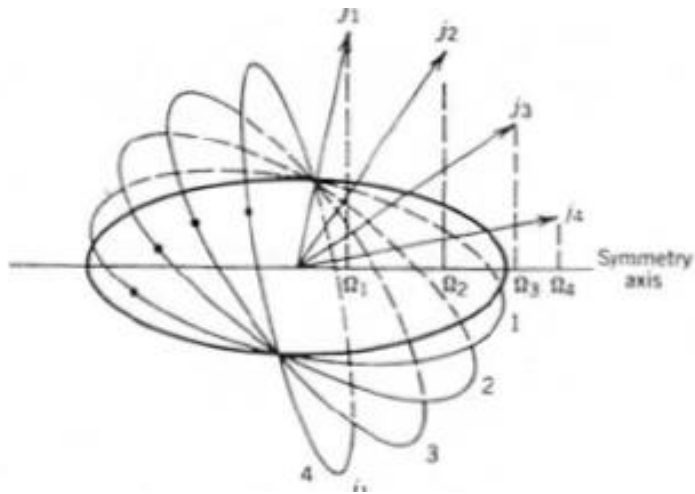
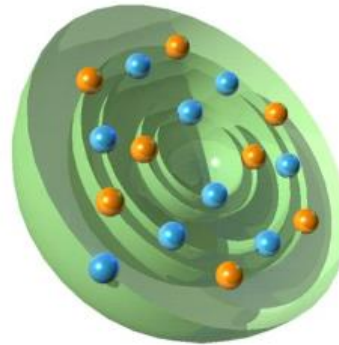
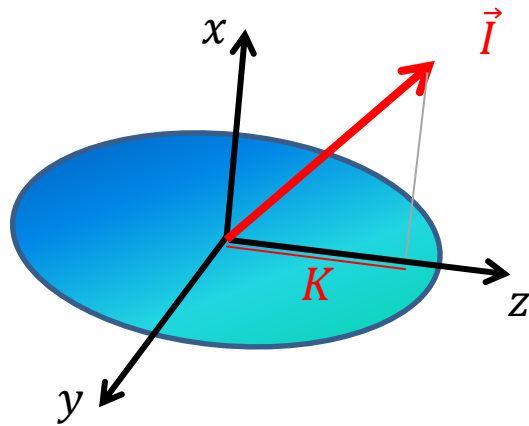


Quadrupole shapes

$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t) Y_{\lambda\mu}(\vartheta, \varphi) \right]$$



Macroscopic shape \Leftrightarrow microscopic structure?



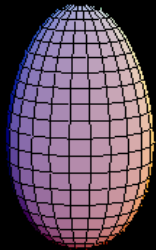
deformation can be dynamic: $a_{\lambda\mu}(t)$

Y_{20} beta Y_{22} gamma

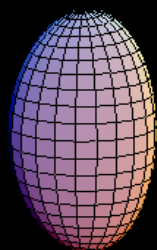


⇒ modes of excitation !

Y_{30}



Y_{31}



Y_{32}



Y_{33}

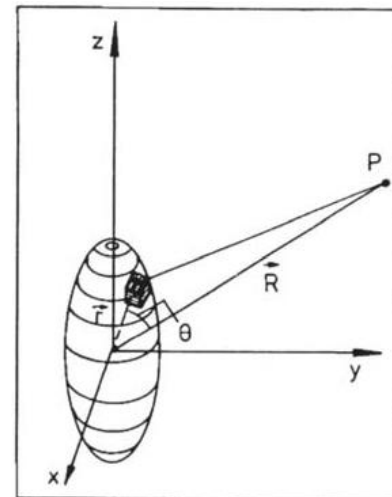


- difficult to measure mass distribution
- electromagnetic interaction sensitive to charge distribution $\rho(\vec{r})$

electric quadrupole tensor $Q_{ij} = \int \rho(\vec{r})(3x_i x_j - r^2 \delta_{ij}) dV$

rotation to principal axis frame $\Rightarrow Q_{ij}$ becomes diagonal

in case of axial symmetry: symmetry axis z

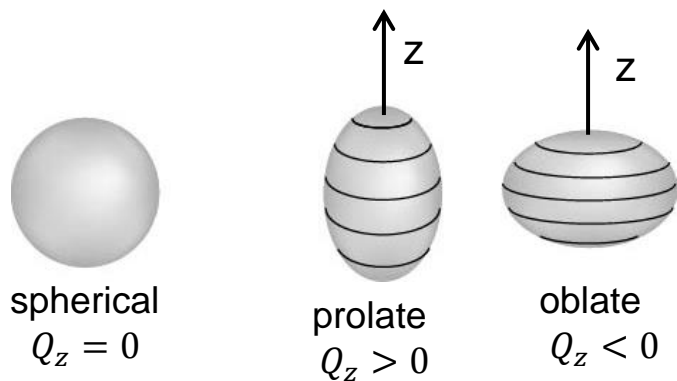


$$Q_z = \int \rho(\vec{r})(3z^2 - r^2) dV = \int \rho(\vec{r}) r^2 (3\cos^2\vartheta - 1) dV = \sqrt{\frac{16\pi}{5}} \int \rho(\vec{r}) r^2 Y_{20}(\vartheta, \varphi) dV = Q_0$$

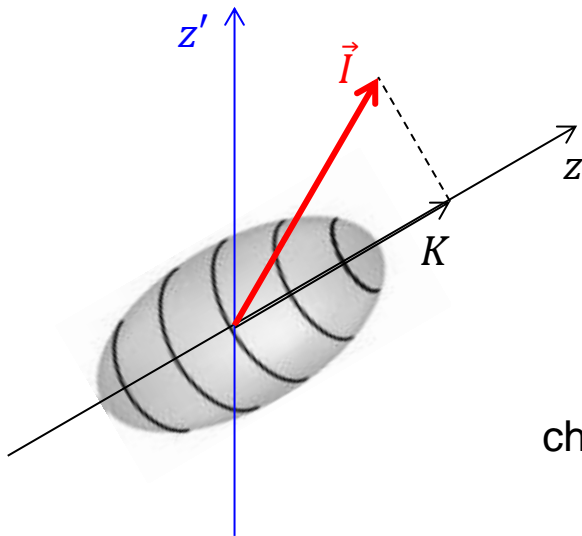
in the same way we can define:

$$Q_2 = \sqrt{\frac{16\pi}{5}} \int \rho(\vec{r}) r^2 Y_{22}(\vartheta, \varphi) dV$$

all quadrupole shapes can be described by Q_0 and Q_2



described in the body-fixed frame of reference !



axial deformation

body-fixed frame (x, y, z) with symmetry axis z

laboratory frame (x', y', z')

angular momentum \vec{I}

angular momentum projection K onto the symmetry axis

charge distribution in laboratory frame \leftrightarrow body-fixed frame

the **spectroscopic quadrupole moment Q_s** observed in the laboratory frame is related to the **intrinsic quadrupole moment Q_0** via:

$$Q_s = \frac{3K^2 - I(I + 1)}{(I + 1)(2I + 3)} Q_0$$

special cases:

$$I = 0 \quad \Rightarrow \quad K = 0 \quad \Rightarrow \quad Q_s = 0$$

$$I = \frac{1}{2} \quad \Rightarrow \quad K = \frac{1}{2} \quad \Rightarrow \quad Q_s = 0$$

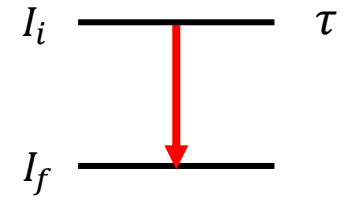
$$I = 2, K = 0 \quad \Rightarrow \quad Q_s = -\frac{2}{7} Q_0$$

It is impossible to measure the quadrupole moment of an even-even nucleus in its ground state



Transition probabilities

$$T_{i \rightarrow f}(\sigma\lambda\mu) = \frac{8\pi(\lambda + 1)}{\lambda\hbar[(2\lambda + 1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} B(\sigma\lambda; I_i \rightarrow I_f) = \frac{1}{\tau}$$

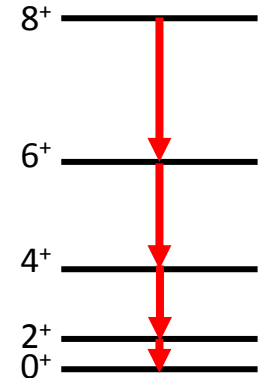


reduced transition probability: $B(\sigma\lambda; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f || \hat{O}(\sigma\lambda) || I_i \rangle|^2$

for a rotational band:

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f || \hat{O}(E2) || I_i \rangle|^2 = \frac{5}{16\pi} Q_t^2 \langle I_i K 2 0 | I_f K \rangle^2$$

transitional quadrupole moment Q_t



- for purely rotational states: $Q_t = Q_0$
- Q_t is a good approximation of the deformation for rotational bands
- we cannot get the sign of the quadrupole moment in this way

Strategies

lifetime measurement \Rightarrow transition probabilities \Rightarrow model-dependent information on shape

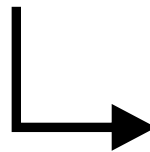


comparison with theoretical models
e.g. beyond-mean field theories

\Rightarrow quantitative test of the model



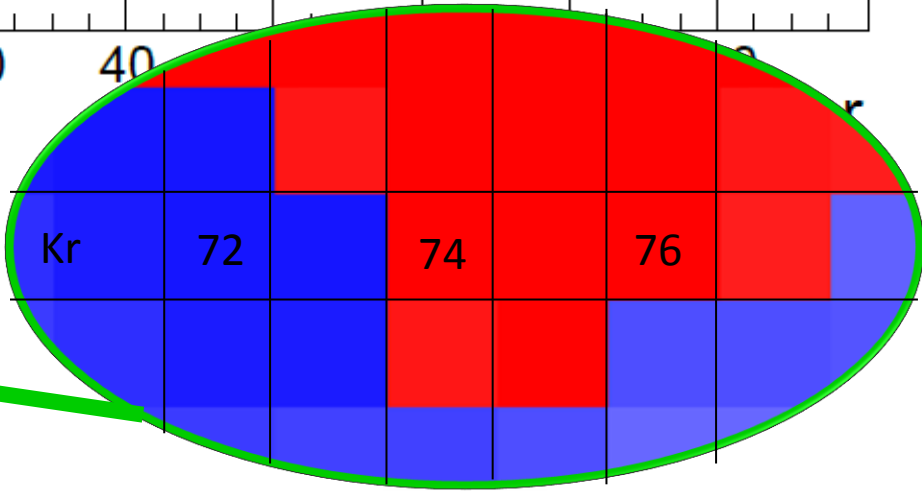
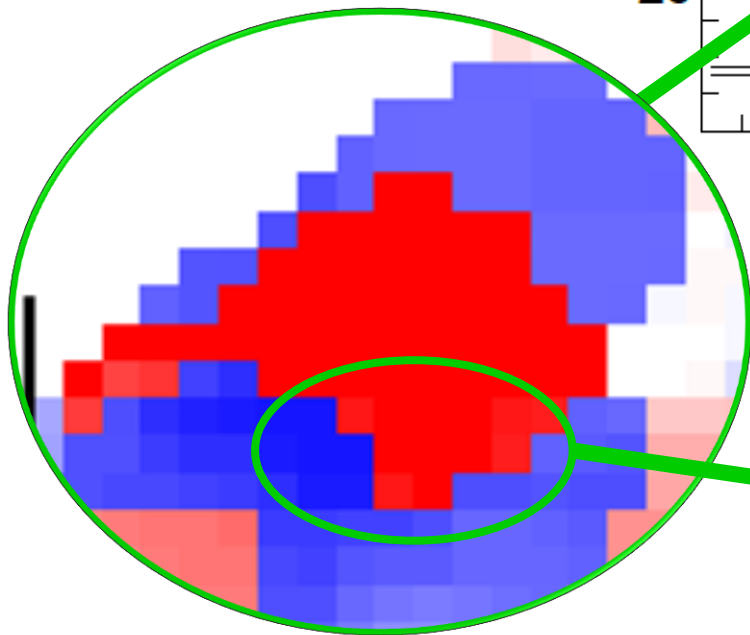
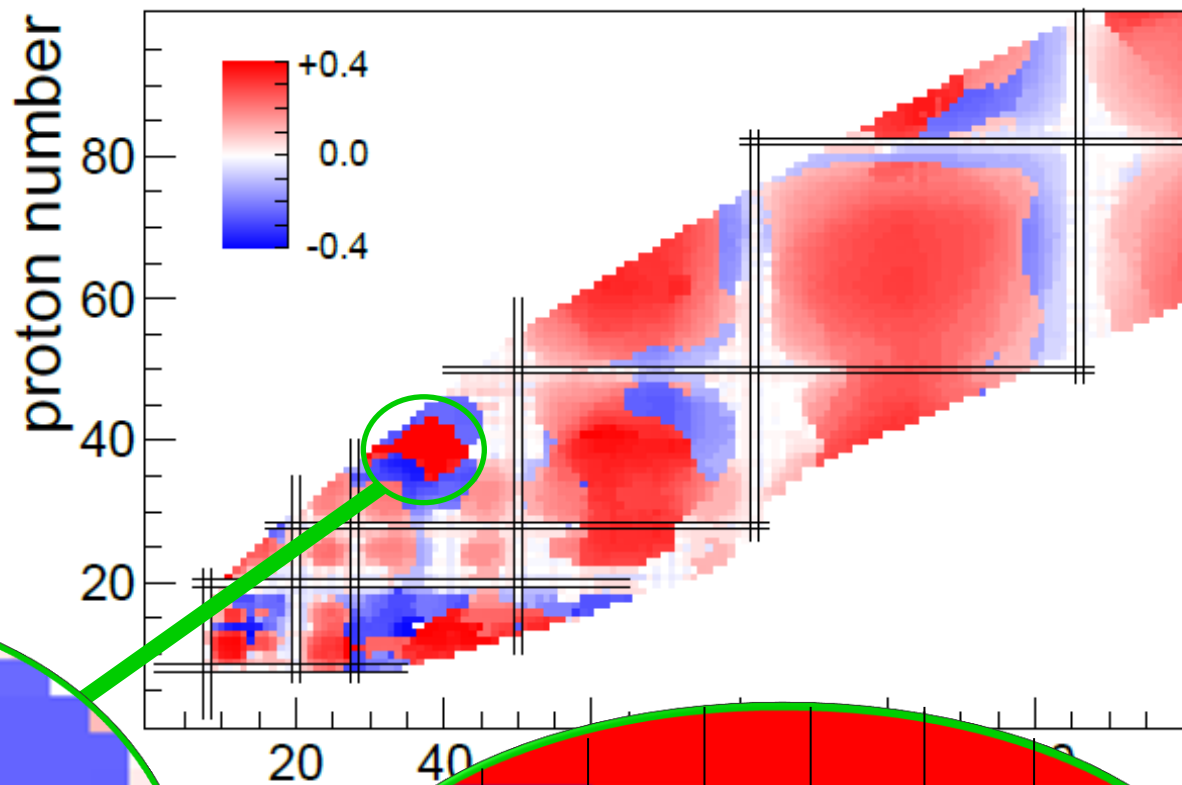
Coulomb excitation \Rightarrow transition probabilities
 \Rightarrow spectroscopic quadrupole moments



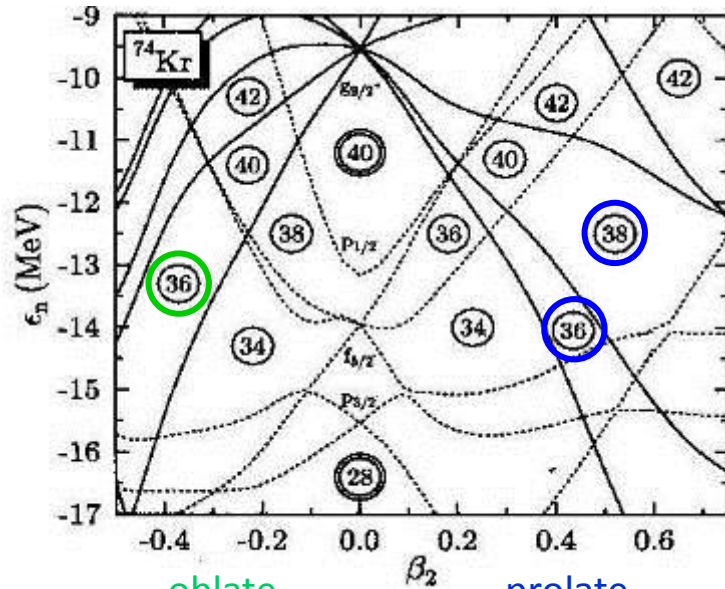
model-independent information on shape
also sensitive to the sign of Q_s

