The new science of complex networks

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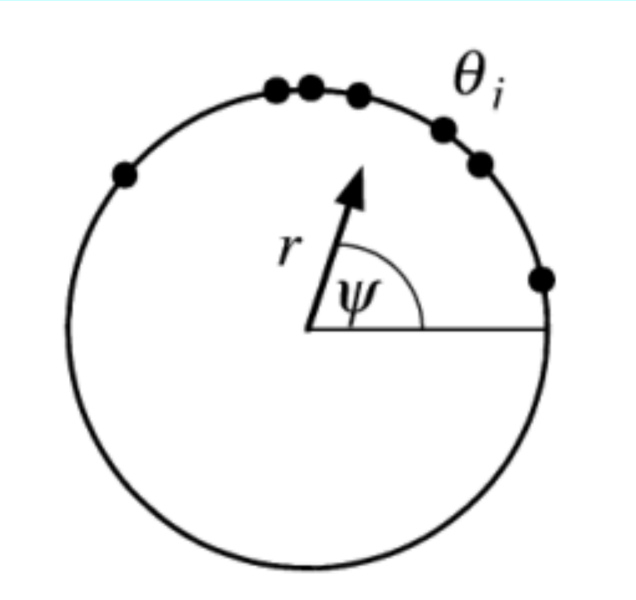
LEVERHULME TRUST

The Alan Turing Institute

COMPLEXITY SCIENCE HUB VIENNA

Interactions

Model of interacting phase oscillators



$$\dot{\vartheta}_i(t) = \omega_i \qquad i = 1, \ldots, N$$

$$\vartheta_i(t) = \omega_i \cdot t$$

$$\dot{\vartheta}_i(t) = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin[\vartheta_j(t) - \vartheta_i(t)]$$

Kuramoto model (1975)

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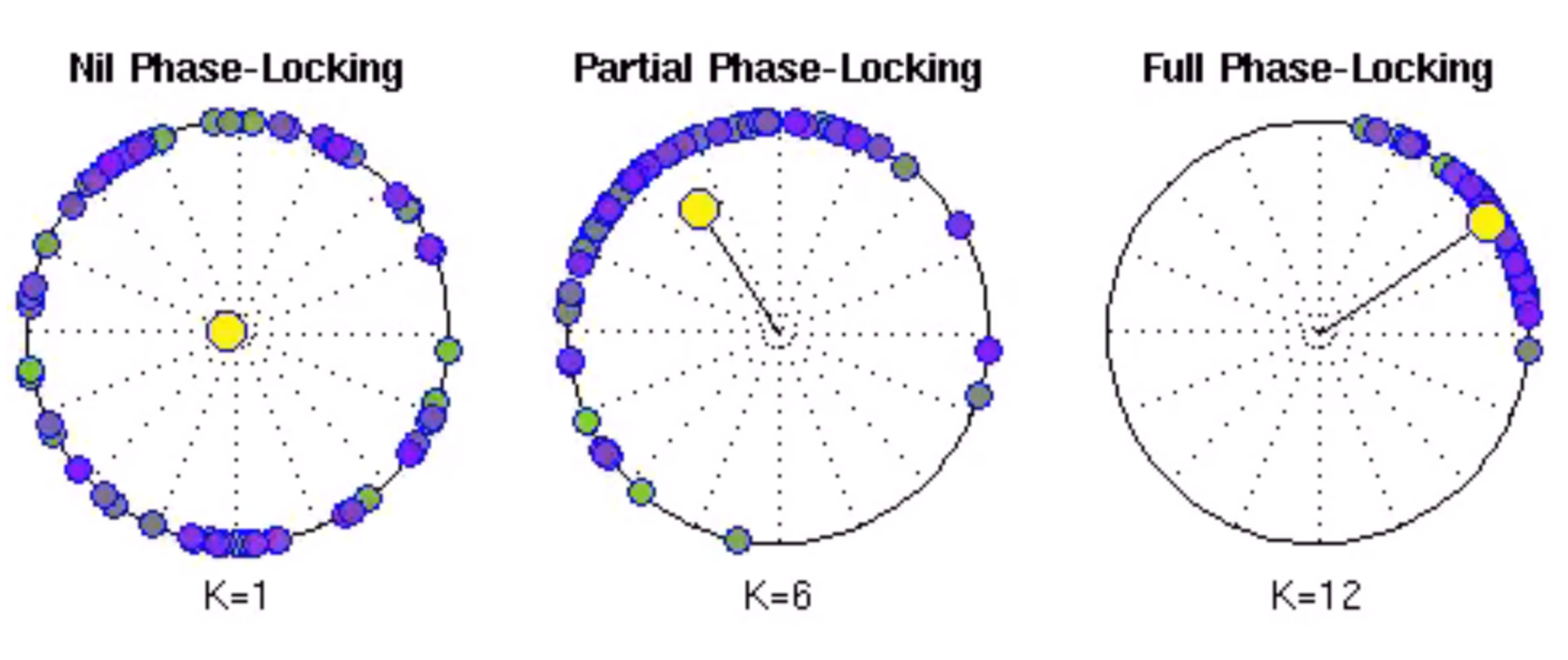
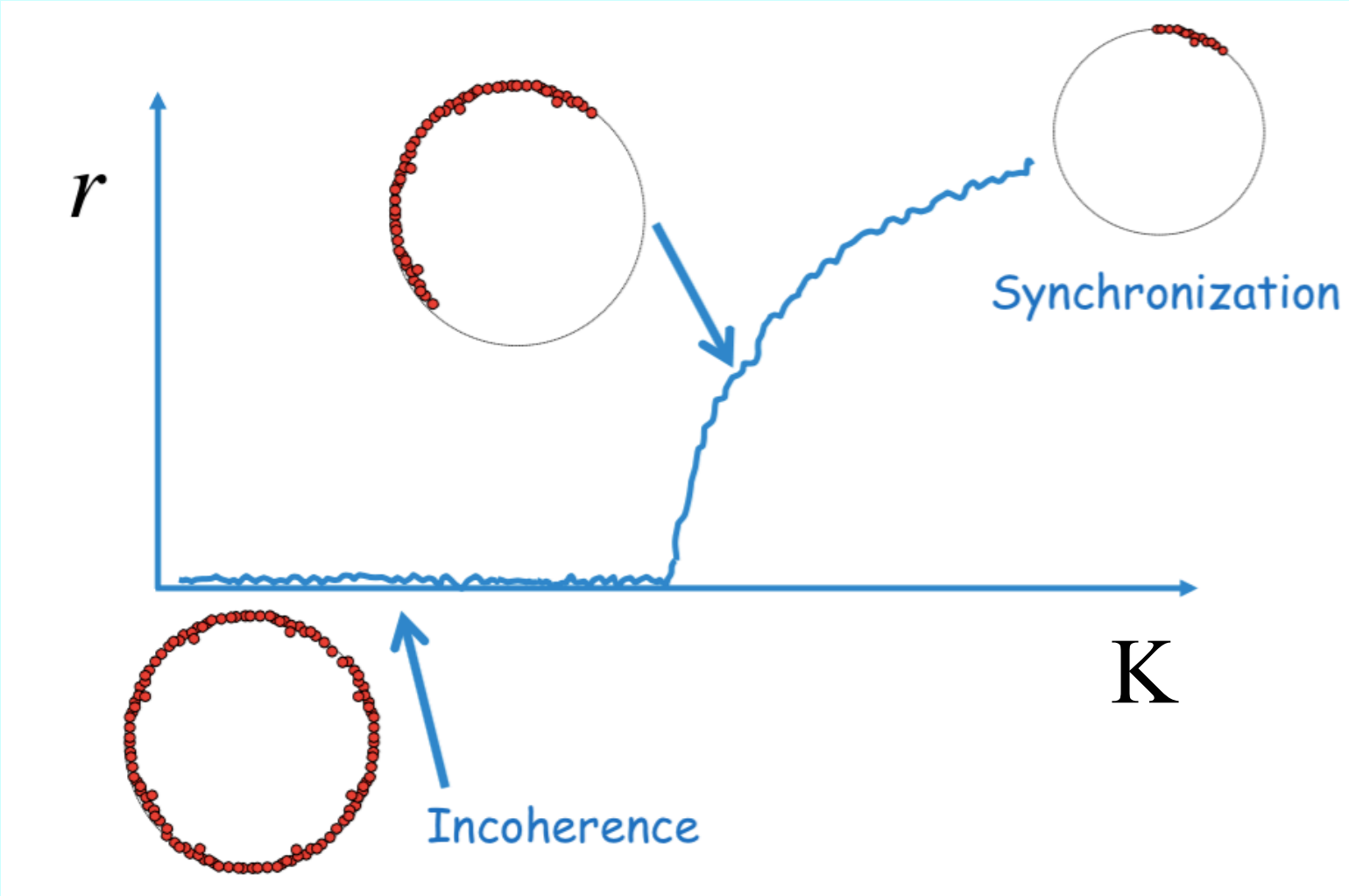
$$\dot{\vartheta}_i(t) = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin[\vartheta_j(t) - \vartheta_i(t)]$$

http://www.complexity-explorables.org/explorables/kuramoto/

Dirk Brockmann: Complexity explorables

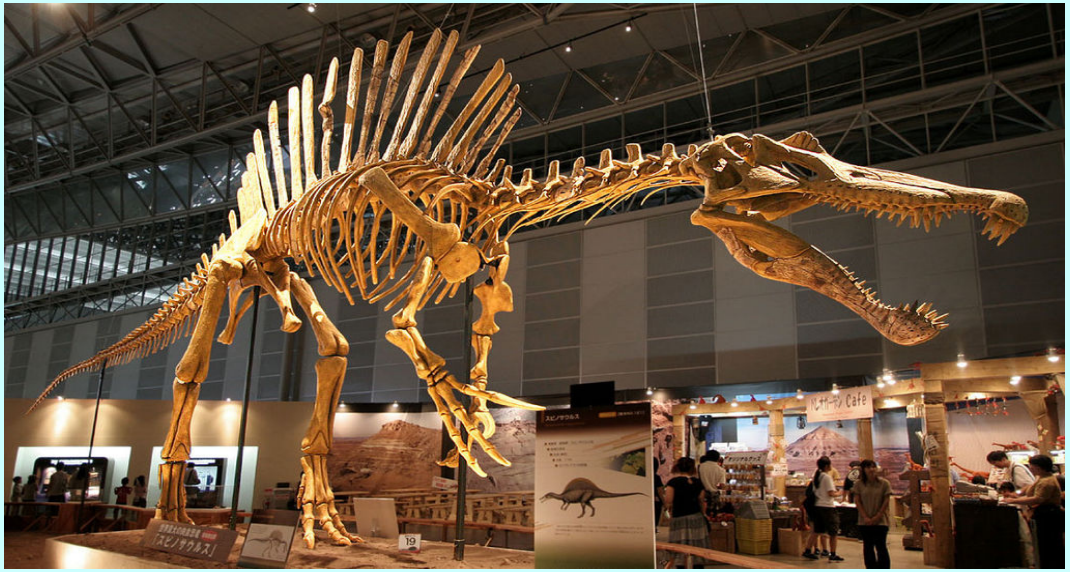




Kuramoto model (1975)

Complex networks…

..i.e. what can we learn of a complex system by looking at the backbone of its interactions ?



Two examples

Example 1: Bird flocks

We have many individuals plus their interactions…..

…..or even more than that !!

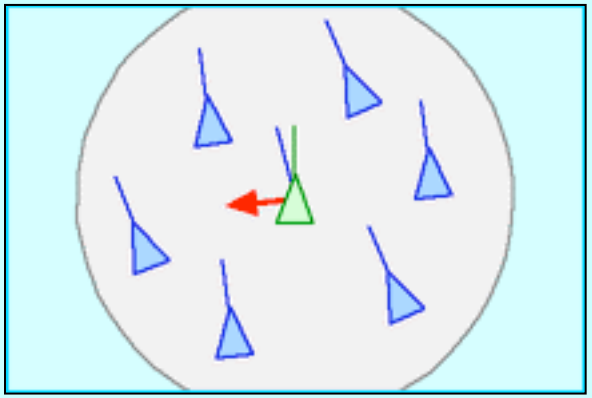
"More is different"
Anderson, Science 1972

In a complex system, simple rules give rise to complex behaviors

In a complex system, simple rules give rise to complex behaviors

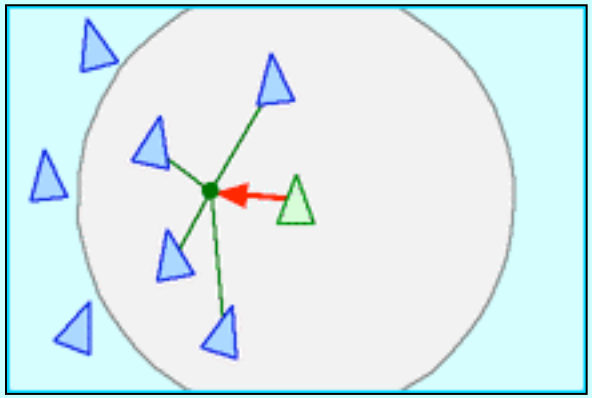
Reynolds (1986): Flocking model

ALIGNMENT



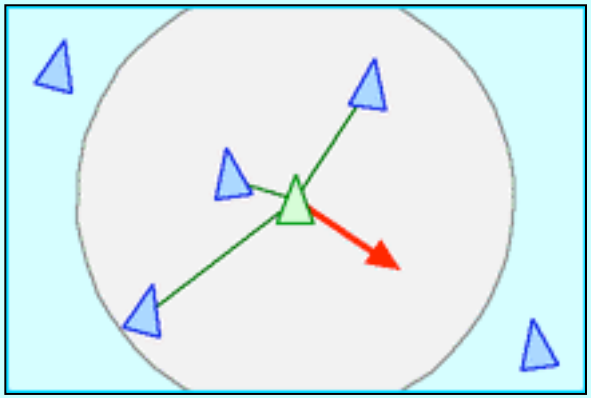
Steer to the average heading of local flockmates

COHESION



to move towards the average position of local flockmates

SEPARATION



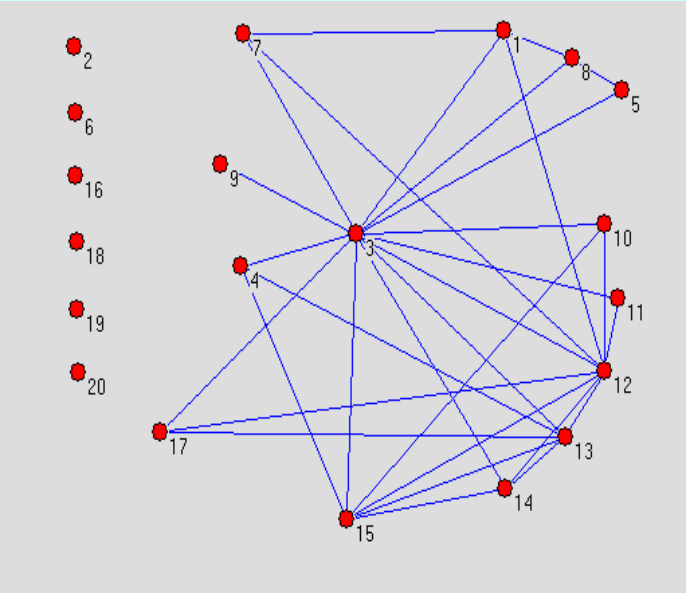
Steer to avoid crowding local flockmates

Vicsek et al. PRL 75(1995) 1226

Ballerini et al. PNAS 105(2008)1232

INTERACTIONS are the MAGIC INGREDIENT !!

We represent the interactions in a complex system as a complex network !!!



Three months of primate interactions
L. Wolfe (1992)

1) The animals are the nodes of the network

2) The interactions are the links of the network (can be weighted, directed, time-varying, etc..)

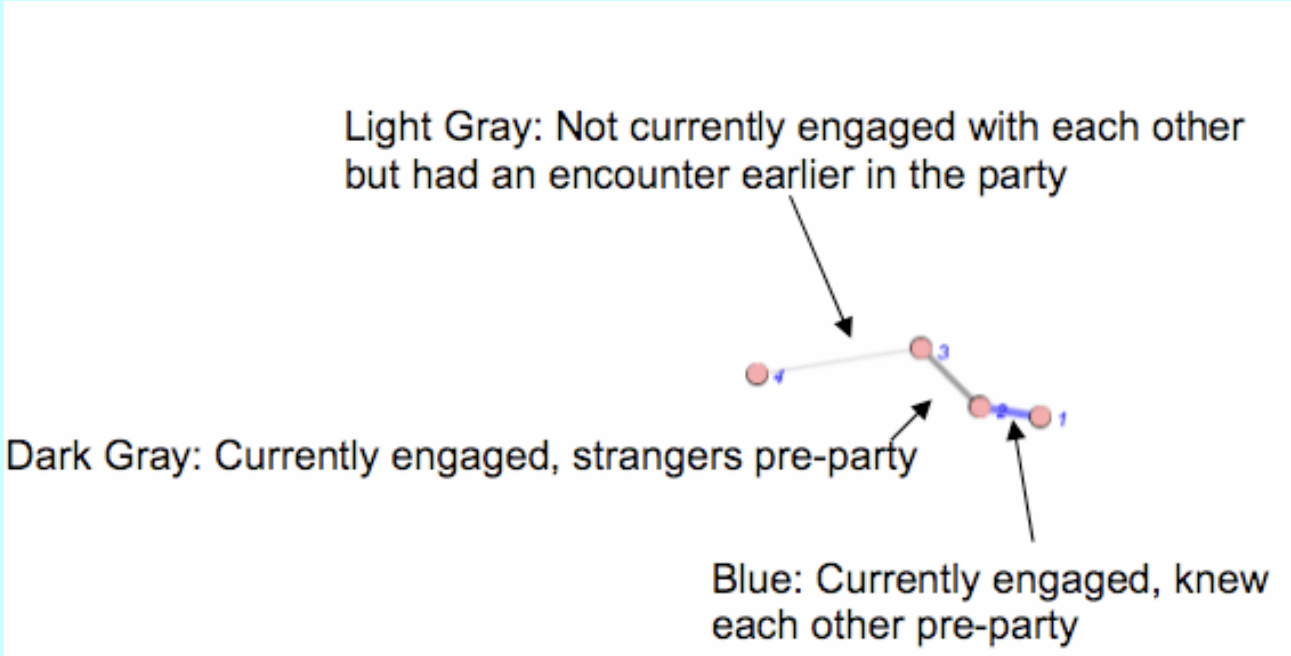
3) Networks are usually sparse, and neither regular nor random

Example 2:

slice:1 time:1.500-31.500
layout:Multiple component Kamada-Kawai layout optimum distance: 30.0minimum epsilon: 1.0cool factor: 0.25initial KK energy: 4265.800017385696
33
11

Example 2: The life of the party (Moody et al. 2005)

- A social network changing over time
- Kamada-Kawai (springs)
- Two types of nodes, three types of links



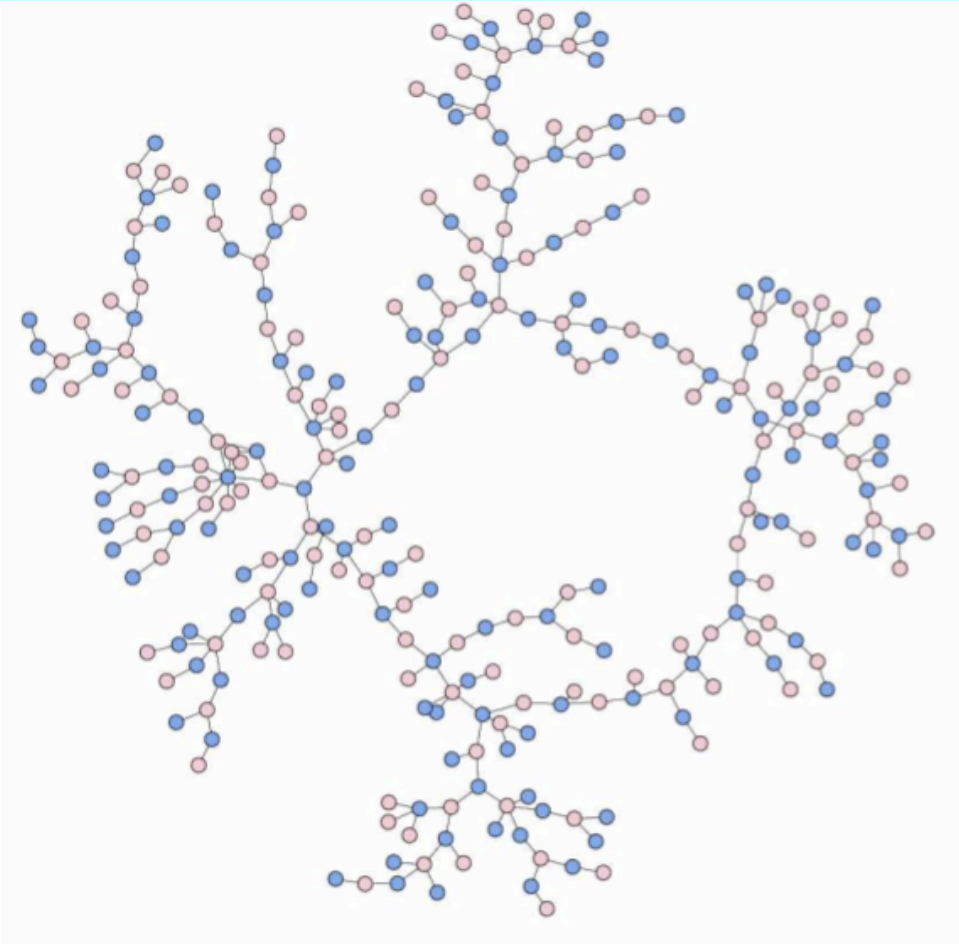
The life of the party (Moody et al. 2005)

- Encounters shrink social distances
- 95% of encounters with pre-known
- Bridging (n=24)
- Outside the party network (n=18)
- n=40 knows only 10 (av=31). Ends up with 23 new
- Homophilous pairings (n=30 + 31)
- High physical attractiveness (n63) vs low (n82)
- High self-monitors (size) (n=79)

It is exciting research to do because…

..networks are everywhere !!

Social Networks

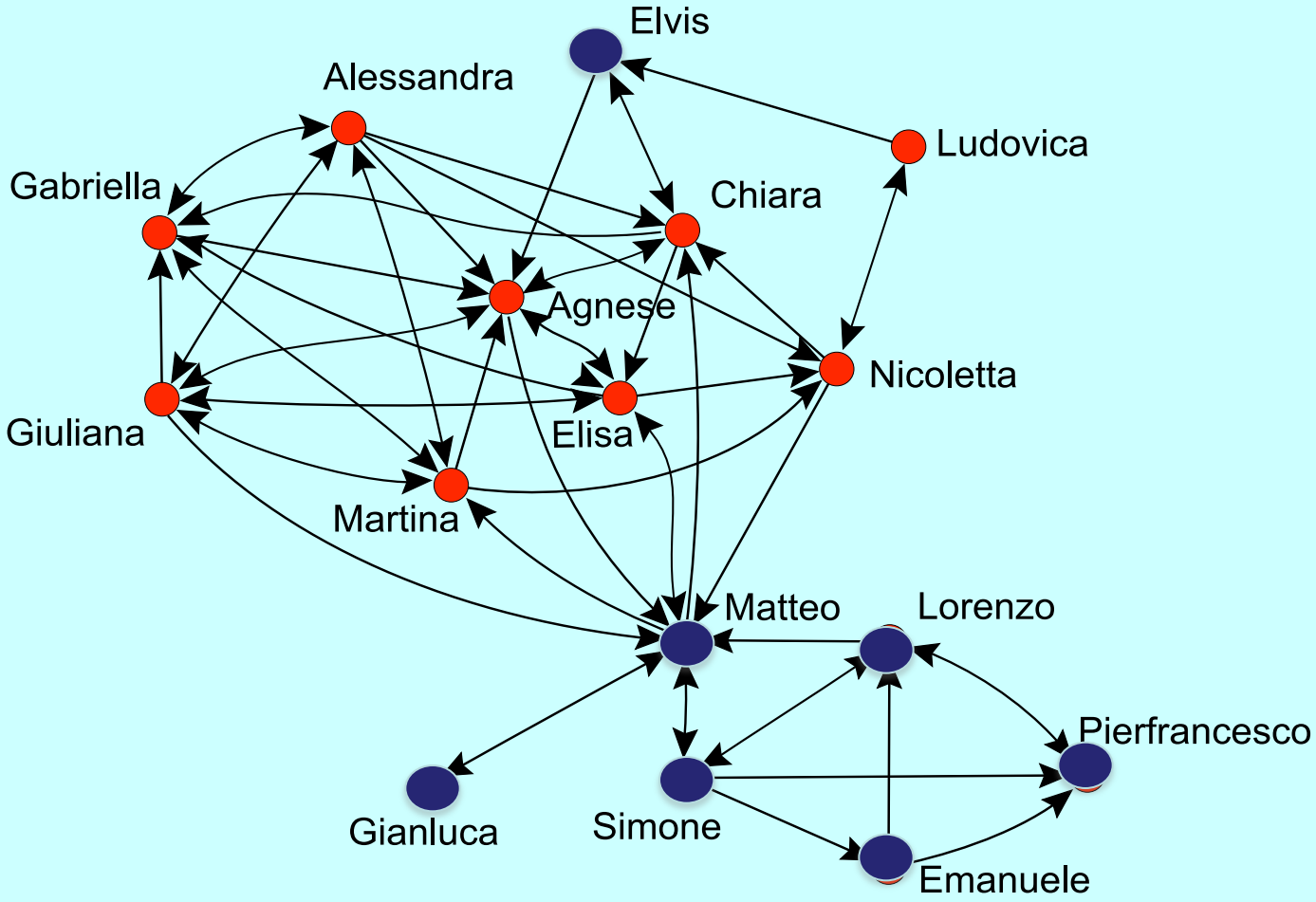


P S Bearman, J Moody, and K Stovel, Am J Soc (2004).

