Renormalons and the Top quark mass measurement

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Outline

- Generality on the top mass measurement
- A reminder slide on renormalons
- What we computed
- How we computed it
- Some results
- Understanding results
- Prospects and conclusions

Disclaimer

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One of his famous quotes: "There are two kind of crazy people"

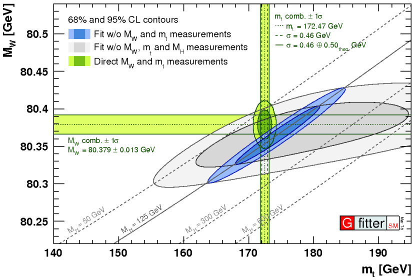
"Those that believe they are Napoleon"

"and those that want to make the trains run on time in Italy"

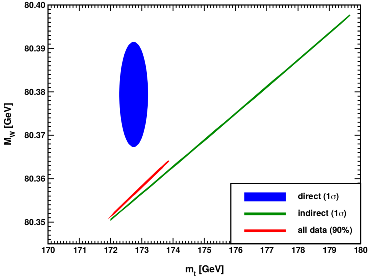
There is a third kind:

There is a third kind:

those that want to make sense out of the top mass measurements at hadron colliders ...



From PDG:



$$\Delta G_\mu / G_\mu = 5 \cdot 10^{-7}; \quad \Delta M_Z / M_Z = 2 \cdot 10^{-5};$$

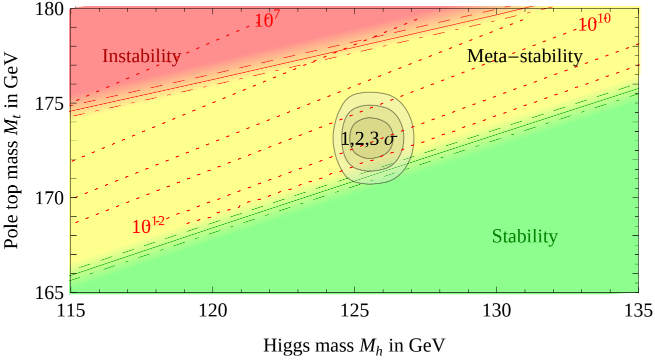
$$\Delta\alpha(M_Z)/\alpha(M_Z) = \begin{cases} 1 \cdot 10^{-4} (\text{Davier et al.; PDG}) \\ 3.3 \cdot 10^{-4} (\text{Burkhardt, Pietrzyk}) \end{cases}$$

Now that M_H is known, tight constraint on M_W-m_t,
(depending on how aggressive is the error on $\alpha(M_Z)$).

But: precision on M_W is more important now ...

Top and vacuum stability

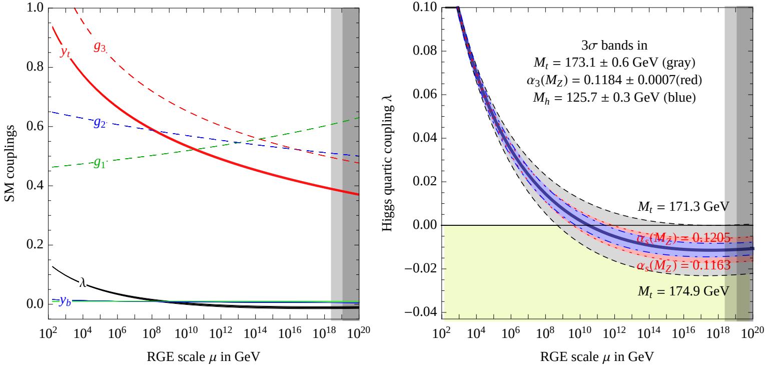
Degrassi et al. 2012



With current value of M_t and M_H the vacuum is metastable.
No indication of new physics up to the Plank scale from this.

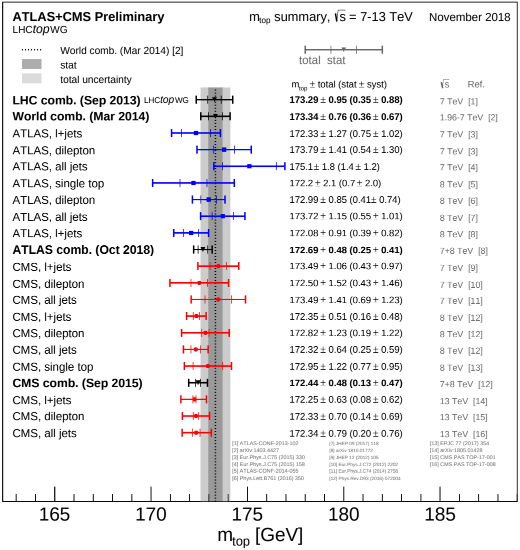
Top and vacuum stability

Degrassi et al. 2012



The quartic coupling λ_H becomes tiny at very high field values, and may turn negative, leading to vacuum instability.
M_t as low as 171 GeV leads to $\lambda_H \to 0$ at the Plank scale.

Top Mass Measurements



DIRECT MEASUREMENTS

(roughly, from the mass of the system of decay products). The most precise method as of now. But: what mass is it? No mass attribute in the plot ...

