TOP-MASS MEASUREMENTS: NLO+PS EFFECTS & RENORMALONS

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Seminario teorico, 19$^{\text{th}}$ March 2019

Based on

- S.F.R., T. Ježo, P. Nason and C. Oleari [arxiv:1801.03944]
- S.F.R., P. Nason and C. Oleari [arxiv:1810.10931]

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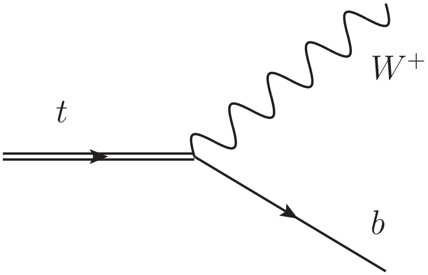

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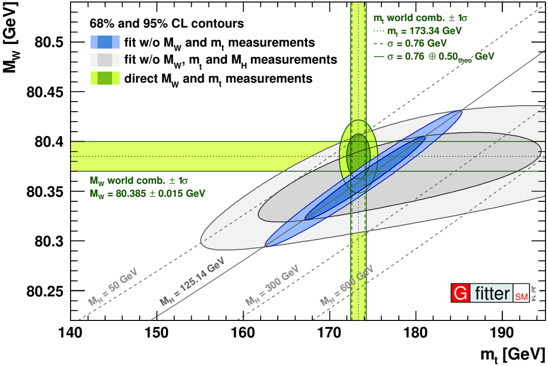

Figure: Global fit to electroweak precision observables [arXiv:1407.3792]

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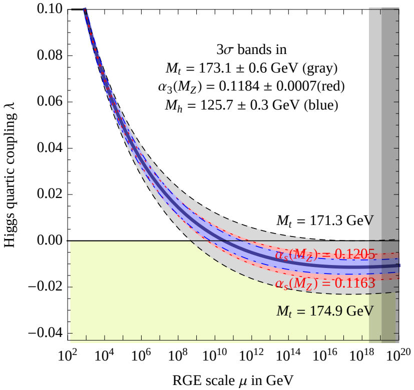

Figure: RG flow of the Higgs quartic couplig for $m_H = 125.7$ GeV [arXiv:1512.01222]

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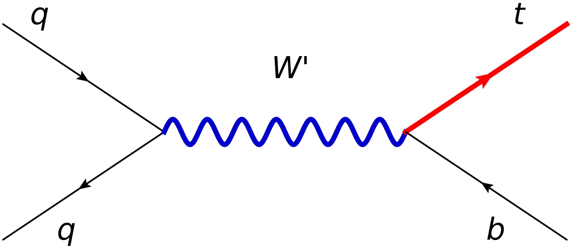

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We want a precise determination of $\mathbf{m_t}$ in a given **renormalization scheme**



Top-quark mass

- Direct measurements give us the most precise determination, provided that the theoretical errors are small and under control.
  - CMS: $m_t = 172.44 \pm 0.13\ \text{(stat)} \pm 0.47\ \text{(syst)}$ GeV
  - ATLAS: $m_t = 172.51 \pm 0.27\ \text{(stat)} \pm 0.42\ \text{(syst)}$ GeV

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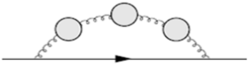
✓ Renormalon ambiguity:



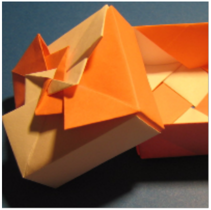
$$\sim m \sum^{\infty} c_n\, \alpha_s^n, \qquad c_n \to \Gamma(n)$$

Resummed series ambiguity $\propto \Lambda_{QCD}$.

- 110 MeV [Beneke, Marquad, Nason, Steinhauser 1605.03609].
- 250 MeV [Hoang, Lepenik, Preisser, 1706.08526].

Although not dramatic now, it is interesting to study the impact of the renormalons on top-mass related observables

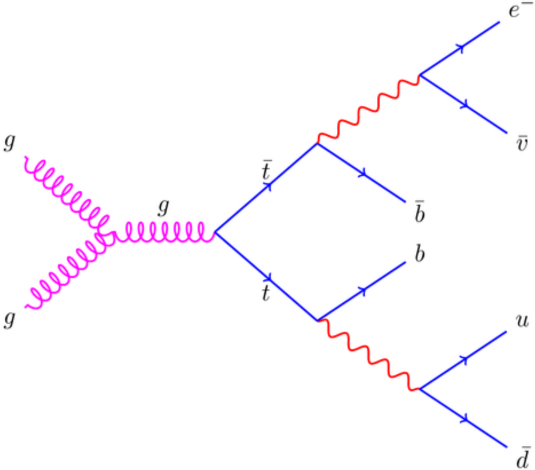
Part I:
Accurate NLO+PS predictions for top-pair production



Based on:
"A Theoretical Study of Top-Mass Measurements at the LHC Using NLO+PS Generators of Increasing Accuracy," with T. Ježo, P. Nason and C. Oleari, **Eur.Phys.J. C78 (2018) no.6, 458**

Direct measurements

- LHC: t mostly produced in pairs



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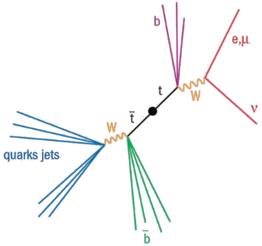

$\Rightarrow$ B-jet;
$\Rightarrow$ W decay products:
$\rightarrow$ charged lepton + neutrino
$\rightarrow$ two light jets

Direct measurements

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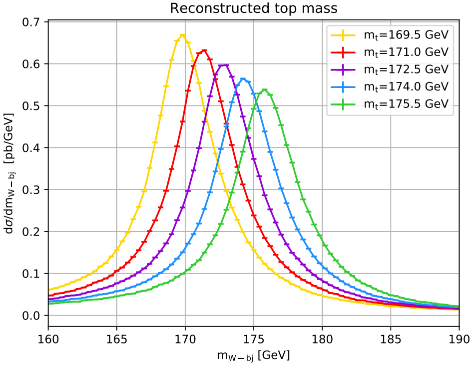
- many ways to infer m_t, the most precise is the template method
  1. Top momentum reconstruction from its decay products.
  2. Given a MC event generator, produce several templates varying the input mass m_t.



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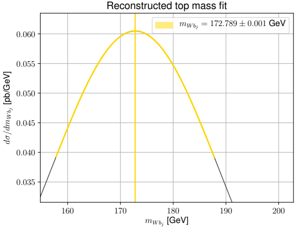
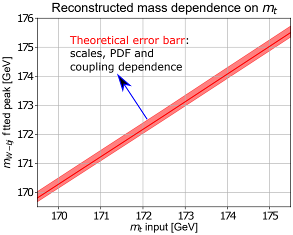
  3. Extract the parametric dependence on the input mass m_t.





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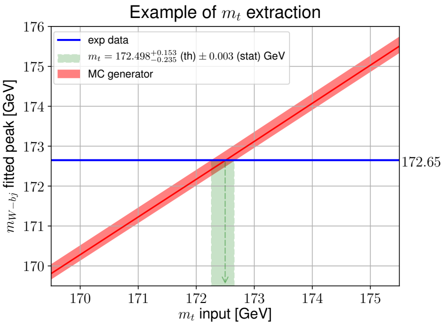
  4. The m_t value that fits the data the best is the extracted mass.



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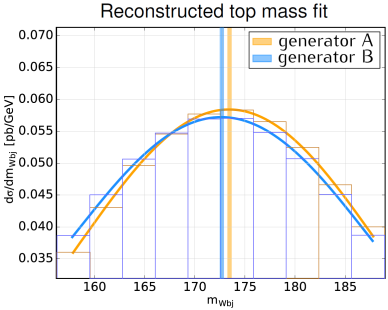
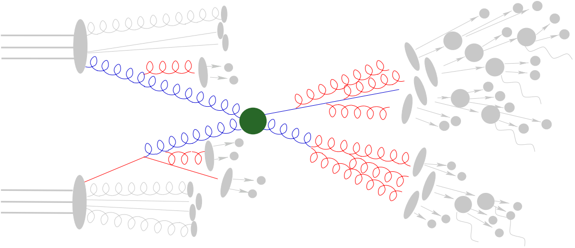
  2. Given a MC event generator, produce several templates varying the input mass m_t.
  3. Extract the parametric dependence on the input mass m_t.
  4. The m_t value that fits the data the best is the extracted mass.
  5. m_t can depend on the MC used



$\Rightarrow$ if A is more accurate than B, use A
$\Rightarrow$ otherwise $|m_t^A - m_t^B|$ contributes to the systematic uncertanty;

Monte Carlo Event generators

- Current standard NLO+PS: hard process described with NLO accuracy, further emissions handled by the PS in the soft and collinear limit.



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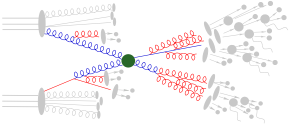

- POWHEG BOX is an NLO event generator, based on the POWHEG method. It generates the hardest emission. The event is then completed by standard SMC that implements the PS.
  [arXiv: hep-ph/0409146]

Sudakov form factor: $$\Delta(\mathbf{k}_\perp) = \exp\left\{-\frac{\int d\phi^{\rm rad}\,\theta(k_\perp^{\rm rad} - \mathbf{k}_\perp)\,R}{B}\right\}$$

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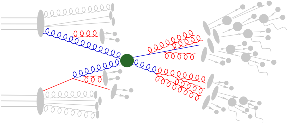

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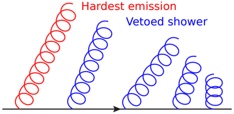
- Pythia8 and Herwig7 include radiation with a $\mathbf{k}_\perp$ smaller than the POWHEG emission one.