Example: neutron-deficient Kr isotopes

P. Möller et al., ADNDT 109, 1 (2016)

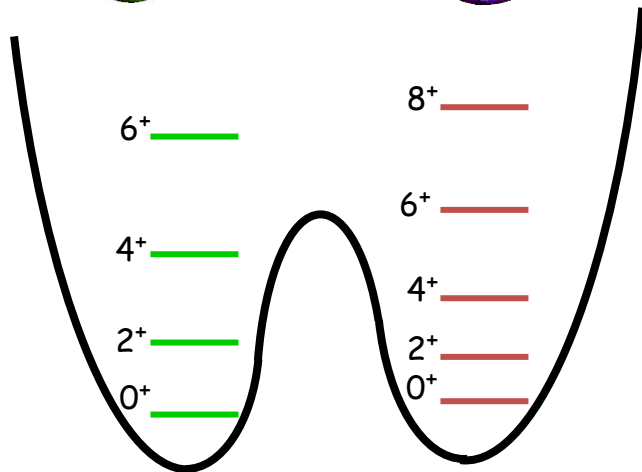
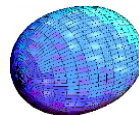
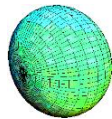


Shape coexistence in light Kr isotopes

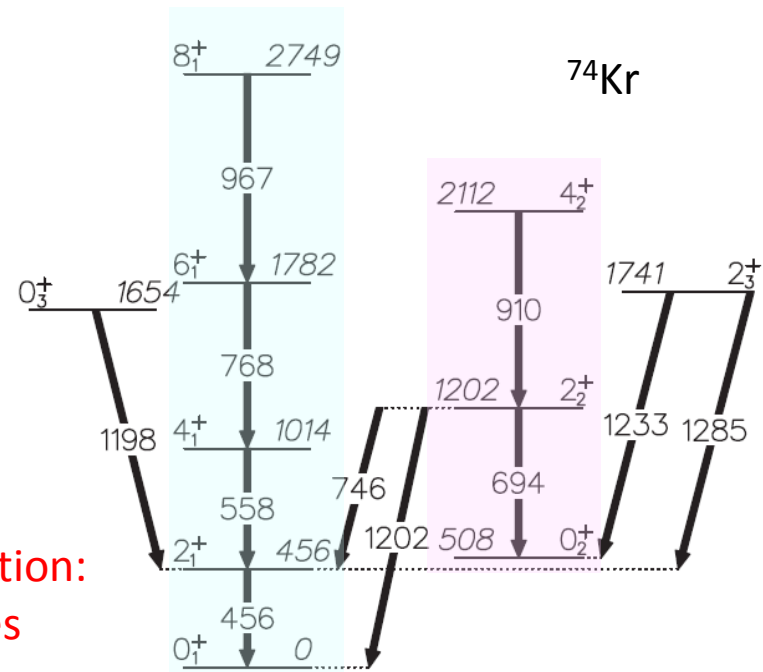
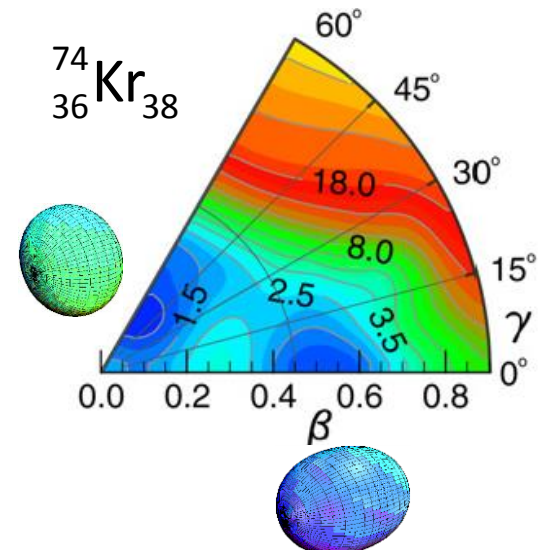


oblate

prolate



potential energy surface
 mean-field calculation
 Gogny D1S interaction

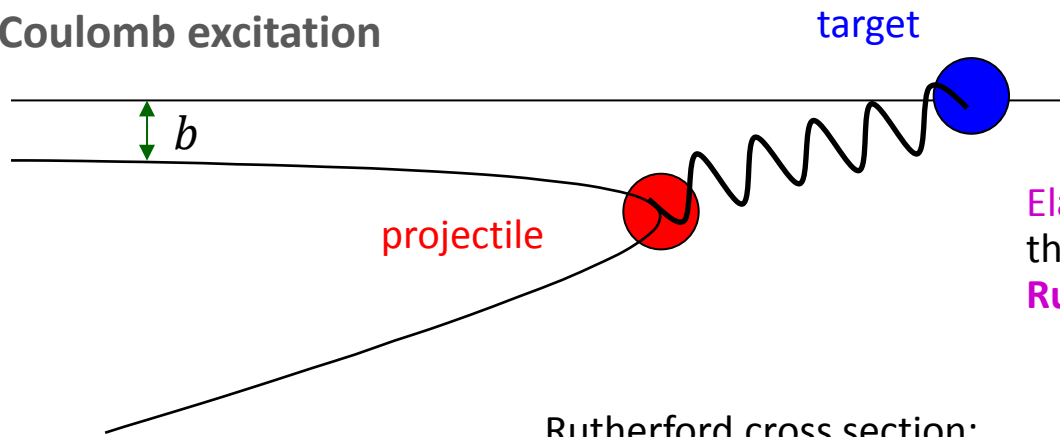


Coulomb excitation:

➤ $B(E2)$ values

➤ Q_s

Coulomb excitation



Elastic scattering of charged particles under the influence of the Coulomb field:
Rutherford scattering

Rutherford cross section:
$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \left(\frac{a_0}{2}\right)^2 \sin^{-4}\left(\frac{\theta_{cm}}{2}\right)$$

with $2a_0$ the distance of closest approach for head-on collisions

Nuclear excitation by the electromagnetic interaction acting between two colliding nuclei:
Coulomb excitation

excitation of both target and projectile possible
often magic nucleus (e.g. ^{208}Pb) as

- projectile \Rightarrow target Coulomb excitation
- target \Rightarrow projectile Coulomb excitation

important technique for radioactive beams

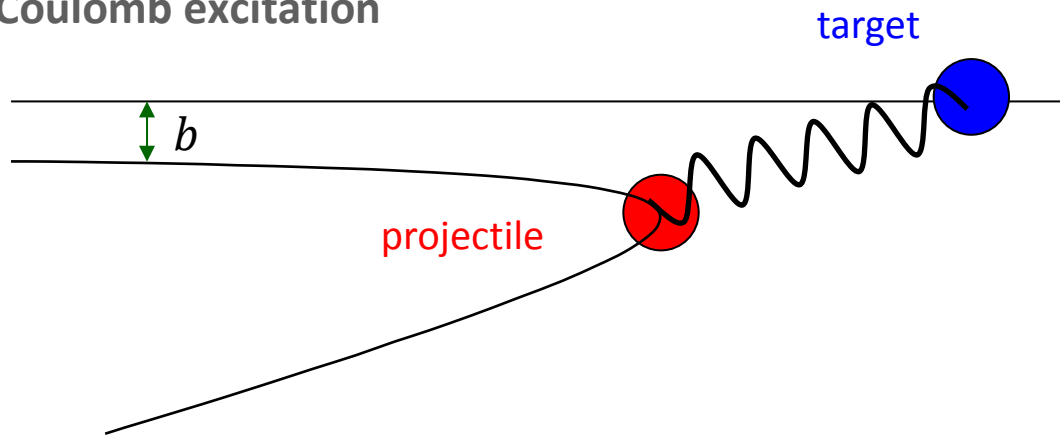
“safe” Coulomb excitation at low energy: purely electromagnetic process

safe condition: distance of closest approach $2a_0 > 1.25 (A_p^{1/3} + A_t^{1/3}) + 5 \text{ fm}$

\Rightarrow choose beam energy and scattering angle (i.e. impact parameter) accordingly

\Rightarrow purely electromagnetic process, can be calculated with high precision

Coulomb excitation



Coulomb excitation cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{CE} = \left(\frac{d\sigma}{d\Omega}\right)_{el} P_{i \rightarrow f}$$

with excitation probability $P_{i \rightarrow f}$

first-order perturbation \Rightarrow excitation amplitude

$$b_{i \rightarrow f} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}(E_f - E_i)t\right) \langle f | V(\vec{r}(t)) | i \rangle dt$$

excitation probability:

$$P_{i \rightarrow f} = \frac{1}{2I_i + 1} |b_{i \rightarrow f}|^2$$

multipole expansion of $V(\vec{r}(t))$:

$$\sigma_{E\lambda} = \left(\frac{Z_t e}{\hbar v}\right)^2 a_0^{-2\lambda+2} B(E\lambda) f_{E\lambda}(\xi)$$

$$\sigma_{M\lambda} = \left(\frac{Z_t e}{\hbar c}\right)^2 a_0^{-2\lambda+2} B(M\lambda) f_{M\lambda}(\xi)$$

magnetic excitations
suppressed by $(v/c)^2$

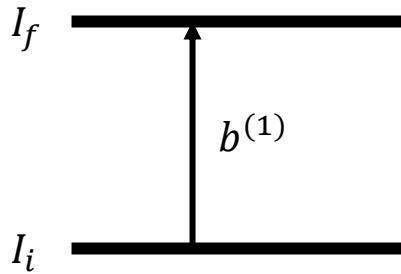
with Coulomb excitation function $f_{E\lambda}(\xi)$
and adiabacity parameter ξ

K. Alder and A. Winther,
*Electromagnetic Excitation, Theory of
Coulomb Excitation with Heavy Ions*
(1975)

to first order: $\sigma_{\pi\lambda} \propto B(\pi\lambda; I_i \rightarrow I_f) \propto |\langle I_f | M(\sigma\lambda) | I_i \rangle|^2$

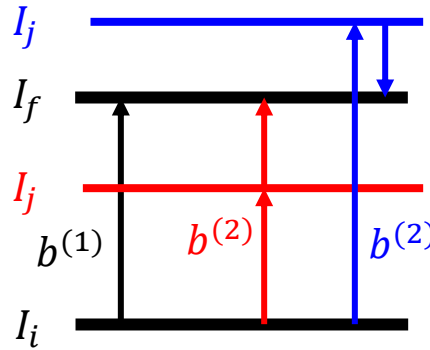
Second-order effects

$$P_{i \rightarrow f} = \frac{1}{2I_i + 1} |b_{i \rightarrow f}^{(1)}|^2$$



1st order:

$$b_{i \rightarrow f}^{(1)} \propto \langle I_f | M(\sigma\lambda) | I_i \rangle$$



2nd order:

$$b_{i \rightarrow f}^{(2)} \propto \sum_j \langle I_f | M(\sigma\lambda) | I_j \rangle \langle I_j | M(\sigma\lambda) | I_i \rangle$$

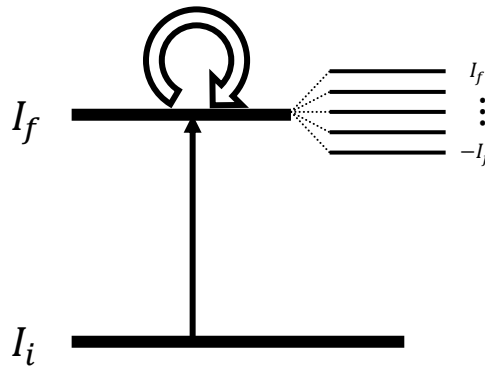
$$P_{i \rightarrow f} = \frac{1}{2I_i + 1} |b_{i \rightarrow f}^{(1)} + b_{i \rightarrow f}^{(2)}|^2$$

\Rightarrow interference terms

special case:
intermediate state
another magnetic substate

“reorientation effect”

$$b_{i \rightarrow f}^{(2)} \propto \langle I_f | M(\sigma\lambda) | I_f \rangle \langle I_f | M(\sigma\lambda) | I_i \rangle$$

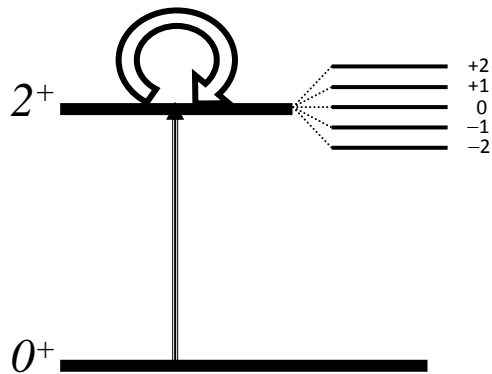


$P_{i \rightarrow f}$ depends on:

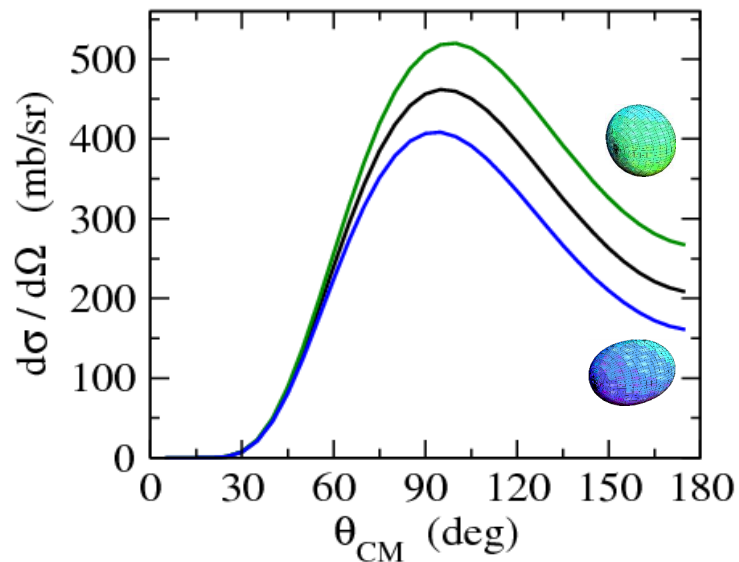
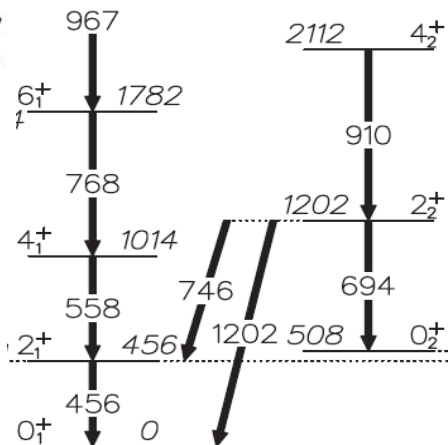
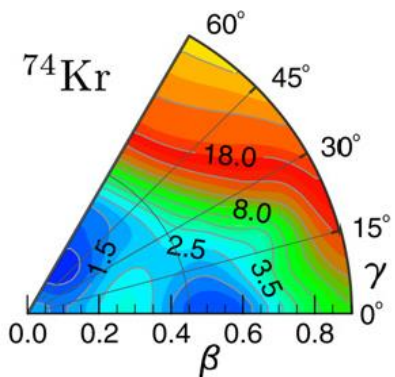
- transitional matrix element
 $B(\sigma\lambda; I_i \rightarrow I_f) = |\langle I_f | M(\sigma\lambda) | I_i \rangle|^2$
- diagonal matrix element
 $\langle I_f | M(\sigma\lambda) | I_f \rangle \propto Q_s(I_f)$

Coulomb excitation sensitive
to spectroscopic quadrupole moments

Reorientation effect



$$b_{0^+ \rightarrow 2^+}^{(2)} \propto \langle 2^+ | M(E2) | 2^+ \rangle \langle 2^+ | M(E2) | 0^+ \rangle$$



to extract $B(E2)$ and Q_s

- differential measurement of cross section (as a function of scattering angle)
- large θ_{cm} most sensitive

Idea: measure $Q_s(2_1^+)$ and $Q_s(2_2^+)$

(remember: $Q_s(0^+) = 0$)

Problem: ^{74}Kr is radioactive ($T_{1/2} = 11.5$ min)

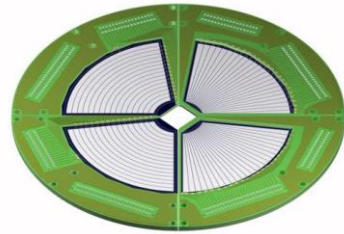
➤ radioactive beam

GANIL – SPIRAL: Isotope separation on-line (ISOL)