Theory issues

- The measurement is performed by reconstructing a top mass peak out of a reconstructed W and a b-jet.
- The reconstructed mass is only loosely related to the top mass (i.e. it cannot be identified with the top mass, for obvious reasons: mass of a colourless system).
- The extracted mass is the mass parameter in the Monte Carlo that yields the best fit to the reconstructed mass distribution.

So:

- ◇ in which renormalization scheme is the MC mass parameter? Pole mass? $\overline{\mathrm{MS}}$ mass?
- ◇ It has been argued that since MC are Leading-Order, they can't distinguish between Pole and $\overline{\mathrm{MS}}$ mass (the difference is around 10 GeV ...).

Selected Th. results relevant to top mass measurements

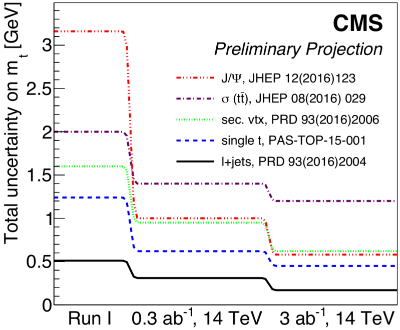
- Narrow width $t\bar{t}$ production and decay at NLO, Bernreuther,Brandenbourg,Si,Uwer 2004, Melnikov,Schulze 2009.
- $l\nu l\nu b\bar{b}$ final states with massive b, Frederix, 2013, Cascioli,Kallweit,Maierhöfer,Pozzorini, 2013.
- NNLO differential top decay, Brucherseifer,Caola,Melnikof 2013.
- NLO+PS in production and decay, Campbell,Ellis,Re,PN
- NNLO production, Czakon,Heymes,Mitov,2015.
- $l\nu l\nu b\bar{b} + \text{jet}$ Bevilacqua,Hartanto,Kraus,Worek 2016.
- Approx. NNLO in production and exact NNLO in decay for $t\bar{t}$. Gao,Papanastasiou 2017.
- Resonance aware formalism for NLO+PS: Ježo,PN 2015;
- Off shell + interference effects+PS, Single top, Frederix,Frixione,Papanastasiou,Prestel,Torielli, 2016
- Off shell + interference effects+PS, $l\nu l\nu b\bar{b}$, Ježo,Lindert,Oleari,Pozzorini,PN, 2016.

Alternative mass-sensitive observables

- Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart,2016 Use boosted top jet mass + SCET.
- Agashe,Franceschini,Kim,Schulze,2016: peak of b-jet energy insensitive to production dynamics.
- Kawabata,Shimizu,Sumino,Yokoya,2014: shape of lepton spectrum. Insensitive to production dynamics and claimed to have reduced sensitivity to strong interaction effects.
- Frixione, Mitov: Selected lepton observables.
- Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ,2013; Bayu etal: M_t from $t\bar{t}j$ kinematics.
- $t\bar{t}$ threshold in $\gamma\gamma$ spectrum (needs very high luminosity), Kawabata,Yokoya,2015

High Luminosity LHC: Expectations

- CMS Projection: 0.1% accuracy at $3ab^{-1}$, 14 TeV;
- Improvements expected from high statistics
- More effective constraints on models using differential distributions
- More possibilities from data driven constraints.

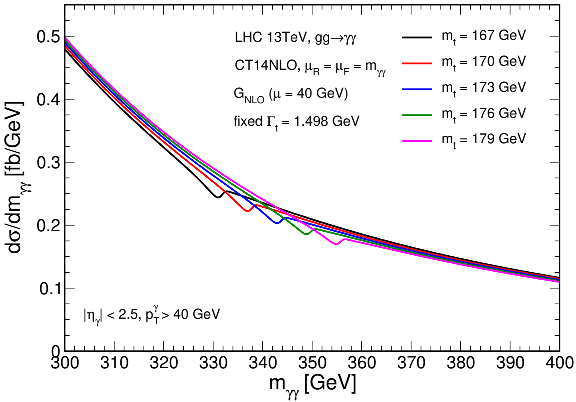


CMS FTR-16-006

Recent discussion of th. issues by Hoang, Corcella, Yokoya, P.N. in SM Physics at the HL-LHC and HE-LHC, arXiv:1902.04070.

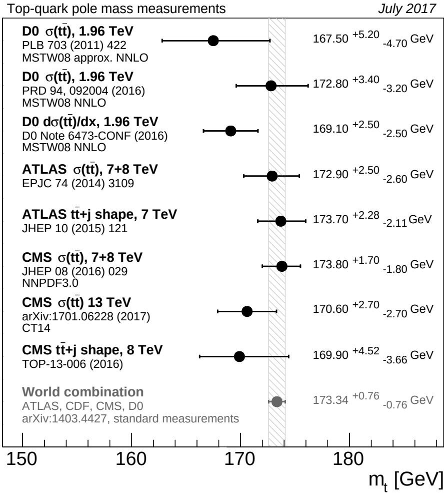
Alternative observables

Rather than $e^+e^- \to t\bar{t}$ at threshold, we may also look at the $\gamma\gamma$ spectrum at LHC Kawabata,Yokoya, 2016. It is unclear whether this can be done even at the High Luminosity LHC ...



It avoids theoretical problems present in direct measurements.

