BACKGROUND EXPERIENCE AND THEORY INNOVATION FOR LHC

Babis Anastasiou
ETH Zurich

PADOVA
20-01-2010

A theory revolution

- Deeper understanding of the structure of gauge theories
- Sharp theoretical predictions for collider experiments
- A new technical revolution and a pace of progress to be very proud of

Highlights

- Revolutionary new methods for one-loop calculations and a promise for precise multi-particle production cross-sections at the Tevatron and the LHC
- Impressive progress on NNLO methods which has lead to precision phenomenology for LEP, HERA, Tevatron and the LHC

One-loop amplitudes

final states with many particles at the LHC

JET ALGORITHMS

NNLO theory

Jet physics at LEP, strong coupling

DIS at HERA

PDFs for the Tevatron and the LHC

Drell-Yan and Higgs @ Tevatron/LHC

The string connection

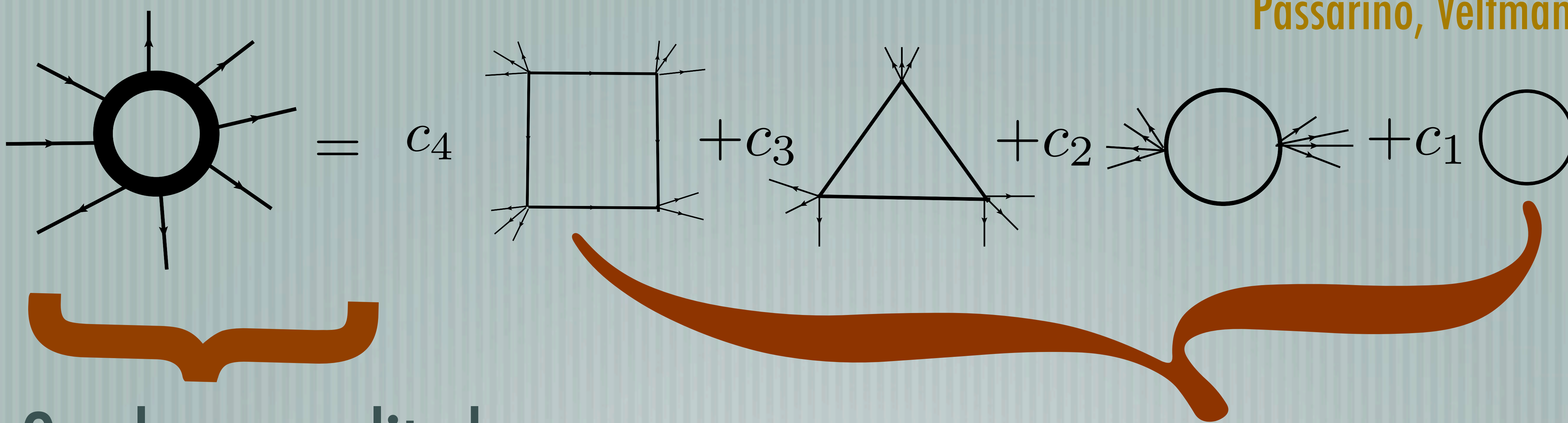
One-loop amplitudes from trees... and masters!!!

Trees in Gauge theory

Loop Master Integrals in scalar field theory

Master Integrals

Passarino, Veltman *1980s*



One-loop amplitude
in Gauge theory

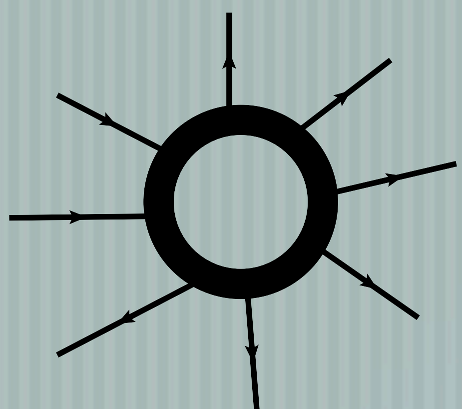
Integrals in scalar field theory

Known method(s) to compute a,b,c,d coefficients
had a (# Legs)! computational cost

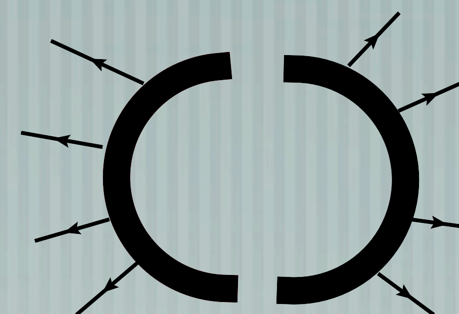
Unitarity

Bern, Dixon, Dunbar, Kosower *1990s*

$$\mathit{Tree} \times \mathit{Tree}$$



$$\approx \int \frac{d^d k}{k^2 (k+p)^2}$$



- Trees as input for the integrand
- Manifest gauge invariance cancelations
- Simplifications by using "natural" spinor variables

- Mismatch between Trees in four dimensions and loop integration in D-dimensions
- Introduction of four dimensional helicity regularization scheme
- Clever theory input (collinear factorization) to recover the full one-loop amplitude

Trees were an essential ingredient. No explicit connection of master integral coefficients to tree amplitudes.

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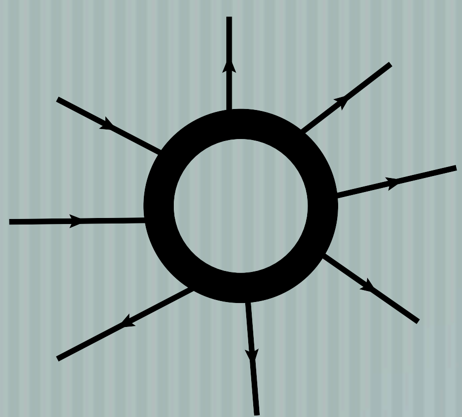

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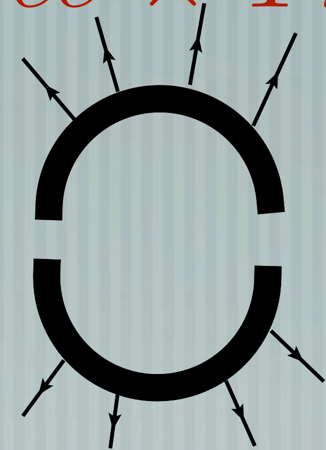
$Tree \times Tree$



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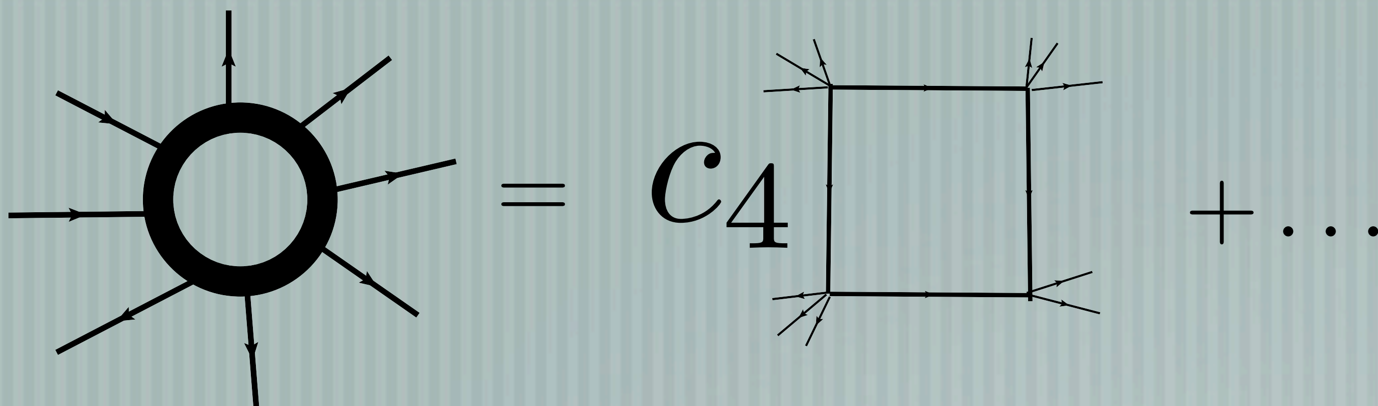
Coefficient of box master !



$c_4 =$

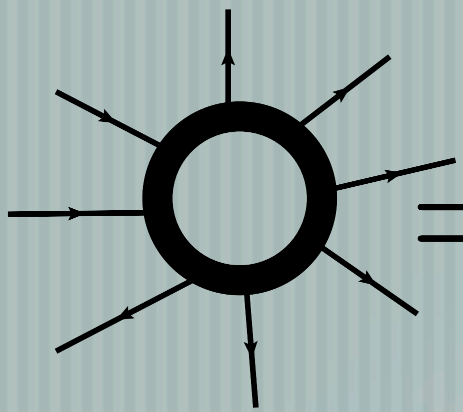
Britto, Cachazo, Feng 2004

- Simple product of four tree amplitudes
- Evaluated at complex momenta
- corresponding to loop momentum values where all propagators of the box master integral are ON-SHELL

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006

(building on del Aguila, Pittau, 2004)



$$= \int \frac{d^d k}{(2\pi)^d} \Big[c_4\, f_4(\vec{k}) + c_3\, f_3(\vec{k}) + c_2\, f_2(\vec{k}) + c_1\, f_1(\vec{k})$$

$$+ \tilde{c}_4\, \tilde{f}_4(\vec{k}) + \tilde{c}_3\, \tilde{f}_3(\vec{k}) + \tilde{c}_2\, \tilde{f}_2(\vec{k}) + \tilde{c}_1\, \tilde{f}_1(\vec{k}) \Big]$$

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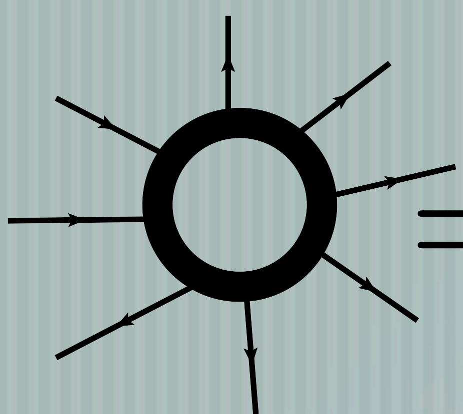

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After Integration:

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Ossola, Papadopoulos, Pittau 2006

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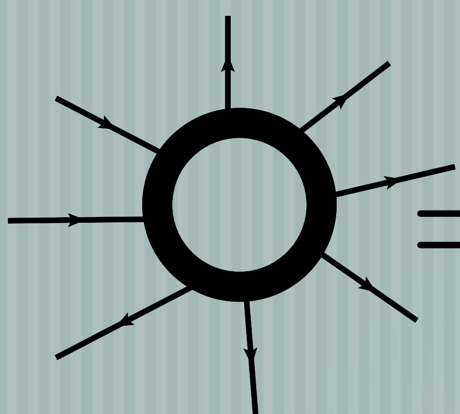

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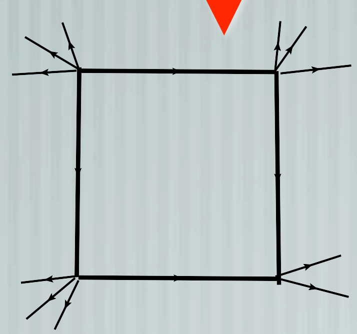
After Integration:

$$= \; c_4$$



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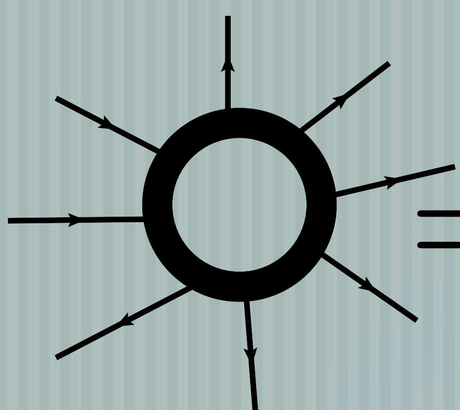

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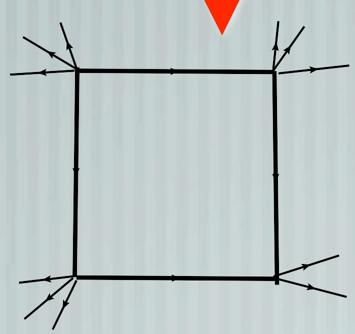
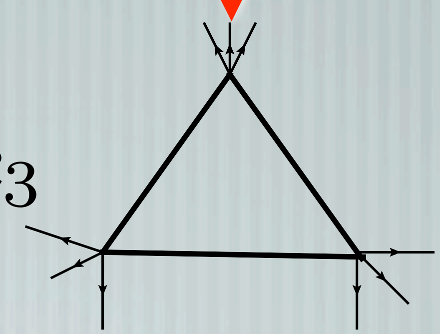
$= \; c_4$  $+ c_3$ 

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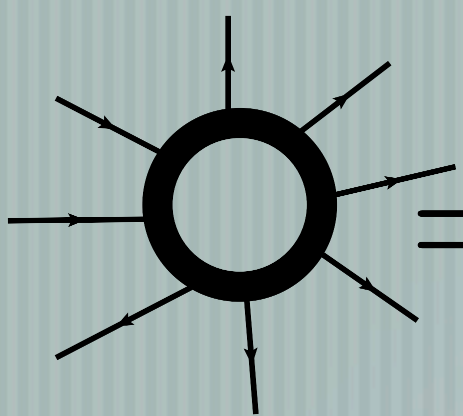

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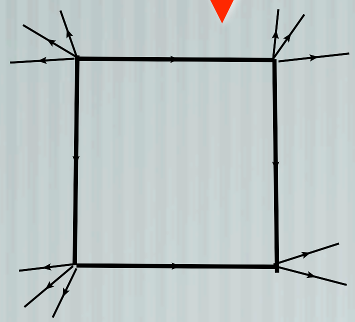
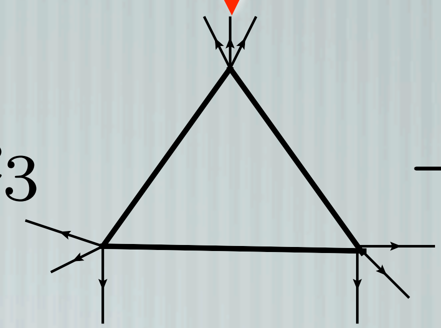
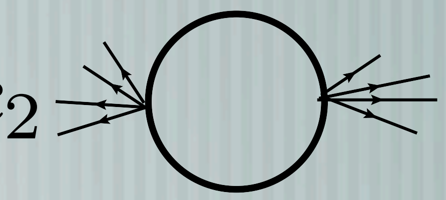
$= c_4$  $+ c_3$  $+ c_2$ 