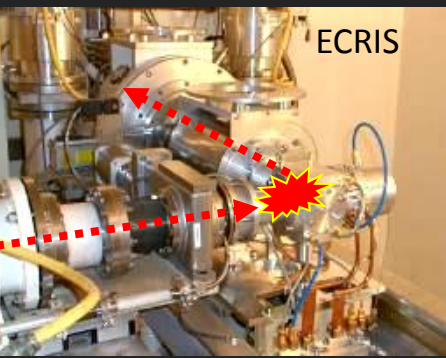
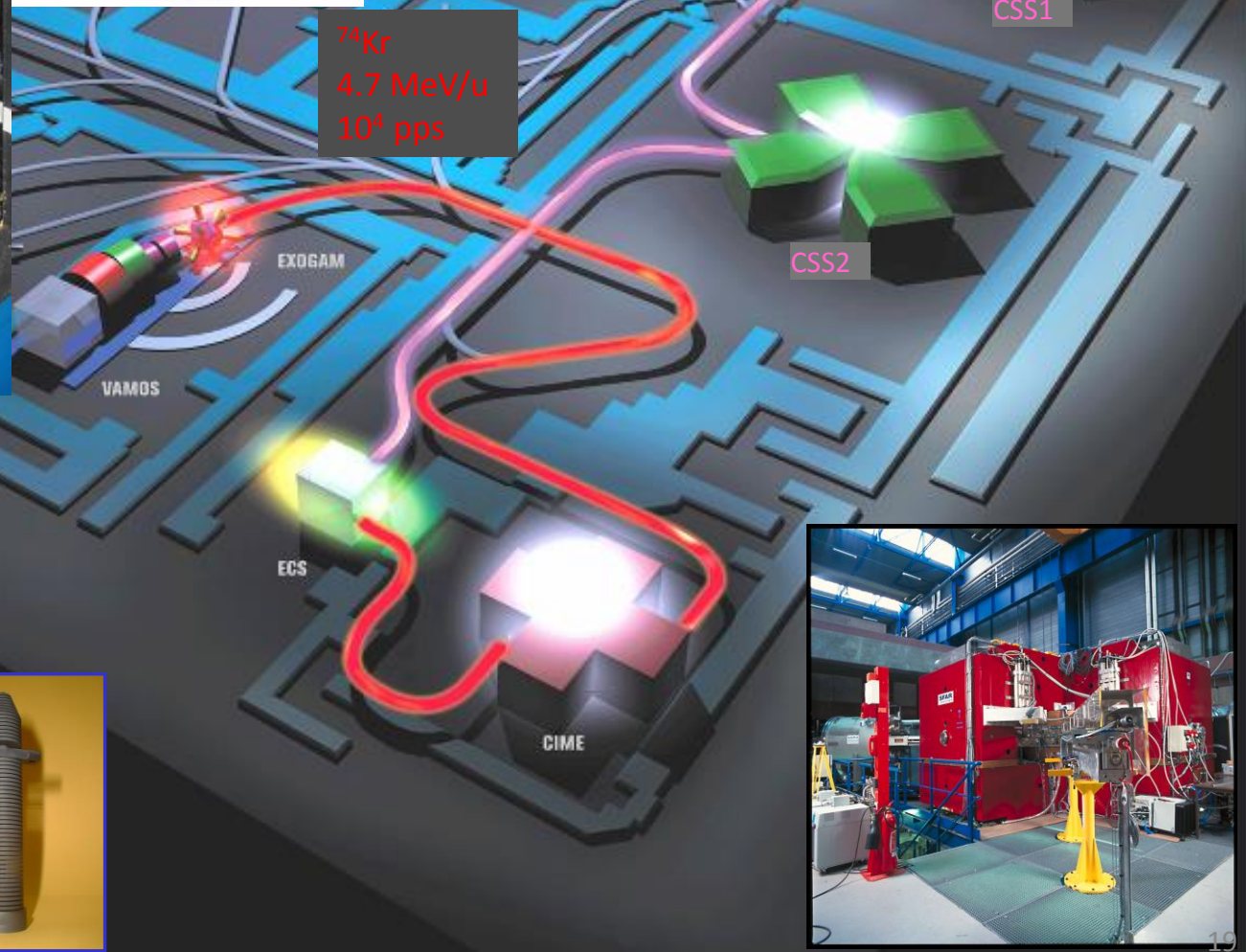
RAL:

Système de Production d'Ions Radioactifs en Ligne

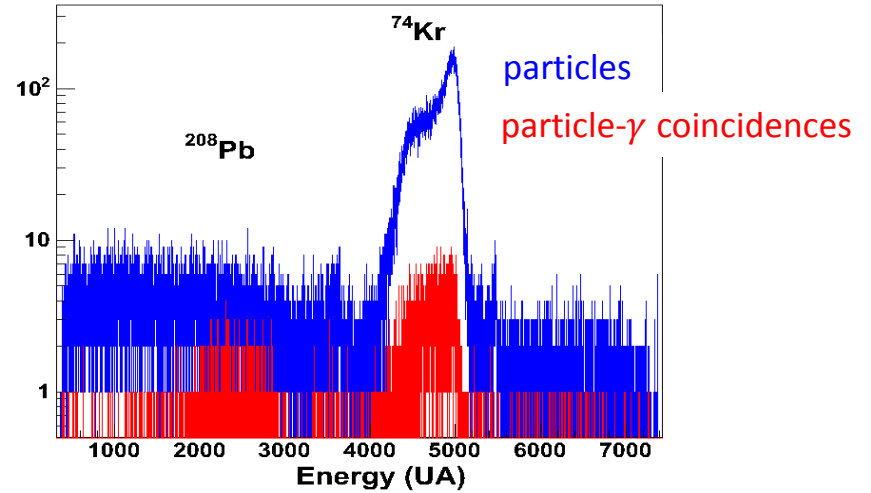
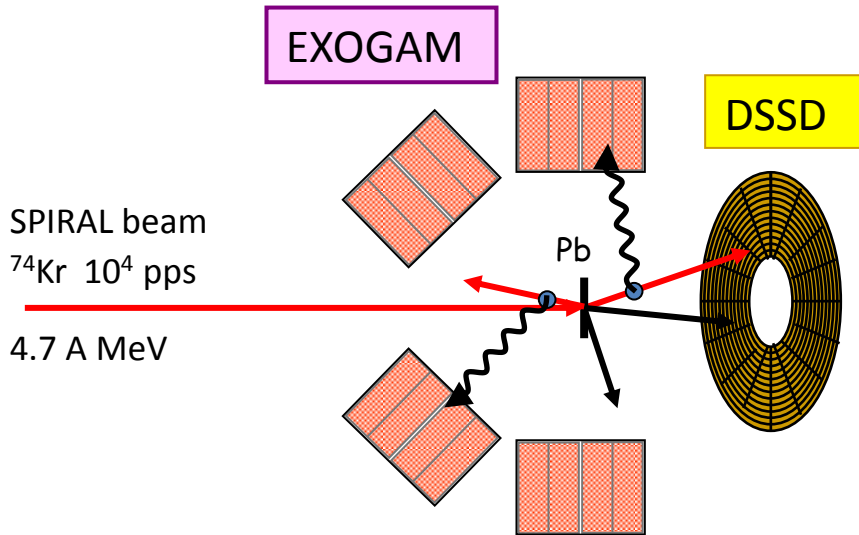


^{78}Kr
68.5 MeV/u
 10^{12} pps

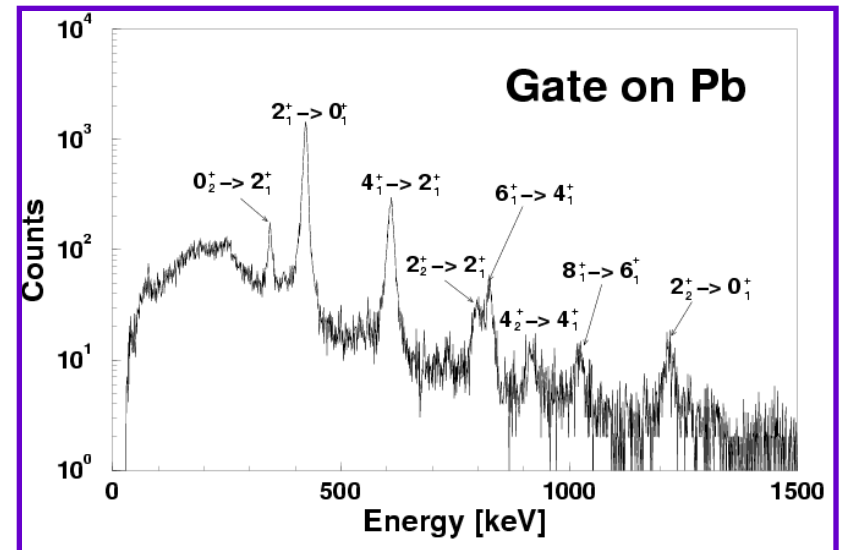
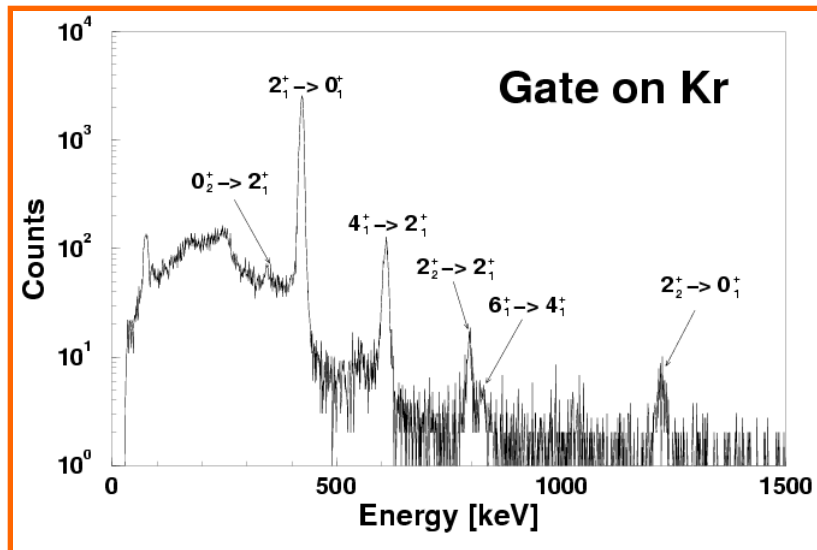
^{74}Kr
4.7 MeV/u
 10^8 pps



Coulomb excitation of ^{74}Kr



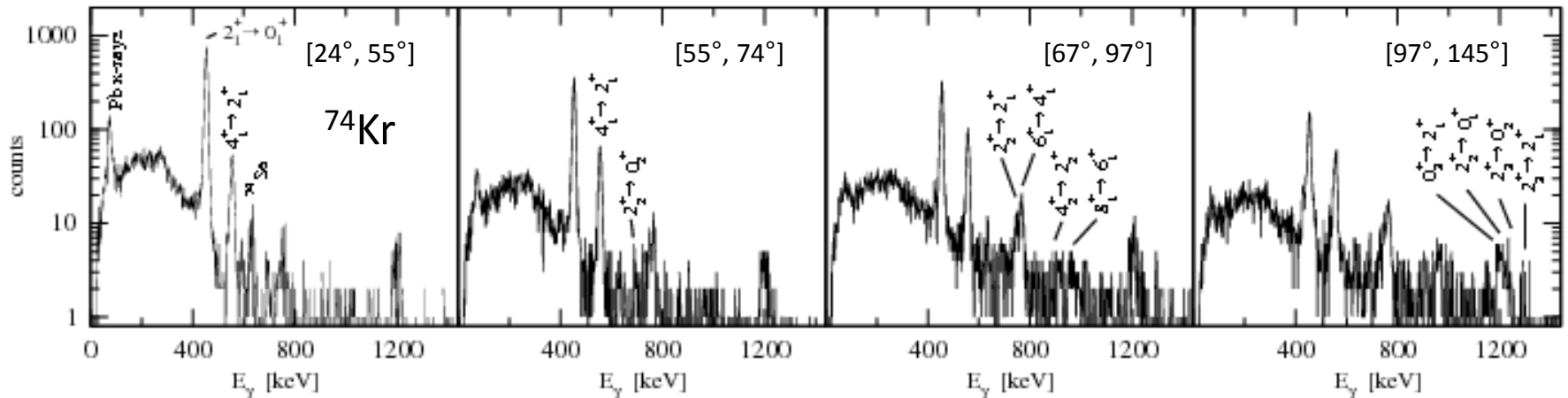
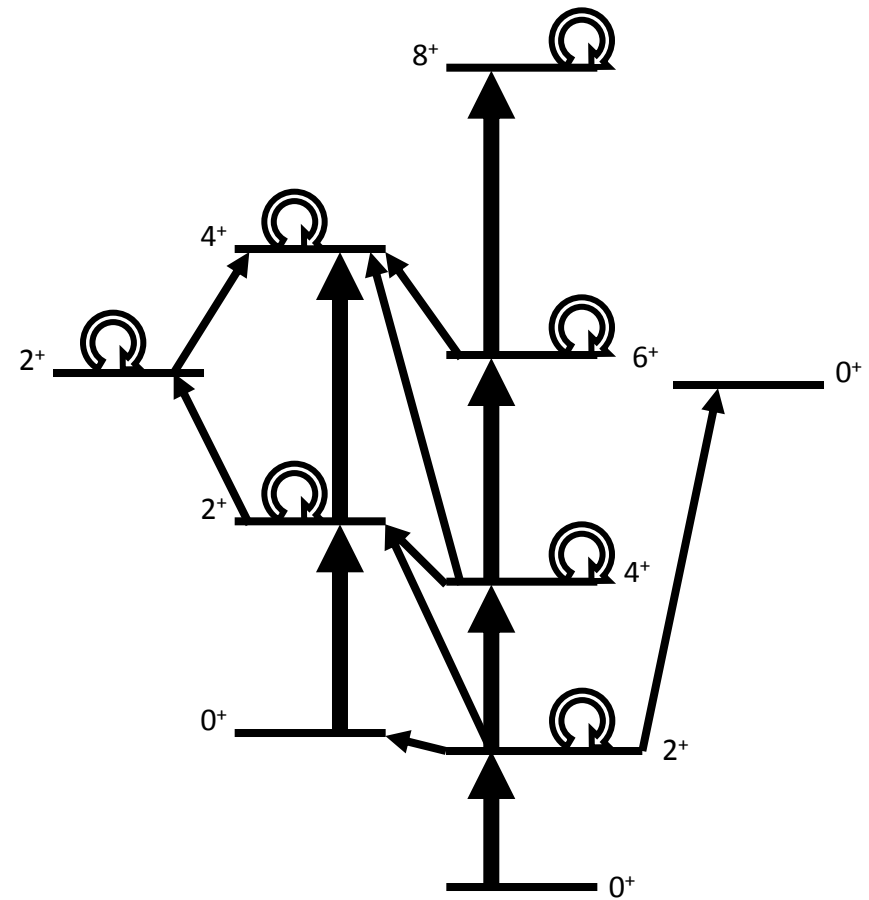
Differential Coulomb excitation cross section for $35^\circ < \theta_{\text{cm}} < 130^\circ$



Multi-step Coulomb excitation of ^{74}Kr

- γ yields as function of scattering angle: differential cross section
- least squares fit of ~ 30 matrix elements (transitional and diagonal)
- experimental spectroscopic data
 - lifetimes, branching and mixing ratios

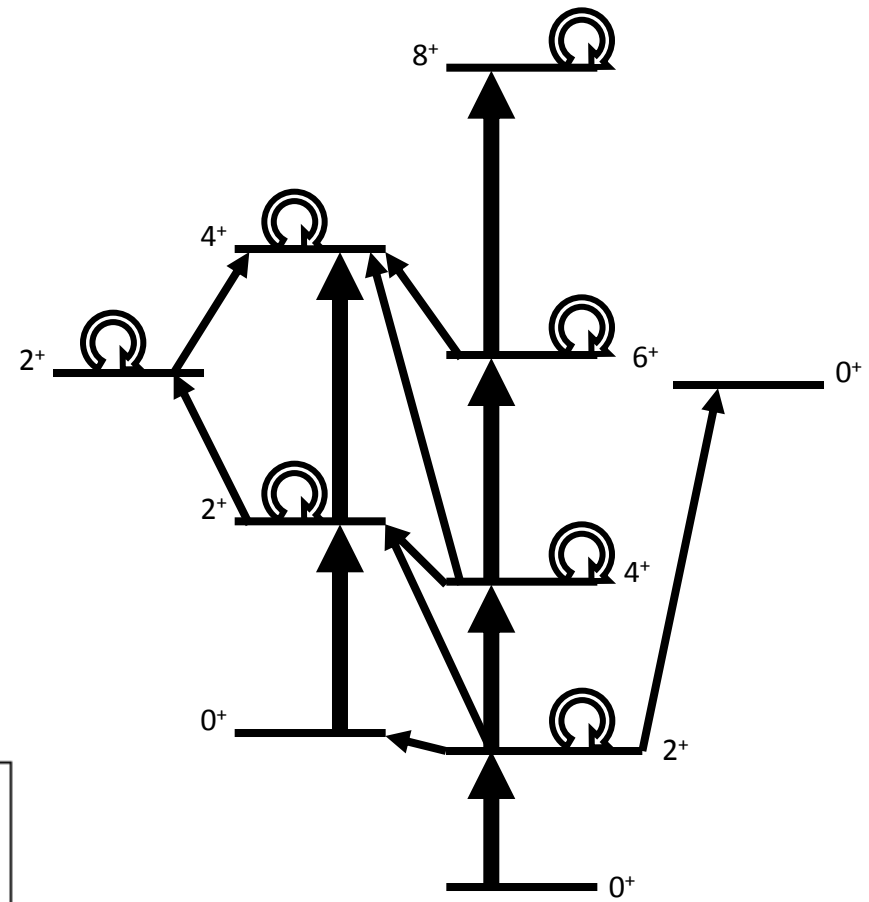
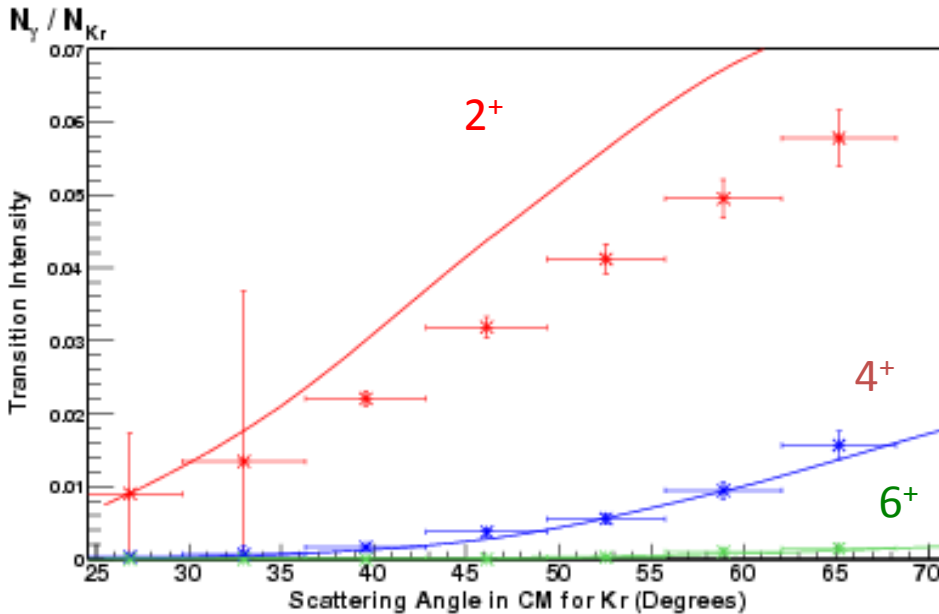
GOSIA: Coulomb excitation and least-squares fitting code