Relationships
in high school

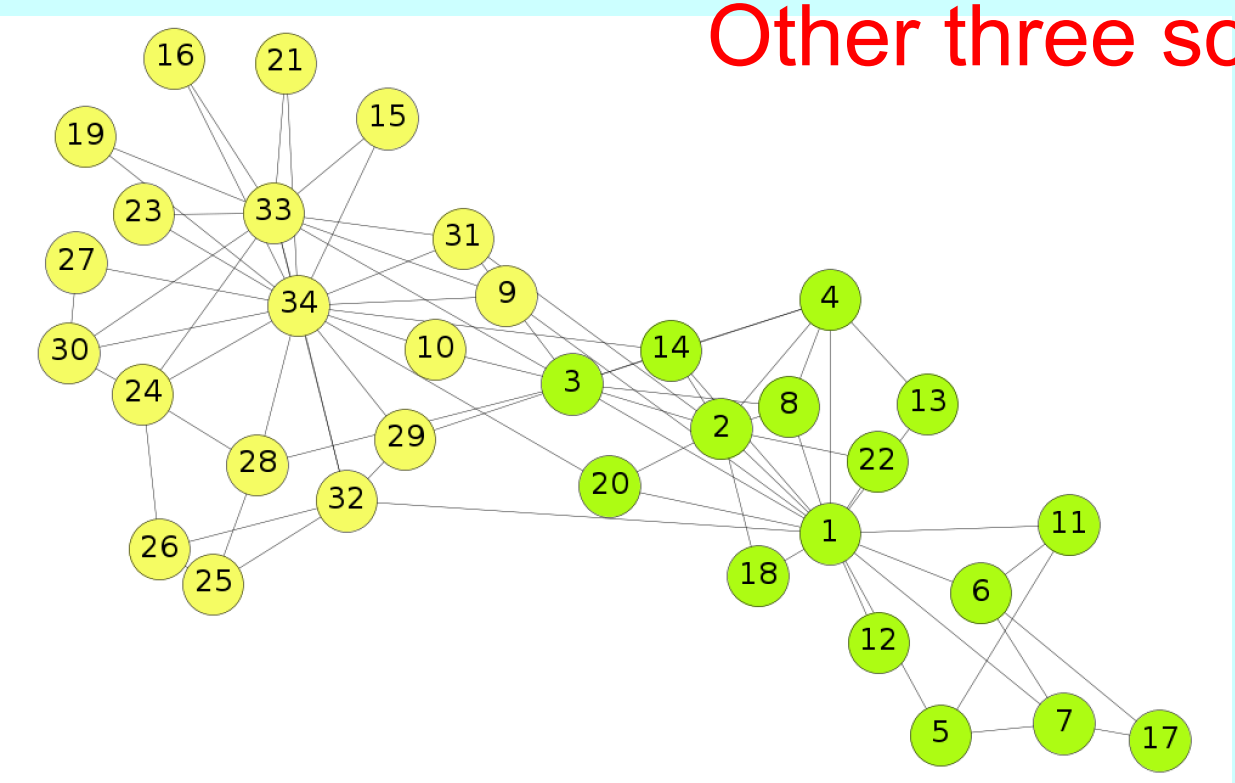
...You can construct yourself your home-made social network !!!

Social Networks

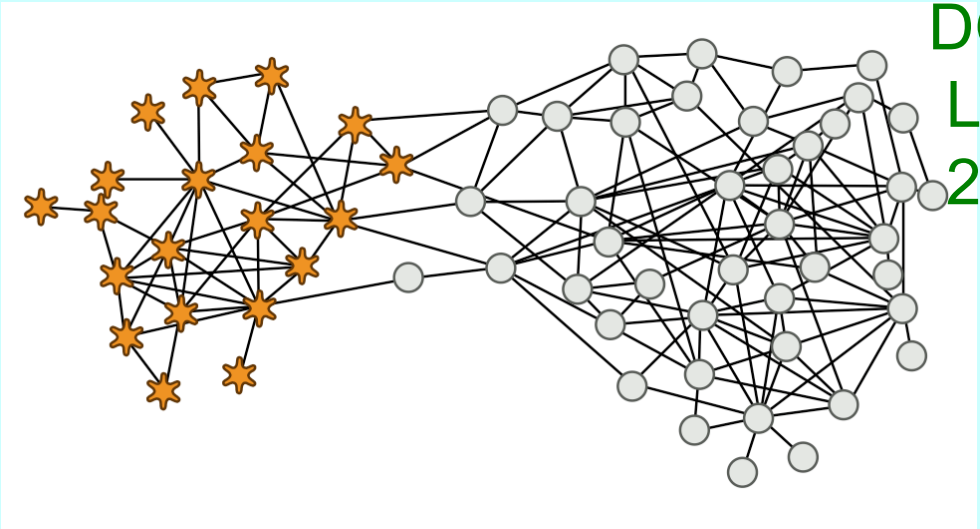


Friendships at the kindergarten of my daughter Elisa (2006)

Other three social examples

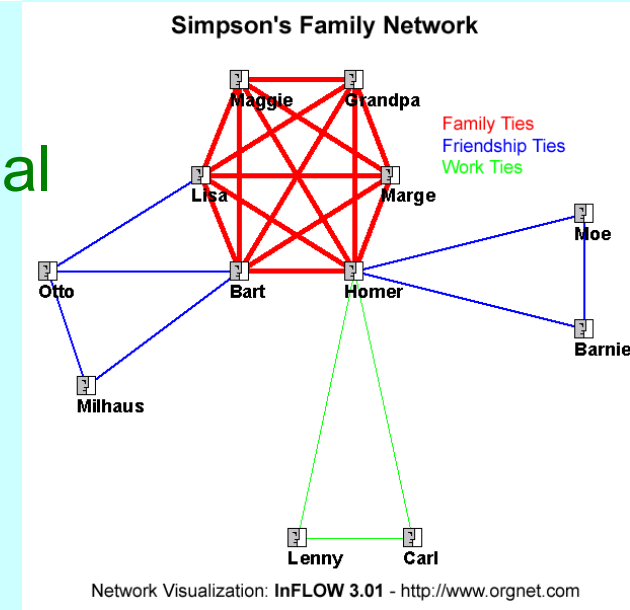


KARATE CLUB
Zachary 1977



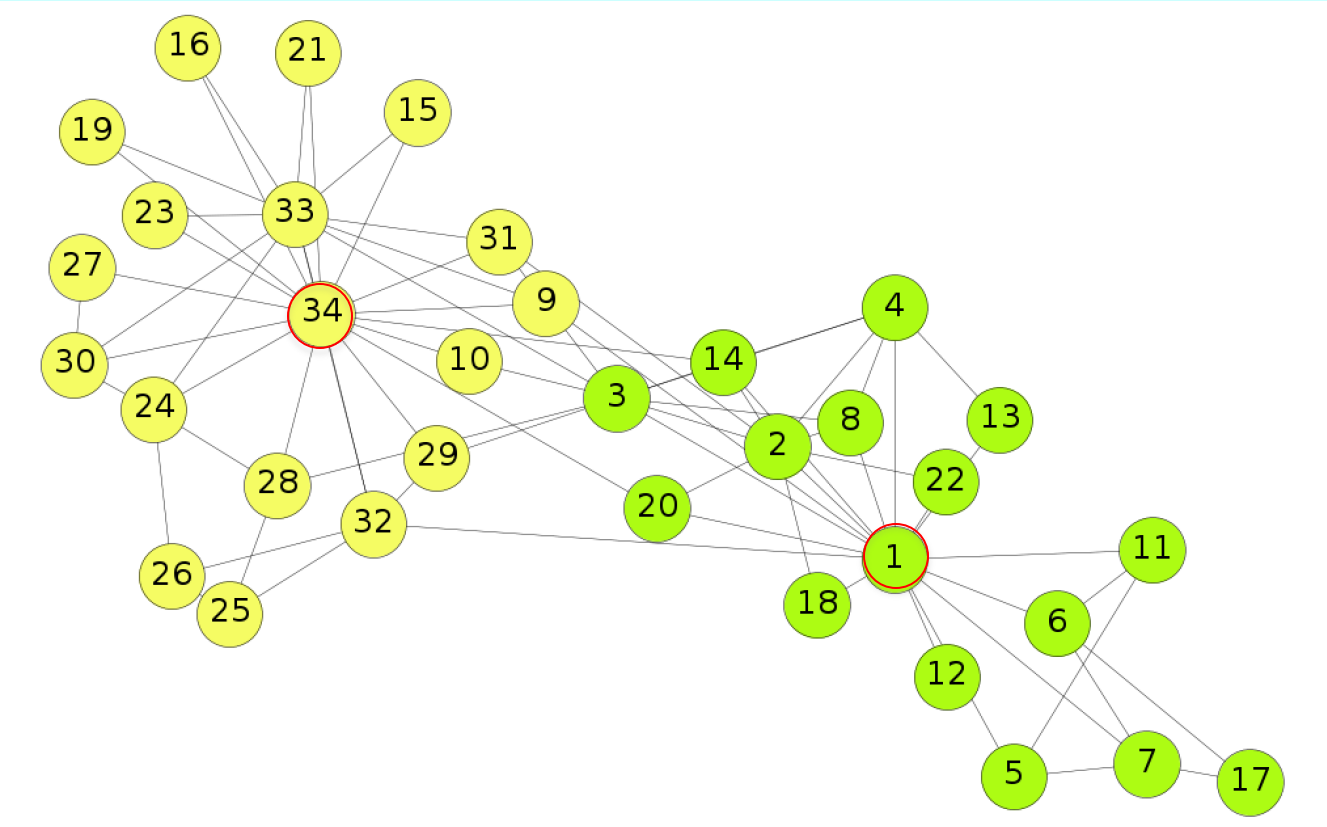
DOLPHINS
Lusseau et al
2003

COMICS



Zachary's karate club

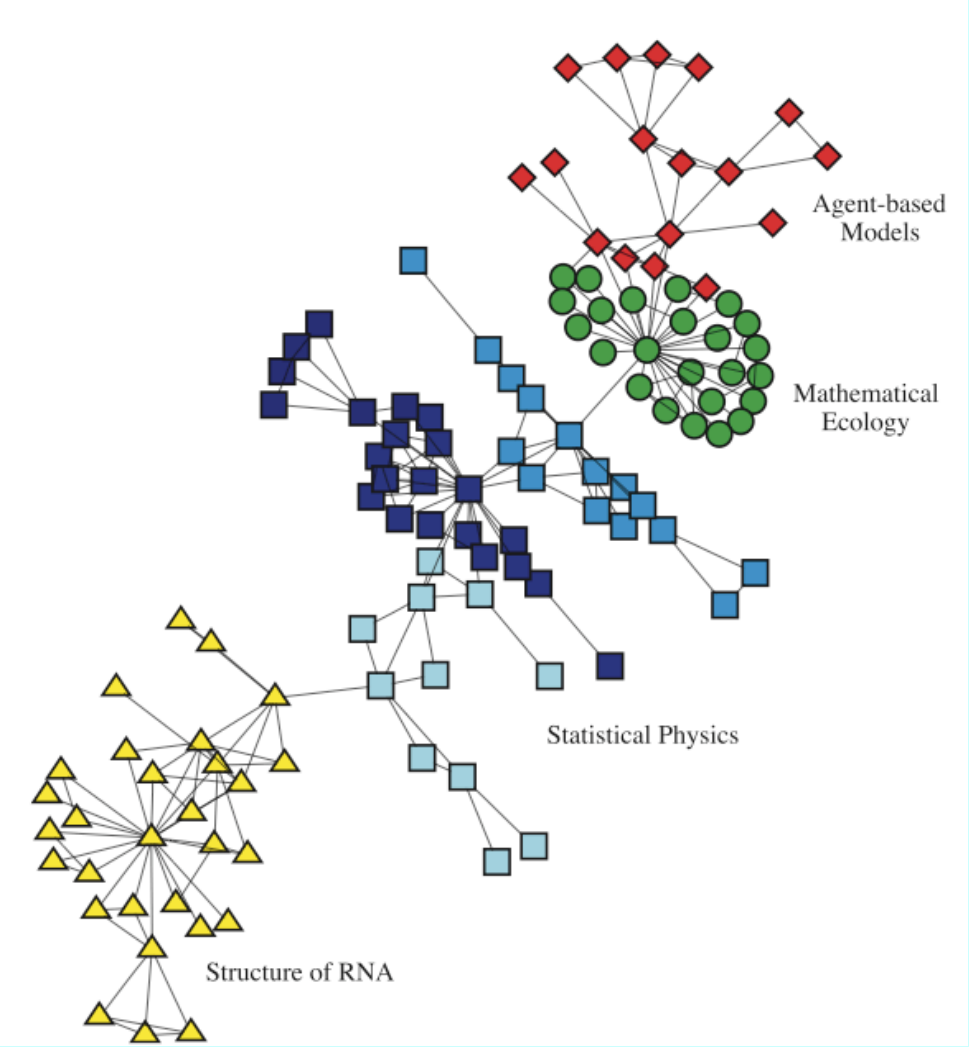
Outside club activities (3 years of observations before fission by anthropologist Wayne Zachary)



Fission:16 members following 1 (the instructor Mr. Hi),

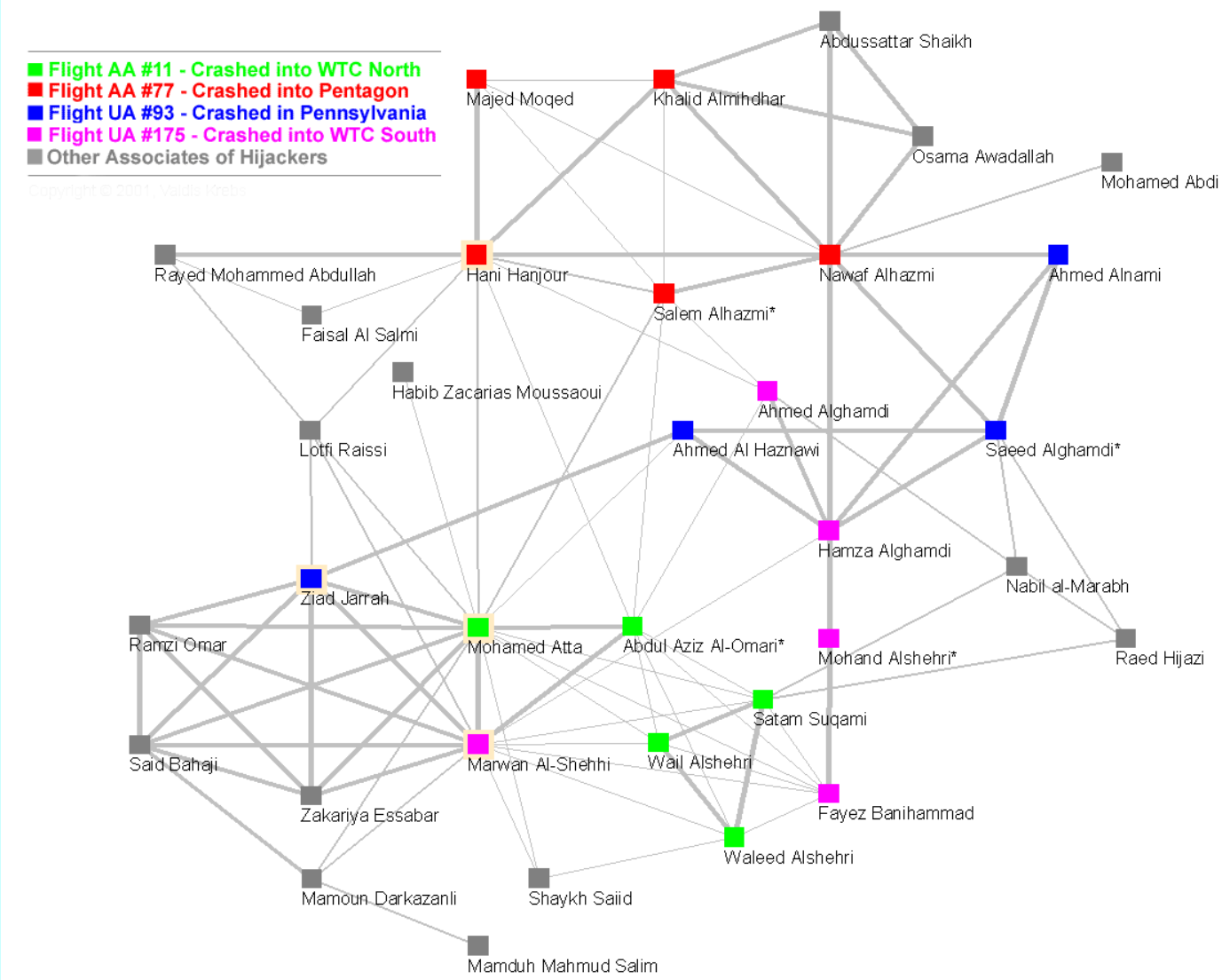
18 members following 34 (Mr. John A. the administrator)

The good and the bad collaboration net



Researchers at the Santa Fe Institute, Newman PNAS 04

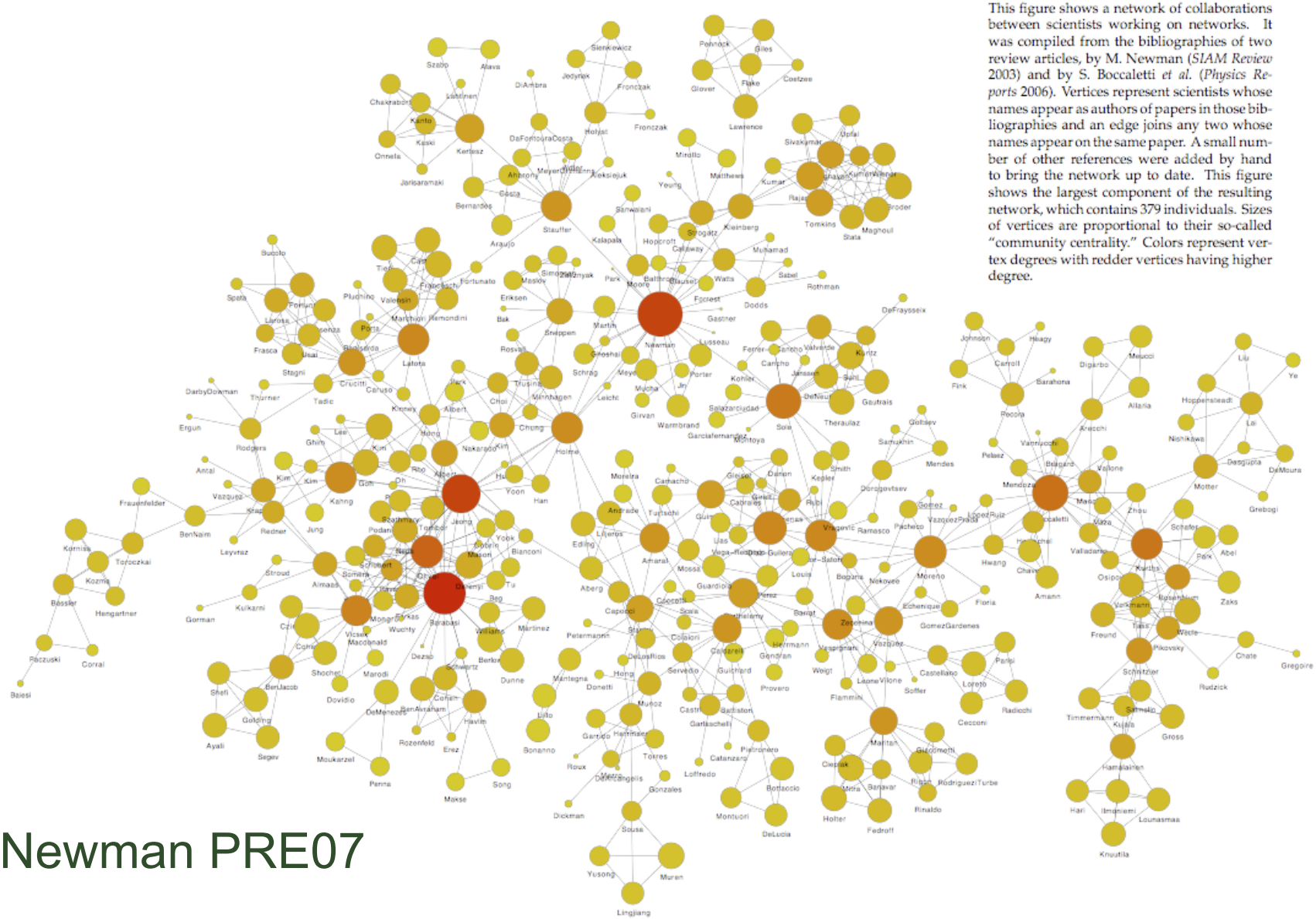
The good and the bad collaboration net



Weighted network

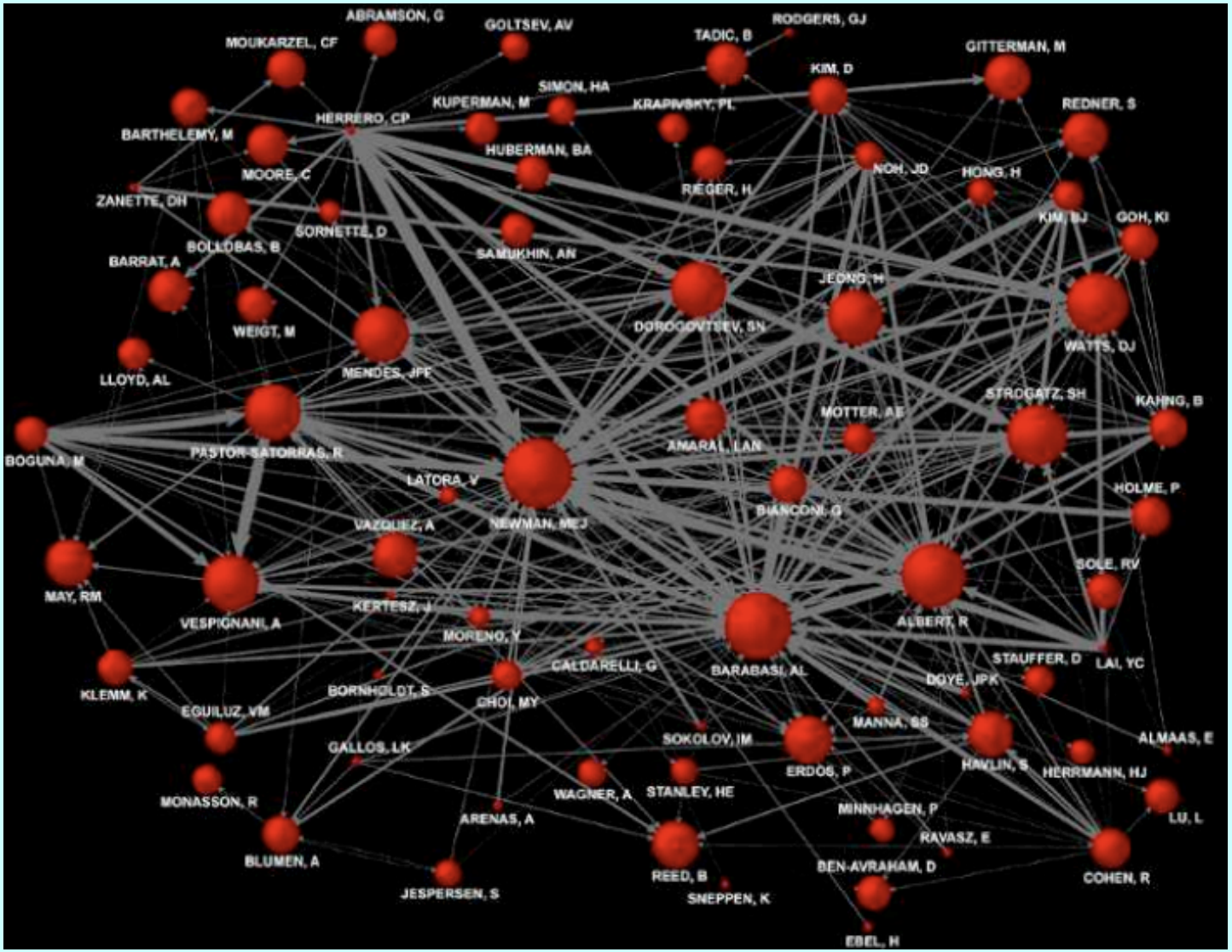
Krebs, Connections 2001

Collaboration network of scientists working on networks !!!



Newman PRE07

Citation network between authors working on networks



Radicchi et al. PRE 80, 056103 (2009)

Happiness is……

Fowler, Christakis. 2008

``*Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.''*

British Medical Journal 337, no. a2338: 1-9

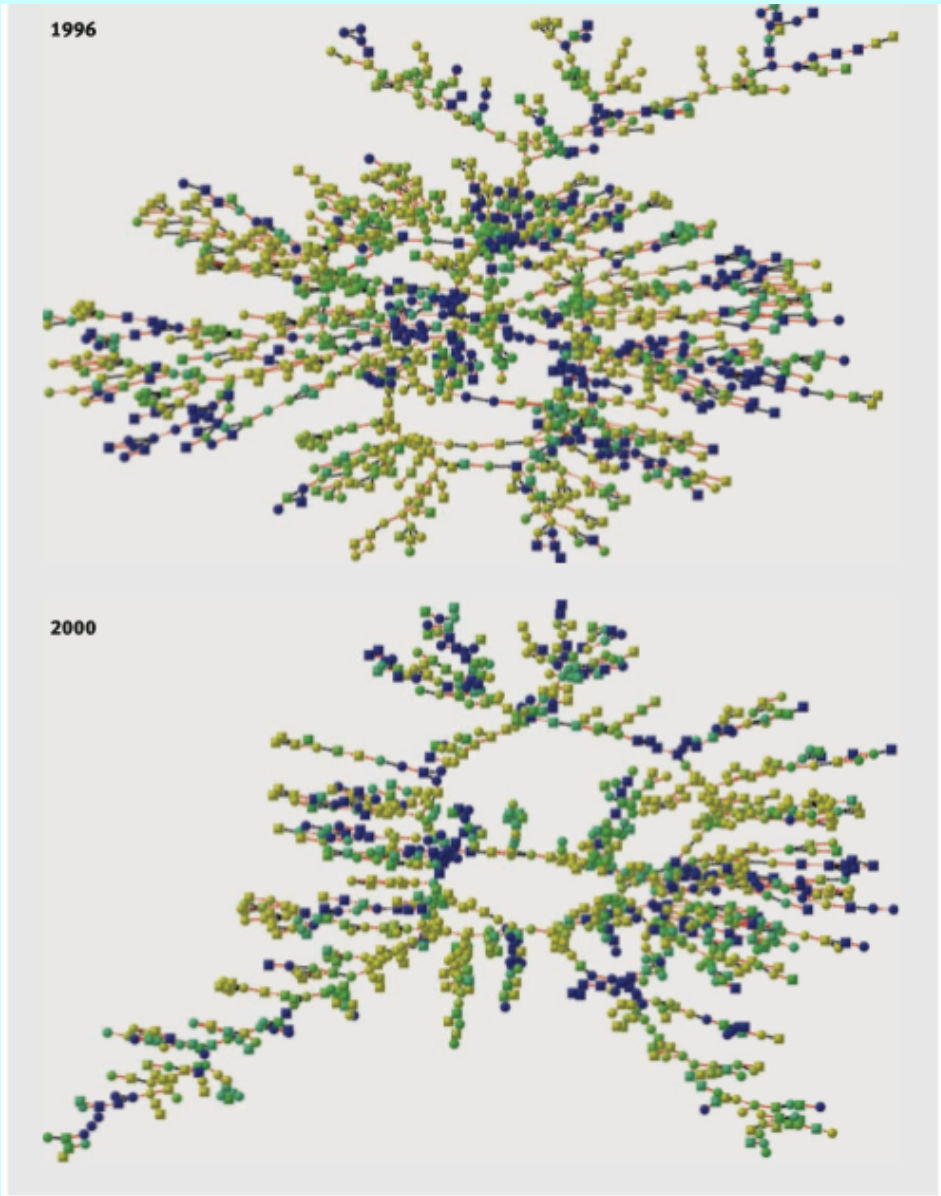


Fig 1 | Happiness clusters in the Framingham social network. Graphs show largest component of friends, spouses, and siblings at exam 6 (centred on year 1996, showing 1181 individuals) and exam 7 (year 2000, showing 1020 individuals). Each node represents one person (circles are female, squares are male). Lines between nodes indicate relationship (black for siblings, red for friends and spouses). Node colour denotes mean happiness of ego and all directly connected (distance 1) alters, with blue shades indicating least happy and yellow shades indicating most happy (shades of green are intermediate)

Happiness is……having happy friends !!