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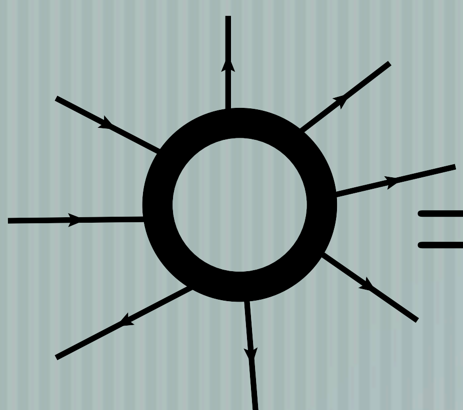

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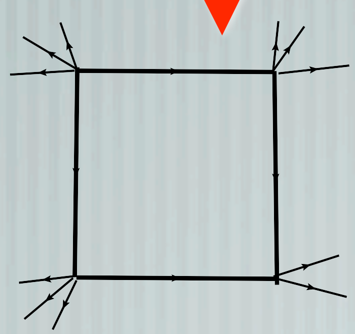
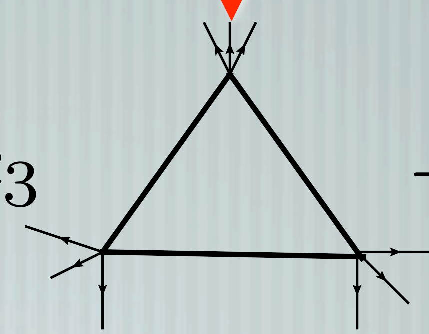
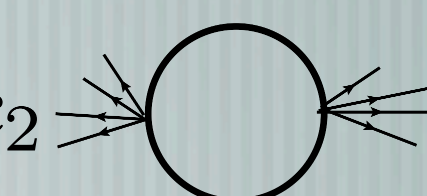
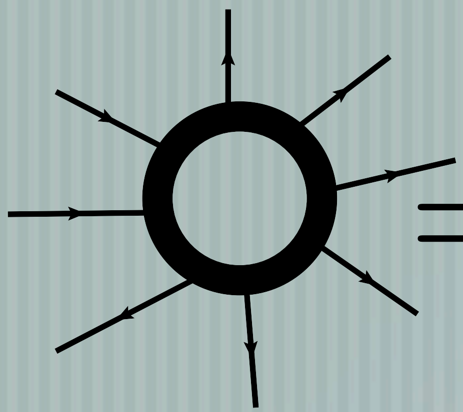
$= \; c_4$  $+c_3$  $+c_2$  $+c_1$ 

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006



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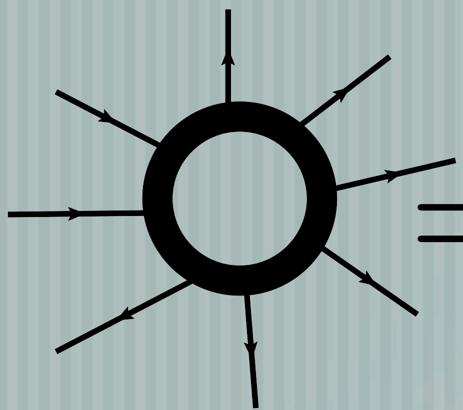

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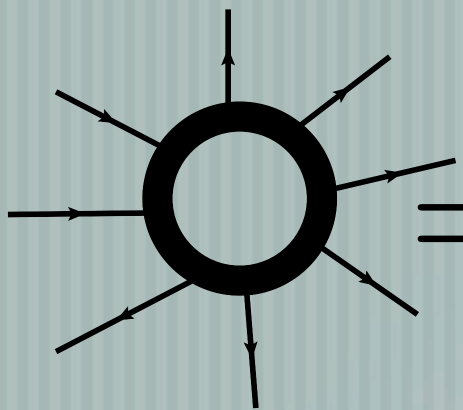
$\tilde{f}_i(\vec{k}), f_i(\vec{k})$: Known rational functions of the loop momentum

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006



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$\tilde{f}_i(\vec{k}), f_i(\vec{k})$: Known rational functions of the loop momentum

$\tilde{c}_i, c_i$: coefficients can be determined algebraically computing the integrand at a sufficient number of values for $\vec{k}$

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^d k}{(2\pi)^d} \Big[c_4\, f_4(\vec{k}) + c_3\, f_3(\vec{k}) + c_2\, f_2(\vec{k}) + c_1\, f_1(\vec{k})$$

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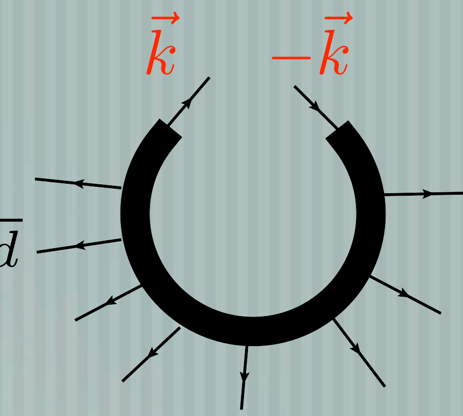

Integrand is "easy", essentially a tree amplitude

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006

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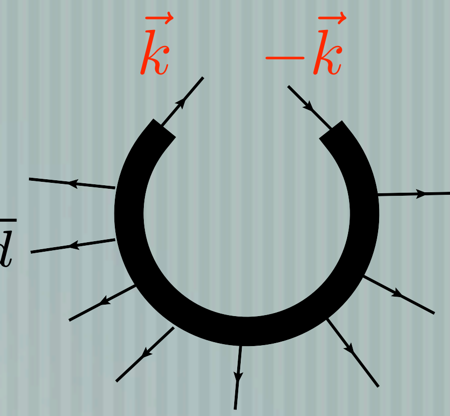


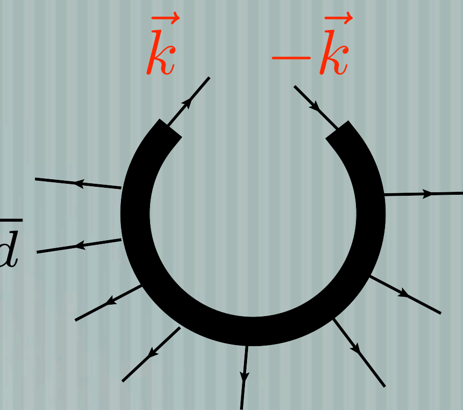
☑ Integrand is “easy”, essentially a tree amplitude

Evaluate integrand at loop momenta values such as loop particles are set ON SHELL

ONE-LOOP INTEGRAND

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^dk}{(2\pi)^d} \left[c_4\, f_4(\vec{k}) + c_3\, f_3(\vec{k}) + c_2\, f_2(\vec{k}) + c_1\, f_1(\vec{k}) + \tilde{c}_4\, \tilde{f}_4(\vec{k}) + \tilde{c}_3\, \tilde{f}_3(\vec{k}) + \tilde{c}_2\, \tilde{f}_2(\vec{k}) + \tilde{c}_1\, \tilde{f}_1(\vec{k}) \right] = \int \frac{d^dk}{(2\pi)^d}$$



- ☑ Integrand is “easy”, essentially a tree amplitude

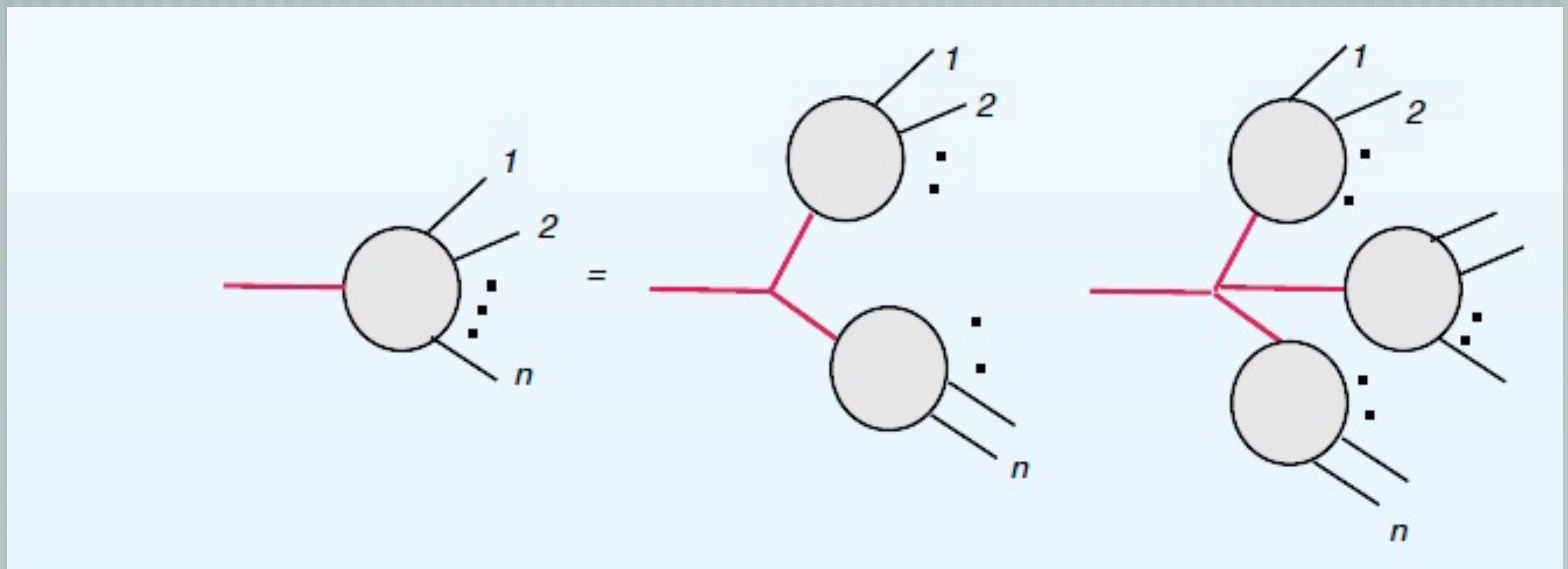
Evaluate integrand at loop momenta values such as loop particles are set ON SHELL

- ☑ ON-SHELL: determines coefficients successively

Coefficients as tree products

Ellis, Giele, Kunszt 2007

- ON-SHELL loop propagators = Product of tree amplitudes
- Evaluation of trees with powerful recursive methods



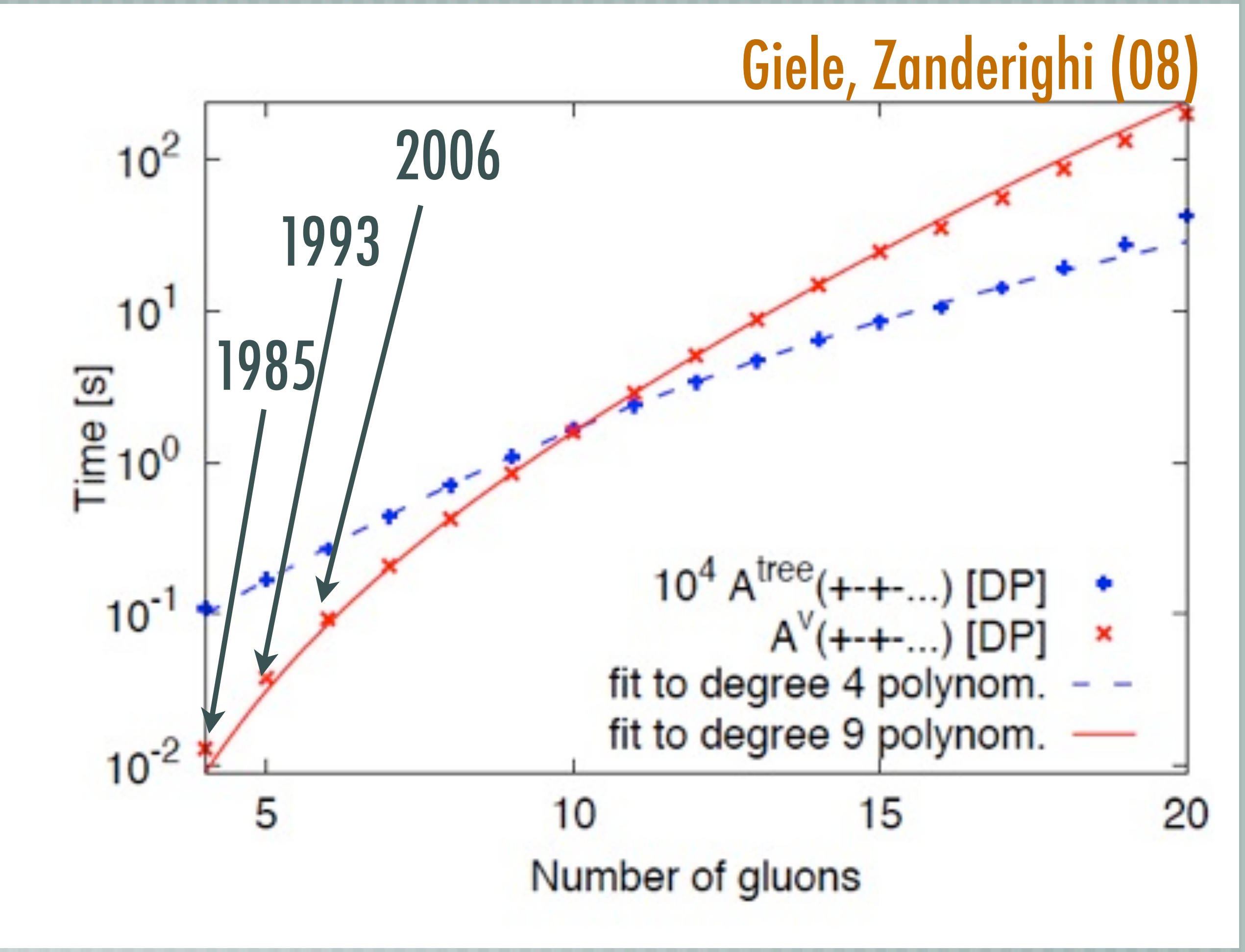
e.g. Berends-Giele, Britto-Cachazo-Feng-Witten, etc