Multi-step Coulomb excitation of ^{74}Kr

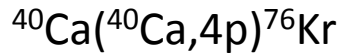
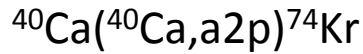
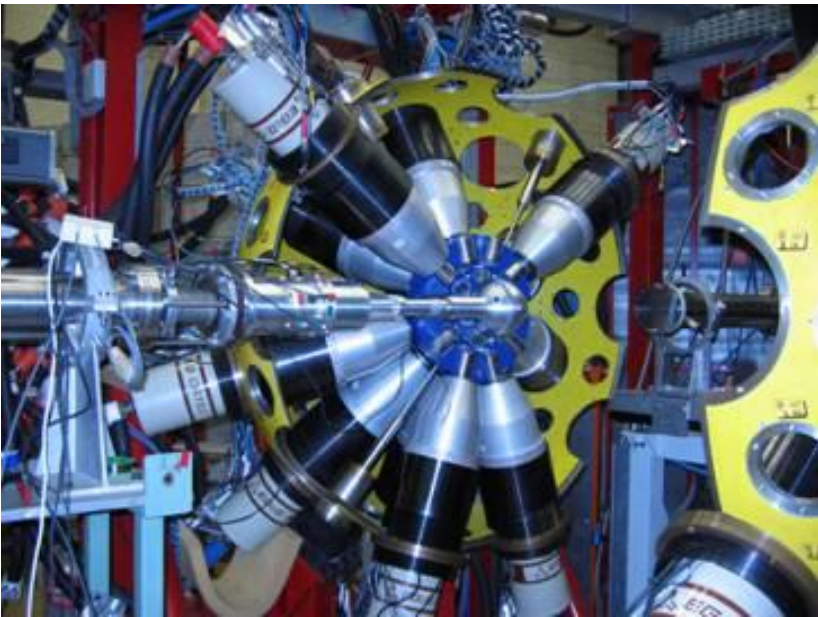
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GOSIA: Coulomb excitation and least-squares fitting code

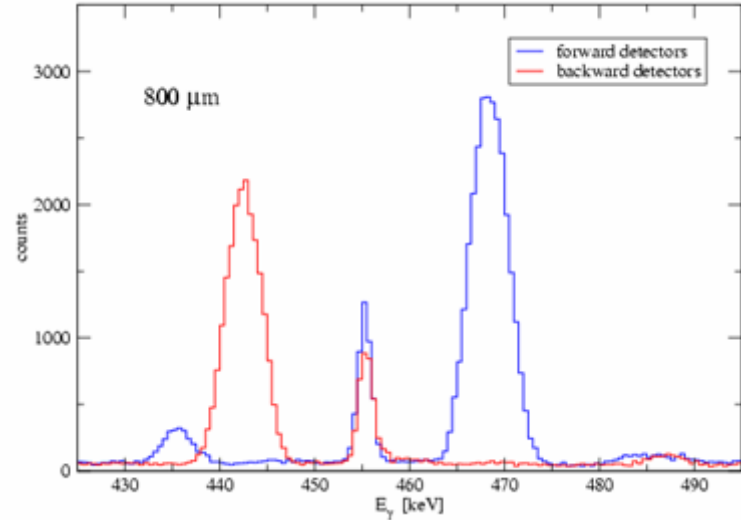
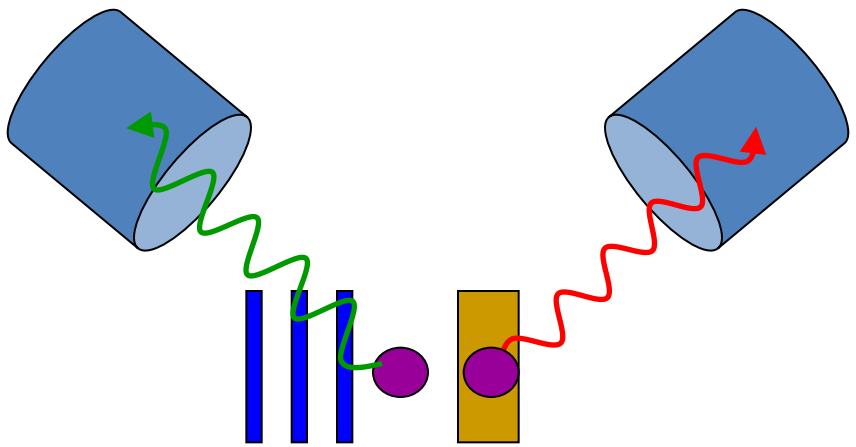


- Results inconsistent with published lifetimes

Lifetime measurement with GASP and the Köln Plunger



124 MeV



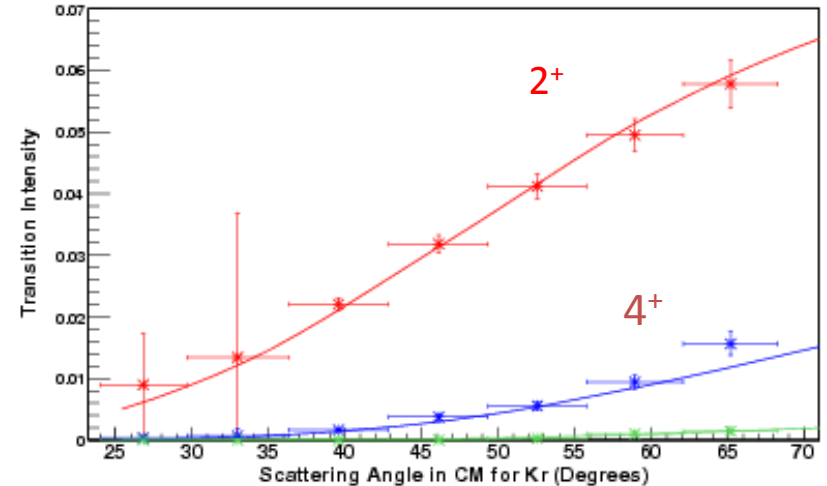
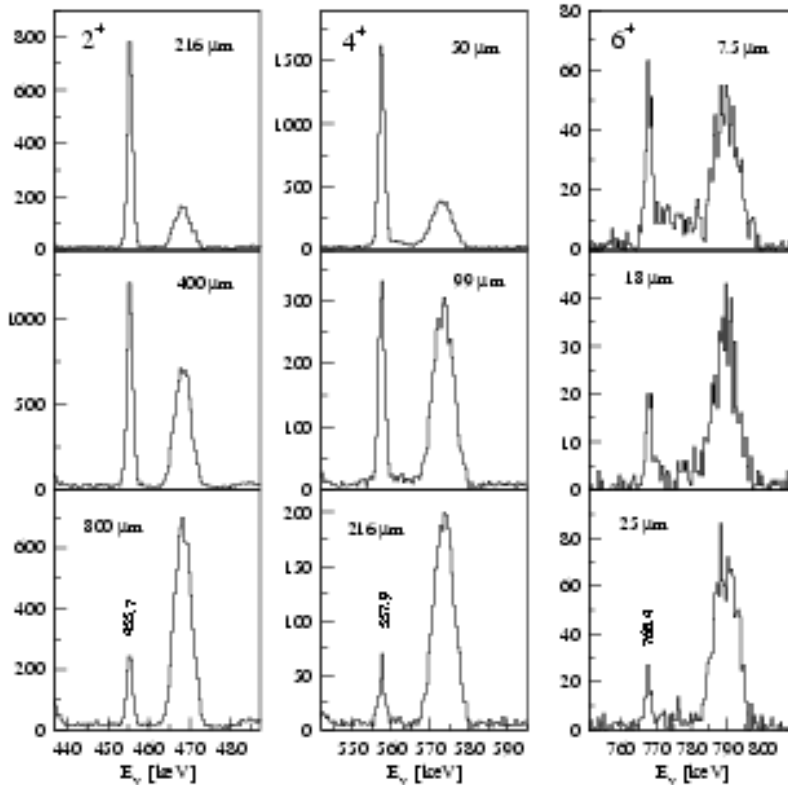
Lifetime results

A. G6rgen, Eur. Phys. J. A 26, 153 (2005)

^{74}Kr	2^+	4^+	^{76}Kr	2^+	4^+
new	33.8(6)	5.2(2)	new	41.5(8)	3.67(9) [ps]
	28.8(57)	13.2(7)		35.3(10)	4.8(5) [ps]
	J. Roth et al., J.Phys.G, L25 (1984)			B. W6rmmann et al., NPA 431, 170 (1984)	

^{74}Kr

- forward detectors (36°)
- gated from above



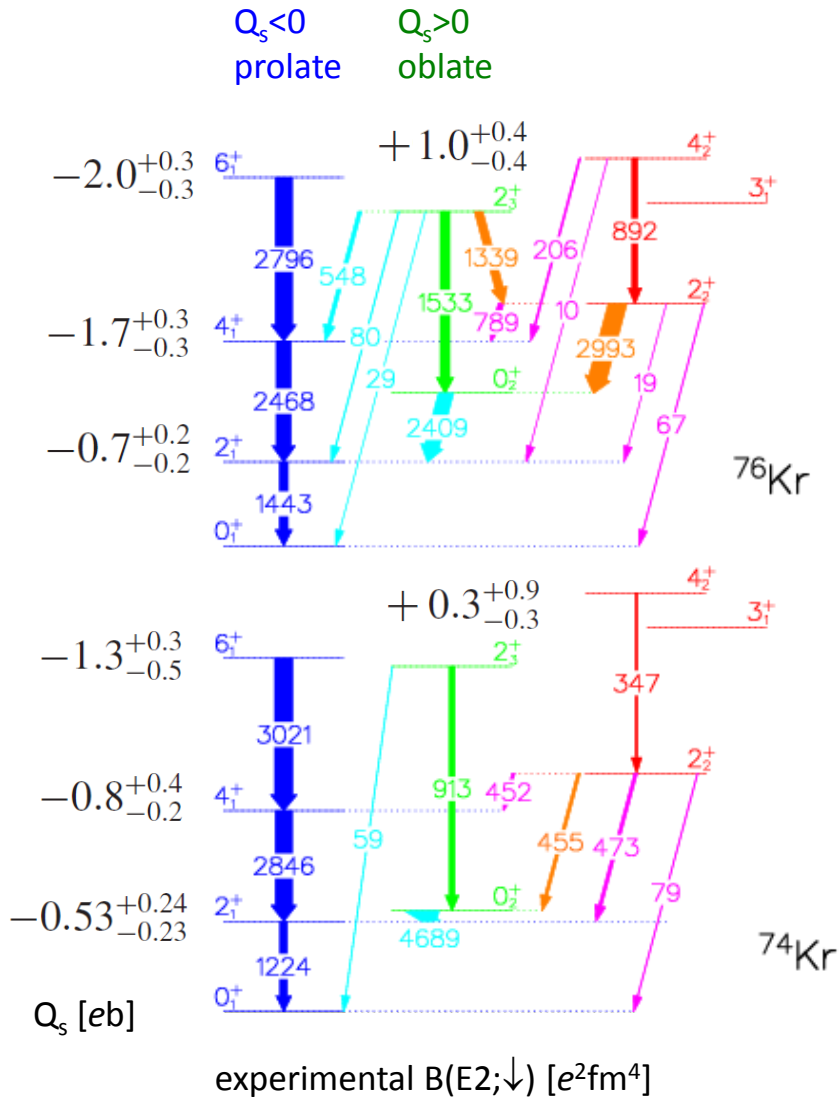
Results consistent with Coulomb excitation.

Lifetimes constrain GOSIA fit.

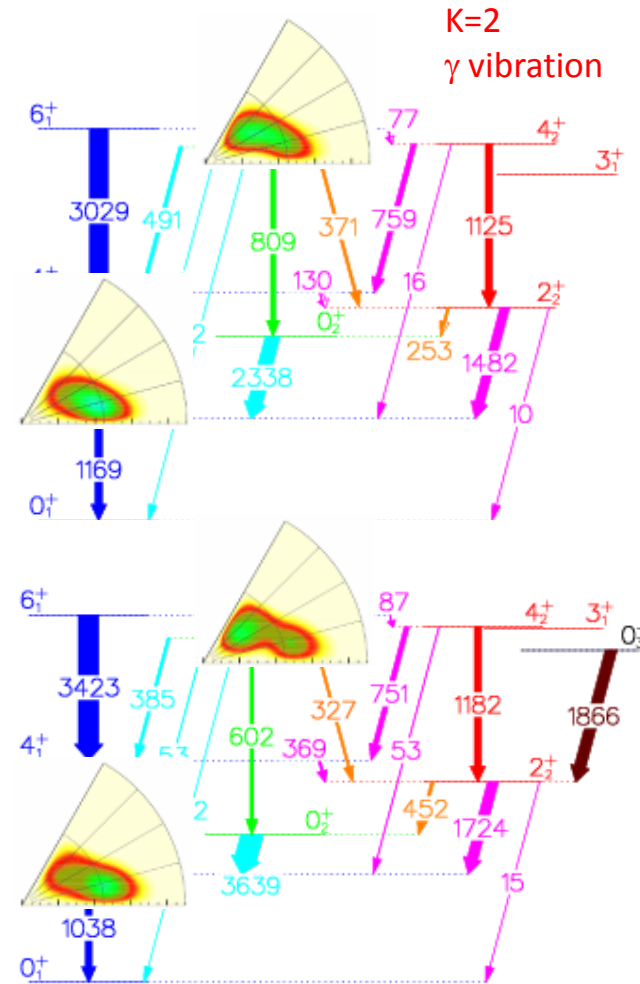
⇒ enhanced sensitivity for non-yrast transitions and diagonal matrix elements

Prolate – oblate shape coexistence

➤ no free parameters in the calculation
(except for the globally derived D1S interaction)

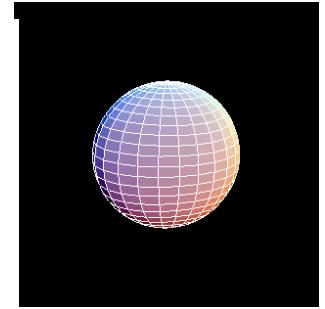


E. Clément et al.,
PRC 75, 054313 (2007)



GCM (GOA) calculation with Gogny D1S
 q_0 q_2 : triaxial deformation

M. Girod et al.,
Phys. Lett. B 676, 39 (2009)



Quadrupole sum rules

We measure $B(E2)$ and Q_s values in the laboratory frame.