- Pythia8 [Sjöstrand et al., arXiv:1410.3012] is a $k_\perp$-ordered shower.



$\Rightarrow$ Natural matching with POWHEG radiation.

Interface between POWHEG BOX and Shower MC

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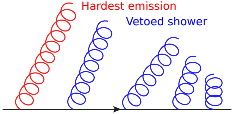
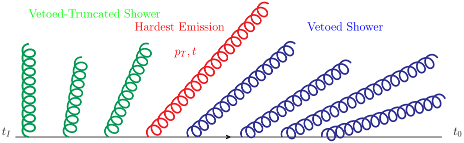


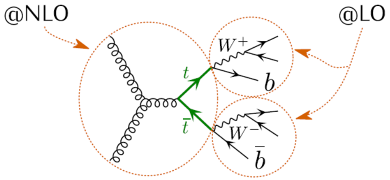
⇒ Natural matching with POWHEG radiation.

- Herwig7 [Bahr et al., arXiv:0803.0883], [Bellm et. al, arXiv:1512.01178] is an angular-ordered shower.



⇒ Truncated-vetoed showers are known to give a contribution; so only a vetoed shower is implemented.

- *hvq* is the first $t\bar{t}$-production generator implemented in `POWHEG BOX`. [arXiv:0707.3088, Frixione, Nason, Ridolfi]

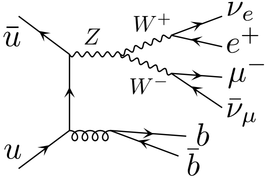


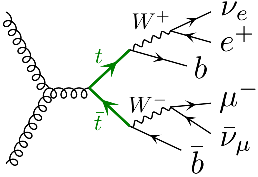
⇒ NLO corrections in production;
⇒ decay performed at LO using reweighting;
⇒ approximate spin correlation and offshell effects.

- Heavily used by the experimental community:
⇒ arXiv:1803.10178, ATLAS
⇒ arXiv:1803.09678, ATLAS
⇒ arXiv:1803.06292, CMS
⇒ arXiv:1803.03991, CMS

$b\bar{b}4\ell$

- $b\bar{b}4\ell$ is the latest $t\bar{t}$-production generator implemented in `POWHEG BOX`.
  [arXiv:1607.04538, Ježo, Lindert, Nason, Oleari, Pozzorini].





$\Rightarrow$ $pp \to b\bar{b}\ell\bar{\nu}_\ell\bar{l}\nu_l$ at NLO;
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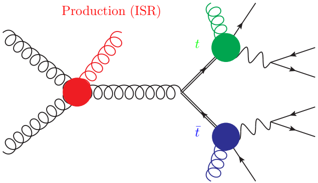
- New resonance-aware formalism that generates emissions preserving the virtuality of the intermediate resonances. This new formalism also offers the opportunity to generate multiple emissions [Ježo, Nason, arXiv:1509.09071].



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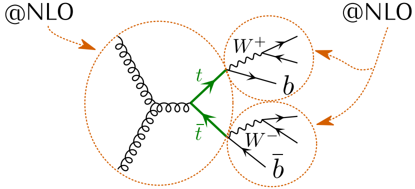
  $\Rightarrow$ interference of radiation in **production and decay**.
- New resonance-aware formalism that generates emissions preserving the virtuality of the intermediate resonances. This new formalism also offers the opportunity to generate multiple emissions [Ježo, Nason, arXiv:1509.09071].
- `Pythia8` and `Herwig7` veto radiation in production harder than the `POWHEG` one. Radiation from resonances is left, by default, unrestricted.
- The user can implement the same veto algorithms acting on radiation off resonances.

ttb_NLO_dec

In this slides I will compare only $b\bar{b}4\ell$ and hvq, but there is also

- ttb_NLO_dec is the precursor of $b\bar{b}4\ell$, [arXiv:1412.1828], Compbell, Ellis, Nason and Re]



⇒ NLO corrections in production and decay using NWA.
⇒ Spin correlation and offshell effects exact at LO.
⇒ Interference with process sharing the same final state at LO.

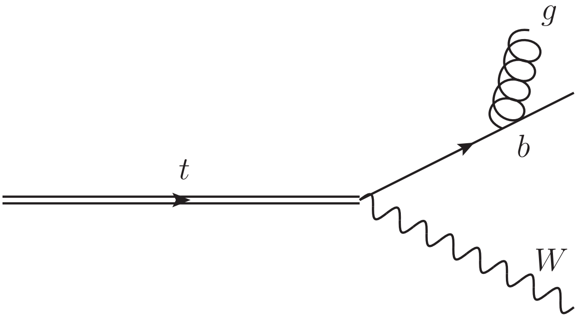
- **Most accurate generator for semi leptonic and hadronic top decay.**

Soon semileptonic decay with full off-shell effects and $b\bar{b}4\ell$-like non-resonant contributions (by Ježo, Pozzorini)

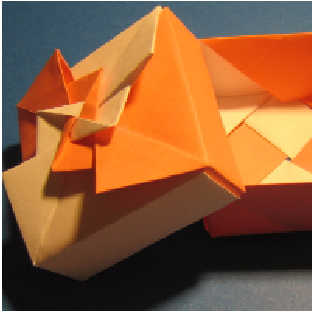
- NLO+PS interface analogous to $b\bar{b}4\ell$

Matrix Element Corrections

- If the t decay is generated at LO, `Pythia8.2` and `Herwig7.1` can modify the shower algorithm in order to generate the hardest emission using the exact Matrix Element for one additional real emission: MEC.
- In this way, also when using hvq, the t decay with an extra emission is described with exact LO matrix elements.



Part I A: comparison among POWHEG generators showered with Pythia8.2



Reconstructed-top mass

- We take m_{Wb_j} as a proxy for all top-mass sensitive observables that rely upon the mass of the decay products.
- Experimental resolution effects are simply represented as a Gaussian smearing ($\sigma =$15 GeV):

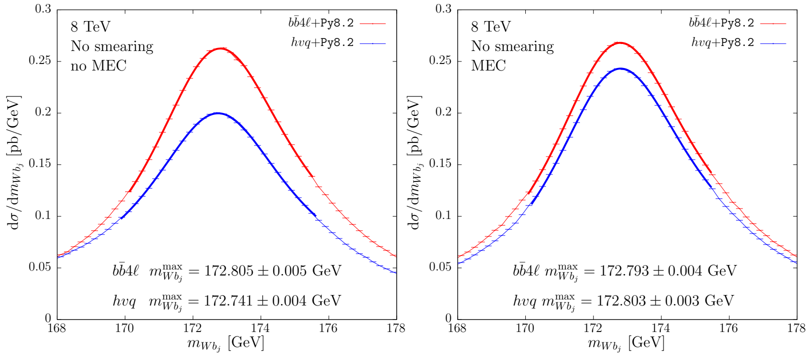
$$\tilde{f}(x) = \mathcal{N} \int \mathrm{d}y \, f(y) \exp\left(\frac{-(x-y)^2}{2\sigma^2}\right) .$$

- We fit the peak position $m_{Wb_j}^{\max}$ using a Skewed Lorentian.
- $\boxed{\Delta m_t \simeq -\Delta m_{Wb_j}^{\max}}$.

reconstructed-top mass: which NLO generator?

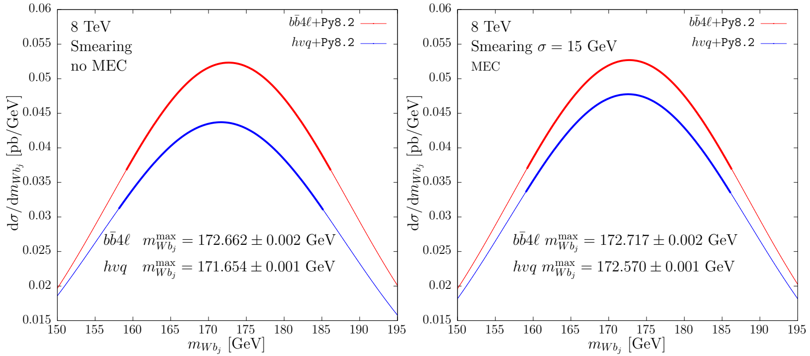
Brief look without smearing:

- Large shape differences with *hvq* if matrix elements corrections (MEC) are off.
- With MEC, differences among the generators of the order of 10-20 MeV.



reconstructed-top mass: which NLO generator?