Conflict of dimensions

- Loop Integrations in D dimensions, Tree amplitudes in four dimensions. Mismatch, i.e. missing terms from amplitude evaluation. Requires a second calculation.
  - Specialized tree-like recursions in D=4 for the missing terms
    Berger, Bern, Dixon, Forde, Kosower 2006
  - Elegant/general solution: Amplitude in a general dimension from results in D=5 and D=6. Ellis, Giele, Kunszt, Melnikov 2008
  - Specialized Feynman rules for missing terms:
    Draggiotis, Garzelli, Papadopoulos, Pittau 2009

Breathtaking developments

- One-loop amplitudes with 22 gluons Giele, Zanderighi (08); Lazopoulos (08); Giele,Winter (09)



- numerical evaluation of all 2 to 4 amplitudes in the Les-Houches 2007 wish-list van Hameren, Papadopoulos, Pittau (09)

$$q\bar{q}, gg \to t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg$$
$$q\bar{q}' \to Wggg, Zggg$$

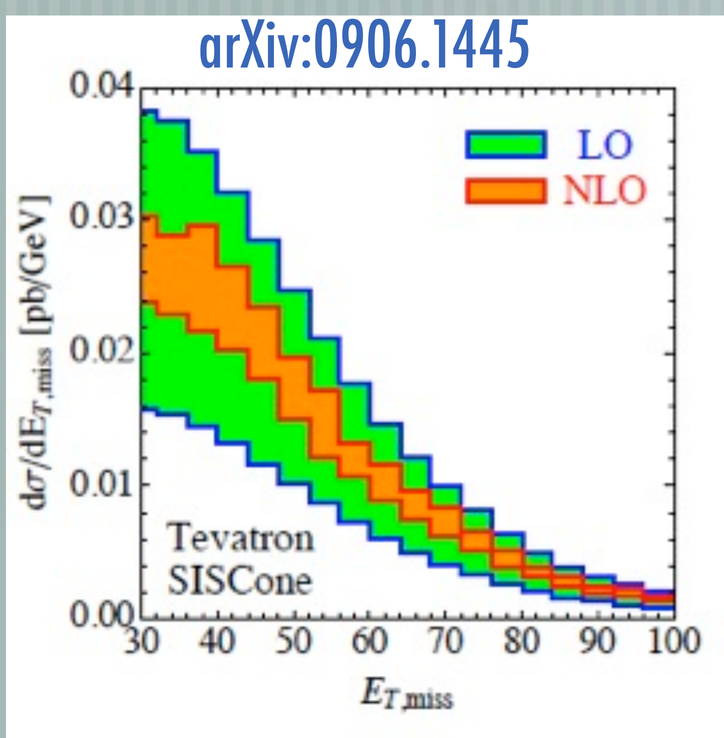
W+3 jets: NLO cross-section

Large Nc approximation

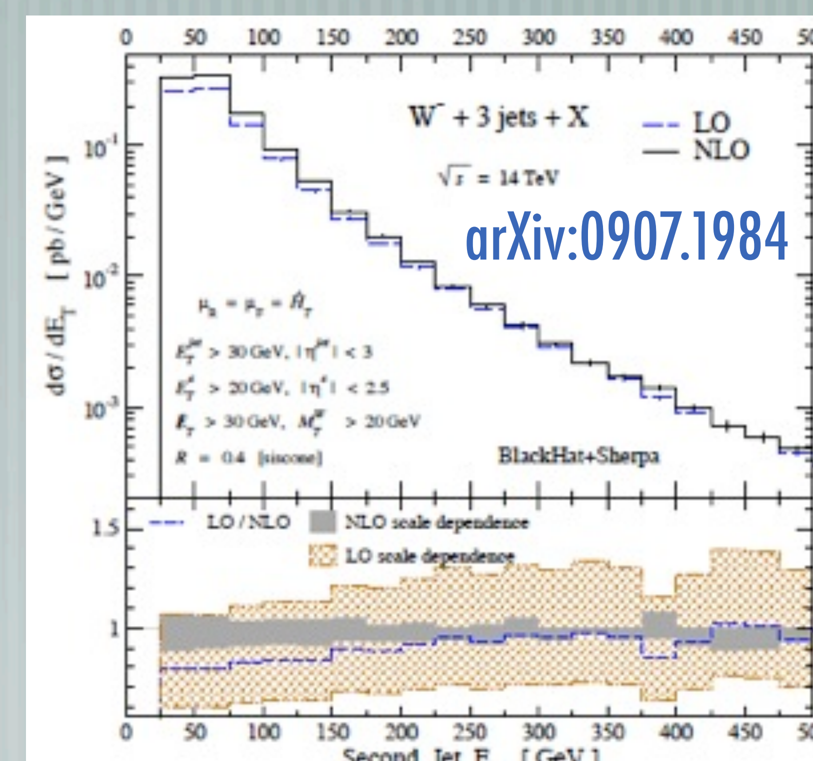
Ellis,Giele,Kunszt,Melnikov,Zanderighi;
Berger,Bern,Dixon,Cordero,Forde,
Gleisberg,Ita,Kosower,Maitre

NEW: complete NLO

Berger,Bern,Dixon,Cordero,
Forde,Gleisberg,Ita,Kosower,
Maitre (arXiv:0907.1984)



Start of a new era, with precise theoretical predictions for multi-particle production at the LHC

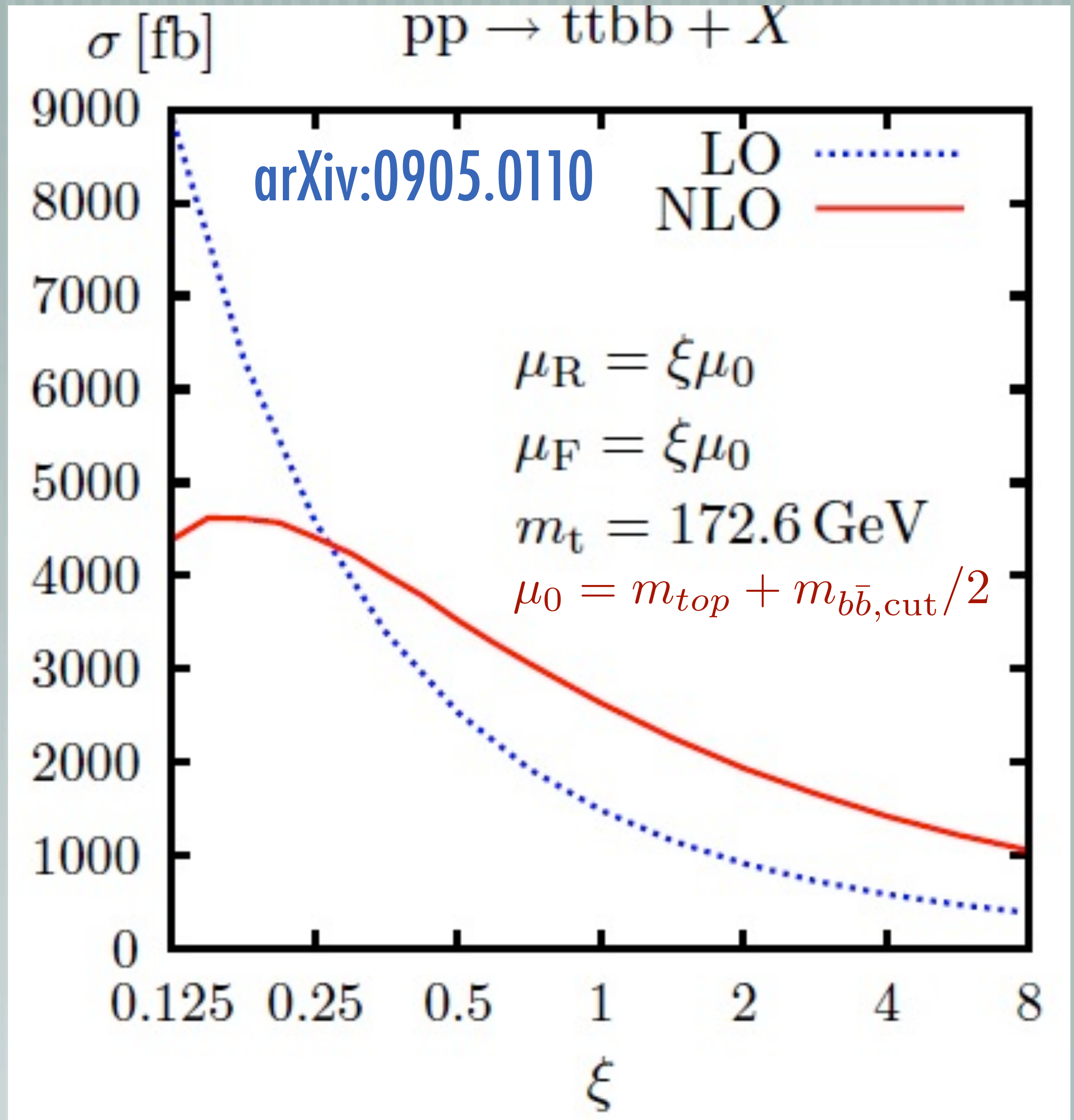


$pp \to t\bar{t}b\bar{b}$:NLO cross-section

Brendenstein,Denner,Dittmaier,Pozzorini

- First full NLO calculation for a 2 to 4 process at a hadron collider
- Important Higgs boson background
- With Feynman diagrams
- intelligent, mostly numerical reduction, to master integrals
- exploits infrared regulators other than the dimension
- And new methods

Bevilacqua,Czakon,Papadopoulos, Pittau,Worek

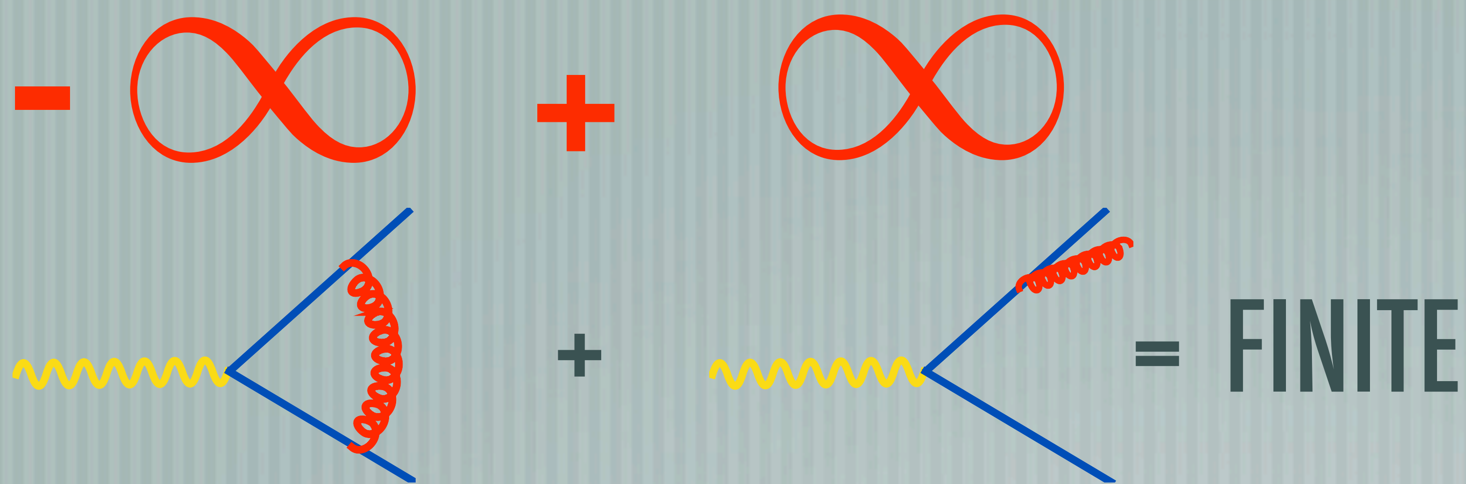


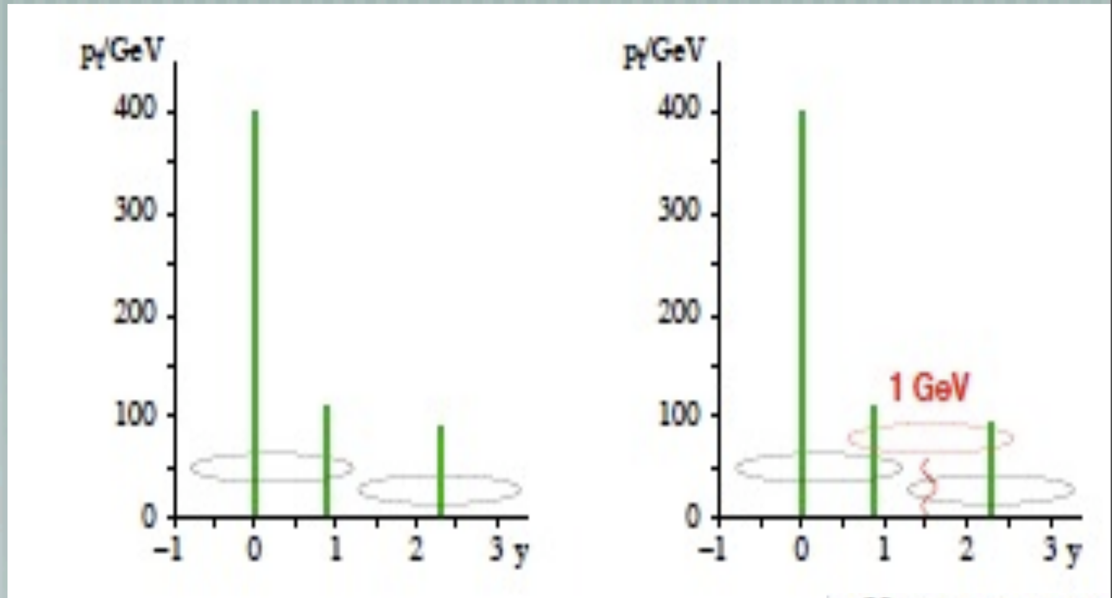
very large NLO corrections

NLO calculations @ LHC

- What can we hope for?
- We cannot do better than tree calculations..., i.e. processes with 7 or 8 particles in the final state.
- All 2 to 4 processes with both Feynman diagrammatic and unitarity methods
- 2 to 5 and perhaps 2 to 6 processes with unitarity methods