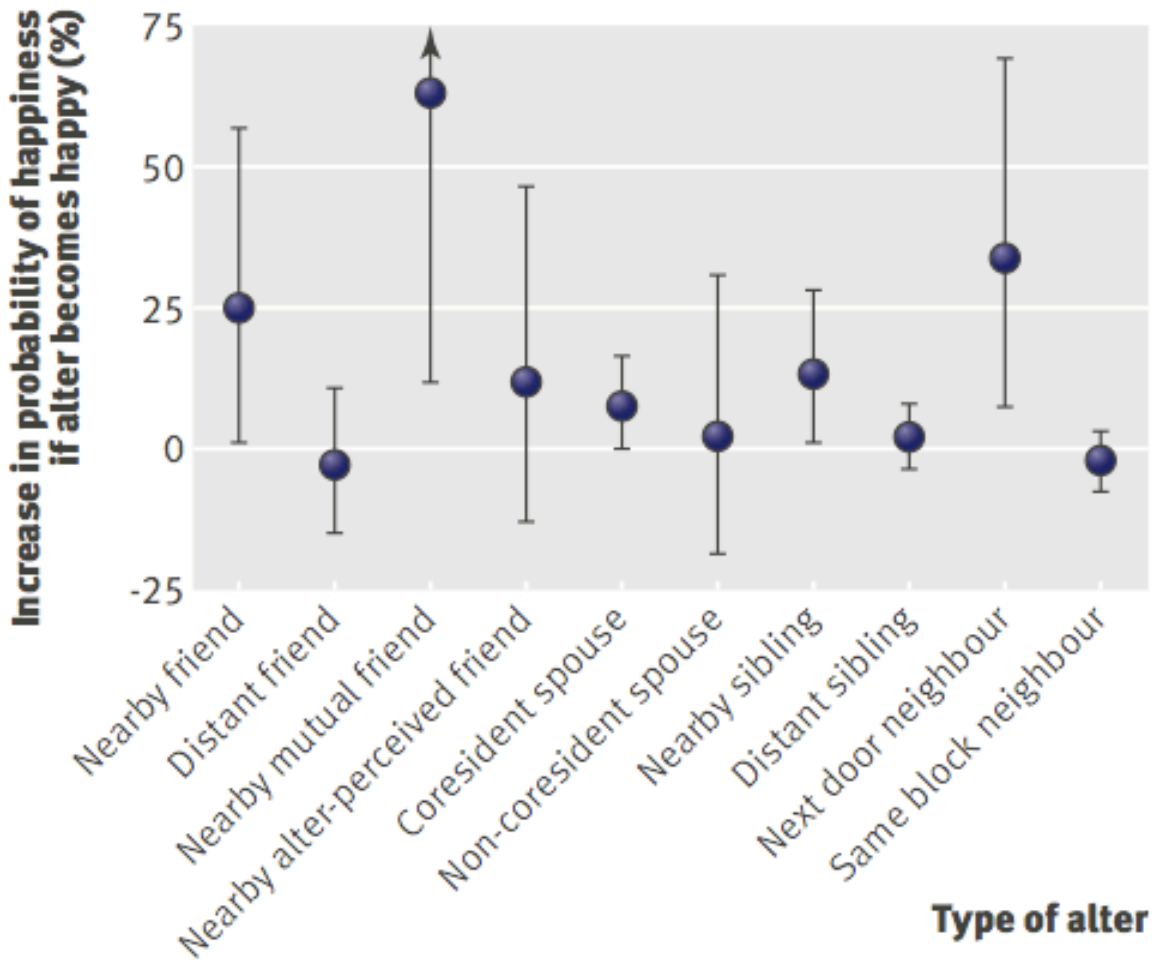


Fig 4 | Alter type and happiness in the Framingham social network. Friends, spouses, siblings, and neighbours significantly influence happiness, but only if they live close to

The NEW ENGLAND JOURNAL of MEDICINE

SPECIAL ARTICLE

The Spread of Obesity in a Large Social Network over 32 Years

Nicholas A. Christakis, M.D., Ph.D., M.P.H., and James H. Fowler, Ph.D.

Obesity is….. having obese friends ??

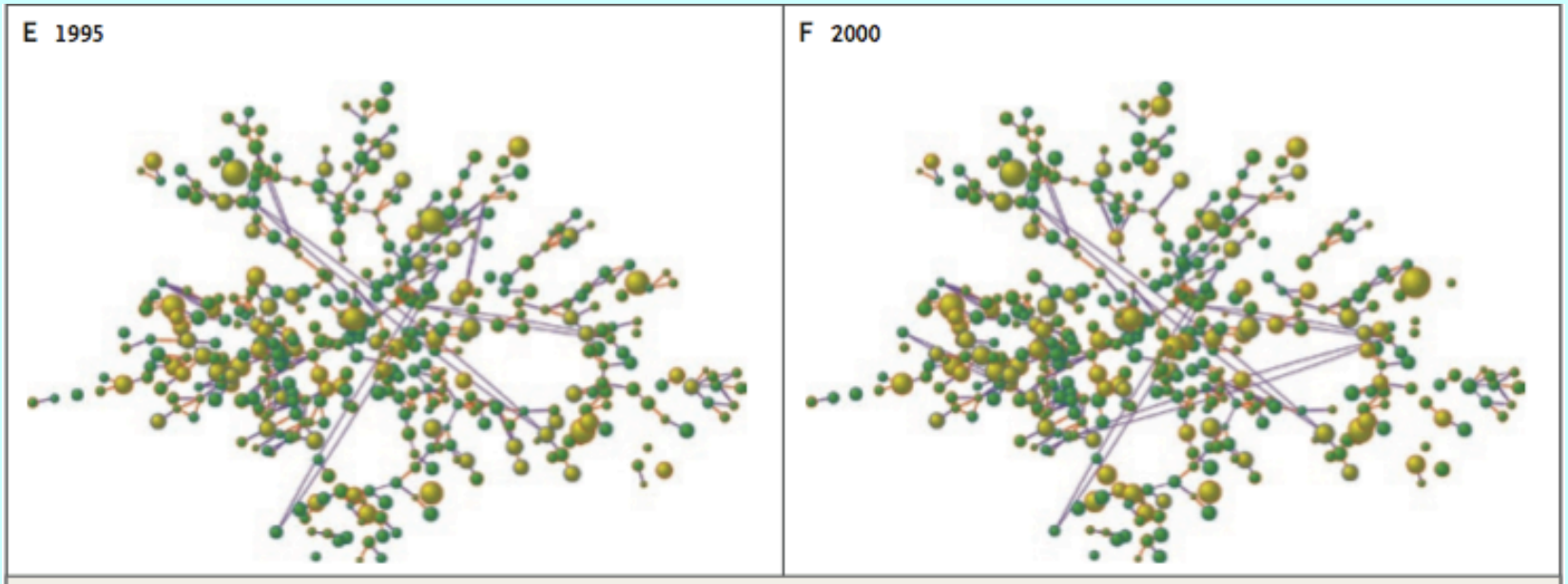
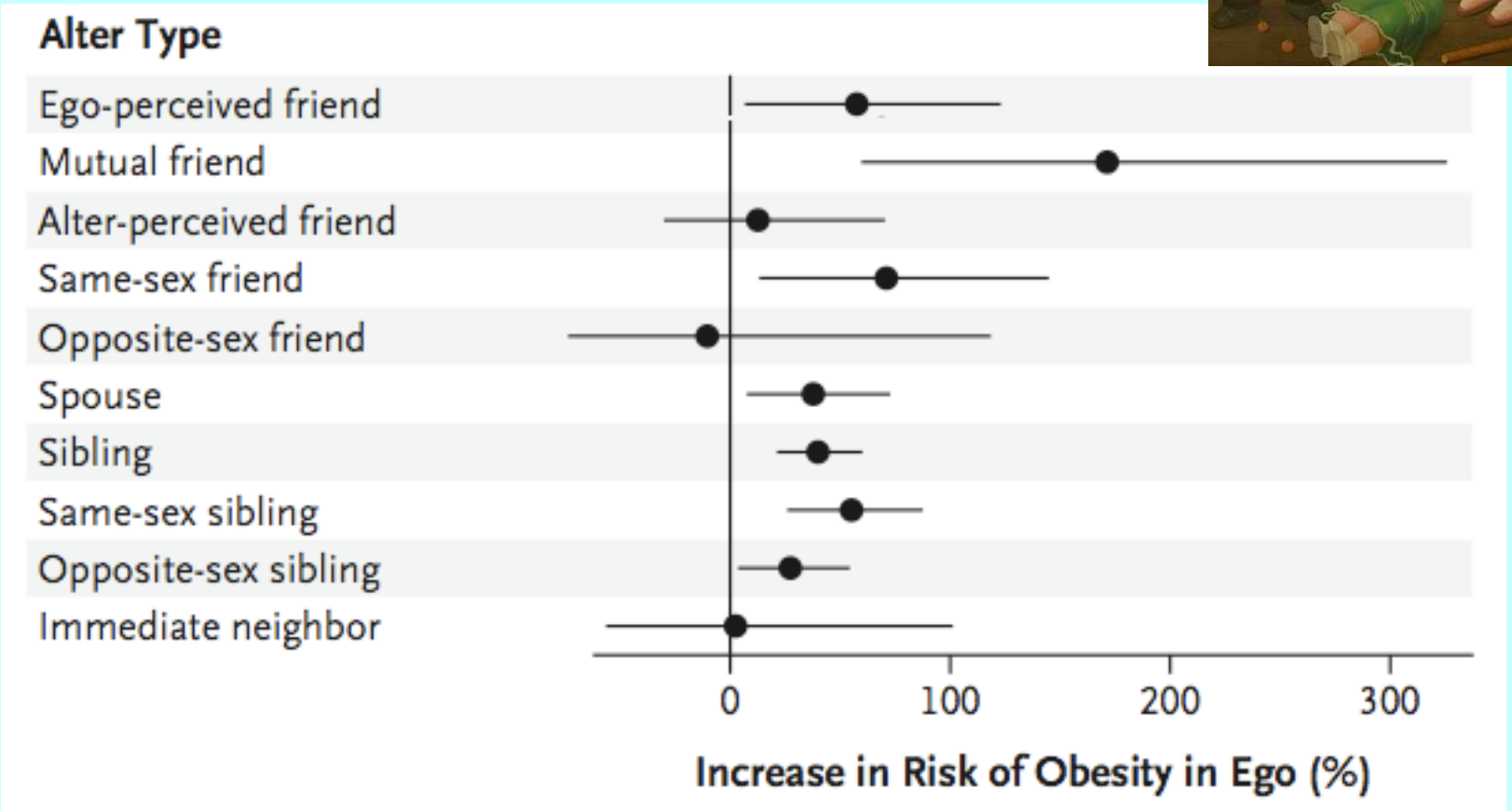
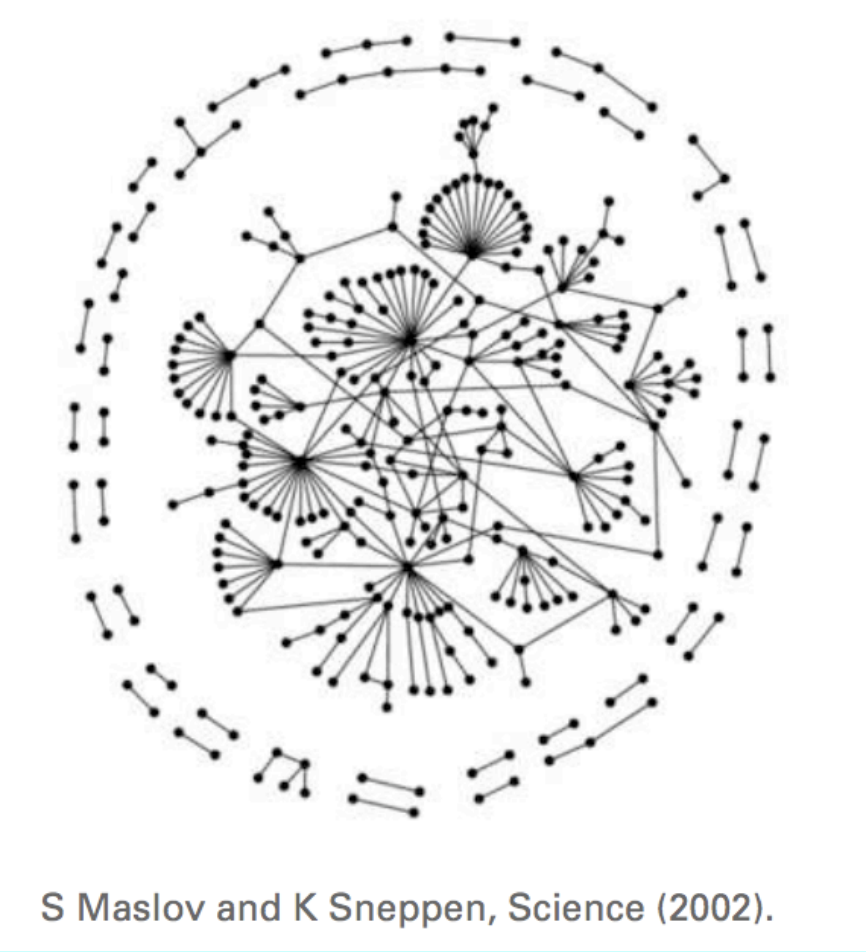


Figure 2. Part of the Social Network from the Framingham Heart Study with Information about Body-Mass Index According to Year.

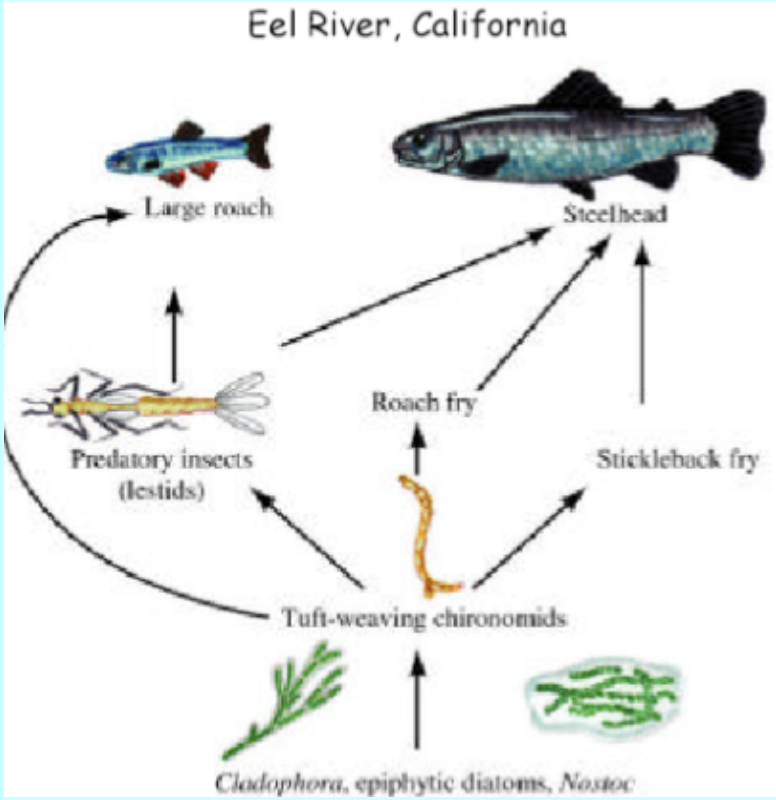
Each circle (node) represents one person in the data set. Circles with red borders denote women, and circles with blue borders denote men. The size of each circle is proportional to the person's body-mass index. The interior color of the circles indicates the person's obesity status: yellow denotes an obese person (body-mass index, ≥30) and green denotes a nonobese person. The colors of the ties between the circles indicate the relationship between them: purple denotes a friendship or a marital tie and orange denotes a familial tie. The disappearance of a circle from one year to another indicates the person's death, and the disappearance of a tie between the circles indicates that the relationship between the two persons no longer exists. The largest connected subcomponent of the whole network and the change in obesity over the 32-year study period are shown in an animation that is available with the full text of this article at www.nejm.org.


Alter Type
Ego-perceived friend
Mutual friend
Alter-perceived friend
Same-sex friend
Opposite-sex friend
Spouse
Sibling
Same-sex sibling
Opposite-sex sibling
Immediate neighbor
0
100
200
300
Increase in Risk of Obesity in Ego (%)

Tens of examples in biology



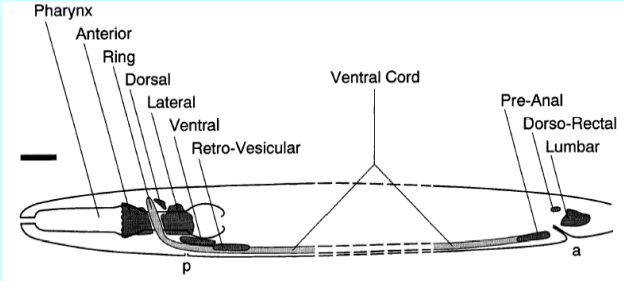
Protein interaction network in the Yeast (S.C.)



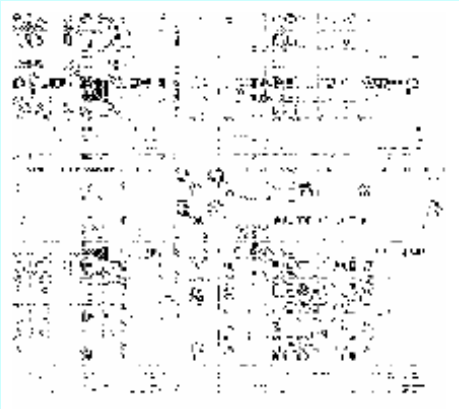
Food Webs

Brain networks: anatomical connectivity

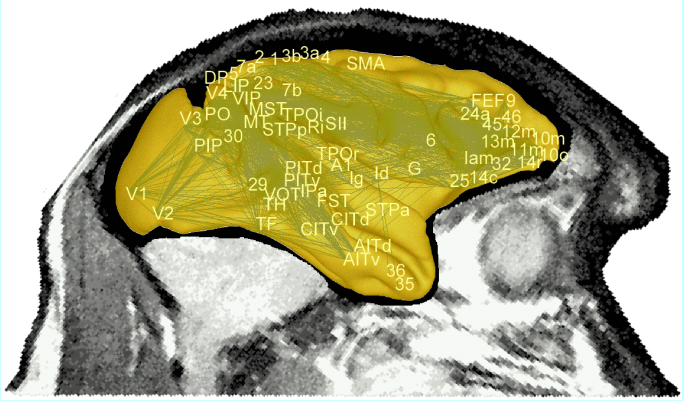
C. elegans: layout of ganglia



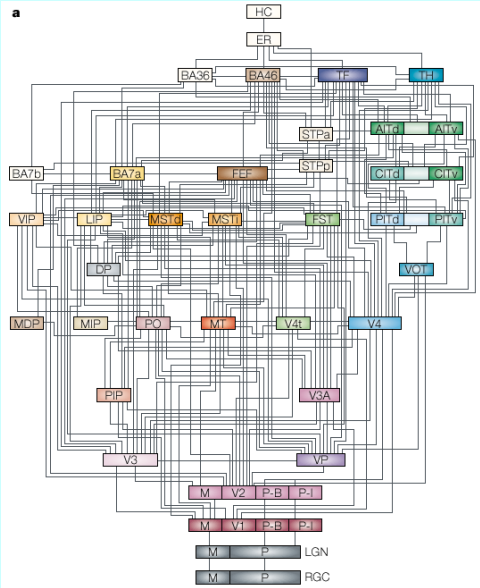
Brenner et al, 1975 mapped every single nervous cell, synapses and gap junctions



Neuron network



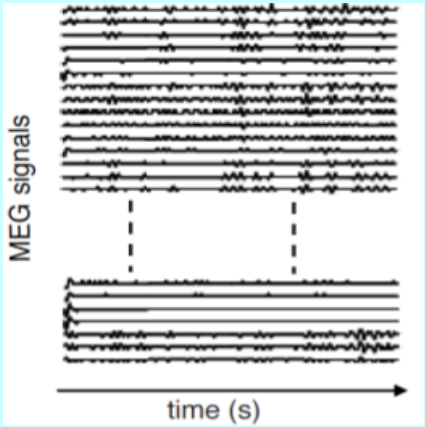
Cortical regions of macaque



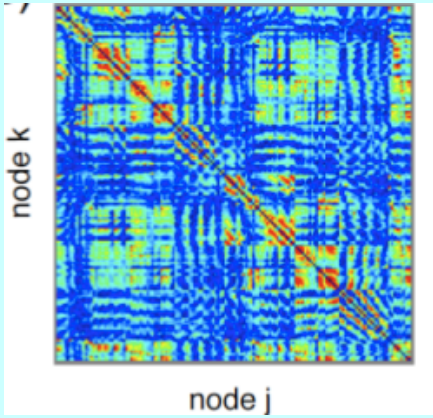
Links between cortical areas

Brain networks: functional connectivity

EEG, MEG, fMRI signals



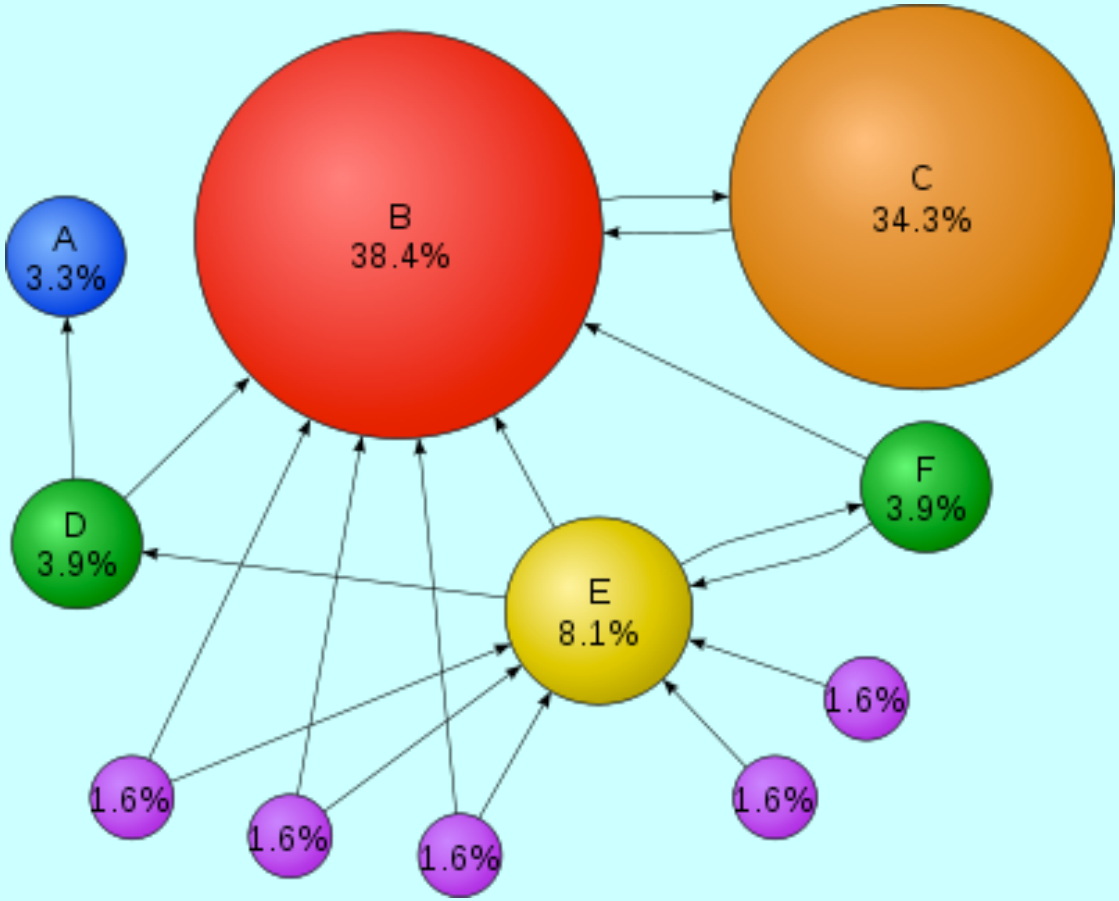
Coherence, synchronization, causality



Functional connectivity

Bullmore, Sporns, Nature Review Neuroscience 2009

Do not forget man-made systems !!