1 GeV difference reduced to 150 MeV when MEC are turned on.



B-jet energy peaks

- Based on arxiv:1603.03445 (Agashe, Kim, Franceschini, Schulze).
- Investigated by CMS in [CMS-PAS-TOP-15-002], that finds

$$m_t = 172.29 \pm 1.17\,(\text{stat}) \pm 2.66\,(\text{syst})\ \text{GeV}\,.$$

- Purely hadronic observable, independent from the top production dynamics.
- At LO, neglecting off-shell effects, in the top frame we have:

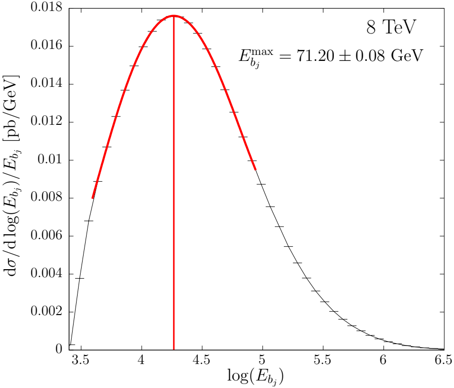
$$\boxed{E_{b_j} = \frac{m_t^2 - m_W^2}{2m_t}}\,.$$

- In the lab frame the distribution is squeezed, but the peak position does not vary.
- After the inclusion of perturbative and non-perturbative effects, for $m_t \approx m_{t,\,c}$, we have:

$$\boxed{E_{b_j}^{\max} = O_c + B(m_t - m_{t,\,c})}\,.$$

B-jet energy peaks

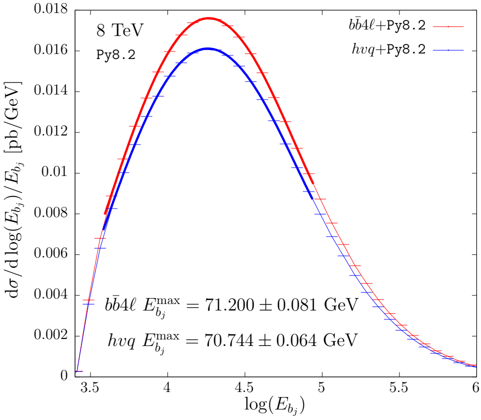
- We fit $\frac{d\,\sigma}{d\,\log E_{b_j}}\frac{1}{E_{b_j}}$ to a fourth order polynomial.



- We find $B \simeq \frac{1}{2} \Rightarrow \boxed{\Delta m_t \simeq -2\Delta E_{b_j}^{\max}}$.

B-jet energy peaks: which NLO generator?

- Large difference between $b\bar{b}4\ell$ and hvq ($\Delta E_{b_j}^{\max} \approx -0.5$ GeV, $\Delta m_t \approx 1$ GeV), but still well below the systematic error quoted by CMS (**2.66 GeV**).



Leptonic observables

- Based on arXiv:1407.2763 (Frixione, Mitov).
- Independent from non-perturbative physics effects.
- Similar analysis performed by ATLAS in arXiv:1709.09407, that finds

$$m_t = 173.2 \pm 0.9\,(\text{stat}) \pm 0.8\,(\text{syst}) \pm 1.2\,(\text{theo})\ \text{GeV}\,.$$

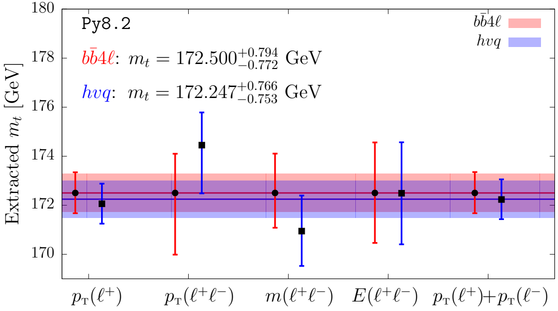
- Measure $\langle O_i \rangle$ for several O_i:

$$\left\{p_\perp(\ell^+),\, p_\perp(\ell^+\ell^-),\, m(\ell^+\ell^-),\, (E(\ell^+)+E(\ell^-)),\, (p_\perp(\ell^+)+p_\perp(\ell^-))\right\}\,.$$

- Assume $\langle O_i \rangle = O_{\mathrm{c},i} + B_i(m_t - m_{t,c})$, where $O_{\mathrm{c},i}$ and B_i can be determined with a MC generator.
- Assuming $\langle O_i \rangle^{\exp} = O_{\mathrm{c},i}^{b\bar{b}4\ell}$, we extract $m_{t,i}$ and $\Delta m_{t,i}$ (due to statistical, scale, PDF etc. variations).

Leptonic observables

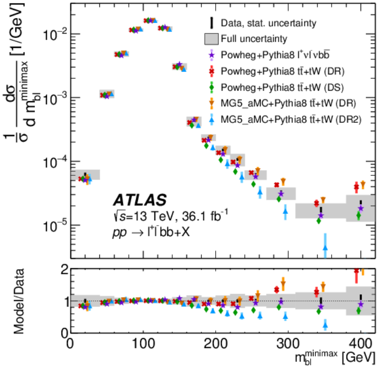
$$\langle O \rangle = O_c^{MC} + B^{MC}(m_t - m_{t,c}) \Rightarrow \boxed{m_t^{MC} = m_{t,c} + \frac{\langle O \rangle^{exp} - O_c^{MC}}{B^{MC}}}$$



– Central $b\bar{b}4\ell$ prediction $= \langle O \rangle^{exp}$

– hvq not able to describe obs depending on spin-correlation effects.

Realistic analyses



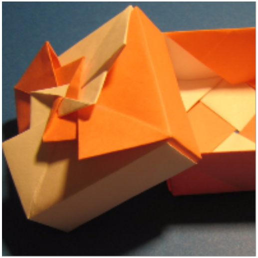
$$m_{bl}^{\min\max} = \min\left[\max\left(m_{b_1 l_1}, m_{b_2 l_2}\right), \max\left(m_{b_1 l_2}, m_{b_2 l_1}\right)\right]$$

Phys. Rev. Lett. **121**, no. 15, 152002 (2018), **ATLAS**

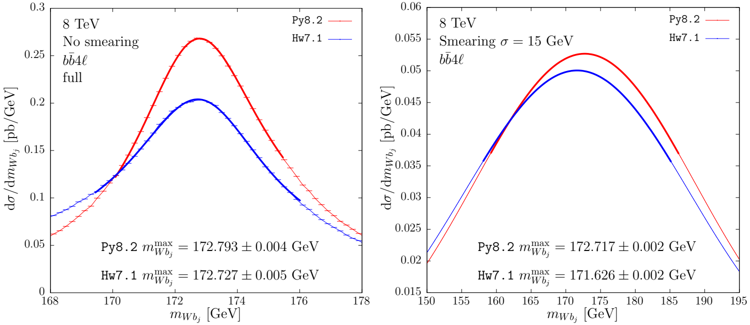
- The generator explicitly including interference (Powheg-Pythia8 lvlvbb) shows **excellent agreement** over the full spectrum.
- *hvq* (+ *Wt* contribution) is not bad, but not as good as $b\bar{b}4\ell$.

The differences with the latest generators are large enough to justify their use but not enough to completely overturn the old measurements based on *hvq*.

Part I B: comparison between Pythia8.2 and Herwig7.1 showers applied to POWHEG BOX events

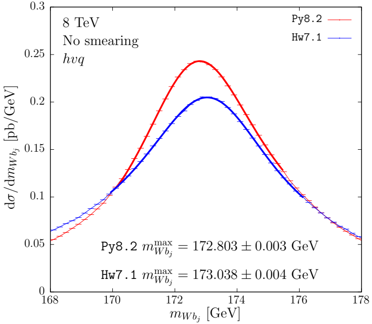


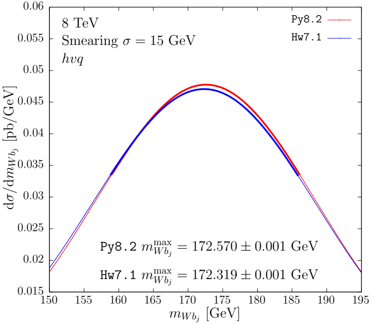
- Large shape-difference between Pythia8.2 and Herwig7.1 leads to a huge displacement after smearing: $\Delta m_t \approx 1$ GeV.



reconstructed-top mass: *hvq*

- Modest difference between Pythia8.2 and Herwig7.1: $\Delta m_t \approx 0.2 - 0.4$ GeV.

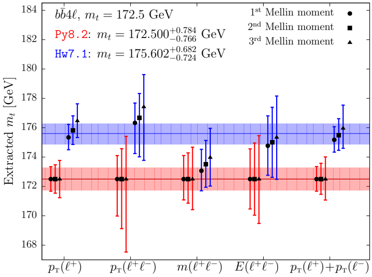




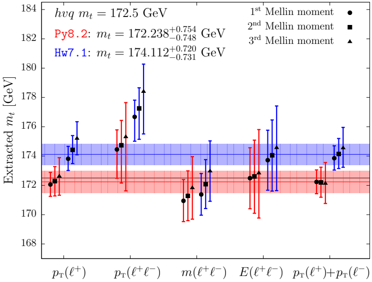
Leptonic observables

- Large difference arises also for purely leptonic observables.

$b\bar{b}4\ell$



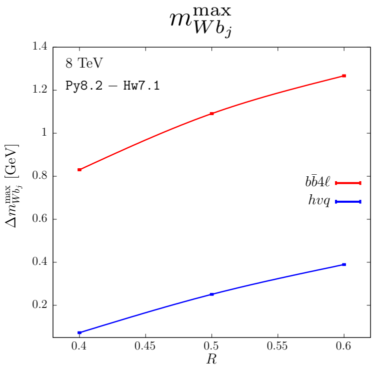
hvq

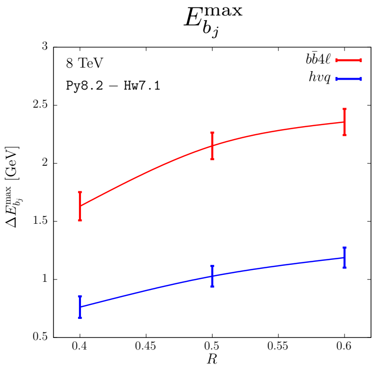


Note: n^{th} Mellin Moment of the Observable O: $\frac{\int O^n d\sigma}{\int d\sigma} = \langle O^n \rangle$.