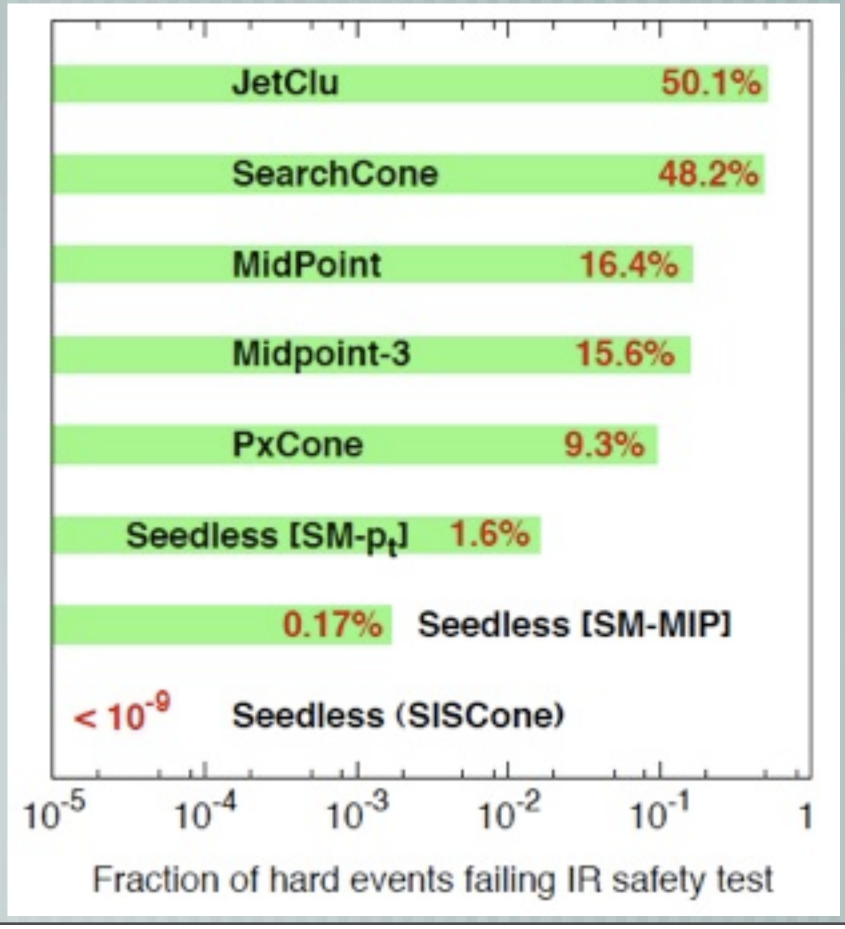
Jets and Infrared safety





arXiv:0704.0292

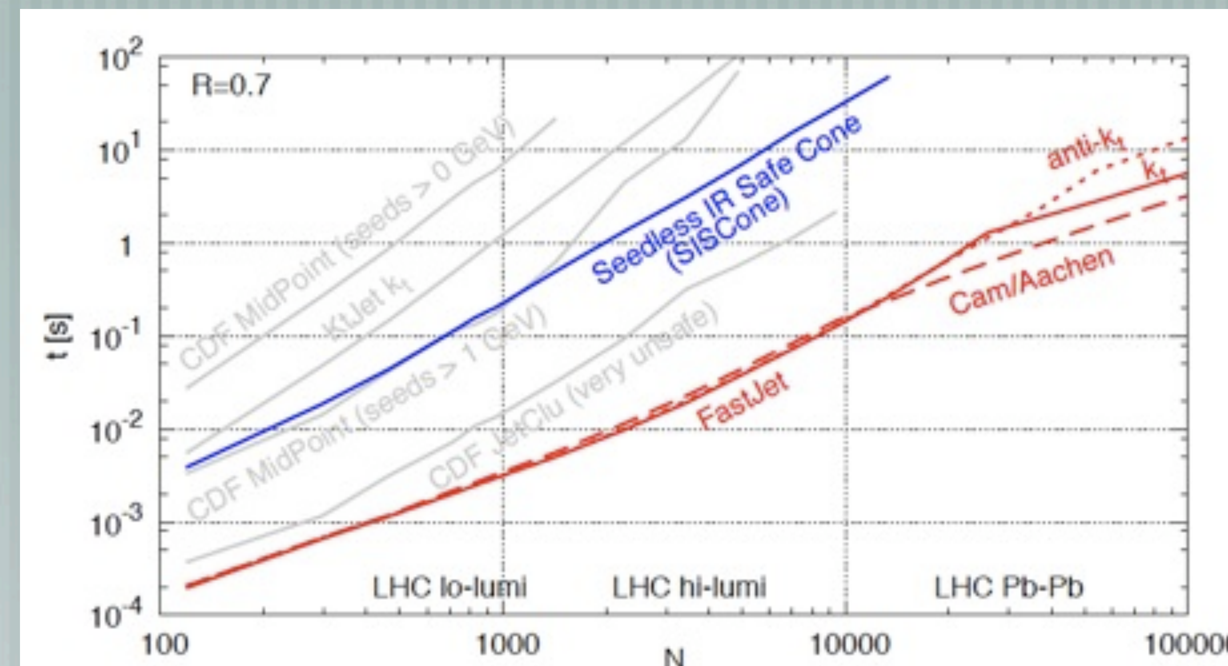
- Soft or Collinear parton emission must not alter the number of jets in an event.
- Many jet measurements are not directly comparable to perturbative calculations (e.g. W+3 jets with JETCLU @ NLO)
- To profit from NLO advances: infrared safe algorithms

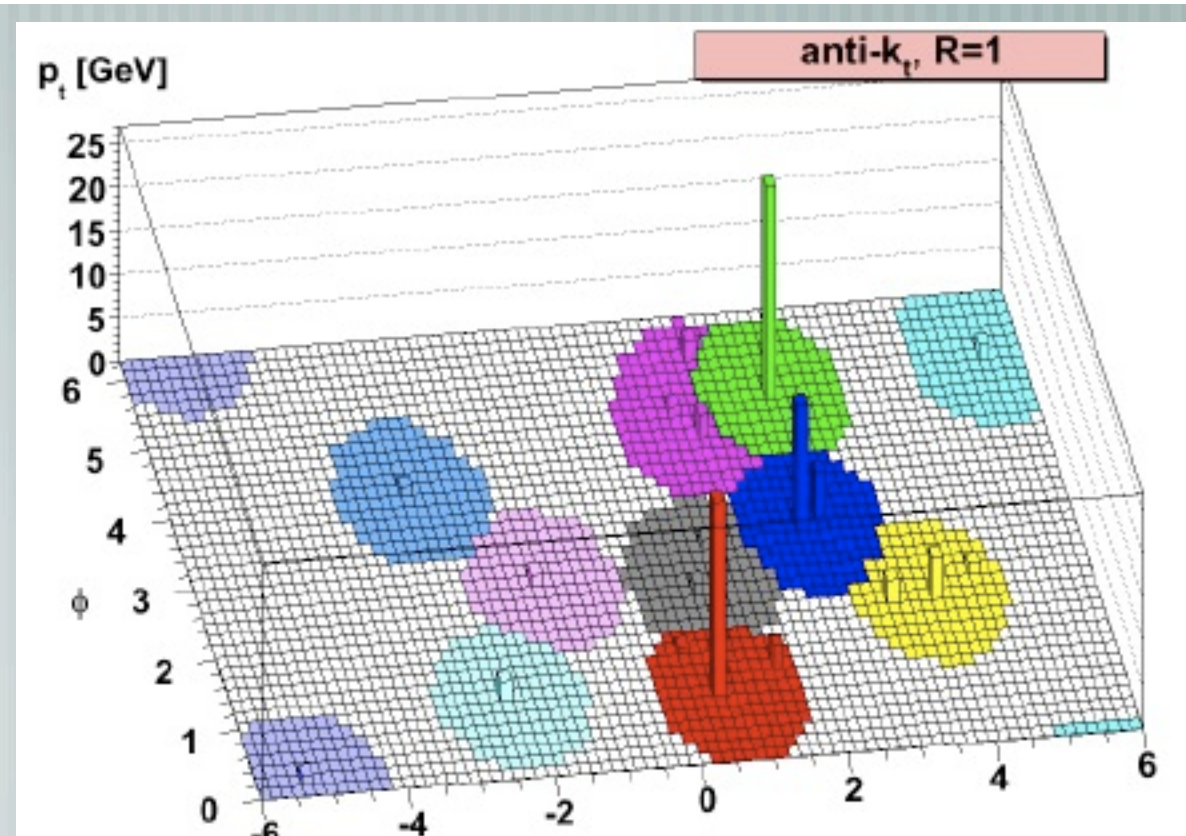


Fast and Safe Jet Finding

Cacciari, Salam, Soyez (2007-2009)

- Fast implementation of recombination algorithms
- New infrared safe cone algorithm (SISCone)
- Better understanding of jet areas
- anti-Kt: recombination algorithm with “perfect cones”

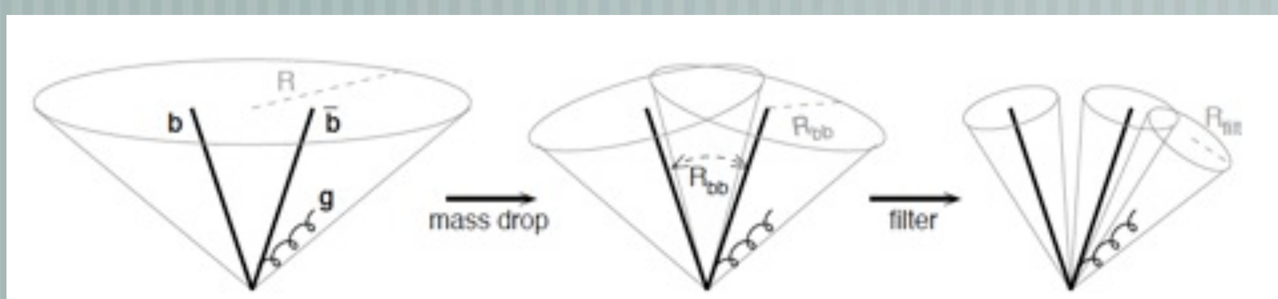


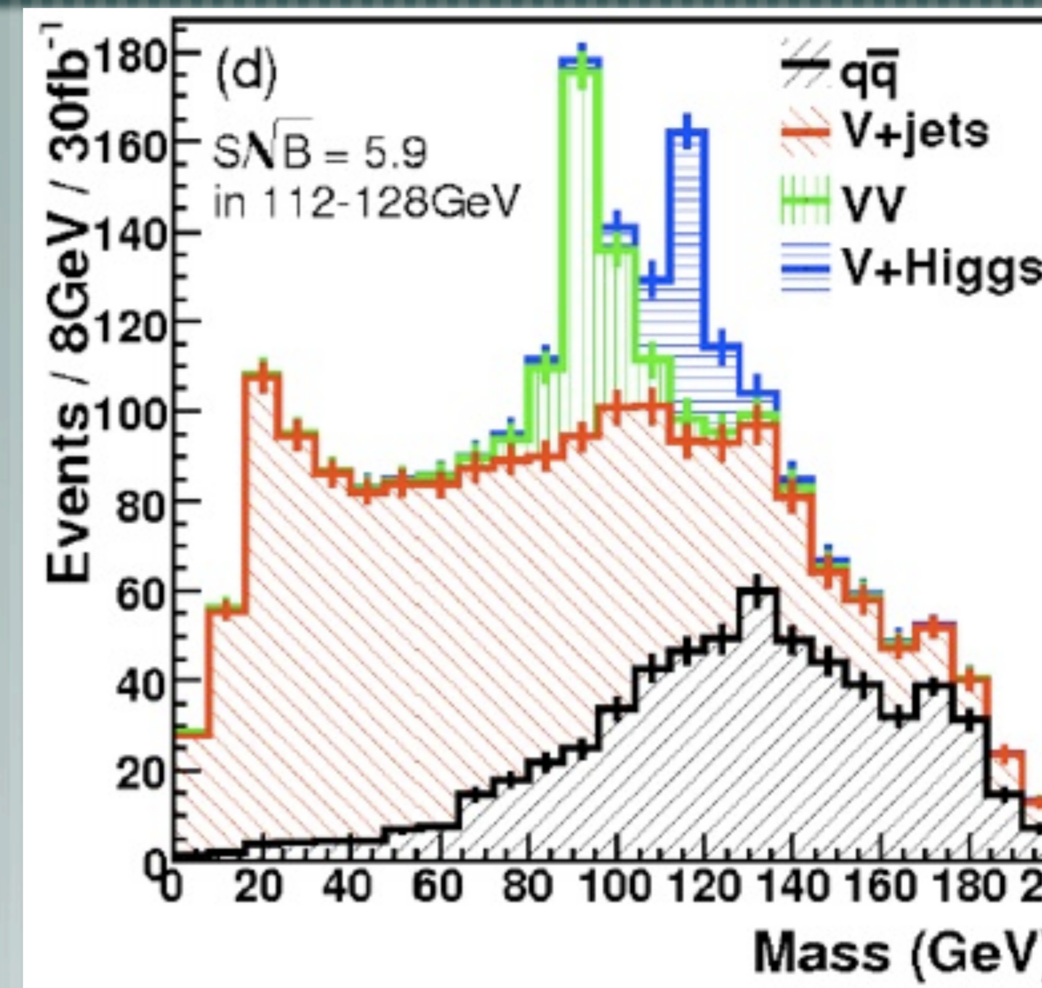


SubJets and Higgs Searches

Butterworth, Davison, Rubin, Salam (2008)

- Heavy Jet from the decay of a high pt Higgs boson has a characteristic substructure





