AD POLOSA — SAPIENZA UNIVERSITY OF ROME

SUB-MEV DARK MATTER AND THE GOLDSTONE EXCITATIONS OF SUPERFLUID HE

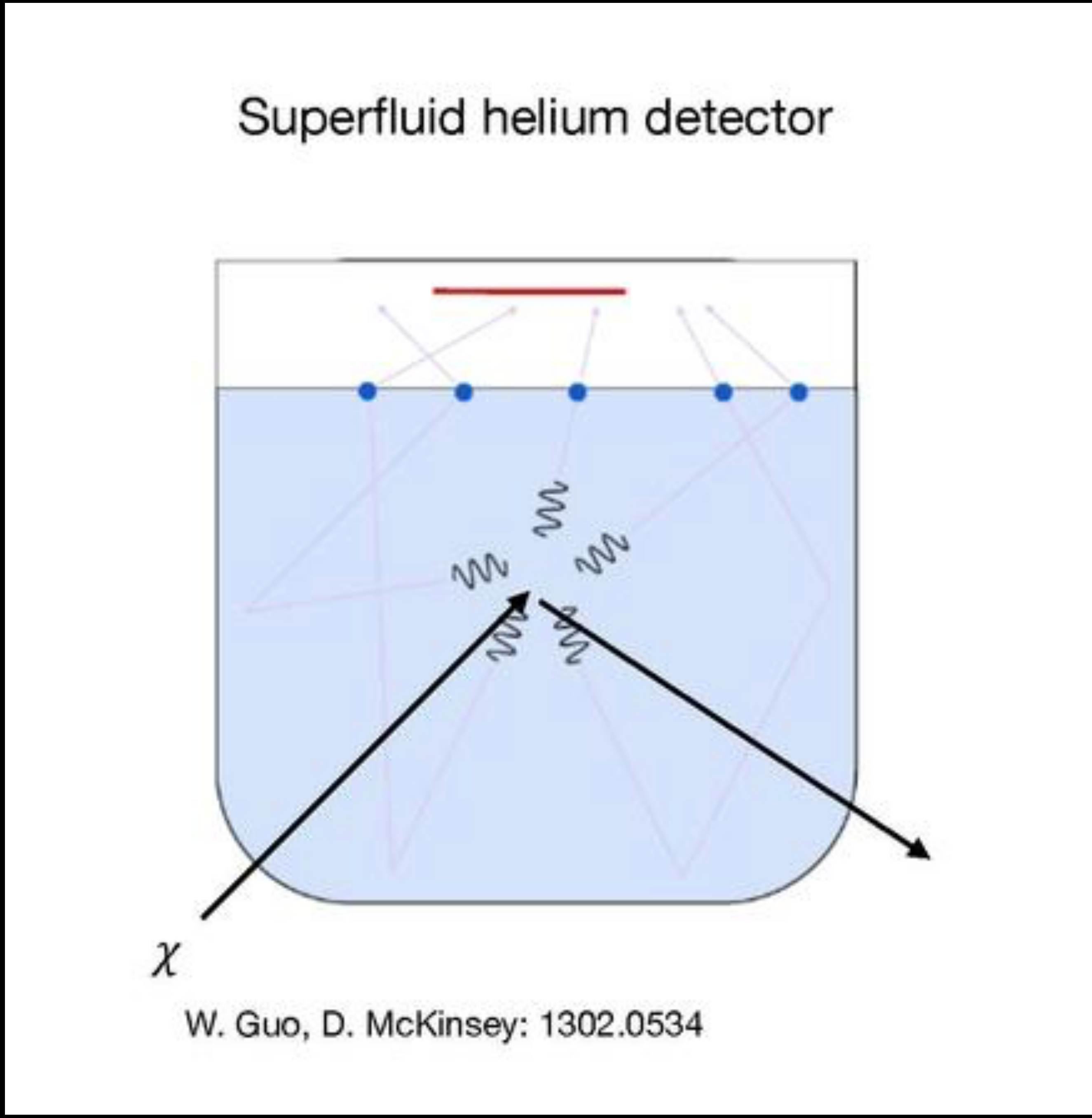
Based on

F. Acanfora, A. Esposito, ADP, EPJC 79 (2019), 549

A. Caputo, A. Esposito, ADP, PRD (2019) in press

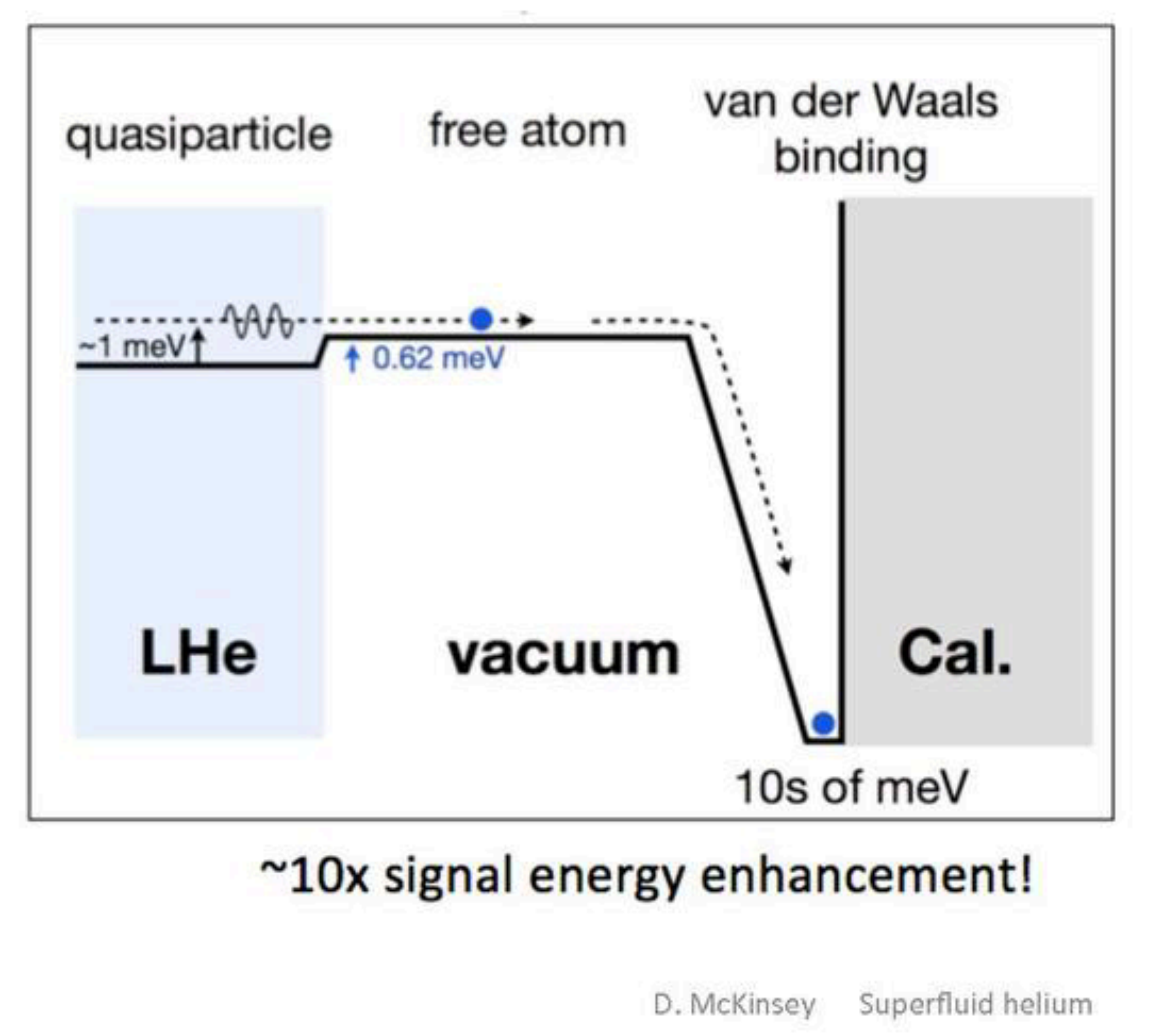
A. Caputo, A. Esposito, E. Geoffray, ADP, S. Sun, 1911.04511

LIGHT DM IN SUPERFLUID-HE



See also, Hertel, Biekert, Lin, Velan, McKinsey, 1810.06283; Maris, Seidel, Stein, PRL2017, 1706.00117