From the total cross section and $t\bar{t}j$ kinematics



It is claimed that since higher order calculations (NNLO for total cross section, NLO for $t\bar{t}j$ shape variables) are used in this determination, one is entitle to specify the scheme used for the mass.

In the figure they are quoted as "pole mass measurement".

- The "pole mass" attribute is not given to direct measurement.
- In some experimental papers and talks, direct measurements are reported as "Monte Carlo Mass" measurements, often stating that they need some theoretical interpretation.
- "Monte Carlo Mass" measurements are often interpreted as pole mass measurements by theorists. See for example
  - Degrassi etal, 2012 on the EW vacuum stability, adding a further 250 MeV error to direct measurements.
  - Ciuchini etal, 2017 in Global EW fits, adding a further 500 MeV error to direct measurements.
- Theorist have done work in proposing alternative methods to avoid the issues on direct measurements; however, the alternative methods are generally inferior in precision.

As a result, the most precise experimental results on m_t are left in a limbo, waiting for some illuminating theoretical interpretation that is not in sight.

High-school quiz on top mass measurement

Tick the correct statements:

- ☐ Direct top mass measurements measure the Pole Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass. but you can pretend that it is the pole mass, just inflate the error a bit.
- ☐ The top is the only SM particle with more than one mass.
- ☐ You should use only leptons to avoid hadronization uncertainty.
- ☐ You should use at least NLO calculations to measure the pole mass.
- ☐ The top pole mass has renormalons, you should stay away from it.
- The MC mass differs from the pole mass by
  ☐ terms of order $m\alpha_S$; ☐ terms of order $\Lambda_{\rm QCD}$; ☐ terms of order $\alpha_S\Gamma_t$.
- The Pole Mass renormalon ambiguity is
  ☐ $\approx$ 1GeV; ☐ $\approx$ 250 MeV; ☐ $\approx$ 200 MeV; ☐ $\approx$ 110 MeV.

What is being done

- Explore MC model sensitivity, for example in colour reconnection (Argyropoulos,Sjöstrand,2014; Christiansen,Skands 2015).
- Model non-perturbative effects using parton showers, and assess their uncertainties by varying shower parameters, hadronization parameters, and MC implementations (Ferrario Ravasio,Ježo,Oleari,P.N.2018).
- Seek observables with reduced MC parameter sensitivity (Corcella,Franceschini,Kim,2017; Andreassen,Schwartz 2017; Hoang,Mantry,Pathak,Stewart 2017).
- Directly address the theoretical problem in field theory, also in simplified contests. This can lead to insights such that at least some questions may be answered.

Directly addressing the theoretical problem ...

- In a very influential paper, Hoang and Stewart (2008) state "It is not the pole mass that is measured at the Tevatron".
- This point of view has been further explored in a sequel of papers; in the last one (Hoang,Plätzer,Samiz 2018) it is claimed that the Monte Carlo mass parameter has a well-defined relation to some short-distance mass.
- All this, and previous publications refer to the mass of an ultra-relativistic top jet, (NOT to the mass of the top decay products; shower MC's are correct only for radiation from ultrarelativistic particles). The mass of an ultrarelativistic top jet can be computed analytically in terms of a well defined mass. This mass can be compared to the MC mass parameter used to describe the same quantity.
- The 2008 paper is often quoted as the origin of the "Monte Carlo Mass" concept.

Directly addressing the theoretical problem ...

- The differences between the MC and Pole mass found by Hoang are parametrically of the order of the Shower cutoff times α_s.
- Although this does not apply directly to the mass measured in direct measurements, it is not unlikely that differences of this order are present also in that case.
- Whether differences of this order justify the original statement, (i.e. direct mass $\neq$ pole mass) is questionable.
- Hoang's 2008 paper is often quoted as the origin of the "Monte Carlo Mass" concept.

For criticism to the "Monte Carlo Mass" concept, see P.N.2018. For a view agreed upon by more authors, see the HE-HL-LHC arXiv:1902.04070.

Our task

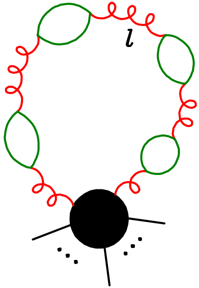
- We desperately need frameworks where the answer to questions are not just *opinions*.
- We want to explore theoretically the non-perturbative corrections in a framework that covers direct measurements.
- Some non-perturbative effects are connected to the factorial growth of the perturbative expansion coefficients. We want to start from these.
- We begin by simplifying the theoretical context as much as we can, but without abandoning the need to study the mass determination from the decay products of an unstable coloured particles at modest velocity.

ABC of I.R. Renormalons

All-orders contributions to QCD amplitude of the form

$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p \frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2)\log\frac{k^2}{m^2}}$$

$$= \alpha_s(m^2)\sum_{n=0}^{\infty}(2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!}.$$



Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p_{\mathrm{QCD}}$.
- Typical scale dominating at order α_s^{n+1}: $m\exp(-np)$.
- OPE connection; for a short distance process:

$$\int \mathrm{d}^4 l(\mathrm{ph.sp})\, l^2(\mathrm{gauge\ inv.})l^{-2}(\mathrm{gluon\ prop.}) \propto \Lambda^4(\mathrm{i.e.\ a\ } G^2 \mathrm{\ VeV}).$$

Motivation

- Linear (i.e. $p = 1$) renormalons may affect top mass measurements at order Λ (near the present experimental accuracy).
- Until now, only the top pole mass renormalon has received some attention.
- Several other sources of linear renormalons come into play in top mass measurements (for example, from jet requirements). What is their structure, and what is their interplay with the pole mass renormalon?
- There is a temptation to use resummation to parametrize linear non perturbative effects in top mass measurement. Is this sound?