- Jet algorithms have varied diagnostic power

$$pp \to VH \to Vb\bar{b}$$

DISCOVERY CHANNEL AT THE LHC

- Similar approach for ttH production

The NNLO front

- Precision of measurements at collider experiments is often excellent
- Perturbation theory is often slow at work, first correction after the leading order too large and too uncertain.
- All “2 to 1” and “2 to 2” hadron collider processes must be computed at NNLO.
- LEP, HERA, TEVATRON, LHC data = NNLO phenomenology

Three-jet events from LEP

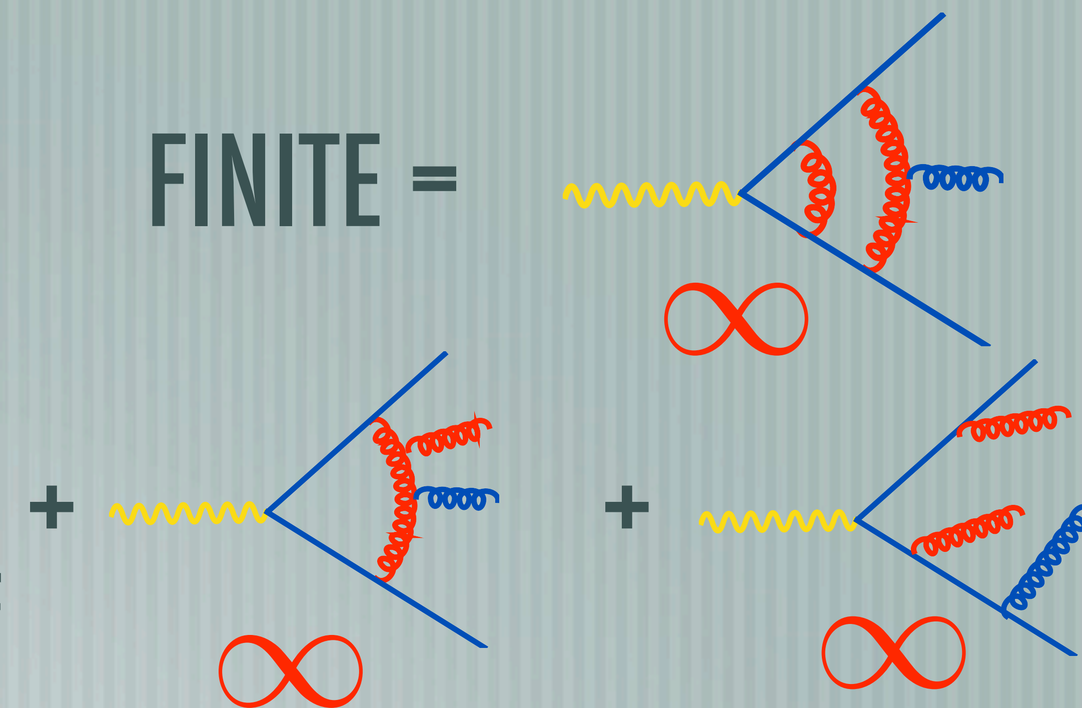
- LEP Legacy: Excellent measurements of three jet cross-sections and jet event shapes at various energies.
- Precise extraction of the strong coupling constant; largest error from theoretical prediction of the cross-section.
- NNLO corrections to $e^+e^- \to 3jets$ was the holy grail of the QCD community for more than a decade.

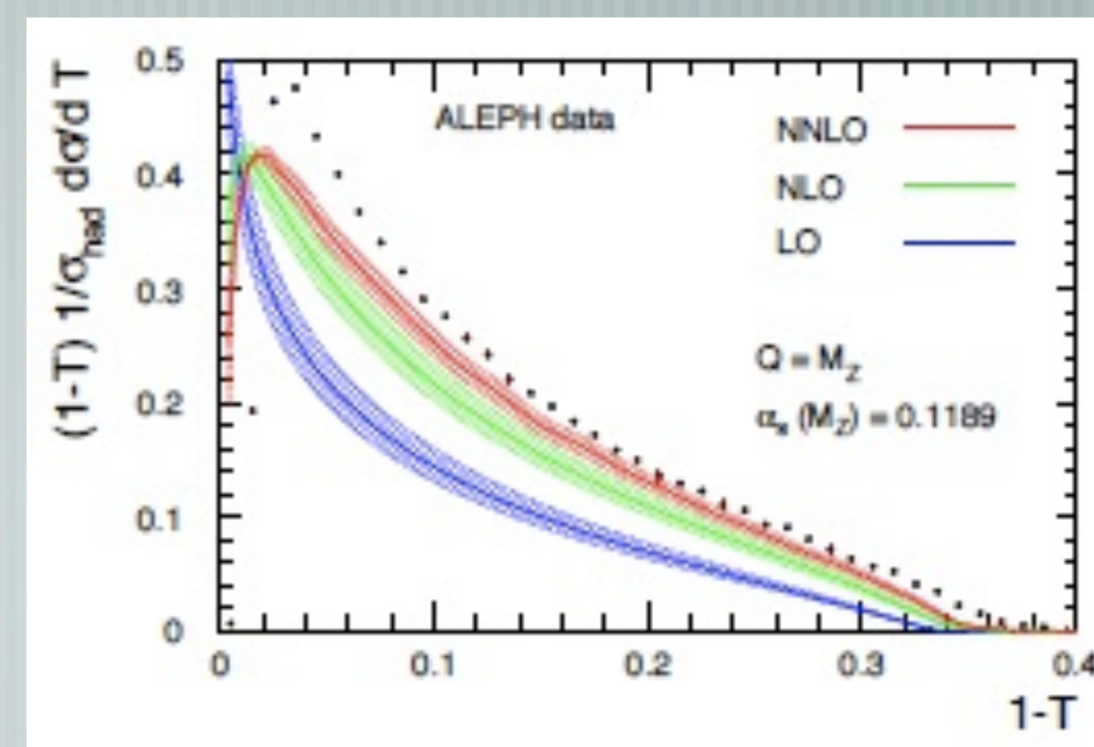
Cancelation of singularities

- Two-loop amplitude computed already in 2001 by Garland, Gehrmann, Glover, Koukoutsakis, Remiddi

- A universal method for the cancelation of matrix element singularities through NNLO for lepton collider processes by Gehrmann-de Ridder, Gehrmann, Glover, Heinrich (2007)

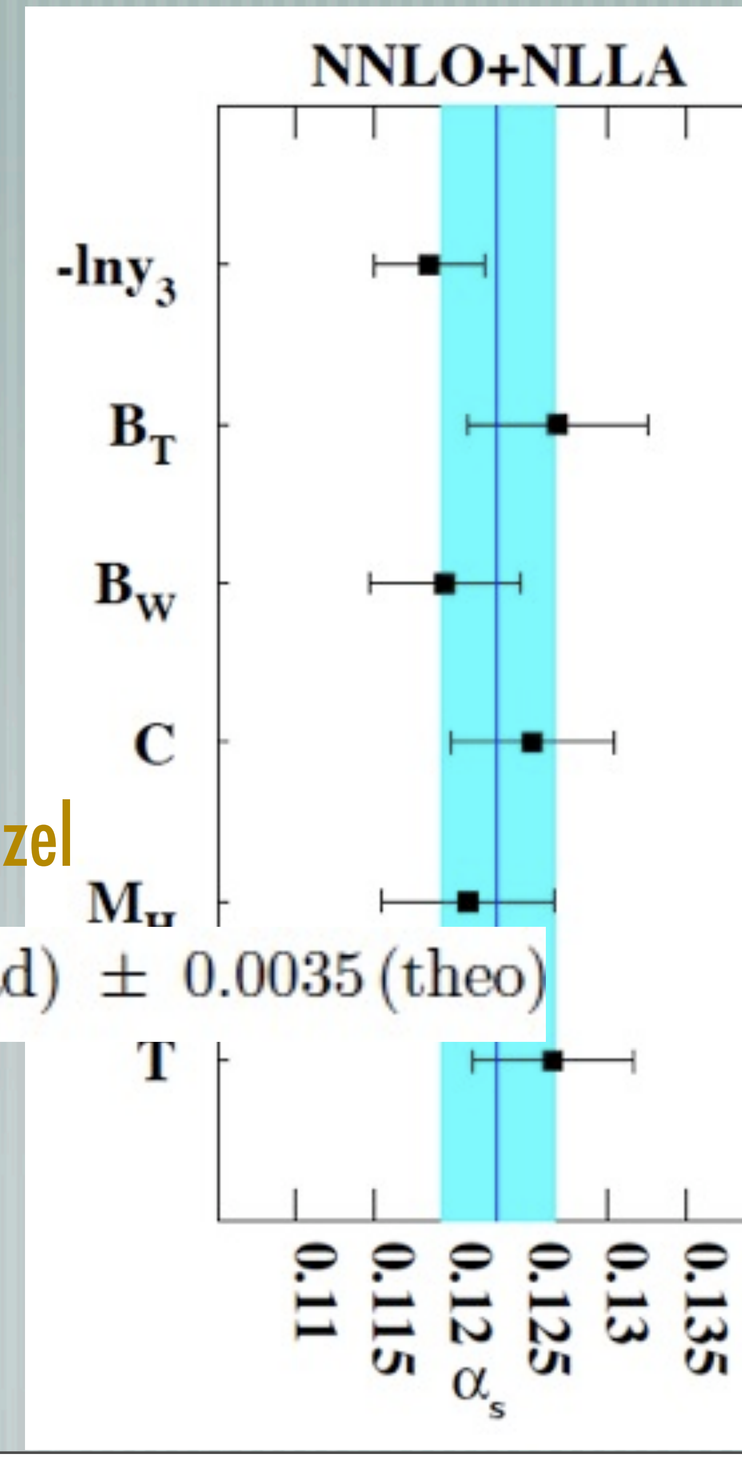
- Revision and an intricate correction by Weinzierl (2008).





α_s from jet event shapes

arXiv:0906.3436



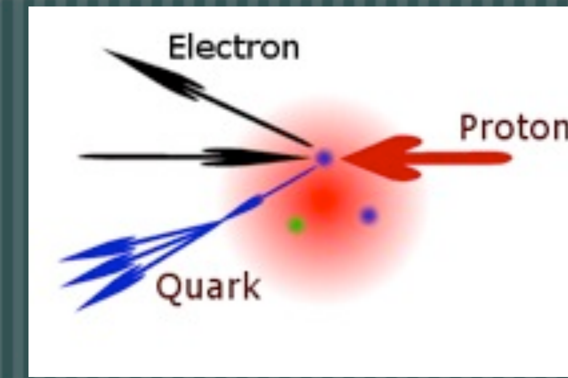
- A synthesis of fixed order QCD, Electroweak corrections , resummation, and hadronization effects describe excellently three jet events at LEP.
- State of the art extraction of alphas with the NNLO result + NLL resummation

Dissertori, Gehrmann-de Ridder, Gehrmann, Glover, Heinrich, Luisoni, Stenzel

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009\,(\text{stat}) \pm 0.0009\,(\text{exp}) \pm 0.0012\,(\text{had}) \pm 0.0035\,(\text{theo})$$

- also from NNLO+"SCET resummation" of the thrust distribution (Becher,Schwarz).

Legacy of HERA

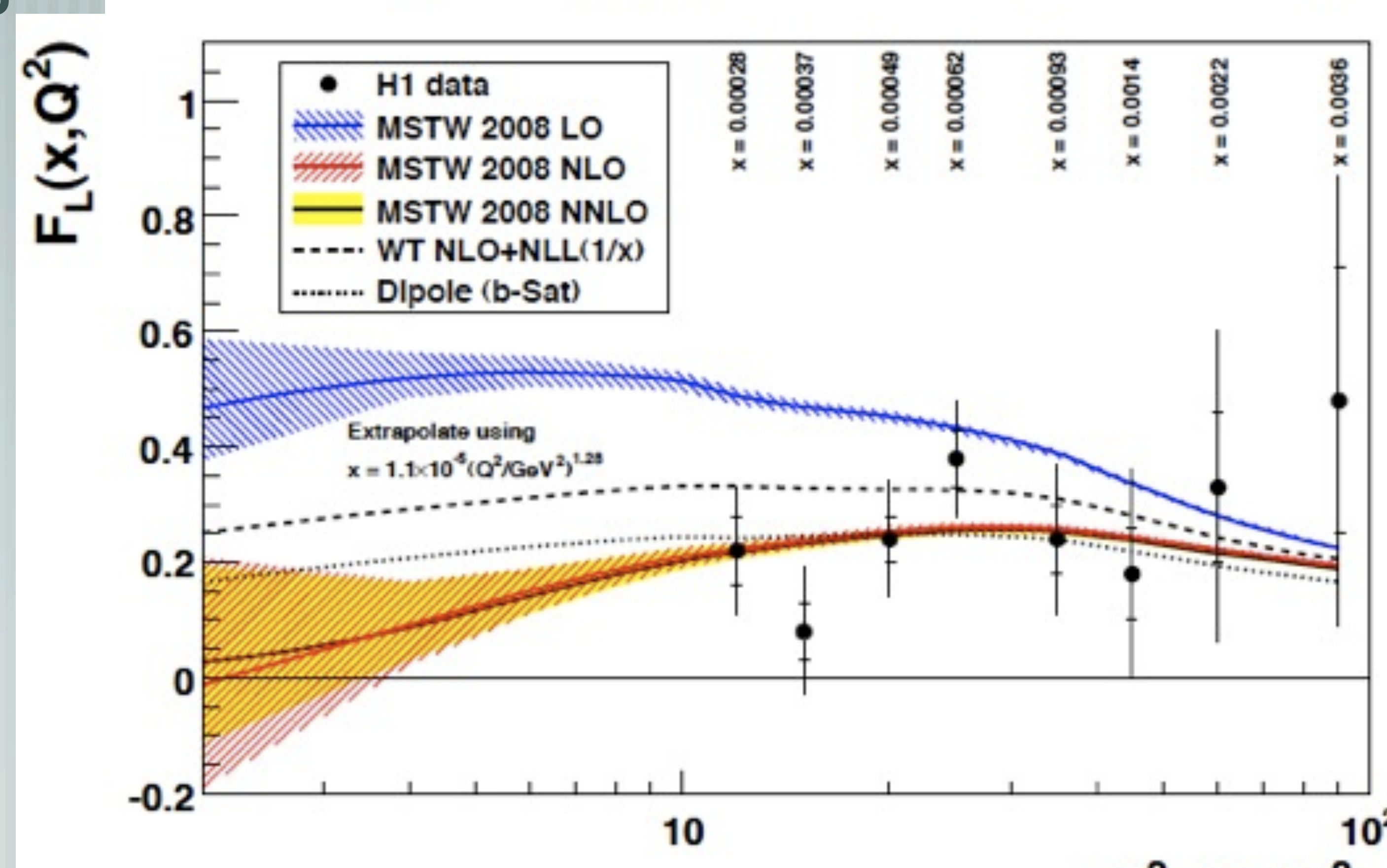


- Tremendous contributions in understanding QCD and the proton

- Altarelli-Parisi evolution kernels computed through NNLO, and structure functions through NNNLO! Moch,Vogt,Vermaseren [2004,2006,2009]

- Experimental highlight: measurement of F_L, directly sensitive to the gluon density.