How can we determine the **intrinsic shape**?

rotate electric quadrupole tensor into principal axis frame

⇒ only two non-zero quadrupole moments

⇒ two parameters (Q, δ) in analogy with Bohr's parameters (β, γ)

$$\mathcal{M}(E2, \mu = 0) = Q \cos \delta$$

$$\mathcal{M}(E2, \mu = \pm 1) = 0$$

$$\mathcal{M}(E2, \mu = \pm 2) = \frac{1}{\sqrt{2}} Q \sin \delta$$

Zero-coupled products of the E2 operators are **rotationally invariant** (the same in the lab and intrinsic frames):

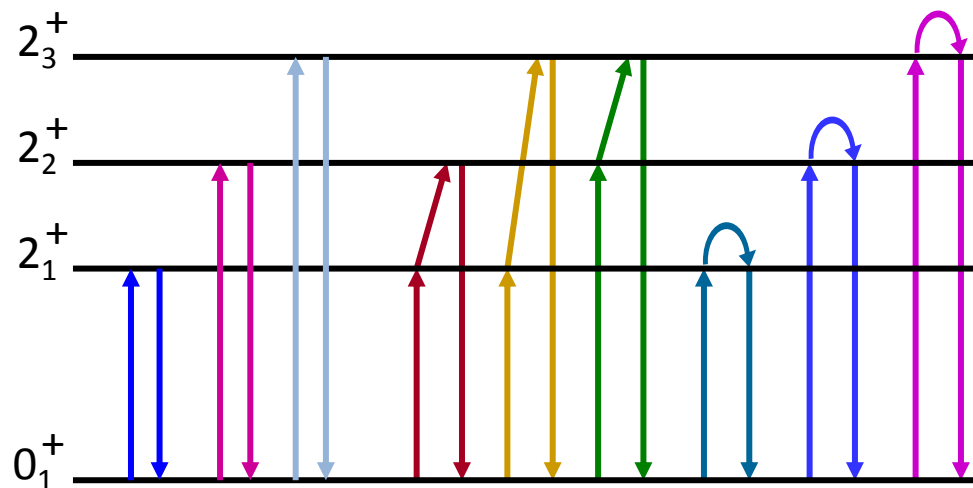
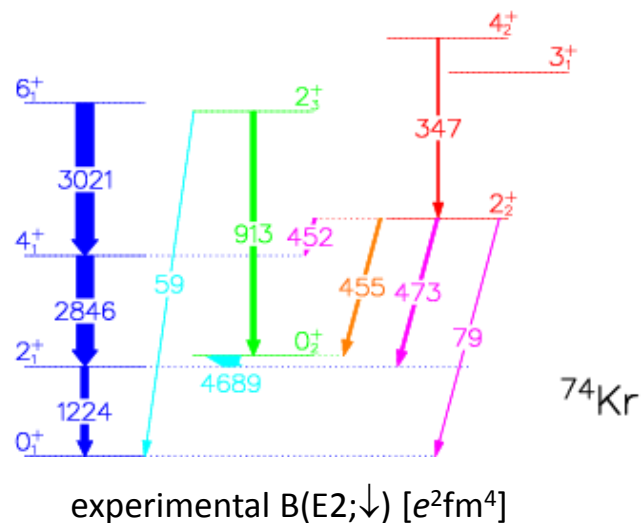
$$\langle s | [E2 \times E2]_0 | s \rangle = \frac{1}{\sqrt{5}} Q^2 = \frac{(-1)^{2s}}{\sqrt{2s+1}} \sum_t \langle s || E2 || t \rangle \langle t || E2 || s \rangle \begin{Bmatrix} 2 & 2 & 0 \\ s & s & t \end{Bmatrix}$$

$$\langle s | [[E2 \times E2]_2 \times E2]_0 | s \rangle = -\sqrt{\frac{2}{35}} Q^3 \cos(3\delta) = \frac{1}{2s+1} \sum_{tu} \langle s || E2 || t \rangle \langle t || E2 || u \rangle \langle u || E2 || s \rangle \begin{Bmatrix} 2 & 2 & 2 \\ s & t & u \end{Bmatrix}$$

D. Cline, Ann. Rev. Nucl. Part. Sci. 36, 683 (1986)

Model-independent method to determine the intrinsic quadrupole shape of the charge distribution from a set of E2 matrix elements

Example for ^{74}Kr , ^{76}Kr

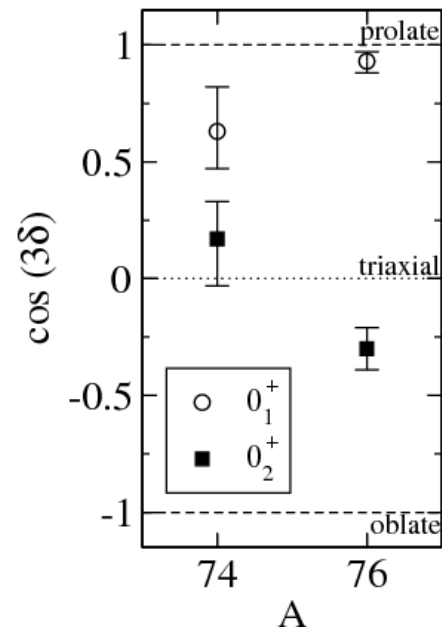
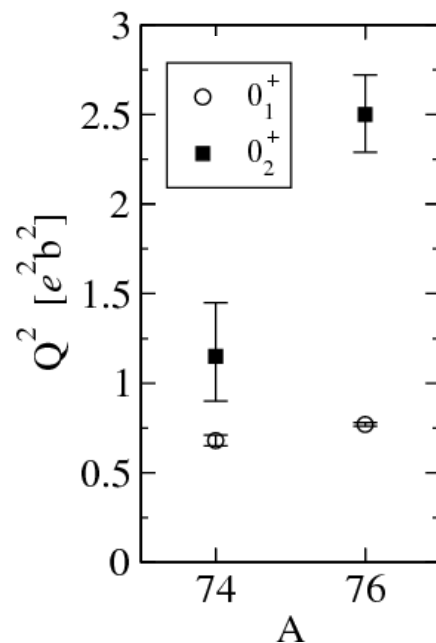


$$\sum_t \langle s || E2 || t \rangle \langle t || E2 || s \rangle \begin{Bmatrix} 2 & 2 & 0 \\ s & s & t \end{Bmatrix}$$

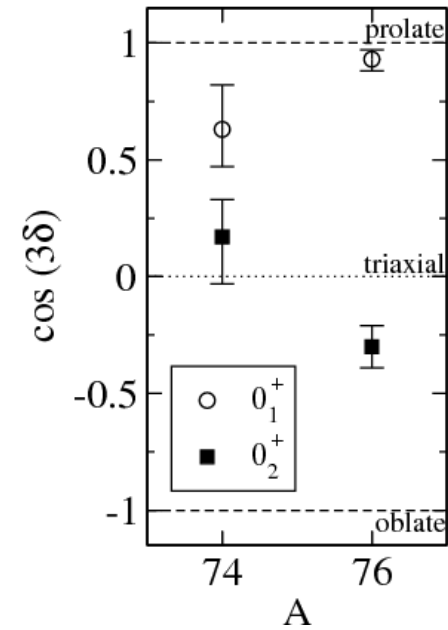
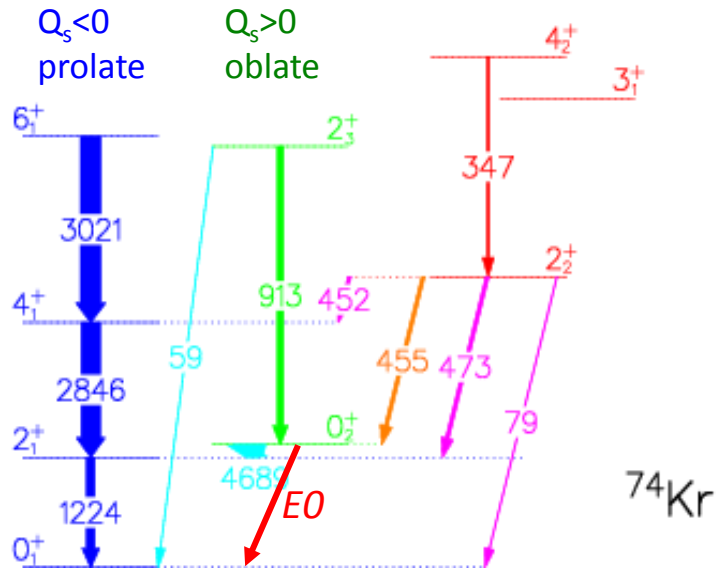
$$\sum_{tu} \langle s || E2 || t \rangle \langle t || E2 || u \rangle \langle u || E2 || s \rangle \begin{Bmatrix} 2 & 2 & 2 \\ s & t & u \end{Bmatrix}$$

needs complete set of matrix elements
 \Rightarrow usually only feasible for 0^+ states

more mixing in ^{74}Kr ?



Prolate – oblate shape coexistence



- evidence for shape coexistence
- indication of mixing
- electric monopole transition?

⇒ conversion electron spectroscopy

electric monopole (E0) transitions

- between states of the same spin and parity, in particular $0^+ \rightarrow 0^+$
- non-radiative: only internal conversion or internal pair creation ($E > 1.022$ MeV) possible
- related to changes in the rms radius of the charge distribution

E0 transition rate:
$$\frac{1}{\tau(E0)} = \rho^2(E0; i \rightarrow f) (\Omega_K + \Omega_{L_1} + \dots + \Omega_{IP})$$

with electronic factors Ω <http://bricc.anu.edu.au/index.php>

and the E0 matrix element
$$\rho(E0; i \rightarrow f) = \frac{|\langle f | \hat{T}(E0) | i \rangle|}{eR^2} = \frac{|\langle f | \sum_k e_k r_k^2 | i \rangle|}{eR^2}$$

with $R = r_0 A^{1/3}$ and sum over all protons
or valence nucleons with effective charge e_k

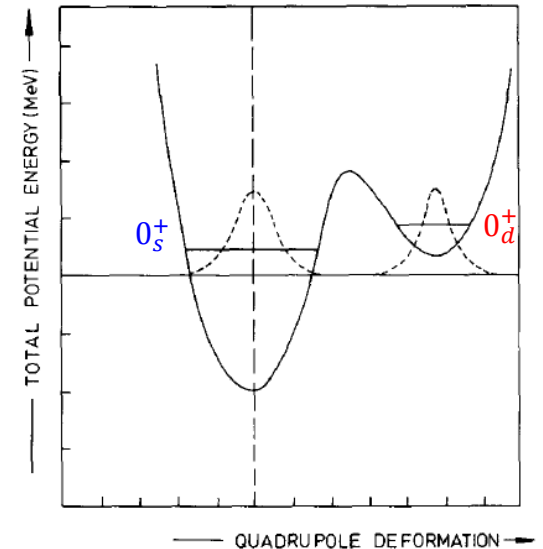
shape coexistence and E0 transitions

two 0^+ states with different intrinsic deformation: $|0_s^+\rangle, |0_d^+\rangle$

mixing of the two configurations gives two eigenstates:

$$|0_2^+\rangle = a|0_s^+\rangle + b|0_d^+\rangle$$

$$|0_1^+\rangle = -b|0_s^+\rangle + a|0_d^+\rangle$$



$$\rho(E0; 0_2^+ \rightarrow 0_1^+) = \frac{1}{eR^2} [ab(\langle 0_d^+ | \hat{T}(E0) | 0_d^+ \rangle - \langle 0_s^+ | \hat{T}(E0) | 0_s^+ \rangle) + (a^2 - b^2)\langle 0_s^+ | \hat{T}(E0) | 0_d^+ \rangle]$$

the wave functions $|0_s^+\rangle$ and $|0_d^+\rangle$ have different deformation $\Rightarrow \langle 0_s^+ | \hat{T}(E0) | 0_d^+ \rangle \approx 0$

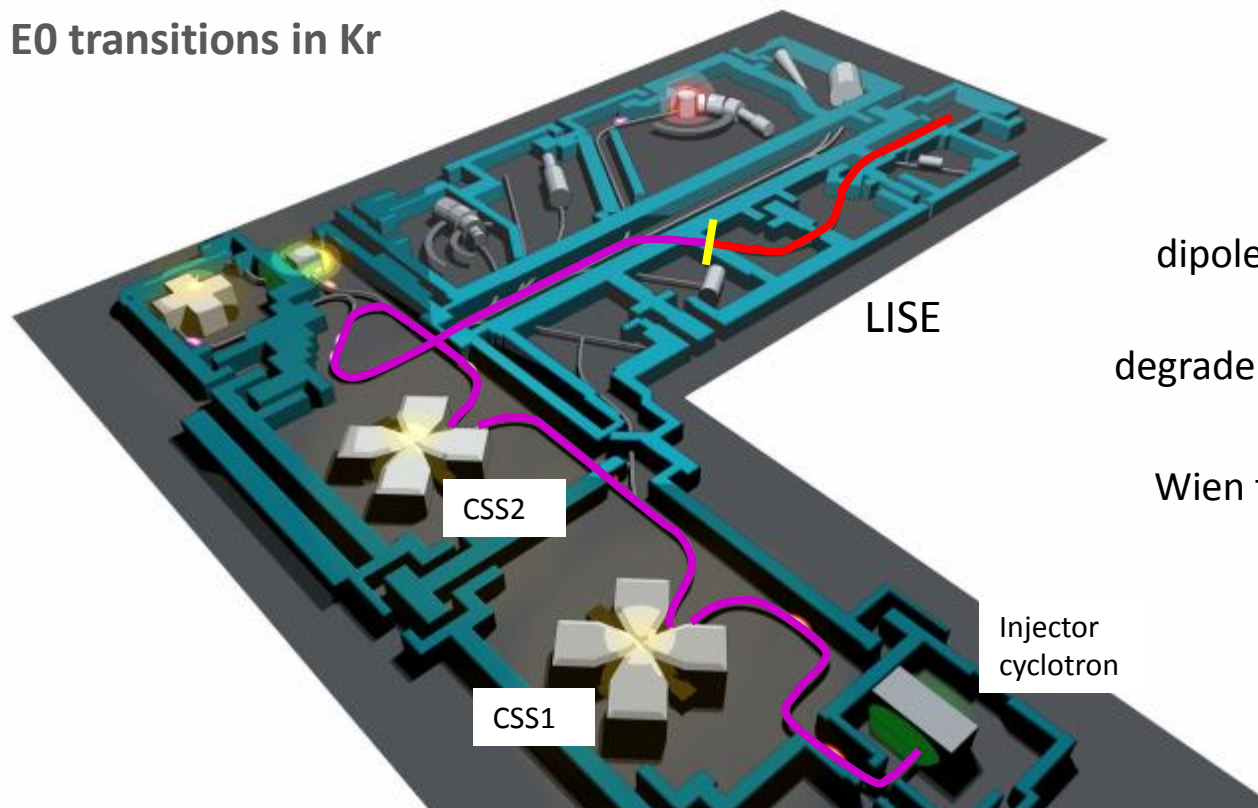
no mixing: $a = 1, b = 0 \Rightarrow ab = 0 \Rightarrow \rho(E0; 0_2^+ \rightarrow 0_1^+) \approx 0$ no E0 transition

mixing: $\rho(E0; 0_2^+ \rightarrow 0_1^+) = \frac{1}{eR^2} [ab(\langle 0_d^+ | \hat{T}(E0) | 0_d^+ \rangle - \langle 0_s^+ | \hat{T}(E0) | 0_s^+ \rangle)]$

$$\rho^2(E0) = \left(\frac{3Z}{4\pi}\right)^2 a^2 b^2 (\beta_2^2 - \beta_1^2)^2$$

E0 transition requires mixing and change in deformation

E0 transitions in Kr



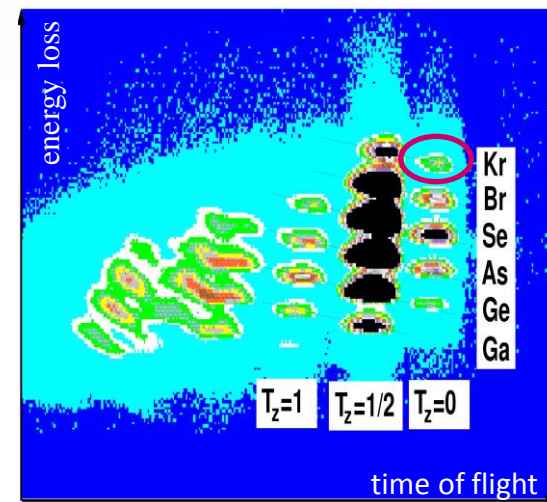
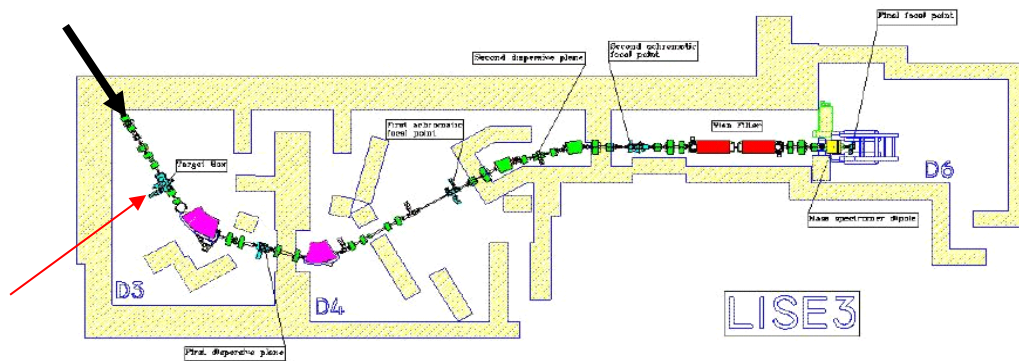
dipole magnets: $B\rho = \frac{mv}{q}$

degrader: Z-dependent energy loss

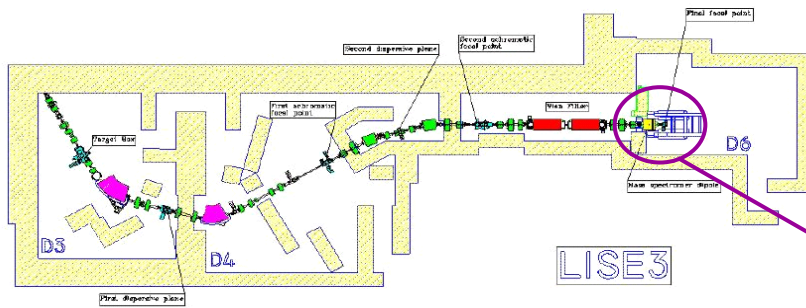
Wien filter: $\vec{E} \perp \vec{B} \Rightarrow qE = qvB$

