Extraction of $\Delta\alpha_{had}$ from pseudodata at LO

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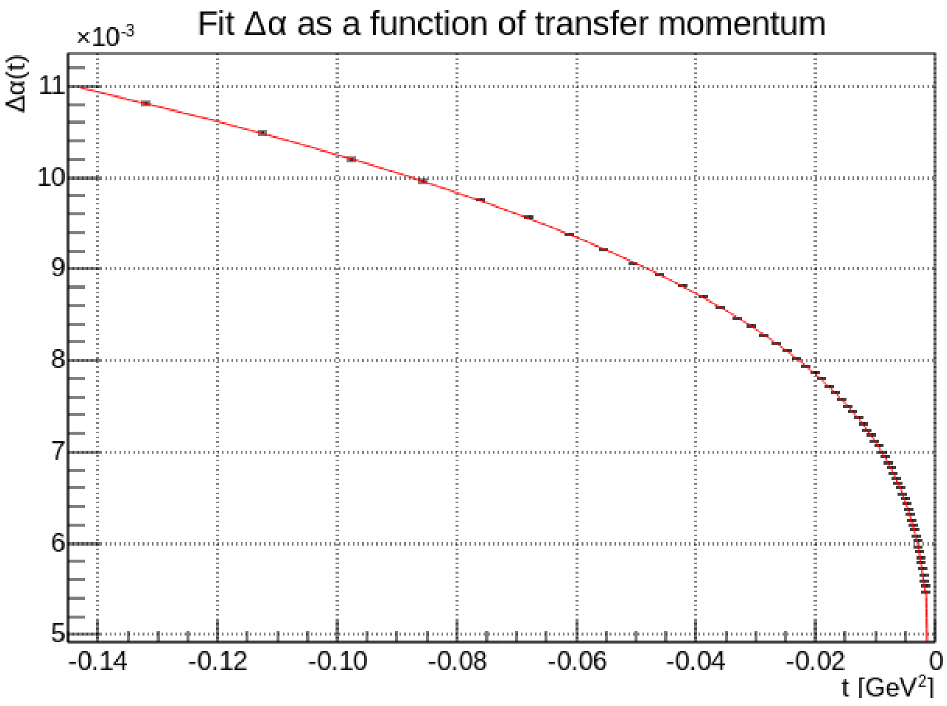
In the previous study we focused directly on Δα, getting the values from Fedor's routine and starting from there perform the fit:

- Get Δα as a function of t from Fedor's routine
- Introduce a statistical (uncorrelated) error on Δα

$$\boxed{\Delta\alpha_i(t_i)} \quad \delta\Delta\alpha_i(t_i) = \frac{1}{2\sqrt{N_i(t_i)}}$$

- Execute the fit on smeared data using a pol3 to model $\Delta\alpha_{had}$



Fit function:

$$\Delta\alpha'(t) = \left[\Delta\alpha_{lep}(t) + [0]t + [1]t^2 + [2]t^3\right]$$

Definition of experimental pseudodata

$$y_i = \frac{N_i(t_i)}{N_0} \frac{\sigma^{0,MC}}{\sigma^{noVP}(t_i)} \qquad \delta y_i = y_i \sqrt{\frac{1}{N_i(t_i)} + \frac{1}{N_0}}$$

$$N_i(t_i) = \int_{\Delta t_i} \frac{d\sigma^{VP}}{dt} dt \cdot L \qquad N_0 = \int_{\Delta t_0} \frac{d\sigma^{VP}}{dt} dt \cdot L$$

Δt_i = bin i in the signal region (x ϵ [0.3, 0.932])

Δt_0 = bin of the normalization region (x ϵ [0.2, 0.3])

L = 1,5 10^7 nb^{-1}

⬇

Full statistics

$$\sigma^{0,MC} = \int_{\Delta t_0} \frac{d\sigma^{VP}}{dt} dt \qquad \sigma^{no\ VP}(t_i) = \int_{\Delta t_i} \frac{d\sigma^{no\ VP}}{dt} dt$$