LIGHT DM IN SUPERFLUID-HE



QUASI PARTICLE W/ AT LEAST 0.62 meV → QUANTUM EVAPORATION OF AN He ATOM → STICK TO A CALOR. SURFACE

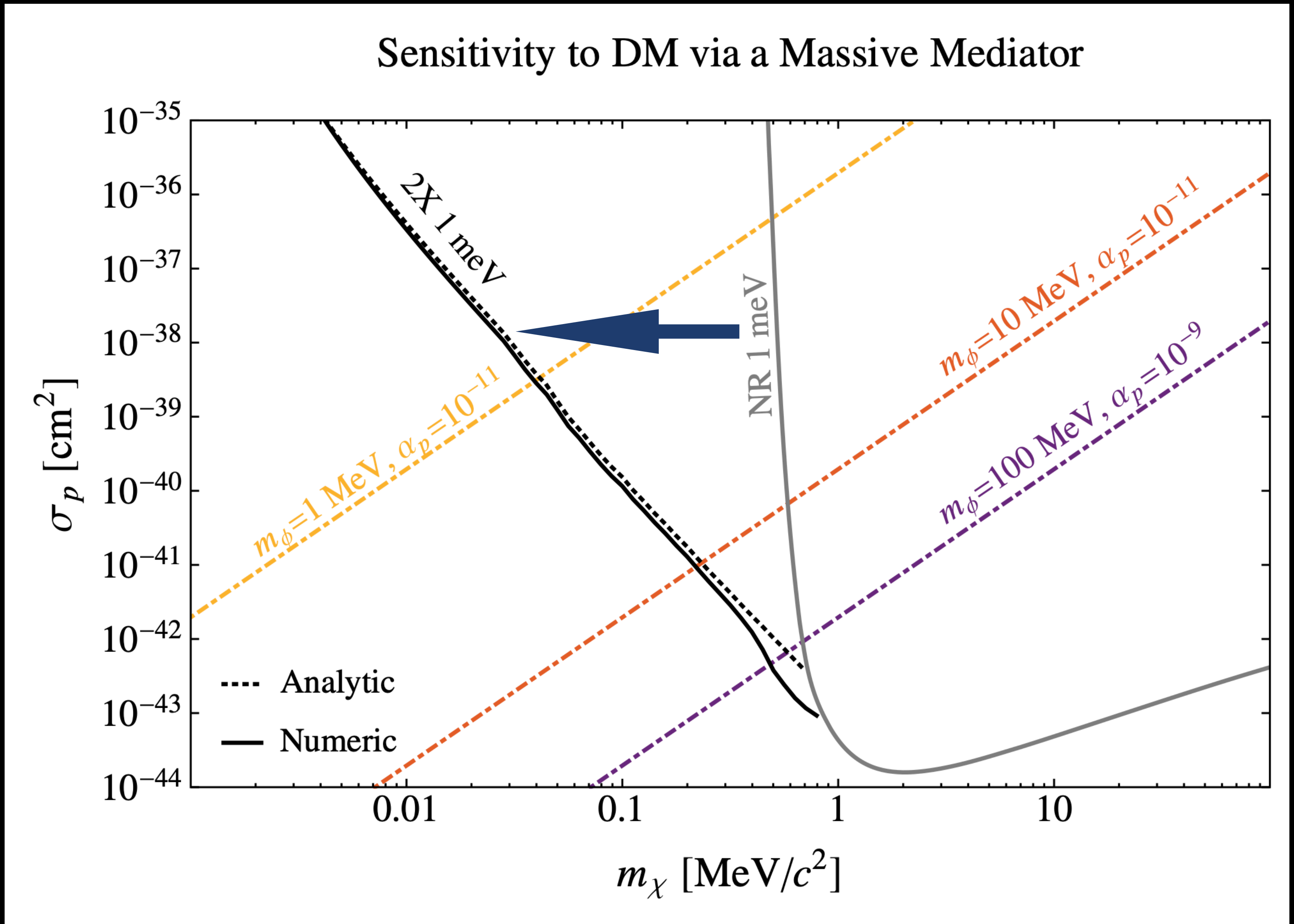
HELIUM STICKS MORE STRONGLY TO ANY SURFACE THAN IT DOES TO ITSELF.

DETECTABILITY OF LDM IN S.F.HE

K. Schutz, K.M. Zurek, PRL (2016), 117

See also S. Knapen, T. Lin, K.M. Zurek, PRD95 (2017) 056019

Use the *microscopic theory* of the superfluid phase of ^{4}He to compute the two-phonon process. *Can probe DM down to KeV.*



SUPERFLUIDS IN QUANTUM FIELD THEORY

BOSONS WITH SHORT RANGE REPULSION

In a gas of ***repelling*** bosons, giving momentum to a particle means producing a ***density wave***, which turn out to obey a ***linear dispersion relation***

$$\omega = \underbrace{\sqrt{\frac{\lambda \bar{\rho}}{2m^3}}}_{c_s} \, k$$

(few low energy excitations)

BOSONS WITH SHORT RANGE REPULSION

A gas of ***repelling*** bosons with finite ***density***, can be described by a QFT (in the NR limit) with a U(1) symmetry

$$\varphi \to e^{i\alpha}\varphi$$

and a Mexican hat potential well, forcing the maginitude of φ to be close to $\sqrt{\bar{\rho}}$

$$\mathscr{L} = i\varphi^\dagger \frac{\partial}{\partial t}\varphi - \frac{1}{2m}\nabla_i \varphi^\dagger \nabla_i \varphi - \frac{\lambda}{4m^2}(\varphi^\dagger \varphi - \bar{\rho})^2$$

BOSONS WITH SHORT RANGE REPULSION

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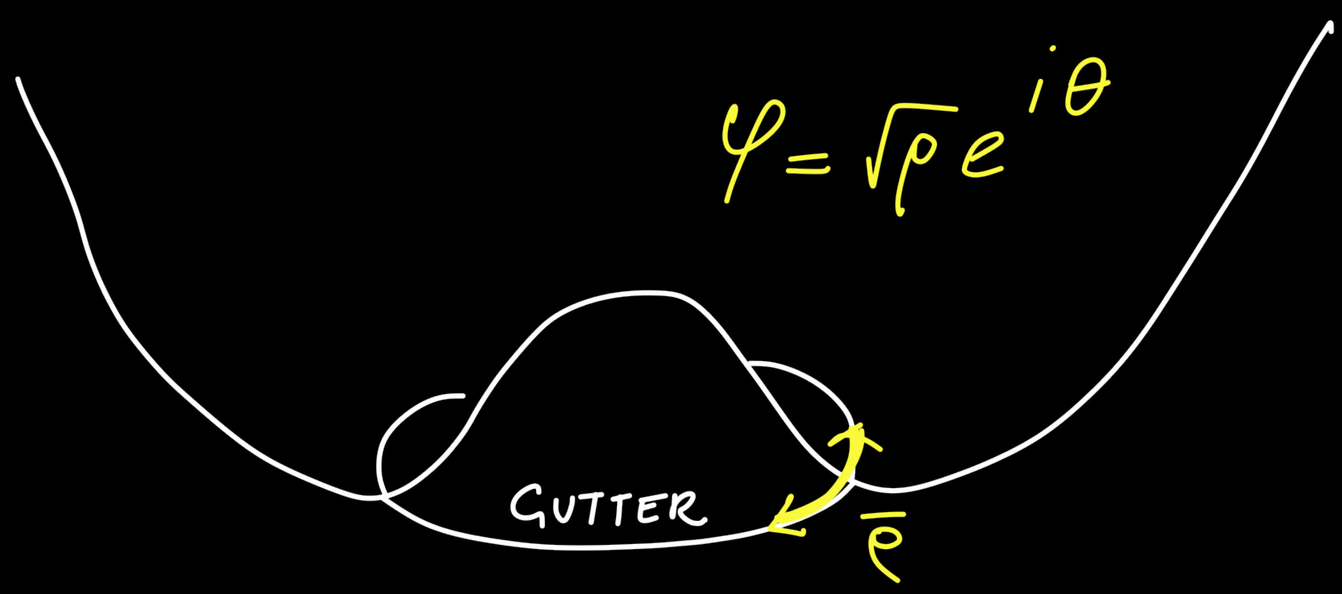


BOGOLUBOV FORMULA

Integrate out the η modes in $\sqrt{\rho} = \sqrt{\bar{\rho}} + \eta\,(\ll \sqrt{\bar{\rho}})$ and get the Lagrangian

$$\mathscr{L} = \frac{(\partial_0 \theta)^2}{\lambda/m^2} - \frac{\bar{\rho}}{2m}(\nabla_i \theta)^2$$

write equations of motion

$$\frac{2}{\lambda/m^2}\partial_0^2\theta - \frac{\bar{\rho}}{m}\Delta\theta = 0$$

and use the solution (for the Goldstone mode)

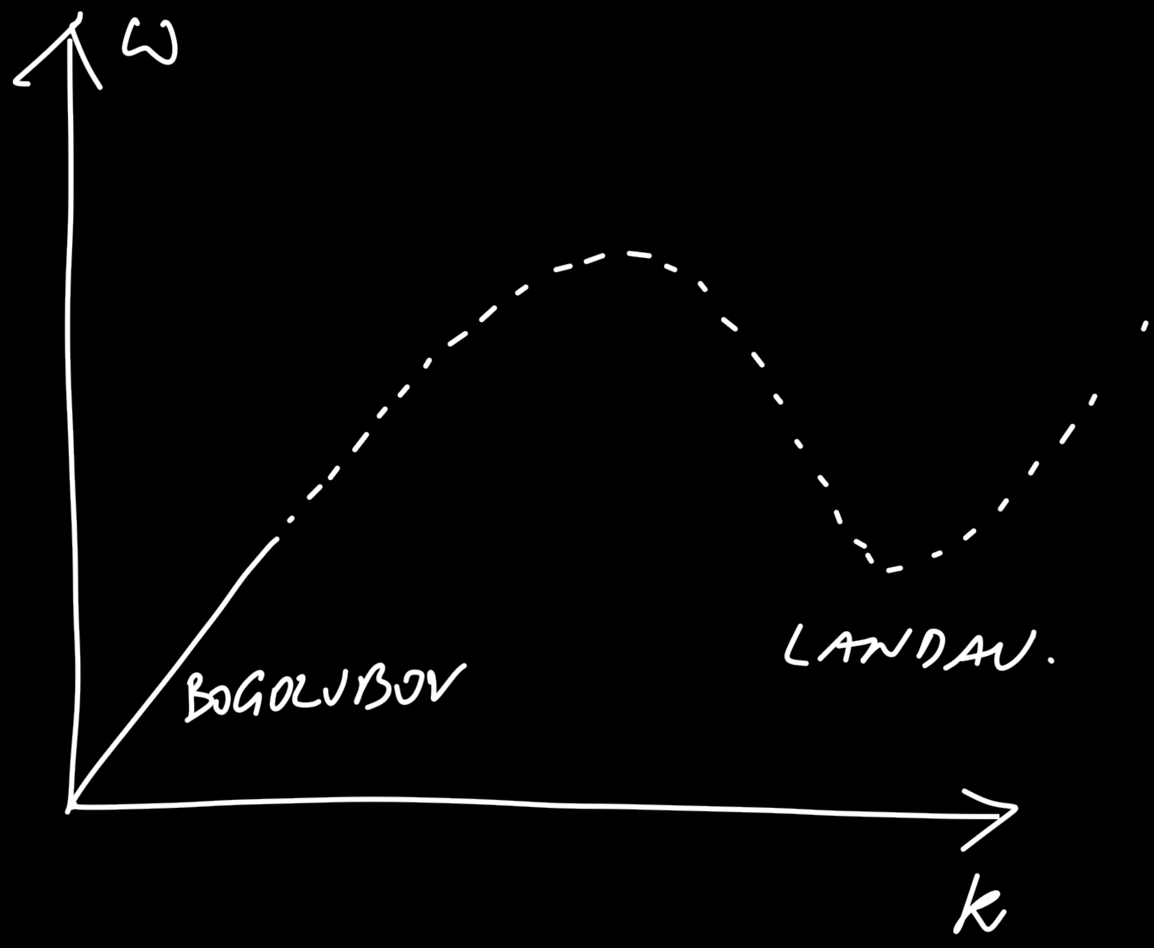
$$\theta \sim e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