-Hyperlinks between Web pages

-- directed network

-- the largest net

20 years of Complex Networks

Watts, Strogatz, Nature 393, 440 (1998)

Barabasi, Albert, Science 286, 509 (1999)

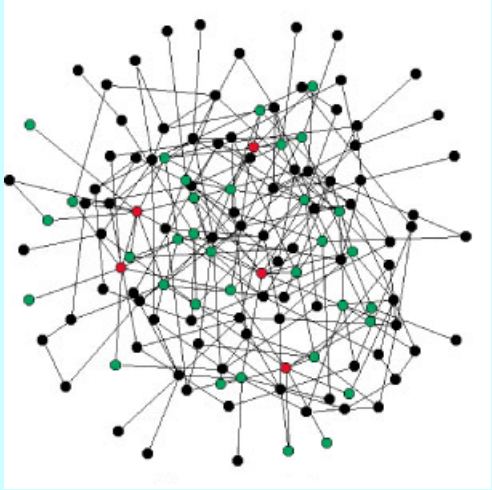
1) Characterize the structure of large real networks

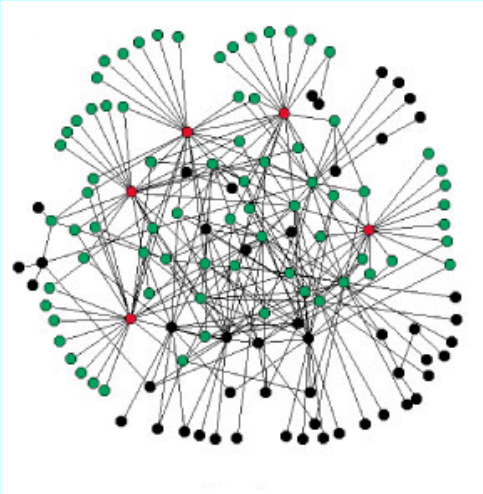
2) Developing new models (growing graphs)

3) Dynamical processes (percolation, diffusion, spreading, games, network of dynamical systems)

Degree distributions

Random graphs

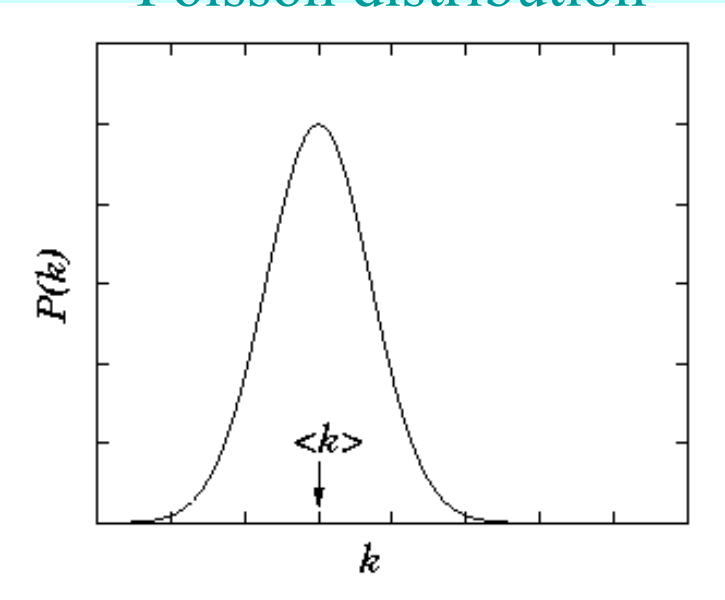




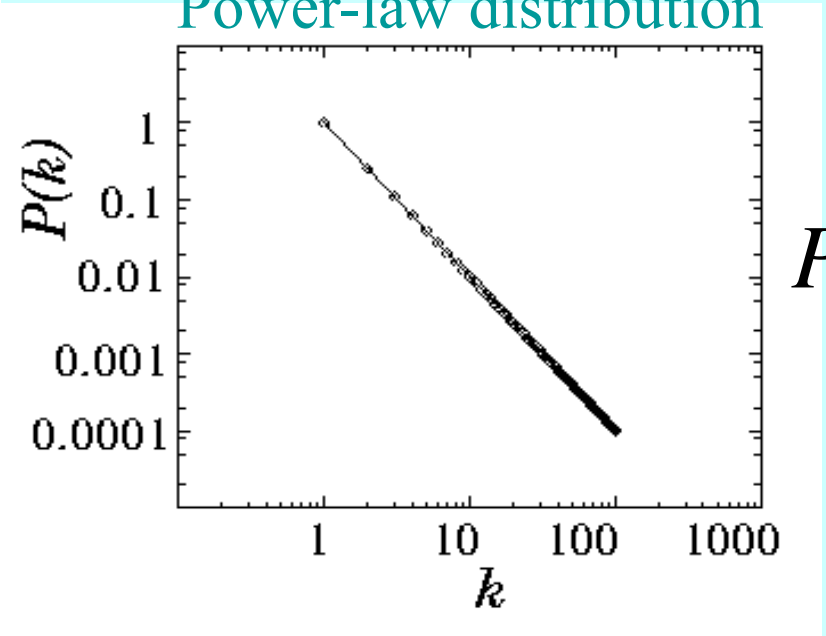
Scale-free networks

Barabasi-Albert
Rev. Mod. Phys 2001

Poisson distribution



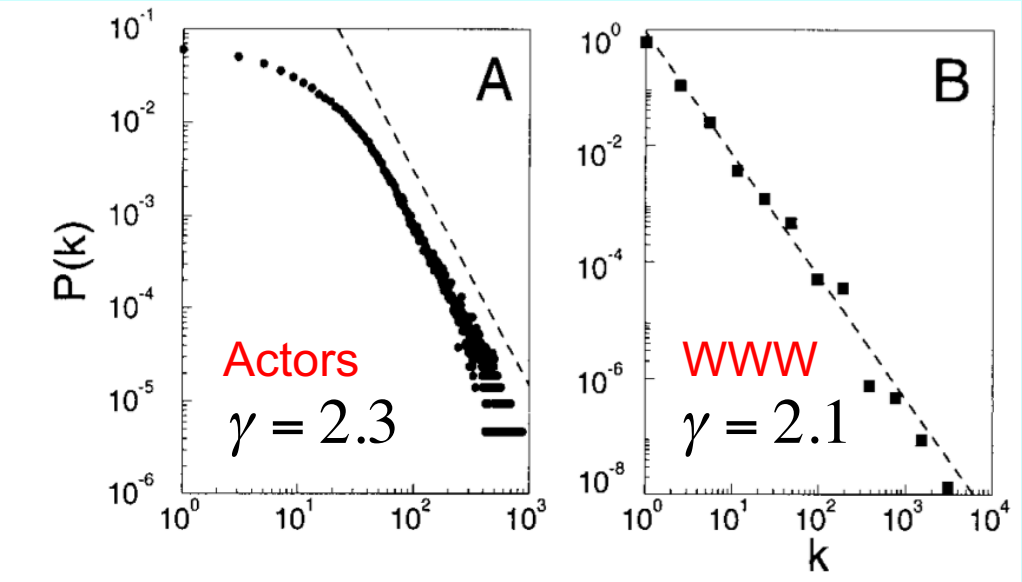
Power-law distribution



$$P(k) \approx k^{-\gamma}$$

Scale-free networks

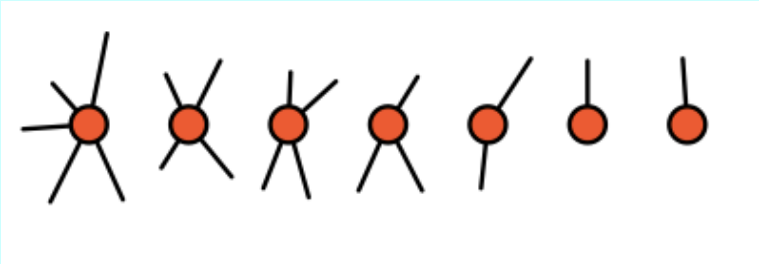
$$P(k) \approx k^{-\gamma}$$

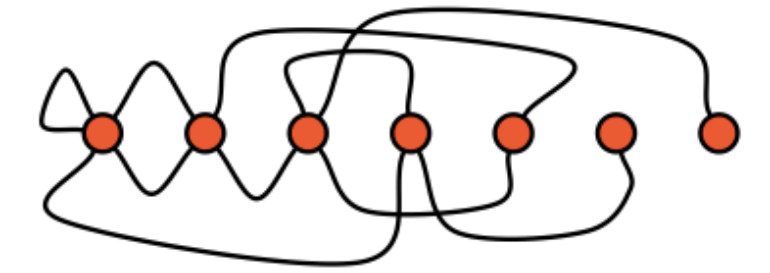


Networks	Exponent	
Internet	2.4	
WWW	2.1	2.7
Protein	2.4	
Metabolic	2.2	2.1
Coauthorship	2.5	
Movie actors	2.3	

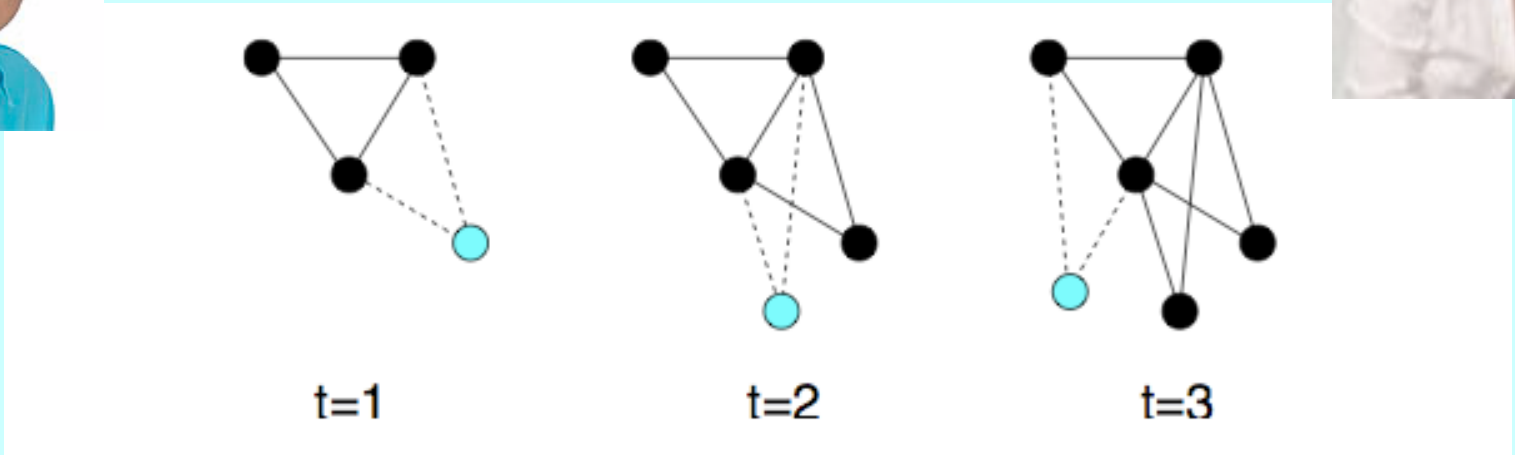
$$2 < \gamma \leq 3 \Rightarrow \begin{cases} \langle k \rangle \; finite \\ \langle k^2 \rangle \to \infty \end{cases}$$

Generalized random graphs





The Barabasi-Albert model



1) Growth : a node (with m links) is added at every time step

2) Linear preferential attachment: $\boxed{\Pi_{n \to i} \propto k_i}$

$$P(k) \xrightarrow[t \to \infty]{} ck^{-3}$$

Barabasi, Albert, Science 286, 509 (1999)

Epidemic spreading

SIS model

S = susceptible
I = infected

Transmission rate

$$S + I \xrightarrow{\lambda} I + I$$

Recovery rate

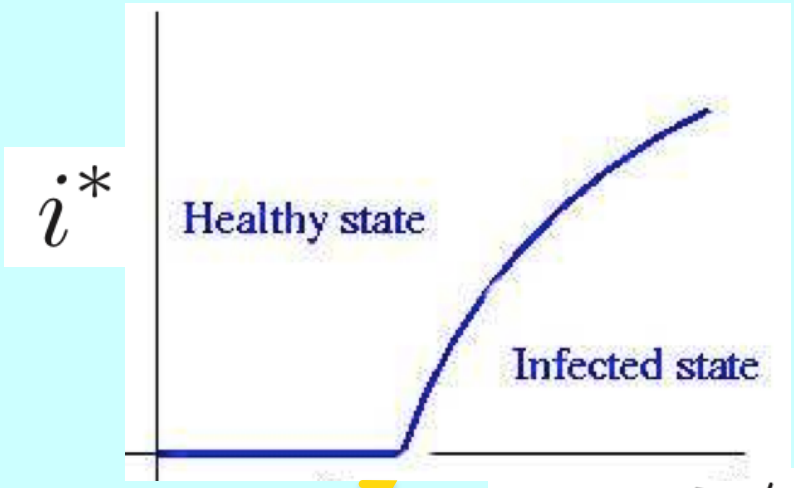
$$I \xrightarrow{\mu} S$$

$$\sigma = \frac{\lambda}{\mu}$$

$$\frac{di(t)}{dt} = -\mu i(t) + \lambda \langle k \rangle i(t)[1 - i(t)]$$

No endemic state (i^*=0) if

$$\sigma < \sigma_c = \frac{1}{\langle k \rangle}$$



$$\sigma = \lambda / \mu$$

Epidemic threshold

$$\sigma_c$$

Epidemic spreading in scale-free nets

Transmission rate

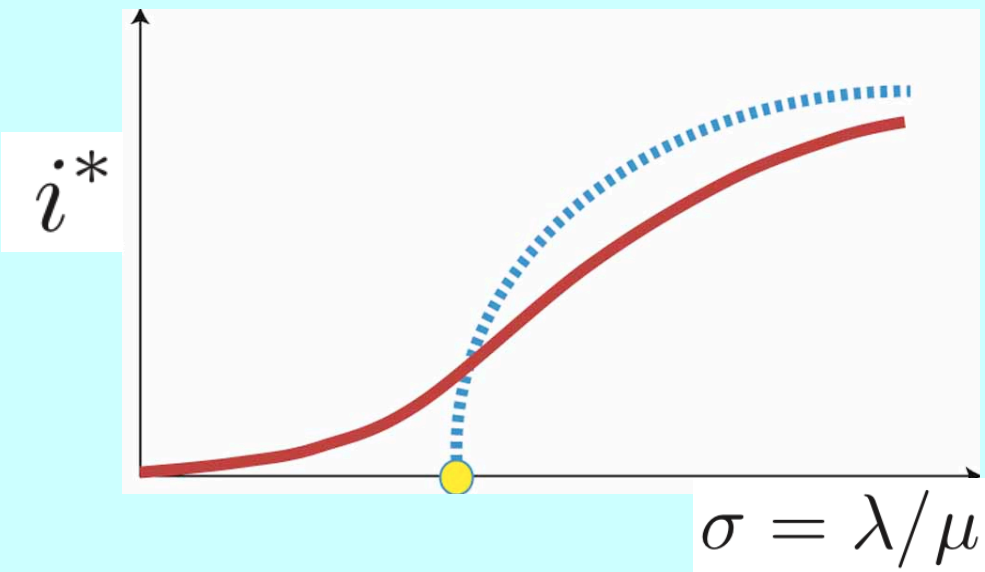
$$S + I \xrightarrow{\lambda} I + I$$

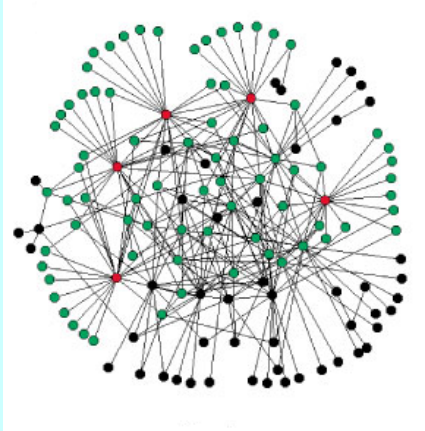
Recovery rate

$$I \xrightarrow{\mu} S$$

$$\frac{di_k(t)}{dt} = -i_k(t) + \lambda k\,[1 - i_k(t)]\,\Theta_k(t) \qquad k = 0, \ldots, N-1$$

$$\Theta_k(t) = \Theta[\{i_{k'}(t)_{k'=0,\ldots,N-1}\}] = \frac{1}{\langle k \rangle} \sum_{k'} k' P(k') i_{k'}(t)$$





$$\sigma_c = \frac{\langle k \rangle}{\langle k^2 \rangle} \xrightarrow[N \to \infty]{} 0$$

No epidemic threshold

Pastor-Satorras, Vespignani, PRL 86, 3200 (2001)

Structure and dynamics of complex nets

- Structural descriptors of networks from the real world

  node properties, degree distributions, degree-degree correlations, motifs, communities

- New graph models

  random graphs with P(k), models of graph growth, correlated graphs

- The structure affects the function

  percolation, diffusion, spreading of diseases, searching information, routing protocols, coupled dynamical systems

Albert, Barabasi, Rev. Mod. Phys. 74 (2002) 47
Newman, SIAM Rev. 45 (2003) 167
Boccaletti, Latora, Moreno, Chavez, Hwang, Phys Rep 424 (2006) 175

A new textbook

October 2017

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COVER DESIGNED BY HART McLEOD LTD

Networks constitute the backbone of complex systems, from the human brain to computer communications, transport infrastructures to online social systems, metabolic reactions to financial markets. Characterising their structure improves our understanding of the physical, biological, economic and social phenomena that shape our world.

Rigorous and thorough, this textbook presents a detailed overview of the new theory and methods of network science. Covering algorithms for graph exploration, node ranking and network generation, among the others, the book allows students to experiment with network models and real-world data sets, providing them with a deep understanding of the basics of network theory and its practical applications. Systems of growing complexity are examined in detail, challenging students to increase their level of skill. An engaging presentation of the important principles of network science makes this the perfect reference for researchers and undergraduate and graduate students in physics, mathematics, engineering, biology, neuroscience and social sciences.

VITO LATORA is Professor of Applied Mathematics and Chair of Complex Systems at Queen Mary University of London. Noted for his research in statistical physics and in complex networks, his current interests include time-varying and multiplex networks, and their applications to socio-economic systems and to the human brain.

VINCENZO NICOSIA is Lecturer in Networks and Data Analysis at the School of Mathematical Sciences at Queen Mary University of London. His research spans several aspects of network structure and dynamics, and his recent interests include multi-layer networks and their applications to big data modelling.

GIOVANNI RUSSO is Professor of Numerical Analysis in the Department of Mathematics and Computer Science at the University of Catania, Italy, focusing on numerical methods for partial differential equations, with particular application to hyperbolic and kinetic problems.

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LATORA
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COMPLEX NETWORKS

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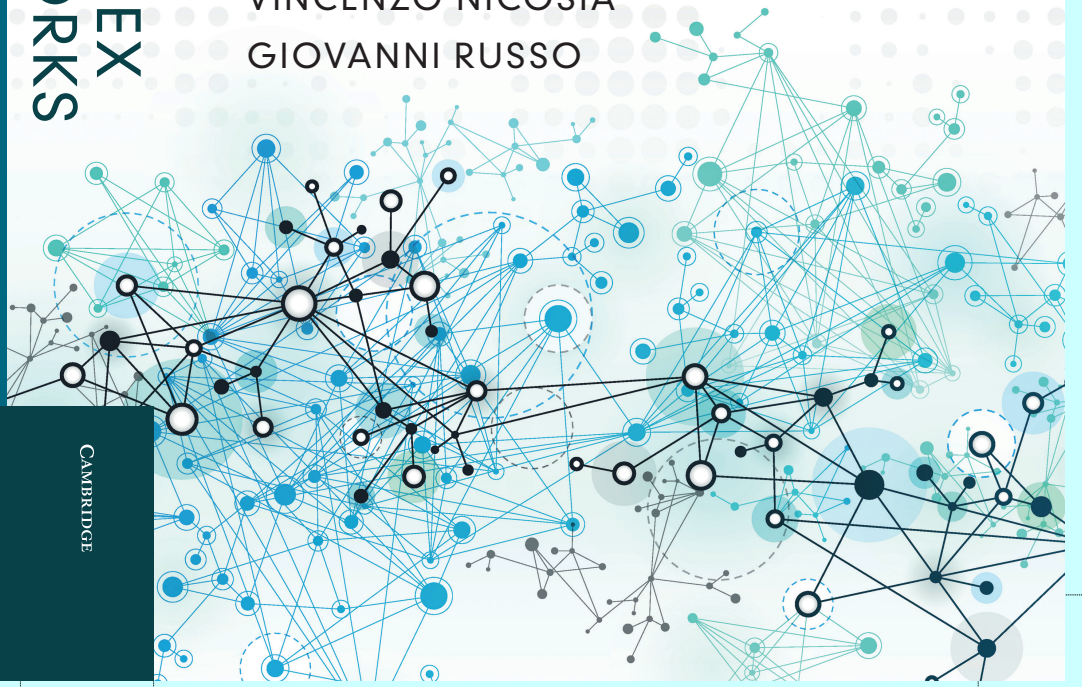
COMPLEX NETWORKS

Principles, Methods and Applications

VITO LATORA
VINCENZO NICOSIA
GIOVANNI RUSSO



Complex Networks: Priciples, Methods and Applications

→ .... another one

→ But different: each chapter contains an idea, a network to explore it, the mathematical background, and an application

→ You are given the ``historically famous'' network data sets + algorithms

The new hot topics in Network Science

``Rich" networks → generalised graphs

Node and links are in space and in time (spatial networks
temporal networks)

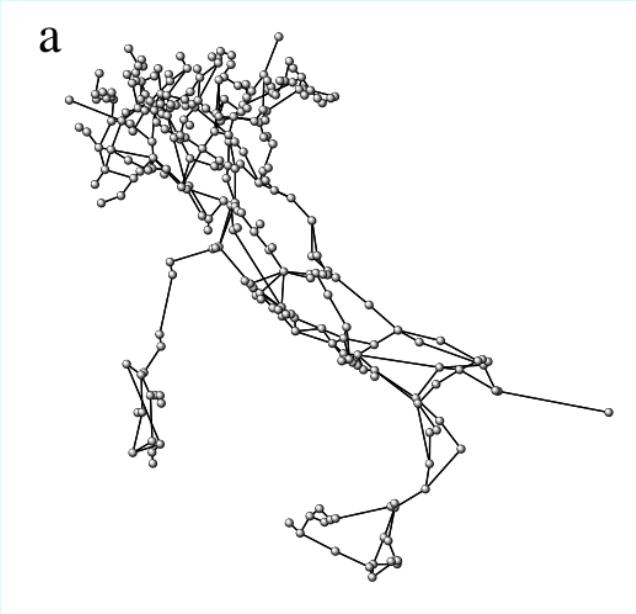
Node and links can be of different types (multiplex,
interconnected networks

The basic units are not only nodes and links (hypergraphs,
simplicial complexes)

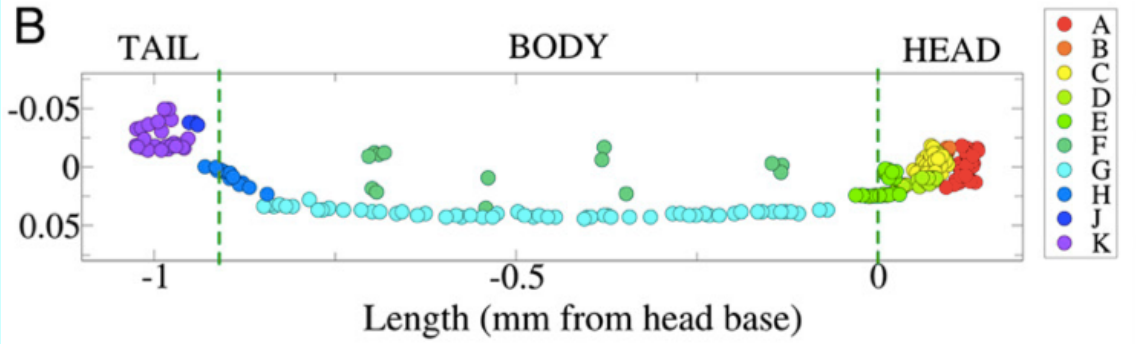
Simplicial models of social contagion,
Iacopini, Petri, Barrat, Latora
Nature Communications 10, 2485 (2019)

New hot topics: Spatial networks

Urban street patterns



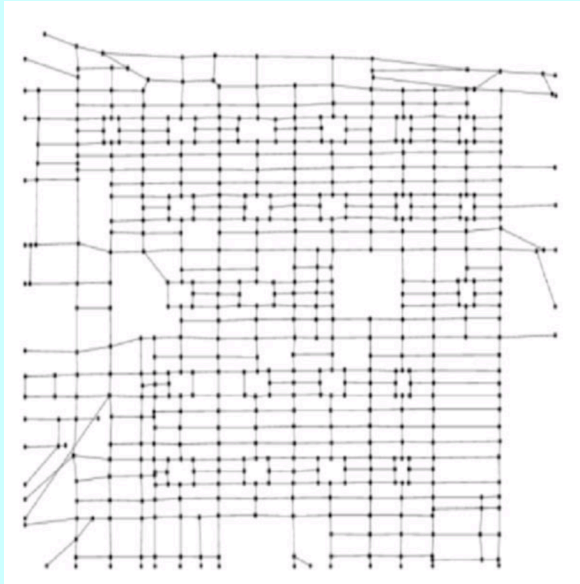
Power grids



Nervous system

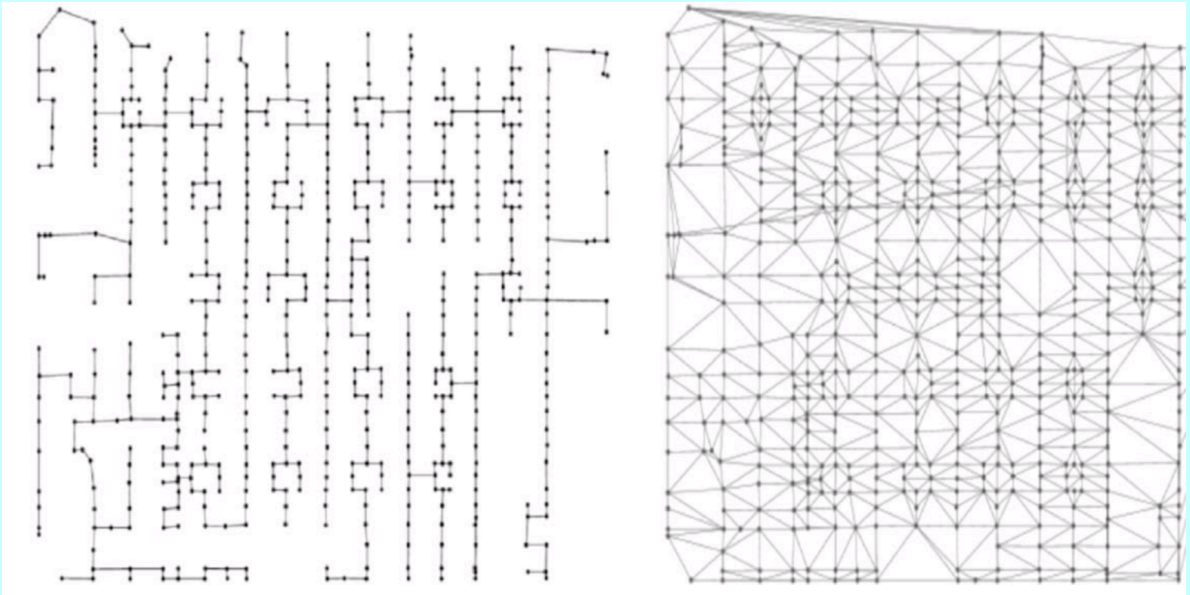
M. Barthelemy, Spatial networks, Phys. Rep. 499 (2011) 1–101