Using these definitions pseuodata simplify as

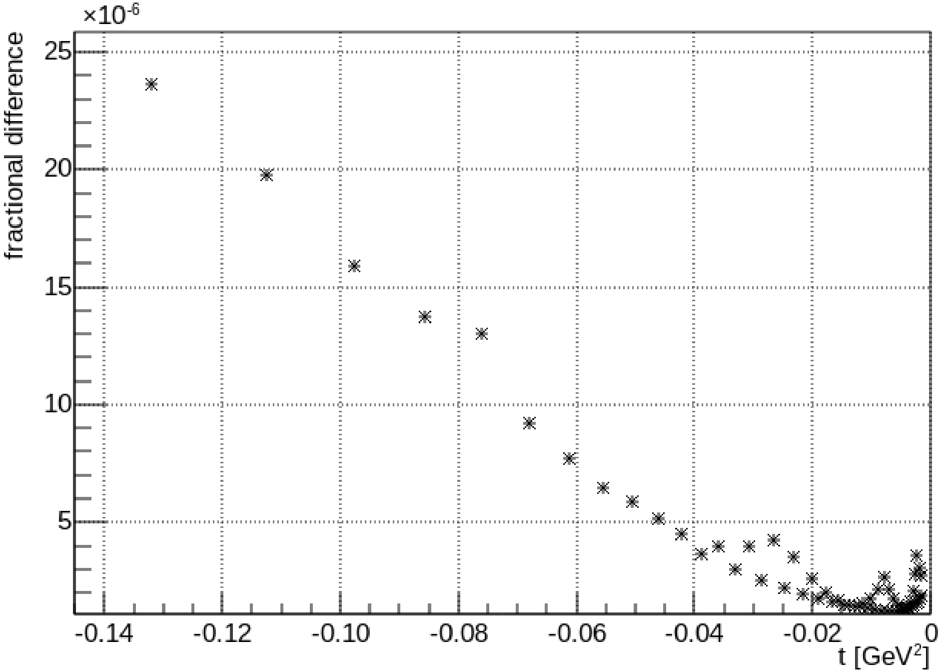
$$y_i = \frac{\int_{\Delta t_i} \frac{d\sigma^{VP}}{dt}}{\int_{\Delta t_i} \frac{d\sigma^{no\ VP}}{dt}}$$

Definition of experimental pseudodata

From this points I want to extract $\Delta\alpha(t)$ → $f(t,\vec{p}) = \frac{1}{|1-\Delta\alpha(t,\vec{p})|^2}$ → I am approximating $\frac{\frac{d\sigma^{VP}}{dt}(t_i)}{\frac{d\sigma^{no\ VP}}{dt}(t_i)} \simeq \frac{\int_{\Delta t_i} \frac{d\sigma^{VP}}{dt}}{\int_{\Delta t_i} \frac{d\sigma^{noVP}}{dt}}$

Which definition of t_i allows an agreement better than 10^{-5} ?

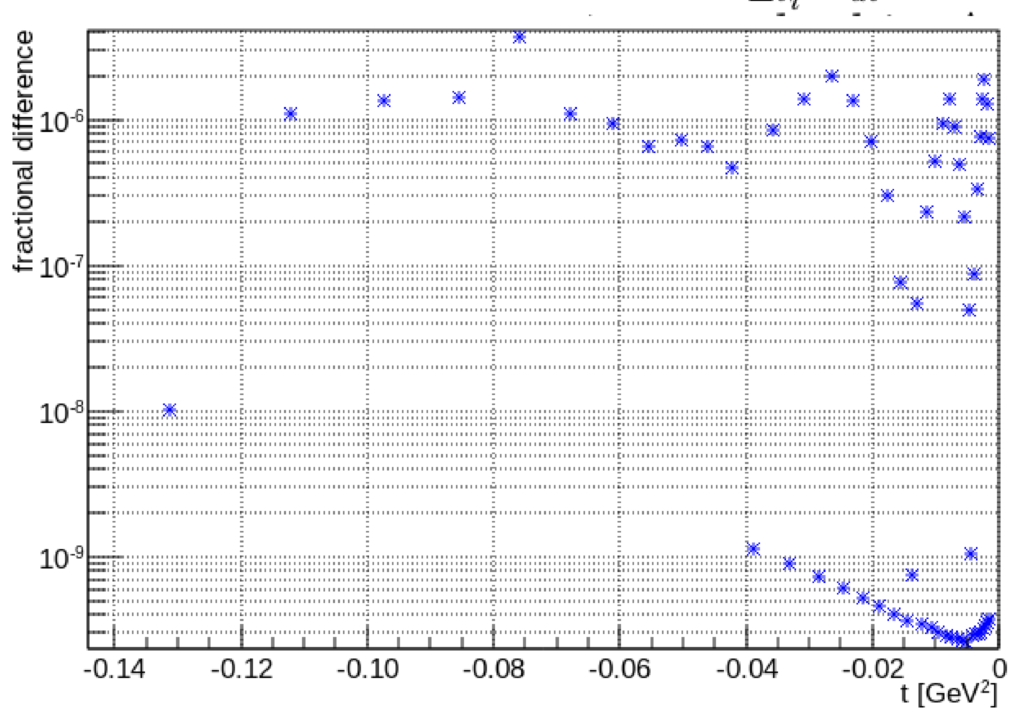
1) t_i is the central point of the bin Δt_i



At the peak the difference is ~ 2,5 10^{-5} : not enough

Thanks to H. Czyz

2) t_i is the mean value on the bin Δt_i : $t_i = \frac{\int_{\Delta t_i} t \frac{d\sigma^{VP}}{dt} dt}{\int_{\Delta t_i} \frac{d\sigma^{VP}}{dt} dt}$