Jet radius dependence

Different R dependence: it is possible that, by tuning the MC in order to fit the data, the discrepancies between `Pythia8.2` and `Herwig7.1` can be reduced.





Summary (I B)

What we have found:

- Pythia8.2: fair **consistency** among the several NLO+PS predictions.
- Herwig7.1:
  1. large difference from Pythia8.2, in particular for $b\bar{b}4\ell$, where vetoed showers are necessary to handle radiation in decay.
  2. large difference between $b\bar{b}4\ell$ and hvq.

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Can we dismiss Herwig7?

- Pythia8.2: MEC and POWHEG very similar for a $k_\perp$-ordered shower.
- Herwig7.1: MEC and POWHEG *technically* different for an angular ordered shower (MEC applied to the hardest emission found at **each step** of the shower). The difference may be due to **higher-order corrections** and thus it should be taken into account.

Conclusions Part I

- Our analysis is really crude.
- Only a realistic analysis performed by a experimental collaboration, after a tuning procedure, can estimate errors on direct measurements of m_t.
- Using several shower generators is the correct way to estimate errors on standard measurements.

Improving Hw 7.1 + POWHEG interface: preliminary!!

- The minimum $p_\perp$ allowed in **Herwig7.1** PS is 1.223 GeV [arXiv 1708.01491, Reichelt, Richardson, Siodmok]. Thus **POWHEG BOX** should not try to generate softer emissions:

$$p_{\perp,\min}^{\texttt{pwhg}} = \sqrt{0.8}\ \text{GeV} \to 1.223\ \text{GeV}$$

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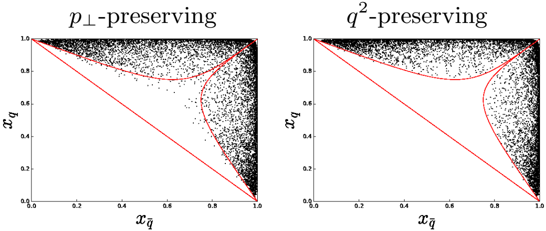
- Let's consider a FSR splitting $a \to b\,c$ performed by Herwig7.1 PS.
  When b or c radiate, the kinematic reconstruction preserves q_a^2.

Not justified by any first principle. By preserving the **virtuality** instead of the transverse momentum, the PS does not overpopulate the dead region and the **agreement with data improves**.



Dalitz plot for $e^+e^- \to q\bar{q}$, from 1708.01491.

Improving Hw 7.1 + POWHEG interface: preliminary!!

- When `Herwig7.1` produces the first emission from a decayed top
$$t \to W\, b \to W\, b\, g$$
the virtuality of the bg pair is preserved in the following steps.
- The $b\bar{b}4\ell$ generator already provides the first emission
$$t \to W\, b\, g$$
`Herwig7.1` is not instructed to preserve the bg-pair virtuality q^2_{bg}.

WWWARN!!!

q^2 is preserved in FSR, but we may have also ISR from the incoming t that can degrade the top mass. Here we are neglecting ISR.

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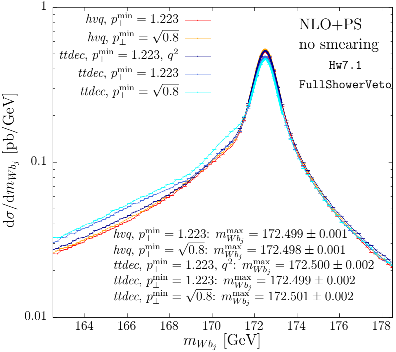
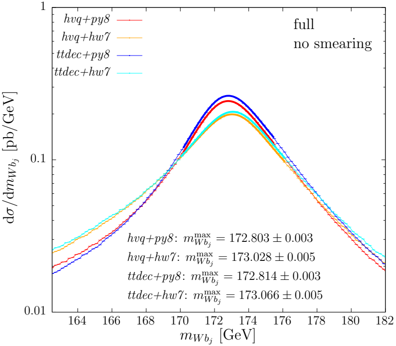
- If we want the same to happen when showering $b\bar{b}4\ell$, we can built the veto in such a way that
  1. Emissions with $p_\perp > p_\perp^{\texttt{pwhg}}$ are vetoed;
  2. At the end of the showering phase, we accept it with probability

$$r = \frac{\sqrt{\lambda(q_t^2, q_W^2, q_{bg}^{2,\mathrm{end}})}}{q_{bg}^{2,\mathrm{end}} - m_b^2} \times \frac{q_{bg}^{2,\texttt{pwhg}} - m_b^2}{\sqrt{\lambda(q_t^2, q_W^2, q_{bg}^{2,\texttt{pwhg}})}}$$

Improving Hw 7.1 + POWHEG interface: preliminary!!

ttdec generator (NLO accurate) plus PS becomes equivalent to *hvq* (LO accurate) + PS + MEC

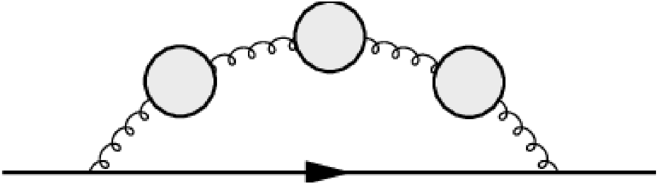




WWWARN!!!

This study is very preliminary, and possibly wrong, what really happens when showering a resonance is currently subject of investigation.

Part II: Renormalons effects in top-mass sensitive observables

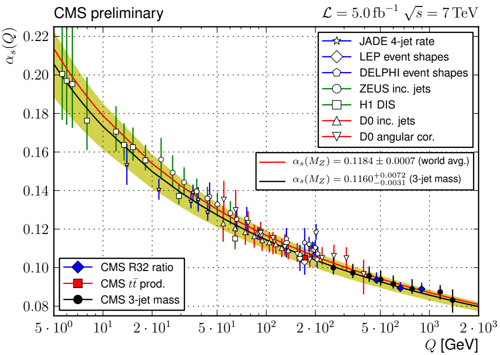


Based on:
"All-orders behaviour and renormalons in top-mass observables" with P. Nason and C. Oleari, arXiv:1801.10931

IR Renormalons

- QCD is affected by **infrared slavery**:

$$\alpha_s(k) = \frac{\alpha_s(Q)}{1 + 2b_0\alpha_s(Q)\log\left(\frac{k}{Q}\right)} = \frac{1}{2b_0\log\left(\frac{k}{\Lambda_{QCD}}\right)}; \quad b_0 = \frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$



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- All orders contribution coming from low-energy region

$$\underbrace{\int_0^Q \mathrm{d}k\, k^{p-1}\alpha_s(Q)}_{\mathrm{NLO}} \Longrightarrow \underbrace{\int_0^Q \mathrm{d}k\, k^{p-1}\alpha_s(k)}_{\text{all orders}} = \boxed{Q^p \times \alpha_s(Q)\sum_{n=0}^{\infty}\left(\frac{2\,b_0}{p}\,\alpha_s(Q)\right)^n n!}$$

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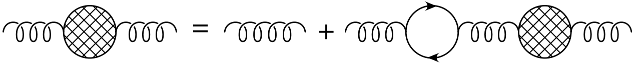
- **Asymptotic series**
  - ⇒ Minimum for $n_{\min} \approx \frac{p}{2b_0\alpha_s(Q)}$
  - ⇒ Size $Q^p \times \alpha_s(Q)\sqrt{2\pi n_{\min}}\, e^{-n_{\min}} \approx \boxed{\Lambda^p_{QCD}}$

  We are interested in $p = 1$, i.e. in **linear renormalons**

Large n_f limit

- All-orders computation can be carried out exactly in the large number of flavour n_f limit



$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} = \alpha_s(\mu)\left(-\frac{n_f \mathrm{T_R}}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right]$$

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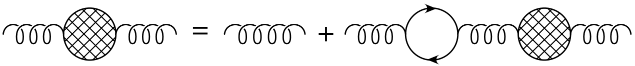

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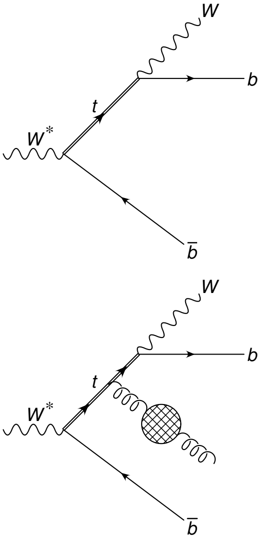
- **naive non-abelianization** at the end of the computation

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} \to \quad \alpha_s(\mu)\underbrace{\left(\frac{11\mathrm{C_A}}{12\pi} - \frac{n_l \mathrm{T_R}}{3\pi}\right)}_{b_0}\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - C\right]$$