BOGOLUBOV FORMULA

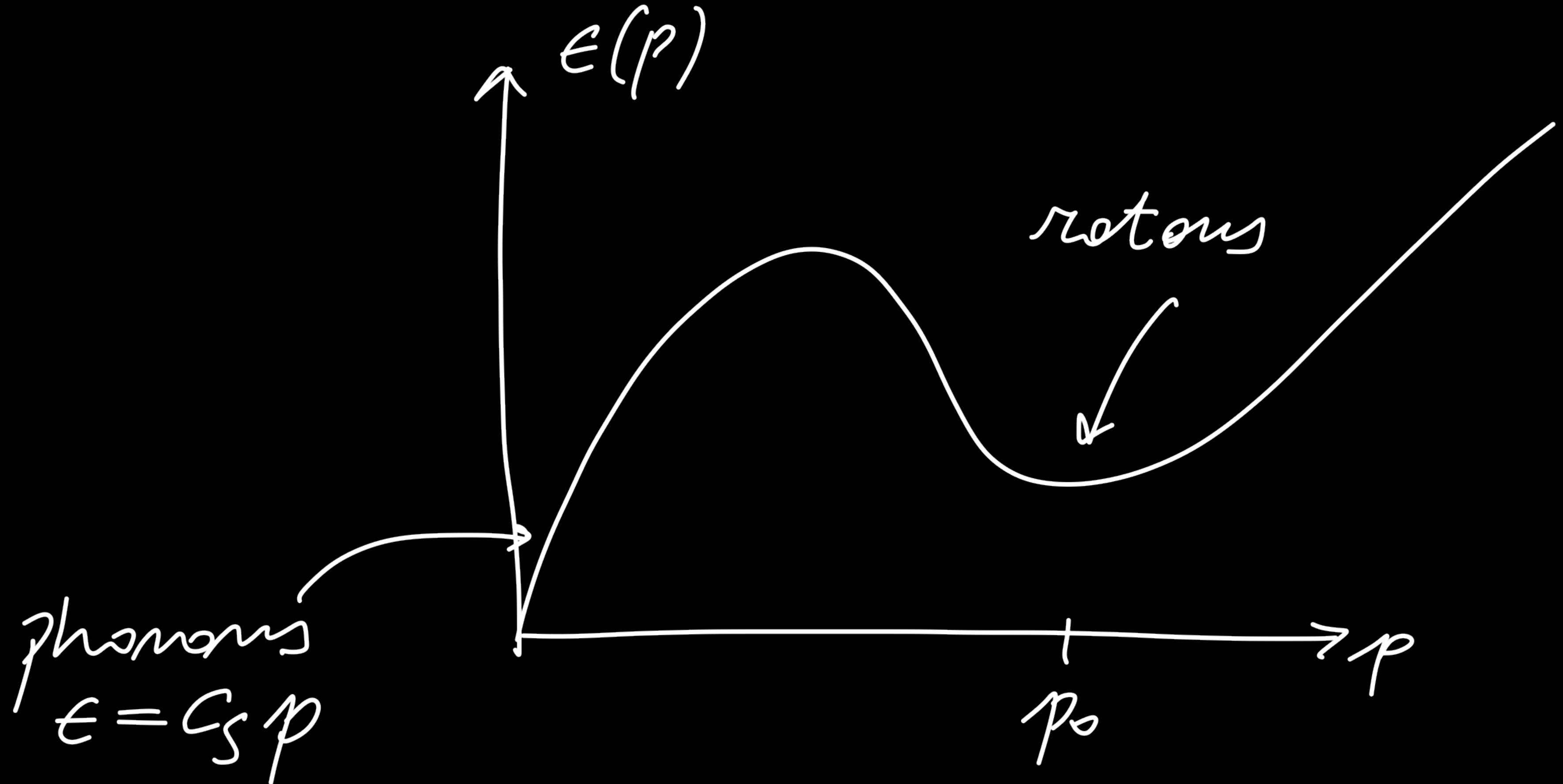
$$-\frac{2\omega^2 m^2}{\lambda} + \frac{\bar{\rho} k^2}{m} = 0$$

$$\omega = \underbrace{\sqrt{\frac{\lambda \bar{\rho}}{2m^3}}}_{c_s} k$$

"Gapless" modes



LANDAU ARGUMENT



In the proximity of the roton

$$\epsilon(p) \simeq \Delta + \frac{1}{2m^*}(p - p_0)^2$$

$\Delta \simeq 0.8\,meV$ $\quad c_s = 248\,m/s \; (\approx 10^{-6})$

$p_0 \simeq 3.94\,keV$ $\quad \bar{n} = 8.5 \times 10^{22}\,cm^{-3}$

$m^* \simeq 0.16\, m_{He}$

GOLDSTONE MODES

Go back to

$$\mathscr{L} = \frac{(\partial_0 \theta)^2}{\lambda/m^2} - \frac{\bar{\rho}}{2m}(\nabla_i \theta)^2$$

Scale/adjust the distance x to write the compact form (back to relativistic)

$$\mathscr{L} = \frac{(\partial_\mu \theta)^2}{\lambda/m^2}$$

with the constraint

$$\theta(x) = \theta(x) + 2\pi$$

GOLDSTONE MODES ($\lambda \to \infty$)

$$\mathscr{L} = (\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) - \lambda(\Phi^\dagger\Phi - \xi^2)^2$$

Set λ to ∞ so that it is (infinitely) more convenient to slide along the gutter than climbing the wall

$$\mathscr{L} = (\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) \quad \text{with} \quad \Phi^\dagger\Phi = \xi^2$$

the constrain is solved by

$$\Phi = \xi e^{i\theta}$$

and we get the massless Goldstone modes (like the one obtained before)

$$\mathscr{L} = \xi^2(\partial_\mu \theta)^2$$

i.e. obtain the NR result by sending $\lambda \to \infty$

PHONON EXCITATIONS

A. Nicolis, 1108.2513 [hep-th]

A. Nicolis, R. Penco, F. Piazza, R. Rattazzi, JHEP (2015)

A. Nicolis, R. Penco, Phys. Rev. B97, 134516 (2018)

PHONONS

Introducing a finite density *ρbar* allows the spontaneous breaking of a global U(1) symmetry (digs the gutter). The low energy dynamics of a superfluid can be described in terms of a scalar field θ*(x)* (that shifts under U(1)).

The phase is the canonical conjugate variable to the density (uncert. relation). In the ground state of the superfluid the number of particles <N> is fixed and the phase θ is "free" to fluctuate.

$$\Delta\theta\,\Delta N \geq \frac{1}{2}$$

The energy of the system is <H>=μ<N>, where μ is the chemical potential. This breaks time translations.

PHONONS

A quantity with the dimensions of a phase, which breaks time translations, is μt; this can be added to the free $\theta(x)$ introducing the new phase field

$$\psi(x) = \langle \psi \rangle + \theta(x) \quad \text{with} \quad \langle \psi \rangle = \mu t$$

The superfluid U(1) is spontaneously broken by the vev.
The **phonon** arises as the the fluctuation of ψ over $\langle\psi\rangle$.

In addition, every condensed matter system breaks Lorentz boosts and defines a special reference frame: the frame in which the system is at rest.

PHONONS

$\theta(x) \propto \pi(x)$ phonon field

The most general low-energy action must be Poincare` and U(1) invariant.

$$S = \int P(X)\, d^4x$$

$$P(X) \equiv P\left(\sqrt{\partial_\mu \psi\, \partial^\mu \psi}\right)$$

In absence of fluctuations θ(x), on the `background`,

$$P(X) = P(\mu)$$

where X is a “local” chemical potential, and P(μ) is an equation of state.