The difference is at least ~4 10^{-6} : OK

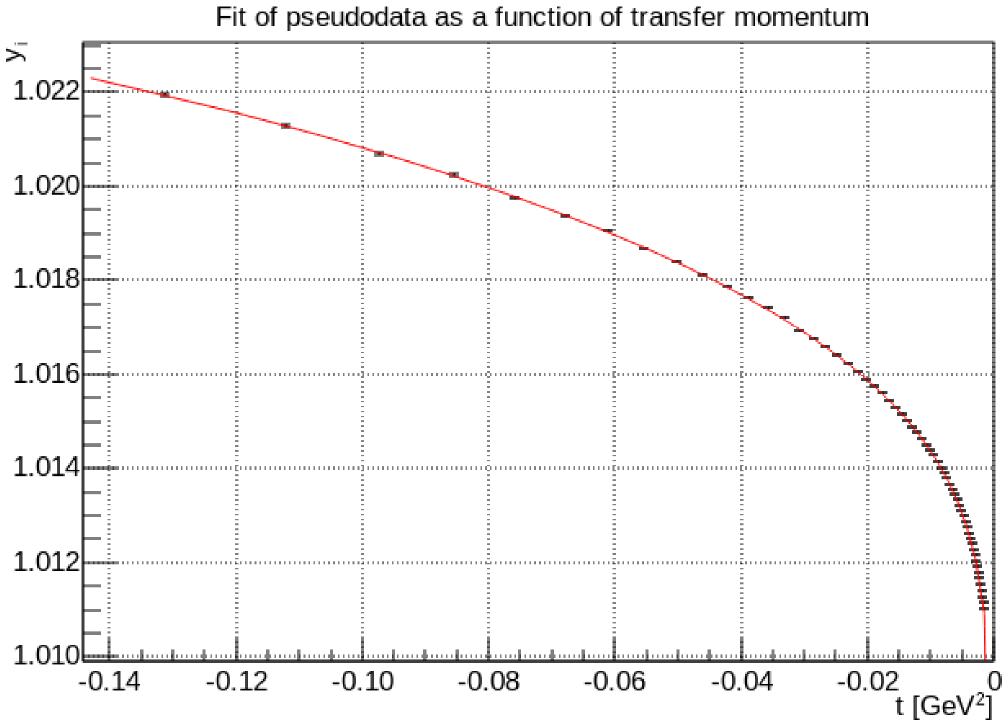
Fit procedure

$\Delta\alpha(t)$ and $\Delta\alpha_{lep}(t)$ known from Fedor's routine

- Apply a statistical smearing to pseudodata y_i

- Extract $\Delta\alpha_{had}$ by fitting pseudodata with $$f(t,\vec{p}) = \frac{1}{|1-(\Delta\alpha_{lep}(t)+\Delta\alpha_{had}(t,\vec{p})|^2}$$

  minimizing χ^2: $$\chi^2 = \sum_{i=0}^{N_{bin}} \left(\frac{y_i - f(t,\vec{p})}{\delta y_i}\right)^2$$

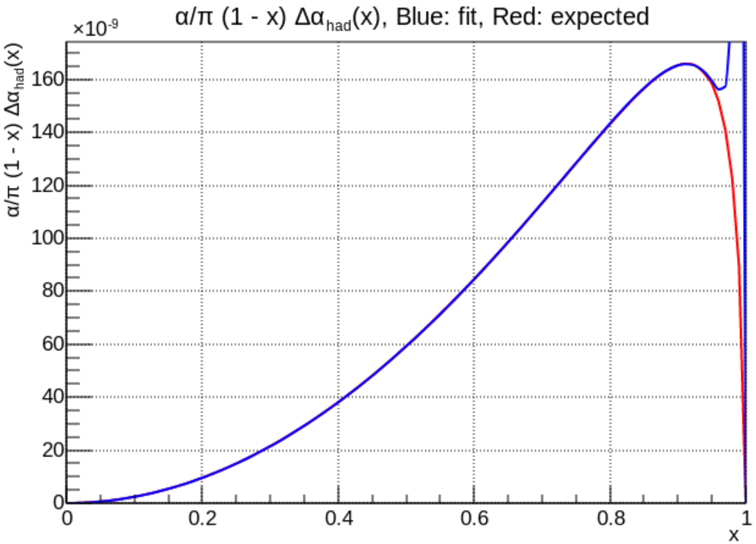


- Known $\Delta\alpha_{had}$ integrate MUonE master formula to obtain a_μ^{HLO}

$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

a_μ^{HLO} distribution can be obtained by a 10k repetition of this fit varying y_i by Gaussian smearing (pseudodata experiment)