Single-top production

$W^* \to t\bar{b} \to Wb\bar{b}$ at all orders using the (complex) pole scheme



Integrated cross section

Integrated cross section (with cuts $\Theta(\Phi)$):

$$\sigma = \int \mathrm{d}\Phi \, \frac{\mathrm{d}\sigma(\Phi)}{\mathrm{d}\Phi} \, \Theta(\Phi)$$

$$= \sigma_{\mathrm{LO}} - \frac{1}{\pi b_0} \int_0^{\infty} \mathrm{d}\lambda \, \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{T(\lambda)}{\alpha_s(\mu)} \right] \arctan \left[\pi \, b_0 \, \alpha_s \left(\lambda e^{-C/2} \right) \right]$$

$\lambda =$ gluon mass

- $T(0) = \sigma_{\mathrm{NLO}}$
- $T(\lambda) = \boxed{\sigma_{\mathrm{NLO}}(\lambda)} + \dfrac{3\lambda^2}{2 T_R \alpha_s} \displaystyle\int \mathrm{d}\Phi_{g^*} \mathrm{d}\Phi_{\mathrm{dec}} \frac{\mathrm{d}\sigma^{(2)}_{q\bar{q}}(\lambda, \Phi)}{\mathrm{d}\Phi} \Big[\Theta(\Phi) - \underbrace{\Theta(\Phi_{g^*})}_{q\bar{q} \to g^*} \Big]$
- $T(\lambda) \xrightarrow{\lambda \to \infty} \dfrac{1}{\lambda^2}$
- $\alpha_s\left(\lambda e^{-C/2}\right) \approx \alpha_s(\lambda) \left[1 + \dfrac{K_g}{2\pi} \alpha_s(\lambda) \right] + \mathcal{O}(\alpha_s^3) = \alpha_s^{\mathrm{MC}}(\lambda)$

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So, if

$$\boxed{\left. \frac{\mathrm{d}T(\lambda)}{\mathrm{d}\lambda} \right|_{\lambda=0} = A \neq 0}$$

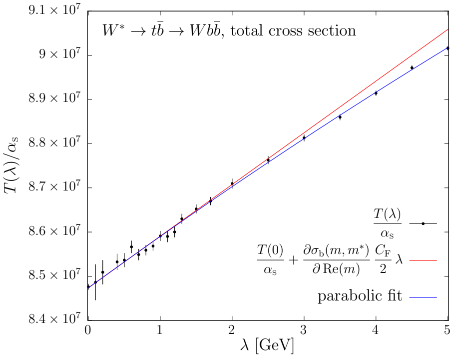
the low-λ contribution takes the form

$$\langle O \rangle \sim -A \sum_{n=0}^{\infty} \int_0^m \mathrm{d}\lambda \left[-2 b_0 \, \alpha_s(m) \log \left(\frac{\lambda^2}{m^2} \right) \right]^n = -Am \sum_{n=0}^{\infty} \left(2 \, b_0 \, \alpha_s(m) \right)^n n!$$

Linear λ term $\leftrightarrow$ Linear renormalons

Total cross section

$\sigma^{\rm tot}(\overline{m}(\mu))$ is renormalon free: $\underbrace{\frac{T(\lambda)}{\alpha_s}}_{\rm pole} \to \underbrace{\frac{T(\lambda)}{\alpha_s} - \frac{\partial \sigma_b}{\partial {\rm Re}(m)} \frac{C_F}{2}\lambda}_{\overline{\rm MS}}$

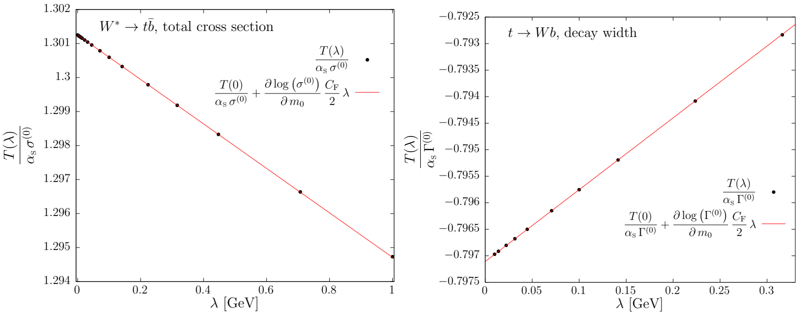


$\Rightarrow$ If a complex mass is used, the top can never be on-shell and the only term that can develop a linear λ sensitivity is the mass counterterm.

Total cross section in NWA

For $\Gamma_t \to 0$ the cross section factorizes

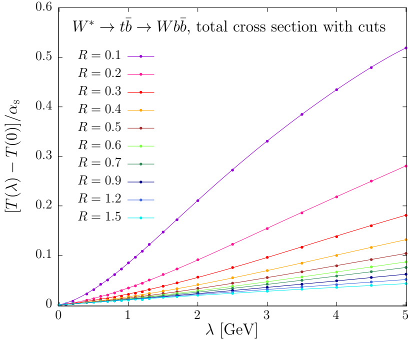
$$\sigma(W^* \to W\, b\, \bar{b}) = \sigma(W^* \to t\bar{b}) \times \frac{\Gamma(t \to W\, b)}{\Gamma_t}$$

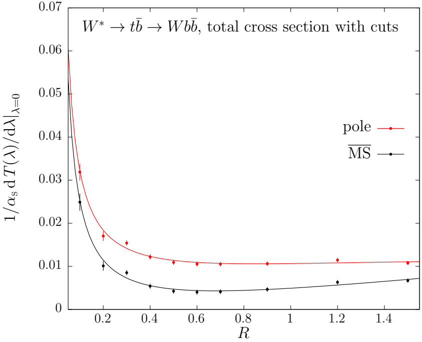


Since both terms are free from linear renormalons, also $\sigma(W^* \to W\, b\, \bar{b})$ is free from linear renormalons.

Total cross section with cuts

Cuts: a b jet and a separate $\bar{b}$ jet with $k_\perp > 25$ GeV (anti-$k_\perp$ jets).



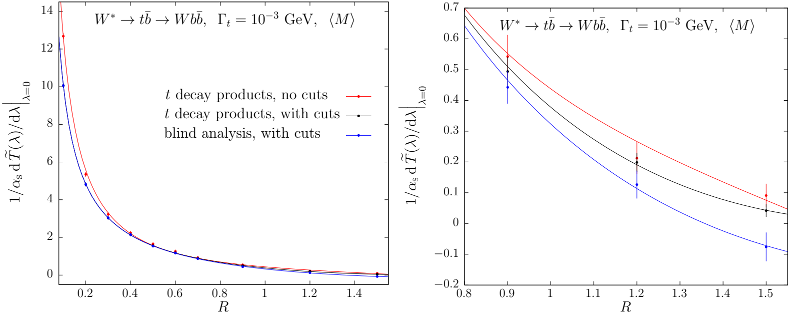


$$\text{Small } R\text{: } \left.\frac{dT(\lambda)}{d\lambda}\right|_{\lambda=0} \propto \frac{1}{R} \Rightarrow \textbf{jet renormalon};$$

Large R: small slope for $\overline{\text{MS}}$.

Reconstructed-top mass in NWA

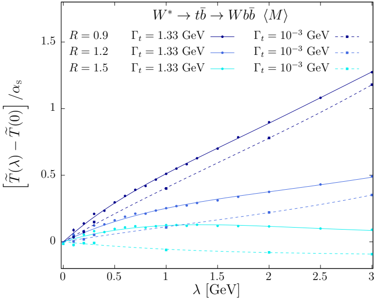
$$\boxed{O = M = \sqrt{(p_W + p_{b_j})^2}}$$

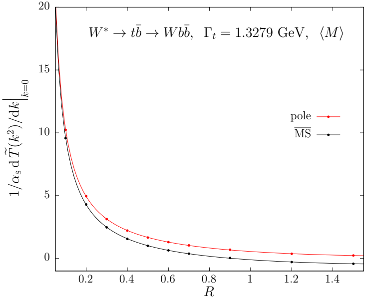


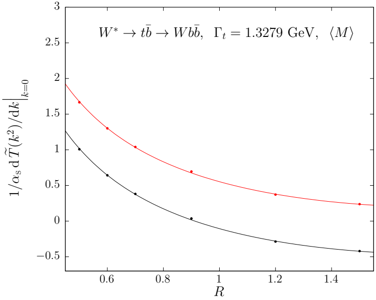
- For $\Gamma_t \to 0$, we can define the "top-decay products"
- For large R, $\langle M \rangle \approx m_{\text{pole}}$ and $T'(0) = 0$: no linear renormalon
- If we move to $\overline{\text{MS}}$ we add $-\frac{C_F}{2} \frac{\partial \langle M \rangle_b}{\partial \text{Re}(m)} \approx -0.67$: physical linear renormalon

Reconstructed-top mass

For the blind analysis, restoring $\Gamma_t = 1.3279$ GeV only slightly changes this picture