PHONONS

We will use in place of θ

$$\psi(x) = \mu t + c_s\sqrt{\frac{\mu}{\bar{n}}}\,\pi(x)$$

in such a way that $\pi(x)$ is canonically normalized

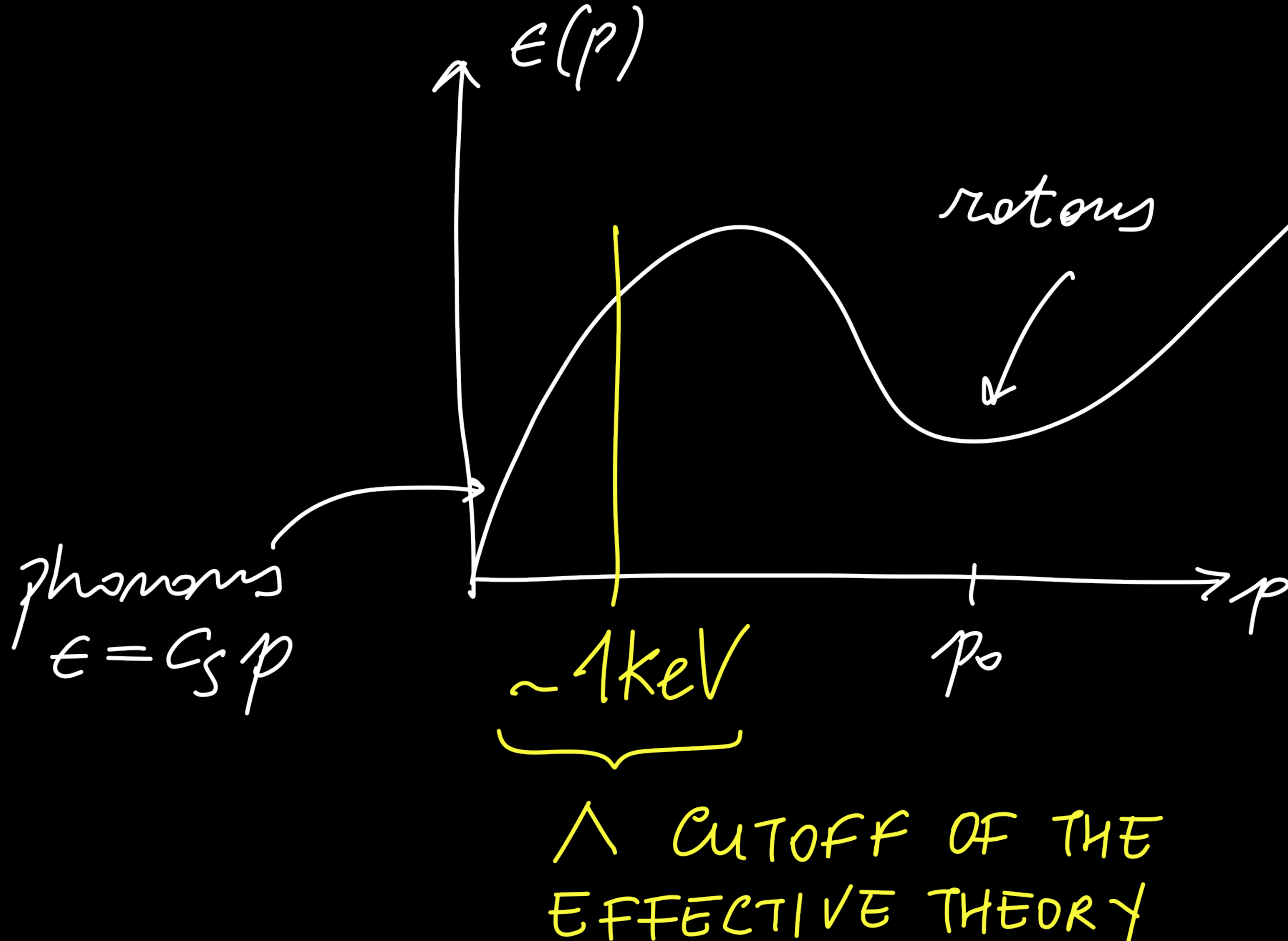
Expanding the Lagrangian up to cubic terms in the phonon

$$\mathscr{L}_{\rm He} = \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\nabla\pi)^2 + \lambda_3 c_s\sqrt{\frac{\mu}{\bar{n}}}\,\dot{\pi}(\nabla\pi)^2 + \lambda_3' c_s\sqrt{\frac{\mu}{\bar{n}}}\,\dot{\pi}^3$$

$c_s^2 = \frac{P'}{\mu P''}$ $\quad \lambda_3 = \frac{c_s^2 - 1}{2\mu}$ $\quad \lambda_3' = \frac{\mu c_s^2}{6\bar{n}}P'''$ and $\bar{n} = P'(\mu)$

$\bar{n}$ is the background number density and $P(\mu)$ is the EOS

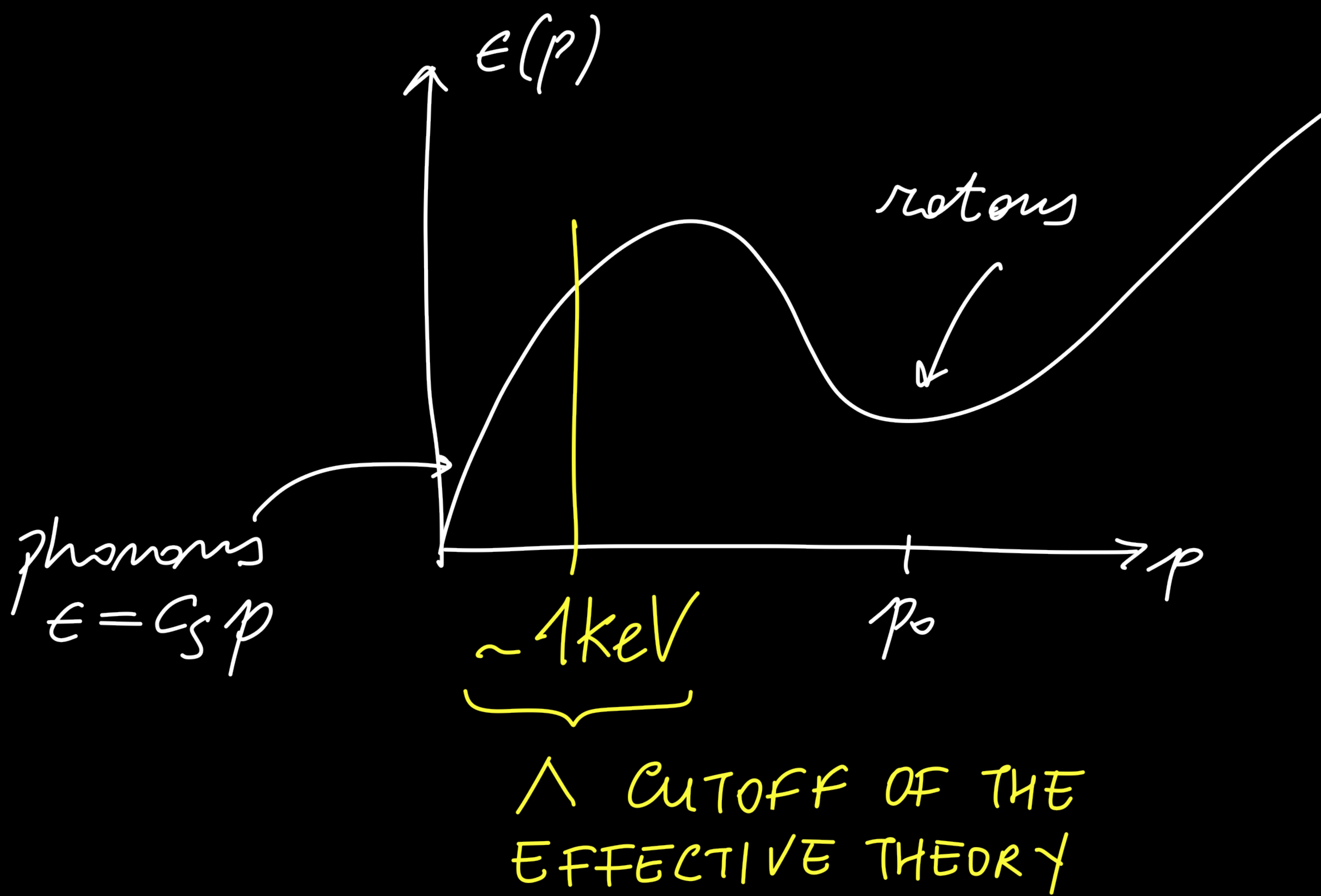
EFT RANGE OF VALIDITY



MOMENTA OF ON-SHELL π's : $|\vec{q}| < \Lambda \simeq 1\,\text{keV}$

ENERGIES " " " " : $E < c_s \Lambda \simeq 1\,\text{meV}$

EFT RANGE OF VALIDITY



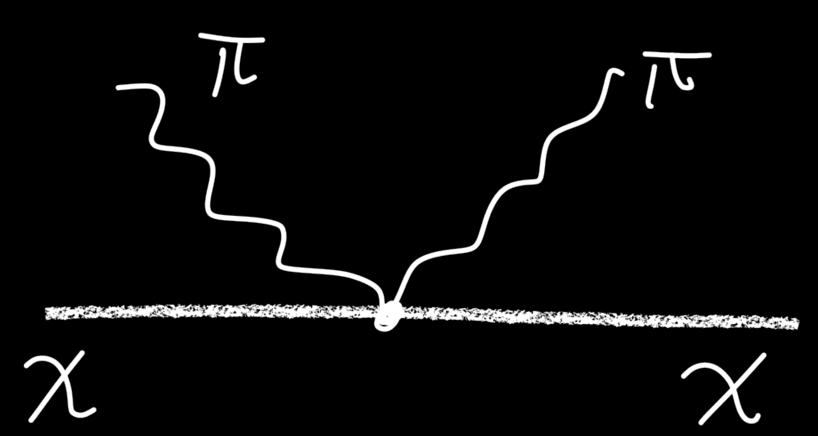
Above 1 KeV the dispersion relation is not linear: collective excitations cannot be described in terms of a phonon dof. The EFT needs higher dimensional operators and loses its predictive power.