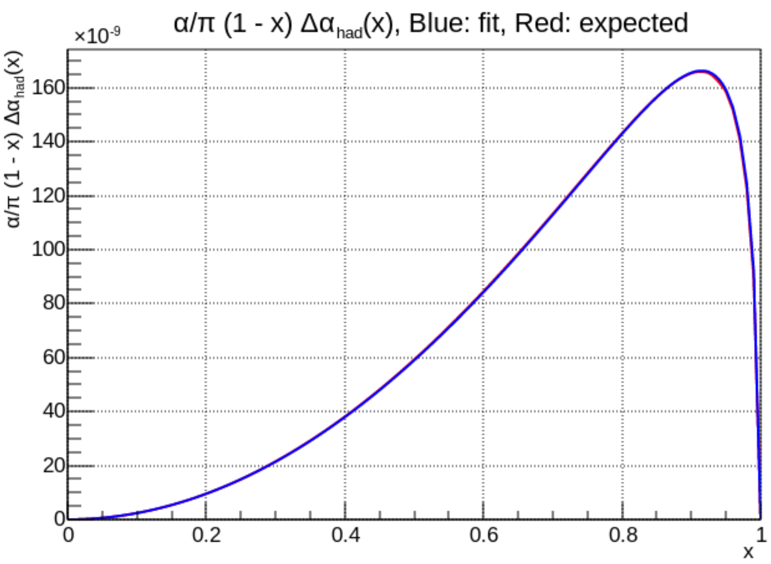
Which parameterization for $\Delta\alpha_{had}$?



$$\Delta\alpha_{had}^{pol}(t,\vec{p}) = p_0 t + p_1 t^2 + p_2 t^3$$

Integration can be performed only in the signal region (x ϵ [0.3, 0.932])

$$\Delta\alpha_{had}^{f.l.}(t,\vec{p}) = \frac{p_0}{3}\left[-\frac{5}{3} - \frac{4p_1}{t} + \frac{\frac{8p_1^2}{t} + \frac{2p_1}{t} - 1}{\sqrt{1-\frac{4p_1}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4p_1}{t}}}{1+\sqrt{1-\frac{4p_1}{t}}}\right|\right]$$

