Asymptotic symmetries: from low to high spins

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28th February 2019

* 1703.01351
* 1712.09591
* 1808.01542
* 1810.05634
* in progress

with:

A. Campoleoni,

C. Heissenberg

BASIC SETUP

* THEORIES WITH MASSLESS QUANTA & LOCAL SYMMETRY

* FOCUS ON THE INTERPLAY BETWEEN

LOCAL SYMMETRY $\longleftrightarrow$ BOUNDARY DATA

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- S-MATRIX ELEMENTS
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LOCAL SYMMETRY ⟷ BOUNDARY DATA

"REDUNDANCY" ?

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Plan

§ I. SOFT THEOREMS & MEMORY EFFECTS: GENERALITIES

§ II. LOW SPINS IN $D = 4$ AND $D > 4$

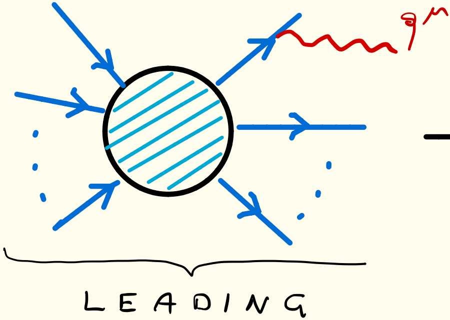
§ III. HIGHER-SPIN ASYMPTOTIC SYMMETRIES

§ IV. ASYMPTOTIC SYMMETRIES & DUALITIES: TWO-FORM LGT AND SCALAR SOFT THEOREMS

§I. Soft theorems & memory effects: generalities

WEINBERG'S SOFT THEOREM

$S_{\alpha\beta}(q) =$



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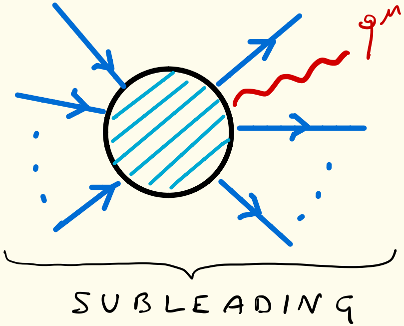
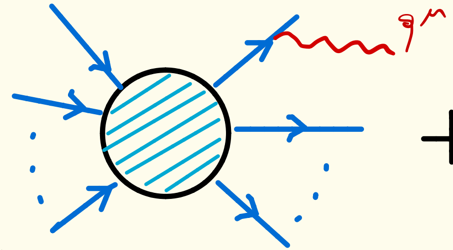


$q^2 = 0, \; q^\mu \to 0$

WEINBERG'S SOFT THEOREM

$$S_{\alpha\beta}(q) = \underbrace{\text{}}_{\text{LEADING}} + \underbrace{\text{}}_{\text{SUBLEADING}}$$



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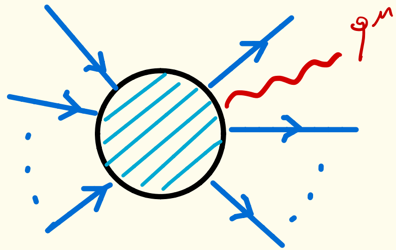

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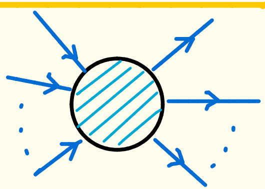
LEADING

SUBLEADING

$$\left\{ \sum_{n \in \{in, out\}}' g_n^{(s)} \, \frac{\epsilon_{\mu_1 \ldots \mu_s}(q)\, p_n^{\mu_1} \cdots p_n^{\mu_s}}{p_i \cdot q} \right\}$$



ORIGIN / MEANING OF THIS SOFT THEOREM ?