Reconstructed-top mass: some numbers

$$M = \sum_{i=0}^{\infty} c_i \alpha_s^i$$

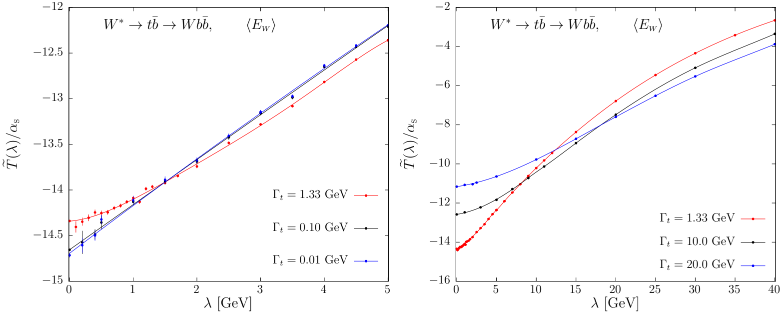
	$c_i \alpha_s^i$ [MeV]		
i	$\mathrm{Re}(m_{\mathrm{pole}} - \overline{m}(\mu))$	$\langle M \rangle_{\mathrm{pole}}$, $R = 1.5$	$\langle M \rangle_{\overline{\mathrm{MS}}}$, $R = 1.5$
5	+89	−10(1)	+79(1)
6	+60	−11(1)	+49(1)
7	+47	−11(1)	+35(1)
8	+44	−12(1)	+31(1)
9	+46	−15(1)	+31(1)
10	+55	−19(1)	+36(1)

More accurate estimates of $m_{\mathrm{pole}} - \overline{m}(\mu)$ (e.g. inclusion of b and c mass effects) can be found in

- [Beneke, Marquad, Nason, Steinhauser, arXiv:1605.03609]: Δm =110 MeV
- [Hoang, Lepenik, Preisser, arXiv:1802.04334]: $\Delta m = 250$ MeV

Energy of the W boson, pole scheme (lab frame)

$\mathbf{E_W}$ = simplified **leptonic observable**. In absence of cuts, is this observable free from renormalons?



When the pole scheme is used we always have renormalons

- Vanishing Γ_t (left): slope ≈ 0.5 near 0;
- Large Γ_t (right): slope ≈ 0.06 near 0;

Energy of the W boson, $\overline{\text{MS}}$ scheme (lab frame)

$\mathbf{E_W}$ = simplified **leptonic observable**. In absence of cuts, is this observable free from physical renormalons?

Γ_t	slope (pole)	$\frac{\partial \langle E_W \rangle_b}{\partial\, \text{Re}(m)}$	$-\frac{C_F}{2} \frac{\partial \langle E_W \rangle_b}{\partial\, \text{Re}(m)}$	slope ($\overline{\text{MS}}$)
NWA	0.53 (2)	0.10 (3)	−0.066 (4)	0.46 (2)
10 GeV	0.058 (8)	0.0936 (4)	−0.0624 (3)	0.004 (8)
20 GeV	0.061 (2)	0.0901 (2)	−0.0601 (1)	0.001 (2)

Yes, if a **finite width** is used, but …

Energy of the W boson (lab frame)

But $\mathcal{O}(\alpha_s^n)$ corrections are dominated by scales of the order $\mu = m_t e^{1-n}$:
we can see the presence of Γ_t only for $\mathbf{n \geq 1 + \log(m_t/\Gamma_t) \approx 6}$

	$\langle E_W \rangle$ [GeV]			
	pole scheme		$\overline{\text{MS}}$ scheme	
i	c_i	$c_i\ \alpha_S^i$	c_i	$c_i\ \alpha_S^i$
0	121.5818	121.5818	120.8654	120.8654
1	$-1.435\,(0) \times 10^1$	$-1.552\,(0) \times 10^0$	$-7.192\,(0) \times 10^0$	$-7.779\,(0) \times 10^{-1}$
2	$-4.97\,(4) \times 10^1$	$-5.82\,(4) \times 10^{-1}$	$-3.88\,(4) \times 10^1$	$-4.54\,(4) \times 10^{-1}$
3	$-1.79\,(5) \times 10^2$	$-2.26\,(6) \times 10^{-1}$	$-1.45\,(5) \times 10^2$	$-1.84\,(6) \times 10^{-1}$
4	$-6.9\,(4) \times 10^2$	$-9.4\,(6) \times 10^{-2}$	$-5.7\,(4) \times 10^2$	$-7.8\,(6) \times 10^{-2}$
5	$-2.9\,(3) \times 10^3$	$-4.4\,(5) \times 10^{-2}$	$-2.4\,(3) \times 10^3$	$-3.5\,(5) \times 10^{-2}$
6	$-1.4\,(3) \times 10^4$	$-2.2\,(4) \times 10^{-2}$	$-1.0\,(3) \times 10^4$	$-1.7\,(4) \times 10^{-2}$
7	$-8\,(2) \times 10^4$	$-1.3\,(4) \times 10^{-2}$	$-5\,(2) \times 10^4$	$-8\,(4) \times 10^{-3}$
8	$-5\,(2) \times 10^5$	$-9\,(4) \times 10^{-3}$	$-2\,(2) \times 10^5$	$-4\,(4) \times 10^{-3}$
9	$-3\,(2) \times 10^6$	$-7\,(4) \times 10^{-3}$	$-1\,(2) \times 10^6$	$-2\,(4) \times 10^{-3}$
10	$-3\,(2) \times 10^7$	$-6\,(5) \times 10^{-3}$	$0\,(2) \times 10^6$	$-1\,(5) \times 10^{-4}$
11	$-3\,(3) \times 10^8$	$-7\,(6) \times 10^{-3}$	$0\,(3) \times 10^6$	$0\,(6) \times 10^{-5}$
12	$-4\,(3) \times 10^9$	$-9\,(9) \times 10^{-3}$	$0\,(3) \times 10^8$	$1\,(9) \times 10^{-3}$

Warning!

Despite the fact the energy of the W boson is not affected by linear renormalons, an accurate determination of the top mass is limited by the reduced sensitivity on the top-mass value:

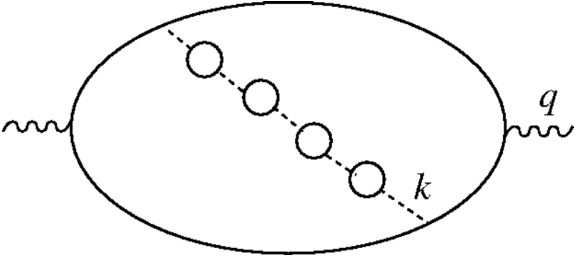
$$2\mathrm{Re}\left[\frac{\partial \langle E_W \rangle_{\mathrm{LO}}}{\partial m}\right] = 0.1$$

$$2\mathrm{Re}\left[\frac{\partial \langle M \rangle_{\mathrm{LO}}}{\partial m}\right] = 1$$

for $E = 300$ GeV, $m_W = 80.4$ GeV, $m_t = 172.5$ GeV $(\beta = 0.5)$

Conclusions

- We devised a simple method that enables us to investigate the presence of linear infrared renormalons in **any infrared safe observable**.
- The **inclusive cross section** and $\mathbf{E_W}$ are free from physical renormalons if $\Gamma_t > 0$ (for σ also in NWA).
- Once jets requirements are introduced, the **jet renormalon** leads to an unavoidable ambiguity.
- For large R, $\langle \mathbf{M} \rangle \approx \mathbf{m}_{\text{pole}}$. This observable has a **physical renormalon**.



THANK YOU FOR THE ATTENTION!

Quantum interference between Wt and $t\bar{t}$ production

The Wt and $t\bar{t}$ contribution do interfere at NLO in the 5f scheme. In Ref. arXiv:1009.2450, (E. Re), two subtraction strategies have been implemented to remove the $t\bar{t}$ contribution from the Wtb predictions, so that we can sum them directly to the *hvq* generator.

1. Diagram Subtraction: $\mathcal{R}^{\mathrm{DS}} = |M_{Wt}|^2$. It is NOT gauge invariant.
2. Diagram Removal: $\mathcal{R}^{\mathrm{DS}} = |M_{Wt} + M_{t\bar{t}}|^2 - C^{\mathrm{sum}}$, with $C^{\mathrm{sum}} = \frac{(m_t\Gamma_t)^2}{[(p_W+p_g)-m_t^2]^2+(m_t\Gamma_t)^2}|M^{t\bar{t}}(\Phi_{\mathrm{dd}})|^2$, with Φ_{dd} a point in the phase space, obtained with reshuffled from the regular real phase space, such as $(p_W + p_b)^2 = m_t^2$.

This trick is not necessary for $b\bar{b}4\ell$, that does include both contributions exactly (in the 4fs, where the quantum interference effects start at LO).

Herwig7.1 = angular-ordered parton-shower.

For $b\bar{b}4\ell$ we can use two different vetoing algorithms:

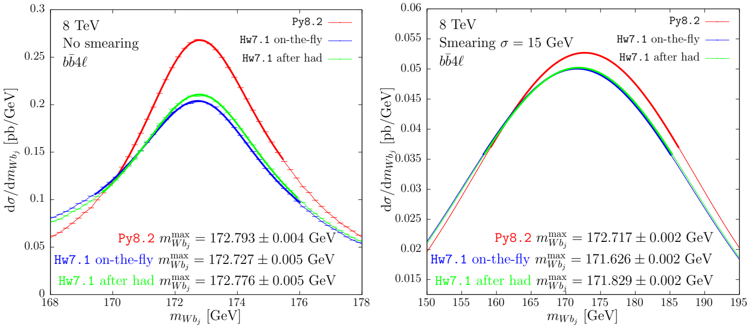
1. on-the-fly: each time an emission is generated. The momenta of the emitted particles have not been generated yet, we must rely on Herwig7.1 definition of $p_\perp$ (our default);
2. before the hadronization: we have access to the momenta of all the particles. These have been reshuffled to ensure 4-momentum conservation.