Urban street patterns



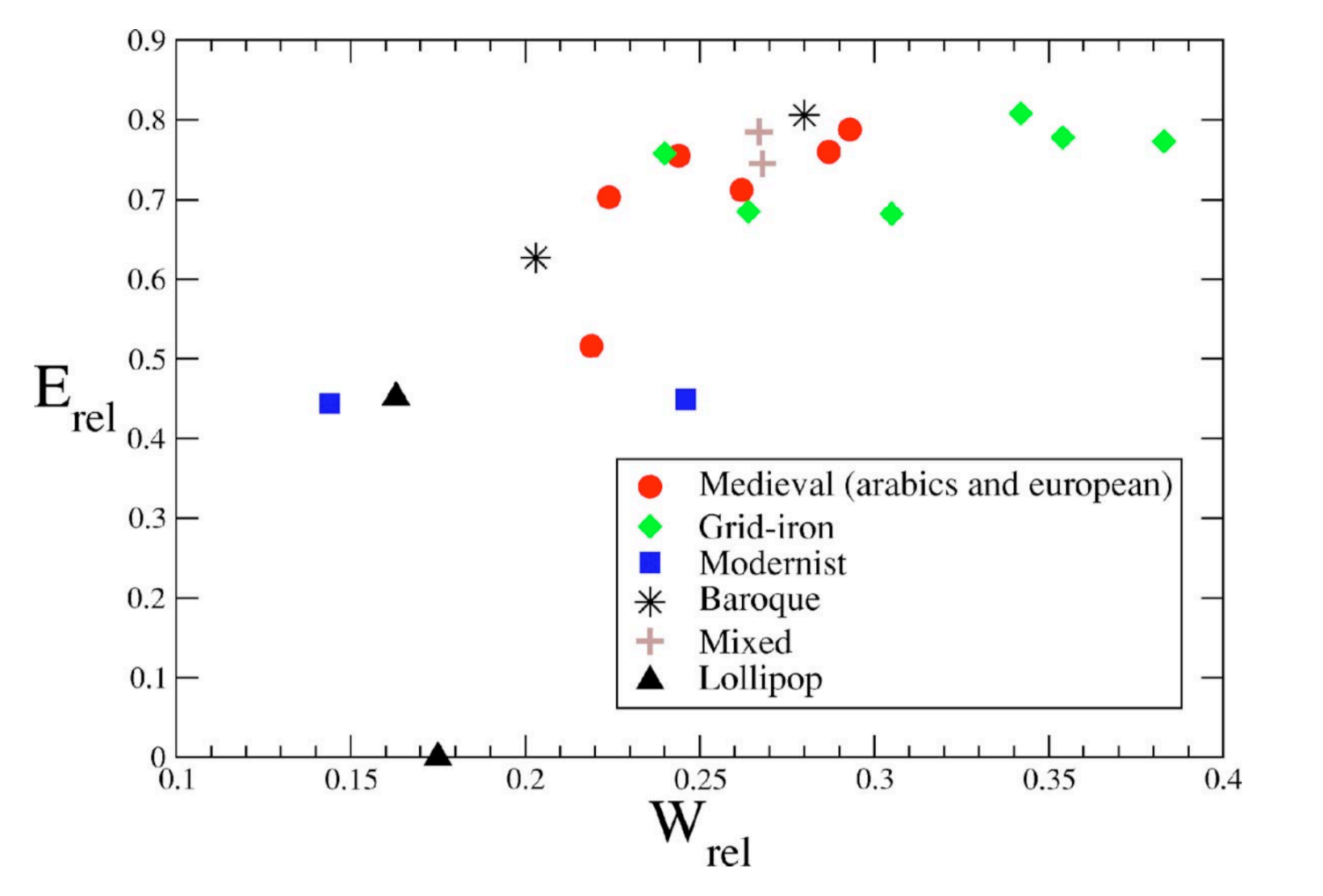
$$W = \sum_{i,j} l_{ij}$$

$$E = \frac{1}{N(N-1)} \sum_{i,j \neq i} \frac{1}{d_{ij}}$$



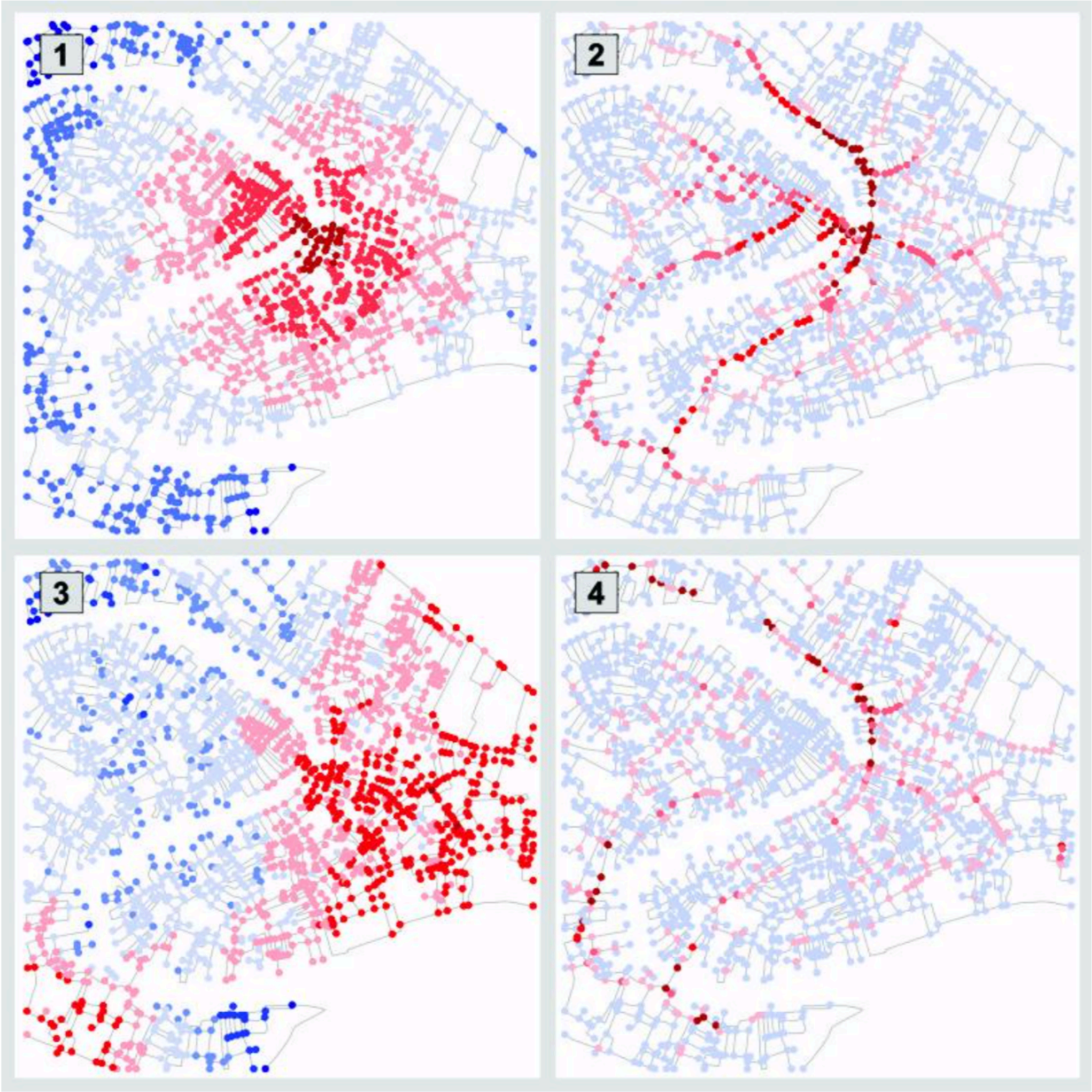
S. Porta, P. Crucitti, V. Latora, Env. and Planning B33, 705 (2006)

Classification of cities

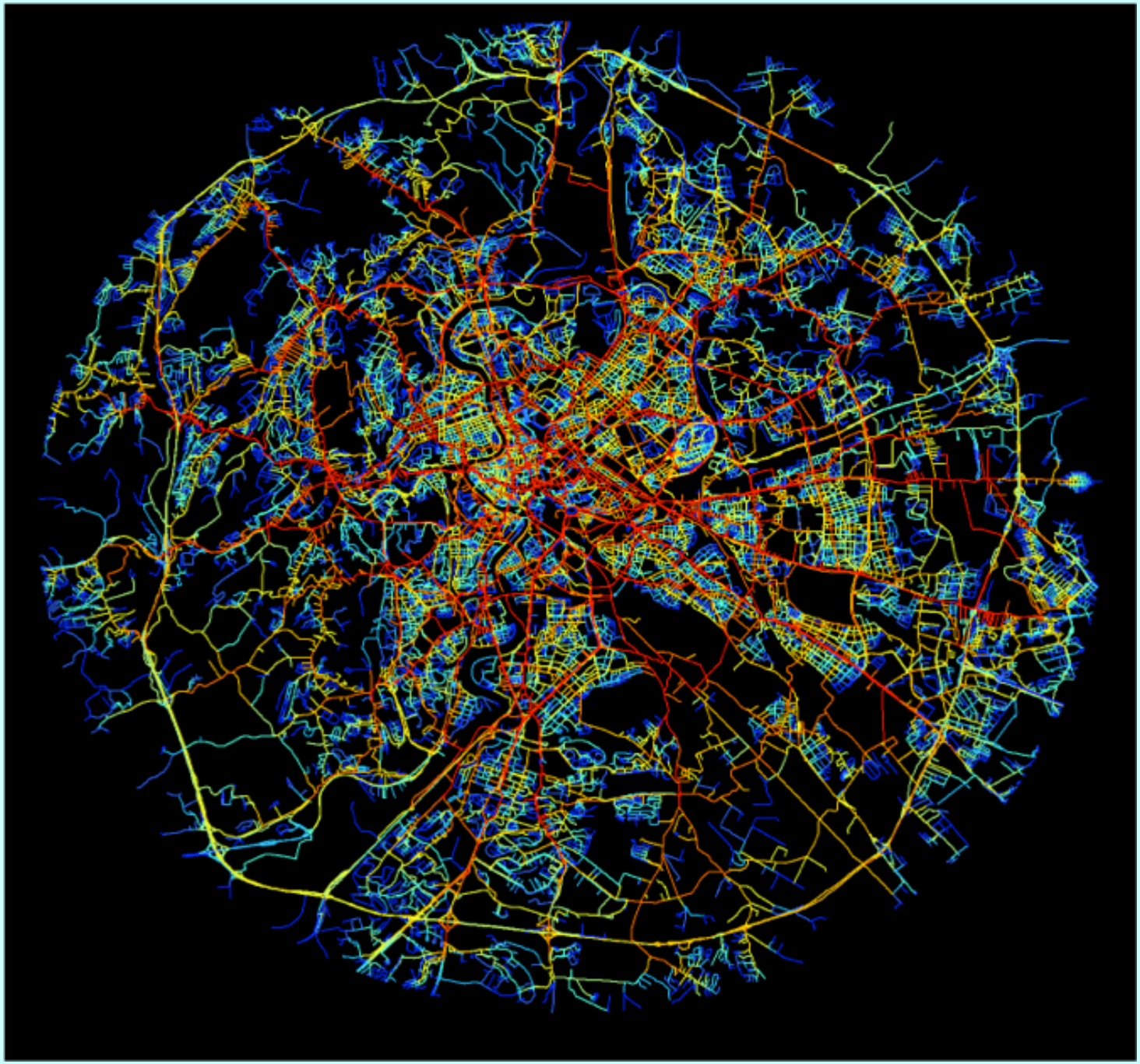


Cardillo, Scellato, Latora, Porta, Phys. Rev. E 73, 066107 (2006)

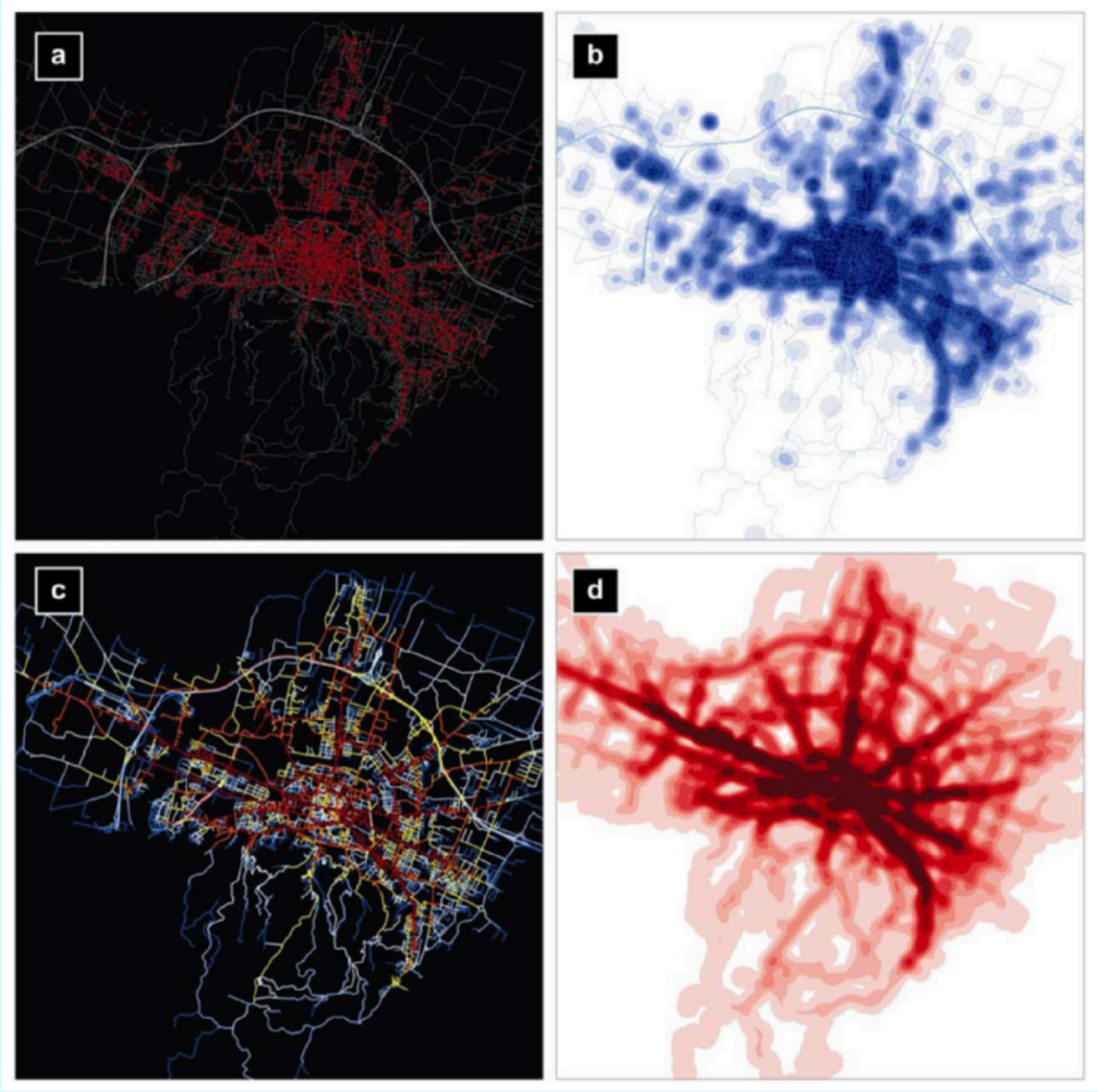
Centrality measures: the hot spots of a city



Crucitti, Latora, Porta, Phys. Rev. E 73, 036125 (2006), and Chaos 16, 015113 (2006)



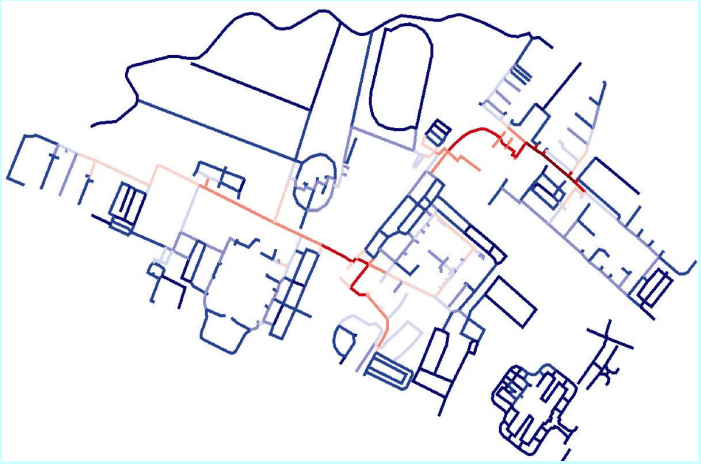
Correlations with human activities

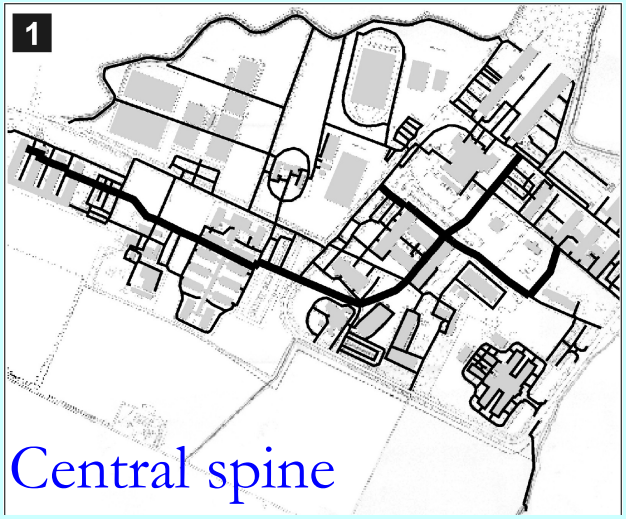


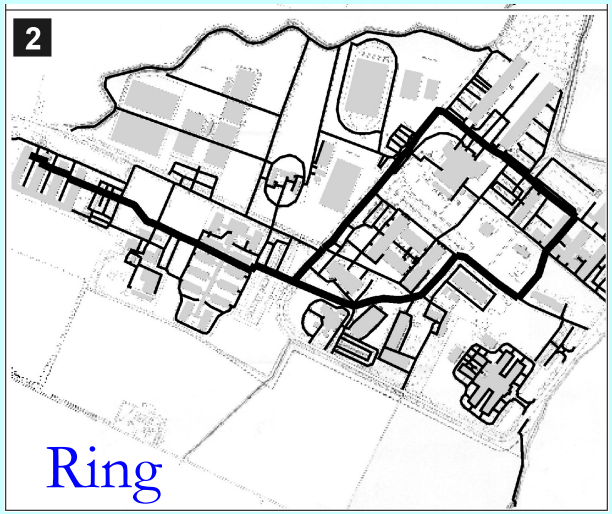
Porta et al., Env. Plann. B 36, 450-465 (2009)

Pass to action !

Parma Uni Campus







Porta et al., Env. Plann. B 36, 450-465 (2009)

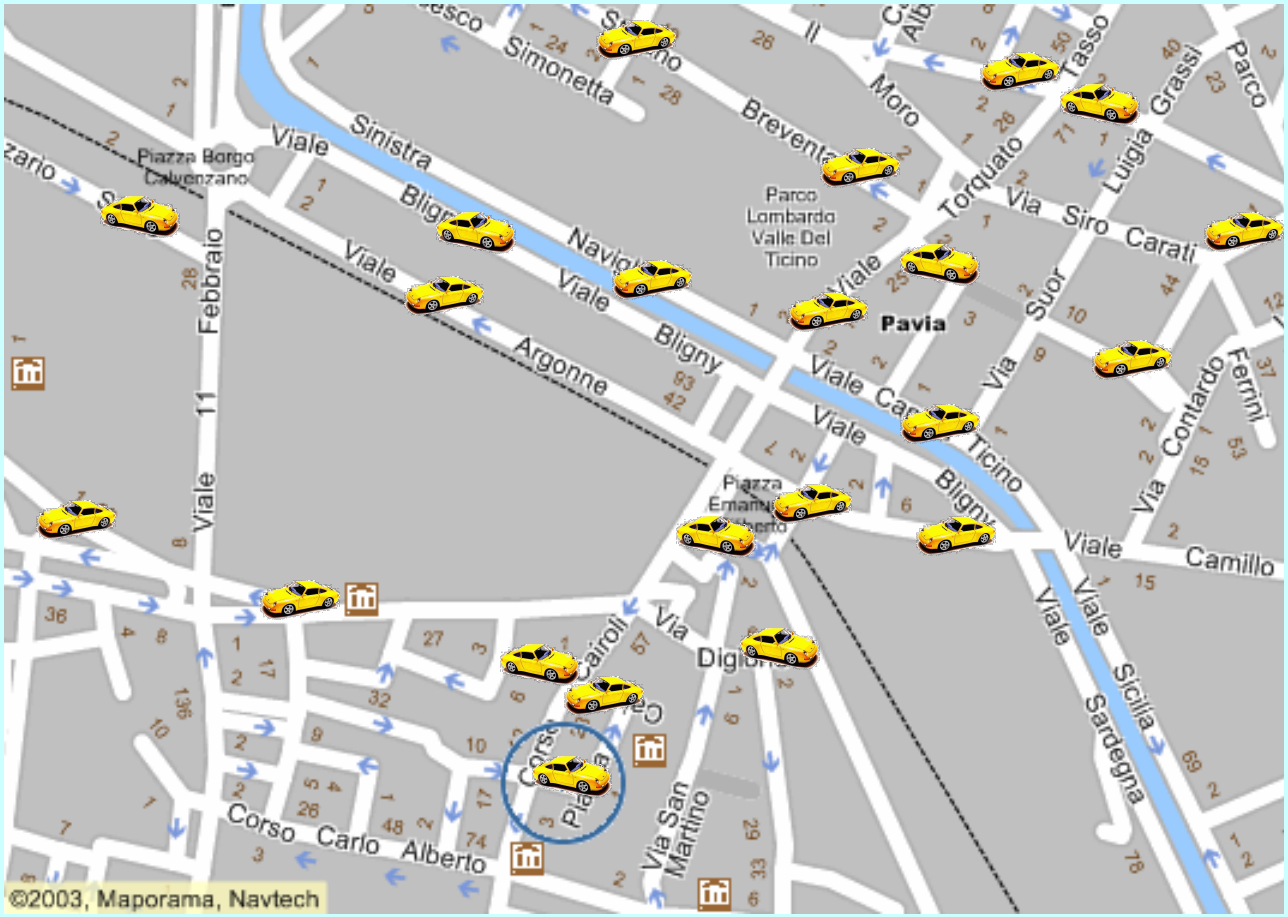
Dynamics: cascading failures

Nodes = Crossings

Arcs = Streets

Arcs weights:

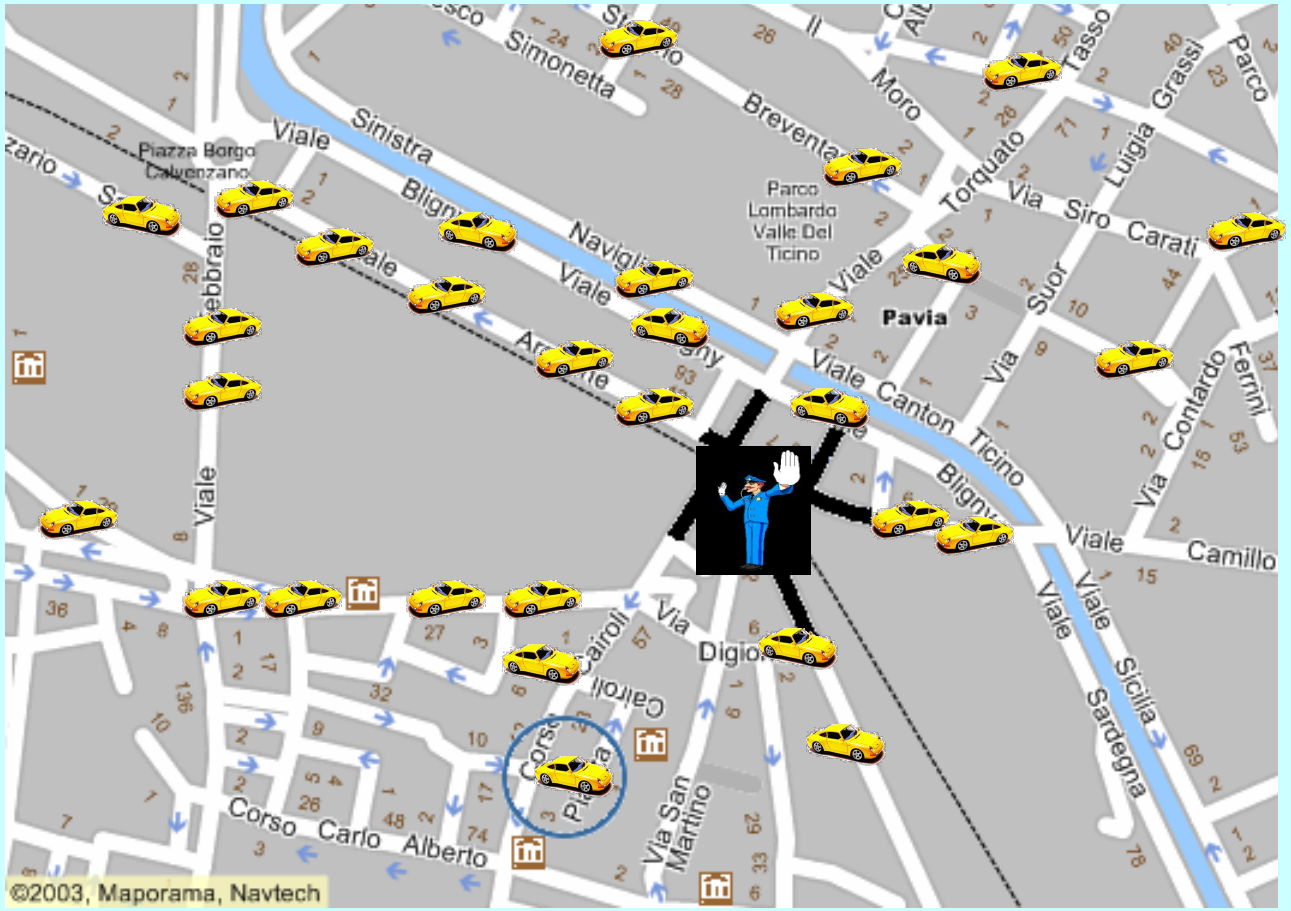
τ_{ij} = time spent in order to go from node i to node j



If today Piazza Emanuele Filiberto is closed to traffic…

People have to find an alternative path.

Load redistribution



Load redistribution can cause traffic in alternative routes.