Abstract:

The resummed Drell-Yan cross section in the double-logarithmic approximation suffers from infrared renormalons. Their presence was interpreted as an indication for non-perturbative corrections of order $\Lambda_{\text{QCD}}/(Q(1-z))$. We find that, once soft gluon emission is accurately taken into account, the leading renormalon divergence is cancelled by higher-order perturbative contributions in the exponent of the resummed cross section.

Their calculation: leading N_f one gluon correction:

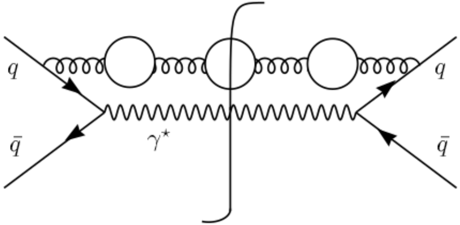
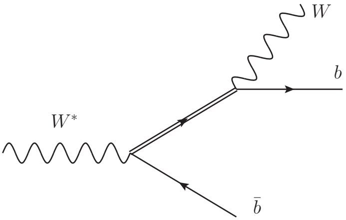


Figure 1: α_s^4-contribution to the partonic Drell-Yan cross section. $\gamma^\star$ represents a photon with invariant mass Q^2 that splits into a lepton pair.

Our work: compute top mass sensitive observables in leading N_f one gluon correction.

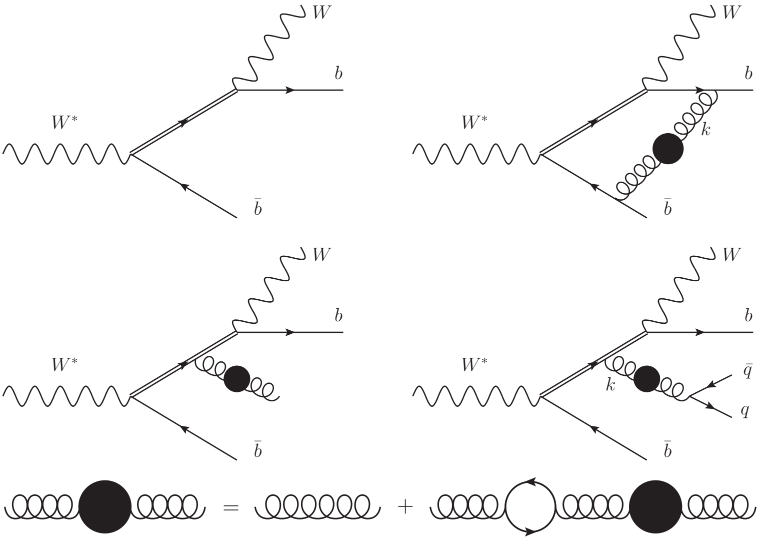
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can examine any infrared safe observable, no matter how complex.

Diagrams up to leading N_f one gluon correction



All-order result

Introducing the notation

- Φ_b, phase space for $Wb\bar{b}$;
- Φ_g, phase space for $Wb\bar{b}g^*$, where g^* is a massive gluon with mass λ,
- $\Phi_{q\bar{q}}$, phase space for $Wb\bar{b}q\bar{q}$

the all-order result can be expressed in terms of

- $\sigma_b(\Phi_b)$, the differential cross section for the Born process;
- $\sigma_v(\lambda, \Phi_b)$, the virtual correction to the Born process due to the exchange of a gluon of mass λ;
- The real cross section $\sigma_{g^*}(\lambda, \Phi_{g^*})$, obtained by adding one massive gluon to the Born final state;
- The real cross section $\sigma_{q\bar{q}}(\Phi_{q\bar{q}})$, obtained by adding a $q\bar{q}$ pair, produced by a massless gluon, to the Born final state;

Consider a (IR safe) final state observable O. Define:

$$N^{(0)} = \left[\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}\right]^{-1}, \quad \langle O\rangle_{\mathrm{b}} = N^{(0)}\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}(\Phi_{\mathsf{b}})\,O(\Phi_{\mathsf{b}}),$$

$$\widetilde{V}(\lambda) = N^{(0)}\int \mathrm{d}\Phi_{\mathrm{b}}\,\sigma_{\mathrm{v}}^{(1)}(\lambda^2,\Phi_{\mathrm{b}})\left[O(\Phi_{\mathsf{b}}) - \langle O\rangle_{\mathrm{b}}\right],$$

$$\widetilde{R}(\lambda) = N^{(0)}\int \mathrm{d}\Phi_{g^*}\,\sigma_{g^*}^{(1)}(\lambda^2,\Phi_{g^*})\left[O(\Phi_{g^*}) - \langle O\rangle_{\mathrm{b}}\right],$$

$$\widetilde{\Delta}(\lambda) = \frac{3\pi}{\alpha_s T_{\mathrm{F}}}\,\lambda^2 N^{(0)}\int \mathrm{d}\Phi_{q\bar{q}}\,\delta(k_{q\bar{q}}^2 - \lambda^2)\,\sigma_{q\bar{q}}^{(2)}(\Phi_{q\bar{q}}) \times [O(\Phi_{q\bar{q}}) - O(\Phi_{g^*})]$$

$\langle O\rangle_{\mathrm{b}} + \widetilde{V}(\lambda) + \widetilde{R}(\lambda)$ is the average value of O in a theory with a massive gluon with mass λ, accurate to order α_s.