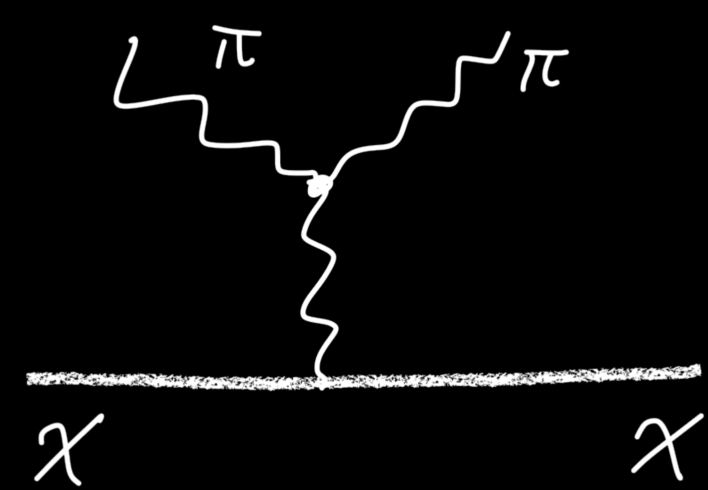
INTERACTION WITH DM: EFT

Consider a **scalar** dark matter χ

$$\mathcal{L}_I = \mathcal{G}_1 \dot{\pi} |\chi|^2 + \mathcal{G}_2 (\nabla \pi)^2 |\chi|^2 + \mathcal{G}_3 \dot{\pi}^2 |\chi|^2$$

$$\mathcal{G}_1 = G_\chi m_\chi P''(\mu)\, c_s \sqrt{\frac{\mu}{\bar{n}}} \qquad \mathcal{G}_2 = -\, G_\chi m_\chi P''(\mu) \frac{c_s^2}{2\bar{n}} \qquad \mathcal{G}_3 = G_\chi m_\chi P'''(\mu) \frac{\mu c_s^2}{2\bar{n}}$$

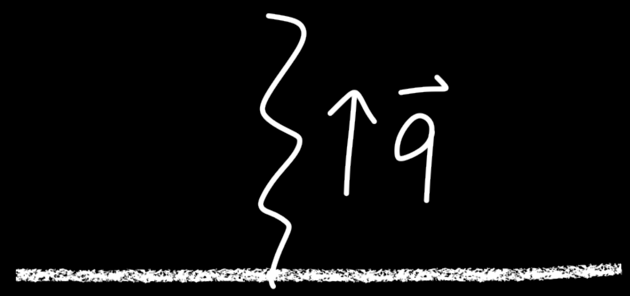




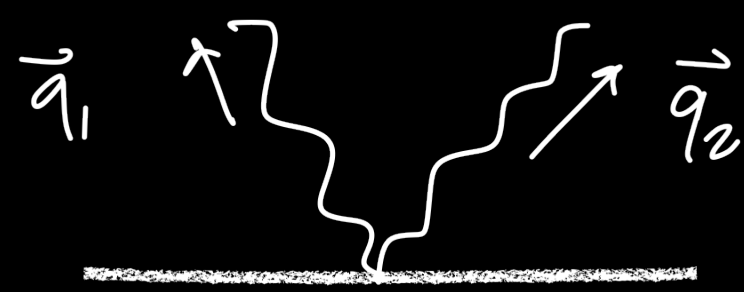
seagull contribution

$$\Delta_\pi(\omega, \mathbf{q}) = \frac{i}{\omega^2 - c_s^2 \mathbf{q}^2 + i\epsilon}$$

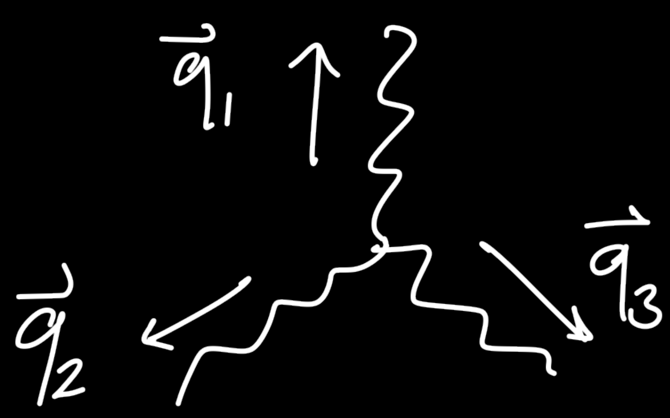
FEYNMAN RULES



$$-\mathscr{G}_1\,\omega$$



$$-2i(\mathscr{G}_3\omega_1\omega_2 - \mathscr{G}_2\mathbf{q}_1\cdot\mathbf{q}_2)$$



$$2c_s\sqrt{\frac{\mu}{\bar{n}}}\Big(\lambda_3\big(\underbrace{\omega_1\mathbf{q}_2\cdot\mathbf{q}_3}_{(123)} + (213) + (312)\big) + 3\lambda_3'\,\omega_1\omega_2\omega_3\Big)$$

$\bar{n}$	$0.65\ \text{keV}^3$	λ_3	$-1.3\times10^{-7}\ \text{keV}^{-1}$
c_s	8.2×10^{-7}	λ_3'	$-8.5\times10^{5}\ \text{keV}^{-1}$
$d\bar{n}/d\mu$	$2.7\times10^{5}\ \text{keV}^2$	$d^2\bar{n}/d\mu^2$	$-1.4\times10^{12}\ \text{keV}$

RESULTS WITH 1&2 PHONONS

SINGLE PHONON EMISSION

Its energy is not enough to be detected with calorimetry (need at least 1 meV).
Quantum evaporation (need 0.62 meV) might work — need ballistic trajectories.

The max. energy of a single phonon is $c_s \times 2m_\chi v_\chi \gtrsim 0.62$ meV $\Rightarrow m_\chi \gtrsim 1$ MeV.

$$\frac{d\Gamma}{d\Omega d\omega} = \frac{\mathcal{G}_1 \omega^2 \mu}{32\pi^2 \bar{n} m_\chi^2 v_\chi} \delta\left(\cos\theta - \frac{c_s}{v_\chi} - \frac{q}{2m_\chi v_\chi}\right)$$

Cherenkhov $\cos\theta \approx 60^\circ$

$$N_{\text{evts}} = \int dv_\chi f_{MB}(v_\chi) \frac{\rho_\chi}{m_{\text{He}}\, \bar{n}\, m_\chi} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \, \frac{d\Gamma}{d\omega}$$

F. Acanfora, A. Esposito, ADP, EPJC 79 (2019), 549

TWO-PHONON PROCESSES

Two-phonon emission processes remain effective also at masses lighter than 1 MeV

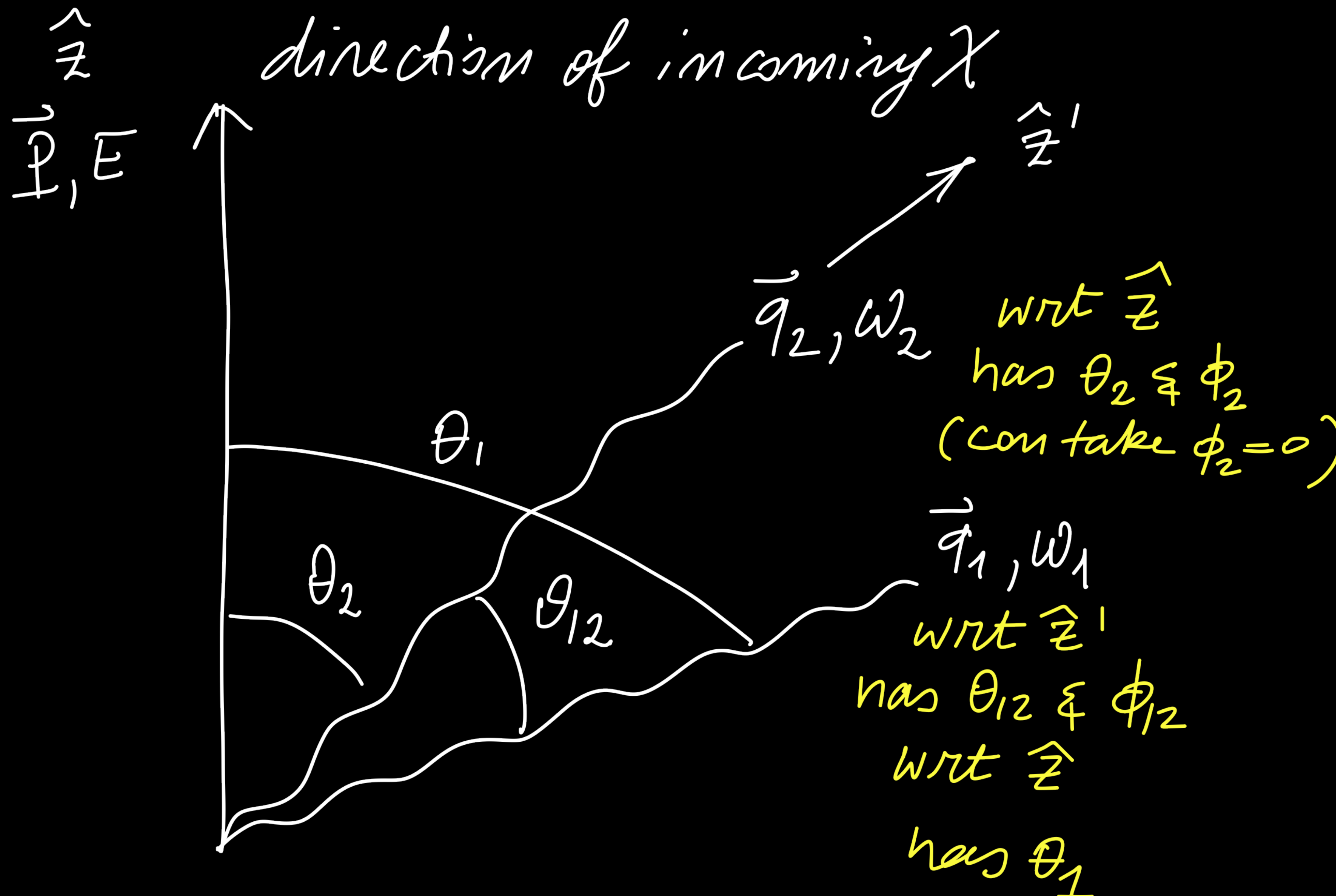
(Qualitative argument) The maximum momentum transfer is still $2m_\chi v_\chi$ but this goes to a virtual $(\omega \neq c_s k)$ rather than to a real phonon