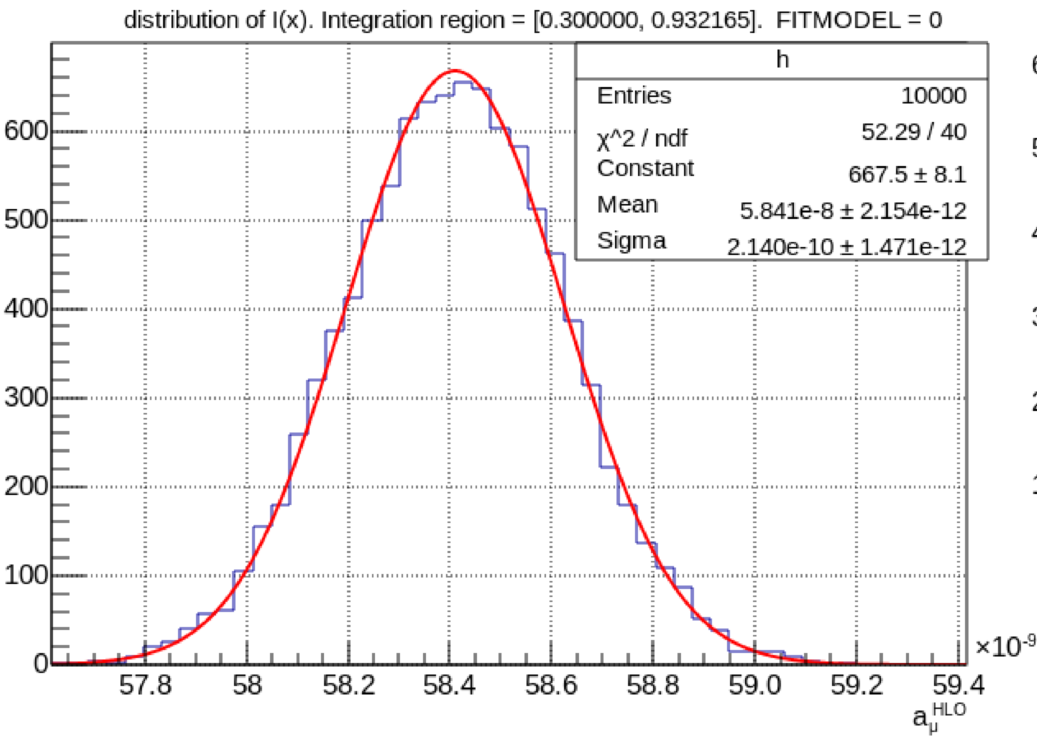


See C.M. Carloni Calame Presentation at Zurich Workshop 4-7 February

Integration can be performed in the entire x range

Results for a_μ^{HLO}

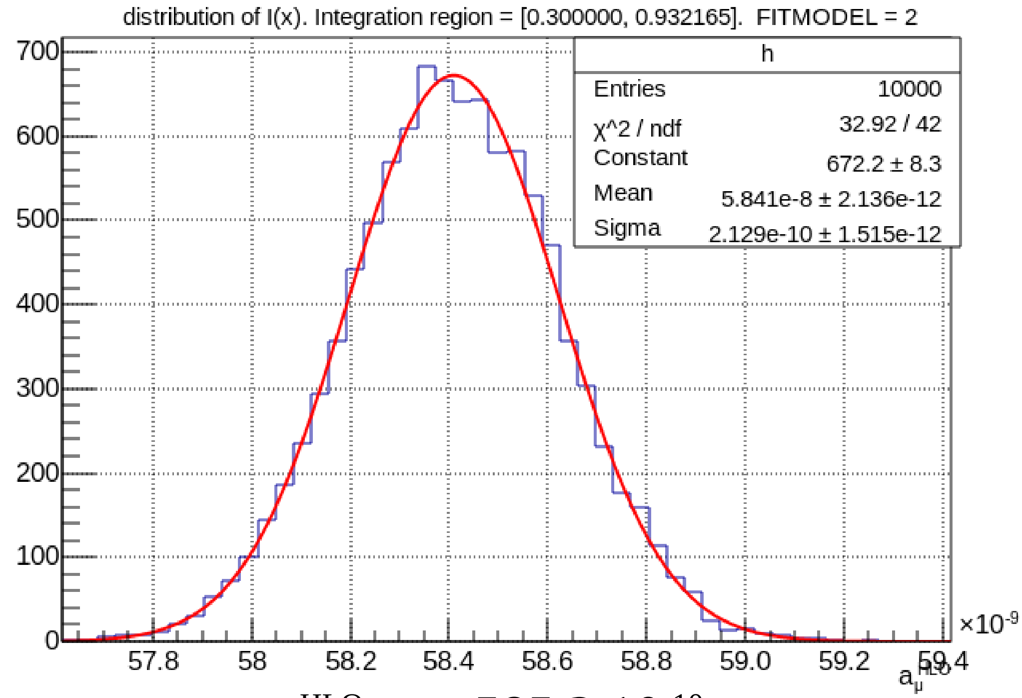
Polynomial parameterization



- $a_\mu^{HLO}{}_{theo} = 585{,}2\ 10^{-10}$
- $a_\mu^{HLO} = (\ 584{,}1 \pm 2{,}1)\ 10^{-10}$
- 0,36% precision

Why do I get a statistical error > 0,3%?

Fermion like parameterization



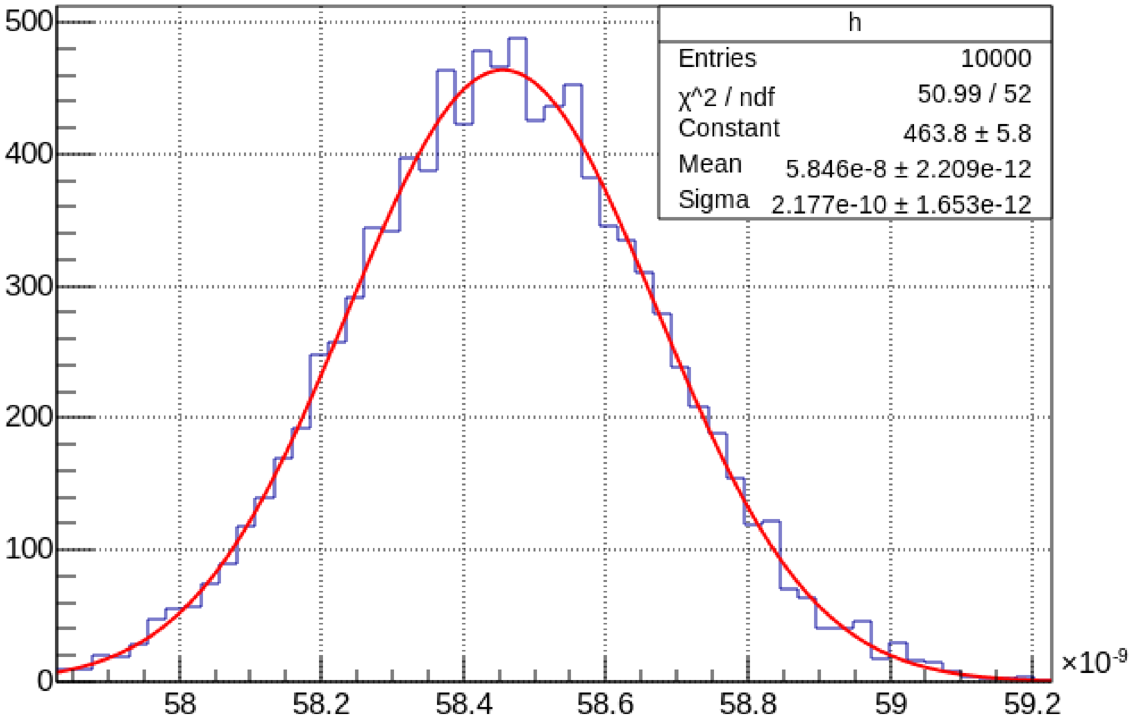
- $a_\mu^{HLO}{}_{theo} = 585{,}2\ 10^{-10}$
- $a_\mu^{HLO} = (\ 584{,}1 \pm 2{,}1)\ 10^{-10}$
- 0,36% precision