For hvq, we can improve the PS description of the hardest emission off the resonances using:

1. MEC (default);
2. Herwig7.1 internal implementation of POWHEG.

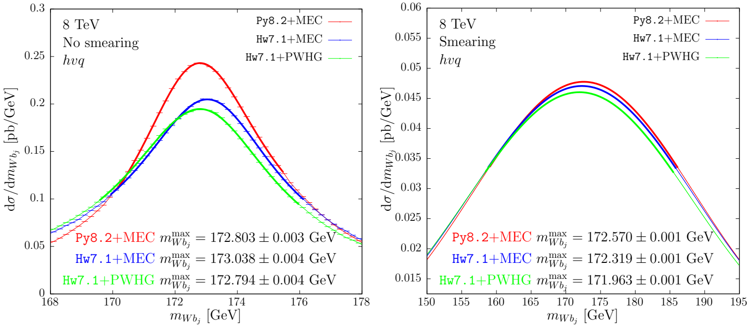
reconstructed-top mass: $b\bar{b}4\ell$

- Large difference between Pythia8.2 and Herwig7.1.
- Small difference between the two matching procedures in Herwig7.1.



reconstructed-top mass: *hvq*

- The difference between Pythia8.2 and Herwig7.1 is comparable with the one between Herwig7.1+MEC and Herwig7.1+POWHEG.
- *hvq*+Herwig7.1+POWHEG quite similar to $b\bar{b}4\ell$+Herwig7.1 ($m_{Wb_j}^{\max} = 172.727$ GeV, smeared $m_{Wb_j}^{\max} = 171.626$ GeV).



B-jet energy peaks

- Based on arxiv:1603.03445 (Agashe, Kim, Franceschini, Schulze).
- Investigated by CMS in [CMS-PAS-TOP-15-002], that finds

$$m_t = 172.29 \pm 1.17\,(\text{stat}) \pm 2.66\,(\text{syst})\ \text{GeV}\,.$$

- Purely hadronic observable, independent from the top production dynamics.
- At LO, neglecting off-shell effects, in the top frame we have:

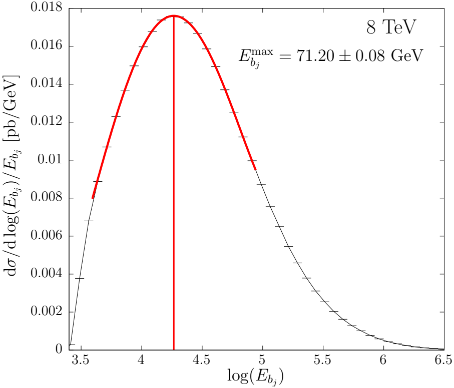
$$\boxed{E_{b_j} = \frac{m_t^2 - m_W^2}{2m_t}}\,.$$

- In the lab frame the distribution is squeezed, but the peak position does not vary.
- After the inclusion of perturbative and non-perturbative effects, for $m_t \approx m_{t,\,c}$, we have:

$$\boxed{E_{b_j}^{\max} = O_c + B(m_t - m_{t,\,c})}\,.$$

B-jet energy peaks

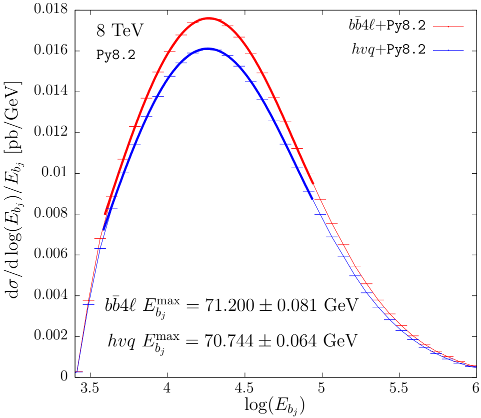
- We fit $\frac{d\,\sigma}{d\,\log E_{b_j}}\frac{1}{E_{b_j}}$ to a fourth order polynomial.



- We find $B \simeq \frac{1}{2} \Rightarrow \boxed{\Delta m_t \simeq -2\Delta E_{b_j}^{\max}}$.

B-jet energy peaks: which NLO generator?

- Large difference between $b\bar{b}4\ell$ and hvq ($\Delta E_{b_j}^{\max} \approx -0.5$ GeV, $\Delta m_t \approx 1$ GeV), but still well below the systematic error quoted by ATLAS (**2.66 GeV**).

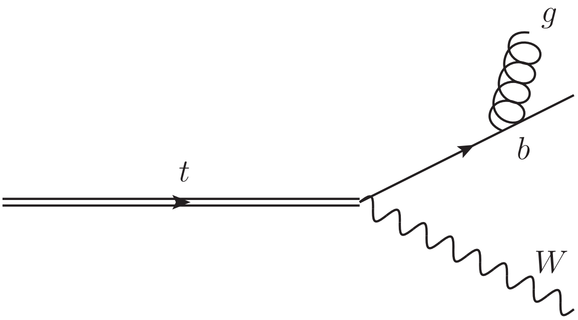


Technical details

- $pp \to b\bar{b}e^+\nu_e\mu^-\bar{\nu}_\mu$ + NLO + PS + underlying event + hadronization.
- $\sqrt{s} = 8$ TeV.
- $\mu = \sqrt[4]{(E_t^2 - p_{z,t}^2)(E_{\bar{t}}^2 - p_{z,\bar{t}}^2)}$. For $Zb\bar{b}$ events $\mu = \frac{\sqrt{p_Z^2}}{2}$.
- `MSTW2008nlo68cl` PDF set.
- `FastJet` implementation of anti-$k_\perp$ jet algorithm, $R = 0.5$.
- $b(\bar{b})$-jet: jet containing the hardest $b(\bar{b})$-flavoured hadron.
- W^+ = hardest e^+ + hardest ν_e.
- W^- = hardest μ^- + hardest $\bar{\nu}_\mu$.
- Selection cuts to suppress the Wt background:
  ⇒ distinct b- and $\bar{b}$-jets with $p_\perp > 30$ GeV, $|\eta| < 2.5$;
  ⇒ e^+ and μ^- with $p_\perp > 20$ GeV, $|\eta| < 2.4$.

Matrix Element Corrections

- If the t decay is generated at LO, `Pythia8.2` and `Herwig7.1` can modify the shower algorithm in order to generate the hardest emission using the exact Matrix Element for one additional real emission: MEC.
- In this way, also when using *hvq*, the t decay with an extra emission is described with exact LO matrix elements.



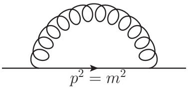
pole-$\overline{\text{MS}}$ mass relation

$\overline{m}(\mu)$ ⇒UV-divergent contribution of self-energy corrections

m_{pole} ⇒UV-divergent + $\underbrace{\text{IR (finite)}}_{\alpha_s^{n+1} n!}$ contributions

- At $\mathcal{O}(\alpha_s)$:

$$m_{\text{pole}} - \overline{m}(\mu) = \text{Fin}\left[i \times \text{(diagram)} \right] = \text{Fin}\left[i\Sigma^{(1)}(\epsilon) \right]$$



$$i\Sigma^{(1)}(\epsilon) = -i\, g^2\, \mathrm{C_F} \left(\frac{\mu^2}{4\pi} e^{\Gamma_E} \right)^{\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\alpha (\not{p} + \not{k} + m)\gamma_\alpha}{[k^2 + i\eta]\,[(k+p)^2 - m^2 + i\eta]} \Bigg|_{\not{p}=m}$$

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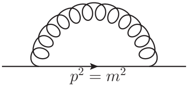

$$\left. \right] = \text{Fin}\left[i\Sigma^{(1)}(\epsilon) \right]$$

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- At all-orders:

$$i\Sigma(\epsilon) = -\, i\, g^2\, \text{C}_\text{F} \left(\frac{\mu^2}{4\pi} \text{e}^{\Gamma_E} \right)^{\epsilon} \int \frac{\text{d}^d k}{(2\pi)^d} \frac{\gamma^\alpha (\not{p} + \not{k} + m) \gamma_\alpha}{[k^2 + i\eta]\, [(k+p)^2 - m^2 + i\eta]} \Bigg|_{\not{p}=m}$$

$$\times \frac{1}{1 + \Pi(k^2 + i\eta, \mu^2, \epsilon) - \Pi_{\text{ct}}}$$

pole-$\overline{\text{MS}}$ mass relation

- At all-orders:

$$i\Sigma(\epsilon) = -\frac{1}{\pi}\int_{0^-}^{+\infty} \frac{d\lambda^2}{2\pi}\left[i \underbrace{\Sigma^{(1)}(\epsilon,\lambda)}_{\lambda=\text{gluon mass}}\right] \text{Im}\left[\frac{1}{\lambda^2 + i\eta}\frac{1}{1+\Pi(\lambda^2+i\eta,\mu^2,\epsilon)-\Pi_{\text{ct}}}\right]$$

$$\text{Fin}\left[i\Sigma(\epsilon)\right] = -\frac{1}{\pi b_0}\int_0^{\infty}\lambda\frac{d}{d\lambda}\left[\frac{r_{\text{fin}}(\lambda)}{\alpha_s(\mu)}\right]\arctan\left[\pi b_0 \alpha_s(\lambda e^{-C/2})\right] + \dots$$

where $\boxed{r_{\text{fin}}(\lambda) \xrightarrow{\lambda\ll 1} -\alpha_s(\mu)\frac{C_F}{2}\lambda}$, $\quad r_{\text{fin}}(\lambda) \xrightarrow{\lambda\to\infty} \mathcal{O}\left(\frac{m^2}{\lambda^2}\right)$

pole-$\overline{\text{MS}}$ mass relation

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- Small λ contribution (independent from C):

$$\frac{C_F}{2}\sum_{n=0}^{\infty}\int_0^m d\lambda\left[-2b_0\,\alpha_s(m)\log\left(\frac{\lambda^2}{m^2}\right)\right]^n = \frac{C_F}{2}m\sum_{n=0}^{\infty}(2\,b_0\,\alpha_s(m))^n\,n!$$