^{78}Kr
70 MeV/u
 $\sim 10^{12}$ pps

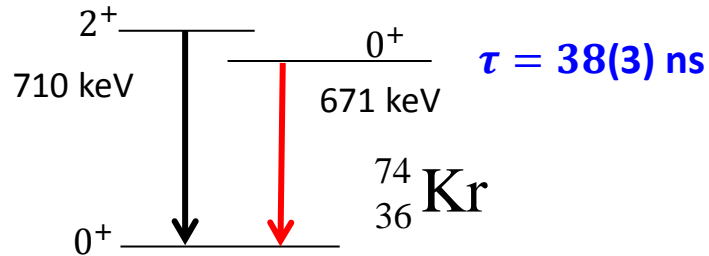
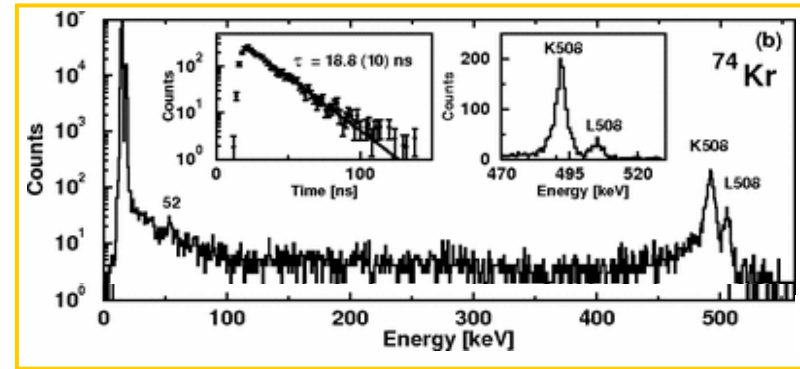
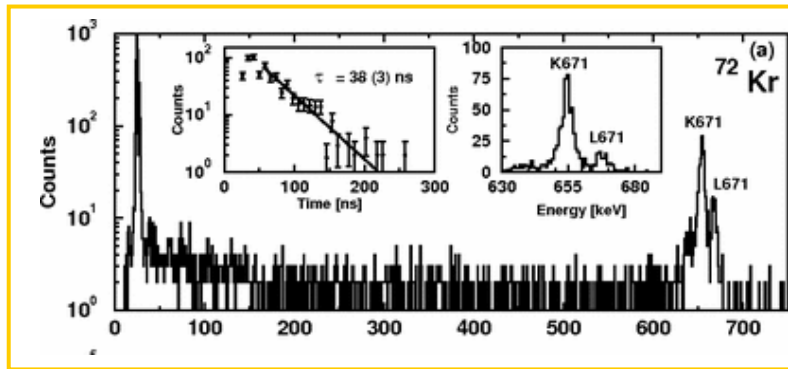
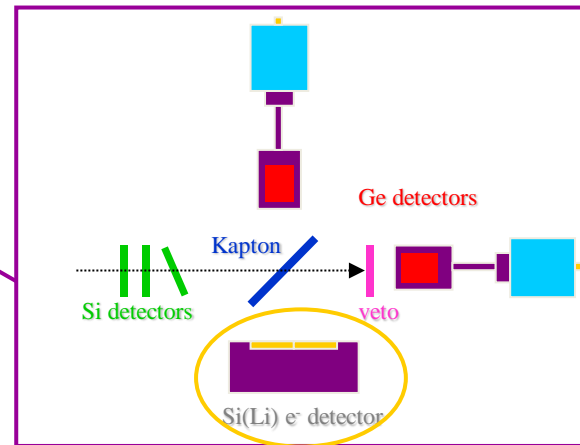
^9Be target



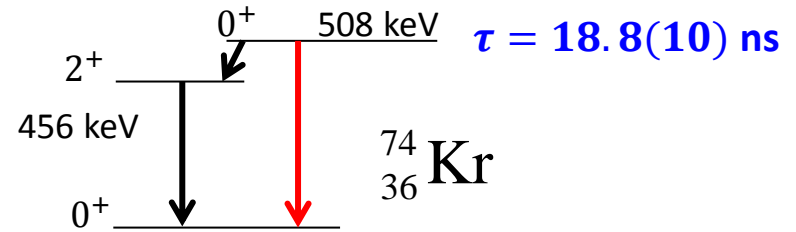
$^{72}\text{Kr} : \sim 2$ ions/sec



E. Bouchez et al.,
PRL 90, 082502 (2003)



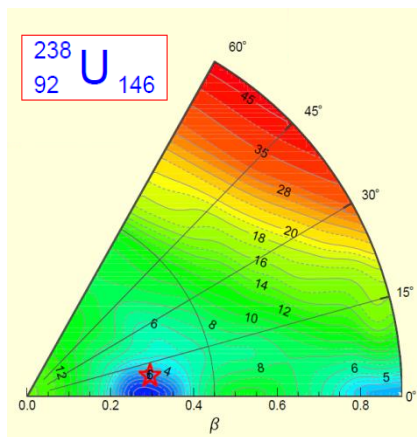
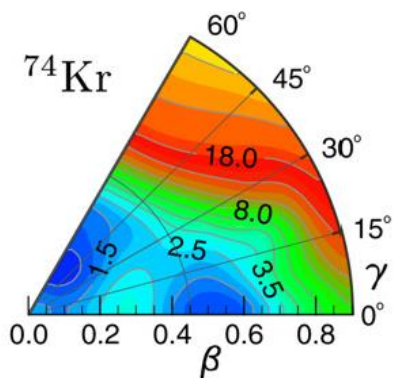
$$\rho(E0; 0_2^+ \rightarrow 0_1^+) = 72(6) \cdot 10^{-3}$$



$$\rho(E0; 0_2^+ \rightarrow 0_1^+) = 85(19) \cdot 10^{-3}$$

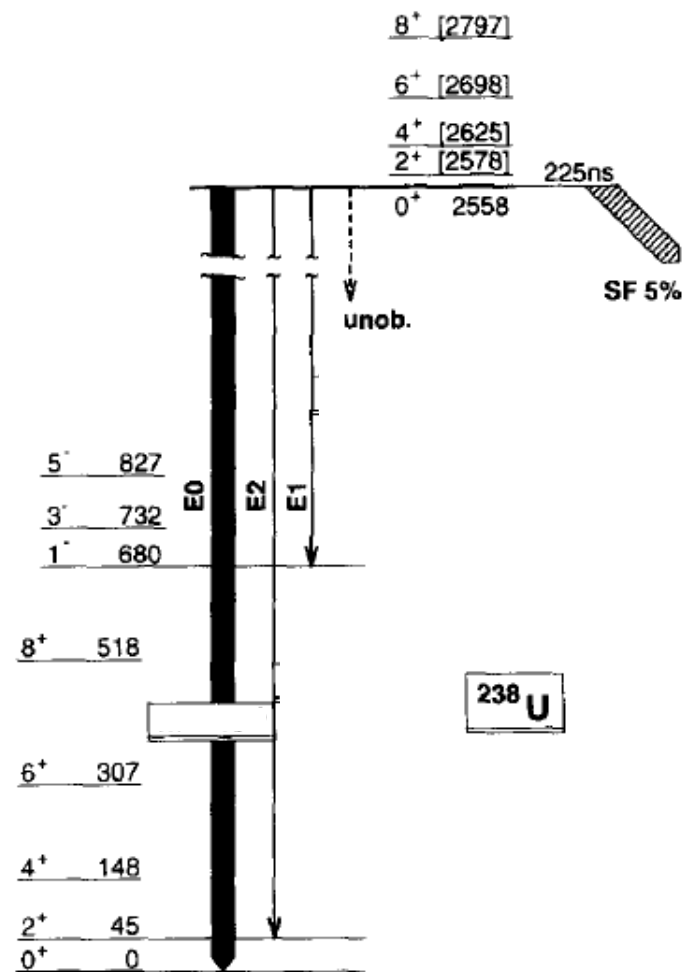
E0 transitions \Rightarrow shape coexistence

\Rightarrow shape isomers



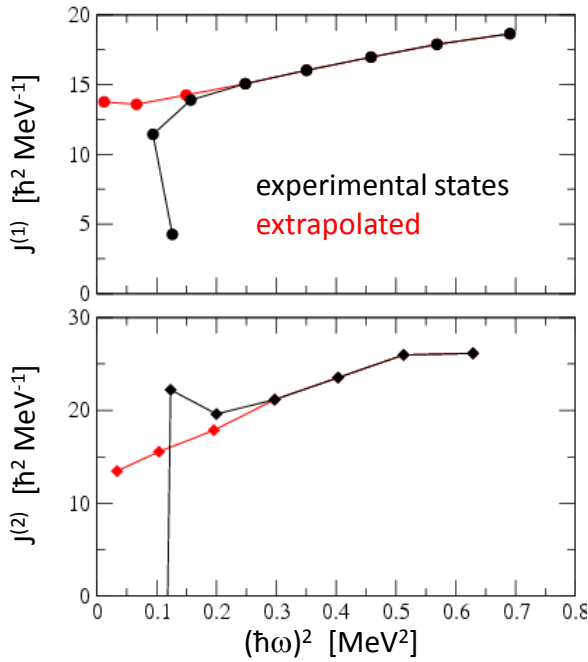
For ^{74}Kr it is: $\rho^2(E0; 0_2^+ \rightarrow 0_1^+) = 85 \cdot 10^{-3}$.
 What E0 strength do you expect for the superdeformed fission isomer at 2.558 MeV in ^{238}U ?

1. much smaller: $1.7 \cdot 10^{-6}$
2. about the same: $85 \cdot 10^{-3}$
3. much larger: 3.8



large difference in deformation
but: very little mixing

Two-level mixing

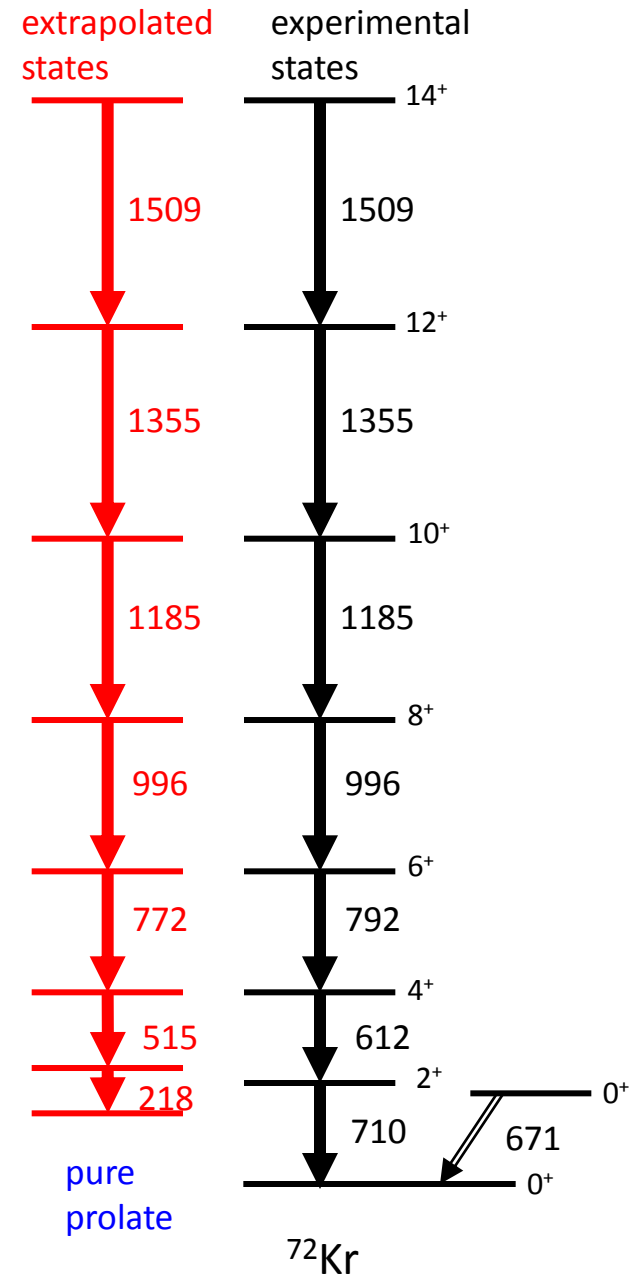
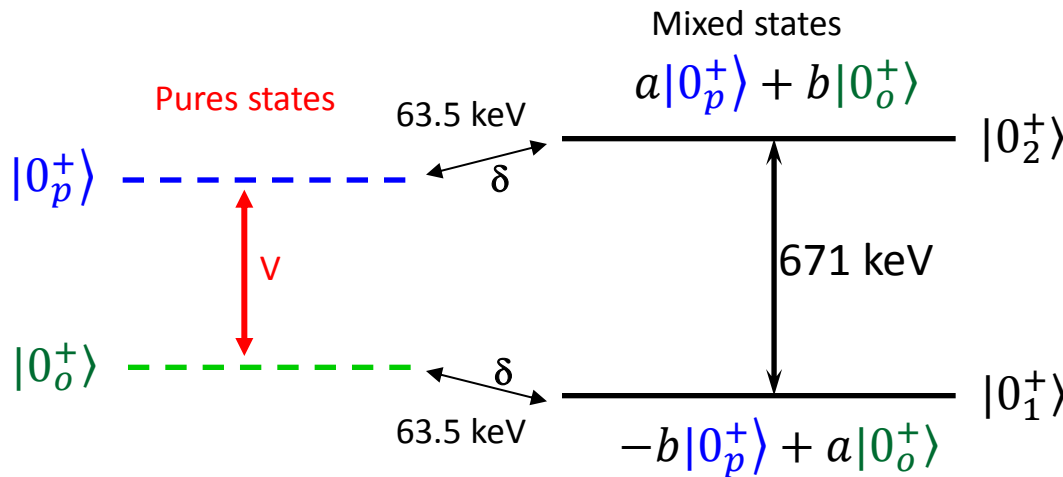


Regular rotational cascade at high spin:

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)$$

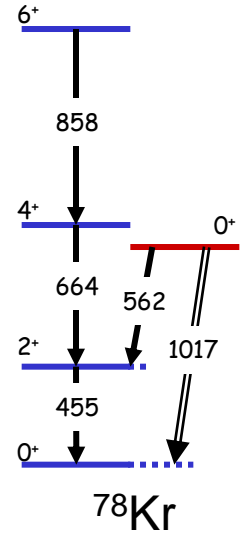
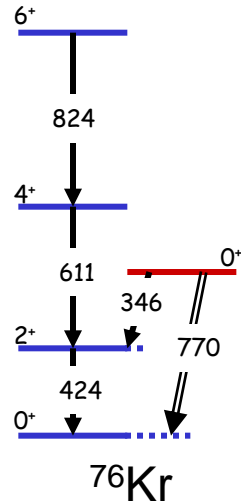
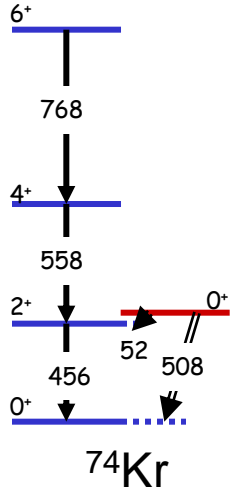
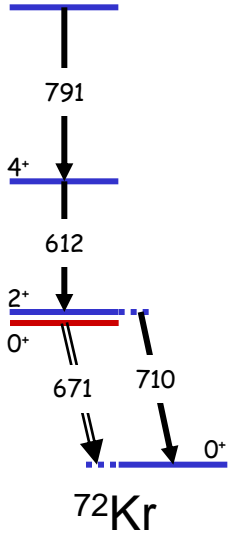
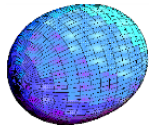
Rotational band is distorted at low spin.
 \Rightarrow influence of mixing

- Interaction V
- mixing amplitudes a, b

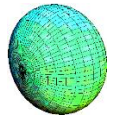


Systematics of the light krypton isotopes

prolate



- energy of excited 0^+
- E0 strengths $\rho^2(E0)$
- configuration mixing
- Inversion of ground state shape for ^{72}Kr



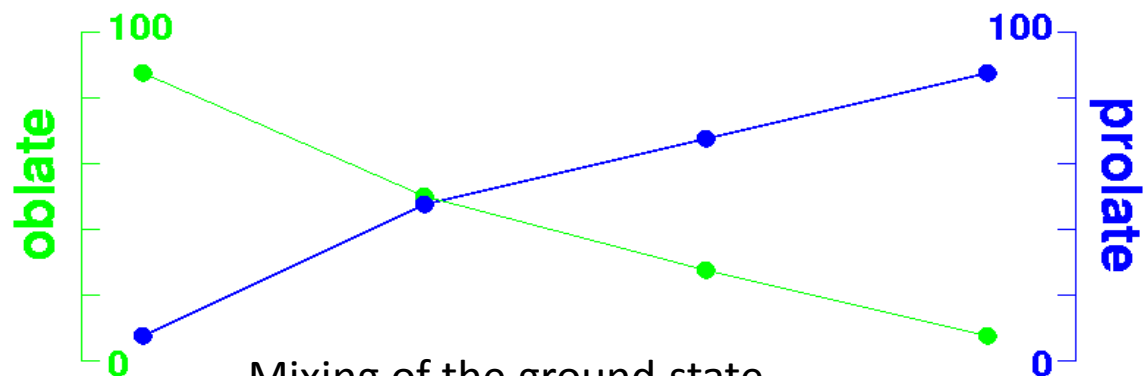
oblate

$\rho^2(E0)$ $70 \cdot 10^{-3}$

$91 \cdot 10^{-3}$

$79 \cdot 10^{-3}$

$47 \cdot 10^{-3}$



Mixing of the ground state (two-level mixing extrapolated from distortion of rotational bands)