- $a_\mu^{HLO}{}_{theo} = 695{,}3\ 10^{-10}$ integrating x [0, 1]
- $a_\mu^{HLO} = (\ 694{,}6 \pm 2{,}2)\ 10^{-10}$
- 0,31% precision

...to compare with results obtained starting from Δα

Polynomial parameterization

60 points

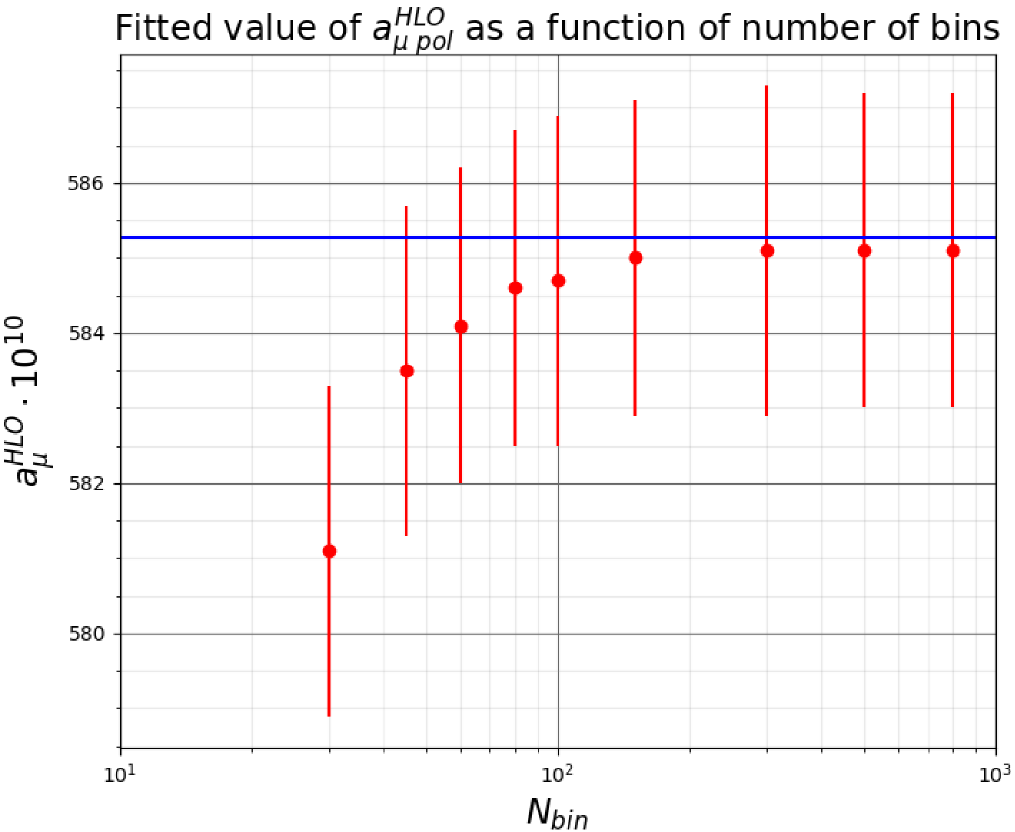


- $a_\mu^{HLO}{}_{theo} = 585{,}2\ 10^{-10}$
- $a_\mu^{HLO} = (\ 584{,}6 \pm 2{,}2)\ 10^{-10}$
- 0,37% precision