The resummed series has an ambiguity proportional to $\mathbf{\Lambda_{QCD}}$:

Linear k term $\leftrightarrow$ Linear renormalons

pole-$\overline{\text{MS}}$ mass relation

- In the pure n_f limit: arxiV:hep-ph/9502300, Ball et all

$$b_0 = -\frac{n_f T_R}{3\pi}\,, C = \frac{5}{3}\,, \qquad \frac{m - \overline{m}(\overline{m})}{m} = \frac{4}{3}\alpha_s(\overline{m})\left[1 + \sum_{i=1}^{\infty} d_i\left(b_0\,\alpha_s(\overline{m})\right)^i\right]$$

i	1	2	3	4	5	6	7	8
d_i	5×10^0	2×10^1	1×10^2	9×10^2	9×10^3	1×10^5	1×10^6	2×10^7

- "Realistic" large b_0 approximation:

$$\alpha_s(\lambda\, e^{-C/2}) = \frac{\alpha_s(\lambda)}{1 - b_0 C \alpha_s(\lambda)} \approx \underbrace{\alpha_s(\lambda)\left[1 + b_0 C \alpha_s(\lambda)\right] = \alpha_s^{\text{CMW}}(\lambda)}_{b_0\,C = \frac{1}{2\pi}\left[\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9} n_l T_R\right]}$$

Pole-$\overline{\text{MS}}$ mass relation

$$m_0 = 172.5 \text{ GeV}, \qquad \Gamma = 1.3279 \text{ GeV}, \qquad m^2 = m_0^2 - i m_0 \Gamma, \qquad \mu = m_0$$

$$m - \overline{m}(\mu) = m \sum_{i=1}^{n} c_i \alpha_s^i(\mu)$$

$m - \overline{m}(\mu)$				
i	$\mathrm{Re}\,(c_i)$	$\mathrm{Im}\,(c_i)$	$\mathrm{Re}\left(m\,c_i\,\alpha_s^i\right)$	$\mathrm{Im}\left(m\,c_i\,\alpha_s^i\right)$
1	4.244×10^{-1}	2.450×10^{-3}	$7.919 \times 10^{+0}$	$+1.524 \times 10^{-2}$
2	6.437×10^{-1}	2.094×10^{-3}	$1.299 \times 10^{+0}$	-7.729×10^{-4}
3	$1.968 \times 10^{+0}$	8.019×10^{-3}	4.297×10^{-1}	$+9.665 \times 10^{-5}$
4	$7.231 \times 10^{+0}$	2.567×10^{-2}	1.707×10^{-1}	-5.110×10^{-5}
5	$3.497 \times 10^{+1}$	1.394×10^{-1}	8.930×10^{-2}	$+1.240 \times 10^{-5}$
6	$2.174 \times 10^{+2}$	8.164×10^{-1}	6.005×10^{-2}	-5.616×10^{-6}
7	$1.576 \times 10^{+3}$	$6.133 \times 10^{+0}$	4.709×10^{-2}	$+2.009 \times 10^{-6}$
8	$1.354 \times 10^{+4}$	$5.180 \times 10^{+1}$	4.376×10^{-2}	-1.031×10^{-6}
9	$1.318 \times 10^{+5}$	$5.087 \times 10^{+2}$	4.608×10^{-2}	$+4.961 \times 10^{-7}$
10	$1.450 \times 10^{+6}$	$5.572 \times 10^{+3}$	5.481×10^{-2}	-2.909×10^{-7}

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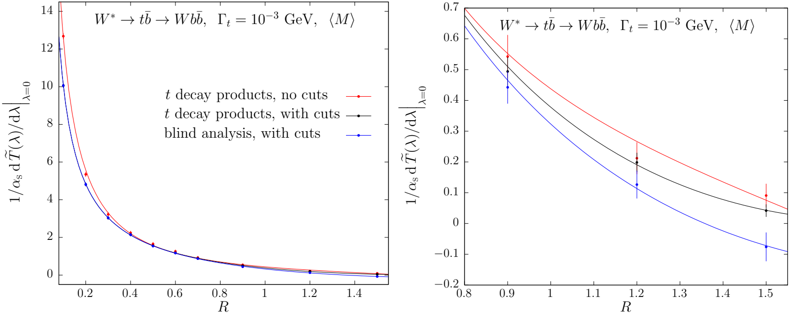
More accurate estimates of $m_{\text{pole}} - \overline{m}(\mu)$ (e.g. inclusion of b and c mass effects) can be found in

- [Beneke, Marquad, Nason, Steinhauser, arXiv:1605.03609]: Δm =110 MeV
- [Hoang, Lepenik, Preisser, arXiv:1802.04334]: Δm = 250 MeV

NB: Actual systematic uncertainty is 500 MeV!

$$O = M = \sqrt{(p_W + p_{b_j})^2}$$



- For $\Gamma_t \to 0$, we can define the "top-decay products"
- For large R, $\langle M \rangle \approx m_{\text{pole}}$ and $T'(0) = 0$: no linear renormalon
- If we move to $\overline{\text{MS}}$ we add $-\frac{C_F}{2}\frac{\partial \langle M \rangle}{\partial \text{Re}(m)} \approx -0.67$: physical linear renormalon

IR-safe observables

Average value of an observable O (e.g. reconstructed-top mass, W-boson energy, ...)

$$\langle O \rangle = \frac{1}{\sigma} \int \mathrm{d}\Phi \, \frac{\mathrm{d}\sigma(\Phi)}{\mathrm{d}\Phi} O(\Phi)$$

$$= \langle O \rangle_{\text{LO}} - \frac{1}{\pi b_0} \int_0^\infty \mathrm{d}\lambda \, \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{\widetilde{T}(\lambda)}{\alpha_s(\mu)} \right] \arctan \left[\pi \, b_0 \, \alpha_s \left(\lambda e^{-C/2} \right) \right]$$

- $\widetilde{T}(0) = \langle O \rangle_{\text{NLO}}$
- $\widetilde{T}(\lambda) = \boxed{\langle O(\lambda) \rangle_{\text{NLO}}} + \frac{3\lambda^2}{2 n_f \mathrm{T_R} \alpha_s} \int \mathrm{d}\Phi_{g^*} \mathrm{d}\Phi_{\text{dec}} \frac{\mathrm{d}\sigma^{(2)}_{q\bar{q}}(\lambda, \Phi)}{\mathrm{d}\Phi} \left[\overline{O}(\Phi) - \underbrace{\overline{O}(\Phi_{g^*})}_{q\bar{q} \to g^*} \right]$

  with $\lambda =$ gluon mass, $\quad \overline{O}(\Phi) = [O(\Phi) - O_{\text{LO}}] \, \Theta(\Phi) / \sigma_{\text{LO}}$
- $\widetilde{T}(\lambda) \xrightarrow{\lambda \to \infty} \frac{1}{\lambda^2}$

Reconstructed-top mass

$$M = \sum_{i=0}^{\infty} c_i \alpha_s^i$$

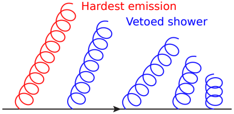
	$c_i\alpha_s^i$ [MeV]		
i	$\mathrm{Re}(m_{\mathrm{pole}} - \overline{m}(\mu))$	$\langle M \rangle_{\mathrm{pole}}$, $R = 1.5$	$\langle M \rangle_{\overline{\mathrm{MS}}}$, $R = 1.5$
5	+89	−10(1)	+79(1)
6	+60	−11(1)	+49(1)
7	+47	−11(1)	+35(1)
8	+44	−12(1)	+31(1)
9	+46	−15(1)	+31(1)
10	+55	−19(1)	+36(1)

More realistic estimate in Beneke et al, 1605.03609:

- neglecting b and c masses: 70 MeV
- including b and c masses: 110 MeV

Interface between POWHEG BOX and Shower MC

- Pythia8 [Sjöstrand et al., arXiv:1410.3012] is a $k_\perp$-ordered shower.



$\Rightarrow$ Natural matching with POWHEG radiation.

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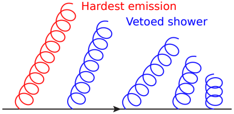
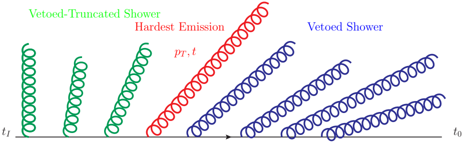


$\Rightarrow$ Natural matching with POWHEG radiation.

- **Herwig7** [Bahr et al., arXiv:0803.0883], [Bellm et. al, arXiv:1512.01178] is an angular-ordered shower.



$\Rightarrow$ Truncated-vetoed showers are known to give a contribution; so only a vetoed shower is implemented.