E. Bouchez et. al., Phys. Rev. Lett. 90, 082502 (2003)

Matrix elements from Coulomb excitation

$$|0_1^+\rangle = a_0|0_p^+\rangle + b_0|0_o^+\rangle$$

$$|2_1^+\rangle = a_2|0_p^+\rangle + b_2|0_o^+\rangle$$

$$|0_2^+\rangle = -b_0|0_p^+\rangle + a_0|0_o^+\rangle$$

$$|2_2^+\rangle = -b_2|0_p^+\rangle + a_2|0_o^+\rangle$$

$$a_0^2 + b_0^2 = 1$$

$$a_2^2 + b_2^2 = 1$$

4 equations:

$$\langle 2_1^+ | E2 | 0_1^+ \rangle = b_0 b_2 \langle 2_o^+ | E2 | 0_o^+ \rangle + a_0 a_2 \langle 2_p^+ | E2 | 0_p^+ \rangle$$

$$\langle 2_1^+ | E2 | 0_2^+ \rangle = a_0 b_2 \langle 2_o^+ | E2 | 0_o^+ \rangle - a_2 b_0 \langle 2_p^+ | E2 | 0_p^+ \rangle$$

$$\langle 2_2^+ | E2 | 0_1^+ \rangle = a_2 b_0 \langle 2_o^+ | E2 | 0_o^+ \rangle - a_0 b_2 \langle 2_p^+ | E2 | 0_p^+ \rangle$$

$$\langle 2_2^+ | E2 | 0_2^+ \rangle = a_0 a_2 \langle 2_o^+ | E2 | 0_o^+ \rangle + b_0 b_2 \langle 2_p^+ | E2 | 0_p^+ \rangle$$

4 unknowns:

2 mixing amplitudes: a_0, a_2

$$Q_{0,p} = \sqrt{\frac{16\pi}{5}} \langle 2_p^+ | E2 | 0_p^+ \rangle$$

$$Q_{0,o} = \sqrt{\frac{16\pi}{5}} \langle 2_o^+ | E2 | 0_o^+ \rangle$$

no transitions between intrinsic prolate and oblate states:

$$\langle I_p^+ | E2 | J_o^+ \rangle = 0$$

using experimental coulomb matrix elements for ^{74}Kr :

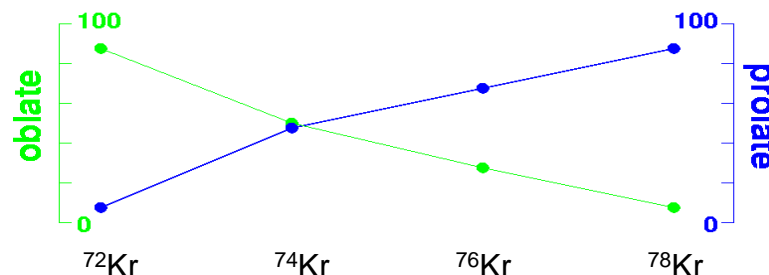
$$a_0^2 = 0.48(2)$$

$$a_2^2 = 0.82(20)$$

$$Q_{0,p} = 3.62(48) \text{ eb}$$

$$Q_{0,o} = -0.66(86) \text{ eb}$$

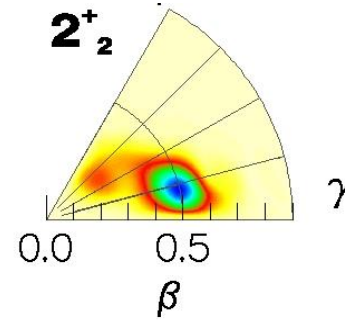
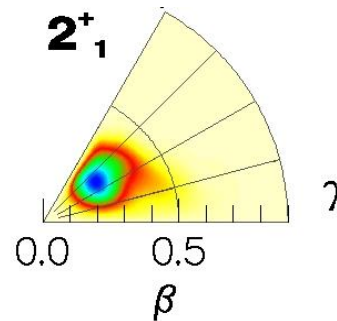
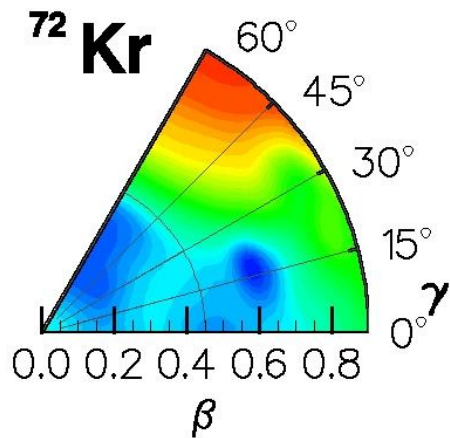
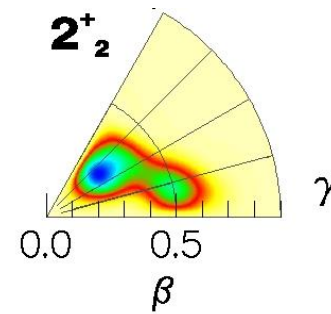
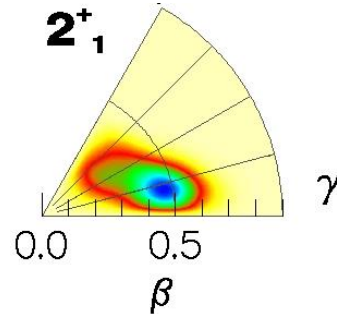
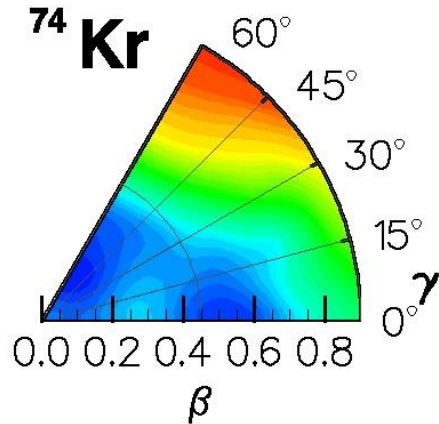
E. Clément et al.,
PRC 75, 054313 (2007)



consistent with mixing obtained from E0 transitions

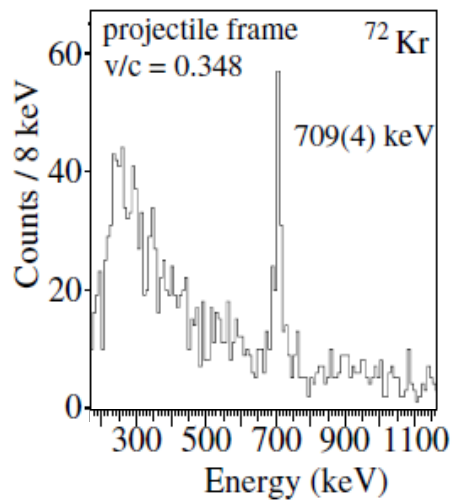
Composition of the wave functions

Inversion of prolate and oblate configurations in ^{72}Kr
reproduced by “beyond mean field” calculations
5DCH / Gogny D1S

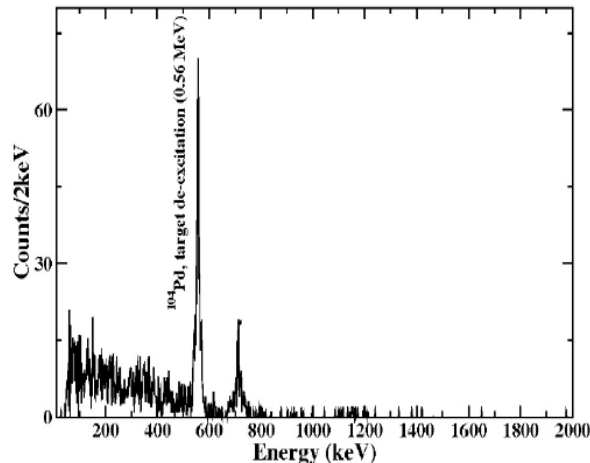


M. Girod et al., Phys. Lett. B 676, 39 (2009)

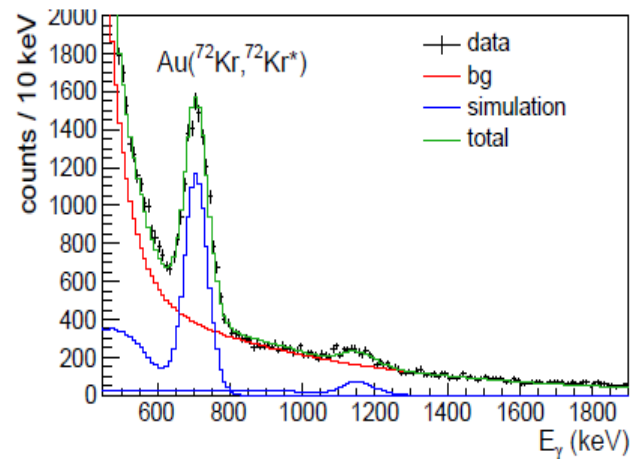
$B(E2; 0^+ \rightarrow 2^+)$ measurements for ^{72}Kr



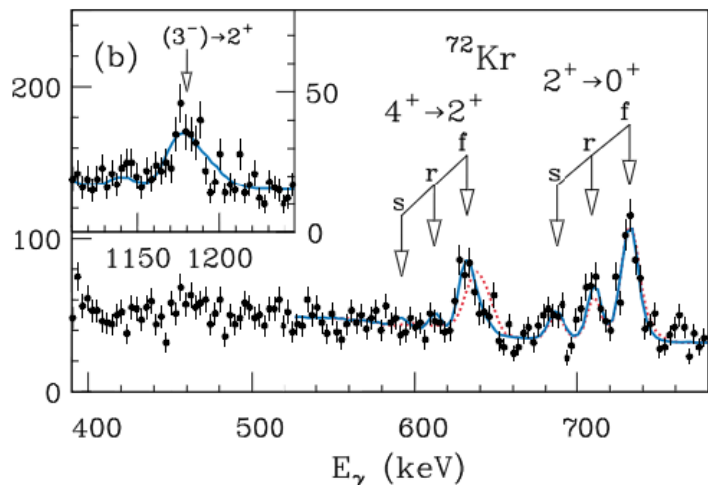
A.Gade et al., PRL 95, 022502 (2005)
 $\text{Au}(^{72}\text{Kr}, ^{72}\text{Kr}')$, 70 MeV/u, MSU



B.S.Nara Singh et al. (unpublished)
 ISOLDE IS478 (2012)
 $^{104}\text{Pd}(^{72}\text{Kr}, ^{72}\text{Kr}')$, 2.8 MeV/u



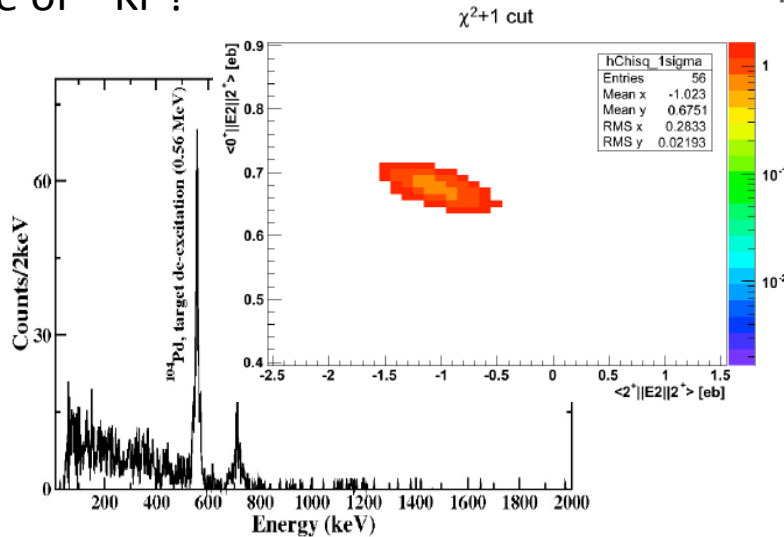
T.Arici, PhD Univ. Giessen (2017)
 $\text{Au}(^{72}\text{Kr}, ^{72}\text{Kr}')$, 150 MeV/u, RIKEN



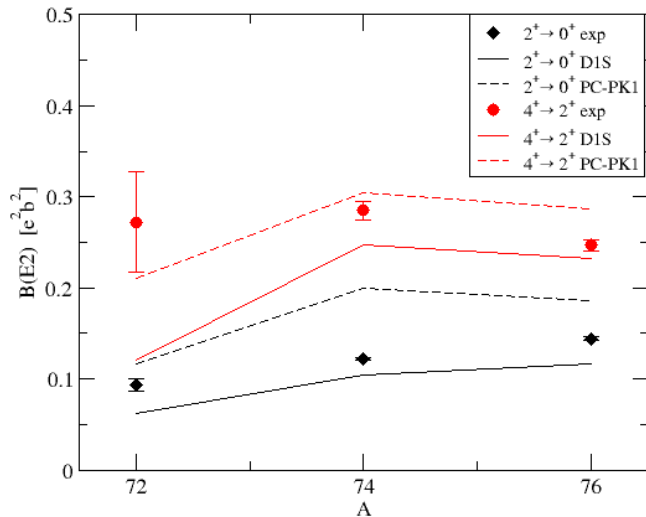
H.Iwasaki et al., PRL 112, 142502 (2014)
 $^9\text{Be}(^{74}\text{Kr}, ^{72}\text{Kr})$, RDDS lifetime, MSU

$B(E2; 0^+ \rightarrow 2^+)$ [$e^2\text{fm}^4$]	Author
4997(647)	Gade
4600(600)	Nara Singh
4050(750)	Iwasaki
4910(700)	Arici

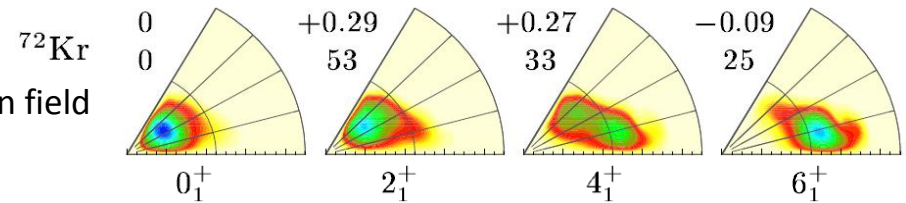
Shape of ^{72}Kr ?



B.S.Nara Singh, HIE-ISOLDE Workshop 2016

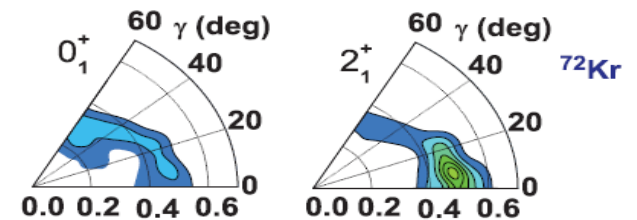


beyond mean field
Gogny D1S
5DCH



M.Girod et al., PLB 676, 39 (2009)

beyond mean field
RMF PC-PK1
5DCH



Y.Fu et al., PRC 87, 054305 (2013)

high-energy Coulomb excitation (MSU, RIKEN)

➤ not sensitive to Q_s

low-energy Coulomb excitation (ISOLDE)

➤ sensitive to $Q_s(2^+)$

➤ very low statistics

➤ favors prolate shape for 2^+ state

lifetime measurement (MSU)

➤ not sensitive to $Q_s(2^+)$

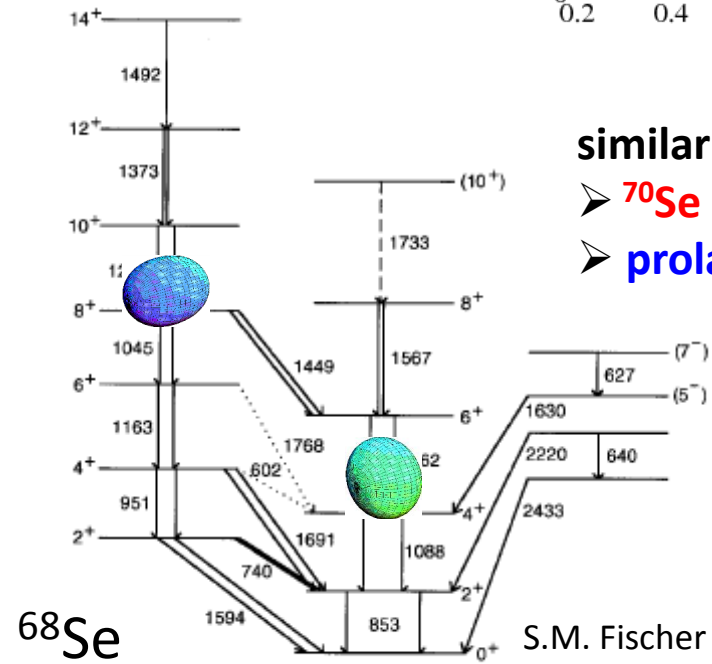
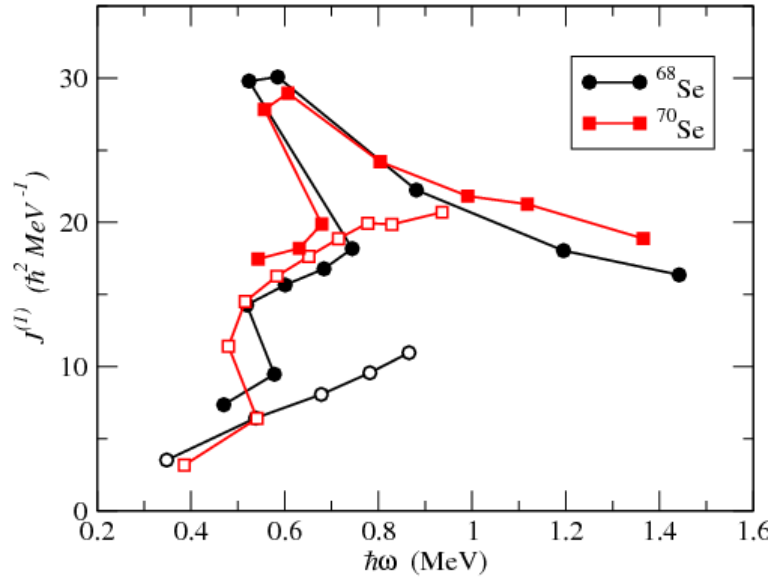
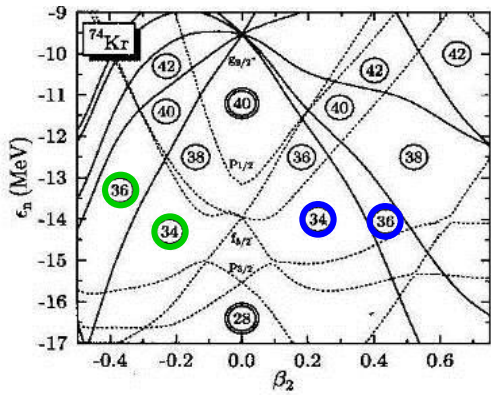
➤ large $B(E2; 4^+ \rightarrow 2^+)/B(E2; 2^+ \rightarrow 0^+)$

$^{74,76}\text{Kr}$: E.Clément et al., PRC 75, 054313 (2007)

^{72}Kr : H.Iwasaki et al., PRL 112, 142502 (2014)

➤ rapid transition from oblate ground state to prolate 2^+ in ^{72}Kr ?