MEMORY EFFECTS

MEMORY: PERMANENT MODIFICATION OF SOME PHYSICAL QUANTITY PERTAINING TO A PROBE CLOSE TO NULL INFINITY, DUE TO INTERACTION WITH A TRANSIENT PERTURBATION OF SOME MASSLESS FIELD

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E.G. VELOCITY KICK OF A CHARGE DUE TO THE PASSAGE OF AN E.M. WAVE :

$$\vec{v}_F - \vec{v}_i \sim \frac{q}{m} \int_{u_i}^{u_F} \vec{E}\, du$$

* suitable falloffs for $\vec{E}$ and $\vec{B}$
* slow motion

Relating to symmetries

Soft theorems:

* Universal relation $\rightarrow$ underlying symmetries?
* Indeed, already in [Ferrari-Picasso '70, '71], for QED

RELATING TO SYMMETRIES

SOFT THEOREMS:

* Universal relation → underlying symmetries?
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MEMORY EFFECTS AS VACUUM TRANSITIONS

* THE MAXWELL TENSOR IS IN THE SAME STATE (SAY: THE VACUUM) BEFORE AND AFTER THE TRANSIENT PERTURBATION OCCURS;

  ⟹ IN ORDER TO INTERPRET THE EFFECT IN TERMS OF DYNAMICS OF THE MAXWELL TENSOR ONE HAS TO ASSUME THAT THE TWO VACUA DIFFER BY A GAUGE TRANSFORMATION

So, we'd like to identify the asymptotic symmetries of a theory

ASYMPTOTIC SYMMETRY GROUP $\Longleftrightarrow$ BOUNDARY DATA

$\Longrightarrow$

HOW TO SELECT "APPROPRIATE" FALLOFFS?

A FEW REMARKS :

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→ NONE OF THEM MAY IMPACT ON THE GAUGE POTENTIALS

* IT IS ONLY AFTER GAUGE FIXING THAT THE COMPONENTS OF THE POTENTIALS (PARTLY) ACQUIRE PHYSICAL MEANING, IT BECOMES SENSIBLE TO DISCUSS THEIR OWN FALLOFFS AND CONSEQUENTLY INVESTIGATE THE ASYMPTOTIC SYMMETRY GROUP:

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ASG: SUBGROUP OF THE FULL GROUP OF (LOCAL) SYMMETRIES THAT PRESERVE

- THE GAUGE FIXING
- THE BOUNDARY DATA

MODULO TRIVIAL SYMMETRIES (:= ZERO CHARGE)

HOWEVER:

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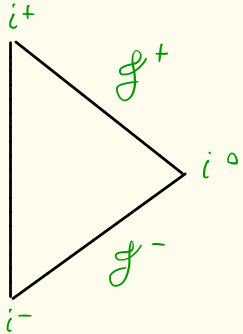
2. AS FOR WHAT CONCERNS THE ASYMPTOTIC SYMMETRY GROUP, IT IS NOT CLEAR HOW TO GUARANTEE, A PRIORI, THE GAUGE INDEPENDENCE OF THE RESULT

§II. Low spins in $D = 4$

Asymptotically-Flat Space Times

* Flat Space



$$ds^2 = -du^2 - 2\,du\,dr + r^2 \gamma_{z\bar z}\, dz\, d\bar z$$

$$(t, r, \vartheta, \varphi) \longrightarrow (u, r, z, \bar z)$$

$\mathscr{I}^+$: Retarded Bondi Coordinates

* $u = t - r$
* $z = e^{i\varphi}\, \mathrm{ctg}\, \vartheta/2$
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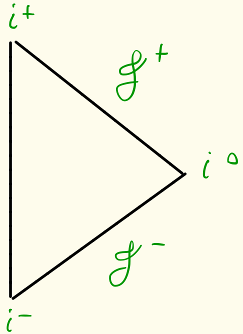

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* **Asymptotically flat spaces*** [Ashtekar - Napoli 1984]

(M, g) whose conformal completion has a null boundary "RESEMBLING" the null boundary of the Carter - Penrose diagram of Minkowski space

*at null infinity

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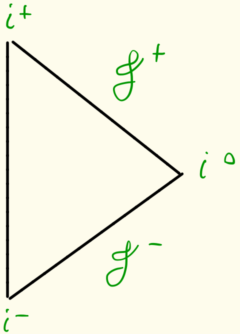

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BMS GROUP

A USEFUL, THOUGH NON-COVARIANT, PARAMETRIZATION OF AFS IS

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

WHERE $h_{\mu\nu}(r, u, z, \bar{z})$ IS SUBJECT TO THE "BONDI GAUGE":

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- $h_{r\mu} = 0$
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- $h_{uu} = \dfrac{2m_B}{r}$
- $h_{uz} = -U_z$
- $h_{zz} = r\, C_{zz}$

* They combine: GAUGE FIXING
FALLOFFS
ANALYSIS OF NONLINEAR EOM

* however, they can be consistently implemented in the LINEAR THEORY too.