Notice: $\widetilde{V}(\lambda) + \widetilde{R}(\lambda)$ has a finite limit for $\lambda \to 0$, while each contribution is log divergent.

defining $\widetilde{T}(\lambda) \equiv \widetilde{V}(\lambda) + \widetilde{R}(\lambda) + \widetilde{\Delta}(\lambda)$, our final result is

$$\langle O \rangle = \langle O \rangle_{\rm b} + \frac{3\pi}{\alpha_S T_{\rm F}} \int_0^\infty \frac{{\rm d}\lambda}{\pi} \frac{\rm d}{{\rm d}\lambda} \left[\widetilde{T}(\lambda) \right] {\rm atan} \left(\frac{\alpha_S \pi b_0}{1 + \alpha_S b_0 log \frac{\lambda^2}{\tilde{\mu}^2}} \right) \tag{1}$$

where $\tilde{\mu} \equiv \mu \exp(5/6)$.

This has the same renormalon structure of the example we considered at the beginning. Now the ${\rm atan}$ function has an unphysical discontinuity near the Landau pole

$$\lambda^2 = \tilde{\mu}^2 \exp \left[\frac{1}{\alpha_S(\mu) b_0} \right] = \Lambda^2_{\rm QCD} \exp(5/3). \tag{2}$$

If we thus have:

$$\widetilde{T}(\lambda) = a + b\,\lambda + \mathcal{O}\left(\lambda^2\right) \tag{3}$$

the integration has an ambiguity of order $b\Lambda_{\rm QCD}$, i.e. a Linear Renormalon.

Comments

- In order to get our results, we need $\lim_{\lambda^2 \to \infty} \widetilde{T}(\lambda^2) = 0\,.$
  This happens if we use the Pole Mass Scheme for m_t.
- The need to include the Δ term has a long story:
  - Seymour,P.N. 1995, I.R. renormalons in e^+e^- event shapes.
  - Dokshitzer,Lucenti,Marchesini,Salam, 1997-1998 Milan factor
- We compute $T(\lambda)$ numerically. The $\lambda \to 0$ limit implies the cancellation of two large logs in V and R. However, the precise value at $\lambda = 0$ can also be computed directly by standard means (which we do).

Changing the mass scheme

The relation of the pole mass as a function of the $\overline{\mathrm{MS}}$ mass in the large N_{F} approximation is well known (Beneke, 1999)

$$\begin{aligned} m &= \bar{m}[1 + R_f(\alpha_s, \mu, \bar{m}) + R_d(\alpha_s, \mu, \bar{m})], \\ R_f &= -\frac{3\pi}{\alpha_s T_F} \int_0^\infty \frac{\mathrm{d}k}{\pi} \frac{\mathrm{d}r_f(\lambda)}{\mathrm{d}\lambda} \mathrm{atan} \frac{-\alpha_s \pi b_0}{1 + \alpha_s b_0 \log \frac{\lambda^2}{\tilde{\mu}^2}} \\ r_f(\lambda) &= -\alpha_s \frac{C_F}{2} \frac{\lambda}{m} + \mathcal{O}\left(\frac{\lambda^2}{m^2}\right) . \end{aligned} \quad (4)$$

We can easily convert our results to the $\overline{\mathrm{MS}}$ scheme:

$$\langle O \rangle_{\mathrm{b}}(m, m^*) = \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*) + \left\{ \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \overline{m}} (m - \overline{m}) + \mathrm{cc} \right\}$$

For the leading renormalon this amounts to

$$\widetilde{T}(\lambda) \to \widetilde{T}(\lambda) - \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \mathrm{Re}(\overline{m})} \frac{C_{\mathrm{F}} \alpha_s}{2} \lambda + \mathcal{O}(\lambda^2) .$$

Changing the mass scheme

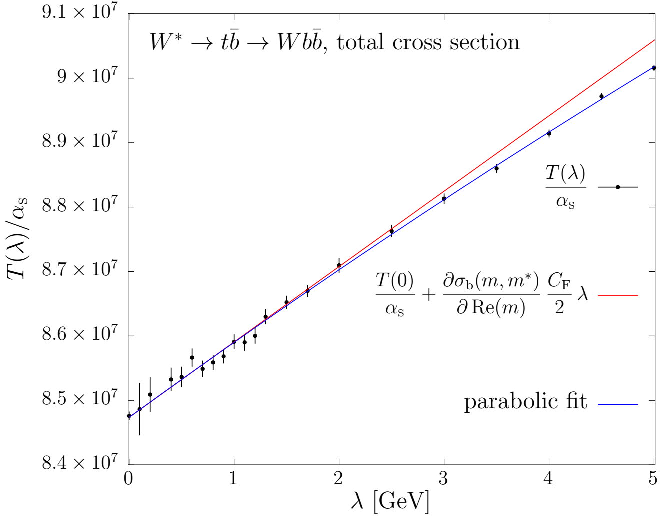
The pole mass, when expressed in terms of the $\overline{\mathrm{MS}}$ mass, has a renormalon, i.e. an uncertainty of order Λ. If the $\overline{\mathrm{MS}}$ mass is to be considered a fundamental parameter of the theory, this means that the pole mass has a *physical* uncertainty.
Can we argue that if we take the pole mass as a fundamental parameter it is the $\overline{\mathrm{MS}}$ mass that has a physical uncertainty?