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3}\left[\frac{8}{3}F_2(y,Q^2) + \frac{40}{9} y g(y,Q^2)(1-\frac{x}{y})\right]$$



Nadolsky DIS2009

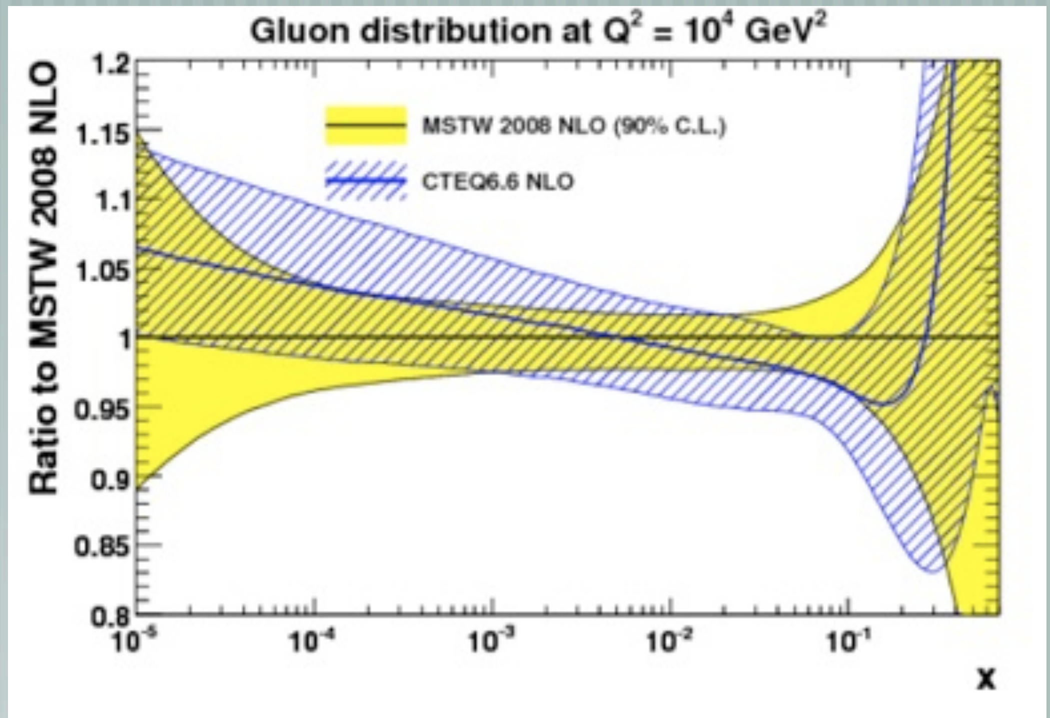
Partons @ TEVATRON/LHC

- Several efforts (CTEQ,MSTW,Alekhin,HERA colloborations) have updated parton densities: input for precise hadron collider phenomenology.
- New ideas on pdf extraction, using Artificial Neural Network methods Ball,Del Debbio,Forte,Guffanti,Latorre,Piccione,Rojo,Ubialli
- Improvements on theoretical treatment, better error estimation, but also important changes from older sets

Parton Densities

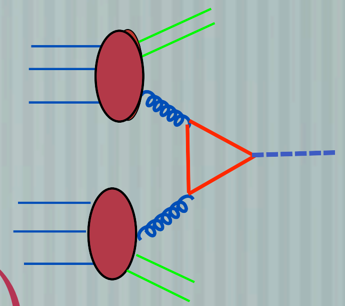
- pdf uncertainties have surprised us at times

MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444



- comparable or bigger uncertainty than scale choice

- new: estimate of alpha_s uncertainty (Martin,Stirling,Thorne, Watt)

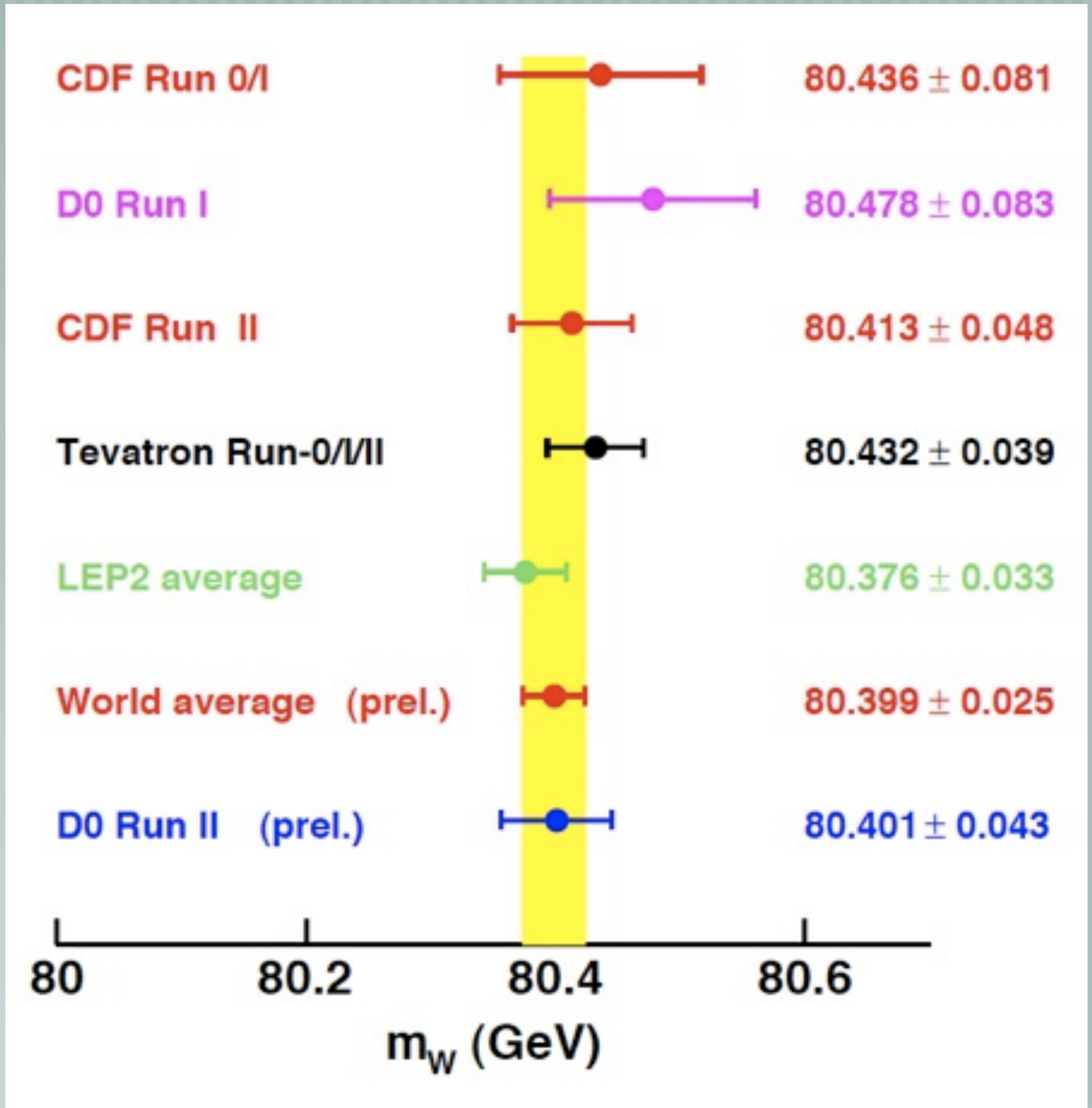


$$389.0\,\mathrm{fb}\,{}^{+8.1\%}_{-11.7\%}(\mathrm{scale})\,{}^{+13.6\%}_{-12.0\%}(\alpha_s + \mathrm{pdf})^{90\%\mathrm{CL}}$$

@ TEVATRON Mhiggs = 165 GeV

Drell-Yan

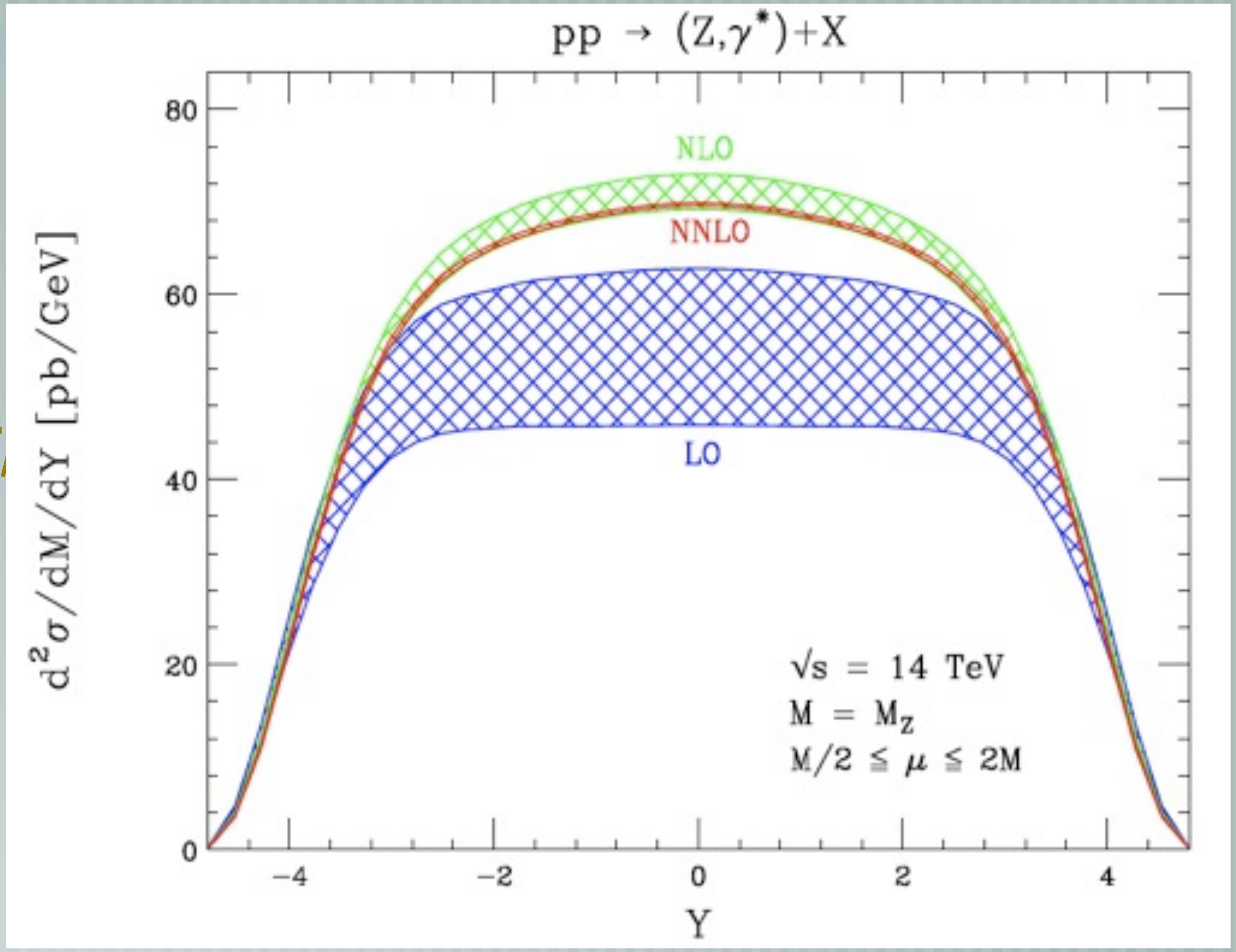
- Clean signal and high precision measurements at a hadron collider environment
- Luminosity Monitor
- Parton densities
- W-mass, Weinberg angle



NEW W-MASS MEASUREMENT FROM D0

Drell-Yan theory

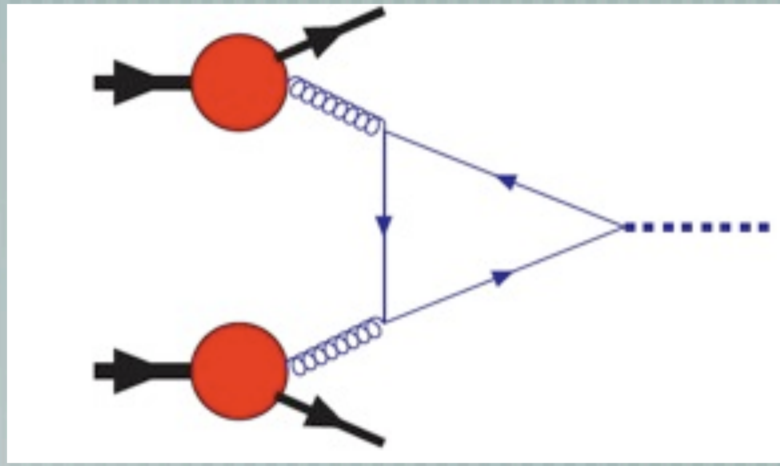
- NNLO total cross-section
  Hamberg, van Neerven 1990; Harlander, Kilgore 2002
- NNLO rapidity distribution
  CA,Dixon,Menikov,Petriello 2004
- Fully differential NNLO
  Melnikov,Petriello 2006;Catani, Cieri,Ferrera,Grazzini 2009