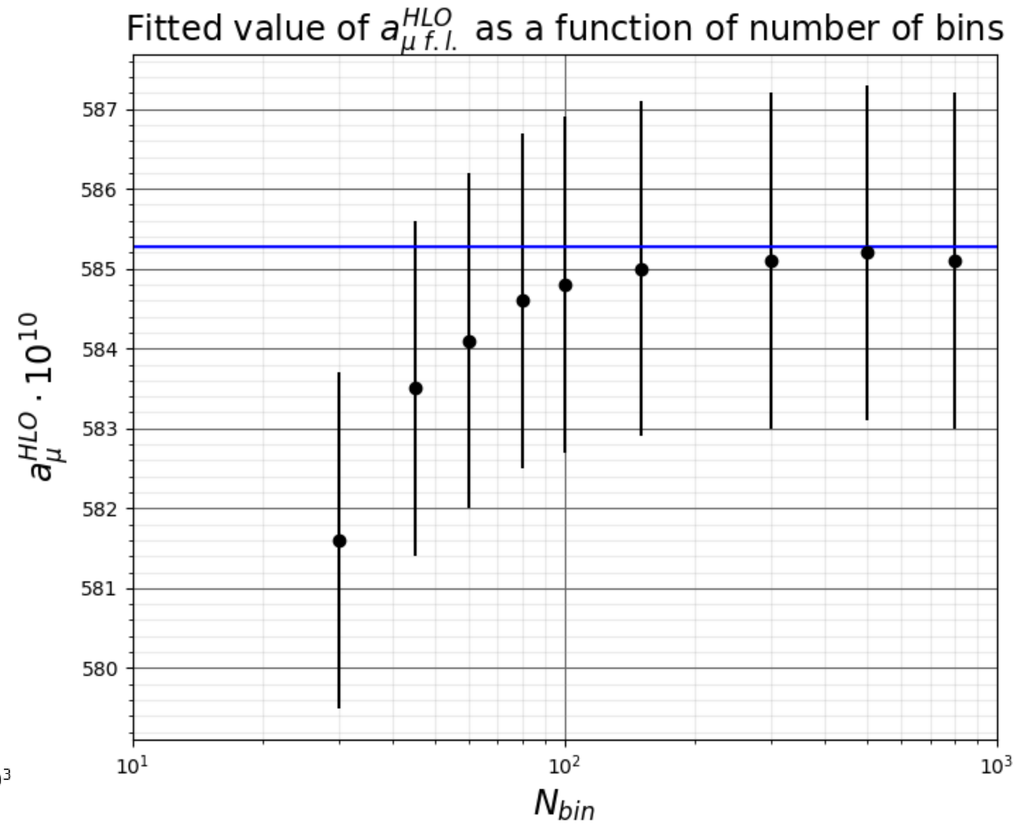
Dependence of a_μ^{HLO} from the number of bins

$a_{\mu\,theo}^{HLO} = 585{,}2\ 10^{-10}$

Polynomial parameterization



Fermion like parameterization



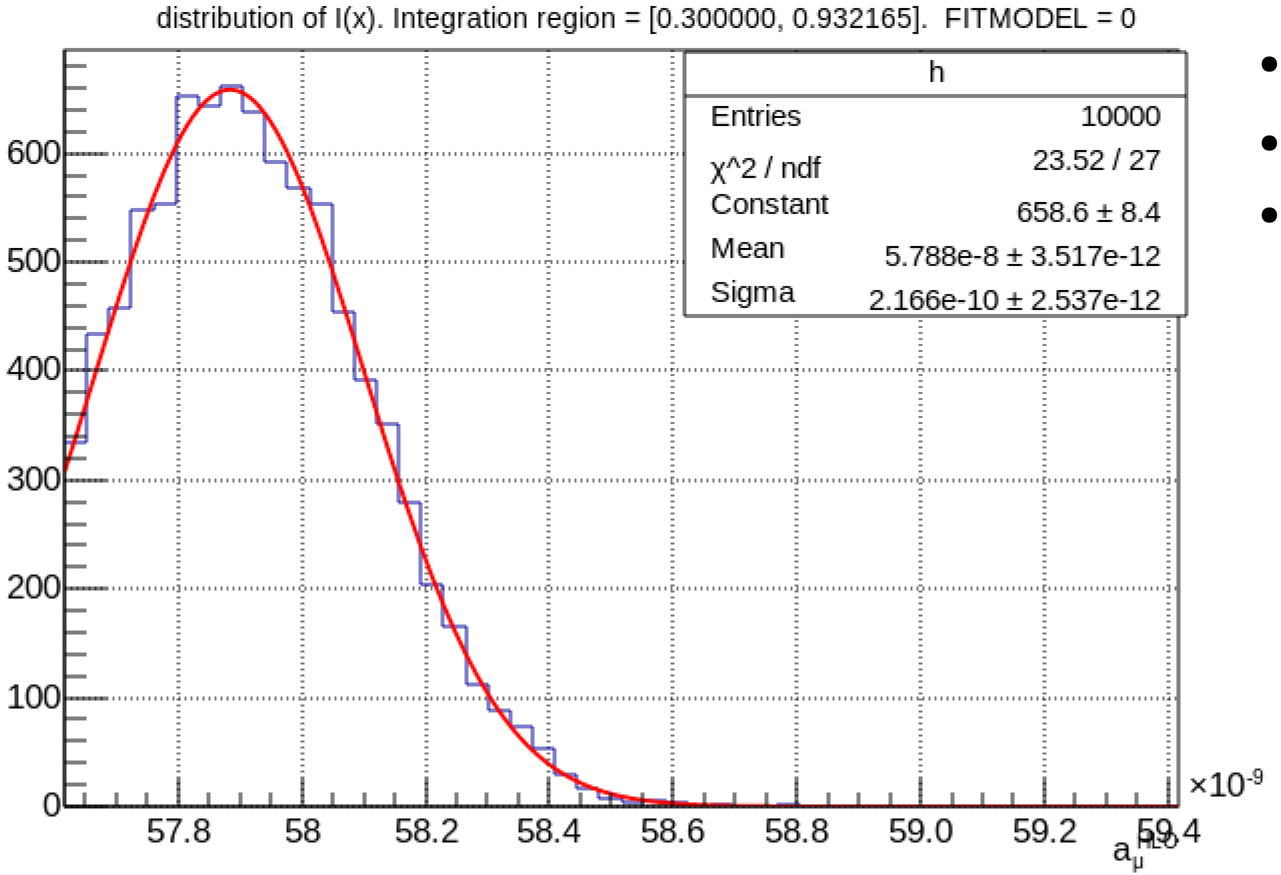
For $N_{bin} > 100$ the value of a_μ^{HLO} is constant: $a_\mu^{HLO} = (\ 585{,}1 \pm 2{,}1)\ 10^{-10}$