↓

Overload

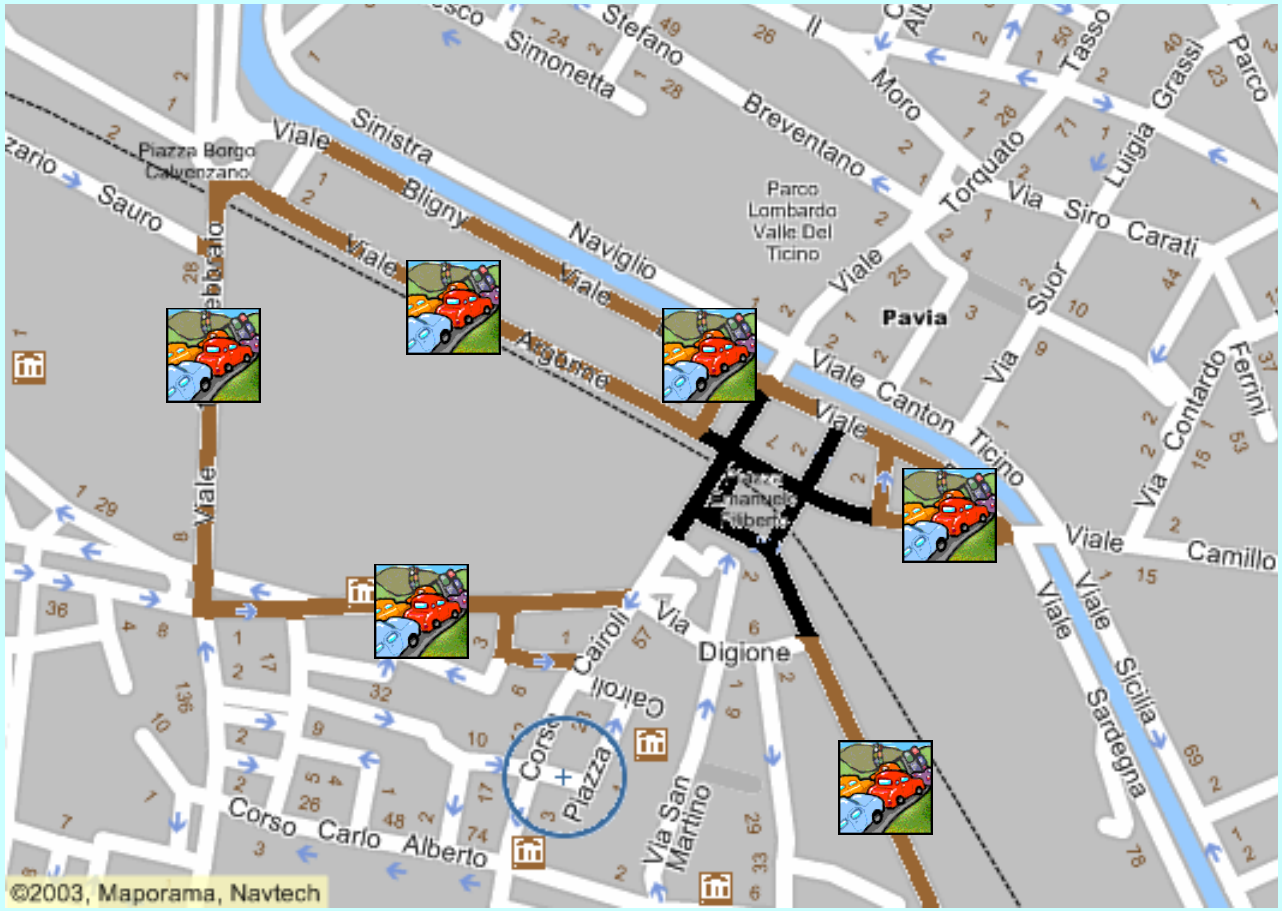
↓

Traffic hold up

↓

Degradation in efficiency

(times τ_{ij} grow longer)



New choice of alternative routes

↓

New overload

↓

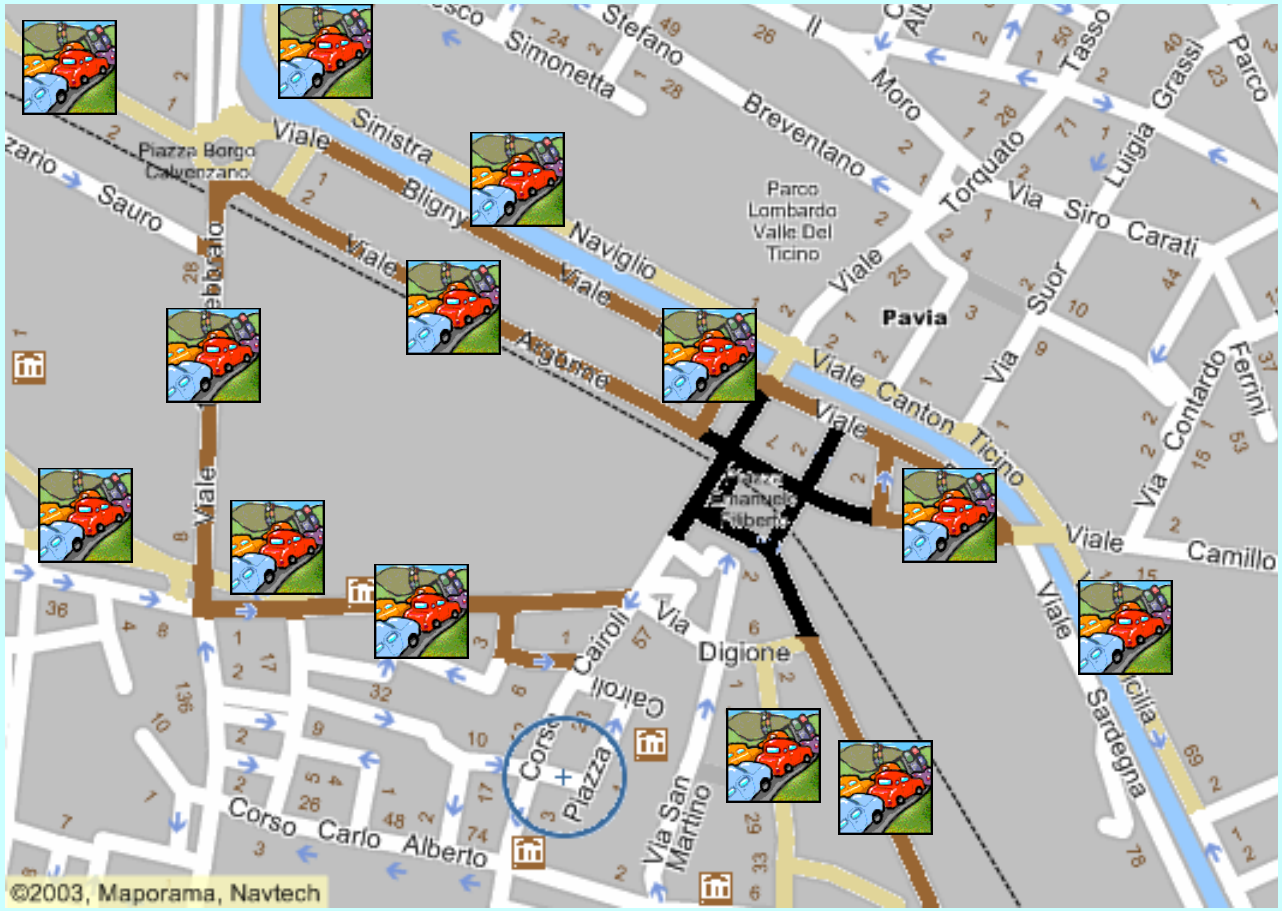
New degradation in efficiency

↓

…

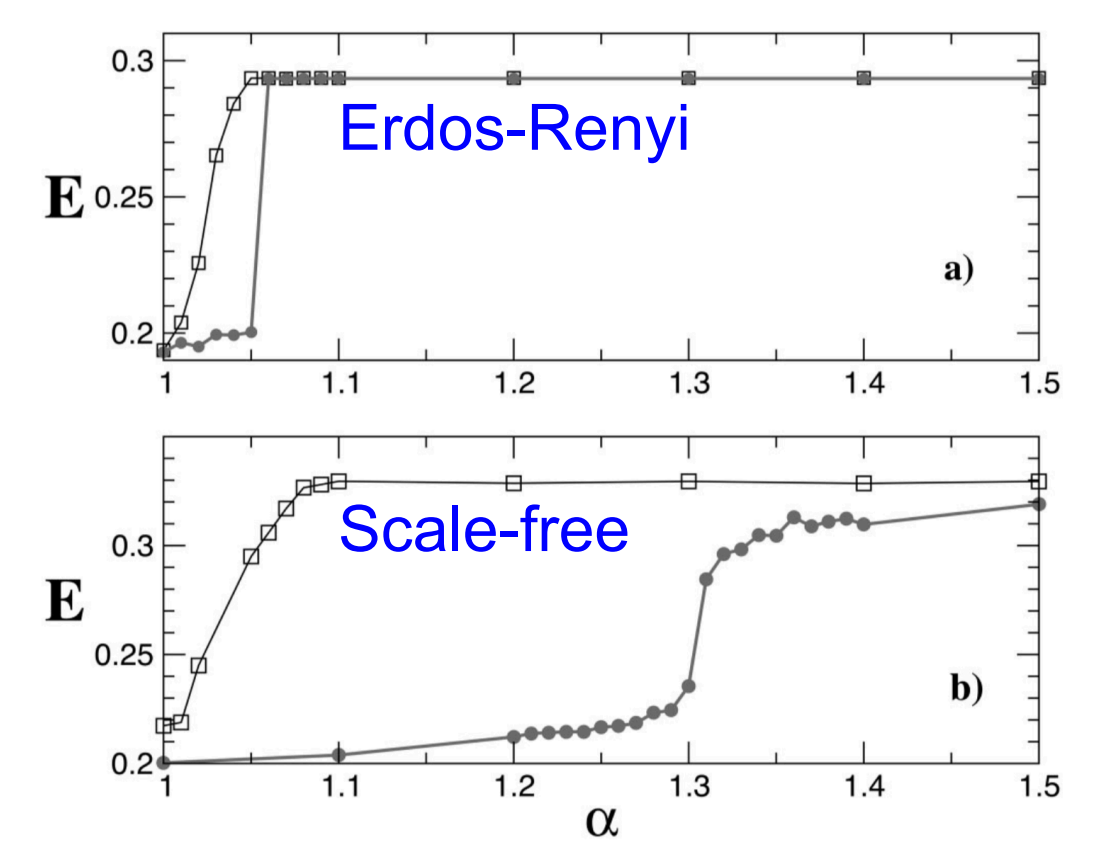
↓

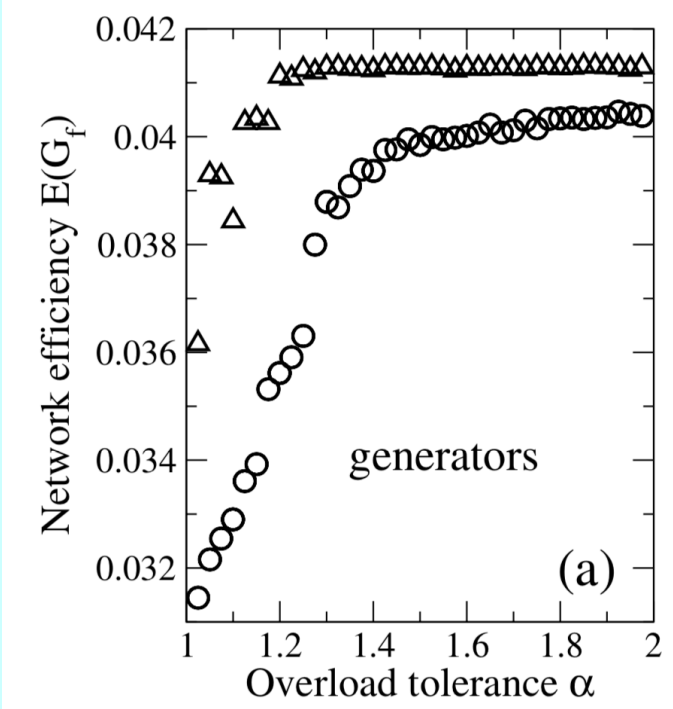
Cascading effect



…and the result is…

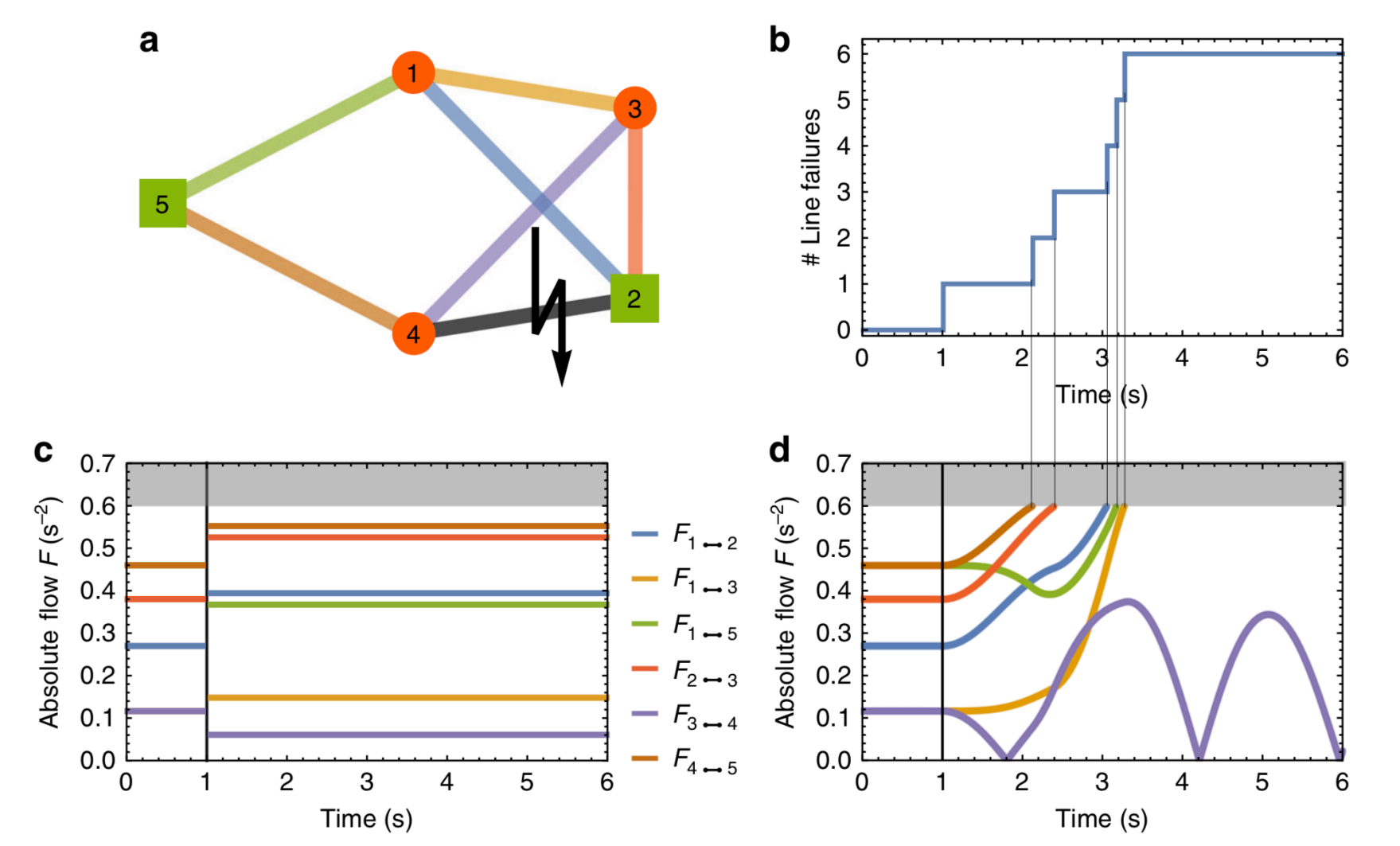
A model for cascading failures





Crucitti, Latora, Marchiori, Phys. Rev. E 69 (2004)045104(R)
Kinney, Crucitti, Albert, Latora, Eur. Phys. J. B 46, 101 (2005)

Cascading failures in power grids



Schafer, Witthaut, Timme, Latora, Nature Comm. 9, 1975 (2018)

Power grids

The swing equation:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\theta_i = \omega_i, \\ I_i \dfrac{\mathrm{d}}{\mathrm{d}t}\omega_i = P_i - \gamma_i \omega_i + \sum_{j=1}^{N} K_{ij} \sin\left(\theta_j - \theta_i\right) \end{cases}$$

$$\text{overload}: \left|F_{ij}(t)\right| > C_{ij} = \alpha K_{ij}$$

Schafer, Witthaut, Timme, Latora, Nature Comm. 9, 1975 (2018)

``Rich" networks → generalised graphs

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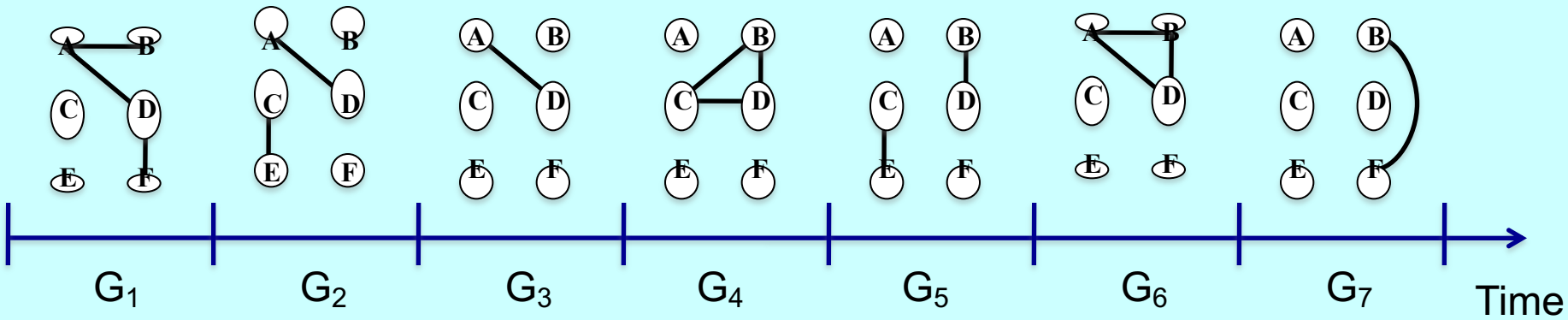
Iacopini, Petri, Barrat, Latora
Nature Communications 10, 2485 (2019)

Temporal networks



Holme and Saramaki, Phys. Rep. 519, 97 (2012)

Temporal networks, Eds Holme and Saramaki, Springer 2013

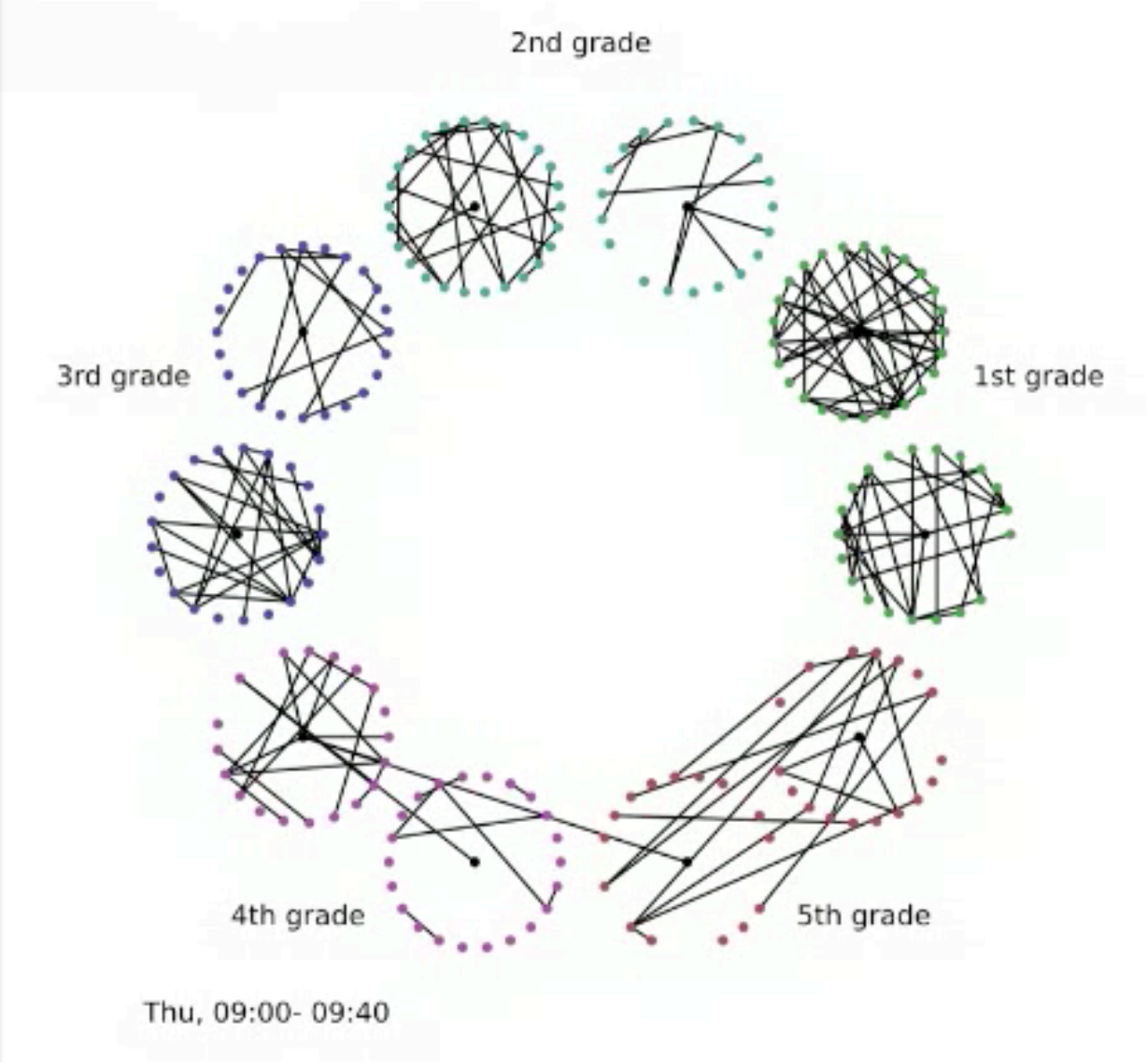
One school day: contacts over time in a French primary school

Stehle et al. PLoS ONE 6, e23176 (2011)

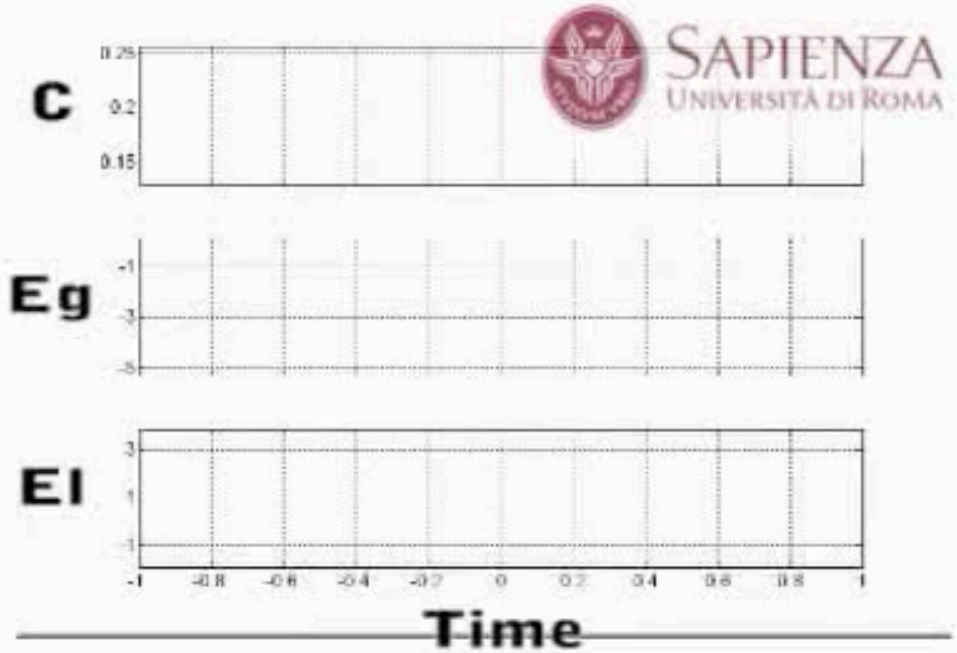
-- 10 classes

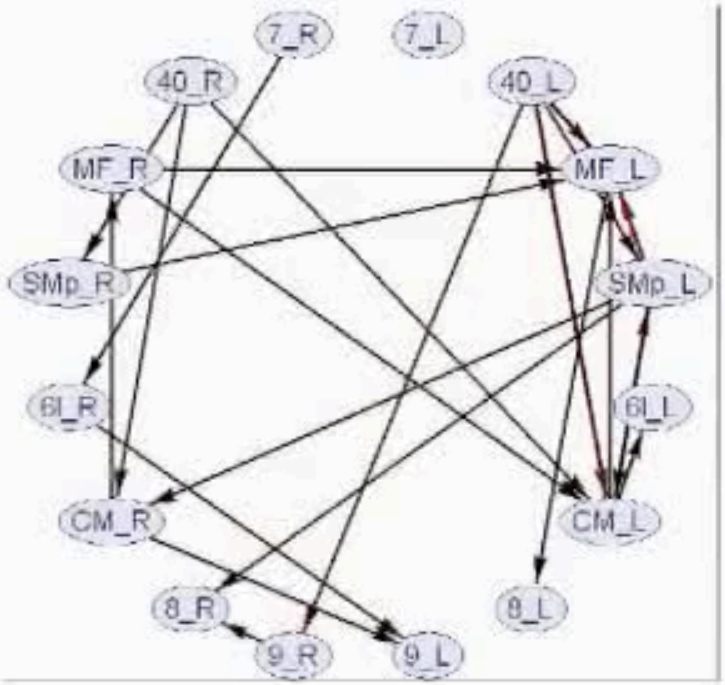
-- 232 children,
10 teachers

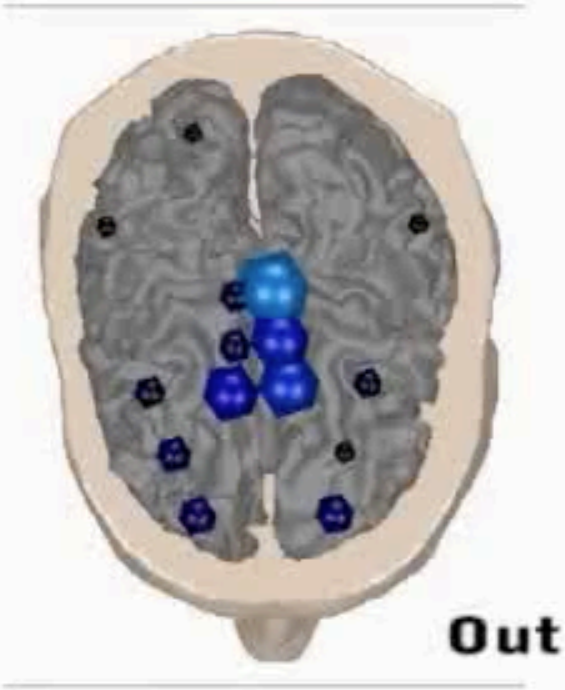
-- face-to-face contacts.
Radio frequency proximity-sensing
(<1.5 m every 20 sec)

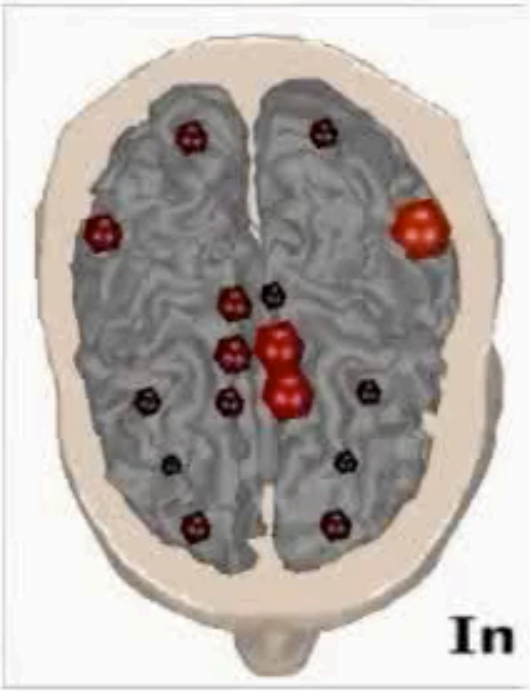


Fondazione
Santa Lucia
IRCCS

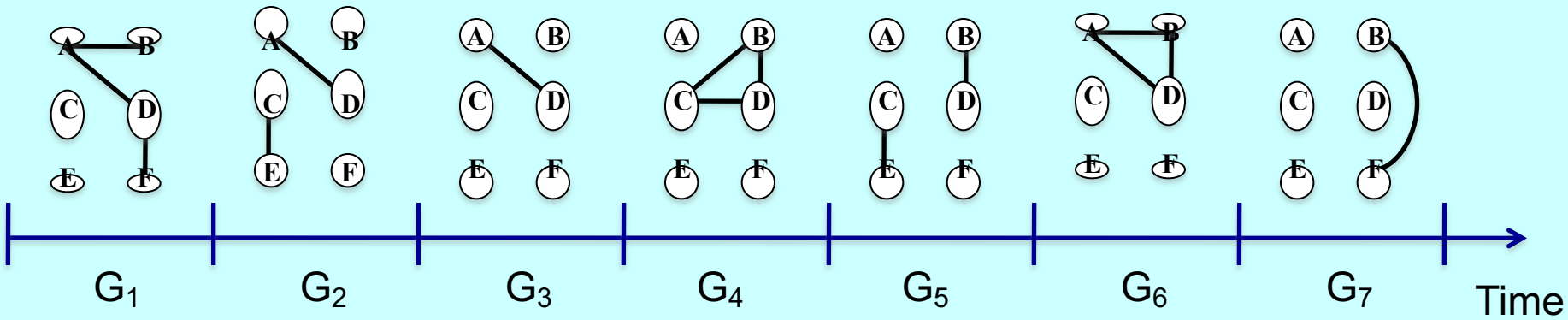

SAPIENZA
UNIVERSITÀ DI ROMA
C
Eg
EI
Time

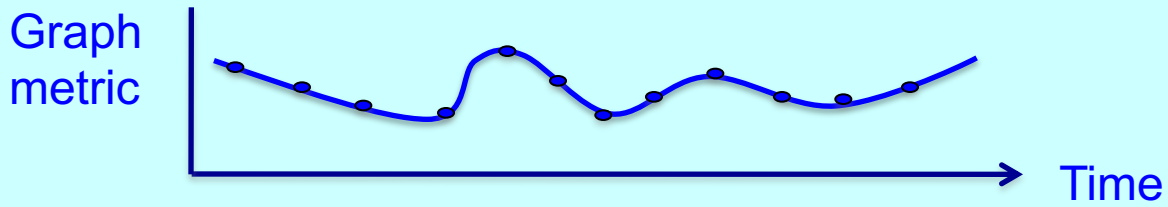

7_R
7_L
40_R
40_L
MF_R
MF_L
SMp_R
SMp_L
6l_R
6l_L
CM_R
CM_L
8_R
8_L
9_R
9_L


Out

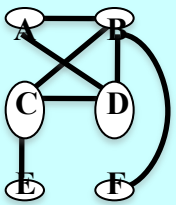

In

Temporal networks

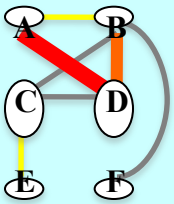




How to characterize the entire sequence G_t as a whole?



