Twisted tori and flux backgrounds of type II and heterotic string theory

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arXiv:0903.0633 by D. A., R. Minasian, M. Petrini
arXiv:0911.**** by D. A., E. Goi, R. Minasian, M. Petrini

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  $\mathcal{M}_{\text{internal}}$ preserving at least $\mathcal{N}=1$ are Generalized CY (GCY)
  hep-th/0406137, hep-th/0505212 by M. Graña, R. Minasian, M. Petrini, A. Tomasiello

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  Relations are: Dualities? Solution generating?

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  - in type II on nilmanifolds, solvmanifolds,
  - in heterotic string.

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Given $\mathfrak{g}$, G, here: $\exists$ a lattice.

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- Nilpotent cases: nilmanifold = iterated fibrations of tori.

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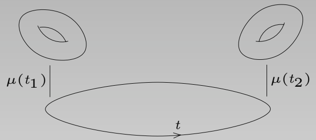

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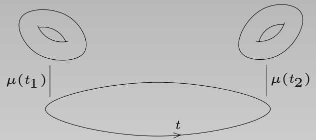

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A_N, A_i : similar formula

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$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) \ , \ A_N = A_p \dots A_1 \ .$$

$A_M = \mu(-t) = e^{-t\ ad_{\partial_t}}$, $de^i = d(e^{-t\ ad_{\partial_t}})^i{}_k \wedge dx^k = \dots = -f^i{}_{tj}\ dt \wedge e^j$.

$A_N,\ A_i$: similar formula, $A_i = \begin{pmatrix} 1 & 0 \\ \mathcal{A}_i(x_{\mathcal{B}^i}) & 1 \end{pmatrix}$.

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Two solutions can be related...

Relating solutions

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Sources, θ	$O3\ ,\ \theta = \frac{\pi}{2}$	$O5\ //\ \mathcal{F}\ ,\ \theta = 0$
RR	$g_s F_5 = e^{4A} * \mathrm{d}(e^{-4A})\ ,\ (F_3)$	$g_s F_3 = -e^{-4A} * \mathrm{d}(e^{2A} J)$
NSNS	$(H = g_s * F_3)$	0
e^{ϕ}	g_s	$g_s e^{2A}$

Twist transformation?
Possible metric transformation, not for $\mathcal{B}$, but on $\mathcal{F}$.
Provide a connection α, $\theta_c^+ = -\frac{\pi}{2}$. B-transform if needed...
$\hookrightarrow$ Twist related ✓
T-duals only for very specific $H \neq 0$.

Explicit non-trivial fibration solutions?$\Rightarrow$ among nilmanifolds .
hep-th/0609124 by M. Graña, R. Minasian, M. Petrini, A. Tomasiello
Another solution on $S^1 \hookrightarrow \mathcal{M} \rightarrow (T^2 \hookrightarrow \mathcal{M}_1 \rightarrow T^3)$.
Obtained by a twist from $(T^2 \hookrightarrow \mathcal{M}_1 \rightarrow T^3) \times S^1$!
Iterated fibration. No T^6 T-dual.

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Solutions on two six-dim. almost abelian solvmanifolds:

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David ANDRIOT

- Introduction
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- Relating solutions
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Solutions not T-dual to T^6.
First IIB solution related to T^6 by the twist transformation.

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