Shape coexistence in light Selenium isotopes

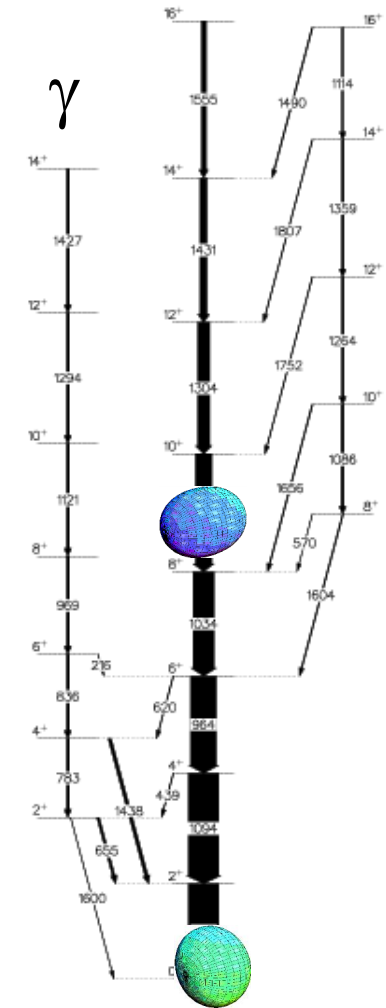


similar $J(1)$ in ^{68}Se and ^{70}Se :

- ^{70}Se oblate near ground state
- prolate at higher spin

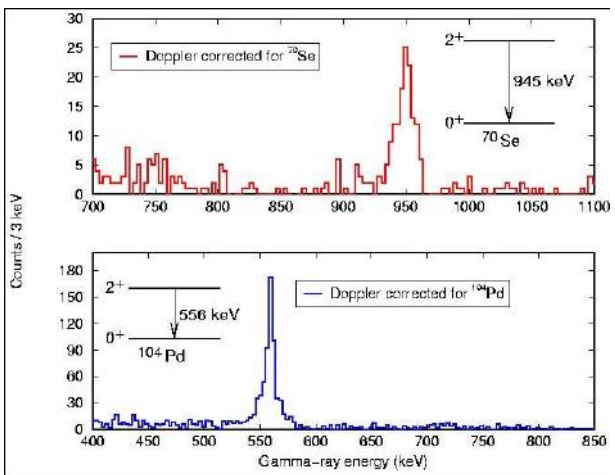
S.M. Fischer et al.,
PRC 67, 064318 (2003)

^{70}Se $\pi(g_{9/2})^2 \otimes \nu(g_{9/2})^2$

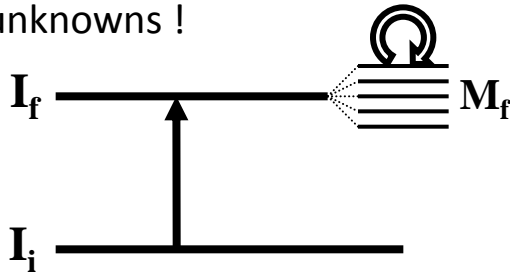


G. Rainovski et al.,
J.Phys.G 28, 2617 (2002)

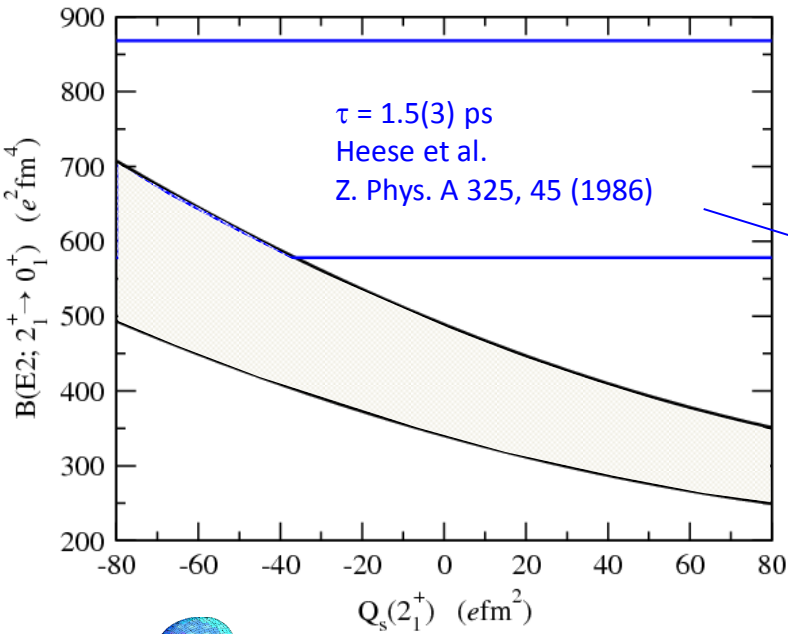
Coulomb excitation of ^{70}Se at CERN / ISOLDE



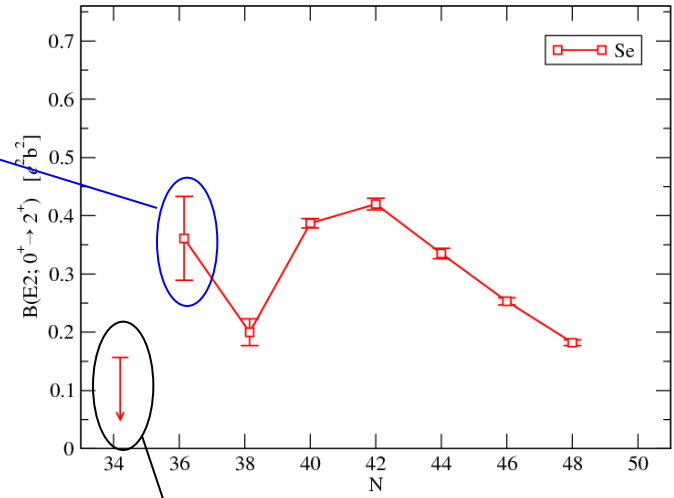
- ^{70}Se on ^{104}Pd at 2.94 MeV/u
- integral measurement of excitation probability $P(2^+)$
- normalization to target
- $P(2^+)$ depends on $B(E2)$ and Q_s
- one measurement, two unknowns !



A.M. Hurst et al.,
PRL 98, 072501 (2007)



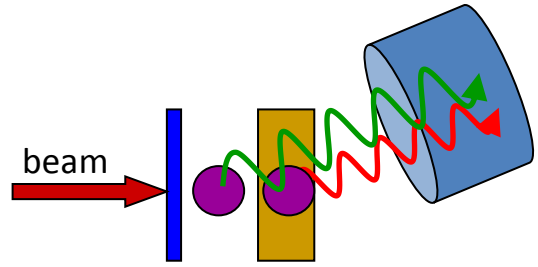
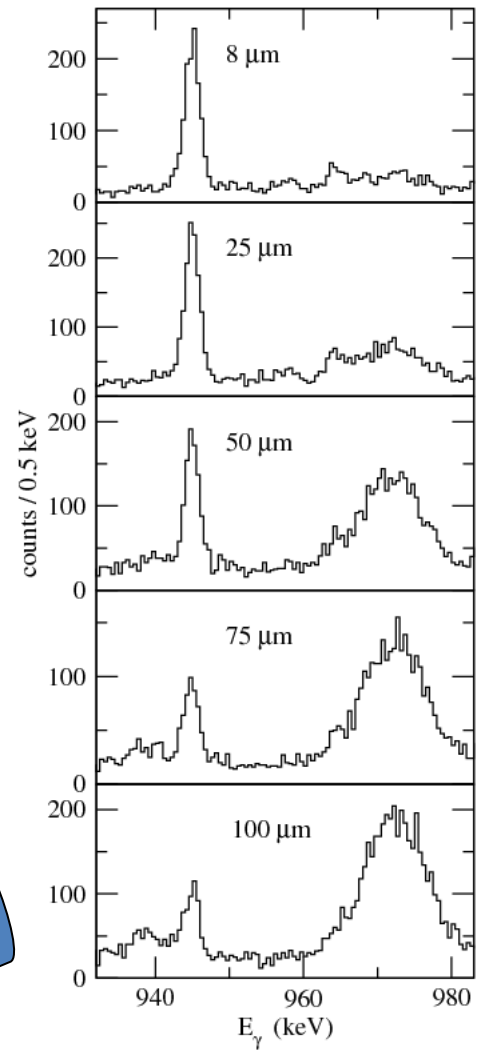
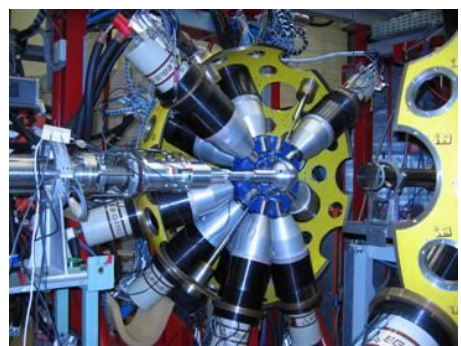
⇒ only **prolate** shape consistent with both Coulex and lifetime measurement



GANIL intermediate-energy Coulex
E. Clément et al., NIM A 587, 292 (2008)

Lifetimes in ^{70}Se and ^{72}Se revisited

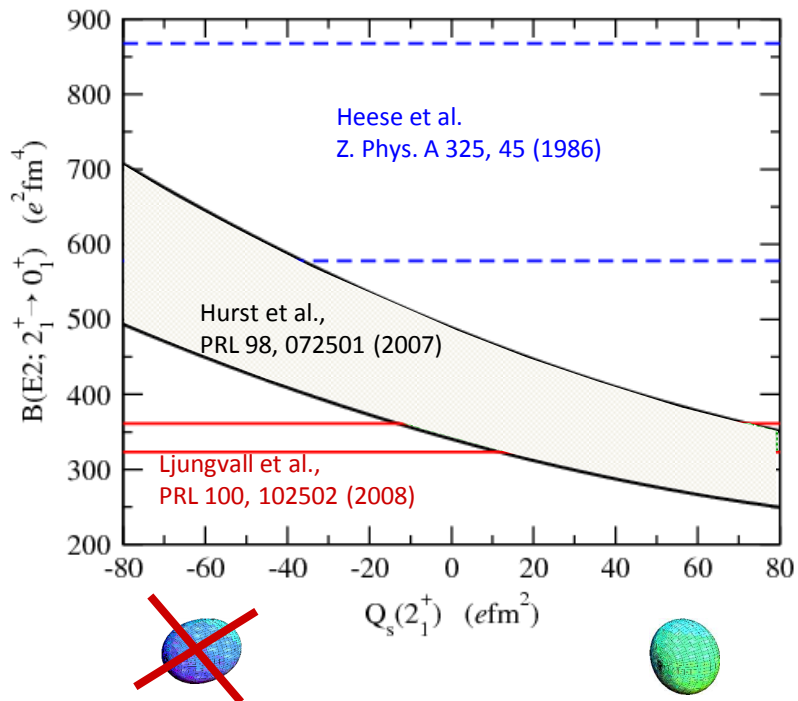
- Recoil-distance Doppler Shift
- GASP and Köln Plunger at Legnaro
- $^{40}\text{Ca}(^{36}\text{Ar},\alpha 2p)^{70}\text{Se}$
- 12 distances between 8 and 400 μm
- gated from above
 - ⇒ side feeding effects eliminated



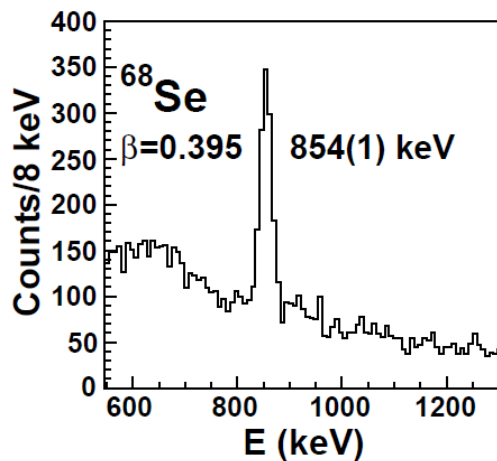
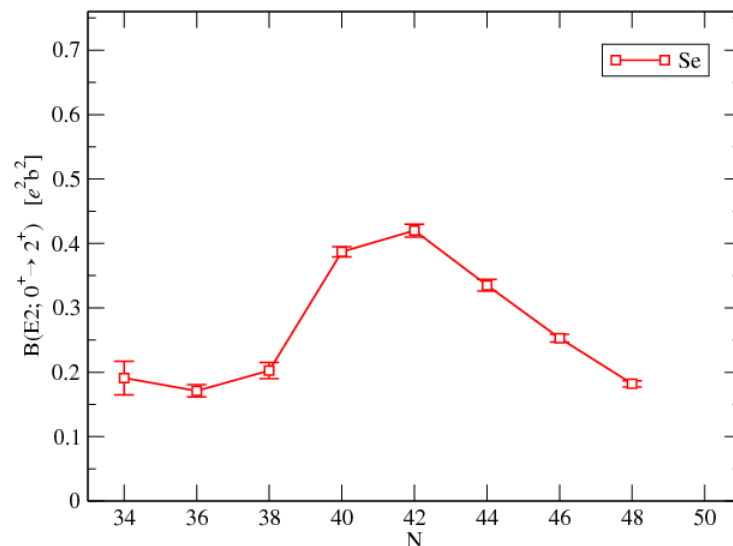
	old [1] τ (ps)	new[2] τ (ps)	$B(E2;\downarrow)$ ($e^2\text{fm}^4$)
^{70}Se	$^{40}\text{Ca}(^{36}\text{Ar},\alpha 2p)^{70}\text{Se}$		
2^+	1.5(3)	3.2(2)	342(19)
4^+	1.4(3)	1.4(1)	370(24)
6^+	3.9(9)	1.9(3)	530(96)
^{72}Se	$^{40}\text{Ca}(^{36}\text{Ar},4p)^{72}\text{Se}$		
2^+	4.3(5)	4.2(3)	405(25)
4^+	2.7(4)	3.3(2)	882(50)
6^+	2.3(2)	1.7(1)	1220(76)

[1] J. Heese et al., Z. Phys. A 325, 45 (1986)
 [2] J. Ljungvall et al., PRL 100, 102502 (2008)

Consequences for the shape of ^{70}Se



- oblate shape in ^{70}Se near the ground state
- Coulomb excitation and lifetime measurements are complementary techniques



intermediate-energy Coulomb excitation MSU
 $B(E2; \uparrow) = 1911(260) e^2\text{fm}^4$

A. Obertelli et al.
 PRC 80 (2009) 031304(R)

Shape evolution in the light Selenium isotopes

- **oblate** rotation prevails only in ^{68}Se
- ⇒ best example for shape coexistence in A=70 region
- experimental quadrupole moments needed

Q_s from Gogny configuration mixing calculation