* m_B, U_z and C_{zz} (and $z \leftrightarrow \bar{z}$) DO NOT DEPEND ON r;

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THUS, IN A SENSE

[Bondi, Metzner and van der Burg; Sachs 1962]

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- BOUNDARY LIMIT

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[Bondi, Metzner and van der Burg; Sachs 1962]

$$GR \;\cancel{\longrightarrow}\; SR$$

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(under stronger conditions $GR \longrightarrow SR$; however, in $D=4$ $\Longrightarrow$ NO GRAV. WAVES)

WHAT ARE THE ASYMPTOTIC SYMMETRIES OF MAXWELL'S THEORY?

IT DEPENDS: WHAT IS THE CLASS OF FALL-OFF CONDITIONS THAT CAN BE CONSIDERED PHYSICALLY SENSIBLE?

SPIN-ONE LGT

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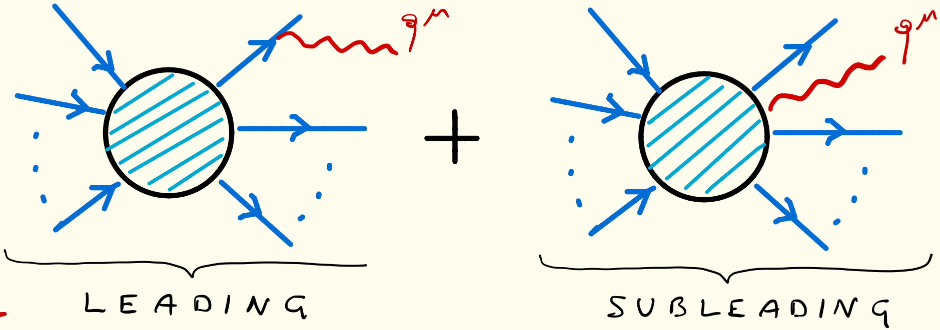

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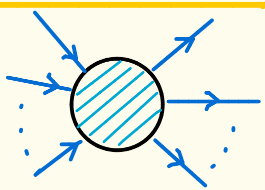

ORIGIN/MEANING OF THIS SOFT THEOREM ?

$\forall D$

No supertranslations nor memory in $D>4$?

[Hollands - Ishibashi - Wald 2016]

Basic assumptions on falloffs:

1. To be assigned directly on the metric
2. Leading falloffs describe D-dim radiation $\sim 1/r^{\frac{D-2}{2}}$

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$\rightarrow$ No supertranslations in $D>4$

* Under these conditions :

$\rightarrow$ No gravitational memory in $D>4$

[Kapec - Lysov - Pasterski - Strominger 2015]
[Mao - Ouyang 2017]
[Pate - Raclariu - Strominger 2017]

Higher-dimensional supertranslations

Alternative falloffs & higher-D gravitational memory

[Kapec - Lysov - Pasterski - Strominger 2015]
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HIGHER - DIMENSIONAL SUPERTRANSLATIONS

ALTERNATIVE FALLOFFS & HIGHER-D GRAVITATIONAL MEMORY

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   (BASICALLY : $h_{uu} \sim 1/r \quad \forall$ D)

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3. MEMORY EFFECTS RELATE TO METRIC COMPONENTS WITH COULOMBIC FALLOFFS : $\sim 1/r^{D-3}$ $\left[= 1/r^{\frac{D-2}{2}} \text{ only in } D=4\right]$; SUBLEADING SYMMETRIES [WITH VANISHING CHARGE AT NULL INFINITY] GRANT FOR $\exists$ OF MEMORY EFFECTS IN ANY (EVEN) D

§ II. HIGHER-SPIN ASYMPTOTIC SYMMETRIES

REMARK

TYPICALLY ONE CONSIDERS ASYMPTOTICALLY UNCHARGED MANIFOLDS:

* FREE E.M. WAVES AT NULL INFINITY — SPIN 1
* ASYMPTOTICALLY FLAT SPACETIMES — SPIN 2
* ASYMPTOTICALLY "HIGHER-SPIN FLAT" SPACETIMES — SPIN S

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→ in principle any asymptotic vacuum amenable to be considered; in practice Minkowski vacuum appears to be special