The off-shell ω gets shared by the two final state phonons

At very low DM masses, one gets

$$\omega_1 \simeq \omega_2 \qquad \mathbf{q}_1 \simeq -\mathbf{q}_2$$

TWO-PHONON PROCESSES



$$\cos\theta_1 = \cos\theta_{12}\cos\theta_2 - \underbrace{A}_{\cos(\phi_{12}-\phi_2)}\sin\theta_{12}\sin\theta_2$$

TWO-PHONONS PROCESSES

We focus on two kinds of events.

Those in which both phonons can produce quantum evaporation

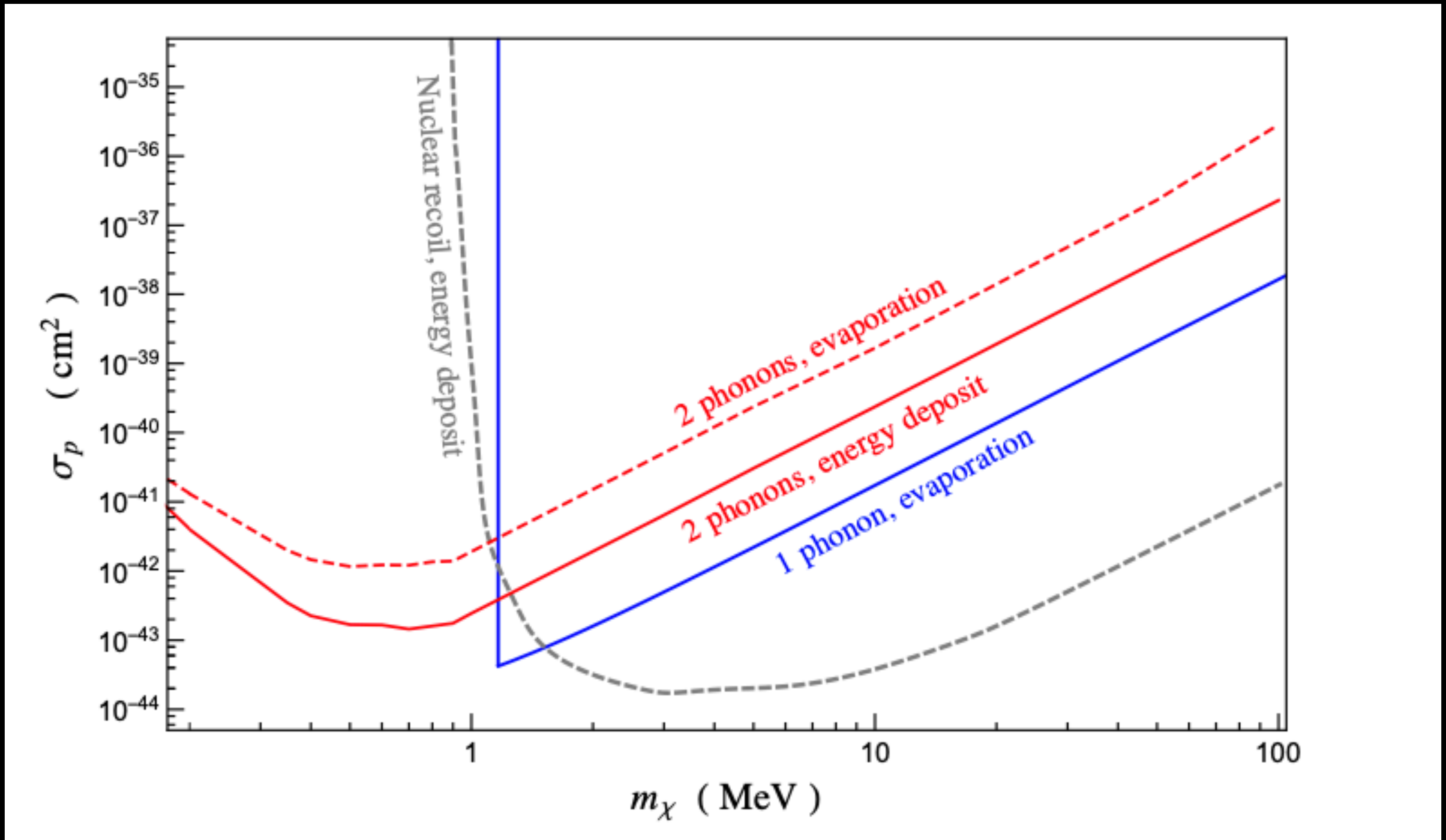
$$\omega_{1,2} \geq 0.62 \text{ meV} \quad (\omega_{1,2} \leq 1 \text{ meV})$$

Those in which phonos deposit energy which can be detected with calorimetric techniques

$$\omega_1 + \omega_2 \geq 1 \text{ meV} \quad (\omega_{1,2} \leq 1 \text{ meV})$$

THE EXCLUSION PLOT (>1MEV)

F. Acanfora, A. Esposito, ADP, EPJC 79 (2019), 549

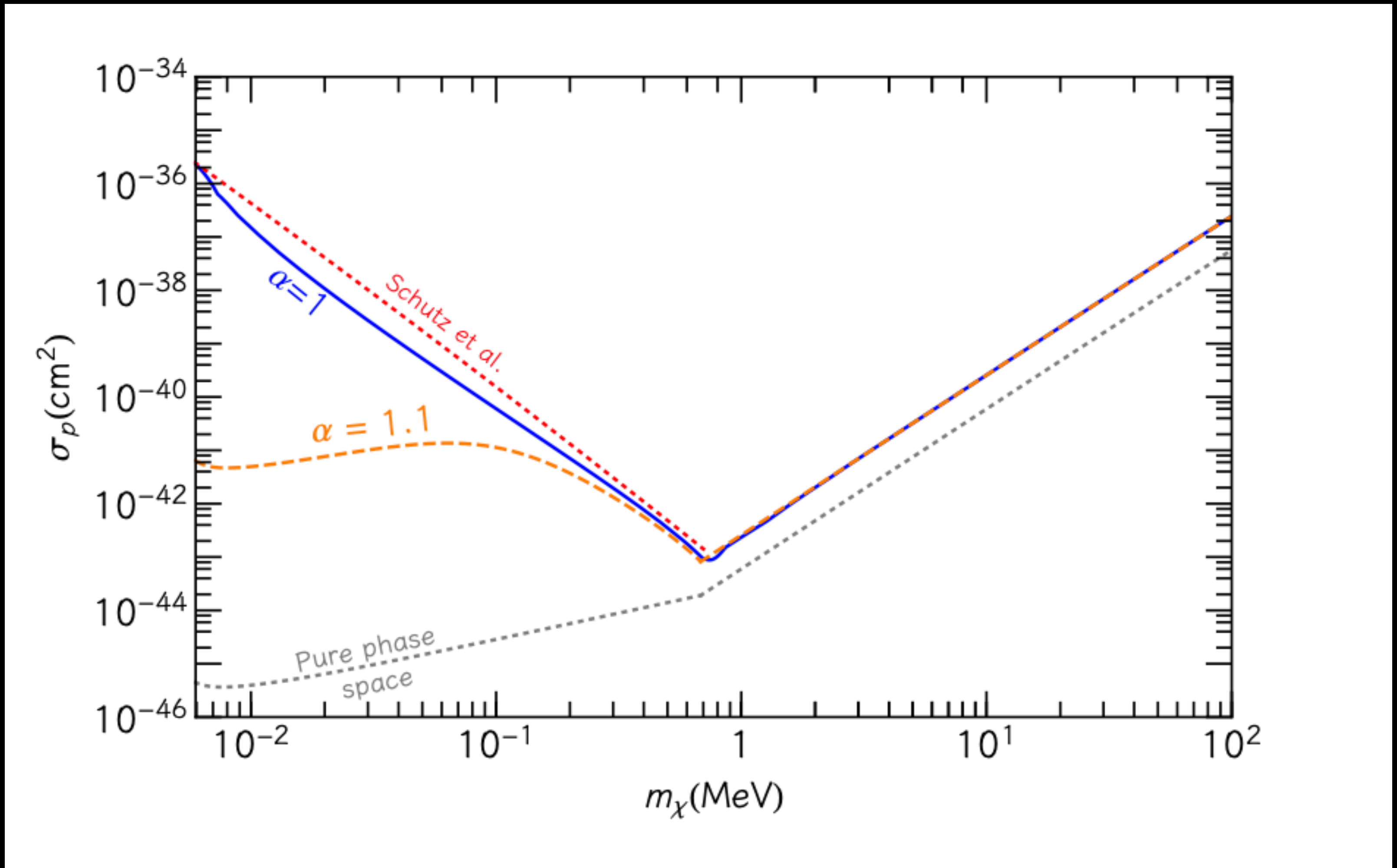


Excluded region corresponding to 3/evts/Kg/year @ zero bckg. Impose total energy released >1meV