Aggregated



Weighted aggregated

We miss time order and correlations between links

New temporal metrics !!

Non-Markovian temporal networks

$$P(G_t = g | G_1 = g_1, G_2 = g_2, \ldots, G_{t-1} = g_{t-1})$$

DAR(p) process $\quad X_t = Q_t X_{(t-Z_t)} + (1 - Q_t) Y_t$

Where

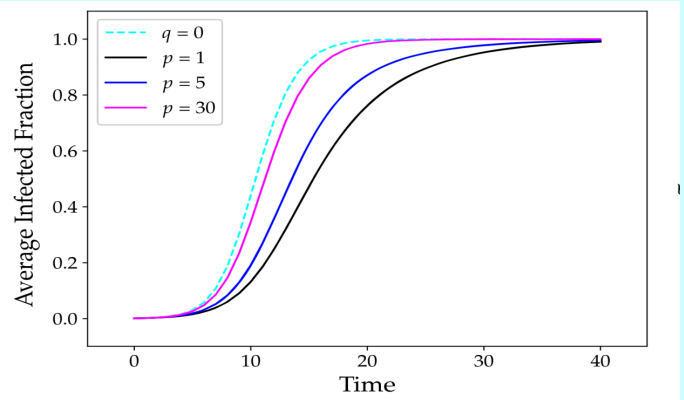
- $Q_t \sim \mathbb{B}(q)$
- $Z_t \in \{1, p\}$
- $Y_t \sim \mathbb{B}(y)$.

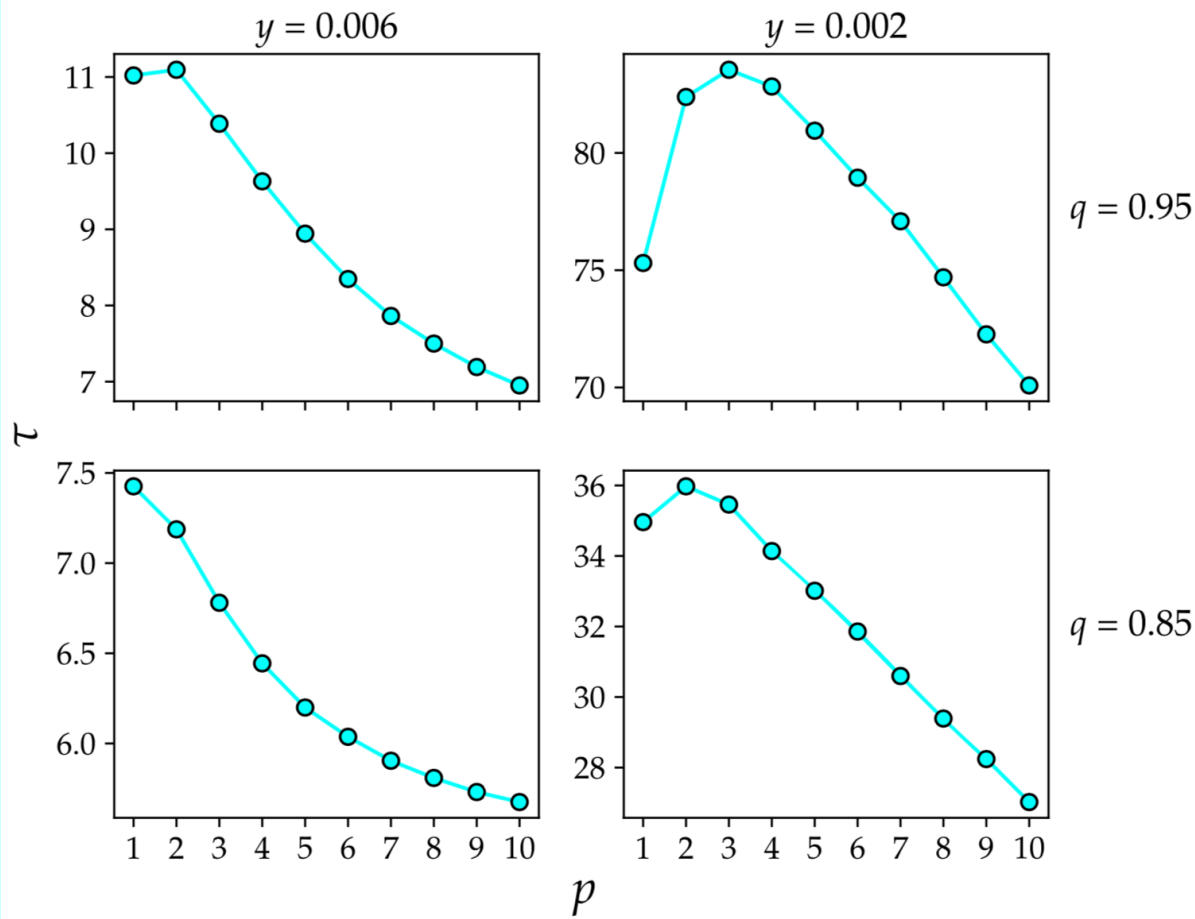
This gives an unweighted temporal network with precisely controlled length (p) and strength (q) of memory.

Williams, Lillo, Latora, New J. Phys. 21 (2019) 043028

Non-Markovian temporal networks

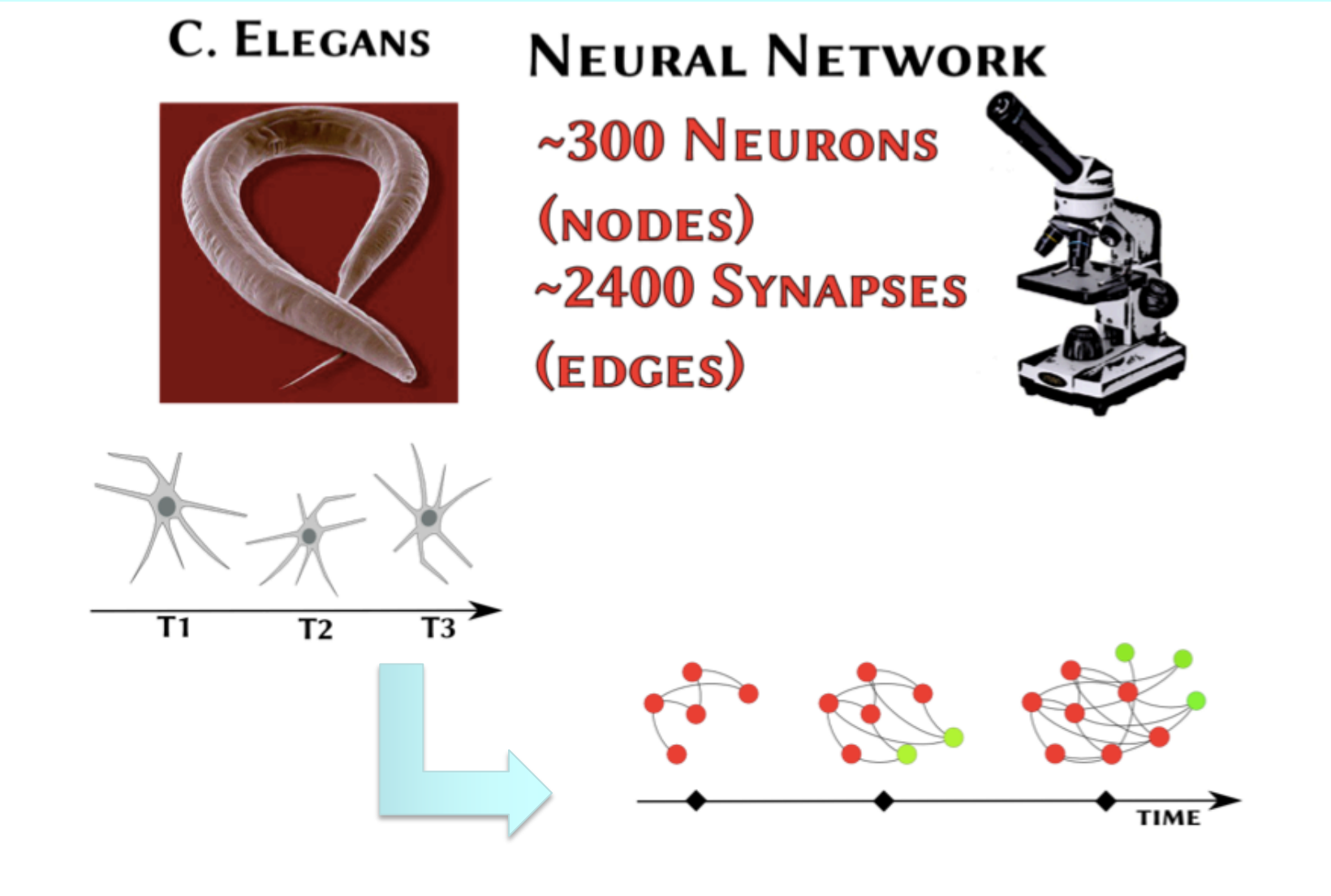
SI model
with $\lambda = 0.5$





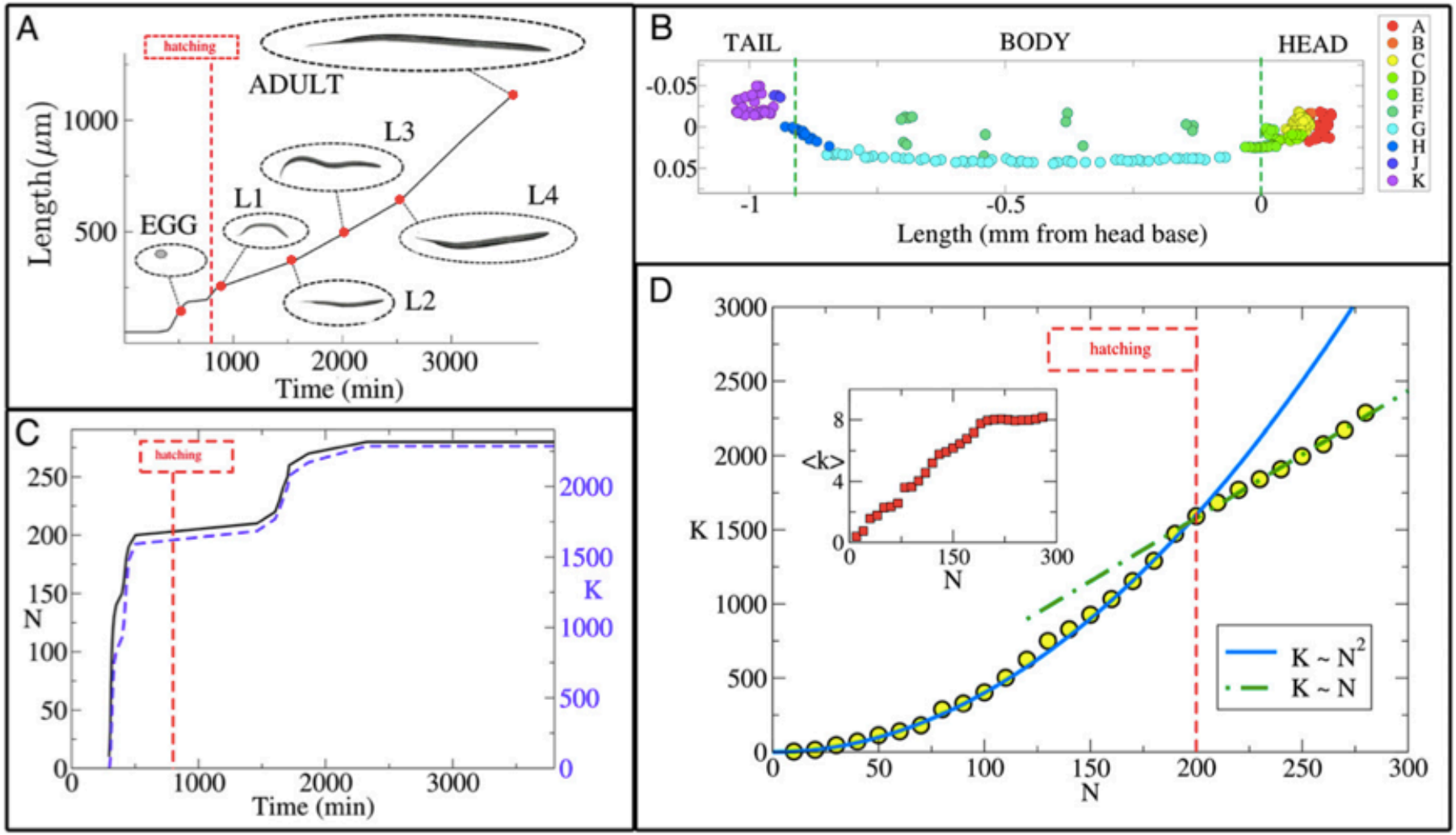
Williams, Lillo, Latora, New J. Phys. 21 (2019) 043028

The growth of a nervous system



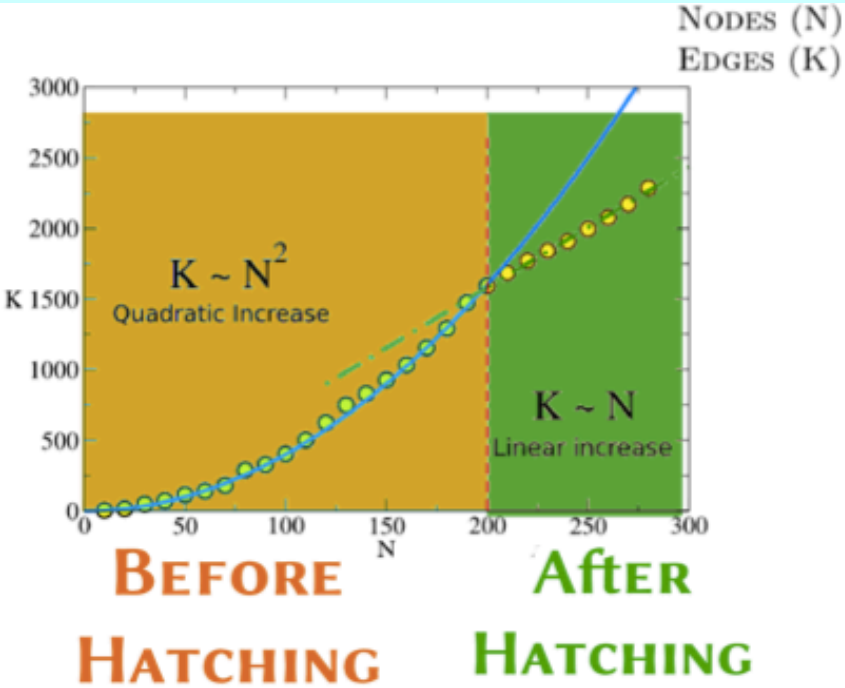
Nicosia, Vertes, Schafer, Latora, Bullmore, Proc. Natl. Acad. Sci. 110, 7880 (2013)

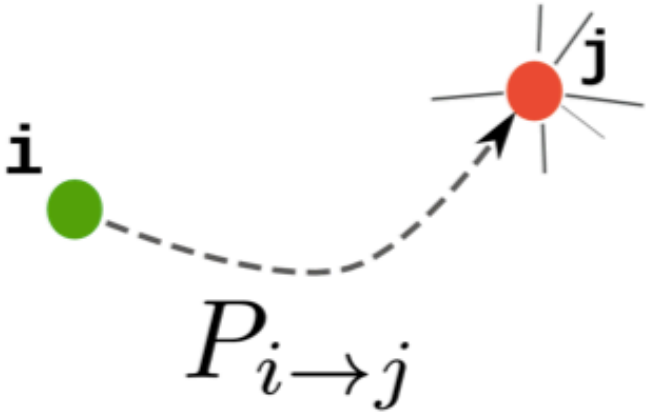
Phase transition in the network growth

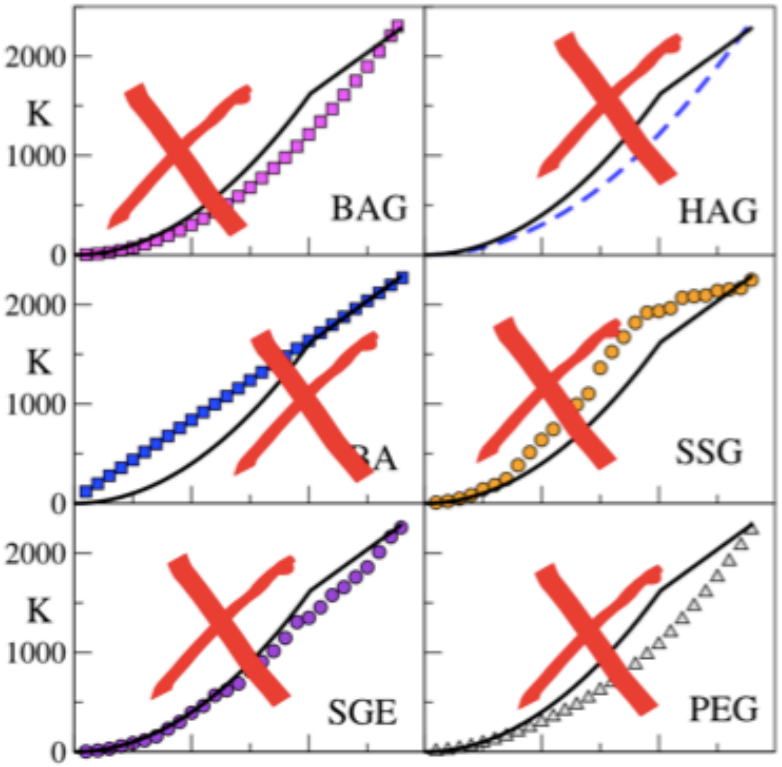


Nicosia, Vertes, Schafer, Latora, Bullmore, Proc. Natl. Acad. Sci. 110, 7880 (2013)

Modeling the temporal phase transition

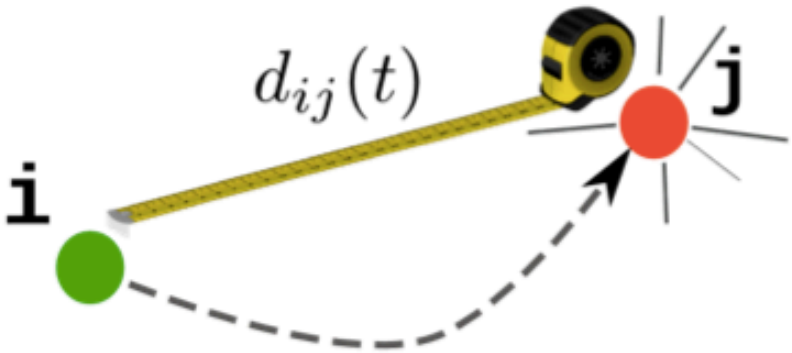






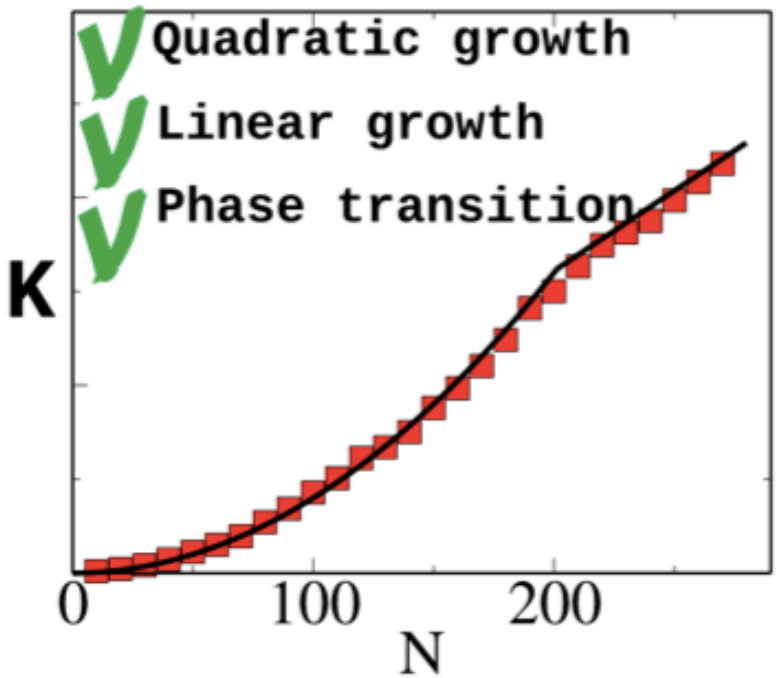
Nicosia, Vertes, Schafer, Latora, Bullmore, Proc. Natl. Acad. Sci. 110, 7880 (2013)

The ESTG model



$$p_{i\to j} \propto h_j e^{-d_{ij}(t)/\delta}$$

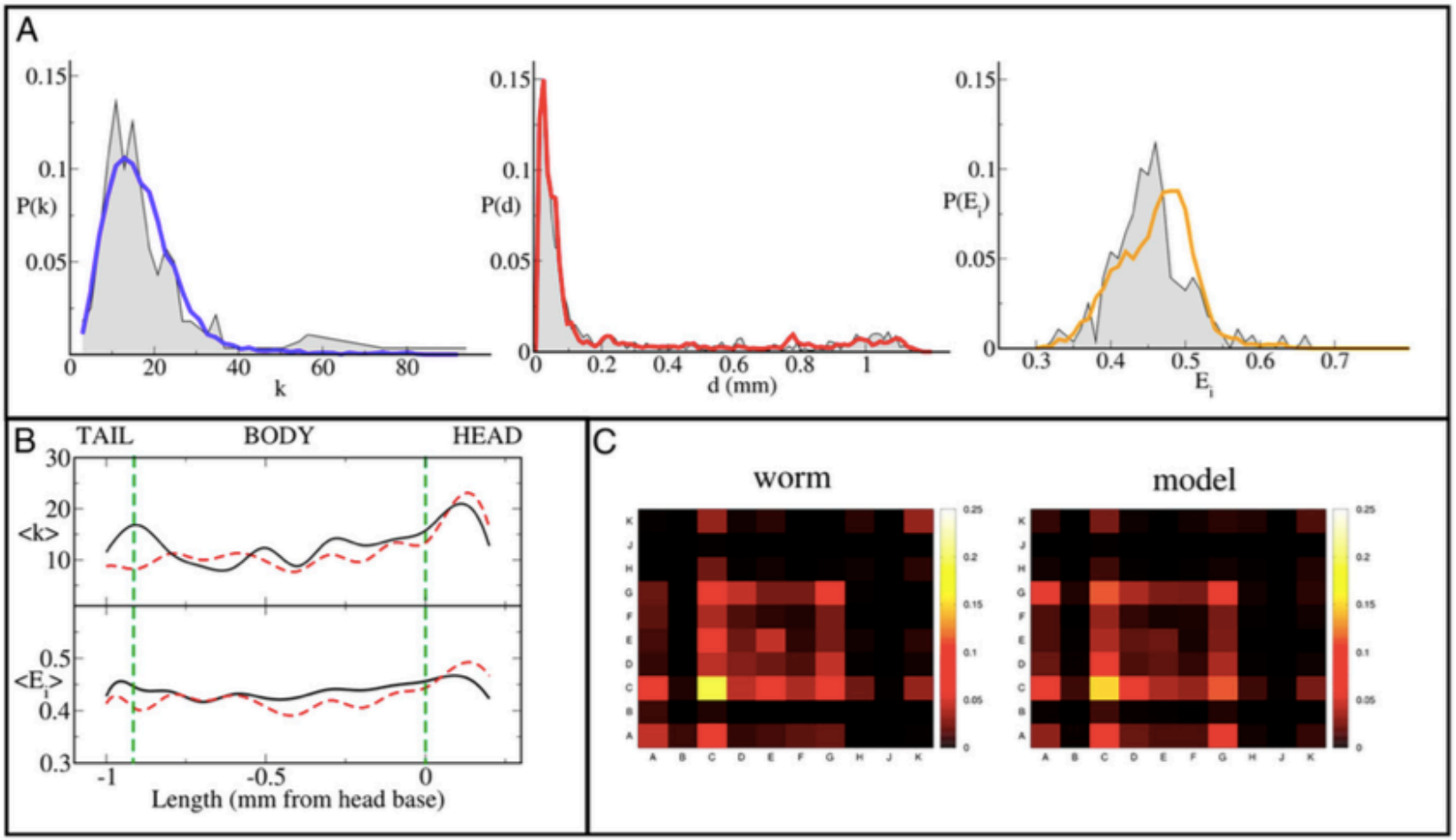
Efficiency Wiring cost



Economical Spatio-temporal growth

Nicosia, Vertes, Schafer, Latora, Bullmore, Proc. Natl. Acad. Sci. 110, 7880 (2013)