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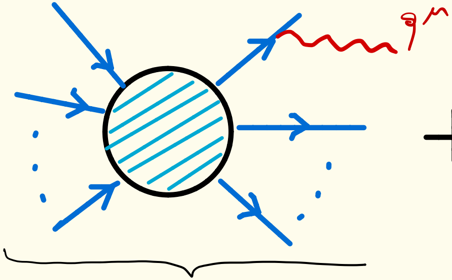
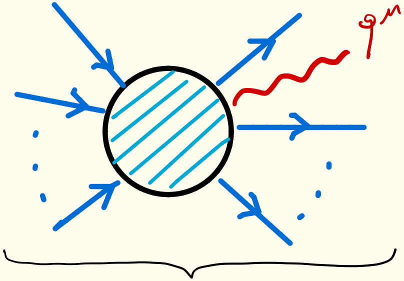
$S_{\alpha\beta}(q) =$ 

$+$ 

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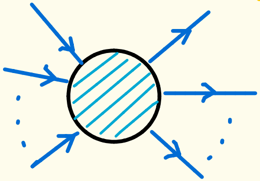
$$\left\{ \sum_{m\in\{in,out\}}{}^{'} \; g_m^{(s)} \, \frac{\epsilon_{\mu_1\dots\mu_s}(q)\, p_m^{\mu_1}\cdots p_m^{\mu_s}}{p_i\cdot q} \right\}$$ 

ORIGIN/MEANING OF THIS SOFT THEOREM ?

$\forall s$

FOR $S \geq 3$:

(AND $D > 4$)

CAN ONE INTERPRET WEINBERG'S RESULT AS THE WARD IDENTITY STEMMING FROM SOME ASYMPTOTIC HIGHER-SPIN SYMMETRY?

FALL-OFFS & BONDI-LIKE GAUGE

CRUCIAL STEP: ASSIGN SPIN-S FALL-OFF CONDITIONS

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- $\varphi_{zzz} = r^2 B_{zzz}$

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* SPIN S :

$$\varphi_{\mu_1 \dots \mu_{s-1} r} = 0$$

$$\varphi_{\mu_1 \dots \mu_{s-2} z\bar{z}} = 0$$

$$\varphi_{u \dots u \underbrace{z \dots z}_{k}} = O(r^{k-1})$$

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$$\varphi_{\mu_1 \dots \mu_{s-1} r} = 0$$

$$\varphi_{\mu_1 \dots \mu_{s-2} z\bar{z}} = 0$$

$$\varphi_{u \dots u \underbrace{z \dots z}_{k}} = O(r^{k-1})$$

* GUIDING LINES :

- ANALOGY WITH GRAVITY
- CONSISTENCY WITH FREE EOM

FALL-OFFS & BONDI-LIKE GAUGE

CRUCIAL STEP: ASSIGN SPIN-S FALL-OFF CONDITIONS

$s = 3$

- $\varphi_{\mu\nu r} = 0$
- $\varphi_{\mu z \bar{z}} = 0$
- $\varphi_{uuu} = B/r$
- $\varphi_{uuz} = U_z$
- $\varphi_{uzz} = r\, C_{zz}$
- $\varphi_{zzz} = r^2 B_{zzz}$

$(z \longleftrightarrow \bar{z})$

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$D = 4$

* GUIDING LINES :

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PROGRAM

- COMPUTE THE RESIDUAL SYMMETRIES

$$\delta \varphi_{\mu_1 \dots \mu_s} = \nabla_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}$$

(IF ANY) THAT KEEP THE BONDI-LIKE "GAUGE"

- SEE WHETHER THEY BEAR ANY RELATION TO WEINBERG'S SOFT THEOREM

HIGHER-SPIN SUPERTRANSLATIONS

SIMPLIFYING ANSATZ: u-INDEPENDENT RESIDUAL SYMMETRY

→ The residual gauge symmetry gets determined by a set of recursive equations:

$$\epsilon_{\underbrace{u\dots u}_{p}\underbrace{z\dots z}_{k}\underbrace{z\bar{z}\dots z\bar{z}}_{c+1}}(z,\bar{z}) \sim \epsilon_{p,k,c} - 2\,\epsilon_{p+1,k,c}$$

$$\epsilon_{p,k,0} \sim r^{k} D_z^{k}\, T_p(z,\bar{z})$$

$$T_{p+1} \sim \alpha(s,p)\, T_p + \beta(s,p)\, D^z D_z\, T_p$$

→ HIGHER-SPIN SUPERTRANSLATIONS $\exists$, and are PARAMETERISED BY A SINGLE FUNCTION ON THE CELESTIAL SPHERE: $T(z,\bar{z})$