The answer is NO!

QCD is defined by its short distance Lagrangian, and its defining fundamental parameters are the short distance ones!

Selected Results

Total cross section



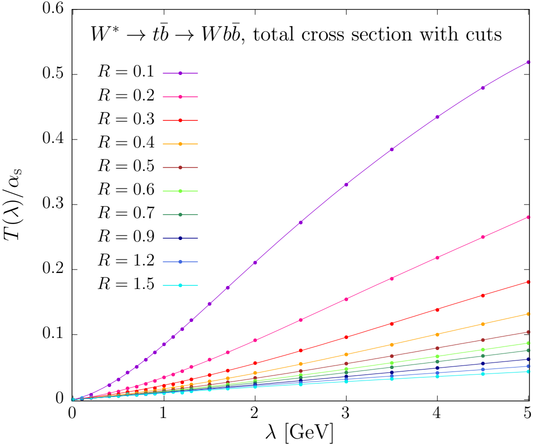
No linear renormalon in $\overline{\mathrm{MS}}$ scheme!

Total cross section

- For $k < \Gamma$: no renormalon in the physics! The top finite width screens the soft sensitivity of the cross section.
  The renormalon is there only if it is present in the mass counterterm; thus, it is not there in the $\overline{\mathrm{MS}}$ scheme.
- What about $k \gg \Gamma$?
  This is the narrow width limit: the cross section factorizes into a production cross section and a partial width.
  The former has no physical renormalons for obviour reasons.
  The latter does not have them for less obvious reasons.

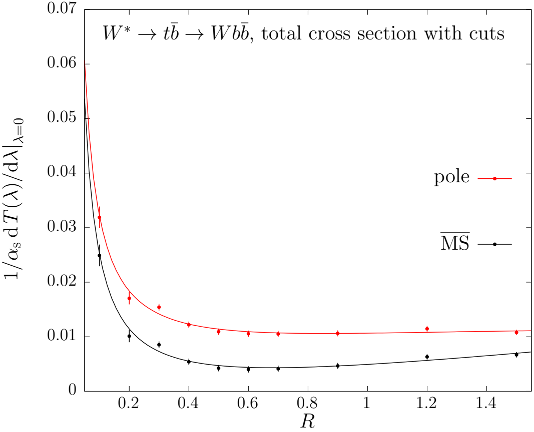
So, the mass from the total σ is free of linear power corrections.

Total cross section with cuts



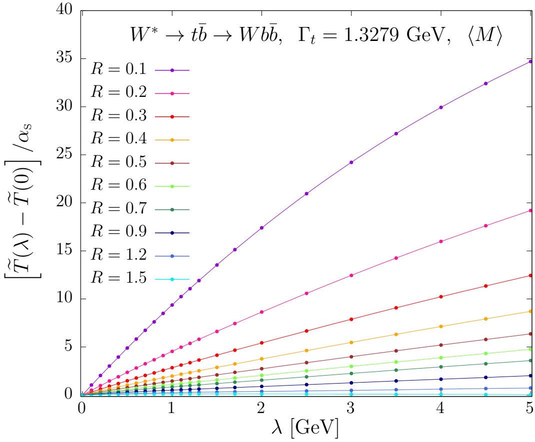
Require a b and a (separated) $\bar{b}$ anti-k_t jet with energy $> 30\,$GeV.

Total cross section with cuts



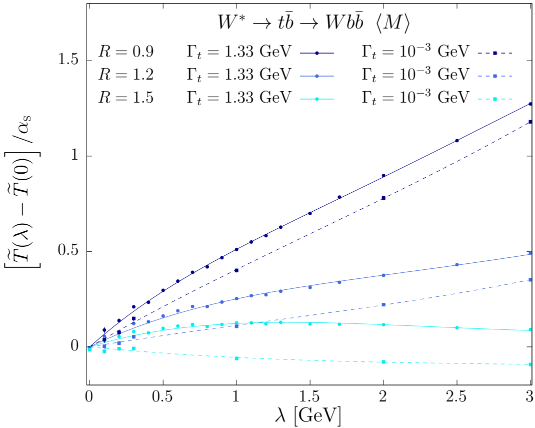
The renormalon is there also in $\overline{\mathrm{MS}}$ scheme!