$$\sigma_{DM-p} = \sigma_p = \frac{G_\chi^2 m_{\chi-He}^2}{256\pi}$$

THE EXCLUSION PLOT

A. Caputo, A. Esposito, ADP, PRD (2019) in press



Excluded region corresponding to 3/evts/Kg/year @ zero bckg. Impose total energy released >1meV

KINEMATICS

$$\Gamma(\chi \to \chi + 2\pi) = \frac{1}{8(2\pi)^4 c_s^5 m_\chi^2 v_\chi} \int_{\mathscr{R}} \frac{|\mathscr{M}|^2}{\sqrt{1-\mathscr{A}^2}} d\theta_{12} d\theta_2 d\omega_1\, \omega_2\, d\omega_2$$

$$\mathscr{A}(\theta_{12}, \theta_2, \omega_1, \omega_2) = \frac{1}{\sin\theta_{12}\sin\theta_2}\left(\cos\theta_{12}\cos\theta_2 + \frac{\omega_2}{\omega_1}\cos\theta_2 - \frac{\omega_2}{c_s P}\cos\theta_{12} - \frac{\omega_1^2+\omega_2^2}{2\omega_1 c_s P}\right)$$

Region $\mathscr{R}$ is defined by $|\mathscr{A}| \leq 1$

As $P \to 0$ the leading term in $\mathscr{A}$ is proportional to

$$\frac{(\mathbf{q}_1 + \mathbf{q}_2)^2}{|\mathbf{q}_1 \times \mathbf{q}_2|}$$

The case back-to-back with same momentum prevails at low m_χ $(P \to 0)$

CUTS FROM DYNAMICS

$$|\mathbf{q}_{1,2}| \leq 1 \text{ KeV}$$

($\omega_{1,2} \leq 1$ meV can be seen only in evaporation)

$$|\mathbf{q}| = |\mathbf{q}_1 + \mathbf{q}_2| \leq 1 \text{ KeV}$$

$$\left(|\mathbf{q}| = |\mathbf{P} - \mathbf{P}'| = \sqrt{P^2 + P'^2 - 2PP'\cos\eta} \right)$$

The bound on momenta coming determine the EFT

$$\Delta = \frac{i}{\omega^2 - c_s^2 \mathbf{q}^2} = \frac{i}{2\omega_1\omega_2(1 - \cos\theta_{12})}$$

The lower cuts on the the phonons energies `cure` the collinear divergence

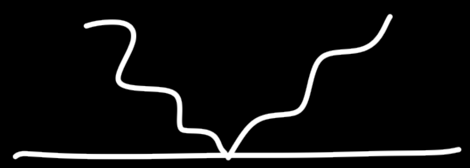
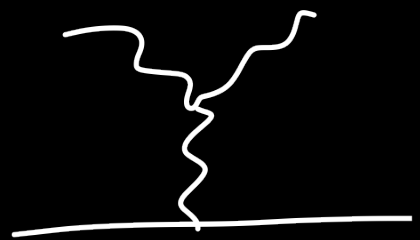
$$\omega_{1,2} \geq 0.62 \text{ meV} \quad (\omega_{1,2} \leq 1 \text{ meV})$$

or

$$\omega_1 + \omega_2 \geq 1 \text{ meV} \quad (\omega_{1,2} \leq 1 \text{ meV})$$

A `LITTLE THEOREM`

A. Caputo, A. Esposito, ADP, PRD (2019) in press

$\mathcal{M}_a + \mathcal{M}_b =$  $+$ 

$$\mathcal{M}_a = -2(\mathcal{G}_3\omega_1\omega_2 - \mathcal{G}_2\mathbf{q}_1\cdot\mathbf{q}_2)$$

$$\mathcal{M}_b = \mathcal{G}_1\omega \times \frac{1}{\omega^2 - c_s^2\mathbf{q}^2} \times 2c_s\sqrt{\frac{\mu}{\bar{n}}}\left(\lambda_3\Big(\underbrace{\omega_1\mathbf{q}_2\cdot\mathbf{q}}_{(12\diamond)} + (21\diamond) + (\diamond 12)\Big) + 3\lambda_3'\,\omega\,\omega_1\omega_2\right)$$

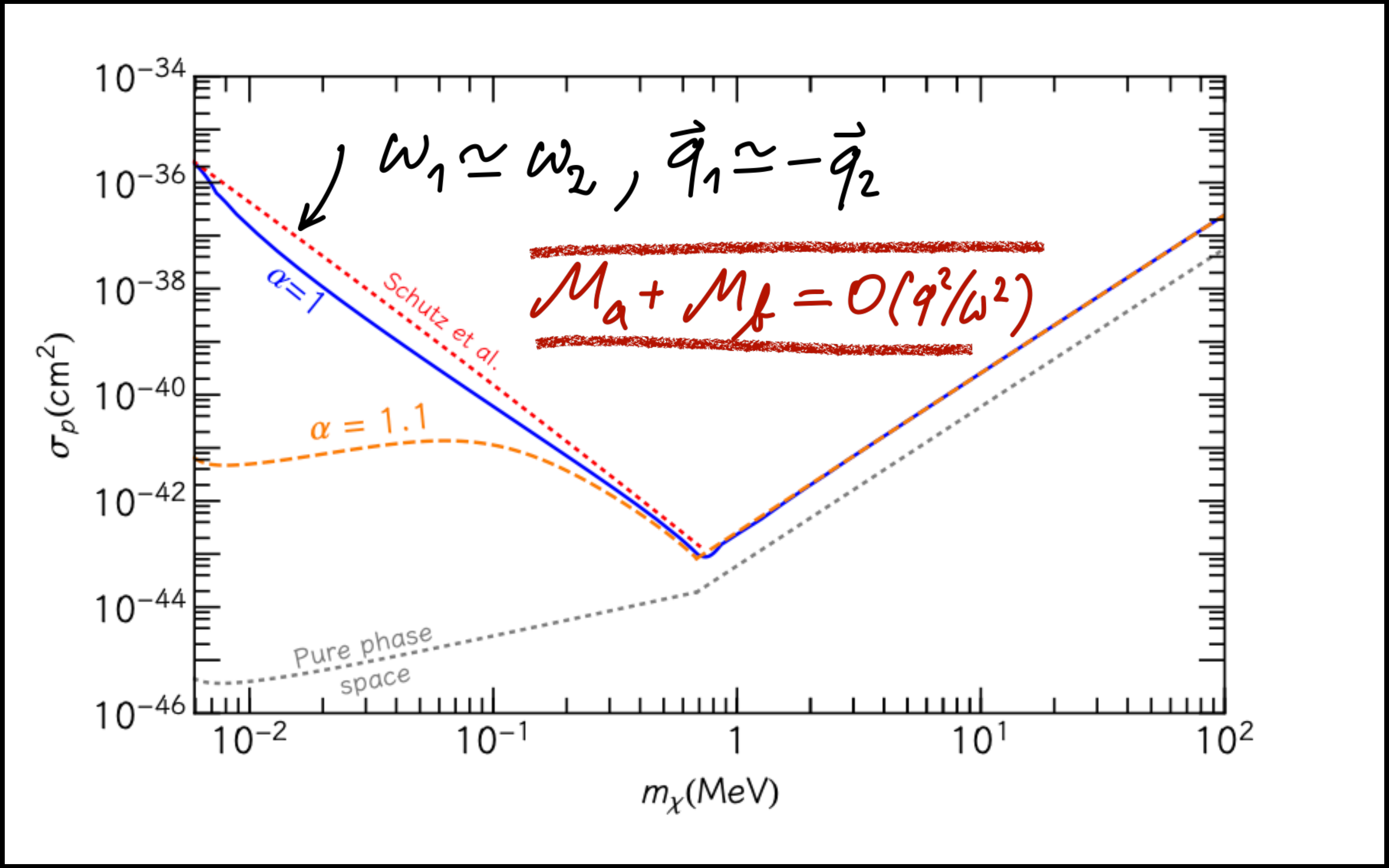
In the back-to-back limit $\mathbf{q}\to 0$, $\mathcal{M}_a$ and the last two terms in $\mathcal{M}_b$ cancel!

$$\boxed{\mathcal{M}_a + \mathcal{M}_b \sim \mathbf{q}^2/\omega^2}$$

The best proof is numerical. More arguments can be crafted.