Conclusions

- In order to have an agreement $< 10^{-6}$ between differential and integrated cross sections ratio it is necessary define t_i as the mean value on th bin Δt_i weighted with the cross section.
- The full statistics available allows to get a ~0,36% precision on a_μ^{HLO} calculated in the signal region.
- The results obtained with two $\Delta\alpha_{had}$ parameterizations used are in agreement
- The fermion-like parameterization of $\Delta\alpha_{had}$ allows to integrate the master formula in the entire x range.
- The results obtained extracting $\Delta\alpha_{had}$ from pseudodata are consistent with the previous extraction from $\Delta\alpha$
- We will now repeat the systematic studies as done on $\Delta\alpha$
- We will move to NLO

BACKUP

Fit of a_μ^{HLO} using as t_i the central value on the bin



- $a_\mu^{HLO}{}_{theo} = 585{,}3\ 10^{-10}$
- $a_\mu^{HLO} = (\ 578{,}8 \pm 2{,}2)\ 10^{-10}$
- 0,37% precision

Not consistent

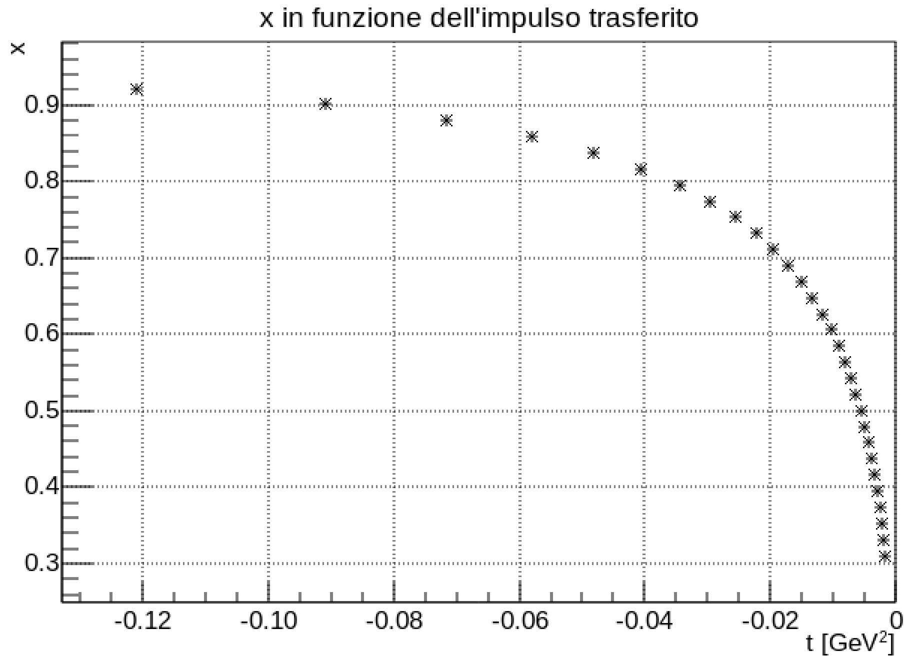
30 equispaced points of

$x_i \in [0.3, 0.932]$

$$t(x) = \frac{x^2 m_\mu^2}{x-1}$$

$t_i \in [-0{,}143, 0]\ GeV^2$

$\boxed{\Delta\alpha_i(t_i)}$



Statistical error on $\Delta\alpha_i$: $\delta\Delta\alpha_i(t_i) = \frac{1}{2\sqrt{N_i(t_i)}}$

N_i: number of events as a function of transfer momentum

$$N_i(t_i) = \int_{\Delta t_i} \frac{d\sigma^{VP}}{dt} dt \cdot L$$

$L = 1{,}5\ 10^7\ nb^{-1}$