The ESTG model

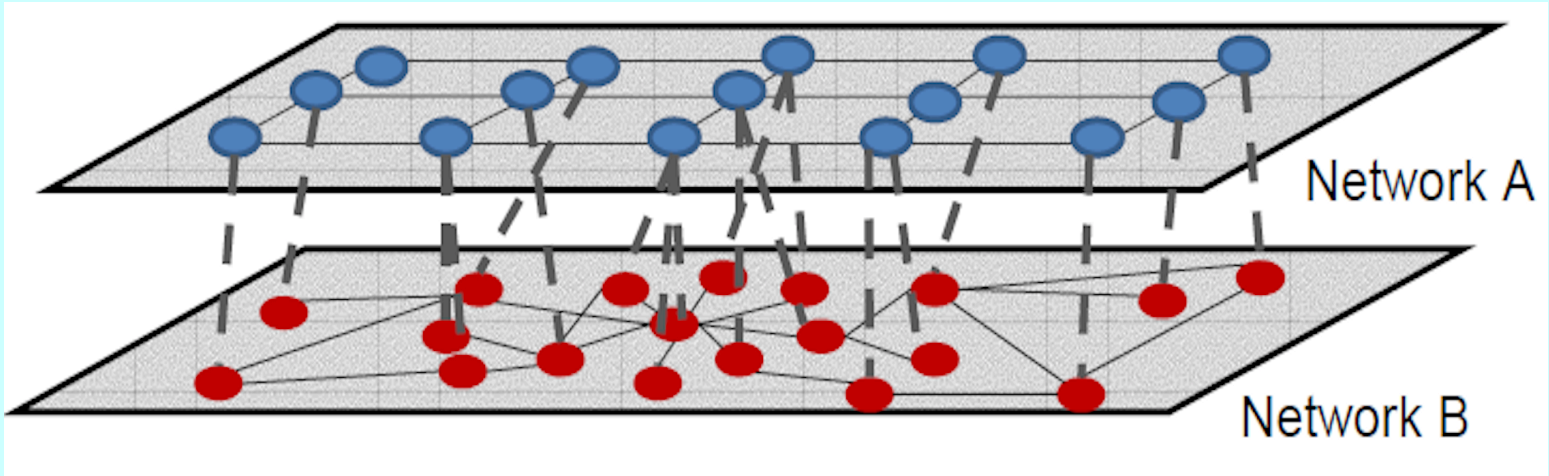


Reproduces distributions of degree, link lengths and node efficiency, and

patterns of connections between different ganglia

Interconnected networks

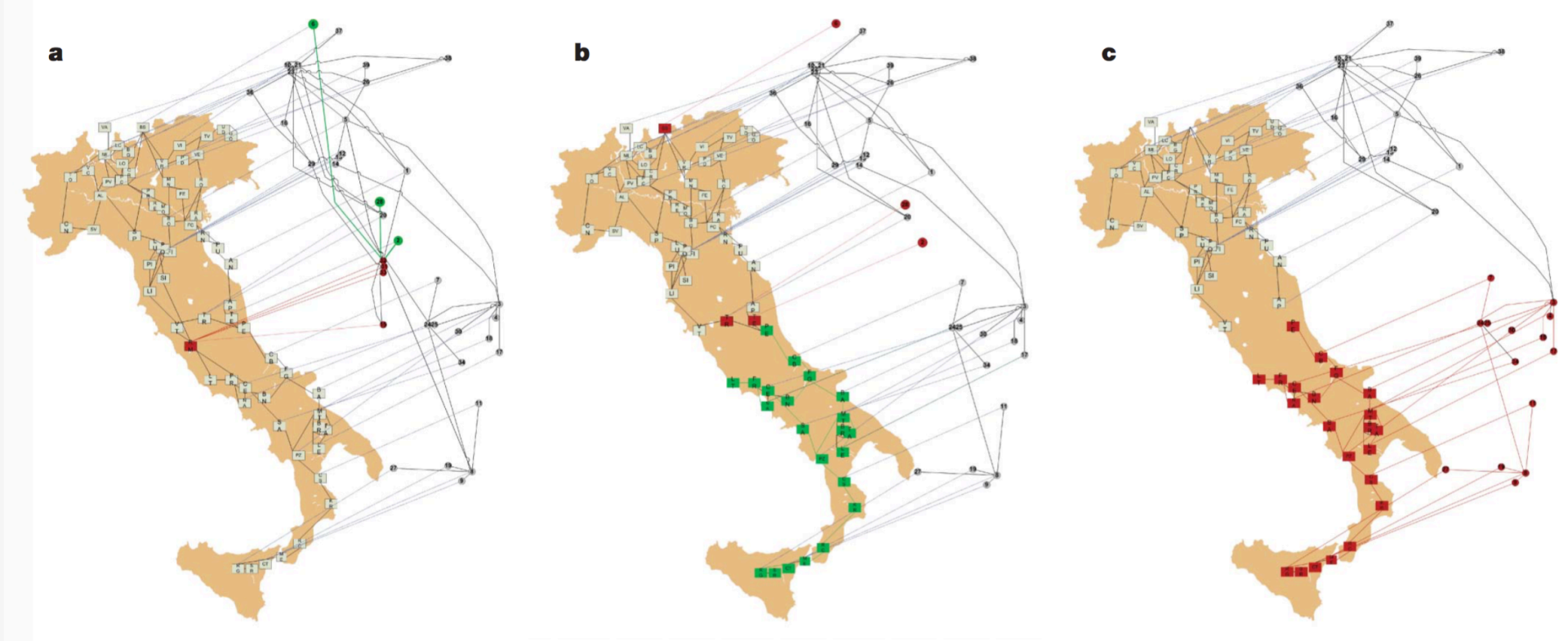
Electric power grid



Communication systems, e.g Internet

Buldyrev et al, Nature 2010

Interconnected networks



Buldyrev et al, Nature 2010

Interconnected networks

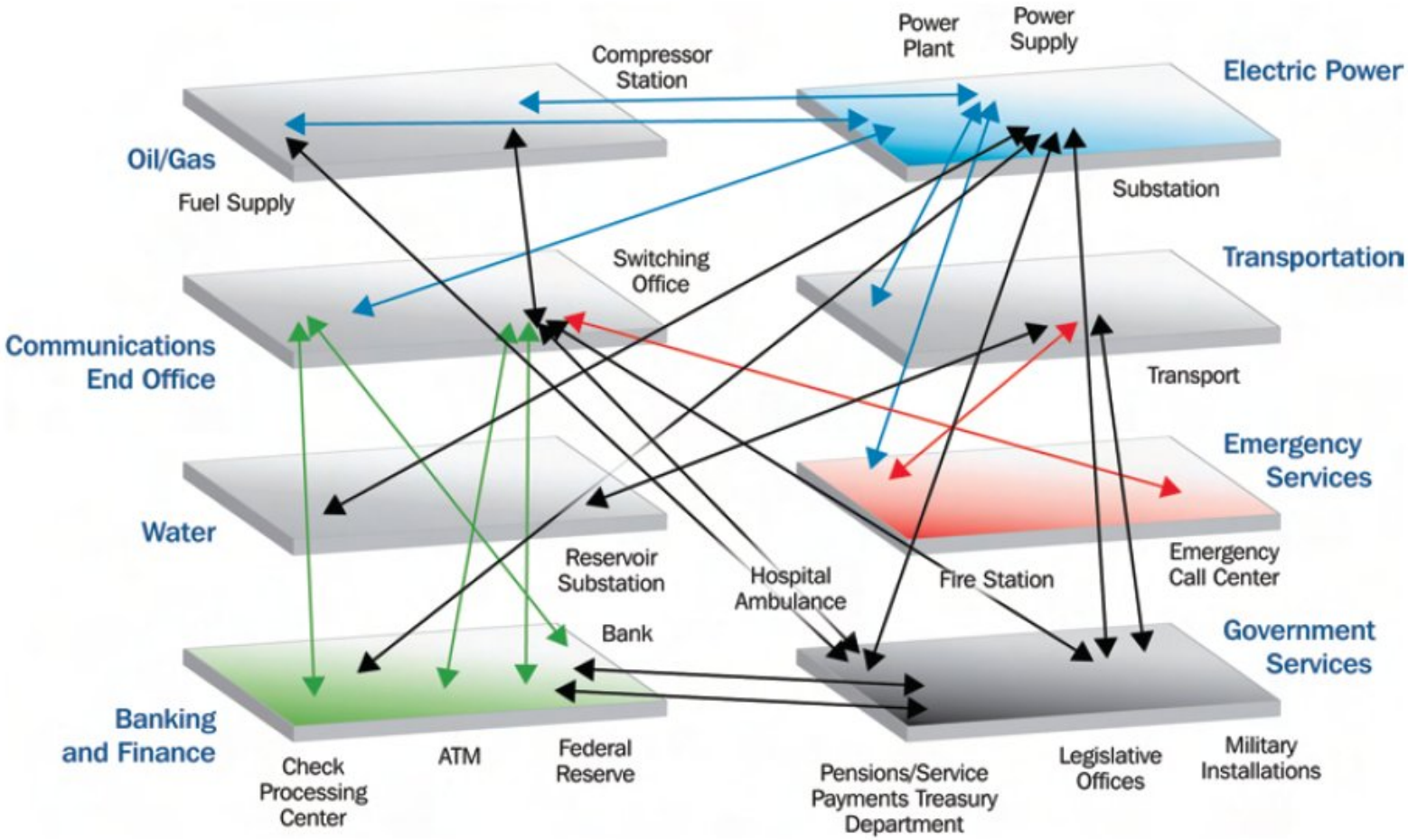
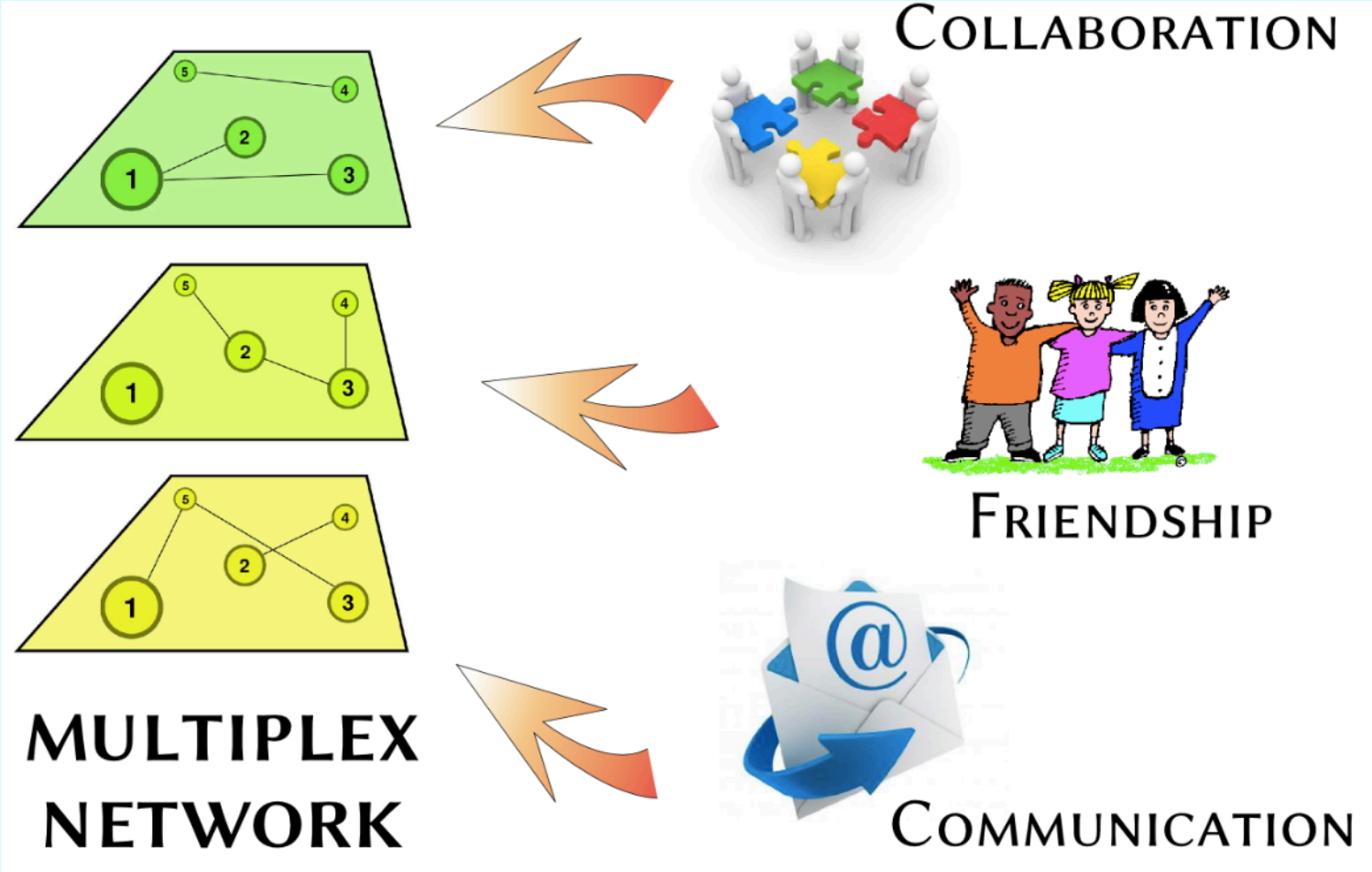


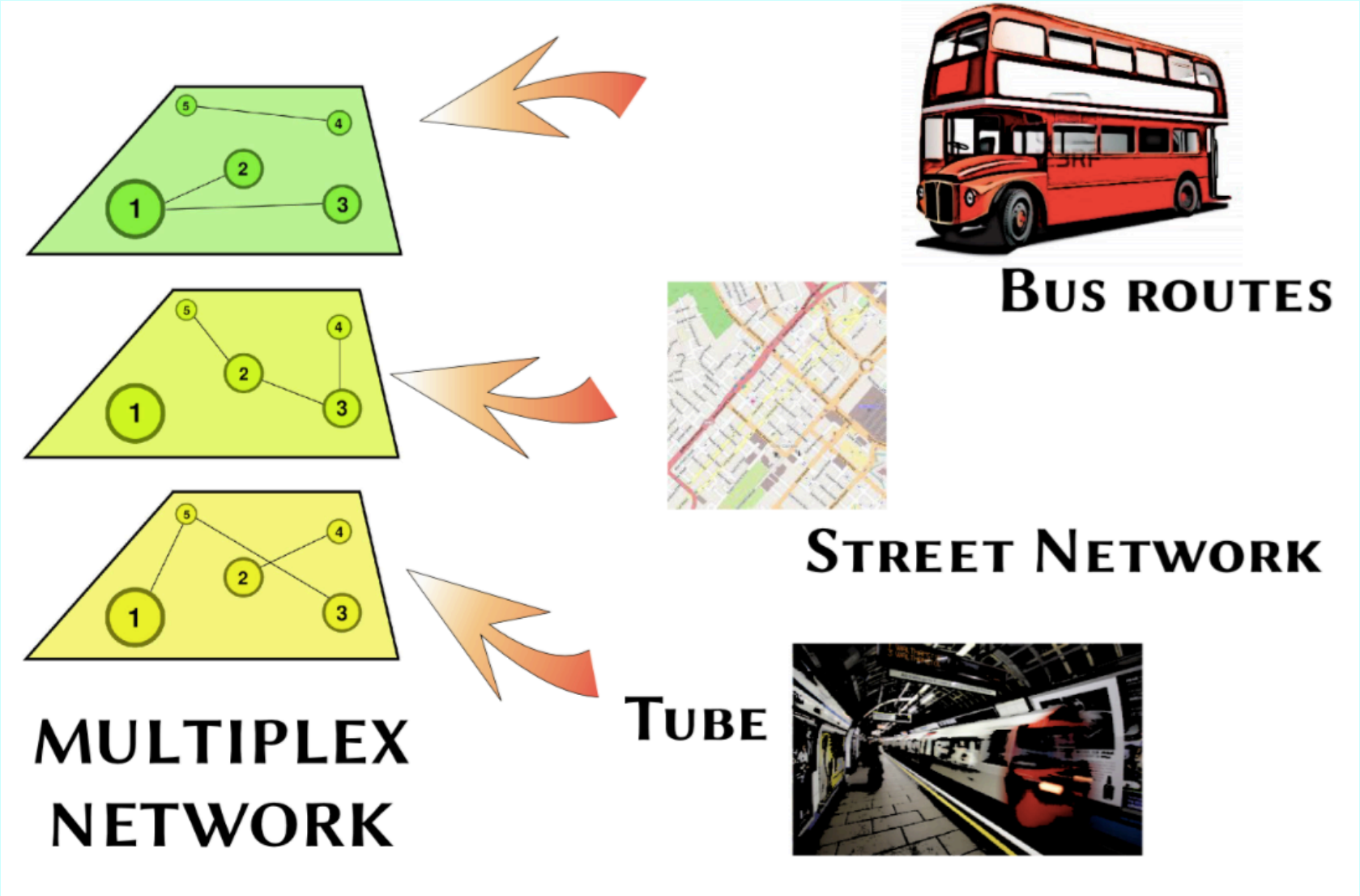
FIGURE 3.1 Connections and interdependencies across the economy. Schematic showing the interconnected infrastructures and their qualitative dependencies and interdependencies. SOURCE: Department of Homeland Security, National Infrastructure Protection Plan, available at http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm.

Multiplex networks



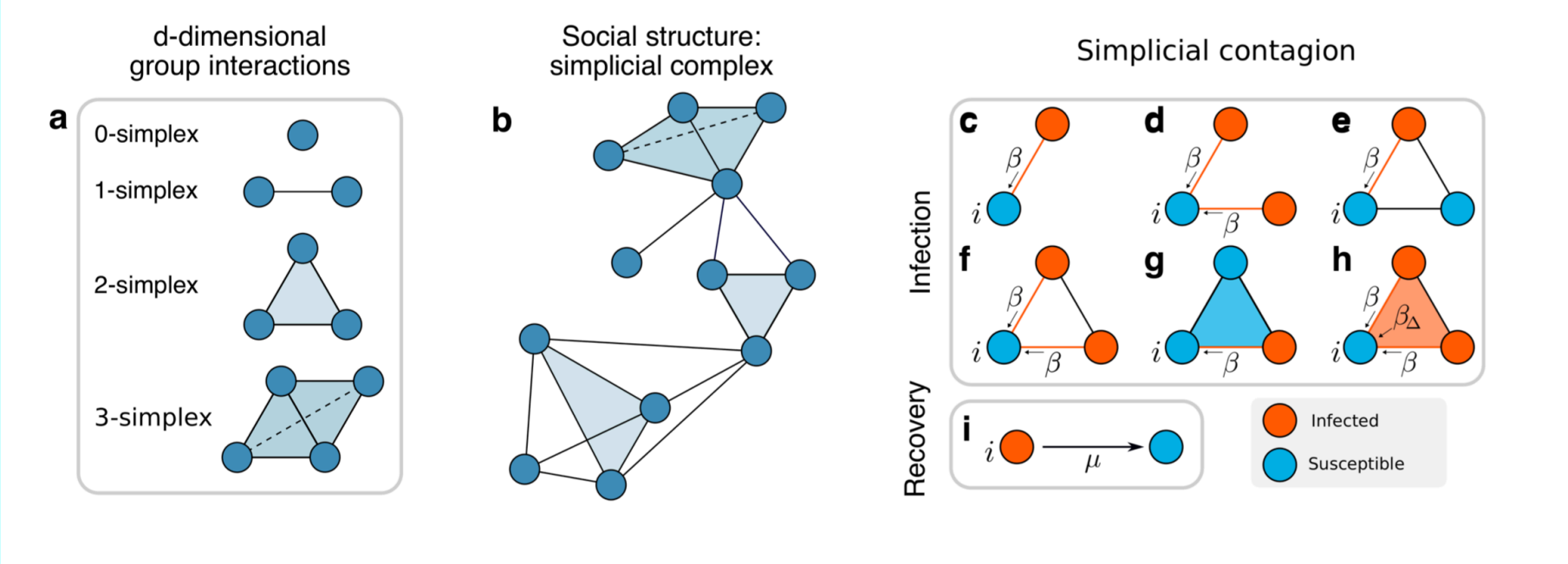
M=3 layers

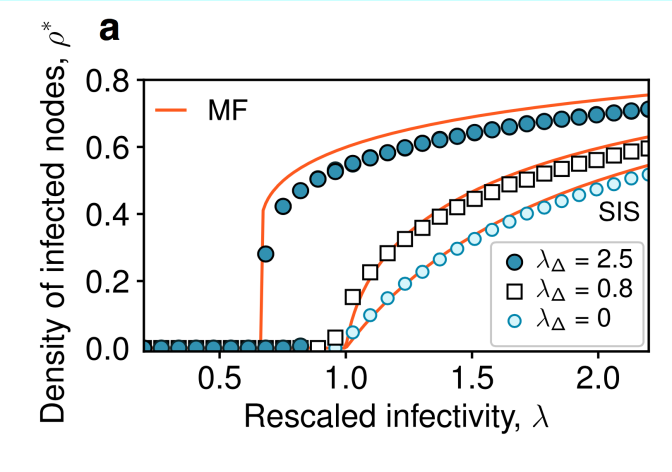
Multiplex networks

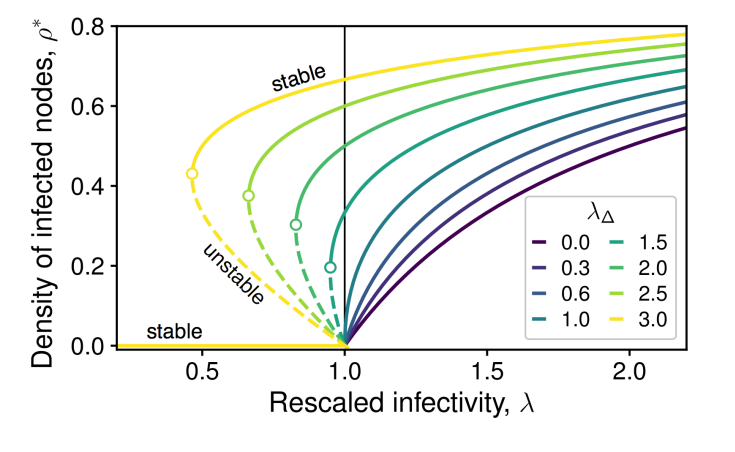


M=3 layers

Simplicial complexes







Iacopini, Petri, Barrat, Latora, arXiv:1810.07031, Nat. Comm.

``Rich" networks → generalised graphs

Node and links are in space and in time (spatial networks
temporal networks)

Node and links can be of different types (multiplex,
interconnected networks

The basic units are not only nodes and links (hypergraphs,
simplicial complexes)

Simplicial models of social contagion,
Iacopini, Petri, Barrat, Latora
Nature Communications 10, 2485 (2019)

Less and more ``**exotic**'' **applications**

Brain, innovation ecosystems, finance,
urban systems.

History, anthropology, archeology, arts.

``Exotic'' network applications

nature human behaviour

LETTERS

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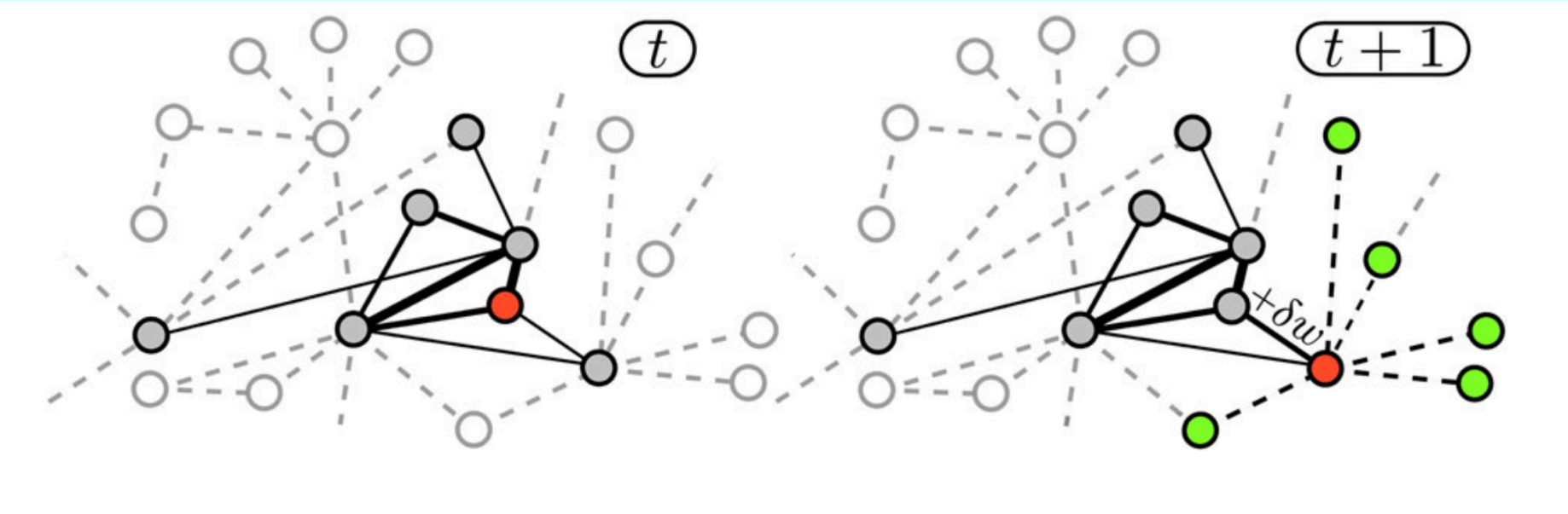
Characterization of hunter-gatherer networks and implications for cumulative culture

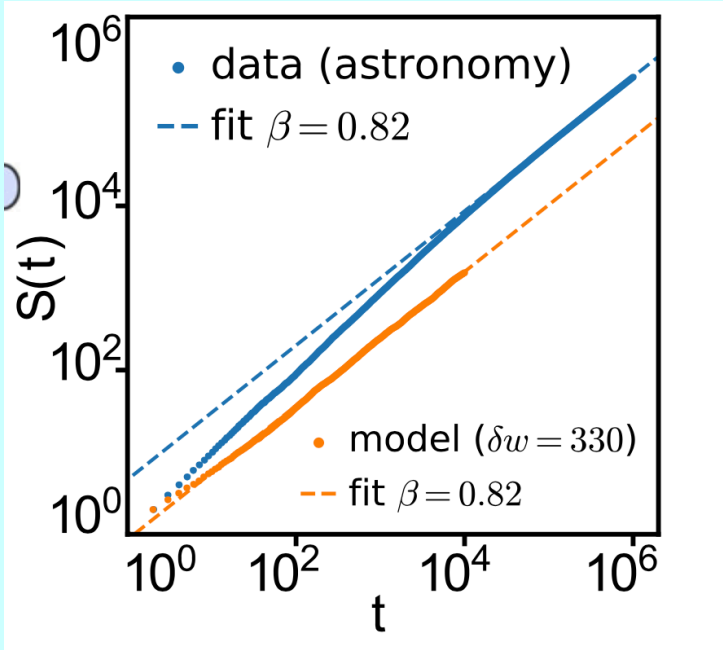
A. B. Migliano[1]*, A. E. Page[1], J. Gómez-Gardeñes[2], G. D. Salali[1], S. Viguier[1], M. Dyble[1], J. Thompson[1], Nikhill Chaudhary[1], D. Smith[1], J. Strods[1], R. Mace[1], M. G. Thomas[3], V. Latora[4] and L. Vinicius[1]

Anthropology
UCL

``Exotic'' network applications



Network Dynamics of Innovation Processes



Iacopini, Milojevic, Latora, Phys. Rev. Lett. 120, 048301 (2018)