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SUPERTRANSLATION WARD IDENTITY

→ LET US EXPLORE THE CONSEQUENCES OF $\left[Q = Q_{SOFT} + Q_{HARD},\, S_{MATRIX}\right] = 0$

$*$ $$Q_s^+ \sim \int_{\mathcal{I}^+} T(z,\bar z)\, \partial_u D_z^s B_{z\ldots z}(u,z,\bar z)\, \gamma_{z\bar z}\, dz\, d\bar z\, du$$

where :
- → Q_s^+ is FINITE and $\neq 0$ (check on admissibility of fall-offs);
- → Analogous expression for Q_s^-;
- → Suitable bd conditions at $\mathcal{I}^\pm_\mp$ are assigned;

$*$ $$\left[Q_H^+,\, \Phi\right] \sim i\, T(z,\bar z)\, \partial_u^{s-1} \Phi$$

$*$ $$\langle OUT |\, Q_s^+ S - S Q_s^- \,| IN \rangle \sim \sum_i g_i^{(s)}\, T(z,\bar z)\, E_i^{s-1} \langle OUT |\, S \,| IN \rangle$$

REPRODUCING WEINBERG'S RESULT

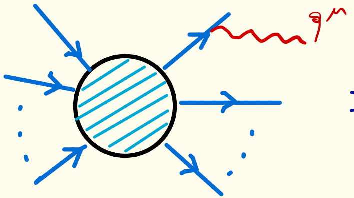
* TECHNICAL STEPS TO GET TO THE RESULT:

→ SUITABLE CHOICE OF $T(z,\bar{z})$;

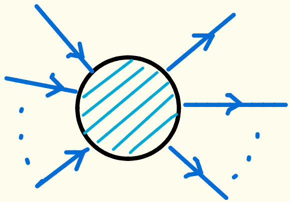
→ IDENTIFY THE INSERTION OF SOFT SPIN-S QUANTA FROM $Q_s^{\pm}$;

→ TRANSFORM FROM MOMENTUM SPACE TO CONFIGURATION SPACE;

→ APPLY D_z^{s-1}, AND EVENTUALLY REPRODUCE WEINBERG'S RESULT:



$$= \left\{ \sum_{m \in \{in,out\}}' g_m^{(s)} \frac{\epsilon_{\mu_1 \dots \mu_s}(q)\, p_m^{\mu_1} \cdots p_m^{\mu_s}}{p_i \cdot q} \right\}$$



Any D, $s = 3$

SPIN 3 : POWER EXPANSION & BRANCHES OF SOLUTIONS

* EVEN D $[n = D-2]$

$$\varphi_{\mu\nu\rho} = \sum_{l=0}^{+\infty} r^{-\frac{n}{2}-l+i} K^{(l)}_{\mu\nu\rho}(u, x^m) ;$$

$i = \#$ indices of m type in $\varphi_{\mu\nu\rho}$

→ non-zero, yet FINITE energy flux at null infinity

→ leading order $\varphi_{uuu} \sim r^{-\frac{n}{2}}$: RADIATION TERM

* ODD D

$$\varphi_{\mu\nu\rho} = \underbrace{\sum_{l=0}^{+\infty} r^{-\frac{n}{2}-l+i} K^{(l)}_{\mu\nu\rho}(u, x^m)}_{\text{contains powers of } \sqrt{r};\ \text{needed to account for radiation}} + \underbrace{\sum_{l=0}^{+\infty} r^{1-n-l} \hat{K}^{(l)}_{\mu\nu\rho}(u, x^m)}_{\text{Contains the Coulomb-type term} \sim r^{3-D}}$$

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SPIN 3 : BONDI-LIKE GAUGE & FALLOFFS

* $\forall\, D$ we chose Bondi-like conditions :

$$\varphi_{r\mu\nu} = 0 \qquad\qquad \gamma^{ij}\varphi_{ij\mu} = 0$$

under which the Fronsdal eqs simplify to

$$\mathcal{F}_{\mu\nu\rho} = \Box\varphi_{\mu\nu\rho} - \nabla_{(\mu}\nabla^{\alpha}\varphi_{\nu\rho)\alpha} = 0$$