Reconstructed top mass



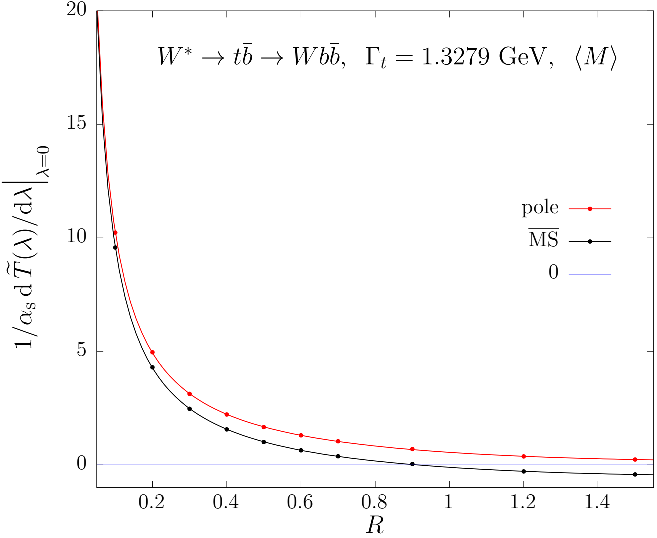
Strong renormalon effects due to jets (coefficient $\propto 1/R$)

Reconstructed top mass



Blowup for large R's; width effects become visible.

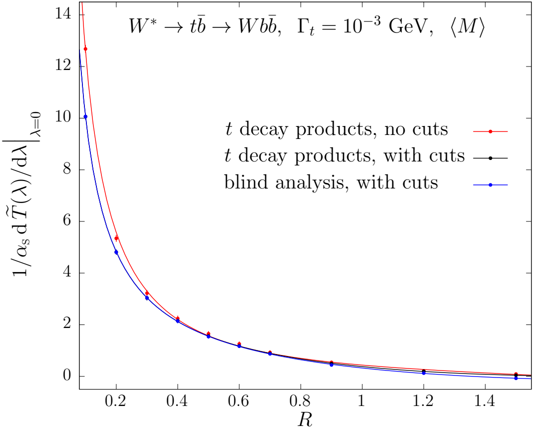
Reconstructed top mass



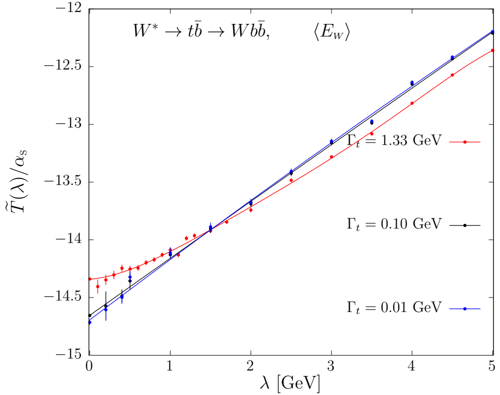
For large radii, m_{pole} seems better!

- In the narrow width limit, we can separate production and decay (they don't interfere). The cross section factorizes, and there exists a "Monte Carlo Truth" for the top decay products.
- We can verify that for large radii, the mass of the reconstructed top using only MC-truth top decay products is very close to the "blind" reconstructed mass.

Reconstructed top mass



Thus a measurement using the reconstructed top mass is closer to the pole mass whatever that means.



Consider $\langle E_W \rangle$.

For $k \gg \Gamma$, the slope is roughly 0.45.

The $\overline{\mathrm{MS}}$ conversion would add -0.067.

It seems that physical linear renormalons are present also in leptonic observables.

But, for $\lambda \ll \Gamma$, the slope of $T(\lambda)$ decreases, approaching 0.067!
The renormalon seems to cancel in the $\overline{\mathrm{MS}}$ scheme!

Two questions:

- Our narrow width result seems to be in contrast with what found in heavy flavour inclusive decays, where no renormalons are present for leptonic observables, if the heavy flavour mass is expressed in the $\overline{\mathrm{MS}}$ scheme (Beneke,Braun,Zakharov,1994; Bigi etal,1994).
  We have verified, however, that if $\langle E_W \rangle$ is computed in the top rest frame, no renormalons are present. So: no contradiction there.
- About the renormalon cancellation for finite width: we have also verified it for larger width values.

Is this an exact statement?
The answer is Yes!!

Back to the basics

Back to the theory of soft cancellation.

- Normally one worries about cancellation of divergences, i.e. terms that go like $\log \lambda$ (where now the gluon mass λ serves as an infrared cutoff.
- We want to check if also terms that are linear in λ do cancel.
- The strategy: if our amplitudes can be analytically continued to complex energies, one can apply the euclidean power counting to examine its infrared sensitivity.
- A very transparent way to carry out such analysis is by going to Old Fashion Perturbation Theory.