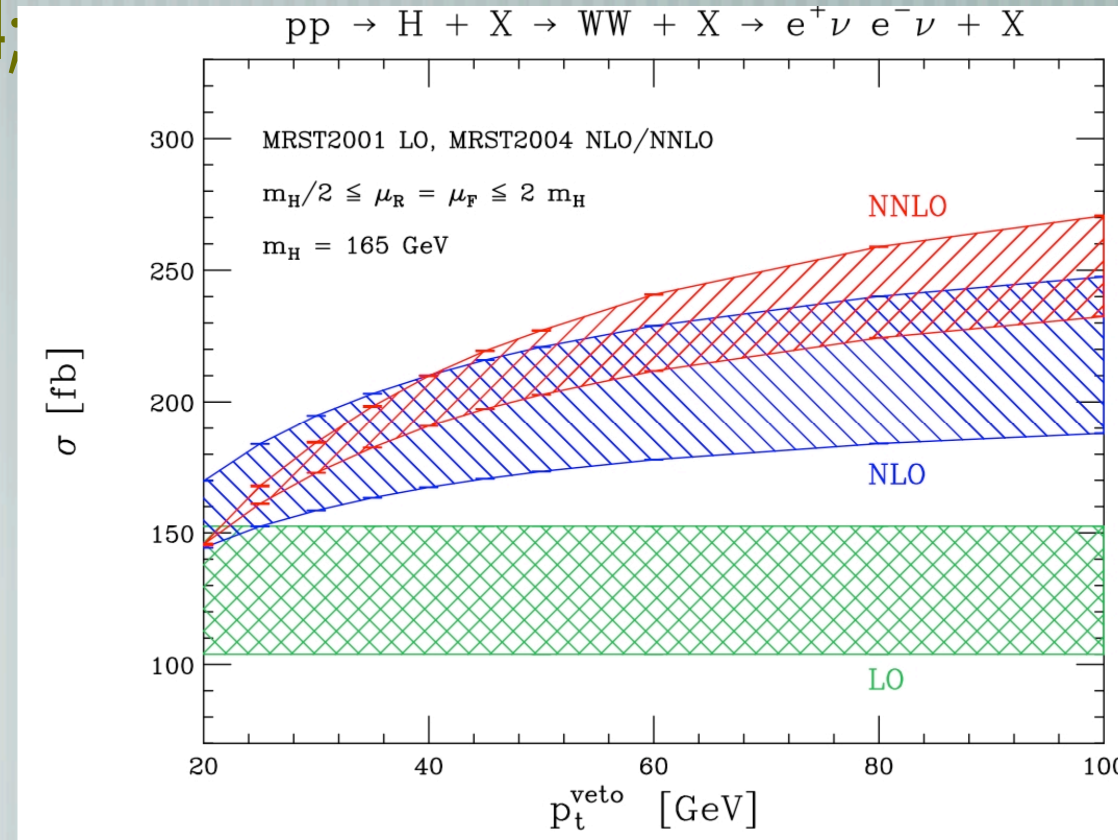
NEXT(?): W-mass measurement requires mixed QCDxQED corrections

Higgs via gluon fusion

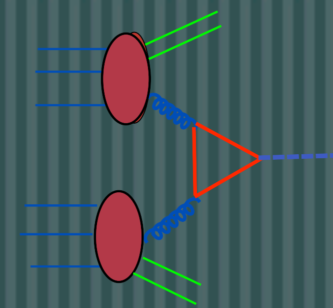
- Total [Harlander,Kilgore 02; CA,Melnikov 02; Ravindran, Smith 03] and fully differential cross-sections through NNLO [CA,Melnikov,Petriello 04; CA,Dissertori, Stockli 07; Catani,Grazzini 07]



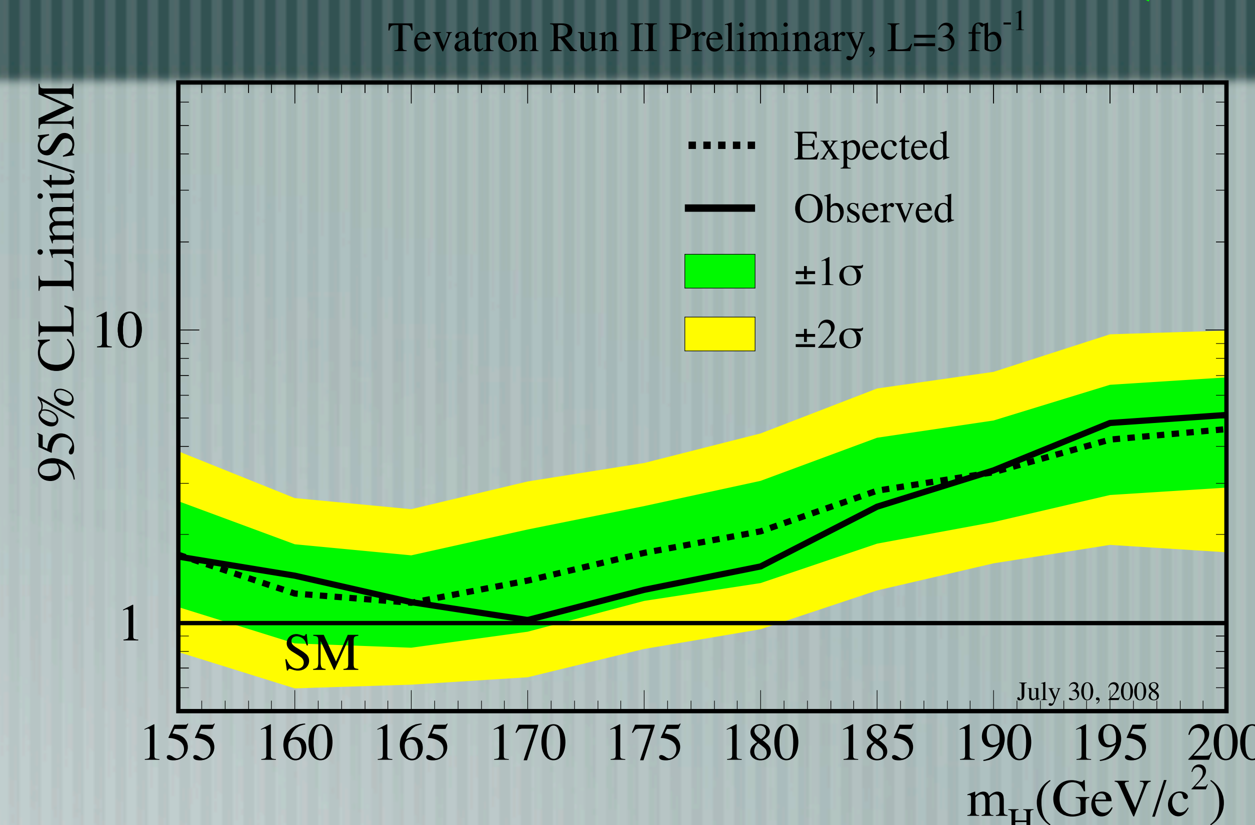
- Very large perturbative corrections, which are sensitive to selection cuts

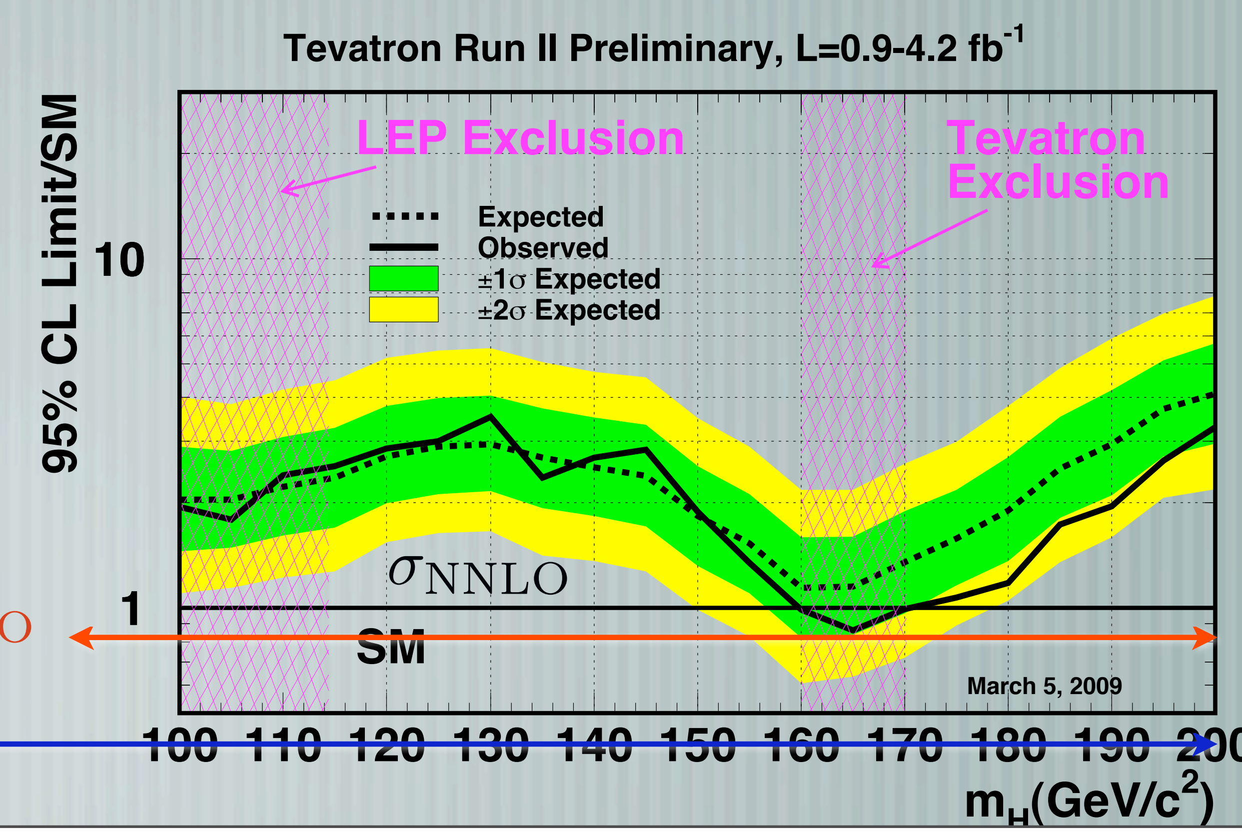


Tevatron Experience



- CDF and D0 start placing limits on higgs boson cross-sections
- Exclusion with a detailed comparison of data with signal and background distributions
- Requires incredible control over qcd effects





$\sigma_{\rm NLO} = 81\%\,\sigma_{\rm NNLO}$

$\sigma_{\rm LO} = 38\%\,\sigma_{\rm NNLO}$

Higgs signal selection

- Break up total nnlo cross-section into 0,1, and 2 jet bins (Pt,jet = 20 gev). Theory precision degrades from the 0-jet to the 1-jet and the 2-jet sample.

$$\frac{\Delta N_{\rm inc}(\text{scale})}{N_{\rm inc}} = 66.5\% \cdot \begin{pmatrix} +5\% \\ -9\% \end{pmatrix} + 28.6\% \cdot \begin{pmatrix} +24\% \\ -22\% \end{pmatrix} + 4.9\% \cdot \begin{pmatrix} +78\% \\ -41\% \end{pmatrix} = \begin{pmatrix} +14.0\% \\ -14.3\% \end{pmatrix}$$

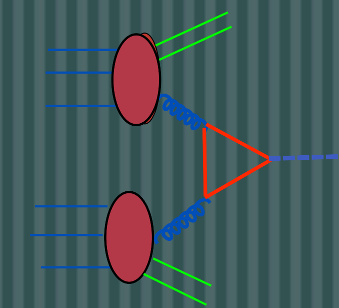
- apply slightly different e.g. lepton selections in the various jet-bins, which are more severe in the 0-jet bin.

$$\frac{\Delta N_{\rm signal}(\text{scale})}{N_{\rm signal}} = 60\% \cdot \begin{pmatrix} +5\% \\ -9\% \end{pmatrix} + 29\% \cdot \begin{pmatrix} +24\% \\ -22\% \end{pmatrix} + 11\% \cdot \begin{pmatrix} +78\% \\ -41\% \end{pmatrix} = \begin{pmatrix} +18.5\% \\ -16.3\% \end{pmatrix}$$

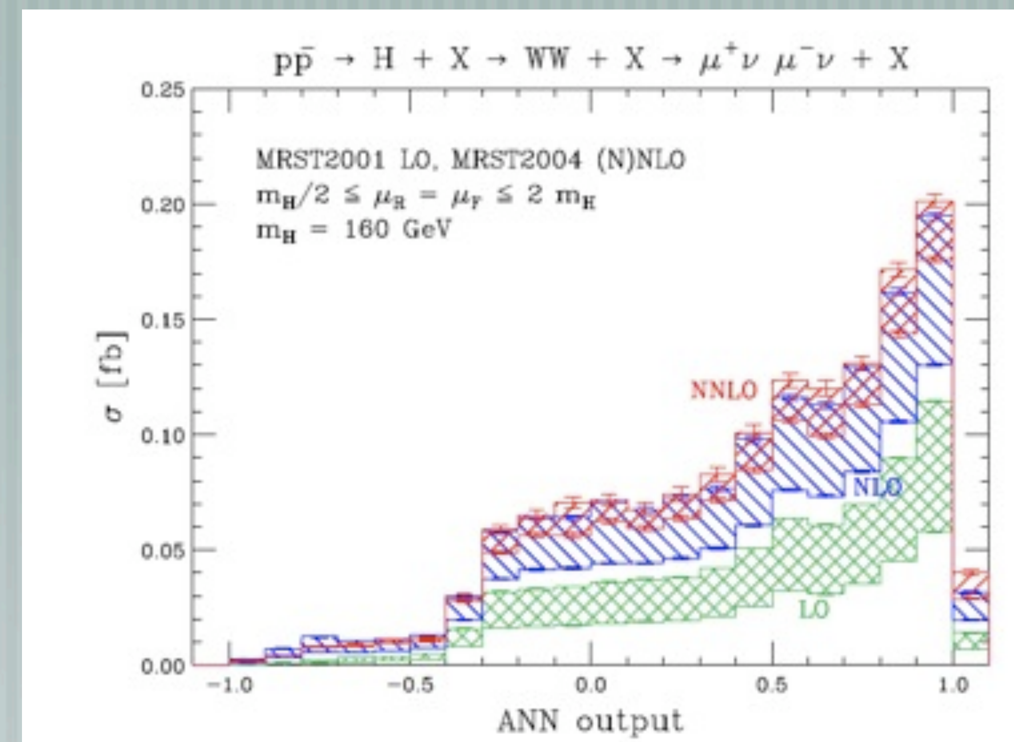
- theory uncertainty for the accepted signal events is different than for the total number before cuts.