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* The solution to the power-like ansatz, plus reasonable assumptions, provides the falloffs (here: leading order) :

$$\varphi_{uuu} = \mathcal{O}(r^{-\frac{n}{2}}), \quad \varphi_{uui} = \mathcal{O}(r^{-\frac{n}{2}+1}), \quad \varphi_{uij} = \mathcal{O}(r^{-\frac{n}{2}+2}), \quad \varphi_{ijk} = \mathcal{O}(r^{-\frac{n}{2}+3})$$

SPIN 3 : ASYMPTOTIC SYMMETRIES & CHARGES

* $\delta \varphi_{\mu\nu\rho} = \nabla_{(\mu} \xi_{\nu\rho)}$ SUCH TO PRESERVE THE FALLOFFS

$$\Longrightarrow \quad \xi_{\mu\nu} = \xi_{\mu\nu}(T, K_{ij}, \rho_i)$$

* $T(x)$, $K_{ij}(x)$, $\rho_i(x)$ BOUND BY DIFFERENTIAL CONSTRAINTS;

THE AMOUNT OF SYMMETRY DEPENDS ON THE SOLUTION SPACE

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HIGHER-SPIN SUPERROTATIONS ($s=3$)

LOOK FOR THE GENERAL SOLUTION, FOR $\epsilon_{\mu\nu} = \epsilon_{\mu\nu}(u, z, \bar{z})$

BESIDES HSP SUPERTRANSLATIONS, PARAMETRISED BY $T(z,\bar{z})$

WE FIND:

→ $K_{ij}(z, \bar{z})$ CONFORMAL KILLING TENSORS ON THE CELESTIAL SPHERE $\left(K(z), \tilde{K}(\bar{z})\right)$

→ $\rho_i(z, \bar{z})$

$$\rho^z = \alpha(z)\partial_z K(z,\bar{z}) + \beta(z)$$

$$\tilde{\rho}^{\bar{z}} = \tilde{\alpha}(\bar{z})\partial_{\bar{z}} K(z,\bar{z}) + \tilde{\beta}(\bar{z})$$

- i,j run over coordinates on the celestial sphere
- K_{ij} and ρ_i satisfy differential constraints.

ENHANCING WHAT ?

$$\mathfrak{bms}: \left\{ \begin{array}{lll} T \quad (ST) & \longleftrightarrow & P^{\mu} \\ & & \\ Y_z \quad (SR) & \longleftrightarrow & M^{\mu\nu} \end{array} \right\}$$

[BARNICH, TROESSAERT]

INFINITE - DIM
ENHANCEMENT OF
POINCARÉ

$1 \div 1$ WITH
SOLUTIONS
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$\mathfrak{bms}$: [BARNICH, TROESSAERT]

$$\begin{cases} T & (ST) \\ Y_z & (SR) \end{cases} \quad \begin{matrix} \longleftrightarrow & P^\mu \\ \longleftrightarrow & M^{\mu\nu} \end{matrix} \Bigg\}$$

1÷1 WITH SOLUTIONS OF $\partial_{(\mu}\epsilon_{\nu)} = 0$

INFINITE - DIM ENHANCEMENT OF POINCARÉ

"$\mathfrak{ahs}_3$" :

$$\begin{cases} T \\ K_{ij} \\ P_i \end{cases} \quad \begin{matrix} \longleftrightarrow & P^{(\mu}P^{\nu)} \\ \longleftrightarrow & M^{\mu(\nu}M^{\rho)\sigma} \\ \longleftrightarrow & P^{(\mu}M^{\nu)\rho} \end{matrix} \Bigg\}$$

TRACELESS PROJECTED

1÷1 WITH SOLUTIONS OF $\partial_{(\mu}\epsilon_{\nu\rho)} = 0$

INFINITE - DIM ENHANCEMENT OF $S=3$ KILLING

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[Kapec, Lysov, Pasterski and Strominger 2015]

- WEAKER FALLOFFS ON $h_{\mu\nu} \rightarrow$ SUPERTRANSLATIONS ANY $D=2M$

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CONCEIVABLY, THIS ATTITUDE MAY PROVIDE THE SOLUTION $\forall\, s, D$

§ IV Two-form LGT & scalar soft theorems

Can scalars have asymptotic symmetries?