Old Fashion Perturbation Theory

The propagator denominators in a Feynmann diagram can be split into an advanced and a retarded part:

$$\frac{i}{\lambda^2 - m^2 + i\epsilon} = \frac{i}{2E_{k,m}}\left[\frac{1}{k^0 - E_{k,m} + i\epsilon} + \frac{1}{-k^0 - E_{k,m} + i\epsilon}\right].$$

The time Fourier transform of the first term vanishes for negative time, while for the second term it vanishes for positive time

$$\begin{aligned}\int \frac{\mathrm{d}k^0}{2\pi}\frac{i\exp(-ik^0 t)}{k^0 - E_{k,m} + i\epsilon} &= \theta(t)\exp(-iE^0_{k,m}t)\\ \int \frac{\mathrm{d}k^0}{2\pi}\frac{i\exp(-ik^0 t)}{-k^0 - E_{k,m} + i\epsilon} &= \theta(-t)\exp(iE^0_{k,m}t)\end{aligned}$$

Old Fashion Perturbation Theory

also true for unstable particles:

$$\frac{i}{\lambda^2 - m^2 + im\Gamma} = \frac{i}{2E_{k,m,\Gamma}} \left[\frac{1}{k^0 - E_{k,m,\Gamma}} + \frac{1}{-k^0 - E_{k,m,\Gamma}} \right],$$

where

$$E_{k,m,\Gamma} = \sqrt{\underline{k}^2 + m^2 - im\Gamma},$$

so that $E_{k,m,\Gamma}$ has a negative imaginary part. As a consequence, we will also have

$$\begin{aligned} \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{k^0 - E_{k,m,\Gamma} + i\epsilon} &= \theta(t) \exp(-iE^0_{k,m,\Gamma} t) \\ \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{-k^0 - E_{k,m,\Gamma} + i\epsilon} &= \theta(-t) \exp(iE^0_{k,m,\Gamma} t) \end{aligned}$$

and both functions will have exponential damping for large positive (negative) time. But the θ functions are there as before.

Old Fashion Perturbation Theory

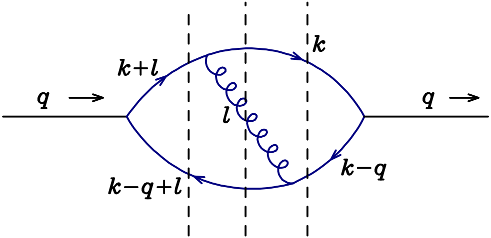
A straightforard manipulation leads to the old fashion perturbation theory rules. In time ordered graphs:

- Split propagators into an advanced and retarded part, and split each Feynmann graph into a sum of time ordered graphs.
- Replace propagators with $1/(2E_{k,m})$
- Put all propagator energies in numerators equal to their on-shell values.
- Include all 3-momentum integrals.
- For each external incoming momentum, include a line coming from $-\infty$ or going to $+\infty$, carrying momentum and energy with corresponding sign.
- For each intermediate state, include an energy denominator

$$\frac{i}{e - e_i + i\epsilon}$$

  where e is the sum of the incoming energy (from the lines to $-\infty$) and e_i is the sum of the energies of the lines in the intermediate state.

Old Fashion Perturbation Theory



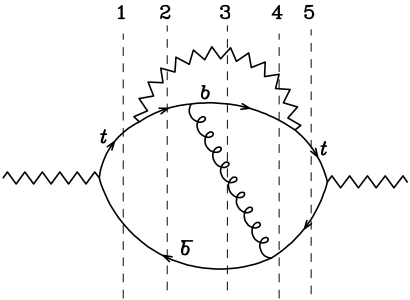
$$\frac{1}{q^0 - E_{k+l} - E_{k-q+l} + i\epsilon} \frac{1}{q^0 - E_k - E_l - E_{k-q+l} + i\epsilon} \frac{1}{q^0 - E_k - E_{k-q} + i\epsilon}$$

Singularities are present only if the momentum integration cannot be displaced in the complex plane away from the poles.
This simply leads to the threshold singularities, and to the Landau conditions for anomalous thresholds.
Away from those, the graph is an analytic function of q^0.

Old Fashion Perturbation Theory

- This leads to KLN cancellation of soft singularities. But it leads to more: the soft sensitivity is the same when q^0 picks up a finite imaginary part!
- The energy denominators do not count any more for the soft sensitivity. Only the $\mathrm{d}^3 l/E_l$ counts (same sensitivity as in Euclidean power counting: $\mathrm{d}^4 l/l^2$. Two more powers of l come from gauge invariance, leading to 4^{th} order power corrections).

back to E_W



Only 2,3,4 cuts should be considered. But:
1 and 5 denominators have opposite imaginary part of order Γ.

$$\mathrm{Im}\left[\frac{1}{E-E_W-E_{b,2}-E_{\bar{b},1}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},1}-E_{g,3}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},4}+i\epsilon}\right]$$

Still analytic, but the imaginary part of q^0 cannot exceed Γ.
So: soft sensitivity higher than linear for scales below Γ.

Conclusions

- This work is addressing theoretical questions having to do with the high-order perturbative structure, in its relation to power corrections.
- Many simplifying assumptions were made; some of them mey be removed in the future.
- In spite of the simplifying assumption, several results have implications even for current measurements:
    - Although there are good reasons to believe that the total cross section is not affected by linear renormalons, as soon as we introduce acceptance cuts, linear corrections, especially due to jets, do appear.
    - For observables that do not depend upon jets, the finite width of the top seems to screen linear renormalon effects. Leptonic observables could benefit from this. In practice, however, this feature does not help at the moment, since it requires very high order calculations.
    - Leptonic observables are also affected by linear renormalons, unless one goes at very high order in their perturbative calculation.

Prospects

There are several directions in which this work can be extended.

- Study observables where jets are calibrated. See if linear renormalons are reduced, and to what extent.
- Does jet trimming reduce linear renormalon effects?
- In general, are there "better" observables from this point of view?