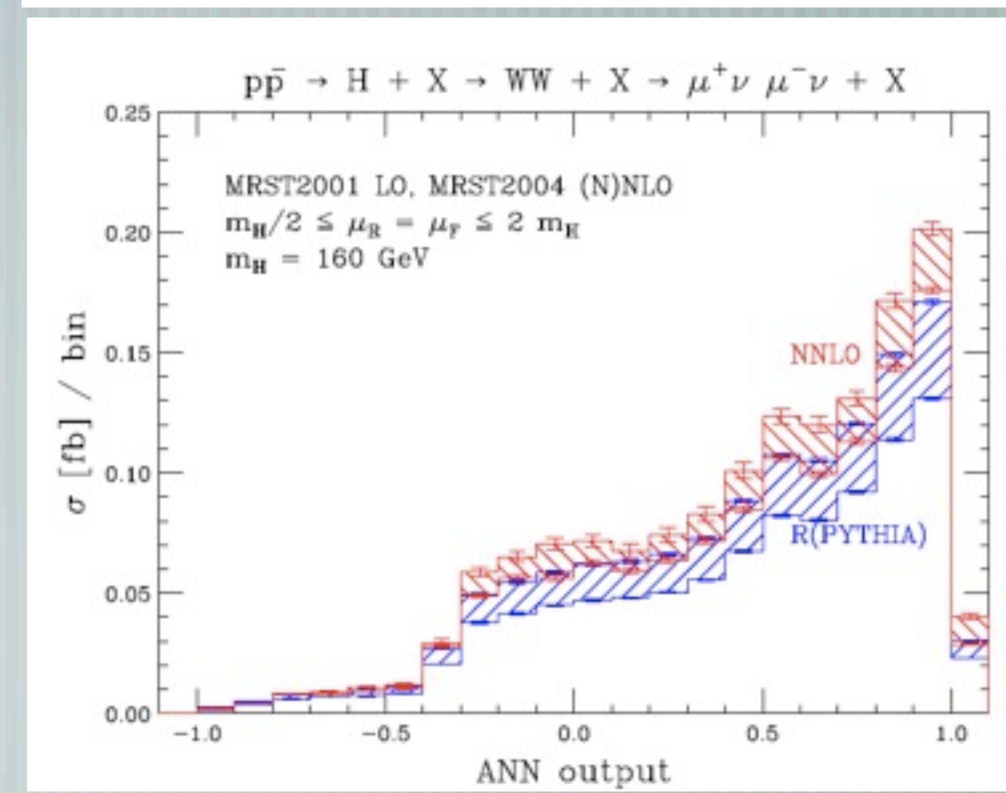
(CA,Dissertori,Grazzini, Stoeckli,Webber)

Differential theory

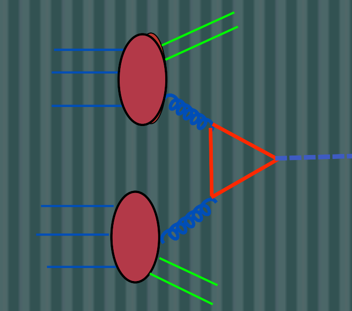


- Lesson: check theory uncertainty on the kinematic bins which drive exclusion
- an NNLO computation of a neural net is as simple as for a rapidity distribution. (CA,Dissertori,Grazzini,Stoeckli, Webber)
- Highly recommended for the CDF and D0 analyses.





Differential theory



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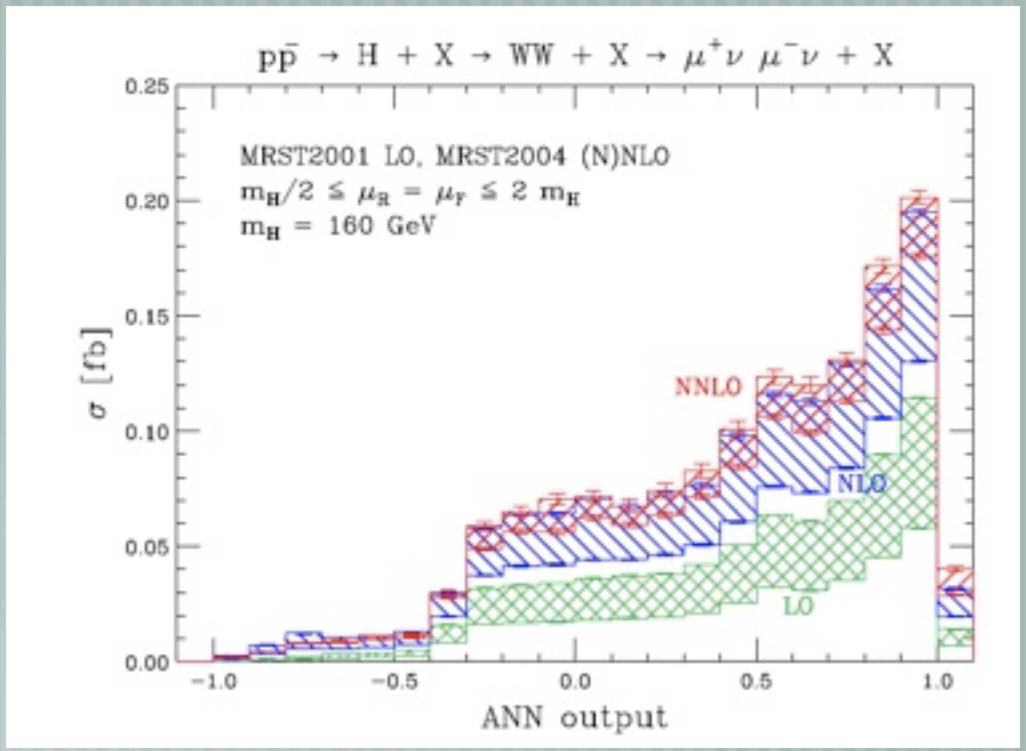
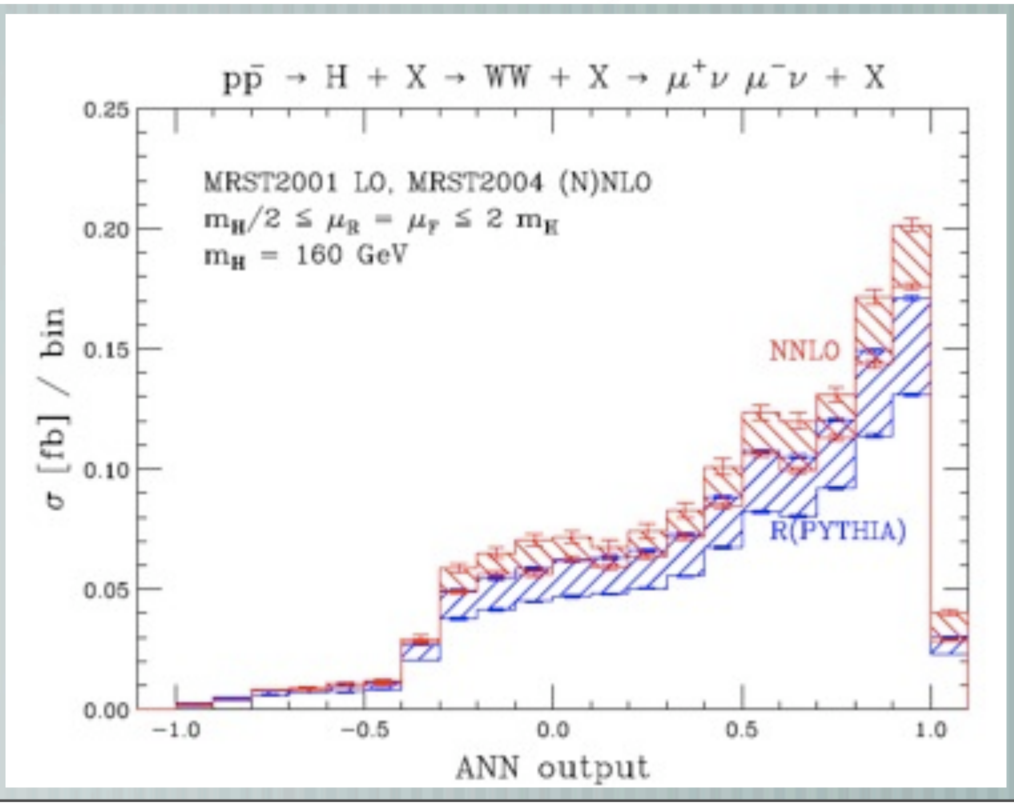
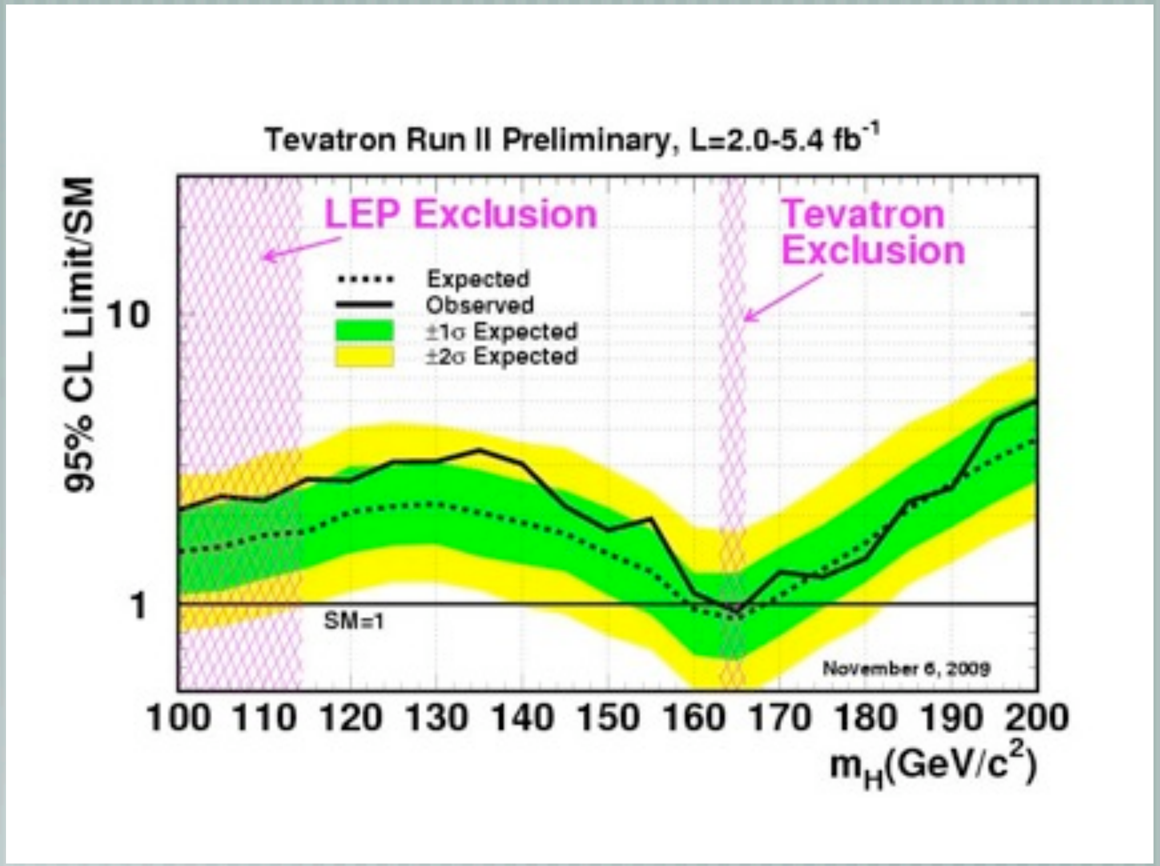




Latest exclusion limits

- Mature analysis, with many improvements concerning the treatment of theory uncertainties
- Space for further improvements
- Using little theory input is a virtue for en experimental study.
- Little theory input should not mean idealized theory input (total cross-section)



Iterative perturbation series

- The perturbation series of gauge theories displays cross-order iterations.
- These are needed to cancel infrared and UV divergences, filtering the superposition principle from ultra short and very large distance effects.
- They are exploited to formulate parton shower algorithms, and resumming large logarithms.
- But, the remainder seems very different at each order in perturbation theory!

An unexpected iteration in N=4 super Yang-Mills theory

$$\mathcal{M}_4^{(2)}(\epsilon) = \frac{1}{2}\left(\mathcal{M}_4^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon)\mathcal{M}_4^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon)$$

CA, Bern, Dixon, Kosower

$$\mathcal{M}_n^{(2)}(\epsilon) = \frac{1}{2}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon)\mathcal{M}_n^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon)$$

$$\mathcal{M}_4^{(3)}(\epsilon) = -\frac{1}{3}\left(\mathcal{M}_4^{(1)}(\epsilon)\right)^3 + \mathcal{M}_4^{(2)}(\epsilon)\mathcal{M}_4^{(1)}(\epsilon) + f^{(1)}(\epsilon)\mathcal{M}_4^{(1)}(3\epsilon) + C^{(3)} + \mathcal{O}(\epsilon)$$

Bern, Dixon, Smirnov

$$\mathcal{M}_n = \exp\left[\sum_{l=1}^{\infty} a^l f^{(l)}(\epsilon)\mathcal{M}_n^{(1)}(l\epsilon) + C^{(l)} + \mathcal{O}(\epsilon)\right]$$

Can be computed in the strong limit with AdS/CFT Alday,Maldacena

$$\ln(1 + \sum_{l=1}^{\infty} a^l \mathcal{M}_n^{(l)}) = \ln(1 + \sum_{l=1}^{\infty} \mathrm{a}^l W_n^{(l)}) + \mathcal{O}(\epsilon)$$

<Wilson Loop> = Amplitude
Sokachev,Korchemsky

Can compute two-loop amplitudes with arbitrary number of legs, using the Wilson-loop duality

CA,Brandhuber,Heslop,Khoze,Spence,Travaglini

Outlook

- Our abilities in simulating precisely collider processes have grown tremendously.
- New computational methods at NLO are extremely powerful. A classic work which will be part of future field theory books.
- Ready to take on the big challenge of finding new physics convincingly in hadron collider data.