D = 4

Miguel Campiglia,[1] Leonardo Coito,[1] and Sebastian Mizera[2,3]
[1]*Instituto de Física, Facultad de Ciencias, Montevideo 11400, Uruguay*
[2]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
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(Received 14 July 2017; published 5 February 2018)

Recently it has been understood that certain soft factorization theorems for scattering amplitudes can be written as Ward identities of new asymptotic symmetries. This relationship has been established for soft particles with spins $s > 0$, most notably for soft gravitons and photons. Here we study the remaining case of soft scalars. We show that a class of Yukawa-type theories, where a massless scalar couples to massive particles, have an infinite number of conserved charges. This raises the question as to whether one can associate asymptotic symmetries to scalars.

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* $\delta B_{\mu\nu} = \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu$ [together with gauge-for-gauge]

* $\epsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma} \sim \partial_\mu \phi$

YES :

[Campiglia, Freidel, Hopfmüller, Soni 1810.04213]

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MAIN POINT : PROVIDE A CANDIDATE SOFT CHARGE Q_S FOR THE SCALAR SOFT THEOREM FROM THE LGT OF $B_{\mu\nu}$:

$$Q_S \sim \int_{S^2} b(u, z, \bar{z})\, \Lambda(z, \bar{z})\, \gamma_{z\bar{z}}\, dz\, d\bar{z}$$

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OK (maybe), BUT WHY THE TWO RESULTS ARE DIFFERENT ?

* IT IS ONLY AFTER GAUGE FIXING THAT THE COMPONENTS OF THE POTENTIALS (PARTLY) ACQUIRE PHYSICAL MEANING, IT BECOMES SENSIBLE TO DISCUSS THEIR OWN FALLOFFS AND CONSEQUENTLY INVESTIGATE THE ASYMPTOTIC SYMMETRY GROUP.

HOWEVER:

1. ONE MAY STILL HAVE TO IMPOSE ADDITIONAL FALLOFF CONDITIONS ON GAUGE-INVARIANT QUANTITIES, e.g. TO COMPLY WITH MATTER FALLOFFS;

2. AS FOR WHAT CONCERNS THE ASYMPTOTIC SYMMETRY GROUP, IT IS NOT CLEAR HOW TO GUARANTEE, A PRIORI, THE GAUGE INDEPENDENCE OF THE RESULT

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BUT THEN WHY "TYPICALLY" IT DOESN'T HAPPEN ?

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- "TYPICALLY" $\rightarrow$ SPIN $1, 2, \ldots, 5$ in $D = 4$
- ALL THESE CASES SHARE ONE RELEVANT FEATURE:

$$\left.\begin{array}{ll} \text{. RADIATION BRANCH} & \sim r^{-\frac{D-2}{2}} \\ \text{. COULOMB BRANCH} & \sim r^{-(D-3)} \end{array}\right\} \text{DEGENERATE} \sim \frac{1}{r}$$

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- "TYPICALLY" $\rightarrow$ SPIN $1, 2, \ldots, 5$ in $D = 4$
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MAYBE, GAUGE DEPENDENT ASG HARDER TO IMPLEMENT IN D_{crit}

§ IV. Remarks & Outlook

R & O

* SPIN-1 MEMORY IN ANY D [Bieri-Garfinkle 2013] [Mao et al 2017, 2018] [Wald et al 2018, 2019]

* ASYMPTOTIC SYMMETRIES ON ASYMPTOTICALLY (A)dS SPACES [Mishra, Sundrum 2017, 2018] [Baumann 2017]

* HIGHER-SPIN ASYMPTOTIC ALGEBRA (FLAT & (A)dS) AT THE NON-ABELIAN LEVEL [Vasiliev 1993, Eastwood 2005, Bekaert 2008, Sleight-Taronna 2016]

* MIXED SYMMETRY FIELDS (BEYOND P-FORMS)

[Di Vecchia, Marotta, Mojaza 2017]
[Afshar, Esmaeili, Sheikh-Jabbari 2018]

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* IN THIS SPIRIT, IT WOULD BE INTERESTING TO SEE WHETHER STRING SCATTERING AMPLITUDES MAY BEAR SOME REMNANTS OF THIS HSP ASYMPTOTIC SYMMETRY