THE EXCLUSION PLOT

A. Caputo, A. Esposito, ADP, PRD (2019) in press



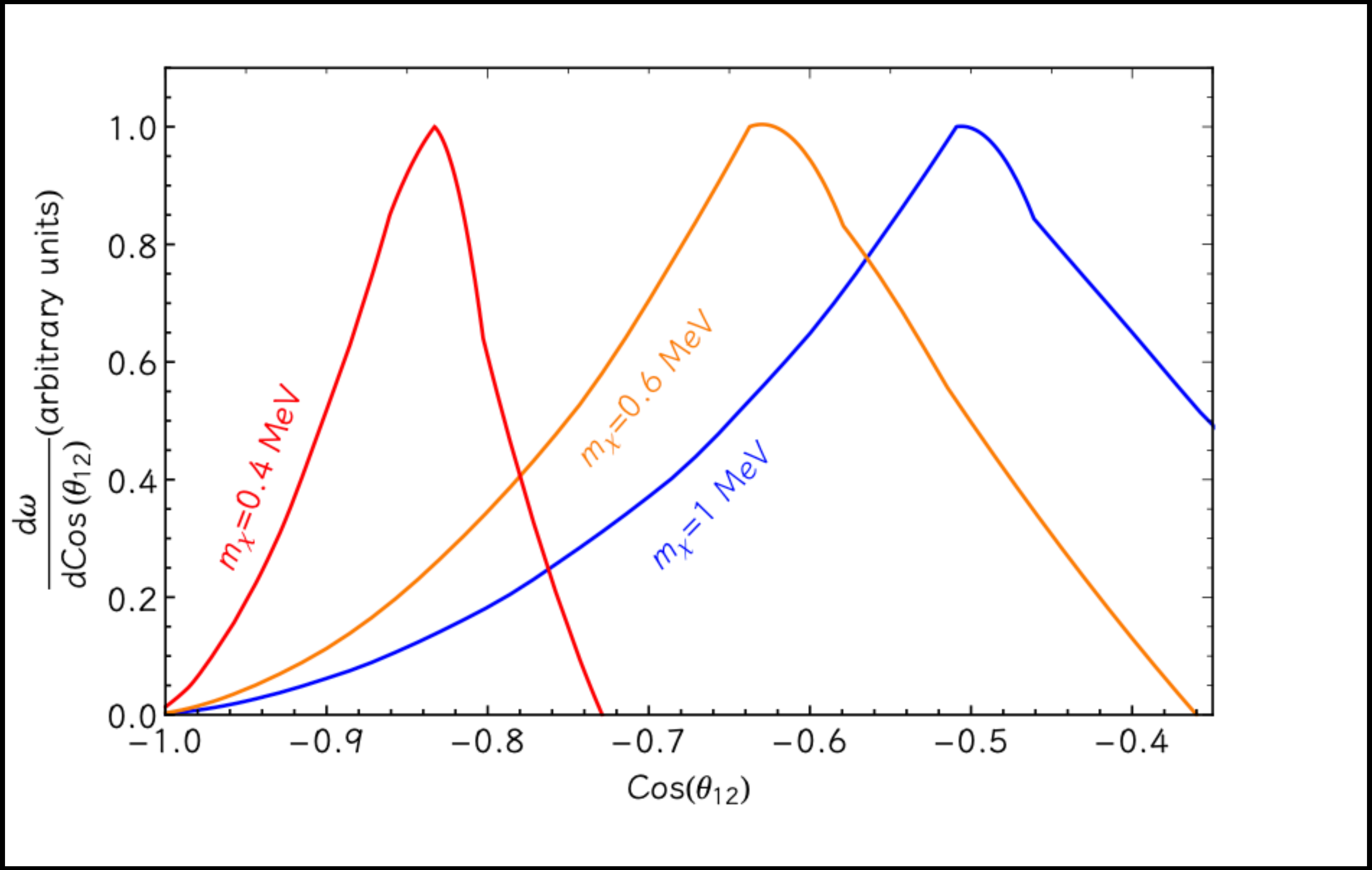
Excluded region corresponding to 3/evts/Kg/year @ zero bckg. Impose total energy released >1meV

$$\sigma_{DM-p} = \sigma_p = \frac{G_\chi^2 m_{\chi-He}^2}{256\pi}$$

DISTRIBUTIONS

F. Acanfora, A. Esposito, ADP, EPJC 79 (2019), 549

A. Caputo, A. Esposito, ADP, PRD (2019) in press



A FEW REMARKS

- We can reproduce the Shutz&Zurek exclusion plot using *only* phonon excitations. We conclude that rotons have marginal role.

- We can compute distributions, e.g. in the relative angle between the two phonons. This might be of relevance in future experimental studies.

- The KeV-MeV mass range exclusion plot (large suppression wrt pure phase-space) can be understood in terms of the cancellation between two contributions to the scattering amplitude. The DM coupling to two phonons is $O(q^2/w^2)$.

- What about processes 2 + (1-*soft* phonon) emission in the final state? …

- We have a method which can be used successfully to solve a a whole class of problems where phonons are the relevant degrees of freedom.

BACKUP SLIDES

MICROSCOPIC THEORY

$$\epsilon(\mathbf{p}) = \frac{\mathbf{p}^2}{2mS(\mathbf{p})}$$

$S(\mathbf{p})$ is the Fourier component of $S(\mathbf{r}-\mathbf{r}')$

$$S(\mathbf{r}-\mathbf{r}') = \frac{\overline{(n(\mathbf{r})-\bar{n})(n(\mathbf{r}')-\bar{n})}}{\bar{n}}$$

$$n(\mathbf{r}) = \frac{1}{m}\rho(\mathbf{r})$$

$$S(\mathbf{p}) = \frac{A}{2\pi\bar{n}}\int \sigma_{n-He}(\omega)\, d\omega$$

where w is the energy transfer due to scattering with neutrons

GOLDSTONE MODES

The hard-core repulsion between bosons is such that a boson moving with momentum *k* will affect all other bosons producing a *density wave* with energy *w* proportional to *k* as in the Bogolubov formula (linear dispersion relation).

The number density of final states per unit energy interval is

$$\rho(E) = \frac{dn}{dE} = \frac{\Pi\, d^3p}{dE} \propto p^2 \frac{1}{dE/dp} \to 0 \quad \text{if} \quad E \to 0$$

Compare the quadratic dispersion case with linear dispersion case at very low energy: paucity of gapless excitation.

"The physics of superfluids lies in the paucity of gapless excitations"

SYSTEM OF REPELLING BOSONS

$$\varphi = \sqrt{\rho}\, e^{i\theta}$$

$$\mathscr{L} = \frac{i}{2}\frac{\partial}{\partial t}\rho - \rho\frac{\partial}{\partial t}\theta - \frac{1}{2m}\left(\rho(\nabla_i\theta)^2 + \frac{1}{4\rho}(\nabla_i\rho)^2\right) - \frac{\lambda}{4m^2}\rho^2$$

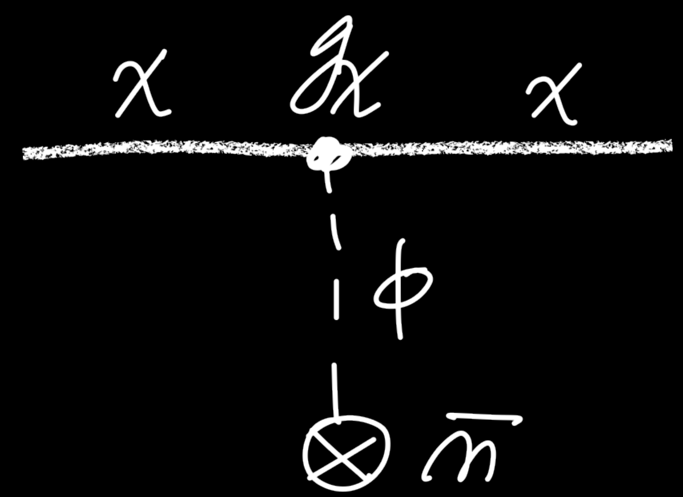
1) the phase θ and the density ρ are conjugate

2) Turn to the Hamiltonian: higher densities correspond to higher energies (if lambda>0). This means that there is an hard core repulsion between bosons — the condition to have a superfluid at T=0.

INTERACTION WITH DM: MICR.

$$\mathscr{L}_I = |\partial\chi|^2 + m_\chi^2|\chi|^2 + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}\phi^2 + g_\chi m_\chi \phi|\chi|^2 + g_{\rm He}\phi\, n$$

On the superfluid background the n aquires a vev $\bar{n}$



$$\boxed{\frac{g_\chi\, g_{\rm He}}{m_\phi^2} = G_\chi}$$

(massive mediator)

$$m^2(\mu) = m_\chi^2 - G_\chi\, \bar{n}(\mu)\, m_\chi$$

INTERACTION WITH DM: MICR.

The effective $\mathscr{L}$ is obtained by promoting $\mu \rightarrow X$

$$m^2(X) = m_\chi^2 - G_\chi m_\chi P'(X)$$

and finally expanding $\mathscr{L}_\text{I}$ in the fluctuations

$$\mathscr{L}_I = |\partial\chi|^2 + m^2(X)|\chi|^2$$

$(Z_\chi = 1$ here)

Besides $\mathscr{L}_I = G_\chi m_\chi |\chi|^2 n(x)$ we also tested

$$\mathscr{L}_I^{\text{Toy}} = G_\chi m_\chi |\chi|^2 n^\alpha(x)\bar{n}^{1-\alpha}$$

with $\alpha = 1, 1.1, \cdots$